Feshbach resonances with large background scattering length: Interplay with open-channel resonances

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Tetsuo Hyodo

Tokyo Institute of Technology

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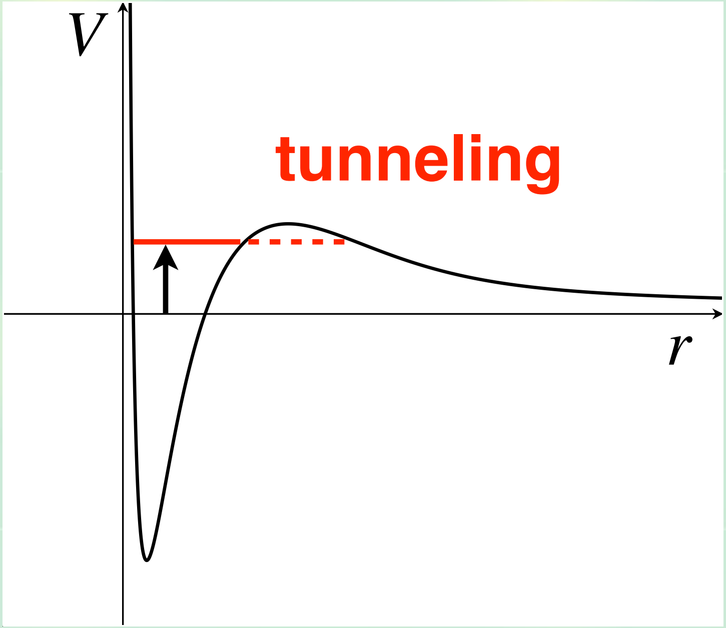
Contents

Resonances

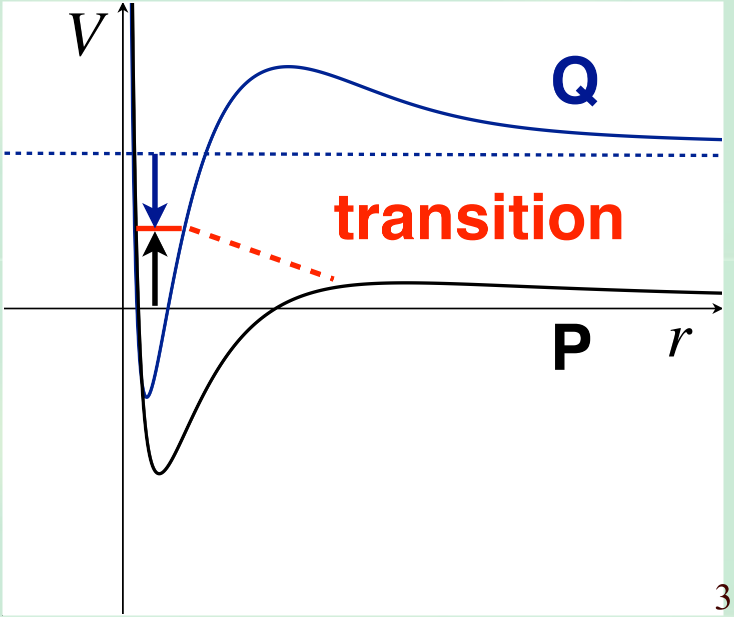
1) Potential resonance

- single channel (P)
- bound by potential barrier
- energy E > 0
- unstable by tunneling
- (dynamically generated state)



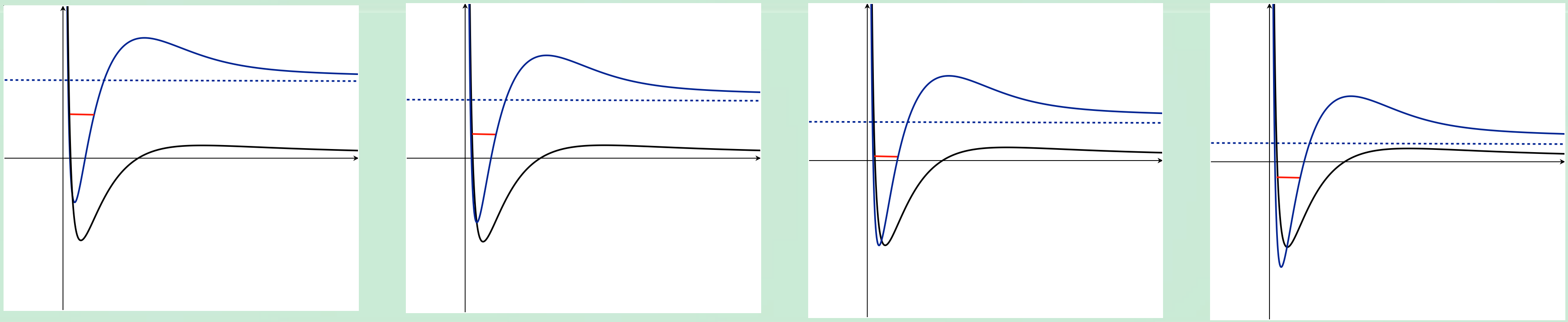
2) Feshbach resonance

- coupled channels (P+Q)
- bound state in Q channel $E_Q < 0$
- above P threshold $E_P > 0$
- unstable by transition
- (CDD pole contribution)



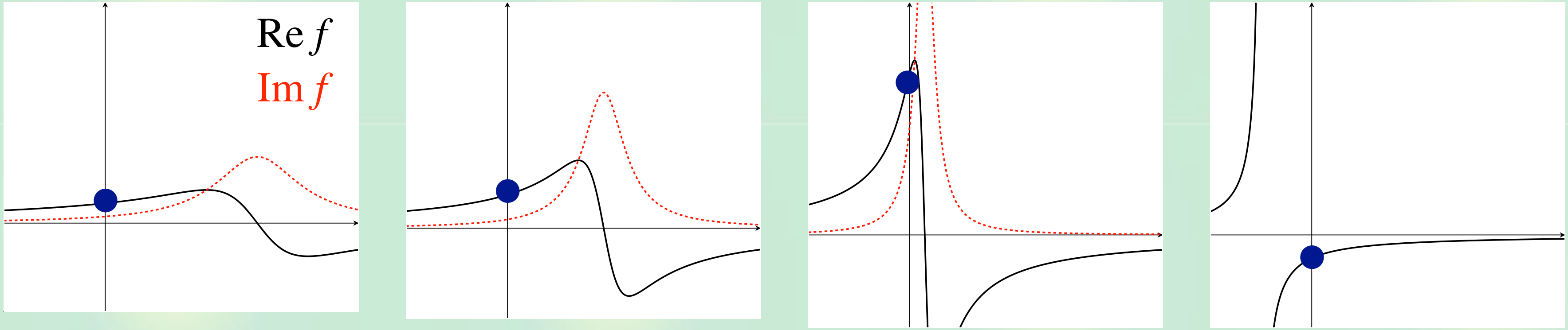
Feshbach resonance in atomic physics

In alkali-metal atoms, (P, Q) are different spin configurations --> magnetic field B modifies the threshold energy difference



In channel P, energy of resonance is changed by B.

scattering amplitude in P channel



Scattering length of P is changed by the magnetic field

Scattering length

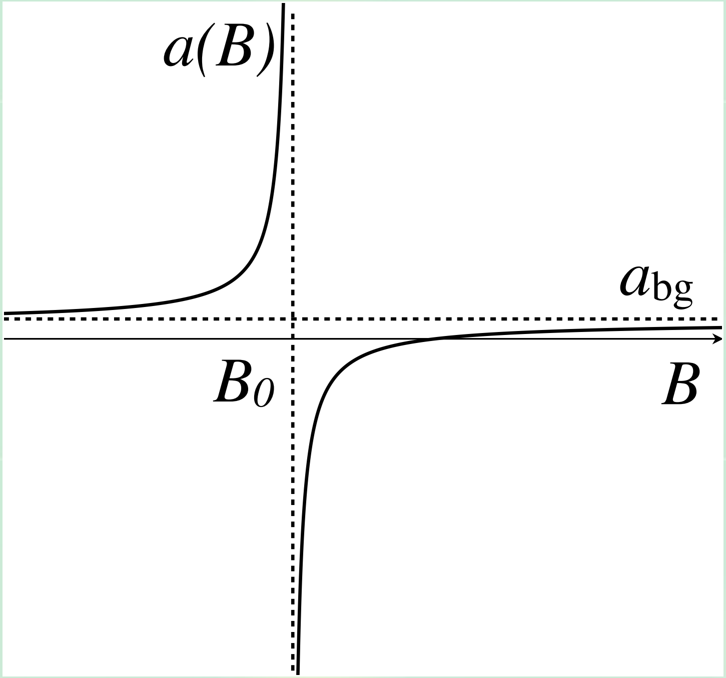
Scattering length of P as a function of magnetic field B

$$a(B) = a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

P interaction

bound state in Q

$B = B_0$: unitary limit



Interaction strength is adjustable by the magnetic field

(Hidden) assumption:
the lower energy P channel is **weakly** interacting ($a_{bg} \sim r_0$).

This is not always the case, e.g., ^{133}Cs, ^{85}Rb, ^{6}Li,...

What happens if $|a_{bg}| \gg r_0$?

Feshbach resonance theory: projection formalisom

Reduction of two-channel problem into a single channel

H. Feshbach, Ann. Phys. (N.Y.) 5, 357 (1958); 19, 287 (1962).

P: open channel, lower energy.
Q: closed channel, higher energy.

$$H|\,\Psi\,\rangle = E|\,\Psi\,\rangle, \quad H = \begin{pmatrix} H_{PP} & H_{PQ} \\ H_{PQ} & H_{QQ} \end{pmatrix}, \quad |\,\Psi\,\rangle = \begin{pmatrix} |\,\Psi_P\,\rangle \\ |\,\Psi_Q\,\rangle \end{pmatrix}$$

(full) Green's operator in Q channel --> $|\psi_Q\rangle$

$$|\,\Psi_Q\,\rangle = \frac{1}{E^+ - H_{QQ}} H_{QP}|\,\Psi_P\,\rangle, \quad E^+ = E + i\delta$$

Eliminating $|\psi_Q\rangle$, we obtain effective Hamiltonian in P

$$E|\,\Psi_P\,\rangle = \left(\underline{H_{PP}} + \underline{H_{PQ}\frac{1}{E^+ - H_{QQ}}H_{QP}} \right)|\,\Psi_P\,\rangle$$
$$\equiv H_{\mathrm{eff}}|\,\Psi_P\,\rangle$$

original P interaction + coupling effect to channel Q

T-matrix for effective single channel interaction

Structure of the effective interaction

$$H_{\text{eff}} = H^0_{PP} + V_{PP} + H_{PQ}\frac{1}{E^+ - H_{QQ}}H_{QP} \equiv H^0_{PP} + V_I + V_{II}$$

t-matrix: Two-potential theorem (c.f. DWBA for nuclear reaction)

$$t = t_I + \langle \Psi^-_P | V_{II} | \Psi_P \rangle$$

$$= \langle \chi_P | V_{PP} | \Psi^+_P \rangle + \langle \Psi^-_P | H_{PQ}\frac{1}{E^+ - H_{QQ}}H_{QP} | \Psi_P \rangle$$

- $|\chi_P\rangle$: free P state
- $|\Psi^+_P\rangle$: full P state with V_{PP}
- $|\Psi_P\rangle$: full P state with V_{eff}

$$|\Psi^\pm_P\rangle = |\chi_P\rangle + \frac{1}{E^\pm - H_{PP}}V_{PP}|\chi_P\rangle, \quad H_{PP} = H^0_{PP} + V_{PP}$$

$$|\Psi_P\rangle = |\Psi^+_P\rangle + \frac{1}{E^+ - H_{PP}}H_{PQ}\frac{1}{E^+ - H_{QQ}}H_{QP}|\Psi_P\rangle$$

Single-pole dominance

Pick up one bound state (relevant for P threshold) from the expansion of Green's operator of Q channel

$$\frac{1}{E^+ - H_{QQ}} = \sum_i \frac{|\,\phi_i\,\rangle\langle\,\phi_i\,|}{E - \epsilon_i^Q} + \int \frac{|\,\phi(\epsilon)\,\rangle\langle\,\phi(\epsilon)\,|}{E^+ - \epsilon} d\epsilon \to \frac{|\,\phi_b\,\rangle\langle\,\phi_b\,|}{E - \epsilon_b^Q}$$

t-matrix for single-pole dominance

$$t = \langle\,\chi_P\,|V_{PP}|\,\Psi_P^+\,\rangle + \frac{\langle\,\Psi_P^-\,|H_{PQ}|\,\phi_b\,\rangle\langle\,\phi_b\,|H_{QP}|\,\Psi_P\,\rangle}{E - \epsilon_b^Q}$$

← **bare pole**

$$= \langle\,\chi_P\,|V_{PP}|\,\Psi_P^+\,\rangle + \frac{|\langle\,\phi_b\,|H_{QP}|\,\Psi_P^+\,\rangle|^2}{E - \epsilon_b^Q - A(E)}$$

← **dressed pole**

coupling to P channel modifies the bare mass (self-energy)

$$A(E) = \langle\,\phi_b\,|H_{QP}\frac{1}{E^+ - H_{PP}}H_{PQ}|\,\phi_b\,\rangle = \Delta_{\mathrm{res}}(E) - \frac{i}{2}\Gamma(E)$$

↑ **mass shift** ↑ **width**

S-matrix

S-matrix (E=k²)

$$\mathcal{S}(E) = S_P(E)\left(1 - 2\pi i \frac{|\langle \phi_b | H_{QP} | \Psi_P^+ \rangle|^2}{E - \epsilon_b^Q - A(E)}\right)$$

↑

single P channel S-matrix

$$A(E) = \langle \phi_b | H_{QP} \frac{1}{E^+ - H_{PP}} H_{PQ} | \phi_b \rangle = \Delta_{\rm res}(E) - \frac{i}{2}\Gamma(E)$$

Near threshold (E ~ 0)

$$\Delta_{\rm res}(E) \sim \text{const.}, \quad \Gamma(E) = 2\pi|\langle \phi_b | H_{QP} | \Psi_P^+ \rangle|^2 \sim 2Ck, \quad \boxed{S_P(k) \sim \exp[-2ika_{\rm bg}]}$$

so the S-matrix is given by

$$S(k) = \exp[-2ika_{\rm bg}]\left(1 - \frac{2iCk}{E - \epsilon_b^Q - \Delta_{\rm res} + iCk}\right)$$

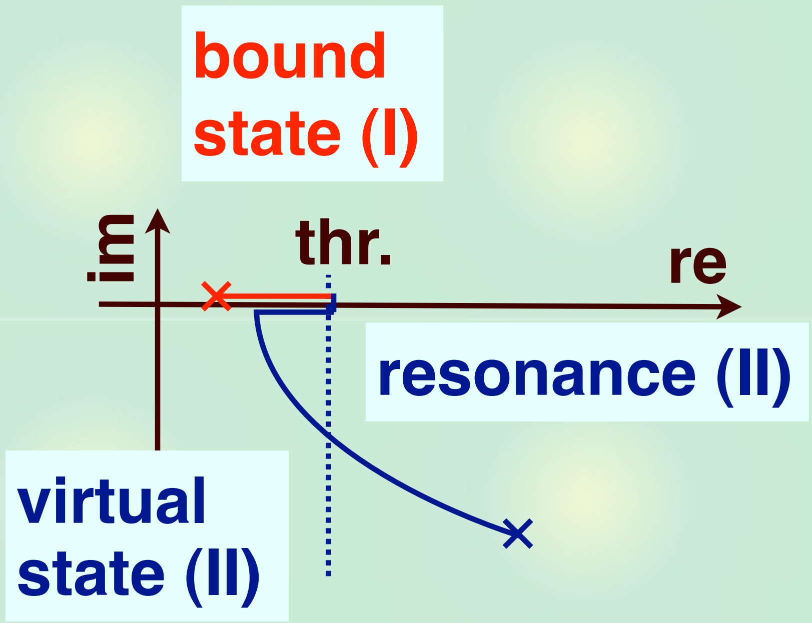
Result of the single-resonance approximation

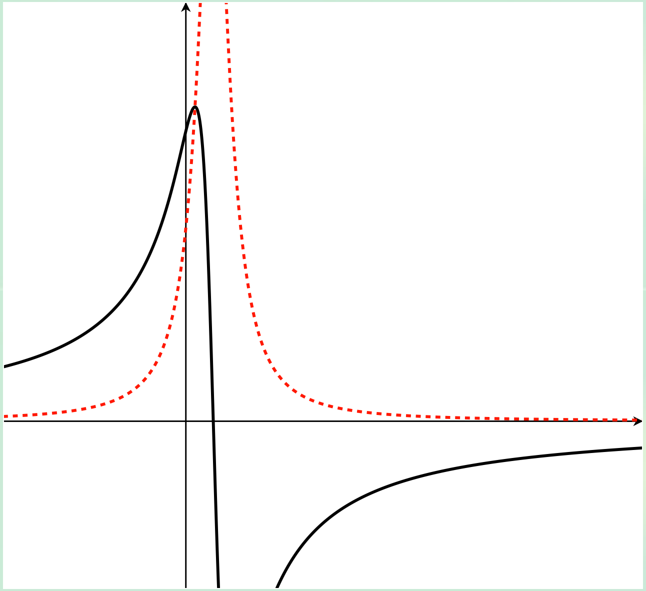
$$\epsilon_b^Q \propto B \quad \Rightarrow \quad a(B) = a_{\rm bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

Open channel singularities

For a while, we consider single P channel.

Bound state/virtual state/resonances: pole of S-matrix





pole close to the threshold

--> large scattering length

--> strong energy dependence

--> affect to the Feshbach resonance?

Significance of virtual state in open channel

Previous assumption: P channel has smooth amplitude
--> scattering length governs the low energy behavior

$$S_P(k) = \exp[-2ika_{\rm bg}]$$

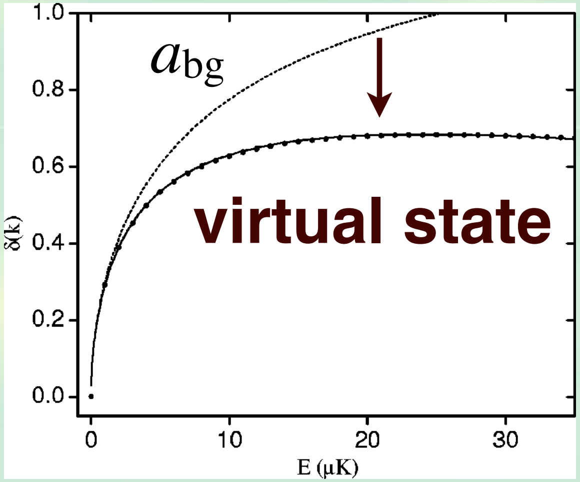
If a pole exists near threshold, then

$$\to S^P_{\rm bg}(k) S^P_{\rm res}(k) = \exp[-2ika^P_{\rm bg}] S^P_{\rm res}(k)$$ ← **potential resonance, not Feshbach**

For a virtual state, it is explicitly written as

$$S_P(k) = \exp[-2ika^P_{\rm bg}] \frac{i\kappa_{\rm vs} - k}{i\kappa_{\rm vs} + k}$$

Example for ^{85}Rb (P-channel only)



``Scattering length parameter only does not fully encapsulate the energy dependence of the scattering physics.''

Mittag-Leffle series

Pole in (full) Green's operator: common with T-matrix

Expansion of Green's operator in P channel: Mittag-Leffle series

$$\frac{1}{E - H_{PP}} = \sum_{n=1}^{\infty} \frac{|\,\Omega_n\,\rangle\langle\,\Omega_n^D\,|}{2k_n(k - k_n)}$$

← **Gamow vector and its dual**

k_n: pole position, arbitrary complex number

- One virtual state

$$\frac{1}{E - H_{PP}} \to \frac{|\,\Omega_{\mathrm{vs}}\,\rangle\langle\,\Omega_{\mathrm{vs}}^D\,|}{2k_{\mathrm{vs}}(k - k_{\mathrm{vs}})}, \quad k_{\mathrm{vs}} = -i\kappa_{\mathrm{vs}}$$

- One virtual state + one bound state

$$\frac{1}{E - H_{PP}} \to \frac{|\,\Omega_{\mathrm{vs}}\,\rangle\langle\,\Omega_{\mathrm{vs}}^D\,|}{2k_{\mathrm{vs}}(k - k_{\mathrm{vs}})} + \frac{|\,\phi_{\mathrm{bs}}\,\rangle\langle\,\phi_{\mathrm{bs}}\,|}{2k_{\mathrm{bs}}(k - k_{\mathrm{bs}})}, \quad k_{\mathrm{vs}} = -i\kappa_{\mathrm{vs}}, \quad k_{\mathrm{bs}} = i\kappa_{\mathrm{bs}}$$

Feshbach resonance with P-channel virtual state

Self-energy function with one virtual state:

$$A(E) = \frac{\langle \phi_b | H_{QP} | \Omega_{\rm vs} \rangle \langle \Omega^D_{\rm vs} | H_{PQ} | \phi_b \rangle}{2k_{\rm vs}(k - k_{\rm vs})} = \Delta_{\rm res}(E) - \frac{i}{2}\Gamma(E)$$

Numerator: no energy dependence

$$A(E) = \frac{-iA_{\rm vs}}{2\kappa_{\rm vs}(k + i\kappa_{\rm vs})}$$

Mass modification in the presence of a virtual state

$$\Delta_{\rm res}(E) = \frac{-A_{\rm vs}/2}{k^2 + \kappa_{\rm vs}^2} = -\frac{A_{\rm vs}}{2\kappa_{\rm vs}^2}\left(1 - \frac{k^2}{\kappa_{\rm vs}^2} + \dots\right)$$

$$\Gamma(E) = \frac{A_{\rm vs}k}{\kappa_{\rm vs}(k^2 + \kappa_{\rm vs}^2)} = \frac{A_{\rm vs}k}{\kappa_{\rm vs}^3}\left(1 - \frac{k^2}{\kappa_{\rm vs}^2} + \dots\right)$$

For $k \ll \kappa_{\rm vs}$, single-resonance approximation is recovered

a-B relation becomes more complicated

$$a(B) \neq a_{\rm bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

Application to physical example

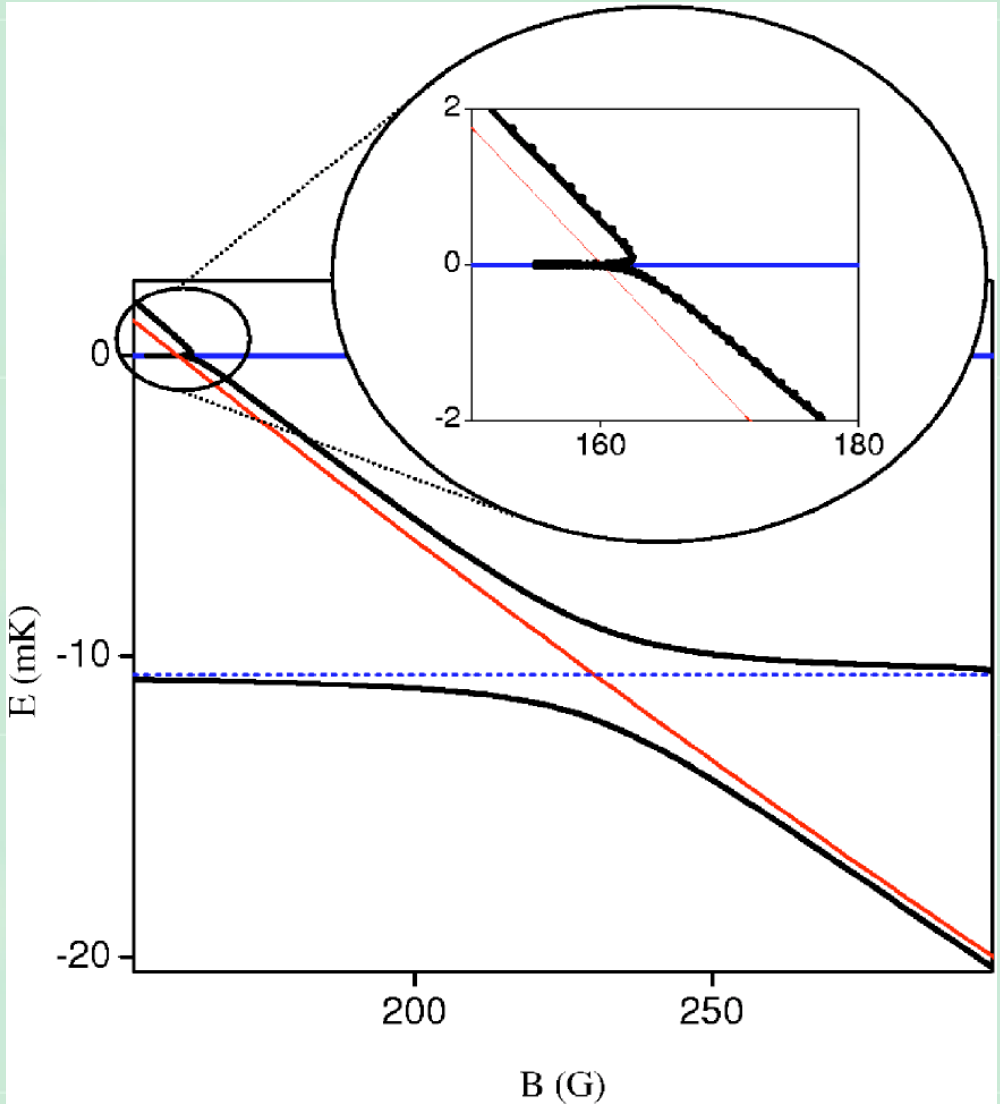
Mass of the Feshbach resonance of ^{85}Rb

$$\epsilon_b - \Delta_{\rm res}(E), \quad \epsilon_b \propto B$$

red
: single-resonance app.

Dotted
: full coupled-channel result with realistic interaction

Solid
: model with 1 virtual and 1 bound state



Summary

We study the Feshbach resonance with near threshold singularity in open channel.

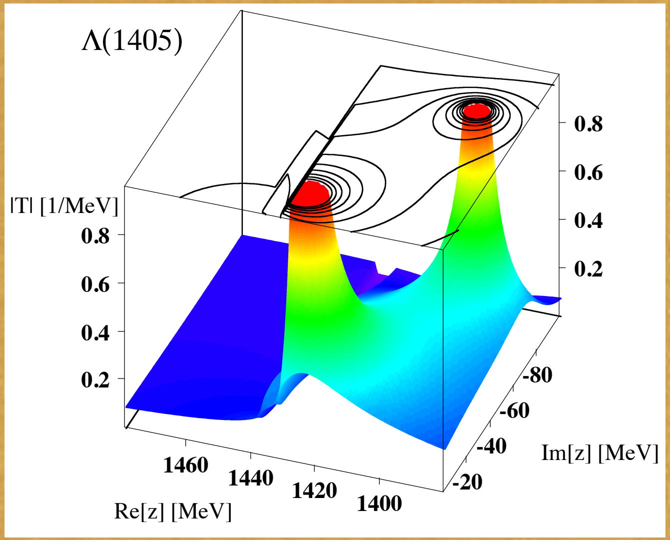
- **Feshbach resonance: a bound state embedded in a continuum**
- **Projection method: effective single-channel interaction**
- **Large a_{bg}: open channel singularity**
- **Open-channel singularity: modifies the linear B dep. of Feshbach resonance**

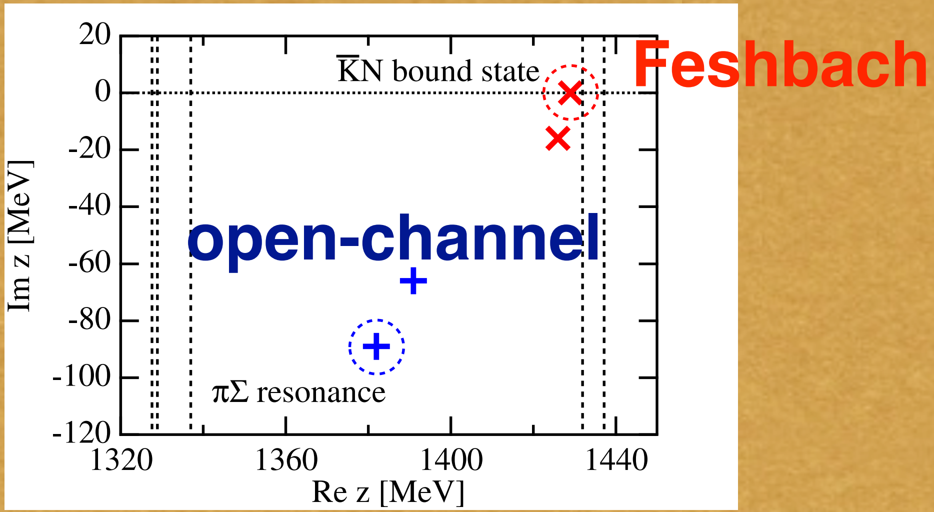
B. Marcelis, et al., Phys. Rev. A 70, 012701 (2004)

Summary

In hadron physics:

- **Λ(1405) in KN-πΣ amplitude?**





T. Hyodo, D. Jido, arXiv:1104.4474, to appear in Prog. Part. Nucl. Phys.

- **Characteristic feature?**
- **Response to open-channel resonance?**