Quark mass dependence of H-dibaryon in $\Lambda\Lambda$ scattering

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Introduction

Formulation
- Effective field theory
- Quark mass dependence

Result
- $\Lambda\Lambda$ amplitude: SU(3) limit / physical point
- Extrapolation in quark mass plane

Summary
H-dibaryon in $\Lambda\Lambda$ scattering

H-dibaryon: $uu dd ss$ bound state predicted in a quark model


Experiments: Negative

- Nagara event: double $\Lambda$ hyper nuclei $\rightarrow$ no deeply bound H

- Belle: $\Upsilon(1S), \Upsilon(2S)$ decay $\rightarrow$ no signal (<< deuteron)
Recent activities

**RHIC-STAR: \( \Lambda \Lambda \) correlation \( \rightarrow \) scattering length**


**H-dibaryon in lattice QCD**

- **Bound with unphysical quark masses**

HAL QCD, T. Inoue *et al.*, Phys. Rev. Lett. 106, 162002 (2011);  
NPLQCD, S. Beane *et al.*, Phys. Rev. Lett. 106, 162001 (2011);  
HAL QCD, T. Inoue *et al.*, Nucl. Phys. A881, 28 (2012); …

- **Physical point simulation is ongoing.**
Near-threshold scaling

Extrapolation: **unbound** at physical point


Near-threshold scaling in s-wave (bound -> unbound)


- unitary limit (infinitely large scattering length)

Unitary limit at unphysical quark masses?
How does the H-dibaryon bound state in the $\Lambda\Lambda$ scattering change along with the variation of the quark masses?

Input: three lightest lattice data in SU(3) limit.

Effective framework which describes the $\Lambda\Lambda$ scattering in a relatively wide range of quark masses.

(Precise $\Lambda\Lambda$ interaction at physical point may be studied by lattice QCD / systematic ChPT.)
Low-energy baryon-baryon scattering

Length scales in the SU(3) limit


- Interaction range by NG boson exchange: \( r_0 \sim 0.24-0.42 \) fm
- large scattering length: \( a \sim 1.2-1.7 \) fm
- large radius \(< \) small binding energy: \( 0.77-1.14 \) fm

At low energy, the interaction can be treated as point like.
Effective Lagrangian

Low energy effective Lagrangian with contact interactions


\[ \mathcal{L}_{\text{free}} = \sum_{a=1}^{4} \sum_{\sigma=\uparrow, \downarrow} B_{a,\sigma}^{\dagger} \left( i \frac{\partial}{\partial t} + \frac{\nabla^2}{2M_a} + \delta_a \right) B_{a,\sigma} + H^{\dagger} \left( i \frac{\partial}{\partial t} + \frac{\nabla^2}{2M_H} + \nu \right) H \]

\[ \mathcal{L}_{\text{int}} = -g[D^{(1)} H + H^{\dagger} D^{(1)}] - \lambda^{(1)} D^{(1)} H^{\dagger} D^{(1)} - \lambda^{(8)} D^{(8)} H^{\dagger} D^{(8)} - \lambda^{(27)} D^{(27)} H^{\dagger} D^{(27)} \]

\[ D^{(F)} = [BB]^{(F)}_{J=0, S=-2, I=0} \]

Length scales at the physical point

- No \( \pi \) exchange in \( \Lambda \Lambda \). \( \pi \) exchange in \( \Xi \Xi (\Lambda \Lambda + 25 \text{ MeV}) \)

\( \rightarrow \) safely applicable below \( \Xi \Xi \) threshold
**Low energy scattering amplitude**

**Coupled-channel scattering amplitude** \((i=ΛΛ, NΞ, ΣΣ)\)

\[
f_{ii}(E) = \frac{\mu_i}{2\pi} \left[ (\mathcal{A}_{\text{tree}}^{\text{tree}}(E))^{-1} + I(E) \right]^{-1}
\]

\[
\mathcal{A}_{ij}^{\text{tree}}(E) = i \quad j + i \quad j
\]

\[
= - \left( V_{ij} + \frac{g^2 d_i^\dagger d_j}{E - \nu + i0^+} \right), \quad V = U^{-1} \begin{pmatrix} \lambda^{(1)} \\ \lambda^{(8)} \\ \lambda^{(27)} \end{pmatrix} U, \quad d = \begin{pmatrix} -\sqrt{\frac{1}{8}} \\ -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{3}{8}} \end{pmatrix}
\]

\[
I_i(E) = \quad i
\]

\[
= \frac{\mu_i}{\pi^2} \left( -\Lambda + k_i \text{artanh} \frac{\Lambda}{k_i} \right), \quad k_i = \sqrt{2\mu_i(E - \Delta_i)}
\]

**EFT describes the low energy scattering for a given \((m_l, m_s)\).**

- scattering length, bound state pole, ...
- Quark mass dep. \(\rightarrow\) baryon masses and couplings \(\lambda\)
Modeling quark mass dependence

“Quark masses” via GMOR relation

\[ B_0 m_l = \frac{m^2_\pi}{2}, \quad B_0 m_s = m^2_K - \frac{m^2_\pi}{2} \]
\[ B_0 = -\frac{\langle \bar{q}q \rangle}{3F^2_0} \]

Baryon masses: linear in \( m_q \)

\[ M_N(m_l, m_s) = M_0 - (2\alpha + 2\beta + 4\sigma)B_0 m_l - 2\sigma B_0 m_s, \]
\[ M_\Lambda(m_l, m_s) = M_0 - (\alpha + 2\beta + 4\sigma)B_0 m_l - (\alpha + 2\sigma)B_0 m_s, \]
\[ M_\Sigma(m_l, m_s) = M_0 - \left(\frac{5}{3}\alpha + \frac{2}{3}\beta + 4\sigma\right)B_0 m_l - \left(\frac{1}{3}\alpha + \frac{4}{3}\beta + 2\sigma\right)B_0 m_s, \]
\[ M_\Xi(m_l, m_s) = M_0 - \left(\frac{1}{3}\alpha + \frac{4}{3}\beta + 4\sigma\right)B_0 m_l - \left(\frac{5}{3}\alpha + \frac{2}{3}\beta + 2\sigma\right)B_0 m_s \]

- three mass difference by \((\alpha, \beta) \rightarrow \text{GMO relation}\)
- fit to experiment/lattice \(\rightarrow\) reasonable

Modeling quark mass dependence

Coupling constants ← scattering length in SU(3) limit

T. Inoue, private communication.

- $a = -f(E=0)$ 1: bound, 8: repulsive, 27: attractive

- This talk: linear in $m_q$, no bare H

$$\lambda^{(F)}(m_l, m_s) = \lambda_0^{(F)} + \lambda_1^{(F)} B_0 (2m_l + m_s)$$

$g(m_l, m_s) = 0$

![Graph showing repulsive and attractive behavior](image)
**Result**

∧∧ amplitude : SU(3) limit

∧∧ scattering amplitude in the SU(3) limit

\[ f_{\Lambda\Lambda}(E) = \frac{1}{8} f^{(1)}(E) + \frac{1}{5} f^{(8)}(E) + \frac{27}{40} f^{(27)}(E) \]

- **bound H** ← bound state in 1
- **attractive scattering length** \( a = -f(E=0) \) ← attraction in 27


- **CDD pole below threshold:** \( f(E)=0 \) \( \rightarrow \) ERE breaks down.

Fig. 10. The invariant-mass-spectrum of $\Lambda\Lambda$ calculated by assuming the S-wave dominance, at several values of the $SU(3)$ breaking parameter $x$.

Shown in Fig. 10 is the invariant-mass-spectrum of the process $\Lambda\Lambda \rightarrow \Lambda\Lambda$ given by

$$\rho_{\Lambda\Lambda}(\sqrt{s}) = |S_l=0_{\Lambda\Lambda}-1|^2 / k$$

with an assumption of S-wave dominance. A peak which corresponds to the $H$-dibaryon can be clearly seen at $x=0.6, 0.8, 1.0$. This demonstrates that there is a chance for experiments of counting two $\Lambda$'s to confirm the existence of the resonant $H$-dibaryon in nature. Deeply bound $H$-dibaryon with the binding energy $B_H > 7$ MeV from the $\Lambda\Lambda$ threshold has been ruled out by the discovery of the double $\Lambda$ hypernucleus, $^6\Lambda\Lambda\text{He}$ [5].

On the other hand, an enhancement of the two $\Lambda$'s production has been observed at a little above $\Lambda\Lambda$ threshold.

Result

- no bound H, but a resonance below $N\Xi$ threshold
- attractive scattering length: $a_{\Lambda\Lambda} = -3.2$ fm
- Ramsauer-Townsend effect near resonance: $\delta=\pi \rightarrow f(E)=0$ <-- remnant of the CDD pole

Extrapolation and unitary limit

Extrapolation in the NG boson/quark mass plane

\[ B_0 m_l = \frac{m_\pi^2}{2}, \quad B_0 m_s = m_K^2 - \frac{m_\pi^2}{2} \]

- unitary limit between SU(3) limit and physical point

- Bound

Unbound

HAL QCD
NPLQCD
physical point
unitary limit

Bound

Unbound

HAL QCD
NPLQCD
physical point
unitary limit
Implication to many-body system

Many-body system of $\wedge$ baryons: BEC-BCS crossover


- “H-matter” may be realized with unphysical quark masses.
We study the quark mass dependence of the H-dibaryon and the ΛΛ interaction using EFT.

SU(3) limit: bound H with attractive scattering length \( \sim \) CDD pole below the threshold.

Physical point: Ramsauer-Townsend effect near resonance H \( \sim \) remnant of the CDD pole.

The ΛΛ scattering undergoes the unitary limit between SU(3) limit and physical point.