Determination of the $\pi\Sigma$ scattering lengths from the weak decays of Λc

Tetsuo Hyodo,

and Makoto Oka

Tokyo Institute of Technology

supported by Global Center of Excellence Program
“Nanoscience and Quantum Physics”

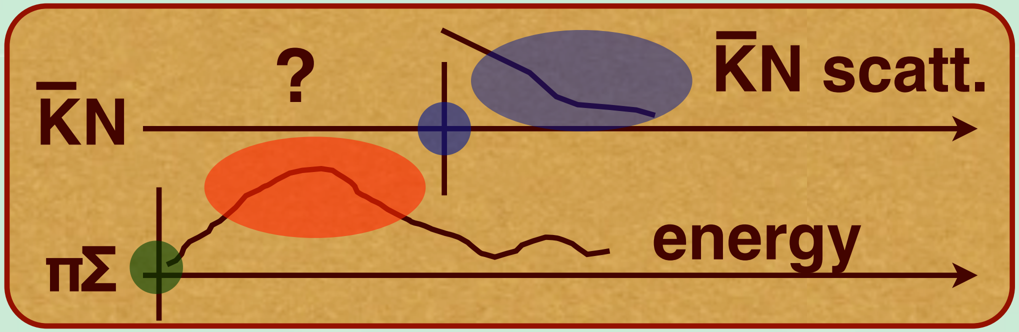
2012, Feb. 11th

Experimental constraints for S=-1 MB scattering

K-p total cross sections

$\bar{K}N$ threshold observables

- threshold branching ratios
- K-p scattering length <-- SIDDHARTA (Talk by Y. Ikeda)



$\pi\Sigma$ mass spectra

- new data is becoming available (LEPS, CLAS, HADES,...)

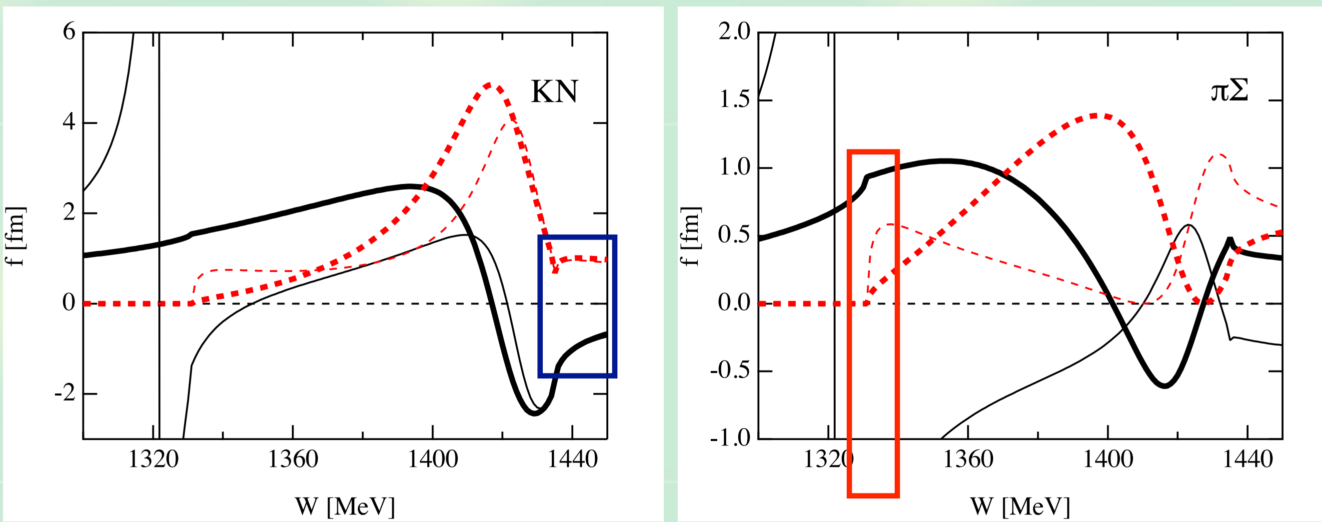
$\boxed{\text{πΣ threshold observables (so far no data) ?}}$

Importance of $\pi\Sigma$ scattering length

Why threshold behavior of $\pi\Sigma$ channel?

Y. Ikeda, T. Hyodo, D. Jido, H. Kamano, T. Sato, K. Yazaki, PTP 125, 1205 (2011)

Simple model extrapolation with $\bar{K}N(I=0)$ being fixed
--> large uncertainty at $\pi\Sigma$ threshold



Determination of $\pi\Sigma$ threshold observables
--> understanding of $\Lambda(1405)$, K nuclei, DISTO result,...

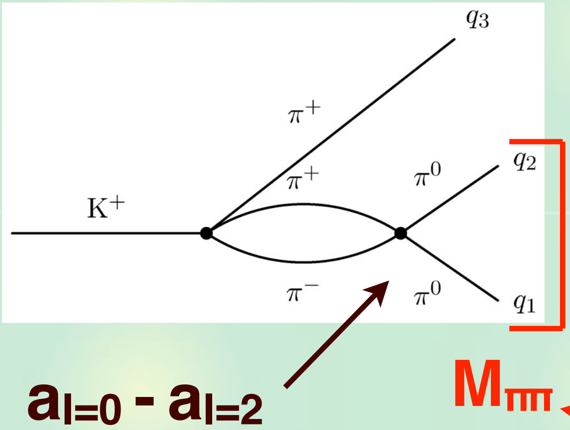
Determination of hadron scattering length

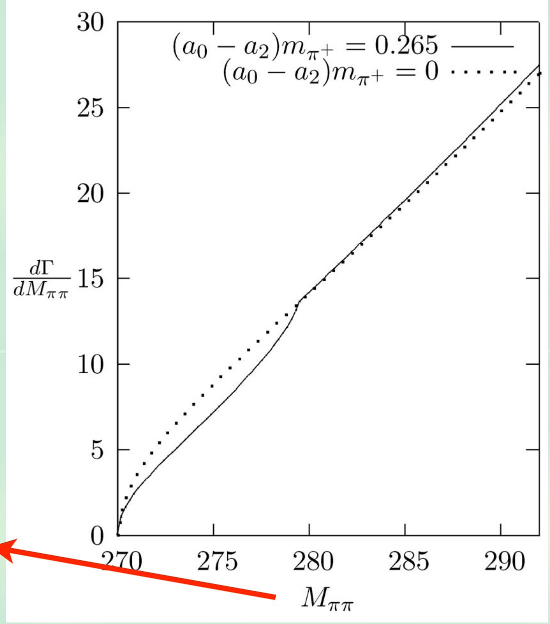
Extraction of hadron scattering length

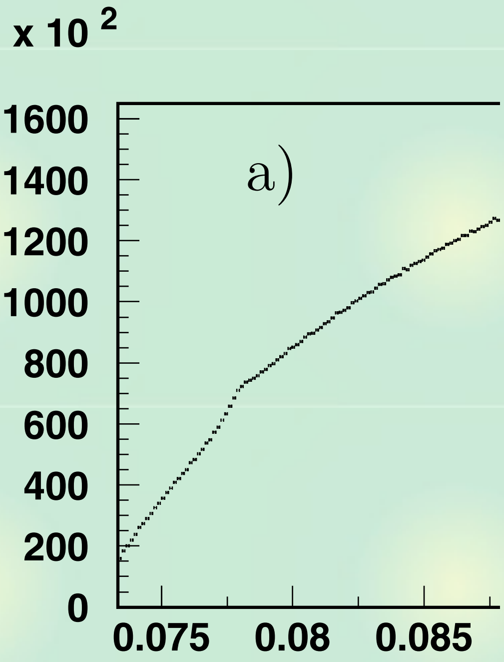
- shift and width of atomic state (c.f. Kaonic hydrogen)
- extrapolation of low energy phase shift
- final state interaction from heavy particle's decay

Isospin violation + threshold cusp + amplitude interference

N. Cabibbo, Phys. Rev. Lett. 93, 121801 (2004)



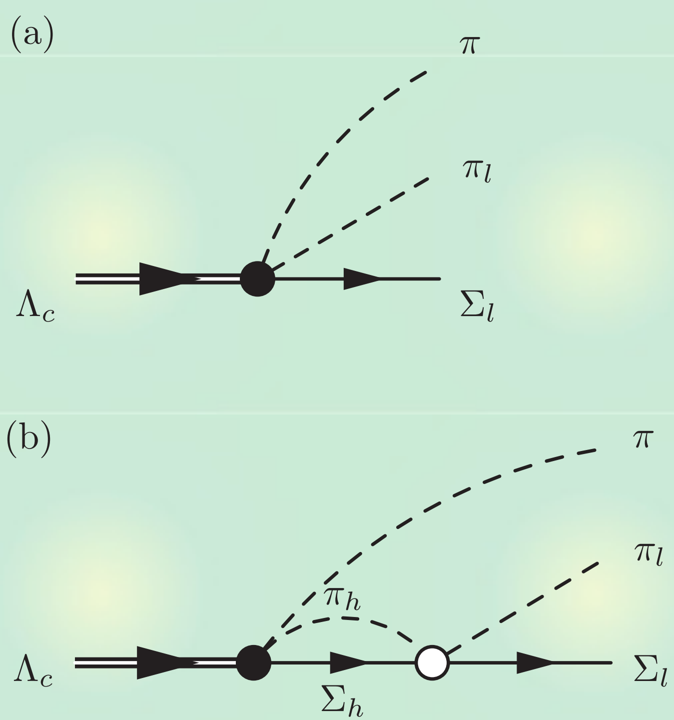


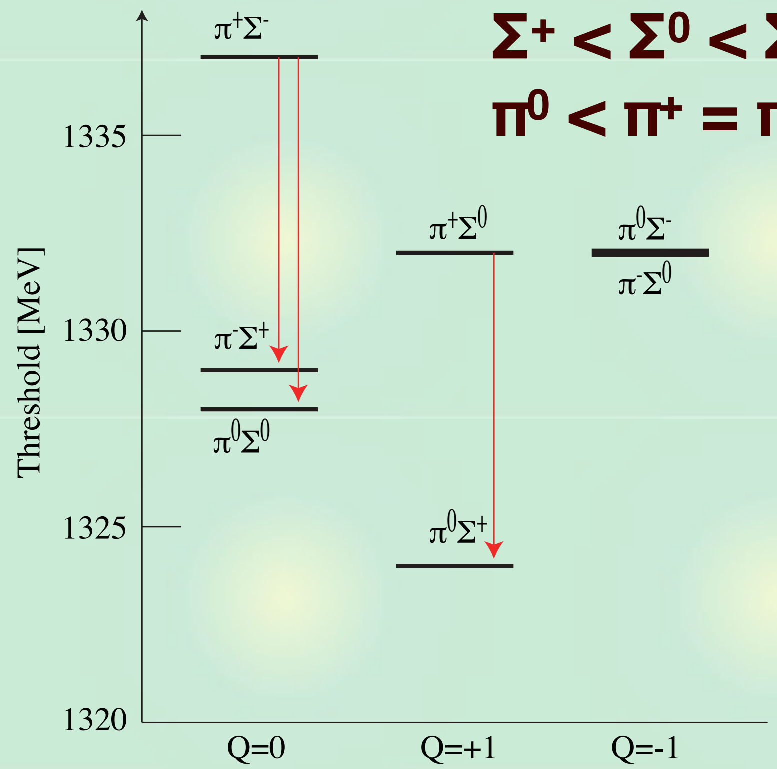


NA48/2, J.R. Batley et al, Phys. Lett. B686, 101 (2010) possible decay modes

Threshold difference of πΣ channels

Isospin violation in πΣ channels





$\Sigma^+ < \Sigma^0 < \Sigma^-$

$\pi^0 < \pi^+ = \pi^-$

To utilize threshold cusp, appreciable mass difference between $(\pi\Sigma)_h$ and $(\pi\Sigma)_l$ is necessary.

$$\pi^+\Sigma^- \to \pi^-\Sigma^+, \quad \pi^+\Sigma^- \to \pi^0\Sigma^0, \quad \pi^+\Sigma^0 \to \pi^0\Sigma^+,$$

Determination of πΣ scattering length

Isospin decomposition of three channels

$$a^{-+} = \tfrac{1}{3}a^0 - \tfrac{1}{2}a^1 + \tfrac{1}{6}a^2 + \cdots ,$$
$$a^{00} = \tfrac{1}{3}a^0 - \tfrac{1}{3}a^2 + \cdots ,$$
$$a^{0+} = -\tfrac{1}{2}a^1 + \tfrac{1}{2}a^2 + \cdots ,$$

mode	$\Lambda_c \to \pi(\pi\Sigma)_h$	$\Lambda_c \to \pi(\pi\Sigma)_l$
a^{-+}	1.7 ± 0.5 %	3.6 ± 1.0 %
a^{00}	1.7 ± 0.5 %	1.8 ± 0.8 %
a^{0+}	1.8 ± 0.8 %	not known

A lot of Λ_c (Belle, Babar, LHC, ...) --> feasible?

Structure around the cusp in $(\pi\Sigma)_l$ + spectrum in $(\pi\Sigma)_h$
--> extraction of the scattering length

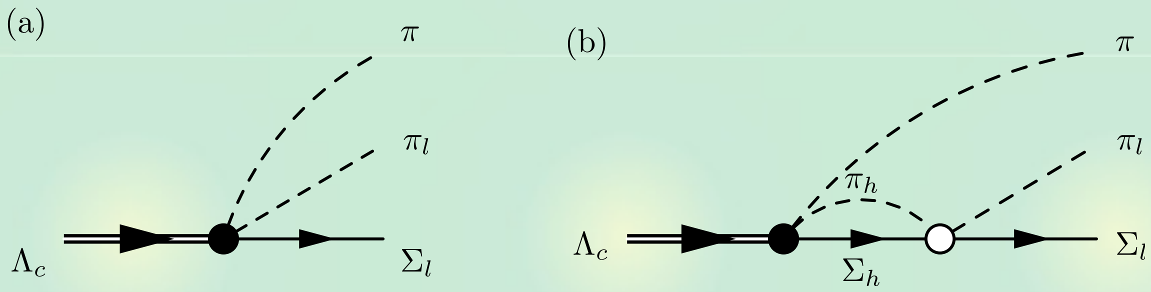
Three unknown scattering lengths, two constraints

$$a^{-+} - a^{00} = a^{0+} + \cdots$$

I=2 scattering length: lattice QCD (HAL QCD, NPLQCD,...)

Decay process and intermediate loop

Decay diagrams for Λ_c --> $\pi\ \pi_l\ \Sigma_l$ process



Spectral representation of the loop function

$$G(W) = \frac{1}{2\pi}\int_{W_{th}}^{\infty} dW' \frac{\rho(W')}{W - W' + i\epsilon} + (\text{subtractions}) \qquad \rho(W) = 2M_h \frac{q(W)}{4\pi W}$$

ρ: phase space, q: three-momentum

Imaginary part of the loop function (on-shell part):

$$\text{Im } G(W) = -\frac{\rho(W)}{2}\Theta(W - W_{th})$$

amplitude (a) : real

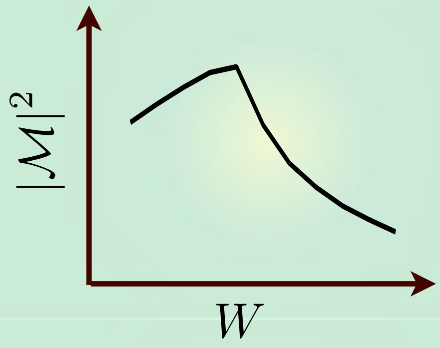
amplitude (b) : real ($W < W_{th}$), complex ($W > W_{th}$)

Threshold cusp in the spectrum

Decomposition of the amplitude

$$\mathcal{M}(W) = \mathcal{M}_0(W) + \tilde{\mathcal{M}}_1(W) m_h \delta$$

δ ~ real ($W < W_{th}$), imaginary ($W > W_{th}$)



π Σ invariant mass spectrum (M_0, M_1: real)

$$|\mathcal{M}|^2 = \begin{cases} (\mathcal{M}_0)^2 + (\tilde{\mathcal{M}}_1 m_h)^2 |\delta|^2 & \text{for} \quad W > W_{\text{th}} \\ (\mathcal{M}_0)^2 + \underline{2\mathcal{M}_0 \tilde{\mathcal{M}}_1 m_h \delta} + (\tilde{\mathcal{M}}_1 m_h)^2 \delta^2 & \text{for} \quad W < W_{\text{th}} \end{cases}$$

- Spectrum is continuous (δ vanishes at threshold)

- Derivative of the spectrum is discontinuous

$$\left.\frac{d|\mathcal{M}|^2}{dW}\right|_{W \to W_{\text{th}}-0} - \left.\frac{d|\mathcal{M}|^2}{dW}\right|_{W \to W_{\text{th}}+0} \propto -\frac{2\mathcal{M}_0 \tilde{\mathcal{M}}_1 m_h M_h}{M_h + m_h} \frac{1}{\delta} + \mathcal{O}(\delta)$$

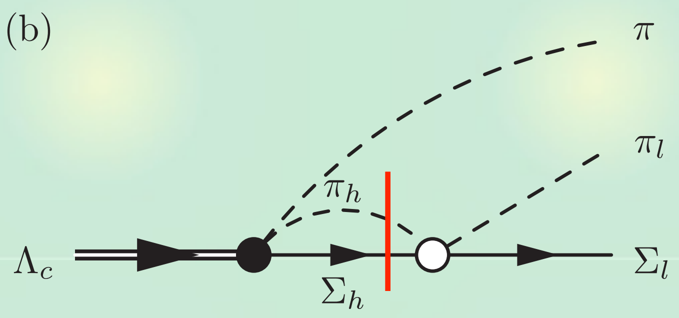
--> threshold cusp

It is purely kinematical effect. General phenomena.

Relation to scattering length

The term which produces the cusp

- Energy is fixed at W = W_{th}
- On-shell kinematics for $\pi_h \Sigma_h$ channel



--> amplitude of $\pi_h \Sigma_h$ --> $\pi_l \Sigma_l$ at threshold: scattering length

General decomposition of the amplitude

$$\mathcal{M}(W) = \mathcal{M}_0(W) + \tilde{\mathcal{M}}_1(W) \boxed{e^{i\theta}} m_h \delta$$

- Cusp appears, but relative phase affect to the structure.
- Relative phase can be calculated by the dynamical model of final state interactions.

Determination of πΣ scattering length

Expansion of the decay spectrum (M_0, M_1: real)

$$|\mathcal{M}|^2 = \begin{cases} A + C'|\delta|^2 + \mathcal{O}(|\delta|^4) & \text{for} \quad W > W_{\rm th} \\ A + B\delta + C\delta^2 + \mathcal{O}(\delta^3) & \text{for} \quad W < W_{\rm th} \end{cases}$$

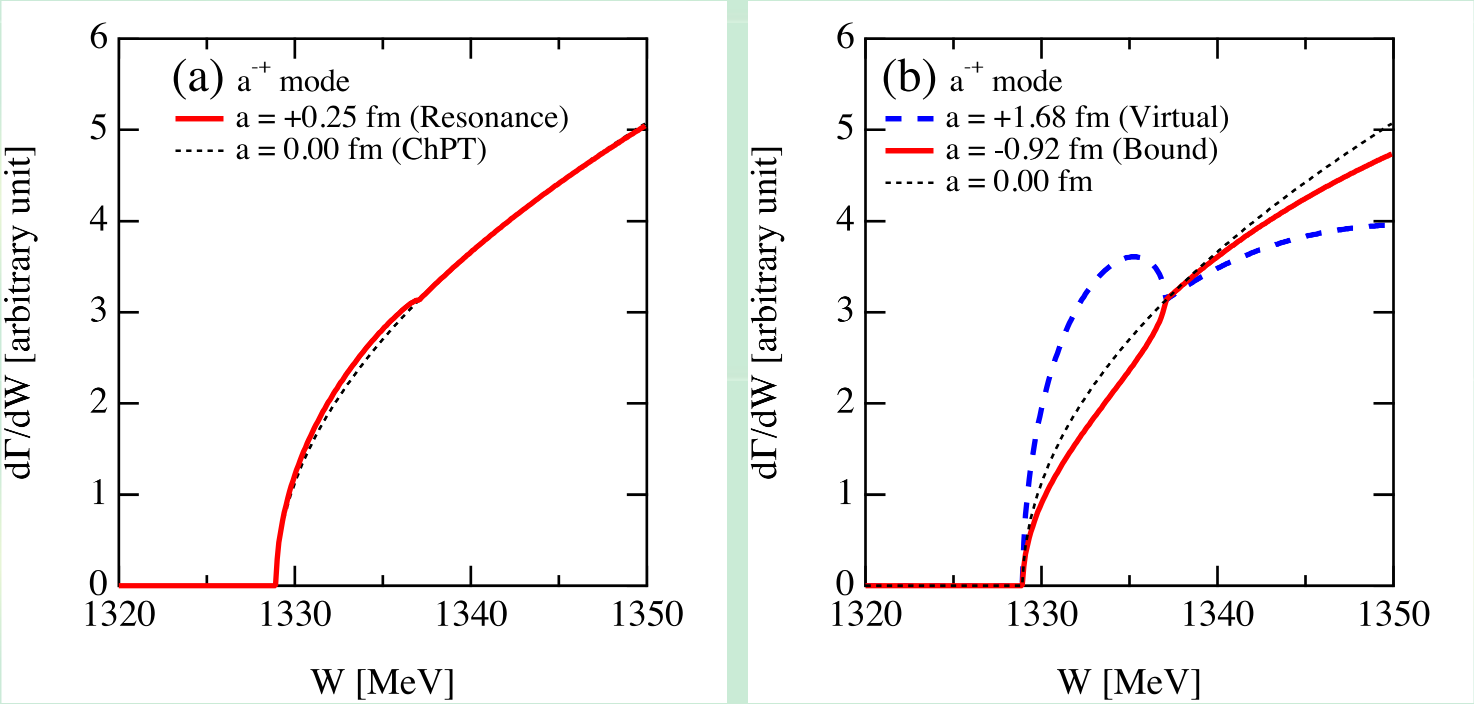
weak process

$$\boxed{M_0^{h(0)}, M_0^{(0)}}$$

$$|a_{h\to l}| = \frac{|B|}{2m_h\sqrt{A}|\mathcal{M}_0^{h(0)}|}, \qquad \frac{a_{h\to l}}{|a_{h\to l}|} = \frac{\mathcal{M}_0^{(0)}}{|\mathcal{M}_0^{(0)}|}\cdot\frac{\mathcal{M}_0^{h(0)}}{|\mathcal{M}_0^{h(0)}|}\cdot\frac{B}{|B|}$$

spectrum

$$\boxed{A, B, C}$$

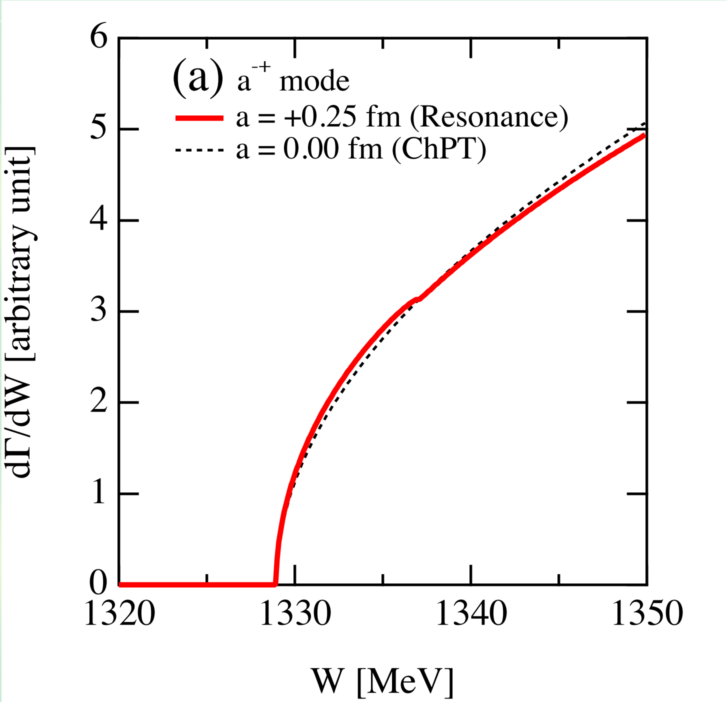


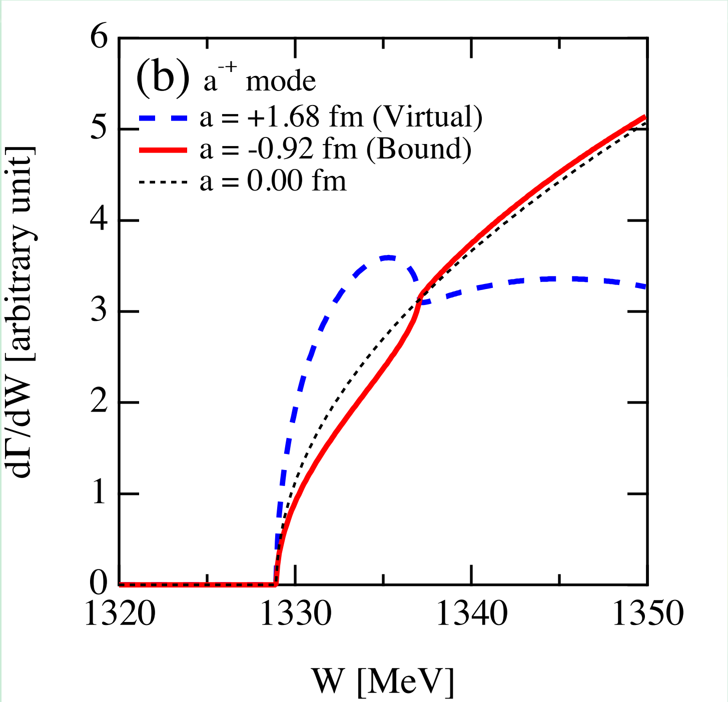
Determination of πΣ scattering length

Expansion of the decay spectrum (relative phase θ= -12 deg)

$$|\mathcal{M}|^2 = \begin{cases} A + B'|\delta| + C'|\delta|^2 + \mathcal{O}(|\delta|^4) & \text{for} \quad W > W_{\text{th}} \\ A + B\delta + C\delta^2 + \mathcal{O}(\delta^3) & \text{for} \quad W < W_{\text{th}} \end{cases},$$

$$|a_{h\to l}| = \frac{\sqrt{B^2 + (B')^2}}{2m_h\sqrt{A}|\mathcal{M}_0^{h(0)}|}, \qquad \frac{a_{h\to l}}{|a_{h\to l}|} = \frac{\mathcal{M}_0^{(0)}}{|\mathcal{M}_0^{(0)}|}\cdot\frac{\mathcal{M}_0^{h(0)}}{|\mathcal{M}_0^{h(0)}|}\cdot\frac{B/\cos\theta}{|B/\cos\theta|}$$





Summary

πΣ scattering length from Λ_c decay

- **πΣ scattering length** is important for low energy $\bar{K}N$-πΣ amplitude.
  --> $\bar{K}$ nuclei and Λ(1405) physics

  Y. Ikeda, T. Hyodo, D. Jido, H. Kamano, T. Sato, K. Yazaki, Prog. Theor. Phys. 125, 1205 (2011)

- **Threshold cusp** : kinematical effect

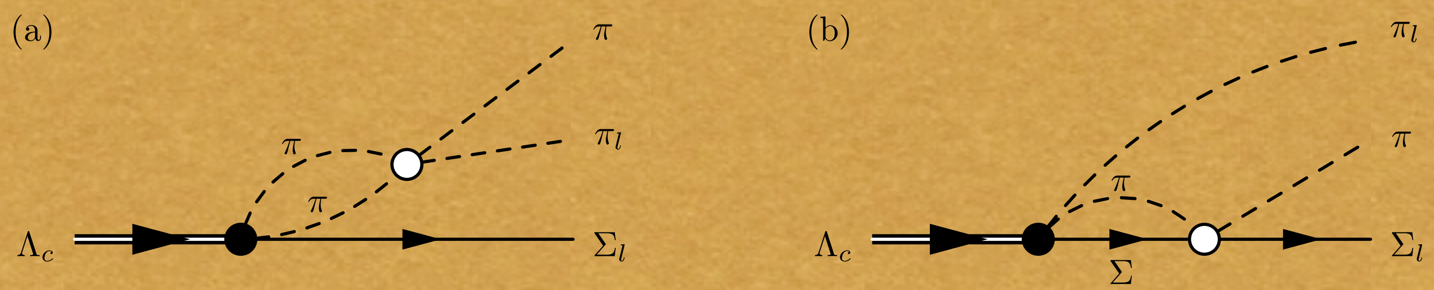
- Cusp in Λ_c --> ππΣ decay is related with the πΣ scattering length.

  T. Hyodo, M. Oka, Phys. Rev. C 83, 055202 (2011)

Summary

Future plans: estimate of amplitude

- **πΣ scattering length** final state interaction



- relative phase between M_0 and M_1

- Cusp in **π$\bar{\text{K}}$N** channel

$$a(\bar{K}^0 n \to K^- p) = \frac{1}{2}(a^{I=0} - a^{I=1})$$

c.f. Kaonic hydrogen: $a(K^- p) = \frac{1}{2}(a^{I=0} + a^{I=1})$