Nature of the sigma meson as revealed by its softening process

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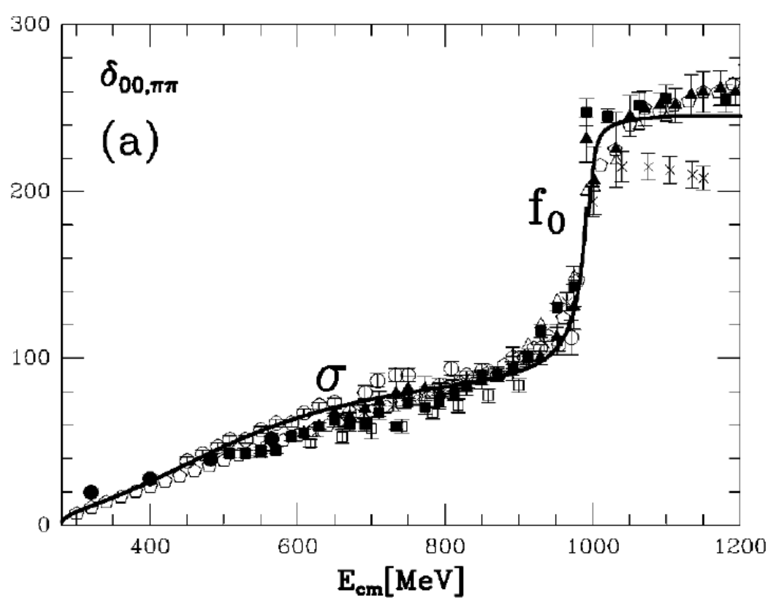
The sigma meson

$f_0(500)$ or σ

$J^P = 0^+$, $I = 0$
Mass : 400-550 MeV
Width : 400-700 MeV



The sigma meson

- **is the lowest resonance in QCD**
- **plays an important role in hadron mass generation due to spontaneous chiral symmetry breaking**
- **provides attraction in phenomenological nuclear force**

Developments in scattering theory + accurate data
--> determination of pole position is now possible.

I. Caprini, G. Colangelo, H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006), ...

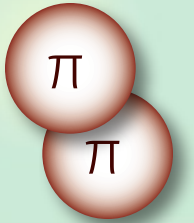
Structure of the sigma meson

Sigma meson in naive constituent quark model ($q\bar{q}$) has some difficulties: light mass (v.s. p-wave excitation), mass ordering of scalar nonet (v.s. $\sigma > \kappa > f_0 \sim a_0$)

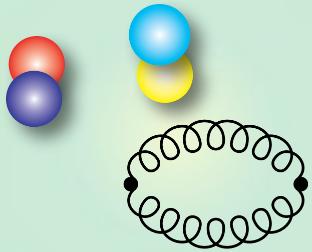
Alternative descriptions of the sigma meson

- **Chiral sigma**
  (e.g. linear sigma model)
  M. Gell-Mann, M. Levy, Nuovo Cim. 16, 705 (1960), ...

- **Dynamical sigma**
  (e.g. mesonic molecule generated by $\pi\pi$ attraction)
  J.I. Basdevant, B.W. Lee, Phys. Rev. D2, 1680 (1970), ...



- **CDD pole contribution (pre-formed state)**
  (e.g. diquark-antidiquark model, glueball, ...)

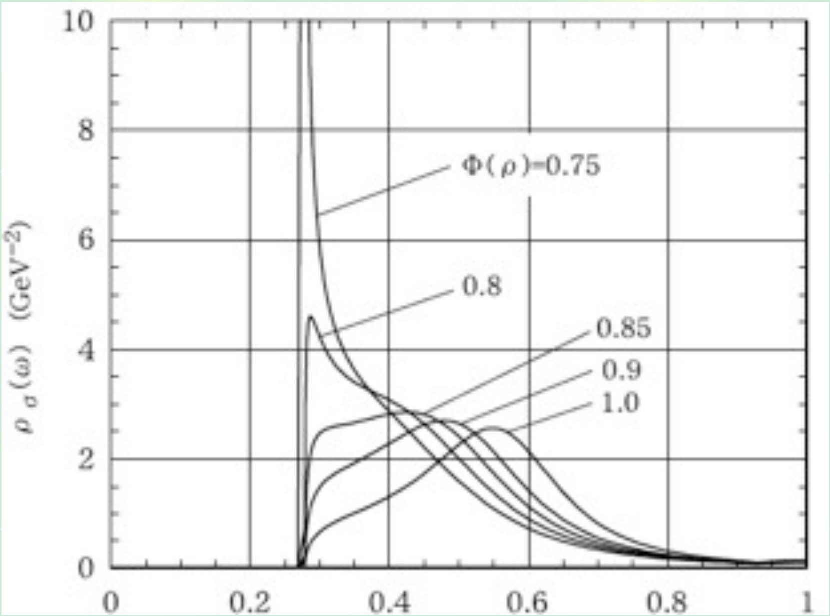


We want to clarify the structure <-- softening

Softening of the sigma meson

Softening of chiral sigma

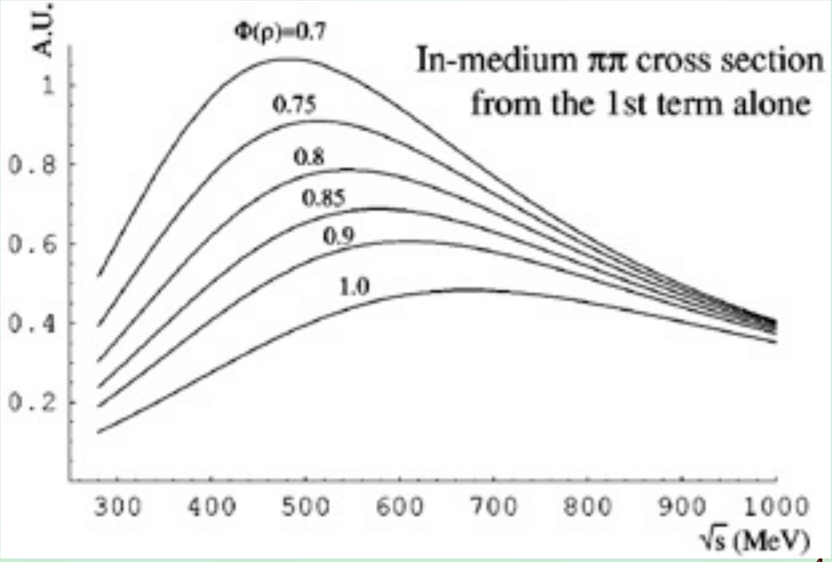
T. Hatsuda, T. Kunihiro, H. Shimizu
Phys. Rev. Lett. 82, 2840 (1999)



Partial restoration of chiral sym.
--> Spectral enhancement in
I=J=0 channel near threshold

Threshold enhancement of $\pi\pi$ cross section, also for the dynamical sigma meson

D. Jido, T. Hatsuda, T. Kunihiro,
Phys. Rev. D63, 011901 (2001)



Mechanism of the softening (chiral sigma)

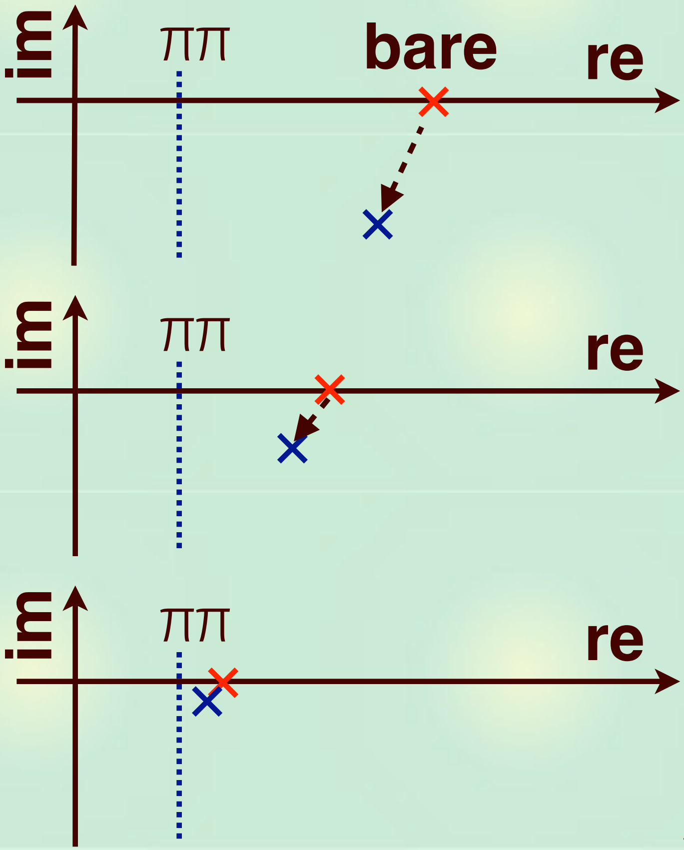
In the previous studies, it seems that the softening takes place, irrespective to the structure of the sigma meson.

Mechanism of the softening?

Softening of the **chiral sigma** (linear sigma model)

Sigma meson:
bare sigma pole acquires finite width through the coupling to $\pi\pi$

Chiral symmetry restoration:
--> **lowering bare sigma mass**
--> reduction of the phase space
--> narrow peak in spectrum



Mechanism of the softening (dynamical sigma)

Softening of the dynamical sigma (ChPT + unitarization)

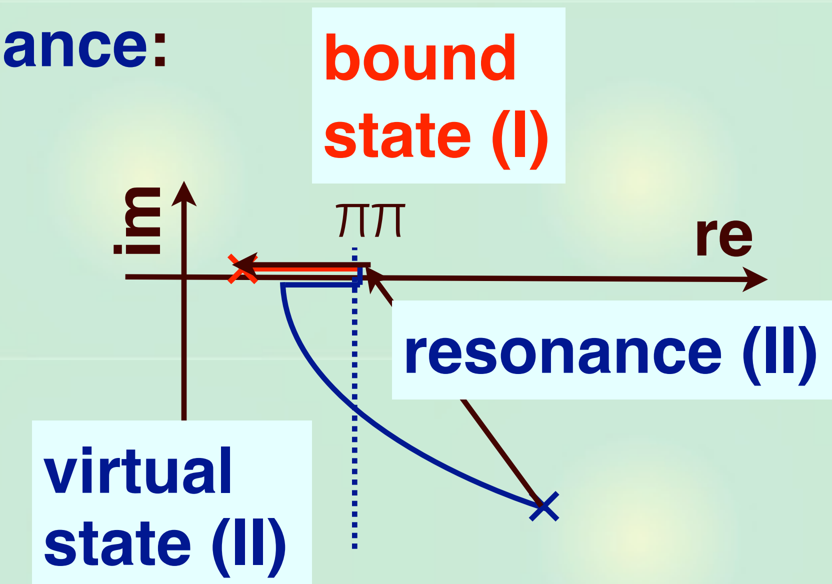
Sigma meson: dynamically generated by $\pi\pi$ attraction

Chiral symmetry restoration:
--> $f_\pi \sim \langle\sigma\rangle$ **decreases**
--> (attractive) interaction $\propto (f_\pi)^{-2}$ **increases**
--> resonance turns into bound state, peak gets narrow

Special nature of the s-wave resonance:

attractive

- resonance
- virtual state
  - pole on the 2nd Riemann sheet below threshold
  - ex.) spin singlet NN
- bound state



--> novel softening pattern?

Tree level interaction

Lagrangian of 2-flavor linear sigma model

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left[\partial M \partial M^\dagger - \mu^2 M M^\dagger - \frac{2\lambda}{4!}(MM^\dagger)^2 + h(M + M^\dagger)\right], \quad M = \sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}$$

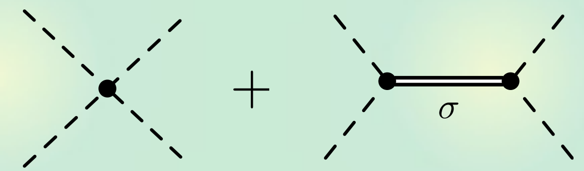
3 parameters <-- m_π, m_σ, $\langle\sigma\rangle$ at mean field level.

ππ scattering amplitude in general (crossing symmetry)

$$T_{\mathrm{tree}}(s,t,u) = A(s,t,u)\delta_{ab}\delta_{cd} + A(t,s,u)\delta_{ac}\delta_{bd} + A(u,t,s)\delta_{ad}\delta_{bc}$$

Tree-level ππ scattering amplitude

$$A(s,t,u) = \boxed{-\frac{m_\sigma^2 - m_\pi^2}{\langle\sigma\rangle^2} - \frac{(m_\sigma^2 - m_\pi^2)^2}{\langle\sigma\rangle^2}\frac{1}{s - m_\sigma^2}}$$



model A

$$= \boxed{\frac{s - m_\pi^2}{\langle\sigma\rangle^2}} - \frac{(s - m_\pi^2)^2}{\langle\sigma\rangle^2}\frac{1}{s - m_\sigma^2}$$

leading order term of ChPT: model B

Unitarization

Unitarity of S-matrix: conservation of probability.
Tree-level amplitude violates unitarity at certain energy.

Optical theorem:

$$\mathrm{Im}\, T^{-1}(s) = -\frac{\Theta(s)}{2} \quad \text{for} \quad s > 4m_\pi^2, \quad \Theta(s) = \frac{1}{16\pi}\sqrt{1-\frac{4m_\pi^2}{s}}$$

Scattering amplitude (N/D method + matching with $T_{\rm tree}$)

J. A. Oller, E. Oset, Phys. Rev. D60, 074023 (1999)

$$T(s;x) = \frac{1}{T^{-1}_{\rm tree}(s;x) + G(s)}$$

$$G(s) = \frac{1}{2}\frac{1}{(4\pi)^2}\left\{\boxed{a(\mu)} + \ln\frac{m_\pi^2}{\mu^2} + \sqrt{1-\frac{4m_\pi^2}{s}}\left[\ln\frac{\sqrt{1-\frac{4m_\pi^2}{s}}+1}{\sqrt{1-\frac{4m_\pi^2}{s}}-1}\right]\right\}$$

Subtraction constant: to exclude the CDD pole

$$G(s) = 0 \quad \text{at} \quad s = m_\pi^2, \quad a(m_\pi) = -\frac{\pi}{\sqrt{3}}$$

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

Amplitude in vacuum

Numerical result in vacuum

Input: $m_\pi = 140$ **MeV,** $m_\sigma = 550$ **MeV,** $\langle\sigma\rangle = 93$ **MeV**

	sigma origin	I=0 scattering length $(m_\pi)^{-1}$	pole position [MeV]
model A	chiral	0.244	423 - 126 i
model B	dynamical	0.174	364 - 356 i
(experiment)		0.222 [1]	441 - 272 i [2]

[1] J.R. Batley *et al.*, Eur. Phys. J. C54, 411 (2008)
[2] I. Caprini, G. Colangelo, H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006)

(we do not aim at fine-tuning of the parameters)

Prescription for symmetry restoration

We introduce the effect of chiral symmetry restoration from the outside of the model, by modifying m_π, m_σ, $<\sigma>$.

1) chiral condensate (pion decay constant): decreases

$$\langle\sigma\rangle = \Phi\langle\sigma\rangle_0, \quad 0 \leq \Phi \leq 1$$

2) mass of pion: no change

$$m_\pi = \text{const.}$$

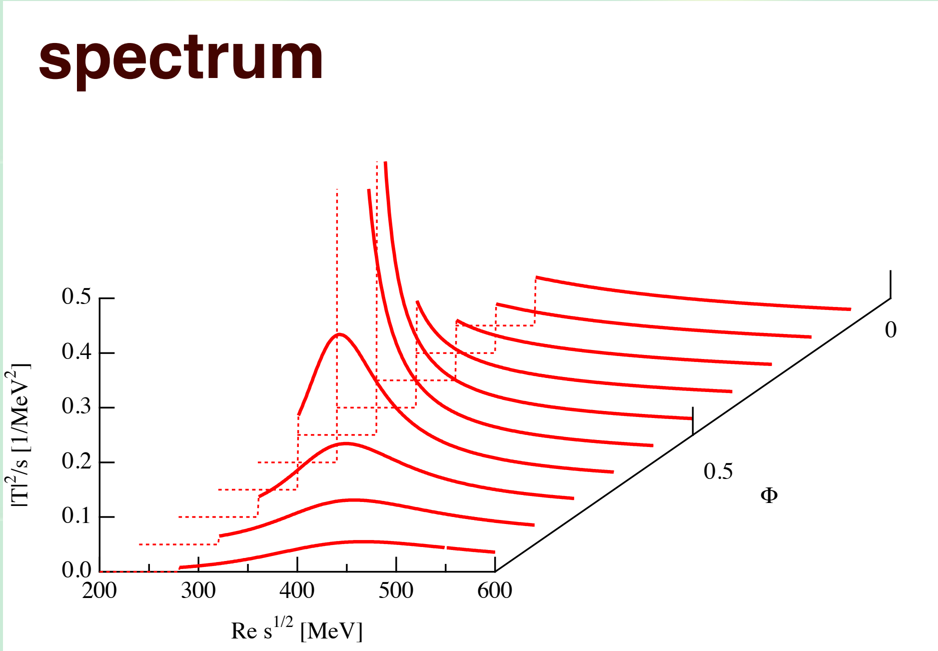
3) mass of chiral sigma for model A: decreases

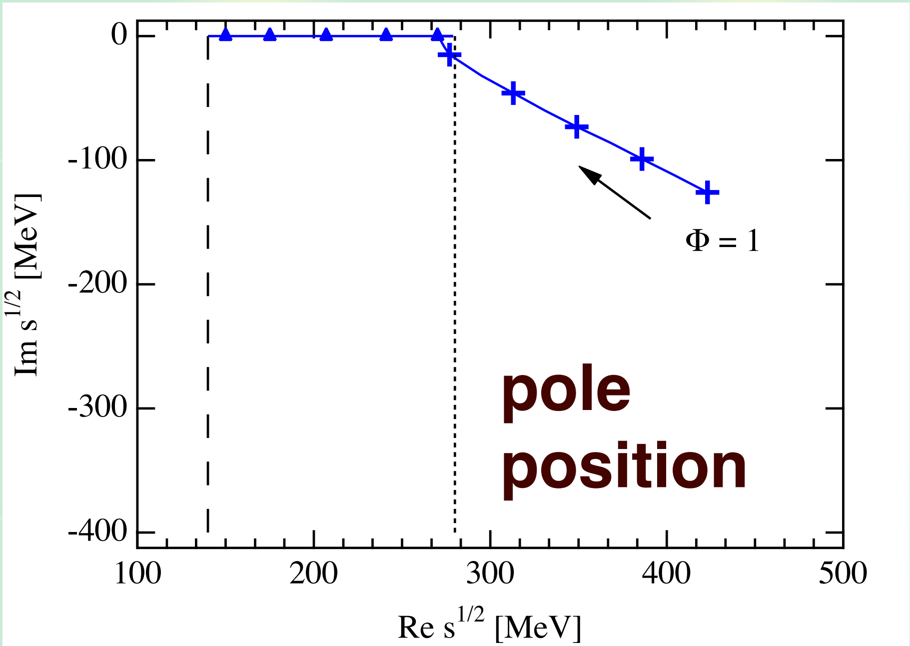
$$m_\sigma\big|_{\langle\sigma\rangle\to 0} = m_\pi, \quad \Rightarrow \quad m_\sigma = \sqrt{\lambda\frac{\langle\sigma\rangle^2}{3} + m_\pi^2}$$

with λ and m_π being fixed.

Symmetry restoration is modeled by the change of Φ.

Softening in model A



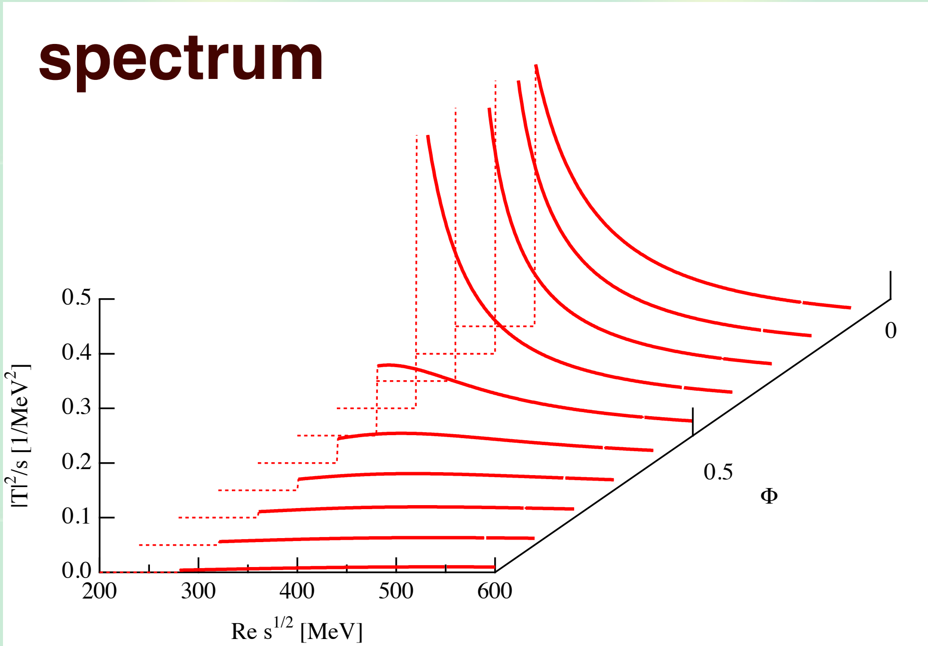


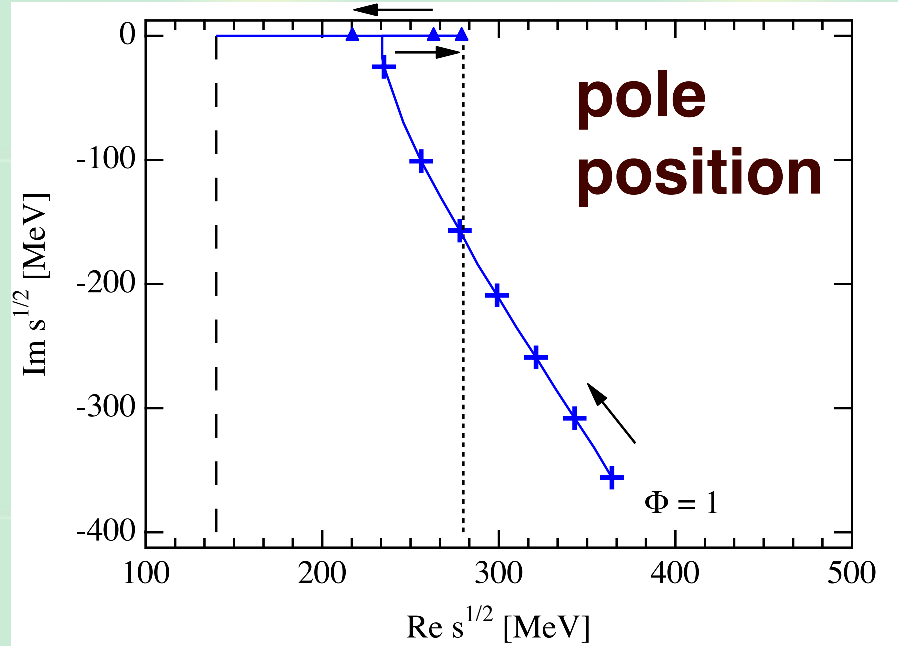
- Linear sigma model + unitarization : **chiral sigma**

- Softening takes place, as expected.

- peak at threshold : $\Phi \sim 0.6$
  <=> bare sigma pole moves below the threshold

Softening in model B





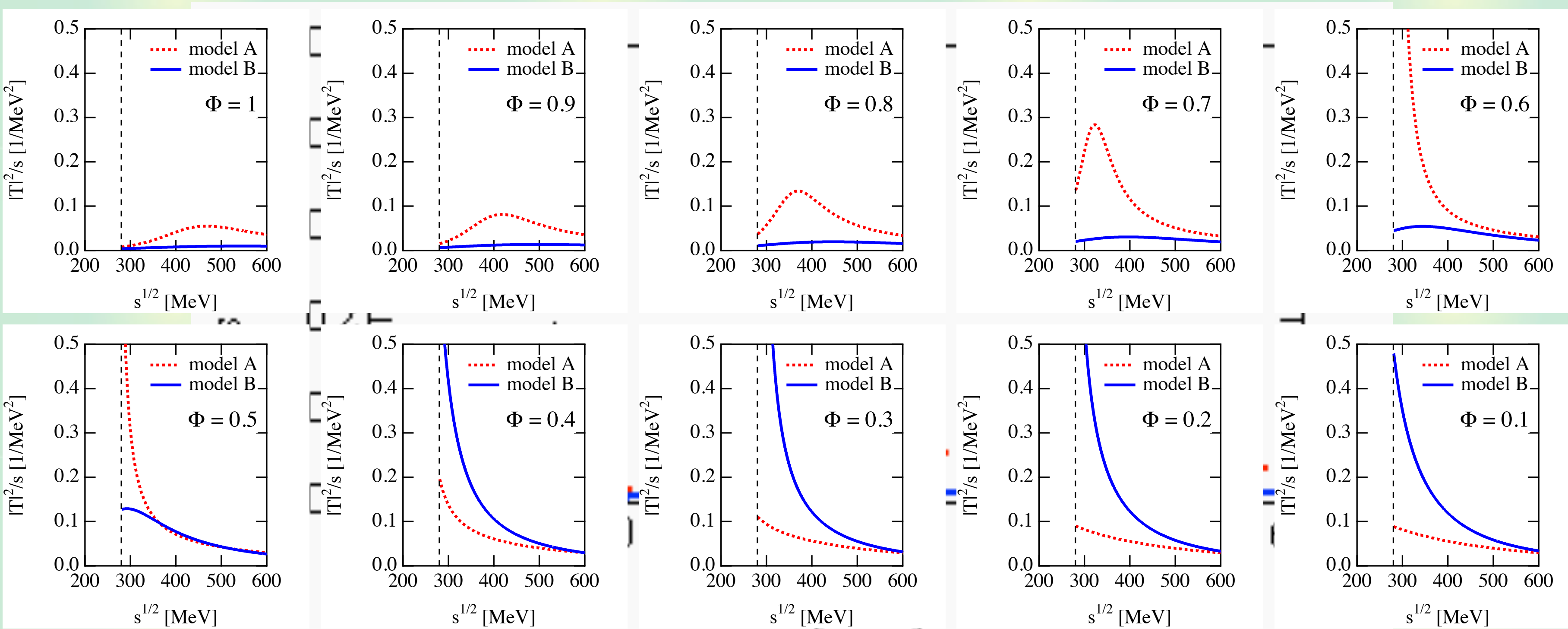
- **ChPT + unitarization : dynamical sigma**

- **Softening takes place, but virtual state appears.**

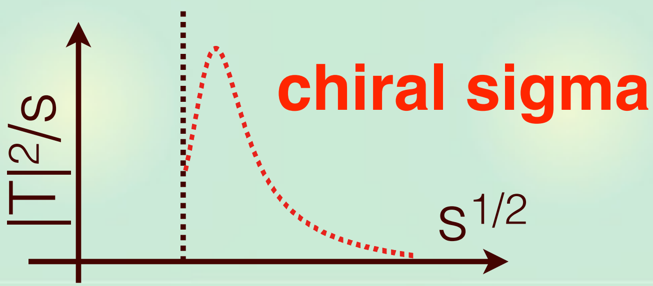
- **at** $\mathrm{Re}[M_{pole}] = 2m_\pi$ **(**$\Phi \sim 0.6$**), due to finite width, spectrum does not show the peak structure**

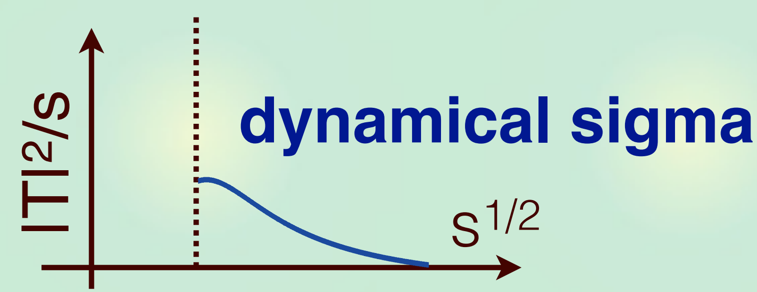
- **peak at threshold :** $\Phi \sim 0.3$ **<=> formation of bound state**

Comparison of model A and model B



- **Strong threshold enhancement : different from each other.**
- **Shape of the spectrum?**





Summary

We study the structure of the sigma meson using chiral symmetry restoration.

- **Dynamical scattering models with**

  (i) chiral sigma

  (ii) dynamical sigma

- **Dynamical sigma softens qualitatively differently from chiral sigma.**

  <-- virtual state (s-wave resonance)

T. Hyodo, D. Jido, T. Kunihiro, Nucl. Phys. A848, 341-365 (2010).