Compositeness of hadrons in field theoretical approach

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Excited hadrons

Fundamental fields in QCD: quarks and gluons

Asymptotic fields: hadrons (color singlet composites)
- mesons \( \sim q\bar{q} \), baryons \( \sim qqq \)

Excitation of hadrons (above two-hadron threshold):
- Internal quark dynamics
- Inter-hadron dynamics (resonances)

Structure of excited hadrons?
Structure of hadron resonances

Example) baryon excited state

Quark model

energy

internal excitation

$q\bar{q}$ pair creation

multiquark

hadronic molecule

What are 3q state, 5q state, MB state, ...?

Clear (model-independent) definition of the structure?
Definition of hadron structure

Number of quarks and *antiquarks* (≠ quark number) ?

\[ |\Lambda(1405)\rangle = + + \ldots \]

may not be a good classification scheme.

Number of *hadrons*

\[ |\Lambda(1405)\rangle = + + \ldots \]

Hadrons are *asymptotic states*  
--> different kinematical structure


**Compositeness of hadrons?**
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S. Weinberg, Phys. Rev. 137, B672 (1965)

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  T. Uchino, T. Hyodo, D. Jido, M. Oka, work in progress

Summary
Weinberg’s compositeness and deuteron

Definition of compositeness

$Z$: probability of finding deuteron in a bare elementary state

S. Weinberg, Phys. Rev. 137, B672 (1965)

$$|\text{deuteron}\rangle = \begin{pmatrix} N \\ N \end{pmatrix} \text{ or } \begin{pmatrix} N \\ N \end{pmatrix} \notin \text{NN model space}$$

$\sim$ elementary particle

$Z = 0$ \hspace{1cm} $Z = 1$

**model independent** relation for weakly bound state

$$a_s = \left[ \frac{2(1 - Z)}{2 - Z} \right] R + \mathcal{O}(m^{-1}), \quad r_e = \left[ \frac{-Z}{1 - Z} \right] R + \mathcal{O}(m^{-1})$$

$a_s$: scattering length

$r_e$: effective range

$R$: deuteron radius (binding energy)

$$a_s = +5.41 \text{ [fm]}, \quad r_e = +1.75 \text{ [fm]}, \quad R \equiv (2\mu B)^{-1/2} = 4.31 \text{ [fm]}$$

$\Rightarrow Z \lesssim 0.2 \quad \Rightarrow$ deuteron is almost composite!
Definition of compositeness

**Definition of the compositeness** $1-Z$

Hamiltonian of two-body system: free + interaction $V$

$$\mathcal{H} = \mathcal{H}_0 + V$$

Complete set for **free** Hamiltonian: bare $|B_0\rangle +$ continuum

$$1 = |B_0\rangle\langle B_0| + \int dk |k\rangle\langle k|$$

$$\mathcal{H}_0|B_0\rangle = E_0|B_0\rangle, \quad \mathcal{H}_0|k\rangle = E(k)|k\rangle$$

Physical bound state $|B\rangle$: eigenstate of **full** Hamiltonian

$$(\mathcal{H}_0 + V)|B\rangle = -B|B\rangle$$

$B$: binding energy

Define $Z$ as the overlap of $B$ and $B_0$:

: probability of finding the physical bound state in the bare state $|B\rangle$

$$Z \equiv |\langle B_0|B\rangle|^2$$

$1 - Z$: **Compositeness** of the bound state
Model-independent but approximated method

With the Schrödinger equation, we obtain

\[ 1 - Z = \int d\mathbf{k} \frac{|\langle \mathbf{k} | V | B \rangle|^2}{|E(\mathbf{k}) + B|^2} \langle k | V | B \rangle : \quad \mathbf{B} \rightarrow \mathbf{k} \]  

\[ = 4\pi \sqrt{2\mu^3} \int_0^\infty dE \frac{\sqrt{E}|G_W(E)|^2}{(E + B)^2} \langle k | V | B \rangle \equiv G_W[E(\mathbf{k})] \quad \text{for s-wave} \]

**Approximation:** For small binding energy \( B << 1 \), the vertex \( G_W(E) \) can be regarded as a constant: \( G_W(E) \sim g_W \)

Then the integration can be done analytically, leading to

\[ 1 - Z = 2\pi^2 \sqrt{2\mu^3} \frac{g_W^2}{\sqrt{B}} \]

**Compositeness \(<-\) coupling \( g_W \) and binding energy \( B \)**

S. Weinberg, Phys. Rev. 137 B672-B678 (1965)

- Model-independent: no information of \( V \)
- Approximated: valid only for small \( B \)
Definition of compositeness

**Z in Yukawa model**

Field theory with Yukawa coupling \((\psi, \varphi, B_0)\)

\[
\mathcal{L}_0 = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}(\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2) + \bar{B}_0(i\not{\partial} - M_0)B_0
\]

\[
\mathcal{L}_{\text{int}} = g_0 \bar{\psi} \varphi B_0 + (\text{h.c.})
\]

Physical bound state \(B\) at total energy \(W = M_B\)

Free (full) propagator of \(B_0\) \((B)\) field (positive energy part)

\[
\Delta_0(W) = \frac{1}{W - M_{B_0}}, \quad \Delta(W) = \frac{Z}{W - M_B}
\]

\(Z\): field renormalization constant

Dyson equation: relation between full and free propagators

\[
\Delta(W) = \Delta_0(W) + \Delta_0(W) g_0 G(W) g_0 \Delta(W)
\]
Definition of compositeness

Master formula of compositeness

Solution of Dyson equation and renormalization

\[ \Delta(W) = \frac{1}{W - M_{B_0} - g_0^2 G(W)} \rightarrow \frac{1}{W - g_0^2 G(W; a)} \]

Renormalization condition, pole at \( W = M_B \): \( M_B = g_0^2 G(M_B; a) \)

The field renormalization constant: residue of the propagator

\[ Z = \lim_{W \rightarrow M_B} \frac{W - M_B}{W - g_0^2 G(W; a)} = \frac{1}{1 - g_0^2 G'(M_B)} \]

Physical coupling constant: residue of T-matrix

\[ g^2 = g_0^2 Z \]

Compositeness in Yukawa theory

\[ 1 - Z = -g^2 G'(M_B) \]
Compositeness: summary

Compositeness of bound states

Method 1: nonrelativistic quantum mechanics

\[ 1 - Z_{NR} = g^2 \frac{M|\lambda^{1/2}(M_B^2, M^2, m^2)|}{16\pi M_B^2(M + m - M_B)} \text{ for } M_B \rightarrow M + m \]

\text{model independent}, but valid only for weak binding

Method 2: field theory with Yukawa coupling

\[ 1 - Z = -g^2 G'(M_B) \]

\text{exact (any } M_B), \text{ but Lagrangian dependent}

Compositeness \leftarrow \text{ mass } "M_B" \text{ and coupling constant } "g"

Experiments, Lattice QCD, Model calculation, ...
Chiral coupled-channel approach: $MB$ scattering, $B^*$

- Interaction $\leftrightarrow$ chiral symmetry
- Amplitude $\leftrightarrow$ unitarity in coupled channels


Test: single-channel scattering of meson $m$ and baryon $M$.

\[
T(W) = \frac{1}{1 - V(W)G(W; a)} V(W) \quad \text{cutoff parameter}
\]

$V$: 4-point interaction, attractive

\[
V(W) = \begin{cases} 
V^{(\text{const})} = Cm & \text{constant interaction} \\
V^{(\text{WT})}(W) = C(W - M) & \text{WT interaction}
\end{cases}
\]
Natural renormalization condition

Mass and coupling of the bound state in dynamical model

Mass: bound state condition (pole at $W=M_B$)

$$1 - V(M_B)G(M_B; a) = 0$$

Coupling constant: residue of the pole

$$g^2 = \lim_{W \to M_B} (W - M_B)T(W) = \begin{cases} 
- [G'(M_B)]^{-1} \\
- \left[ G'(M_B) + \frac{G(M_B; a)}{M_B - M} \right]^{-1}
\end{cases}$$

constant interaction

WT interaction

Apply the master formula of compositeness

$$1 - Z = -g^2 G'(M_B)$$
Compositeness of bound states

Compositeness in Yukawa theory

\[ 1 - Z = -g^2 G'(M_B) = \begin{cases} 1 & \text{constant interaction} \\ 1 + \frac{G(M_B; a)}{(M_B - M)G'(M_B)} & \text{WT interaction} \end{cases} \]

- constant interaction --> purely composite bound state
- WT interaction --> mixture of composite and elementary

- Purely composite bound state for WT interaction:

\[ G'(M_B) = -\infty \quad \text{or} \quad G(M_B; a) = 0 \]

\[ M_B = M + m \quad \text{or} \quad C \rightarrow -\infty \]

1) zero energy bound state
2) infinitely strong two-body attraction

Model space ≠ structure of generated resonances
Application to hadron models

Check of natural renormalization scheme

WT Natural renormalization condition
\[ \langle \text{to exclude elementary contribution from the loop function} \]


\[ G(W = M; a_{\text{natural}}) = 0 \]

1) \( a = a_{\text{natural}}, \text{ vary } B \)

2) \( B = 5 \text{ MeV, vary } a \)

natural scheme \( \rightarrow Z \sim 0 \)

large deviation \( \rightarrow Z \sim 1 \)
Naive generalization: input pole position and residue

\[ 1 - Z = - g^2 G'(M_B) \]

- no completeness: \( Z \) is not normalized
- \( g, M_B \) are complex: \( Z \) is complex

Number of degrees of freedom

Bound states: \( g, M_B \) \( \leftrightarrow \) \( g_0, a \)

| 2 | 2 |

Resonances: \( \text{Re } g, \text{Im } g, \text{Re } z_R, \text{Im } z_R \) \( \leftrightarrow \) \( g_0, a \)

| 4 | 2 |

Definition with simple Yukawa model is insufficient?
Yukawa model:

\[ g_0 \text{ controls both } B_0\psi\varphi \text{ coupling and } \psi\varphi \rightarrow \psi\varphi \]

Add contact interaction to control \( \psi\varphi \rightarrow \psi\varphi \)

\[ V = \frac{g_0^2}{W - M_0} + V_{\text{con}} \]

- wavefunction renormalization + vertex renormalization

\[ g^2 = Z_3 g_0^2 \]

Origin of the phase of the residue?

T. Uchino, T. Hyodo, D. Jido, M. Oka, work in progress
Compositeness of the bound state

Field renormalization constant $Z$: compositeness

Model independent formula

\[ 1 - Z_{NR} = g^2 \frac{M|\lambda^{1/2}(M_B^2, M^2, m^2)|}{16\pi M_B^2(M + m - M_B)} \text{ for } M_B \rightarrow M + m \]

S. Weinberg, Phys. Rev. 137 B672 (1965)

Exact formula

\[ 1 - Z = -g^2 G'(M_B) \]


Expressed in terms of physical quantities
Application to hadron models

Bound state by energy-indep. int.

--> purely composite state

Bound state by energy-dep. (chiral) int.

--> mixture of composite and elementary

Natural scheme corresponds to $Z \sim 0$

--> composite particle is generated