Structure of near-threshold s-wave resonances

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Weinberg’s compositeness and deuteron

$Z$: probability of finding deuteron in a bare elementary state

$S. \text{ Weinberg, Phys. Rev. 137, B672 (1965)}$

$|\text{deuteron}\rangle = \left| \begin{array}{c} N \cr N \end{array} \right| \text{ or } \left| \begin{array}{c} N \end{array} \right|

\not\in NN \text{ model space}

\sim \text{ elementary particle}

Z = 0 \quad Z = 1

Model-independent relation for a shallow bound state

\[ a_s = \left[ \frac{2(1 - Z)}{2 - Z} \right] R + \mathcal{O}(m^{-1}_\pi), \quad r_e = \left[ \frac{-Z}{1 - Z} \right] R + \mathcal{O}(m^{-1}_\pi) \]

$a_s \sim 5.41 \text{ [fm]}$: scattering length

$r_e \sim 1.75 \text{ [fm]}$: effective range

$R \sim (2\mu B)^{-1/2} \sim 4.31 \text{ [fm]}$: deuteron radius (binding energy)

$\Rightarrow Z \approx 0.2$: Deuteron is almost composite!
Application to near-threshold resonances

Features of the Weinberg's argument:

- Model-independent approach (no potential, wave-fn, ... )
- Relation with experimental observables
- Only for **bound states with small binding**

Interesting (exotic) hadrons: **resonances**

--> application to resonances by analytic continuation

\[
1 - Z = \int dq \frac{|\langle q | V | B \rangle|^2}{[E(q) + B]^2} \sim -g^2 \frac{dG(W)}{dW} \bigg|_{W=M_B}
\]


- \(Z\) can be **complex and larger than unity**. Interpretation?

What about **near-threshold resonances** (~ small binding)?
Application to near-threshold resonances

**Effective range expansion**

S-wave scattering amplitude at low momentum

\[ f(k) = \frac{1}{k \cot \delta - ki} \rightarrow \left( \frac{1}{a} - ki + \frac{r_e}{2} k^2 \right)^{-1} \]

Truncation is valid only at small \( k \).

**Scattering length** \( a \)

- strength of the interaction
- cross section at zero momentum: \( 4\pi a^2 \)

**Effective range** \( r_e \)

- typical length scale of the interaction
- can be negative (energy-dep., Feshbach resonance, ...)

Application to near-threshold resonances

The amplitude has two poles

\[
f(k) = \left( \frac{1}{a} - ki + \frac{r_e}{2}k^2 \right)^{-1}
\]

\[
k^\pm = i \frac{1}{r_e} \pm \frac{1}{r_e} \sqrt{-\frac{2r_e}{a} - 1}
\]

Pole trajectories with a fixed \( r_e < 0 \)

Positions of poles \(<-->\) scattering length + effective range

\[
a = \frac{k^+ + k^-}{ik^+ k^-}, \quad r_e = \frac{2i}{k^+ + k^-}
\]

\((a, r_e)\) are real for resonances
Application to near-threshold resonances

Field renormalization constant

Eliminate $R$ from the Weinberg’s relations

$$Z = 1 - \sqrt{1 - \frac{1}{1 + a/(2r_e)}} = \frac{2k^-}{k^- - k^+}$$

$Z$ (residue) is determined by the pole position

$1 - Z$ is pure imaginary and $0 \leq |1 - Z| \leq 1$
Application to near-threshold resonances

Validity of the effective range expansion

A model calculation

- solid lines: pole position in a scattering model
- dashed lines: position deduced from \((a, r_e)\)

If the effective range is large, the expansion works well.
Application to near-threshold resonances

**Example: \( \Lambda_c(2595) \)**

Pole position of \( \Lambda_c(2595) \) with \( \pi\Sigma_c \) threshold in PDG

<table>
<thead>
<tr>
<th>( \mathbb{E} ) [MeV]</th>
<th>( \Gamma ) [MeV]</th>
<th>( a ) [fm]</th>
<th>( r_e ) [fm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.67</td>
<td>2.59</td>
<td>10.5</td>
<td>-19.5</td>
</tr>
</tbody>
</table>

- Isospin symmetry is assumed.
- \( \pi\pi\Lambda \) channel is not taken into account.
- Decay of \( \Sigma_c \) is not taken into account.

\[ |1-Z| \sim 0.6 \quad \text{Interpretation ?} \]

**Larger effective range** than typical hadronic scale

Chiral interaction gives \( r_e \sim -4.6 \) fm

\( \Rightarrow \Lambda_c(2595) \) is not likely a \( \pi\Sigma_c \) molecule
Near-threshold s-wave resonances

Effective range expansion:
Resonance structure $\leftrightarrow (a, r_e)$

Compositeness $1-Z$:
pure imaginary and normalized

Application to $\Lambda_c(2595)$

Large $r_e$ $\rightarrow$ not likely a molecule