Structure and compositeness of exotic hadrons

Tetsuo Hyodo

Yukawa Institute for Theoretical Physics, Kyoto Univ.

2018, Oct. 25th
Introduction: exotic hadron resonances

- “Structure” of unstable state is nontrivial!

Compositeness of hadron resonances

S. Weinberg, Phys. Rev. 137, B672 (1965);

- Weak binding relation from EFT

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);

- Implication from nearby CDD zero

Introduction: exotic hadron resonances

Classification of hadrons

Observed hadrons

PDG2018: http://pdg.lbl.gov/

~ 150 baryons

~ 210 mesons

All ~ 360 hadrons emerge from single QCD Lagrangian. All flavor quantum numbers are described by $qqq/\bar{q}\bar{q}$. 
Introduction: exotic hadron resonances

Exotic candidates beyond $qqq/\bar{q}\bar{q}$

Tetraquark candidate (Belle)  
$\mathcal{Z}_b(10610), \mathcal{Z}_b(10650)$

$Y(5S) \rightarrow \pi^\pm + \mathcal{Z}_b$

$\rightarrow Y(nS)(b\bar{b}) + \pi^\pm(u\bar{d}/d\bar{u})$


Pentaquark candidate (LHCb)  
$P_c(4450), P_c(4380)$

$\Lambda_b \rightarrow K^- + P_c$

$\rightarrow J/\psi(c\bar{c}) + p(uud)$


Only a few are observed. Why only a few?
Various hadronic excitations

Description of excited baryons

Conventional structure

Exotic structures

In QCD, non-qqq structures naturally arise.

- Baryons: superposition of qqq + exotic structures

→ How can we disentangle different components?
Introduction: exotic hadron resonances

Unstable states via strong interaction

Hadron resonances

PDG2018: http://pdg.lbl.gov/

- stable/unstable via strong interaction
- Excited states are mostly unstable. —> resonances
**Introduction: exotic hadron resonances**

**Difficulty of resonances**

**Resonance as an “eigenstate” of Hamiltonian**

- **complex energy**
  G. Gamow, Z. Phys. 51, 204 (1928)

  **Zur Quantentheorie des Atomkernes.**
  Von G. Gamow, z. Zt. in Göttingen.
  Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

- **diverging wave function** ($\text{Im} \ k < 0$)

  \[
  \langle R \mid R \rangle = \int dr |\psi_R(r)|^2 \sim \int_0^\infty dr e^{-2\text{Im}[k]r} \to \infty
  \]

  **Bi-orthogonal basis (Gamow vectors): normalizable!**

  N. Hokkyo, Prog. Theor. Phys. 33, 1116 (1965)

  \[
  |\tilde{R}\rangle = |R^*\rangle, \quad \langle\tilde{R} \mid R\rangle = \left| \int dr [\psi_R(r)]^2 \right| < \infty
  \]

  - **Complex expectation value** (norm, $<r^2>$) $\rightarrow$ interpretation?
Introduction: exotic hadron resonances

Classification of resonances

1) Potential (shape) resonance
   - 1 channel (P)
   - potential barrier: \( E > 0 \)
   - unstable via tunneling
   - (composite of \( P \)-channel)

2) Feshbach resonance
   - coupled-channel (\( P + Q \))
   - bound state of \( Q \): \( E_Q < 0, \ E_P > 0 \)
   - unstable via transition
   - ("elementary": other than \( P \))

Classification by their origin
Structure of unstable state is nontrivial.

Weak binding relation for stable bound states

S. Weinberg, Phys. Rev. 137, B672 (1965)

Compositeness × threshold channel

or

“Elementariness” Z

other contributions

↑

observables

Effective field theory —> description of low-energy scattering amplitude, generalization to unstable resonances
Weak binding relation for stable states

**Weak binding relation from EFT**

**Compositeness $X$ of s-wave weakly bound state ($R \gg R_{\text{typ}}$)**

S. Weinberg, Phys. Rev. 137, B672 (1965);

\[ |d\rangle = \sqrt{X} |NN\rangle + \sqrt{1 - X} |\text{others}\rangle \]

\[ a_0 = R \left\{ \frac{2X}{1 + X} + \mathcal{O} \left( \frac{R_{\text{typ}}}{R} \right) \right\}, \quad r_e = R \left\{ \frac{X - 1}{X} + \mathcal{O} \left( \frac{R_{\text{typ}}}{R} \right) \right\} \]

$a_0$: scattering length, $r_e$: effective range
$R = (2\mu B)^{-1/2}$: radius of wave function
$R_{\text{typ}}$: length scale of interaction

- Deuteron is NN composite ($a_0 \sim R \gg r_e$) $\rightarrow X \sim 1$
- Internal structure from observable

Problem: applicable only for stable states.
Low-energy scattering with near-threshold bound state

- Nonrelativistic EFT with contact interaction


\[
H_{\text{free}} = \int dr \left[ \frac{1}{2M} \nabla \psi^\dagger \cdot \nabla \psi + \frac{1}{2m} \nabla \phi^\dagger \cdot \nabla \phi + \frac{1}{2M_0} \nabla B_0^\dagger \cdot \nabla B_0 + \nu_0 B_0^\dagger B_0 \right],
\]

\[
H_{\text{int}} = \int dr \left[ g_0 \left( B_0^\dagger \phi \psi + \psi^\dagger \phi^\dagger B_0 \right) + \nu_0 \psi^\dagger \phi^\dagger \phi \psi \right]
\]

- **cutoff** : \( \Lambda \sim 1/R_{\text{typ}} \) (interaction range of microscopic theory)

- At low energy \( p \ll \Lambda \), interaction \( \sim \) contact
Eigenstates

\[ H_{\text{free}} |B_0\rangle = \nu_0 |B_0\rangle, \quad H_{\text{free}} |p\rangle = \frac{p^2}{2\mu} |p\rangle \]

free (discrete + continuum)

\[ (H_{\text{free}} + H_{\text{int}}) |B\rangle = -B |B\rangle \]

full (bound state)

- normalization of \(|B\rangle\) + completeness relation

\[ \langle B |B\rangle = 1, \quad 1 = |B_0\rangle\langle B_0| + \int \frac{dp}{(2\pi)^3} |p\rangle\langle p| \]

- projections onto free eigenstates

\[ 1 = Z + X, \quad Z \equiv |\langle B_0 | B\rangle|^2, \quad X \equiv \int \frac{dp}{(2\pi)^3} |\langle p | B\rangle|^2 \]

"elementariness" compositeness

\[ Z, X: \text{real and nonnegative} \rightarrow \text{interpreted as probability} \]
Weak binding relation from EFT

**Weak binding relation**

ψΦ scattering amplitude (exact result)

\[
f(E) = - \frac{\mu}{2\pi} \frac{1}{[v(E)]^{-1} - G'(E)}
\]

\[
v(E) = v_0 + \frac{g_0^2}{E - v_0}, \quad G(E) = \frac{1}{2\pi^2} \int_0^\Lambda dp \frac{p^2}{E - p^2/(2\mu) + i0^+}
\]

Compositeness \(X \leftarrow v(E), G(E)\)

\[
X = \frac{G'(-B)}{G'(-B) - [1/v(-B)]'}
\]

\(1/R = (2\mu B)^{1/2}\) expansion: leading term \(\leftarrow X\)

\[
a_0 = -f(E = 0) = R \left\{ \frac{2X}{1 + X} + \mathcal{O} \left( \frac{R_{\text{typ}}}{R} \right) \right\}
\]

renormalization dependent

renormalization independent

If \(R \gg R_{\text{typ}}\), correction terms neglected: \(X \leftarrow (B, a_0)\)
Weak binding relation from EFT

**Introduction of decay channel**

**Introduce decay channel**

\[
H'_{\text{free}} = \int dr \left[ \frac{1}{2M'} \nabla \phi'^\dagger \cdot \nabla \phi' - \nu \psi \psi'^\dagger \psi' + \frac{1}{2m'} \nabla \phi'^\dagger \cdot \nabla \phi' - \nu \phi \phi'^\dagger \phi' \right],
\]

\[
H'_{\text{int}} = \int dr \left[ g' \left( B_0 \phi' \psi' + \psi'^\dagger \phi'^\dagger B_0 \right) + v' \psi'^\dagger \phi' \phi' \psi' + v^t \left( \psi'^\dagger \phi' \phi' \psi' + \psi'^\dagger \phi'^\dagger \phi' \psi' \right) \right],
\]

**Quasi-bound state: complex eigenvalue**

\[
H = H_{\text{free}} + H'_{\text{free}} + H_{\text{int}} + H'_{\text{int}}
\]

\[
H |QB\rangle = E_{QB} |QB\rangle, \quad E_{QB} \in \mathbb{C}
\]

**Generalized relation: correction term \leftarrow threshold difference**

\[
a_0 = R \left\{ \frac{2X}{1 + X} + \mathcal{O} \left( \frac{R_{\text{typ}}}{R} \right) + \mathcal{O} \left( \frac{l^3}{R^3} \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}} \in \mathbb{C}, \quad l \equiv \frac{1}{\sqrt{2\mu \nu}}
\]


**If \( |R| \gg (R_{\text{typ}}, |l|) \) correction terms neglected:**

\[
X \leftarrow (E_{QB}, a_0)
\]
Complex compositeness

Unstable states $\rightarrow$ complex $Z$ and $X$

\[ Z + X = 1, \quad Z, X \in \mathbb{C} \]

- Probabilistic interpretation?

New definition

\[ \tilde{Z} = \frac{1 - |X| + |Z|}{2}, \quad \tilde{X} = \frac{1 - |Z| + |X|}{2} \]

- interpreted as probabilities $\tilde{Z} + \tilde{X} = 1, \quad \tilde{Z}, \tilde{X} \in [0, 1]$
- reduces to $Z$ and $X$ in the bound state limit

\[ U : \text{uncertainty of interpretation} \]

\[ U = |Z| + |X| - 1 \]


- Sensible interpretation only for small $U$ case
Weak binding relation from EFT

Application: \( \Lambda(1405) \) properties

Generalized weak binding relation

\[
a_0 = R \left\{ \frac{2X}{1 + X} + \mathcal{O} \left( \left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O} \left( \left| \frac{l}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu \nu}}
\]

- Consider \( \Lambda(1405) \) in \( \bar{K}N \) scattering
- To determine \( X \), we need \((E_{QB}, a_0)\)

Recent analysis with chiral SU(3) dynamics

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)

- Chiral interaction up to NLO order
- Fit to whole set of experimental data (with SIDDARTA)

\[
\rightarrow E_{QB} = -10 \text{ -26i MeV}, \quad a_0 = 1.39-0.85i \text{ fm}
\]

(position of the “high-mass pole”)
Weak binding relation from EFT

Application: result

Estimation of correction terms: $|R| \sim 2 \text{ fm}$

$$a_0 = R \left\{ \frac{2X}{1 + X} + O \left( \left| \frac{R_{\text{typ}}}{R} \right| \right) + O \left( \left| \frac{l}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu \nu}}$$

- vector meson exchange picture: $|R_{\text{typ}}/R| \sim 0.12$
- energy difference from $\pi\Sigma$: $|l/R|^3 \sim 0.16$

Neglecting the correction terms, we obtain

$$X_{\bar{K}N} = 1.2 + i0.1, \quad \tilde{X}_{\bar{K}N} = 1.0, \quad U = 0.5$$

- comparison with other $(E_{QB}, a_0)$ determinations in PDG

| Ref. | $E_{QB}$ (MeV) | $a_0$ (fm) | $X_{\bar{K}N}$ | $\tilde{X}_{\bar{K}N}$ | $U$ | $|r_e/a_0|$ |
|------|----------------|------------|----------------|-----------------|-----|----------|
| [43] | $-10 - i26$    | 1.39 $- i0.85$ | 1.2 $+ i0.1$  | 1.0             | 0.5 | 0.2      |
| [44] | $-4 - i18$     | 1.81 $- i0.92$ | 0.6 $+ i0.1$  | 0.6             | 0.0 | 0.7      |
| [45] | $-13 - i20$    | 1.30 $- i0.85$ | 0.9 $- i0.2$  | 0.9             | 0.1 | 0.2      |
| [46] | $2 - i10$      | 1.21 $- i1.47$ | 0.6 $+ i0.0$  | 0.6             | 0.0 | 0.7      |
| [46] | $-3 - i12$     | 1.52 $- i1.85$ | 1.0 $+ i0.5$  | 0.8             | 0.6 | 0.4      |

$\Lambda(1405)$ is $\bar{K}N$ composite dominance $\leftarrow$ observables

systematic error
Compositeness of near-threshold bound state can be determined only by observables.

S. Weinberg, Phys. Rev. 137, B672 (1965)

Weak binding relation can be generalized to unstable states with effective field theory.

\[
\alpha_0 = R \left\{ \frac{2X}{1 + X} + \mathcal{O} \left( \left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O} \left( \left| \frac{l}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu}}
\]

Recent determination of \( R \) and \( \alpha_0 \) shows that high-mass pole of \( \Lambda(1405) \) is dominated by \( \bar{K}N \) composite component.

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016);
Analytic structure of scattering amplitude

Pole of scattering amplitude \( f(E_{\text{pole}}) = \infty \)

- represents (complex) eigenvalue of Hamiltonian
- unique through analytic continuation

CDD (Castillejo-Dalitz-Dyson) zero

- pole of inverse amplitude, zero of amplitude \( f(E_{\text{CDD}}) = 0 \)
- unique through analytic continuation
- role of CDD zero in hadron scattering, resonances, etc.


CDD zero \( \leftarrow \) elementary/composite?
Fate of pole in zero coupling limit

Zero coupling limit (ZCL): switching off the channel coupling


Implication from nearby CDD zero

Pole behavior in the ZCL

- Composite (P origin): pole remains in P amplitude
- “Elementary” (Q origin): pole decouples from P amplitude

Is this reflected in the CDD zero?
General discussion

**Scattering amplitude** $f(E)$ is meromorphic in energy


$$\frac{1}{2\pi} \oint_C dz \frac{d \arg f(z)}{dz} = n_Z - n_P \equiv n_C$$

- $n_Z$ ($n_P$) : number of zeros (poles) in contour $C$
- Topological invariant of $\pi_1(U(1)) \cong \mathbb{Z}$

Pole cannot decouple without merging with CDD zero

$\Rightarrow$ existence of nearby CDD zero indicates “elementary” (i.e. origin is not in this channel).
Example: $\Lambda(1405)$

Poles and zeros in the $\bar{K}N$ and $\pi\Sigma$ amplitudes

- In $\pi\Sigma$ amplitude, CDD zero exists near the high-mass pole, and merges with it to decouple in the ZCL.

- In $\bar{K}N$ amplitude, CDD zero exists near the low-mass pole, and merges with it to decouple in the ZCL.

Low- (high-)mass pole is not $\bar{K}N$ ($\pi\Sigma$) composite.
“Elementary” pole decouples from the amplitude in the zero coupling limit.

For a pole to decouple from the amplitude, there must be a nearby CDD zero.

\[
\frac{1}{2\pi} \oint_C dz \frac{d \arg f(z)}{dz} = n_Z - n_P
\]

The dynamical (composite) component of the eigenstate is small if a CDD zero exists near the eigenstate pole.