Status of $\Lambda(1405)$ in chiral dynamics

Tetsuo Hyodo

Yukawa Institute for Theoretical Physics, Kyoto Univ.

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Λ(1405) in chiral SU(3) dynamics
- Precise experimental constraint
- Determination of pole positions
  Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)

Kaonic nuclei
- Local $\bar{K}N$ potential and $\Lambda(1405)$ wave function
  K. Miyahara, T. Hyodo, PRC93, 015201 (2016)
- Density of kaonic nuclei
- $\bar{K}N$ v.s. $NN$ correlations
Two aspects of $K(\bar{K})$ meson

- **NG boson** of chiral $SU(3)_R \otimes SU(3)_L \rightarrow SU(3)_V$
- **Massive** by strange quark: $m_K \sim 496$ MeV

$\rightarrow$ **Spontaneous/explicit** symmetry breaking

$\bar{K}N$ interaction ...


- is coupled with $\pi\Sigma$ channel
- generates $\Lambda(1405)$ below threshold

- is fundamental building block for $\bar{K}$-nuclei, $\bar{K}$ in medium, ...
**SIDDHARTA measurement**

Precise measurement of the kaonic hydrogen X-rays

M. Bazzi, et al., PLB 704, 113 (2011); NPA 881, 88 (2012)

- **EM int.**
  - **p**

- **K-**
  - **strong int.**

- **exp.**
  - **Γ**
  - **ΔE**

- **binding energy**
  - **EM value**

- Shift and width of atomic state $\leftrightarrow$ K-p scattering length


Quantitative constraint on the $\bar{K}N$ interaction at fixed energy

Fig. 7. Comparison of the present result for the strong-interaction $1s$-energy-level shift and width of kaonic hydrogen with the two experimental results: KEK-PS E228 (1997) [14] and DEAR (2005) [15]. The error bars correspond to quadratically added statistical and systematic errors. The right panel shows the error in the energy shift as a function of the width (vertical axis) for each experiment. The dashed lines represent the SIDDHARTA precision calculated assuming the same statistics but with differing width.

As a result, the $1s$-level shift $\epsilon_{1s}$ and width $\Gamma_{1s}$ of kaonic hydrogen were determined by SIDDHARTA to be

$$\epsilon_{1s} = -283 \pm 36 \text{ (stat)} \pm 6 \text{ (syst)} \text{ eV}$$

$$\Gamma_{1s} = 541 \pm 89 \text{ (stat)} \pm 22 \text{ (syst)} \text{ eV}$$

The quoted systematic error is a quadratic summation of the following contributions: the SDD gain shift, the SDD response function, the ADC linearity, the low-energy tail of the kaonic-hydrogen higher transitions, the energy resolution, and the procedural dependence shown by an independent analysis [31].

**4. Conclusion**

We have determined the strong-interaction energy-level shift and width of the kaonic-hydrogen atom $1s$ state with the best accuracy up to now [31]. The obtained shift and width are plotted in Fig. 7 along with the other two recent results [14,15]. It should be noted that the smaller the width, the better the accuracy of determining the energy. The right panel of Fig. 7 shows the errors on the energy shift as a function of the width (vertical axis) for each experiment, together with guide lines representing SIDDHARTA precision calculated assuming the same statistics but with differing width. In comparison with the DEAR result, the accuracy of determining the energy in SIDDHARTA is obviously improved.
Strategy for $\bar{K}N$ interaction

Above the $\bar{K}N$ threshold: direct constraints
- $K-\rho$ total cross sections (old data)
- $\bar{K}N$ threshold branching ratios (old data)
- $K-\rho$ scattering length (new data: SIDDHARTA)

Below the $\bar{K}N$ threshold: indirect constraints
- $\pi\Sigma$ mass spectra (new data: LEPS, CLAS, HADES, …)
Construction of the realistic amplitude

Chiral coupled-channel approach with systematic $\chi^2$ fitting

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)

Chiral perturbation theory

1) TW term
2) Born terms
3) NLO terms

$O(p)$

6 cutoffs

TW model

$O(p)$

7 LECs

NLO model
Branching ratios

<table>
<thead>
<tr>
<th></th>
<th>TW</th>
<th>TWB</th>
<th>NLO</th>
<th>Experiment</th>
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</thead>
<tbody>
<tr>
<td>$\Delta E$ [eV]</td>
<td>373</td>
<td>377</td>
<td>306</td>
<td>$283 \pm 36 \pm 6$ [10]</td>
</tr>
<tr>
<td>$\Gamma$ [eV]</td>
<td>495</td>
<td>514</td>
<td>591</td>
<td>$541 \pm 89 \pm 22$ [10]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.36</td>
<td>2.36</td>
<td>2.37</td>
<td>$2.36 \pm 0.04$ [11]</td>
</tr>
<tr>
<td>$R_n$</td>
<td>0.20</td>
<td>0.19</td>
<td>0.19</td>
<td>$0.189 \pm 0.015$ [11]</td>
</tr>
<tr>
<td>$R_c$</td>
<td>0.66</td>
<td>0.66</td>
<td>0.66</td>
<td>$0.664 \pm 0.011$ [11]</td>
</tr>
<tr>
<td>$\chi^2$/d.o.f</td>
<td>1.12</td>
<td>1.15</td>
<td>0.96</td>
<td></td>
</tr>
</tbody>
</table>

Accurate description of all existing data ($\chi^2$/d.o.f. $\sim 1$)
Comparison with SIDDHARTA

<table>
<thead>
<tr>
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<th>TW</th>
<th>TWB</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2/d.o.f.$</td>
<td>1.12</td>
<td>1.15</td>
<td>0.957</td>
</tr>
</tbody>
</table>

TW and TWB are reasonable, while best-fit requires NLO.
Subthreshold extrapolation

Uncertainty of $\bar{K}N \rightarrow \bar{K}N (I=0)$ amplitude below threshold


- c.f. without SIDDHARTA


SIDDHARTA is essential for subthreshold extrapolation.
Extrapolation to complex energy: two poles

Two poles: superposition of two states


- Higher energy pole at 1420 MeV, not at 1405 MeV
- Attractions of TW in 1 and 8 ($\bar{K}N$ and $\pi\Sigma$) channels

NLO analysis confirms the two-pole structure.
PDG changes

PDG particle listing of \( \Lambda(1405) \)


- Our analysis (+ 2 other groups) included
- Pole positions are now tabulated, prior to mass/width.
Construction of $\bar{K}N$ potential

Local $\bar{K}N$ potential is useful for

- extraction of the wave function of $\Lambda(1405)$
- application to few-body Kaonic nuclei/atoms

Strategy

Fit to experimental data (chiral SU(3) EFT)

equivalent amplitude

Single-channel complex $\bar{K}N$ potential [1] (used in $\bar{K}$-nuclei calculation)

Coupled-channel real $\bar{K}N-\pi\Sigma-\pi\Lambda$ potential [2]

[1] K. Miyahara, T. Hyodo, PRC 93, 015201 (2016);
Realistic $K\bar{N}$ potentials

**Structure of $\Lambda(1405)$**

$K\bar{N}$ wave function at $\Lambda(1405)$ pole

K. Miyahara. T. Hyodo, PRC93, 015201 (2016)

- substantial distribution at $r > 1$ fm
- root mean squared radius $\sqrt{\langle r^2 \rangle} = 1.44$ fm

The size of $\Lambda(1405)$ is much larger than ordinary hadrons.
**Recent experiment for $\bar{K}NN$ (J-PARC E15, $^3$He(K-,Λp)n)**


$B = 47 \pm 3^{+3}_{-6} \text{ MeV}, \quad \Gamma = 115 \pm 7^{+10}_{-9} \text{ MeV}$

**Theoretical calculation with realistic $\bar{K}N$ interaction**

- Fit to $K-p$ cross sections and branching ratios
- SIDDHARTRA constraint of Kaonic hydrogen


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<tbody>
<tr>
<td>$B$ [MeV]</td>
<td>53.3</td>
<td>47.4</td>
<td>32.2</td>
<td>25-28</td>
</tr>
<tr>
<td>$\Gamma_{\pi^YN}$ [MeV]</td>
<td>64.8</td>
<td>49.8</td>
<td>48.6</td>
<td>31-59</td>
</tr>
</tbody>
</table>

- $2N$ absorption ($\Gamma_{YN}$) is NOT included.
Kaonic nuclei

Rigorous few-body approach to $\bar{K}$ nuclear systems


- Stochastic variational method with correlated gaussians

$\hat{V} = \hat{V}^{KN} (\text{Kyoto } \bar{K}N) + \hat{V}^{NN} (\text{AV4'})$  \hspace{1cm} (single channel)

Results for $A = 2, 3, 4, 6$

<table>
<thead>
<tr>
<th>B [MeV]</th>
<th>$\bar{K}NN$</th>
<th>$\bar{K}NNN$</th>
<th>$\bar{K}NNNN$</th>
<th>$\bar{K}NNNNNN$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-28</td>
<td>45-50</td>
<td>68-76</td>
<td>70-81</td>
<td></td>
</tr>
<tr>
<td>31-59</td>
<td>26-70</td>
<td>28-74</td>
<td>24-76</td>
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- quasi-bound state below the lowest threshold
- decay width (without multi-$\bar{N}$ absorption) $\sim$ binding energy
High density?

Nucleon density distribution in four-nucleon system

Applications

- central density increases (not substantially $\rightarrow$ NN core)
- $B = 68$-$76$ MeV (Kyoto $\bar{K}N$)
- $B = 85$-$87$ MeV (AY)

Central density is not always proportional to $B \leftarrow$ tail of w.f.
Interplay between $NN$ and $\bar{K}N$ correlations

Two-nucleon system

$^1S_0$ ($I_{NN}=1$)

unbound

$^3S_1$ ($I_{NN}=0$)

bound (d)

$\bar{K}N(I=0):\bar{K}N(I=1) = 3:1$

$\Lambda(1405)$

$\bar{K}N(I=0):\bar{K}N(I=1) = 1:3$

$NN$ correlation $< \bar{K}N$ correlation (also in $A=6$)

Applications
Applications

Interplay between $\text{NN}$ and $\overline{\text{KN}}$ correlations 2

Four-nucleon system with $J^\pi=0^-, I=1/2, I_3=+1/2$

\[
|\overline{\text{KNNNNN}}\rangle = C_1 \left( \begin{array}{c} p \\ p \\ p \\ n \\ n \\ n \end{array} \right) + C_2 \left( \begin{array}{c} p \\ p \\ \overline{K} \\ n \\ n \\ n \end{array} \right)
\]

- $\overline{\text{KN}}$ correlation
  - $I=0$ pair in $K-p$ (3 pairs) or $\overline{K}n$ (2 pairs) : $C_1 > C_2$

- $\text{NN}$ correlation
  - $\text{ppnn}$ forms $\alpha$ : $C_1 < C_2$

- Numerical result
  - $|C_1|^2 = 0.08, \ |C_2|^2 = 0.92$

$\text{NN}$ correlation $> \overline{\text{KN}}$ correlation
SIDDHARTA measurement of kaonic hydrogen reduces the ambiguity of $\bar{K}N$ amplitude.

Pole positions of $\Lambda(1405)$ are determined by fitting all existing data with $\chi^2$/d.o.f. $\sim 1$.

$$z_1 = (1424^{+7}_{-23} - i26^{+3}_{-14}) \text{ MeV}, \quad z_2 = (1381^{+18}_{-6} - i81^{+19}_{-8}) \text{ MeV}$$

Summary: $\Lambda(1405)$

Realistic $\bar{K}N$ potential is constructed.

Structure of few-body kaonic nuclei reflects the interplay between $NN$ and $\bar{K}N$ correlations.

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)

K. Miyahara, T. Hyodo, PRC93, 015201 (2016)