Quantum chaos and revival dynamics in coupled Sachdev-Ye-Kitaev models

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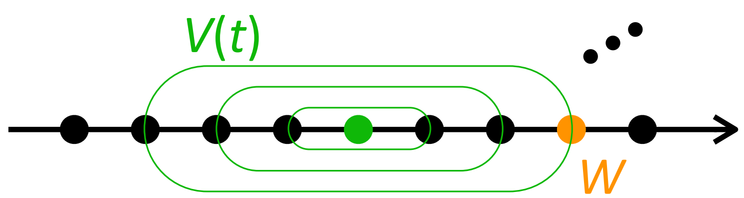
Quantum chaos = operator growth

$$\mathcal{O} \to e^{-iHt}\mathcal{O}e^{iHt} = \mathcal{O} - i[H,\mathcal{O}] - \frac{1}{2}[H,[H,\mathcal{O}]] + \cdots$$

example: $H = \sum_{a=1}^{L}(S_{z,a}S_{z,a+1} + S_{x,a} + S_{z,a})$

$$\mathcal{O} \to [H,\mathcal{O}] \to [H,[H,\mathcal{O}]] \longrightarrow [H,[H,[H,\mathcal{O}]]] \longrightarrow \cdots$$

$S_{z,a}$; $S_{y,a}$; $S_{x,a}S_{z,a+1}, S_{z,a-1}S_{x,a}, S_{z,a}, S_{x,a}$; $S_{z,a-1}S_{y,a}S_{z,a+1}, S_{y,a}S_{z,a+1}, S_{z,a-1}S_{y,a}, S_{y,a}$; $\cdots$



O(*t*) expands rapidly over operator basis

$$\langle -[V(t),W(0)]^2\rangle \sim e^{\lambda_L t}$$

⇒ use this as definition of quantum chaos

This is reasonable because

Quantum analog of classical chaos (=initial value sensitivity)

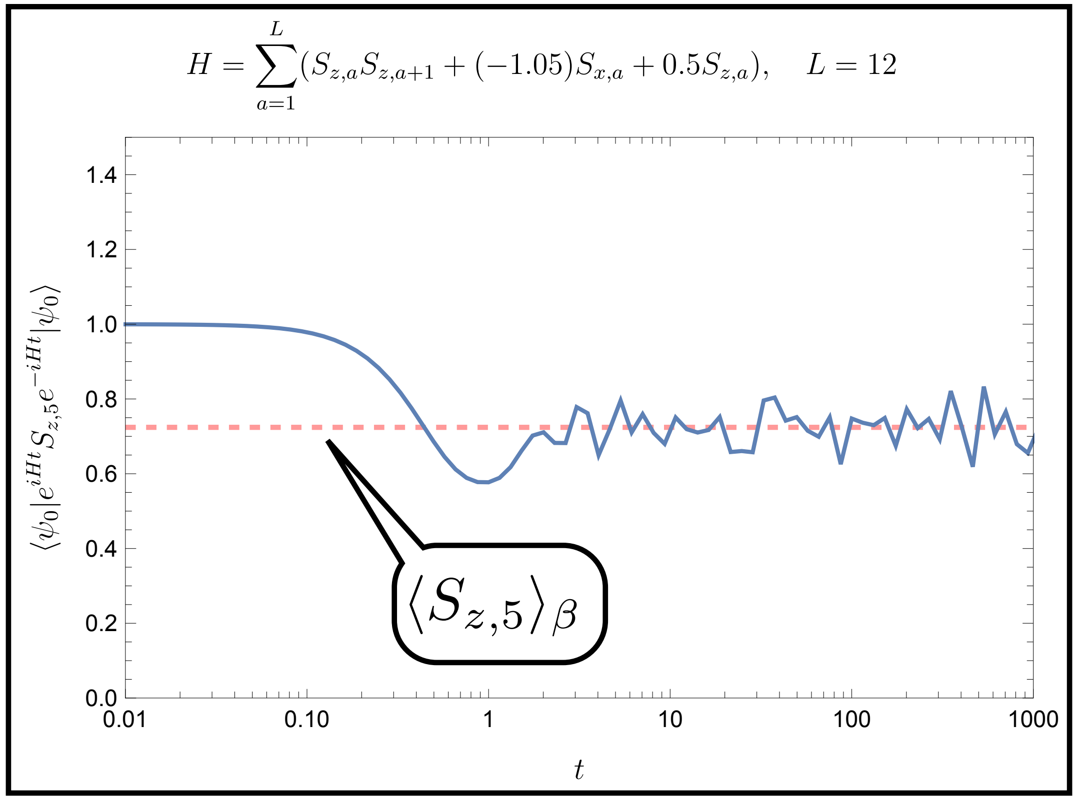
$$\frac{\partial x(t)}{\partial x(0)} = \{x(t),p(0)\}_{\text{P.B.}} = e^{\lambda_L^{(\text{cl})}t}$$

Also related to thermalization

Thermalization in isolated system

$$U|\psi_0\rangle\langle\psi_0|U^\dagger \stackrel{?}{=} Z(\beta)^{-1}\sum_n e^{-\beta E_n}|n\rangle\langle n| \qquad U = e^{-iHt}$$

➡ contradict to unitary time evolution...



Thermalization = initial pure state becomes indistinguishable by small operators from thermal mixed state

⟺ $U|\psi_0\rangle$ is maximally entangled (scrambled) state [Sekino,Susskind,'08]

U can create entanglement only when *U* is not tensor product

➡ operator growth in $UVU^\dagger$

OTOC and bound on chaos

Out-of-time-ordered correlator: $\langle V(t)W(0)V(t)W(0)\rangle$

For a system with large *N* (local degrees of freedom), OTOC behaves as

$$\langle V(t)W(0)V(t)W(0)\rangle = \langle V(t)^2\rangle\langle W(0)^2\rangle + 2\langle V(t)W(0)\rangle^2 + \langle V(t)W(0)V(t)W(0)\rangle^{\rm conn}$$

≈ 1 $\quad$ $\sim e^{-\Gamma t}$ $\quad$ $\sim -\frac{1}{N}e^{\lambda_L t}$ ⇔ operator growth

λ_L quantifies strength of quantum chaos

Bound on chaos: [Maldacena,Shenker,Stanford,'15]

For thermal OTOC, analyticity in *t* requires

$$\langle e^{\frac{3\beta H}{4}}V(t)e^{-\frac{\beta H}{4}}W(0)e^{-\frac{\beta H}{4}}V(t)e^{-\frac{\beta H}{4}}W(0)\rangle_\beta \Rightarrow \lambda_L \le \frac{2\pi}{\beta}$$

Sachdev-Ye-Kitaev model

Q. Is there a model which saturate chaos bound? If yes, how can we see it?

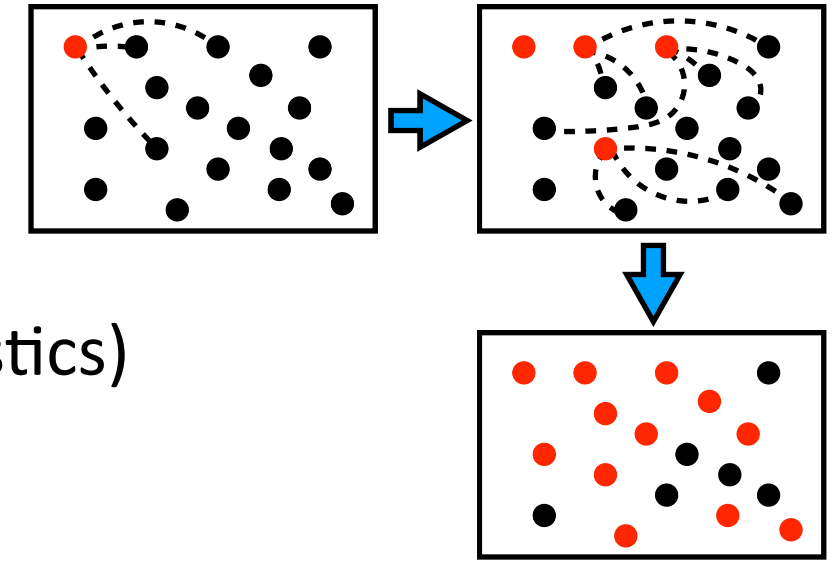
In general, solvable ⟷ simple (i.e. highly symmetric) ➡ not chaotic

A: Sachdev-Ye-Kitaev (SYK) model

$$H_{\mathrm{SYK}} = \sum_{i<j<k<\ell}^{N} J_{ijk\ell}\psi_i\psi_j\psi_k\psi_\ell$$

N fermions in 0d with random non-local few body interactions (*k*-local)

- **solvable** in large *N* limit
- **saturates chaos bound** at low temperature
- Satisfy other quantum chaos criteria (RMT-like level statistics)
- dual to AdS_2 dilaton gravity at low energy



Applications of SYK model

Various phenomena related to quantum chaos can be studied by SYK and its variants

- thermalization by quantum quench [Bhattacharya,Jatkar,Sorokhaibam,'18]
- chaos/integrable transition [Garcia-Garcia,Loureiro,Romero-Bermudez,Tezuka,'17]
- entanglement entropy [Chen,Zhang,'19][Zhang,'20]
- SYK chain → spatial direction [Gu,Qi,Stanford,'16]
- Hayden-Preskill protocol [Nakata,Tezuka,'23]
- non-Hermitian [Garcia-Garcia,Zheng,Ziogas,'20]
- RMT universality classes [Kanazawa,Wettig,'17]
- sparse disorder [Garcia-Garcia,Jia,Rosa,Verbaarschot,'20][Xu,Susskind,Su,Swingle,'20]
- operator growth vs spreading [Rberts,Stanford,Streicher,'18][Carrega,Kim,Rosa,'20]
- Krylov complexity [Parker,Cao,Avdoshkin,Scaffidi,Altman,'18]
- projection measurement [Milekhin,Popov,'22]

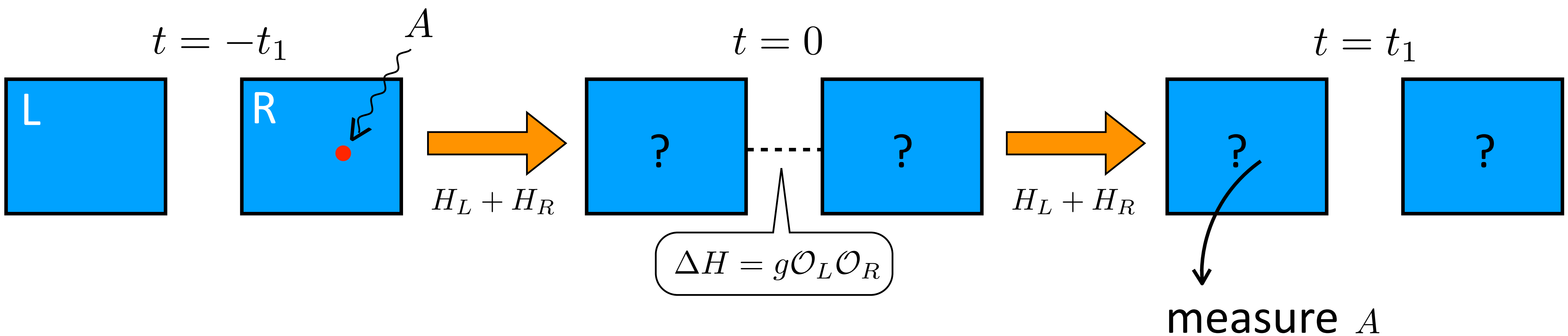
This tallk: **regenesis**

Regenesis

Setup: L/R systems are the same and strongly chaotic

Set initial state to $|\psi(0)\rangle = |\mathrm{TFD}\rangle_\beta = \sum_n e^{\beta(H_L+H_R)/4}|n\rangle_L \otimes |n\rangle_R$

[Maldacena,Stanford,Yang,'17][Gao,Liu,'18]



Question: Do we observe a large value of $\langle A\rangle$ in L-system at late time?

Naive expectation: NO, because...

- only weak & instantaneous LR interaction
- strong chaos may erase input excitation in R-system quickly

True answer: YES, but how?

Regenesis

Regenesis is measured by

$$\langle \mathrm{TFD}|[A_R(-t_1), e^{ig\mathcal{O}_L\mathcal{O}_R\Delta t}A_L(t_1)e^{-ig\mathcal{O}_L\mathcal{O}_R\Delta t}]|\mathrm{TFD}\rangle$$

$$= g\Delta t\langle \mathrm{TFD}|[A_L(t_1), \mathcal{O}_L][A_R(-t_1), \mathcal{O}_R]|\mathrm{TFD}\rangle$$

$$= g\Delta t \mathrm{Tr}_L(e^{-\beta H_L/2}[A_L(t_1), \mathcal{O}_L(0)]^2)$$

$${}_R\langle n|\mathcal{O}_R|n\rangle_R = {}_L\langle n|\mathcal{O}_L^*|n\rangle_L$$

becomes large at $t_1 \gtrsim t_{scr} = \lambda_L^{-1}\log N$

[Maldacena,Stanford,Yang,'17][Gao,Liu,'18]

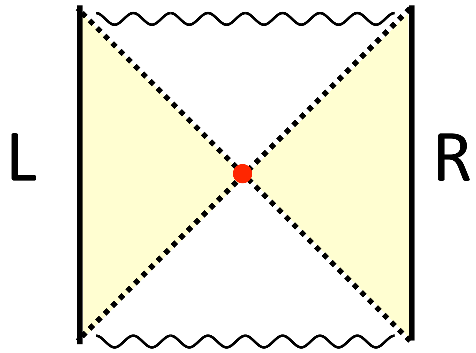
Signal reformulates because it is scrambled!

Note: regenesis takes place for any *A* and *O*, but the entanglement structure of $|\psi(0)\rangle = |\mathrm{TFD}\rangle$ is crucial (c.f. quantum teleportation in LOCC circuit)

Regenesis = traversable wormhole

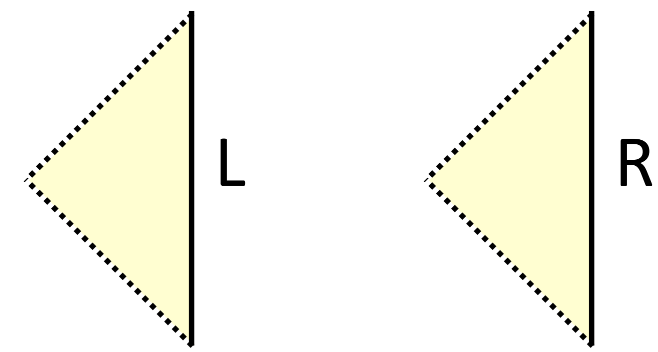
Maximally extended black hole is dual to mixed state $e^{-\beta H}$ purified by the other exterior

[Maldacena,Susskind,'13]



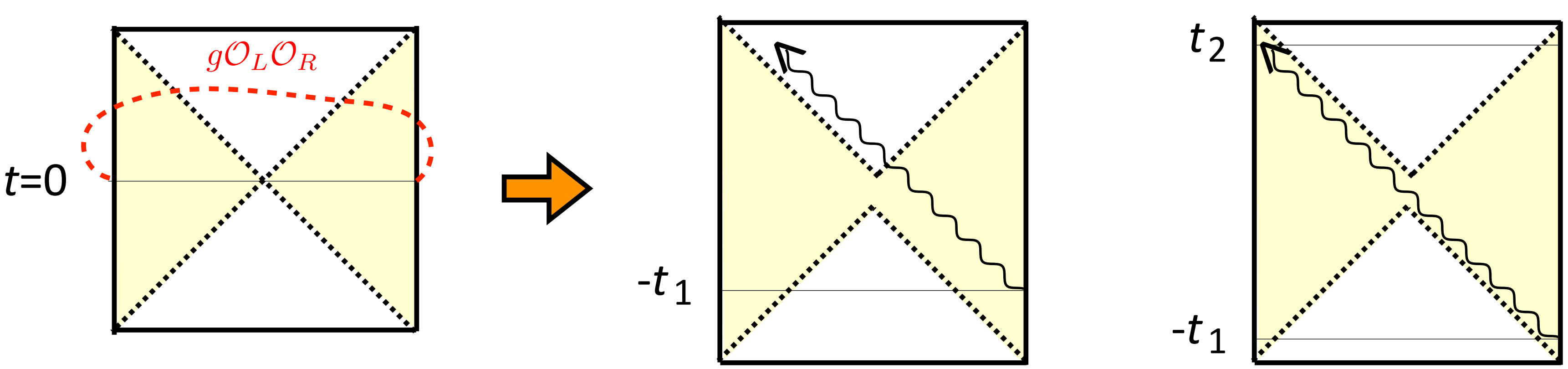
$\Leftrightarrow$ $\rho_{LR} = |\mathrm{TFD}\rangle\langle\mathrm{TFD}|$

(different from



$\Leftrightarrow$ $\rho'_{LR} = e^{-\beta H_L} \otimes e^{-\beta H_R}$)

Left case can be made traversable by non-nocal LR interaction



[Gao,Jafferis,Wall,'16]

Signal on R at $t = -t_1$ can propagate to L if $t_1 \gtrsim t_{scr}$, which reaches L at $t_2 \sim t_1$

This talk: regenesis in field theory side simulated by SYK model.

Short Summary & Contents

1. SYK model in large N limit [reviews]

2. Regenesis (revival) in two coupled SYK [TN,Numasawa,'20] + [reviews]

3. Imperfectly correlated disorder [TN,Numasawa,'19][TN,Numasawa,'22]

4. Summary & Future problems

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1. SYK model in large *N* limit [reviews]

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high *T* phase:	low *T* phase:
no revival	revival oscillation
$\lambda_L \sim 2\pi T$	$\lambda_L \ll 1$
similar to uncoupled SYK	gapped ➡ quasi-particle picture is good

3. Imperfectly correlated disorder [TN,Numasawa,'19][TN,Numasawa,'22]

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- $\lambda_L \ll 1$
- gapped ➡ quasi-particle picture is good

3. Imperfectly correlated disorder [TN,Numasawa,'19][TN,Numasawa,'22]

- less gapped ➡ less revival
- No phase transition when correlation is below a finite value

4. Summary & Future problems

Sachdev-Ye-Kitaev model

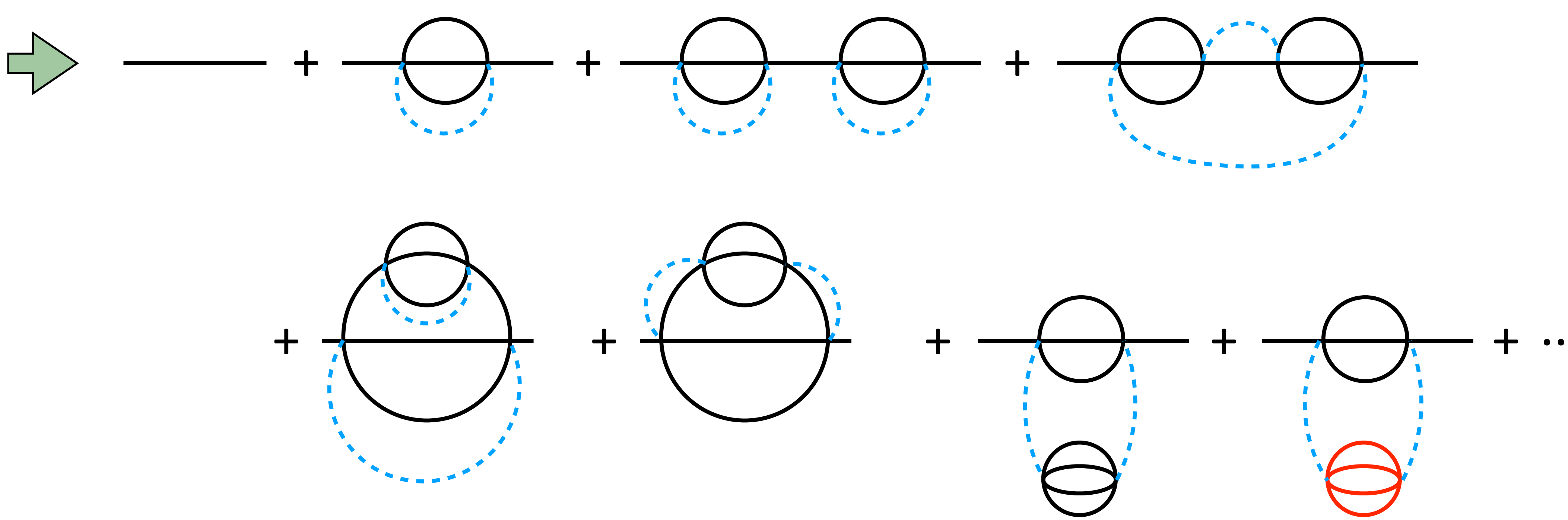
$$H_{\mathrm{SYK}} = \sum_{i<j<k<\ell}^{N} J_{ijk\ell}\psi_i\psi_j\psi_k\psi_\ell \qquad \{\psi_i, \psi_j\} = \delta_{ij}$$

$J_{ijk\ell}$: Gaussian disorder $\quad \langle J_{ijk\ell}^2 \rangle = \mathcal{J}^2 N^{-3} \qquad (\mathcal{J} = 1)$

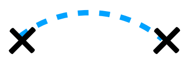
Physical quantities are defined as (quenched) average

$$\langle \mathcal{O} \rangle_{\mathrm{quench}} = \left\langle \frac{\mathrm{Tr}\mathcal{O}e^{-\beta H}}{\mathrm{Tr}e^{-\beta H}} \right\rangle_{J_{ijk\ell}}$$

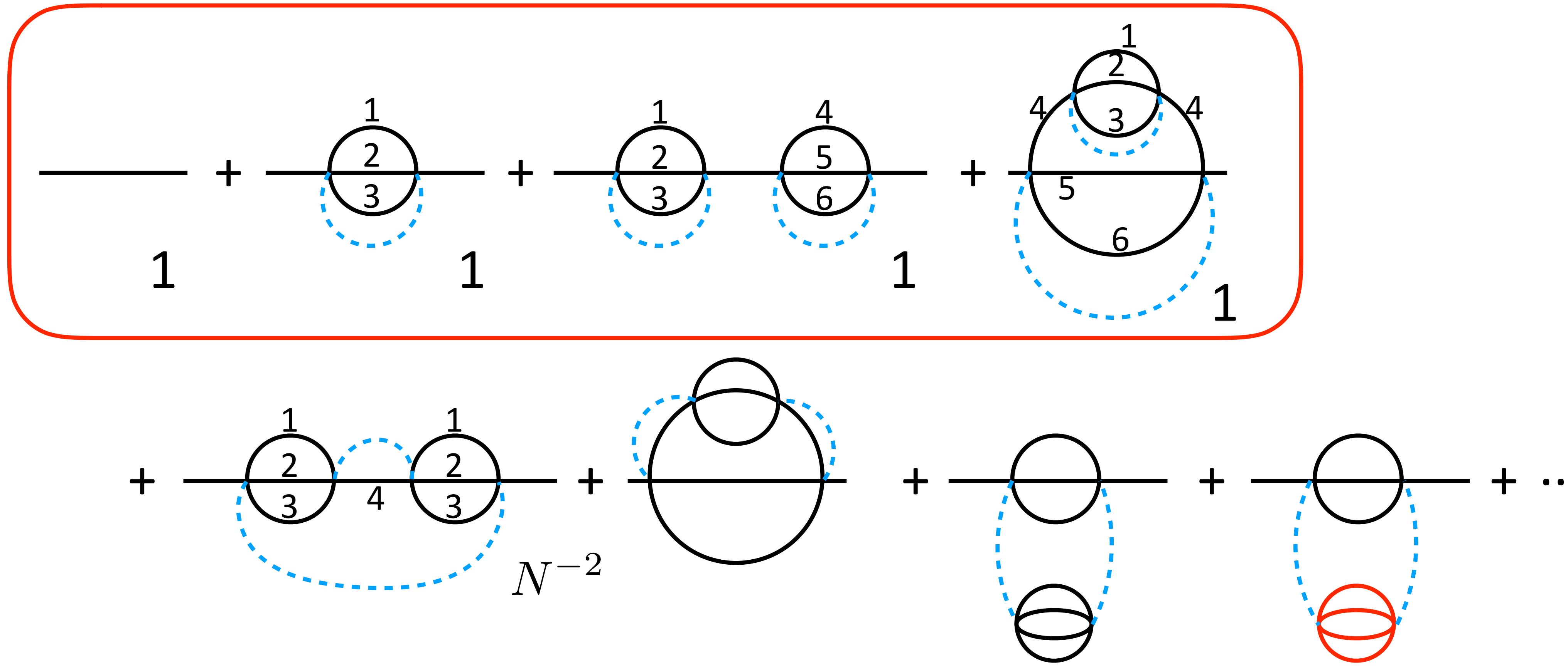
$\mathcal{O} = \psi_i(\tau)\psi_i(0)$ ⇨



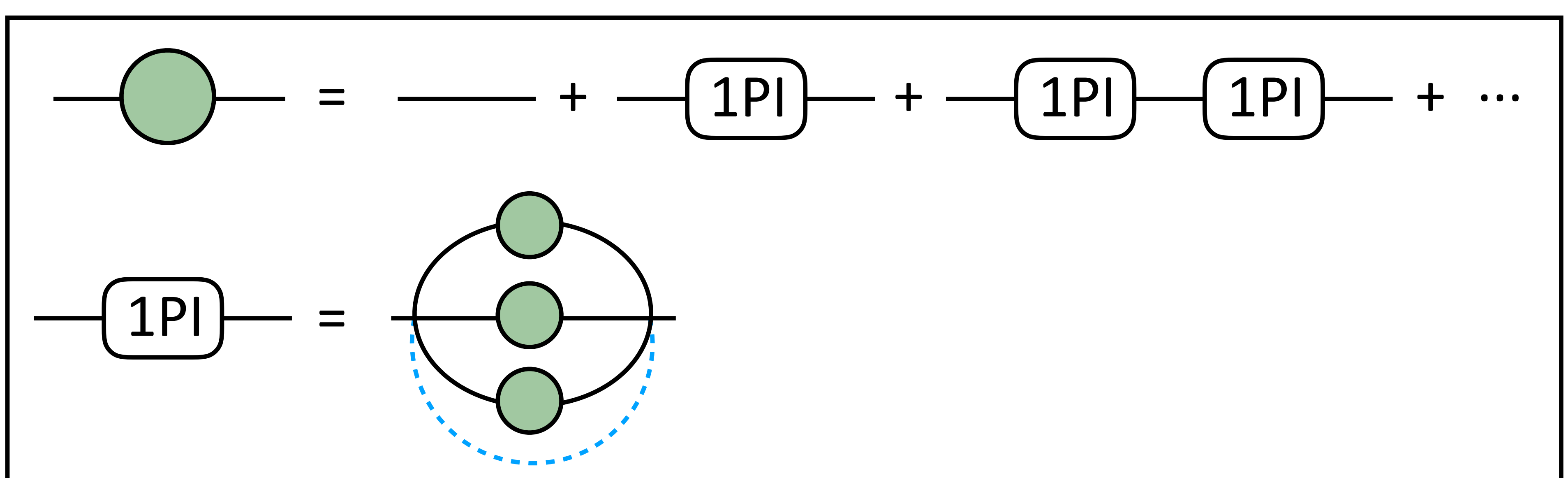
Large *N* simplification

 : N^{-3}

index sum: N



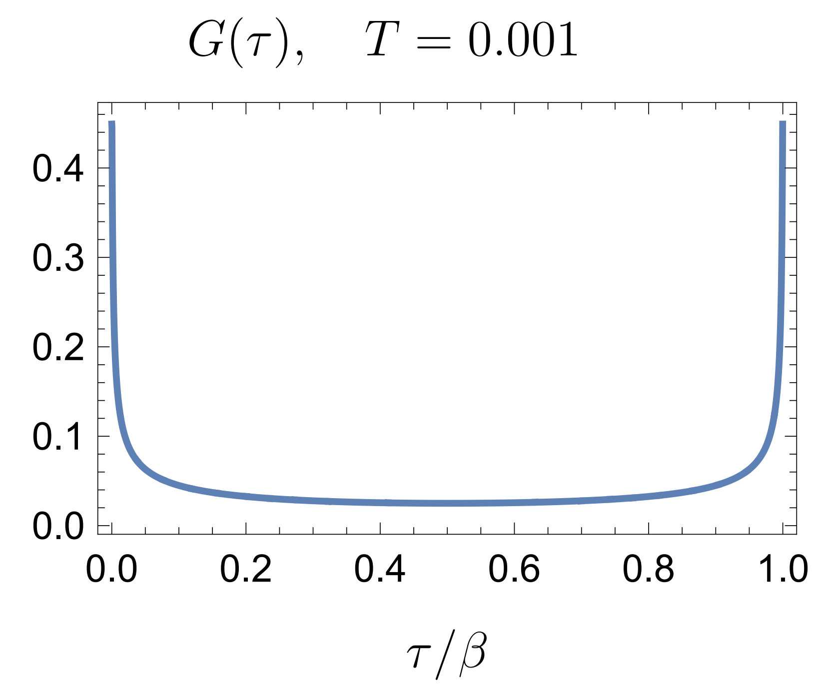
Only melonic diagrams contributes at large *N*



Two-point function

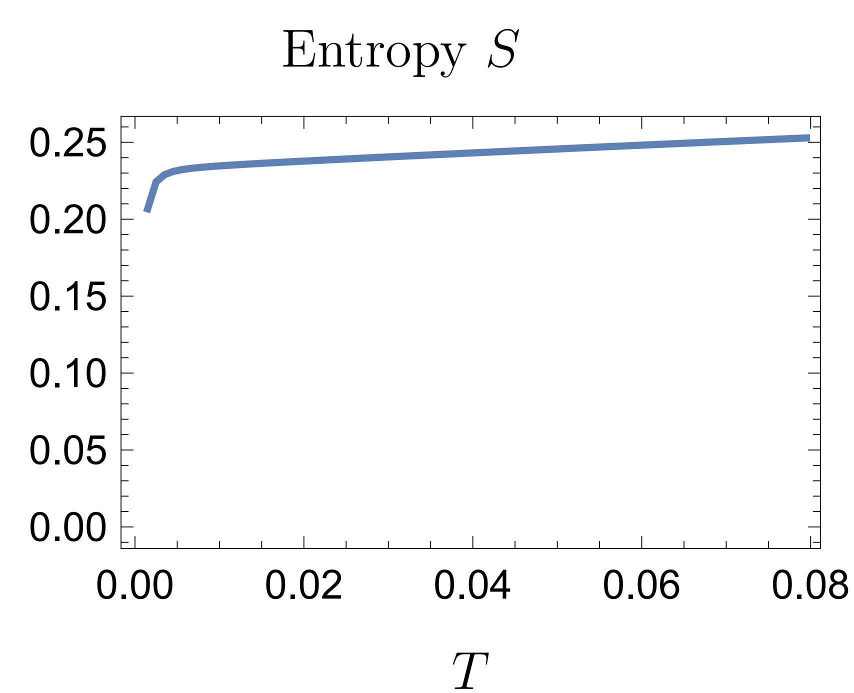
Schwinger-Dyson equation:

$$\partial_\tau G(\tau_1,\tau_2) - \int_0^\beta d\tau' \Sigma(\tau_1,\tau')G(\tau',\tau_2) = \delta(\tau_1-\tau_2)$$
$$\Sigma(\tau_1,\tau_2) = G(\tau_1,\tau_2)^3$$



At low temperature, G(τ) decays in power-law $G(\tau) \sim \tau^{-\frac{1}{2}}$

Not gapped (if gapped, $G(\tau) \sim e^{-E_{\text{gap}}\tau}$)



O(N) entropy even at $T \to 0$

Gravity dual in low temperature limit

SYK at low temperature:

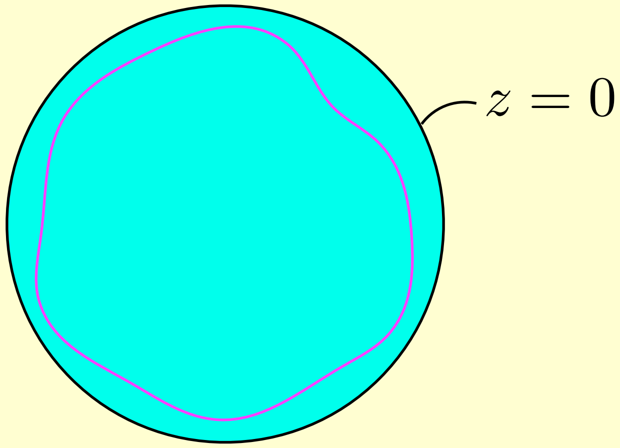
Emergent reparametrization symmetry $G(\tau_1,\tau_2) \to (f'(\tau_1)f'(\tau_2))^{\frac{1}{4}} G(f(\tau_1),f(\tau_2))$

After choosing a solution $G(\tau_1,\tau_2) = |\tau_1-\tau_2|^{-\frac{1}{2}}$, there is still SL(2,R) symmetry

Jackiw-Teitelboim gravity (2d gravity + dilaton): [Teitelboim,'83][Jackiw,'85]

$$S = \frac{1}{16\pi G_N}\left[\int_M \phi(R+2) + \phi_b \int_{\partial M} K\right]$$

$\int \mathcal{D}\phi$ ⇨ $R+2=0$: AdS$_2$ $ds^2 = \frac{dt^2+dz^2}{z^2}$



No bulk graviton ⇨ dynamical d.o.f. = shape of boundary $(t(u),z(u))$ $z\sim 0$

f(τ) in SYK = reparametrization of boundary in JT gravity

∵ Same symmetry and spontaneous breaking pattern

Same effective action $S = N\int d\tau \frac{2f'f''' - 3(f'')^2}{2(f')^2}$

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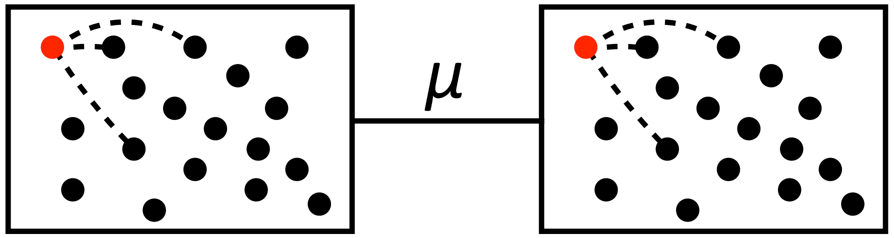
Two-coupled SYK

$$H = H_{\rm SYK}(\psi_i^L) + H_{\rm SYK}(\psi_i^R) + \mu H_{\rm int}$$

$$H_{\rm int} = i\sum_i \psi_i^L \psi_i^R$$



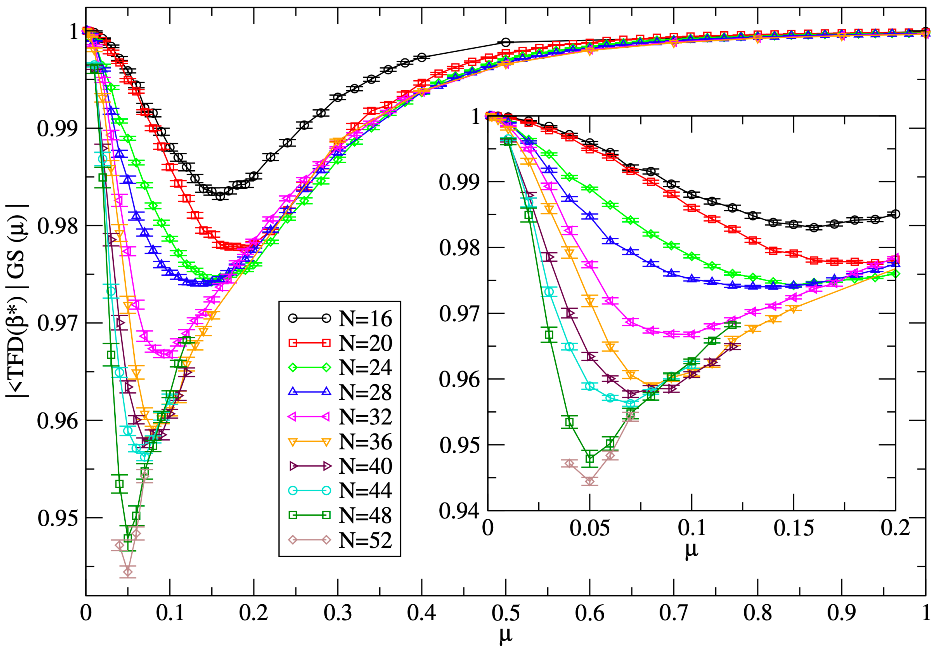
[Maldacena,Qi,'18]

$H_{\rm int}$ is relevant and gapped → *H* is gapped

Ground state of *H* $\approx e^{-\beta_*(\mu) H^L_{\rm SYK}/2}|I\rangle = |{\rm TFD}\rangle_{\beta_*(\mu)}$

$|I\rangle$: ground state of $H_{\rm int}$

$(\psi_i^L - i\psi_i^R)|I\rangle = 0 \longrightarrow |I\rangle = \sum_n |n\rangle \otimes |n\rangle$



(figure from [Alet,Hanada,Peng,'20])

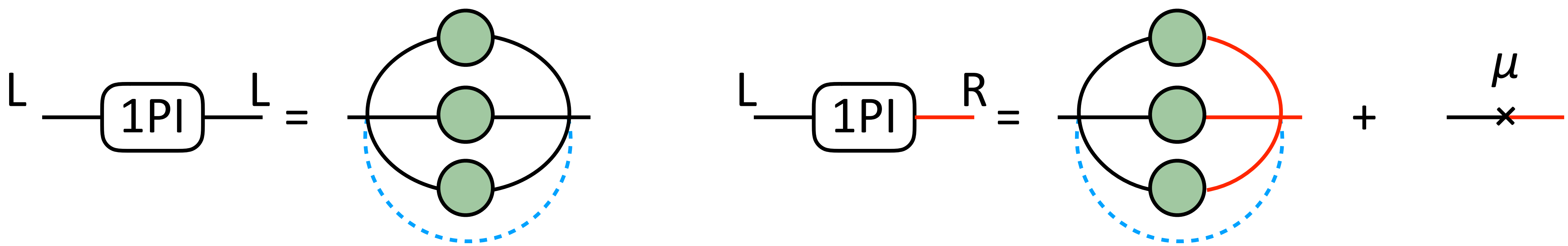
↓

At low temperature,

$\langle[\phi^L(t), \phi^R(0)]\rangle_\beta \approx {}_{\beta^*}\langle{\rm TFD}|[\phi^L(t), \phi^R(0)]|{\rm TFD}\rangle_{\beta^*}$: same as regenesis

Large N phases

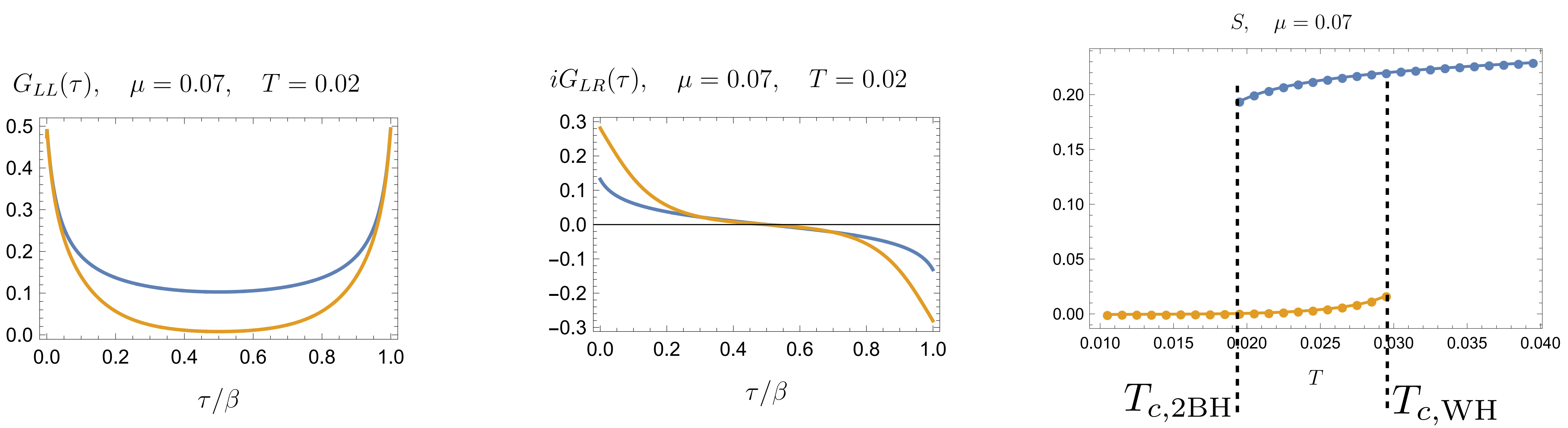
Schwinger-Dyson equations



$$\partial_\tau G_{LL}(\tau) - \int d\tau' \Sigma_{L\cdot}(\tau-\tau')G_{\cdot L}(\tau') = \delta(\tau), \quad \Sigma_{LL}(\tau) = G_{LL}(\tau)^3$$

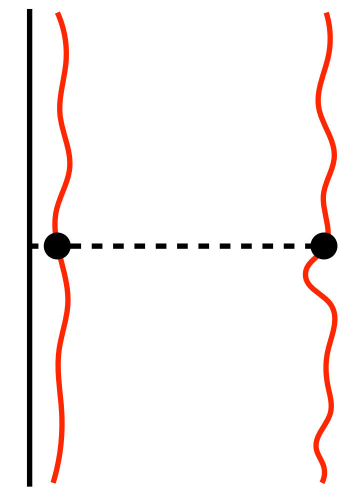
$$\partial_\tau G_{LR}(\tau) - \int d\tau' \Sigma_{L\cdot}(\tau-\tau')G_{\cdot R}(\tau') = 0, \quad \Sigma_{LR}(\tau) = G_{LR}(\tau)^3 + i\mu\delta(\tau)$$

Have two solutions, one exists only for $T < T_{c,\mathrm{WH}}$, and the other only for $T > T_{c,\mathrm{2BH}}$

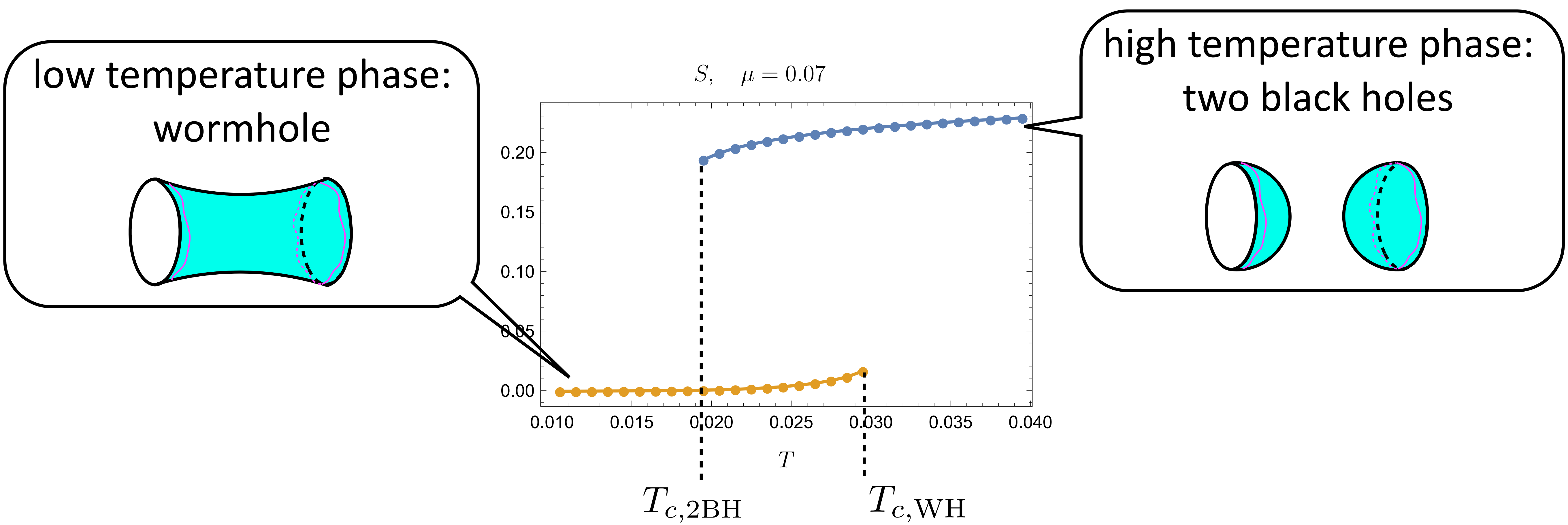


Gravity dual = Hawking-Page transition in AdS_2

$$S = \int_M \phi(R+2) + \phi_b \int_{\partial M} K + \mu \sum_i \int du \mathcal{O}_i(u_L)\mathcal{O}_i(u_R)$$

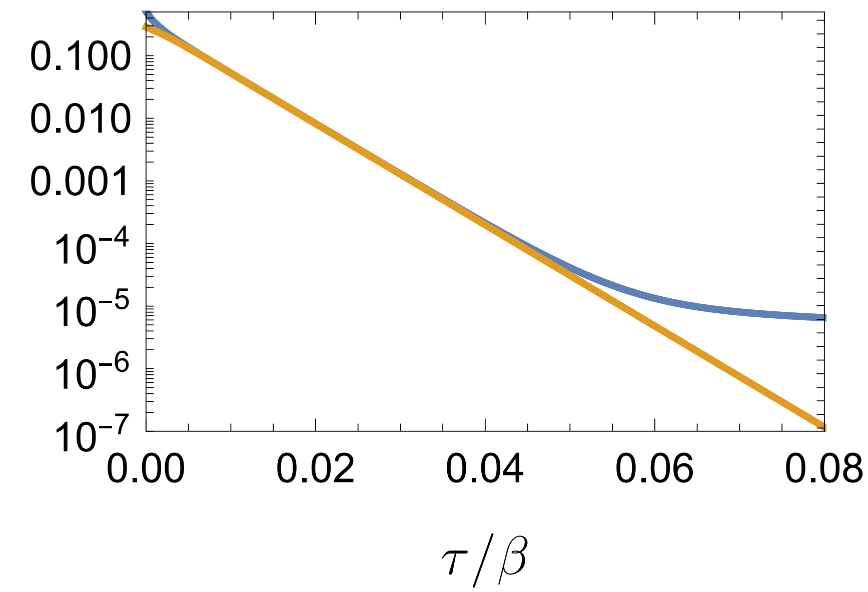


➡ dynamical d.o.f. = shape of **two boundaries**



$T < T_{c,\mathrm{WH}}$: gapped phase

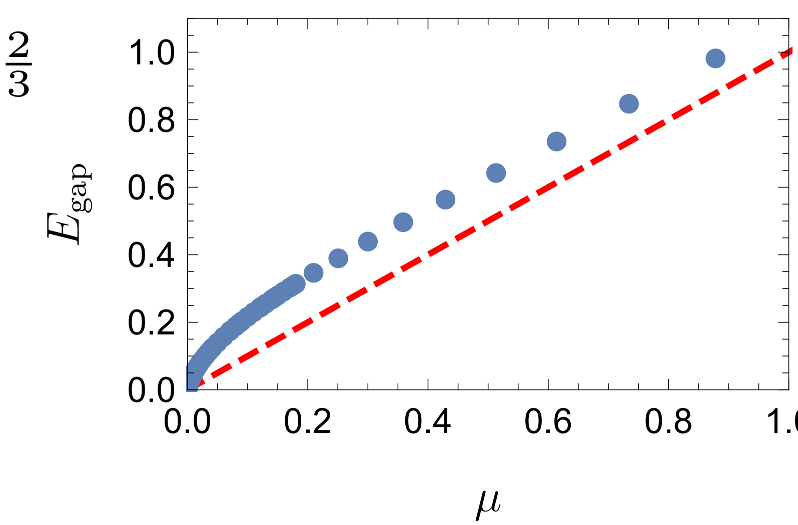
$G_{ab}(\tau) \sim e^{-\nu\tau}$ ➡ gapped



$$G_{ab}(\tau) = \langle \mathrm{Tr} e^{\tau\hat{H}} \hat{\psi}_i^a e^{-\tau\hat{H}} \hat{\psi}_i^b e^{-\beta\hat{H}} \rangle$$
$$= \sum_{m,n} \langle m|\hat{\psi}_i^a|n\rangle\langle n|\hat{\psi}_i^b|m\rangle e^{E_m(\tau-\beta)-E_n\tau}$$

Since $\langle 0|\psi_i^a|0\rangle = 0$, (*m*,*n*)=(0,1) is dominant ➡ $G_{ab}(\tau) \sim e^{-E_{\mathrm{gap}}\tau}$

SYK interactions enhance E_{gap} from naive one $(\sim \mu)$ to $\mu^{\frac{2}{3}}$



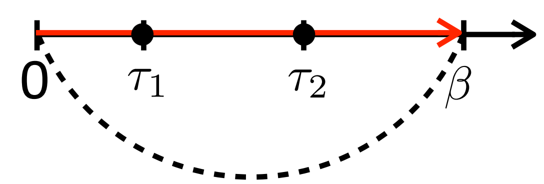
Real time: Schwinger-Keldysh formalism

$$\hat{\psi}(u) = e^{\hat{H}u}\hat{\psi}e^{-\hat{H}u}$$

$$\langle \mathcal{T}\psi_1(u_1)\psi_2(u_2)\rangle_\beta = \begin{cases} \mathrm{Tr}(e^{-\hat{H}(\beta-u_1)}\hat{\psi}_1 e^{-\hat{H}(u_1-u_2)}\hat{\psi}_2 e^{-\hat{H}u_2}) & (\mathrm{Re}[u_1] > \mathrm{Re}[u_2]) \\ -\mathrm{Tr}(e^{-\hat{H}(\beta-u_2)}\hat{\psi}_2 e^{-\hat{H}(u_2-u_1)}\hat{\psi}_1 e^{-\hat{H}u_1}) & (\mathrm{Re}[u_2] > \mathrm{Re}[u_1]) \end{cases}$$

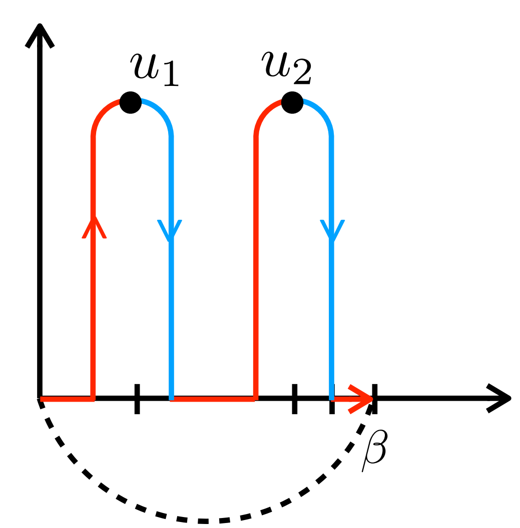
Euclidean $(u=\tau)$

$$\langle \mathcal{T}\psi_1(\tau_1)\psi(\tau_2)\rangle_\beta = \int \mathcal{D}\psi\, \psi_1(\tau_1)\psi_2(\tau_2) e^{-\int_0^\beta d\tau L}$$



Real time $(u = \tau + it)$

$$\langle \mathcal{T}\psi_1(u_1)\psi(u_2)\rangle_\beta = \int \mathcal{D}\psi\, \psi_1(u_1)\psi_2(u_2) e^{-\int_{C_{1+2}} du L}$$



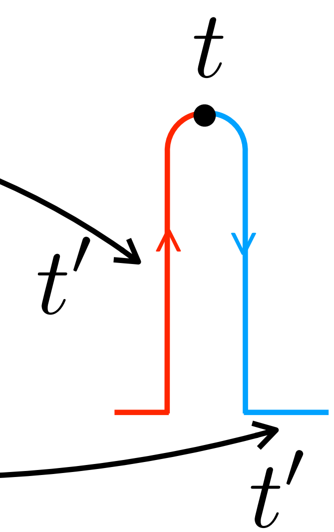
C_{1+2} : Keldysh contour

Independent path integrable d.o.f. on Red/Blue lines.

Real time two-point functions

$$G_{ab}(\tau-\tau') = \langle \mathcal{T}\psi_i^a(\tau)\psi_i^b(\tau')\rangle_\beta$$

$$\Rightarrow \begin{cases} G^{>}_{ab}(t-t') = -i\langle \mathcal{T}\psi_i^a(\epsilon+it)\psi_i^b(-\epsilon+it')\rangle_\beta \\ G^{<}_{ab}(t-t') = -i\langle \mathcal{T}\psi_i^a(-\epsilon+it)\psi_i^b(\epsilon+it')\rangle_\beta \end{cases}$$



Symmetries from operator formalism $\mathrm{Tr}(e^{-\hat{H}(\beta-u_1)}\hat{\psi}_1 e^{-\hat{H}(u_1-u_2)}\hat{\psi}_2 e^{-\hat{H}u_2})$:

$$G_{ab}(u) = -G_{ba}(-u) \qquad G^{>}_{ab}(t) = -G^{<}_{ba}(t)$$

$$(G_{ab}(u))^* = -G_{ab}(-u^*) \qquad \Rightarrow \qquad G^{>}_{ab}(t)^* = G^{<}_{ab}(t)$$

$$G_{ab}(u+\beta) = -G_{ab}(u) \quad \text{(KMS relation)} \qquad G^{>}_{ab}(t-i\beta) = -G^{<}_{ab}(t)$$

Real time EoM

Real-time EoM are obtained by replacing $\int_0^\beta d\tau$ in SD eq with Keldysh contour

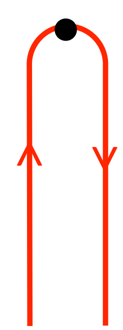
$$\partial_\tau G_{ab}(\tau) - \int d\tau' \Sigma_{ac}(\tau - \tau') G_{cb}(\tau') = \delta_{ab}\delta(\tau)$$

$$\Sigma_{ab}(\tau) = G_{ab}(\tau)^3 + i\mu\epsilon_{ab}\delta(\tau)$$

$$-i\partial_t G^R_{ab}(t) + \int_{-\infty}^{\infty} dt' \Sigma^R_{ac}(t-t') G^R_{cb}(t') + i\mu\epsilon_{ac} G^R_{cb}(t) = -\delta_{ab}\delta(t)$$

$$\Sigma^>_{ab}(t) = -G^>_{ab}(t)^3$$

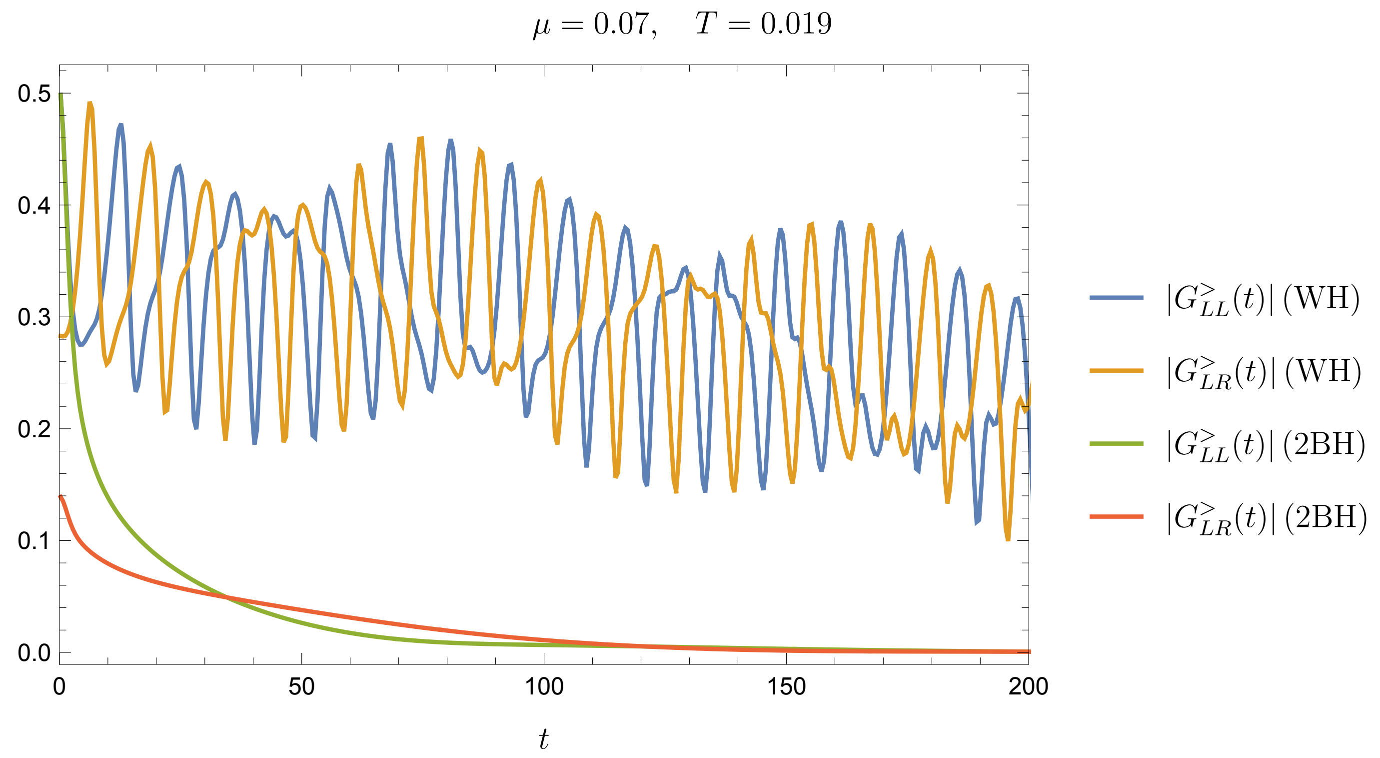
$$G^R_{ab}(t) = \theta(t)(G^>_{ab}(t) - G^<_{ab}(t))$$

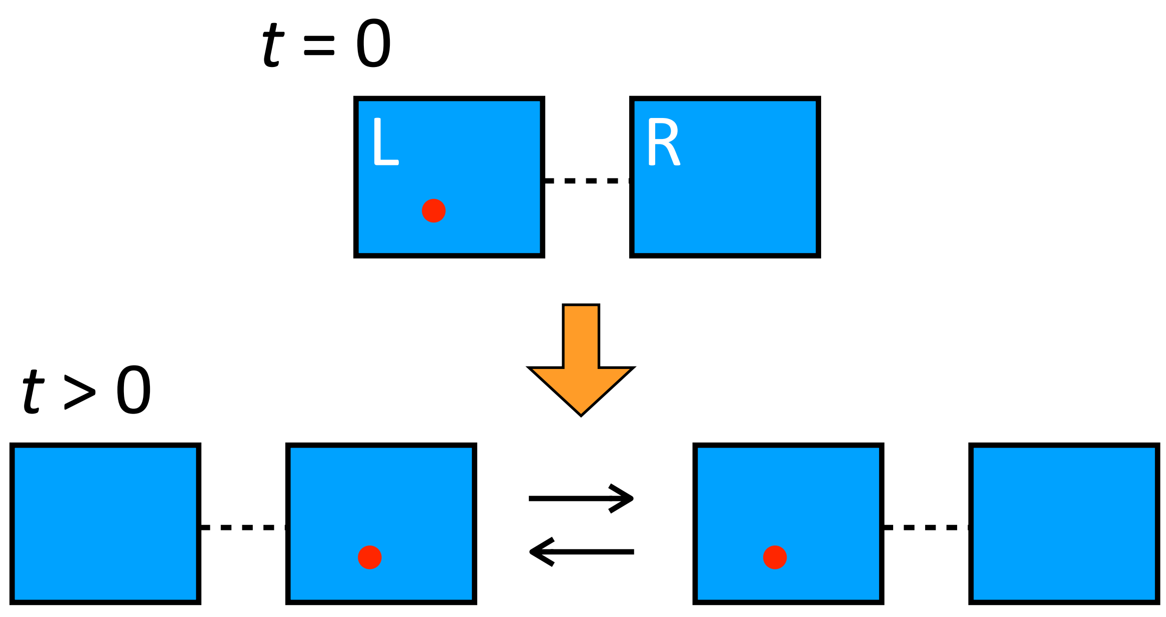


Together with $\widetilde{G}^>_{ab}(\omega) = \dfrac{\widetilde{G}^R_{ab}(\omega) - (\widetilde{G}^R_{ba}(\omega))^*}{1 + e^{-\beta\omega}}$ (KMS) form closed set of equations.

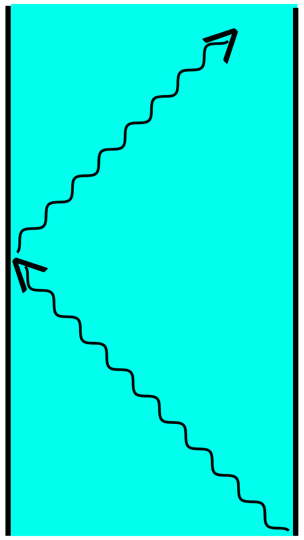
Revival oscillation

In low temperature phase $G^{>}_{LL}(t)$ and $G^{>}_{LR}(t)$ show out-of-phase oscillation





[Plugge,Lantagne-Hurtubise,Franz,'20]



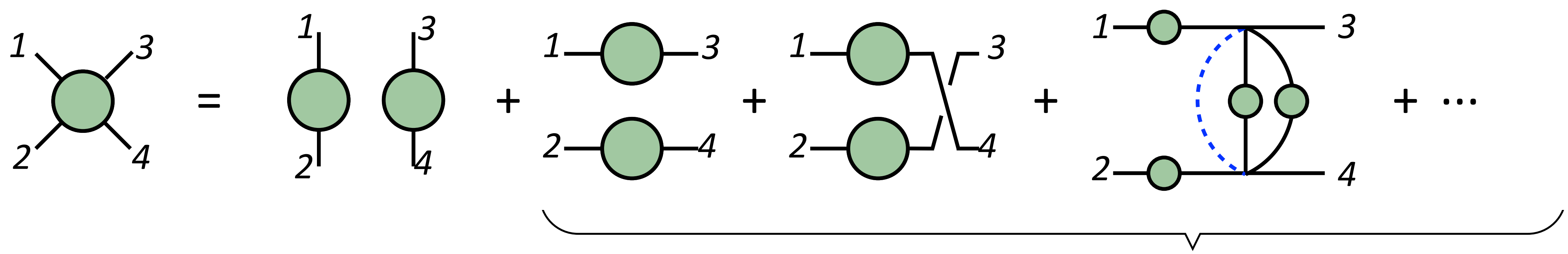
No revival oscillation in high temperature phase.

Out-of-time-ordered 4-pt fcn

(Euclidean) 4-point function:

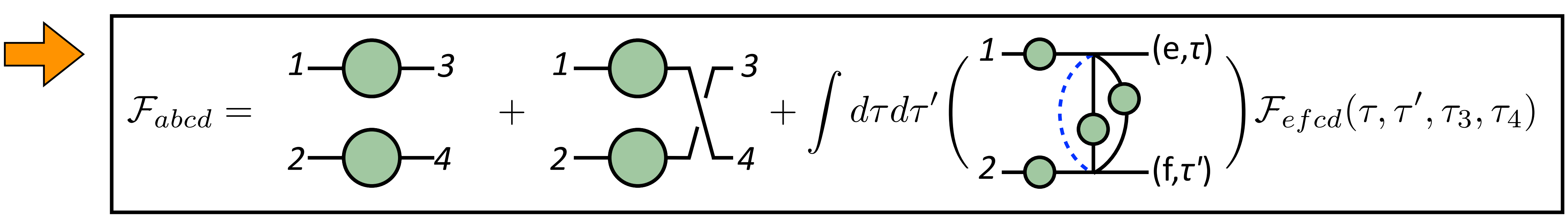
$$\frac{1}{N^2}\sum_{i,j}\langle \mathcal{T}\psi_i^a(\tau_1)\psi_i^b(\tau_2)\psi_j^c(\tau_3)\psi_j^d(\tau_4)\rangle_\beta$$

[Maldacena-Stanford,'16]



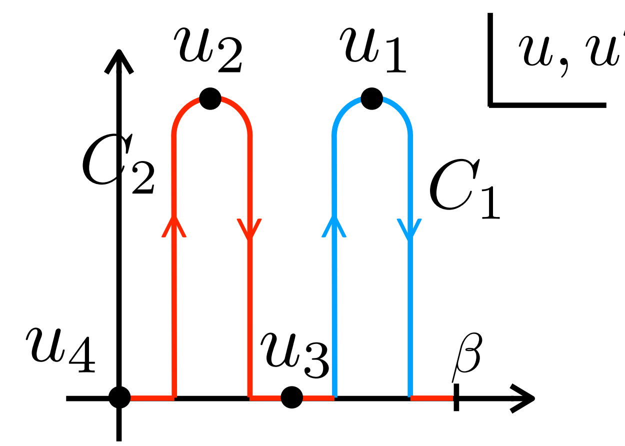
$$= \frac{1}{N}\mathcal{F}_{abcd}(\tau_1,\tau_2,\tau_3,\tau_4)$$

Melonic dominance in large *N*



Real-time continuation

$$(\tau_1,\tau_2,\tau_3,\tau_4)\to(u_1,u_2,u_3,u_4)=\Big(\frac{3\beta}{4}+it_1,\frac{\beta}{4}+it_2,\frac{\beta}{2},0\Big)$$



$$\mathcal{F}_{abcd}=-G_{ac}(\tfrac{\beta}{4}+it_1)G_{bd}(\tfrac{\beta}{4}+it_2)+G_{ad}(\tfrac{3\beta}{4}+it_1)G_{bc}(-\tfrac{\beta}{4}+it_2)$$

$$+\Big[\int_{C_1}du\int_{C_2}du'+\cdots\Big]G_{ae}(\tfrac{3\beta}{4}+it_1-u)G_{bf}(\tfrac{\beta}{4}+it_2-u')G_{ef}(u-u')^2\mathcal{F}_{efcd}(u,u')$$

We are only interested in the growing behavior at $t_{1,2}\gg 1$ ⇨ keep only ▭

$$\mathcal{F}_{abcd}=e^{\frac{\lambda_L(t_{1+2})}{2}}f_{abcd}(t_{1-2})$$

⇨

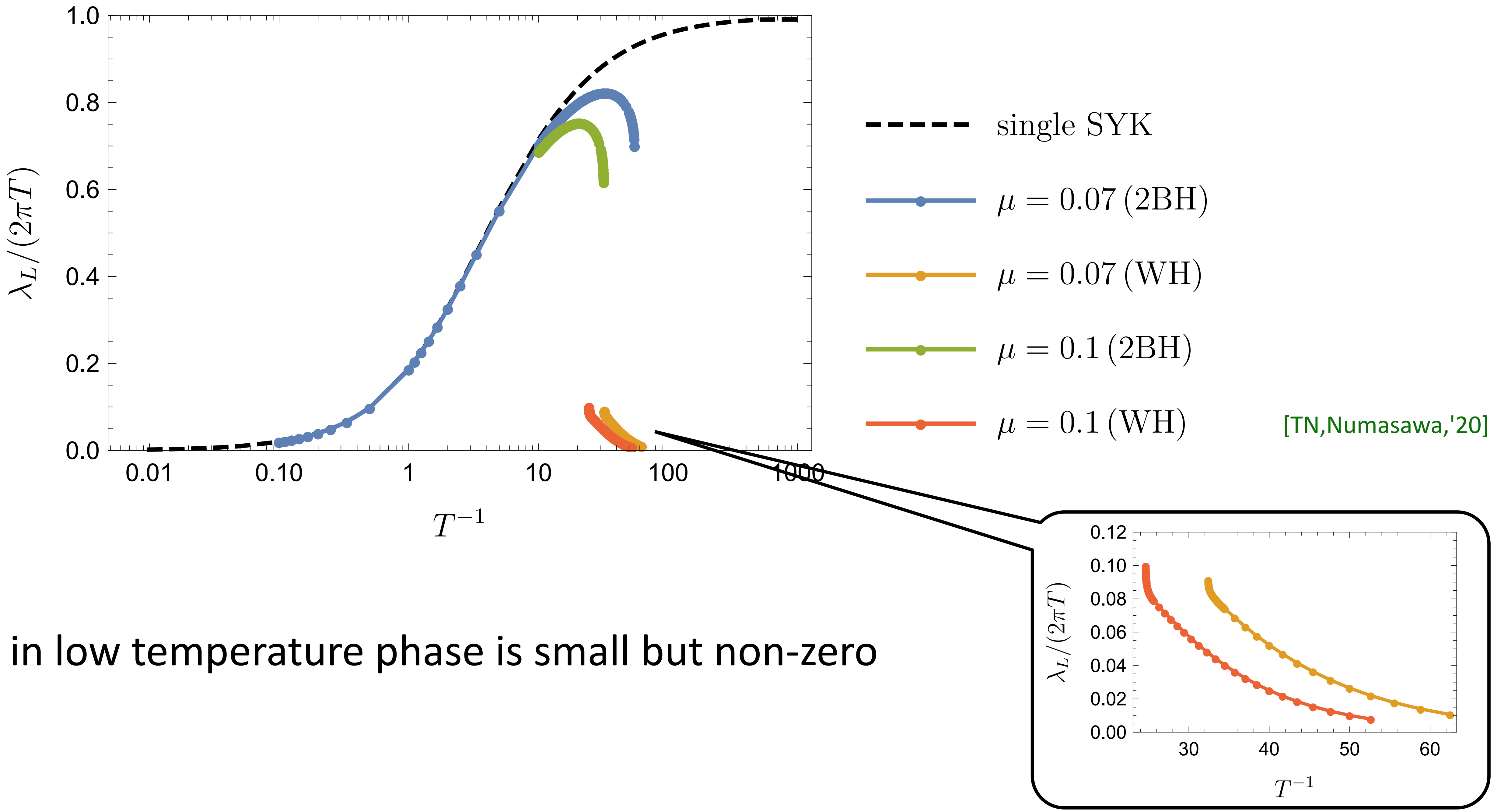
$$f_{abcd}(t_{1-2})=\int dt_{5-6}\mathcal{K}^R_{abef}(\lambda_L;t_{1-2},t_{5-6})f_{efcd}(t_{5-6})$$

$$\mathcal{K}^R_{abef}=\int d\Big(\frac{t_{5+6}}{2}\Big)e^{-\frac{\lambda_L t_{1+2}}{2}}G^R_{ae}(t_{1-5})G^R_{bf}(t_{2-6})G_{ef}\Big(\frac{\beta}{2}+it_{5-6}\Big)^2e^{\frac{\lambda_L t_{5+6}}{2}}$$

Quantum chaos exponent

$$f_{abcd}(t_{1-2}) = \int dt_{5-6} \mathcal{K}^R_{abef}(\lambda_L; t_{1-2}, t_{5-6}) f_{efcd}(t_{5-6})$$

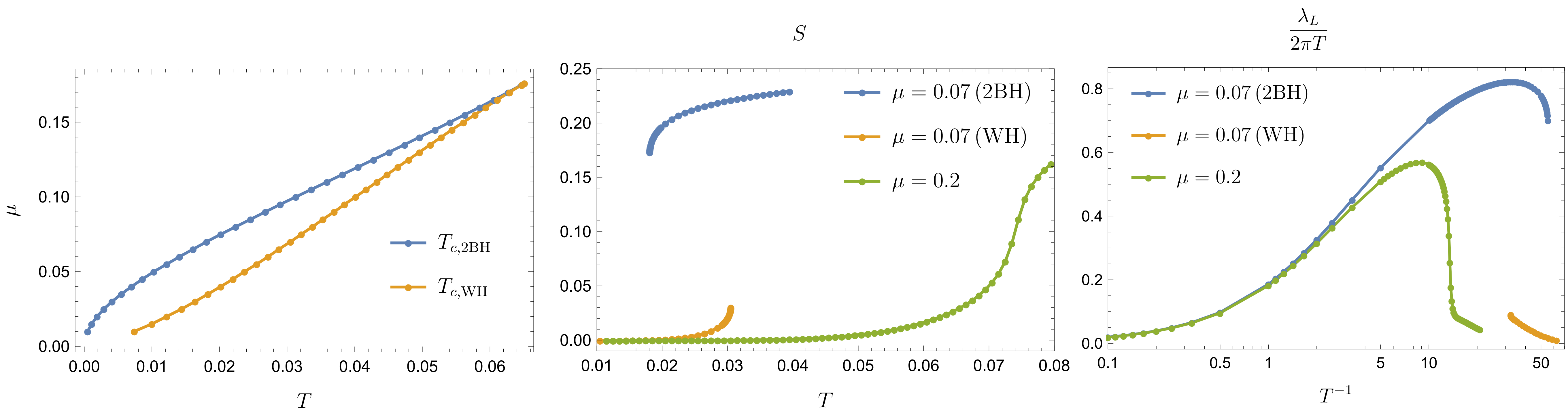
➡ Chaos exponent is λ_L such that the largest eigenvalue of $\mathcal{K}^R$ is 1



[TN,Numasawa,'20]

λ_L in low temperature phase is small but non-zero

Phase diagram



Phase transition exists when $\mu < \mu_c \approx 0.177$

High temperature phase: *O*(*N*) entropy, $\lambda_L \sim 2\pi T$

➡ described well by uncoupled SYK

Low temperature phase: small (but nonzero) λ_L, revival oscillation

➡ described well by quasiparticle picture

Quasiparticle approximation

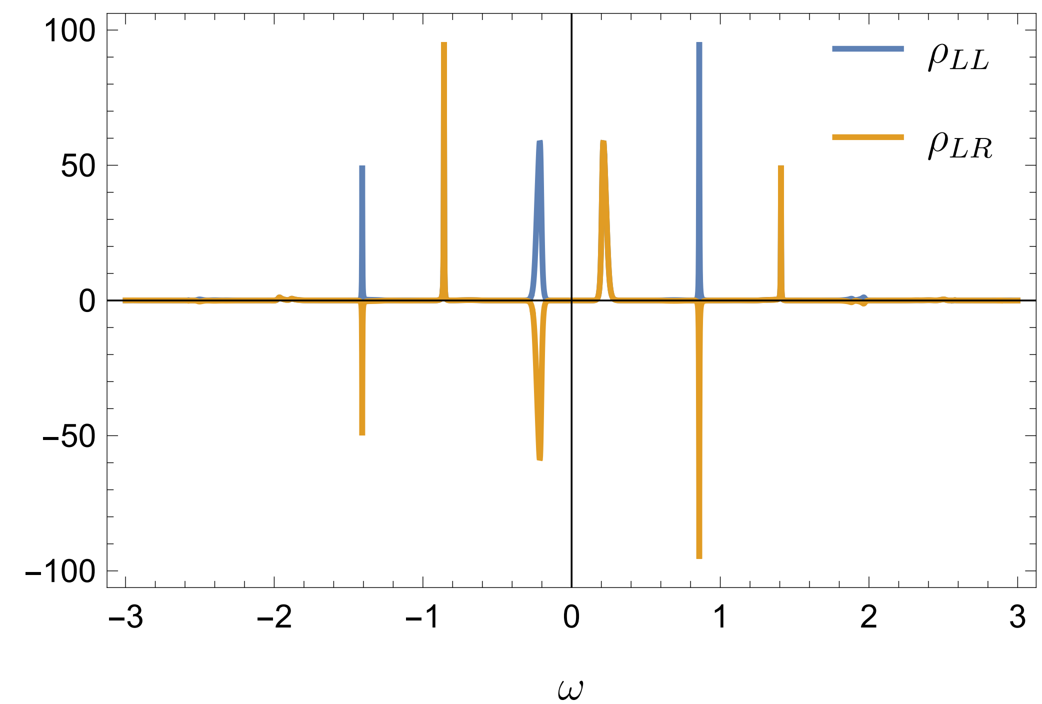
All 2-pt fcns are encoded in spectral functions $\rho_{LL} = 2\mathrm{Im}[\widetilde{G}^R_{LL}(\omega)]$ and $\rho_{LR} = 2\mathrm{Re}[\widetilde{G}^R_{LR}(\omega)]$

$$\because \; G^>_{ab}(t) = \int d\omega e^{-i\omega t} \frac{(i)\rho_{ab}(\omega)}{1+e^{-\beta\omega}}$$ (KMS relation)

In low temperature phase, ρ_{ab} = sharp peaks

$$\rho_{ab}(\omega) = \sum_i A_{ab,i} \mathrm{Im}\Big[\frac{1}{\omega - (\pm\omega_i + i\Gamma_i)}\Big]$$

$$\Rightarrow \; G^>_{ab}(t) = e^{-i\omega_1 t - \Gamma_1 t} + \cdots$$



ρ_{ab} is dominated by first a few peaks

	ω_i	A_i	$\Gamma_i(T=0.03)$	$\Gamma_i(T=0.016)$
1st	0.217	2.28	0.0115	0.00109
2nd	0.869	0.508	0.00153	0.000173
3rd	1.42	0.243	0.00115	0.000108

$$\Big(\frac{2A_1+2A_2+2A_3}{2\pi} = 0.964\Big)$$

Revival = 1st peak + 2nd peak

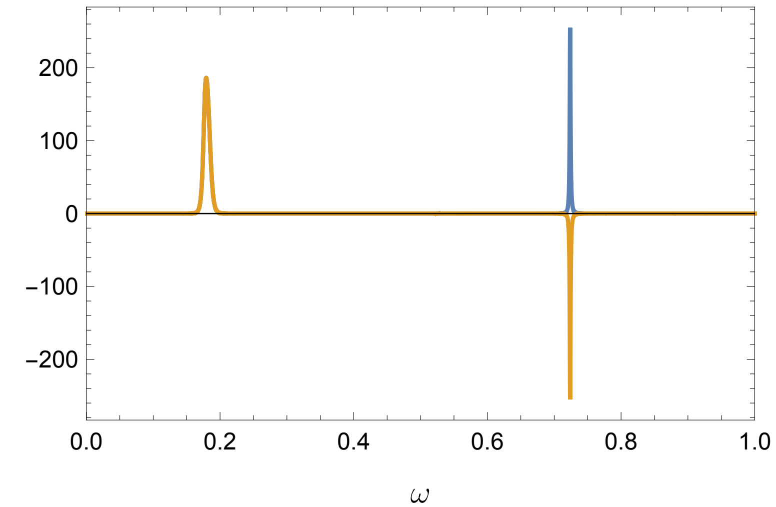
If we use only 1st peak,

$$G^{>}_{ab}(t) \sim e^{-i\omega_1 t - \Gamma_1 t} \Rightarrow |G^{>}_{ab}(t)| = e^{-\Gamma_1 t}$$: no oscillation

To see revival oscillation, we also include 2nd peak

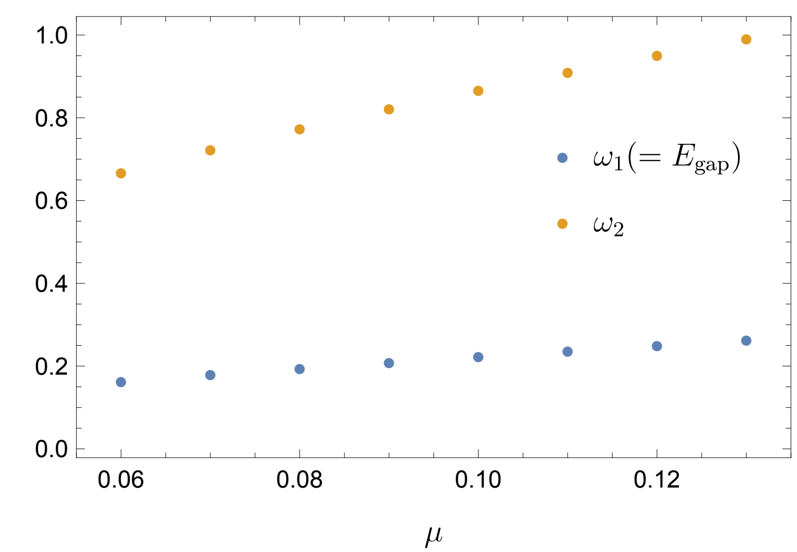
$$G^{>}_{LL}(t) \sim A_1 e^{-i\omega_1 t - \Gamma_1 t} + A_2 e^{-i\omega_2 t - \Gamma_2 t}$$

$$G^{>}_{LR}(t) \sim A_1 e^{-i\omega_1 t - \Gamma_1 t} - A_2 e^{-i\omega_2 t - \Gamma_2 t}$$



$A_{LR,2} = -A_{LL,2}$ ⇒ out-of-phase oscillation with period $\frac{2}{\omega_2 - \omega_1}$

revival is successful if $\omega_2 - \omega_1 \gg \Gamma_1$

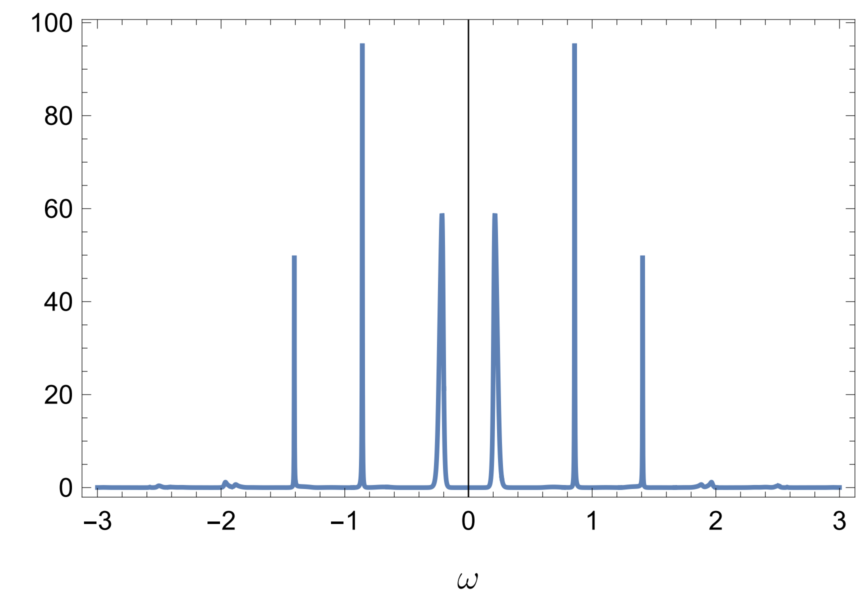


Note: From operator formalism it follows $\omega_1 = E_{\rm gap}$

We also observe $\omega_2 \approx 4E_{\rm gap}$ (c.f. [Qi,Zhang,'20])

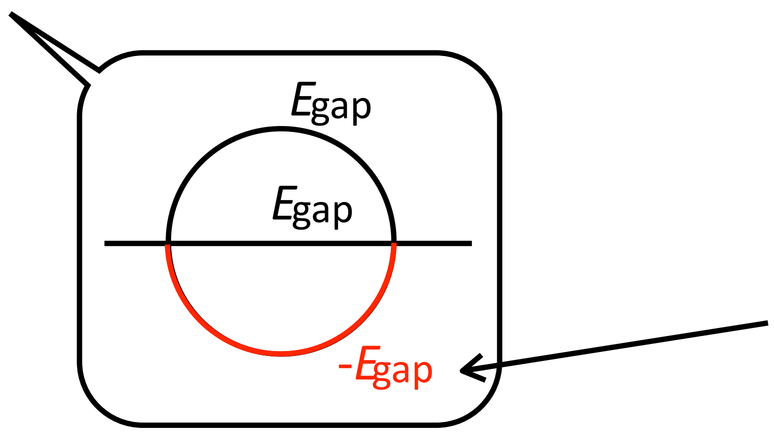
Γ_1 by first peak

From SD eq $\widetilde{G} = \dfrac{1}{\omega - \widetilde{\Sigma}}$,



$$\Gamma_1 = \mathrm{Im}[\widetilde{\Sigma}_{ab}(\omega = E_{\mathrm{gap}})]$$

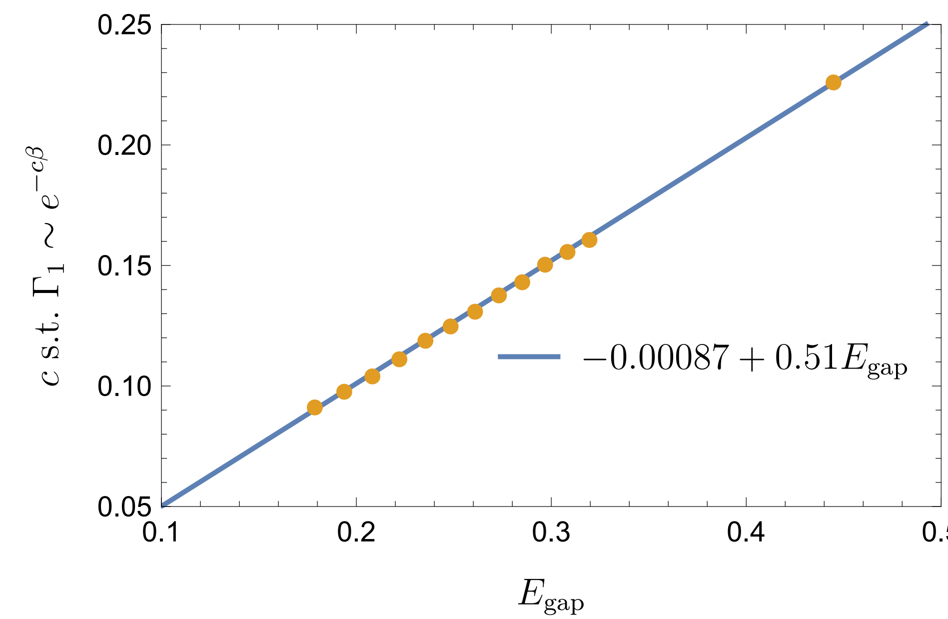
$$= \frac{1}{3\Gamma_1} e^{-\beta E_{\mathrm{gap}}}$$



$$G_{ab}(t) \sim e^{-iE_{\mathrm{gap}}t - \Gamma_1 t} + e^{-\beta E_{\mathrm{gap}}} e^{iE_{\mathrm{gap}}t - \Gamma_1 t}$$

$$\Gamma_1 \sim e^{-\frac{\beta E_{\mathrm{gap}}}{2}}$$

[Qi,Zhang,'20]



Large E_{gap} helps revival in two ways: transmission speedup & decay suppression

λ_L by first peak

Ladder eq:

$$f_{abcd}(t_{1-2}) = \int dt_5 dt_6 e^{-\frac{\lambda_L t_{1+2}}{2}} G^R_{ae}(t_{1-5}) G^R_{bf}(t_{2-6}) G_{ef}\left(\frac{\beta}{2} + it_{5-6}\right)^2 e^{\frac{\lambda_L t_{5+6}}{2}} f_{efcd}(t_{5-6})$$

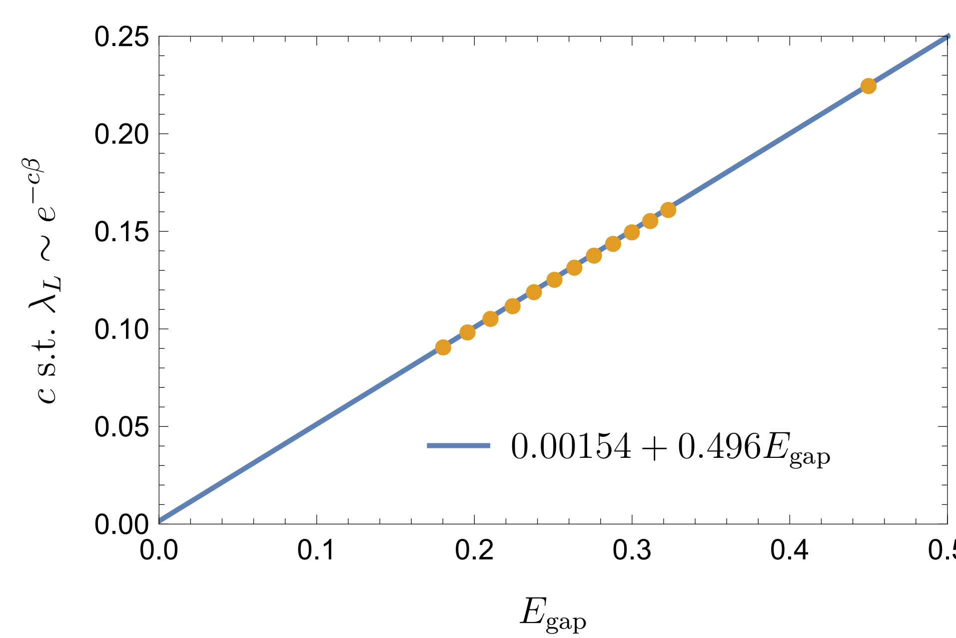
$$G^R_{ab}(t) \sim \theta(t) e^{\pm i E_{\rm gap} t - \Gamma_1 |t|}$$ ⇒ are cancelled by $\partial_{t_{1(2)}} \pm i E_{\rm gap} + \Gamma_1 + \frac{\lambda_L}{2}$

$$G_{ef}\left(\frac{\beta}{2} + it_{5-6}\right)^2 \sim e^{-\beta E_{\rm gap} - 2\Gamma_1 |t|} \times (\text{rapid oscillation})^2$$

$$\left[-\frac{d^2}{dt'^2} + \frac{1}{4}\left(\frac{\lambda_L}{2\Gamma_1} + 1\right)^2 - 6e^{-|t'|}\right] f_{abcd}(t') = 0 \qquad (t' = \Gamma_1 t)$$

$\frac{\lambda_L}{\Gamma_1}$ = O(1) const. independent of β, μ (≈ 2.706)

fixed by b.c. at $t' = \pm\infty$ and smoothness at t'=0



[TN,Numasawa,'20]

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✓ 1. SYK model in large N limit [reviews]

✓ 2. Regenesis (revival) in two coupled SYK [TN,Numasawa,'20] + [reviews]

3. Imperfectly correlated disorder [TN,Numasawa,'19][TN,Numasawa,'22]

4. Future problems

Imperfectly correlated disorders

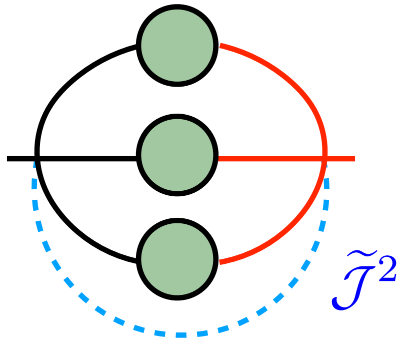
$$H = H_{\rm SYK}(J^L_{ijk\ell};\psi^L_i) + H_{\rm SYK}(J^R_{ijk\ell};\psi^R_i) + i\mu\sum_i \psi^L_i\psi^R_i$$

$$\langle {J^L_{ijk\ell}}^2\rangle = \langle {J^R_{ijk\ell}}^2\rangle = N^{-3} \qquad \langle J^L_{ijk\ell}J^R_{ijk\ell}\rangle = \widetilde{\mathcal{J}}^2 N^{-3} \qquad (\widetilde{\mathcal{J}} < 1)$$

LR entanglement structure of ground state is modified:

$$|gs\rangle \approx e^{-\beta^*(\mu)(H^L_{\rm SYK}+H^R_{\rm SYK})}|I\rangle \qquad (\psi^L_i - i\psi^R_i)|I\rangle = 0$$

Diagrams contributing to G_{LR}, Σ_{LR} are suppressed

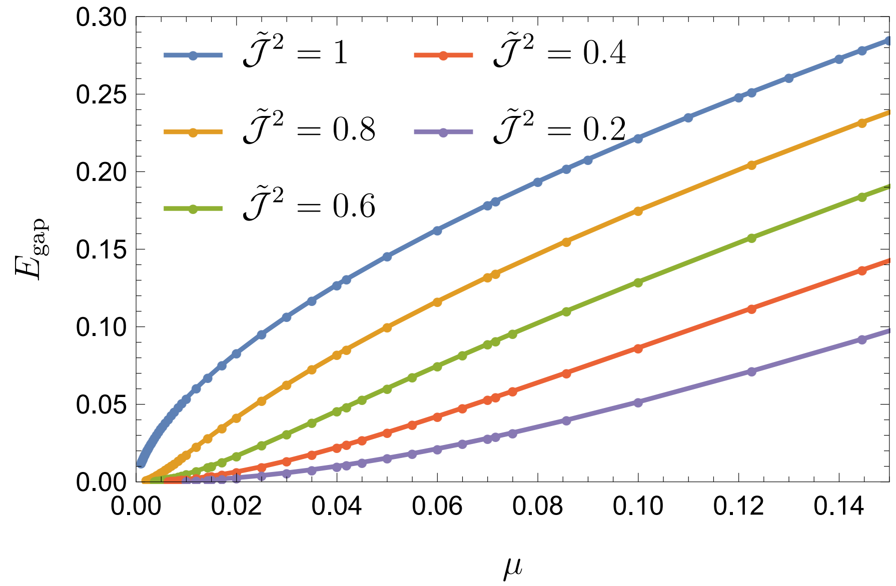


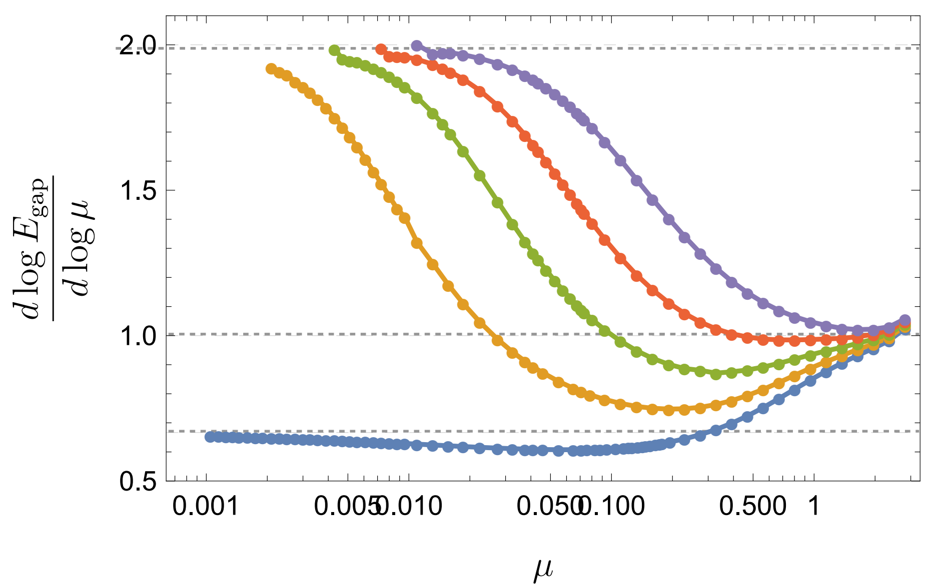
Expectation: $E_{\rm gap}$ suppressed

⇓

revival suppressed due to quasiparticle picture

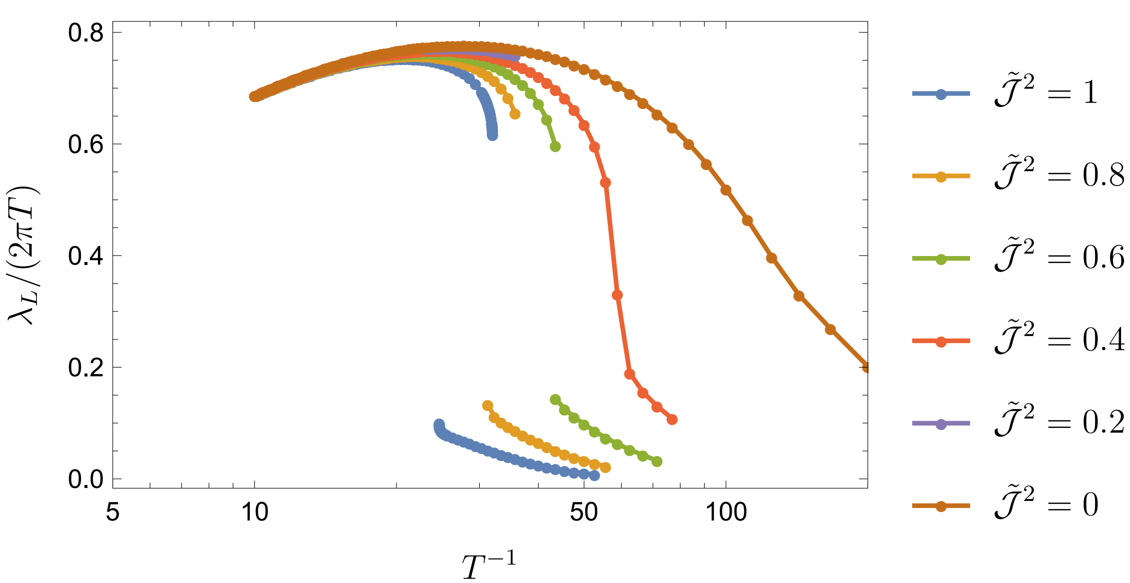
Suppressed E_{gap} , increased λ_L





E_{gap} decreases monotonically in $\tilde{\mathcal{J}}^2$

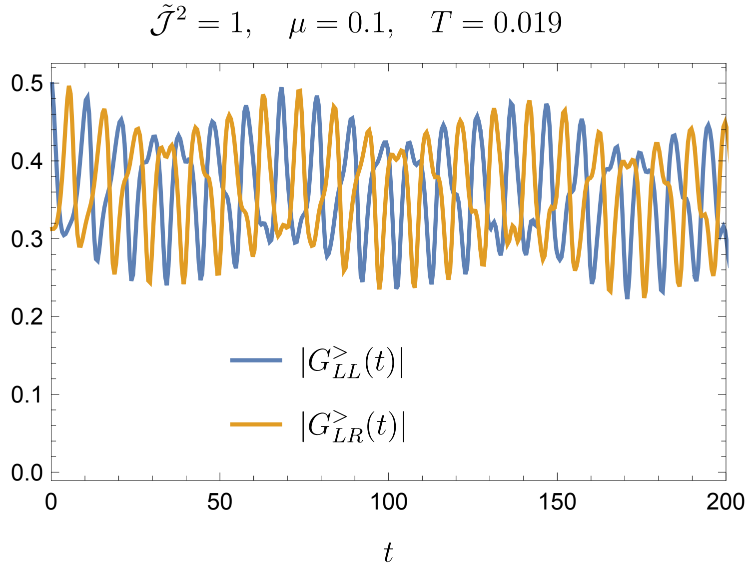
In $\mu\to 0$ limit, $E_{\text{gap}}\sim\mu^{\frac{2}{3}}$ only when $\tilde{\mathcal{J}}^2=1$. Otherwise $E_{\text{gap}}\sim\mu^2$

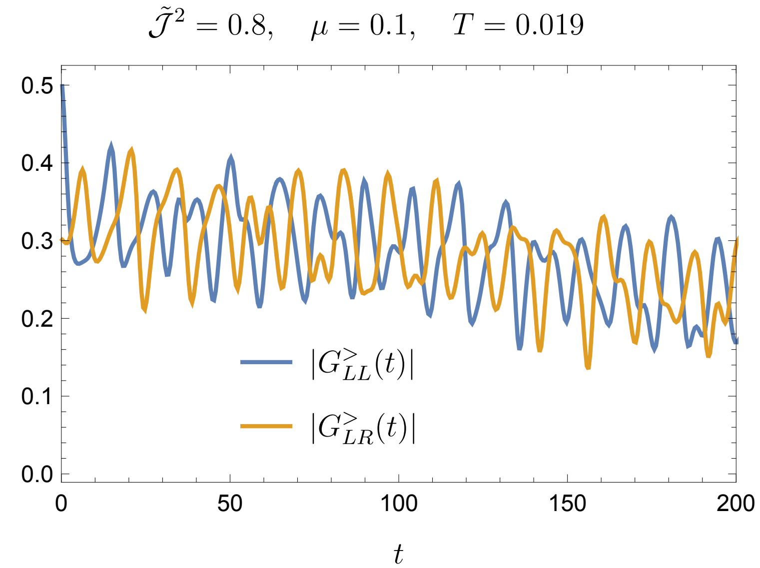


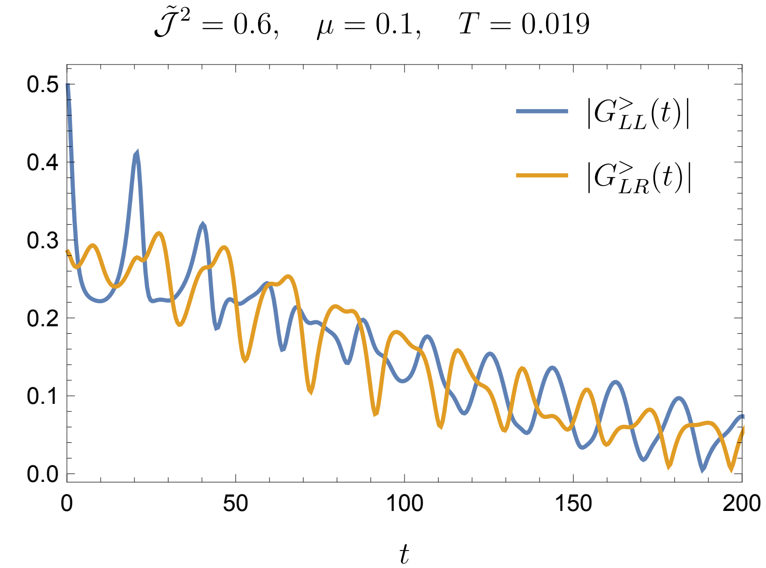
λ_L increase monotonically in $\tilde{\mathcal{J}}^2$

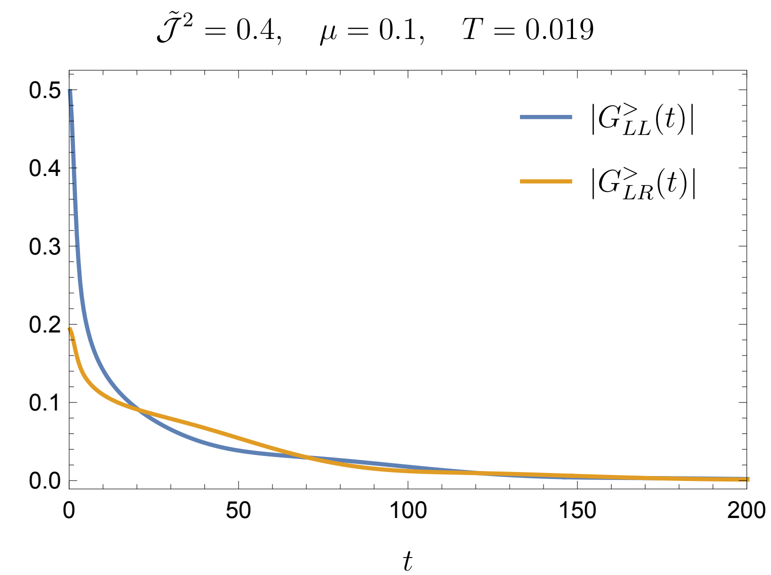
⇨ at low temperature, consistent with quasiparticle picture $\lambda_L\sim e^{-\frac{\beta E_{\text{gap}}}{2}}$

Suppressed revival oscillation









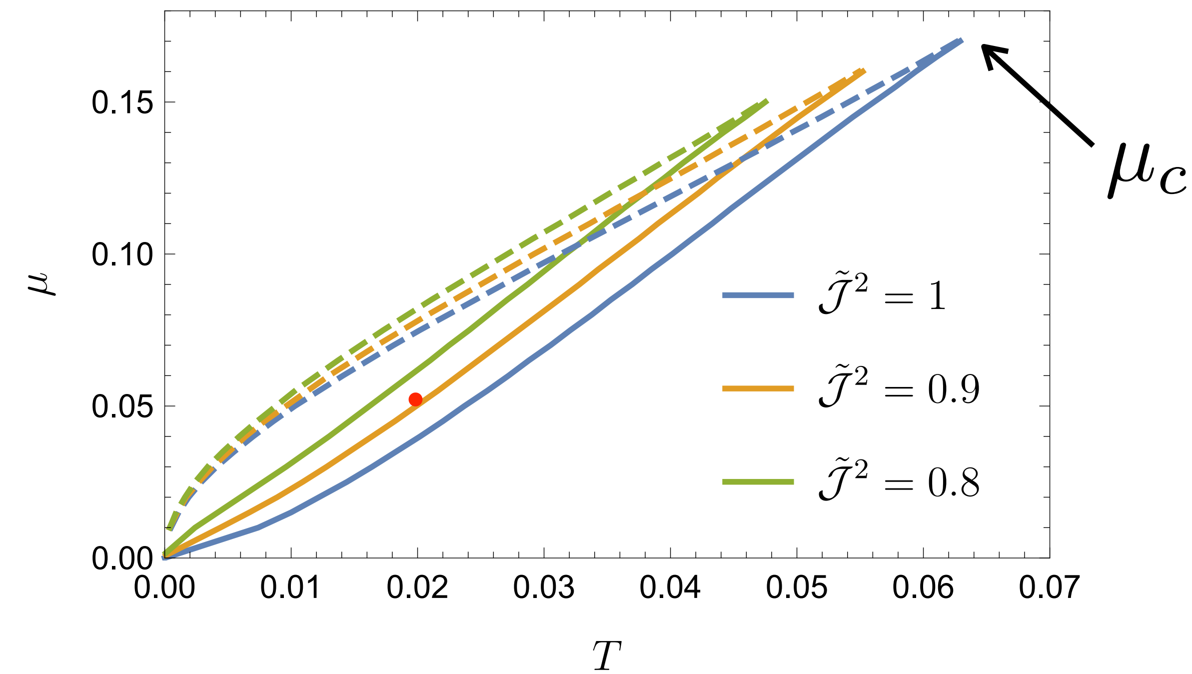
Slower oscillation & faster decay ⇨ consistent with quasiparticle picture

No revival for $\tilde{\mathcal{J}}^2 \leq 0.4$ at this (μ,T), but for smaller μ,T there may be.

Question:

Does revival completely disappear from (μ,T) plane at some finite $\tilde{\mathcal{J}}^2$?

Phase diagram



As $\tilde{\mathcal{J}}^2$ is decreased, $T_{c,\mathrm{WH}}(\mu)$ decreases

ex. • (μ=0.05, T=0.02) is in low temperature phase (revival) for $\tilde{\mathcal{J}}^2 = 1, 0.9$,

but in high temperature phase (no revival) for $\tilde{\mathcal{J}}^2 \leq 0.8$

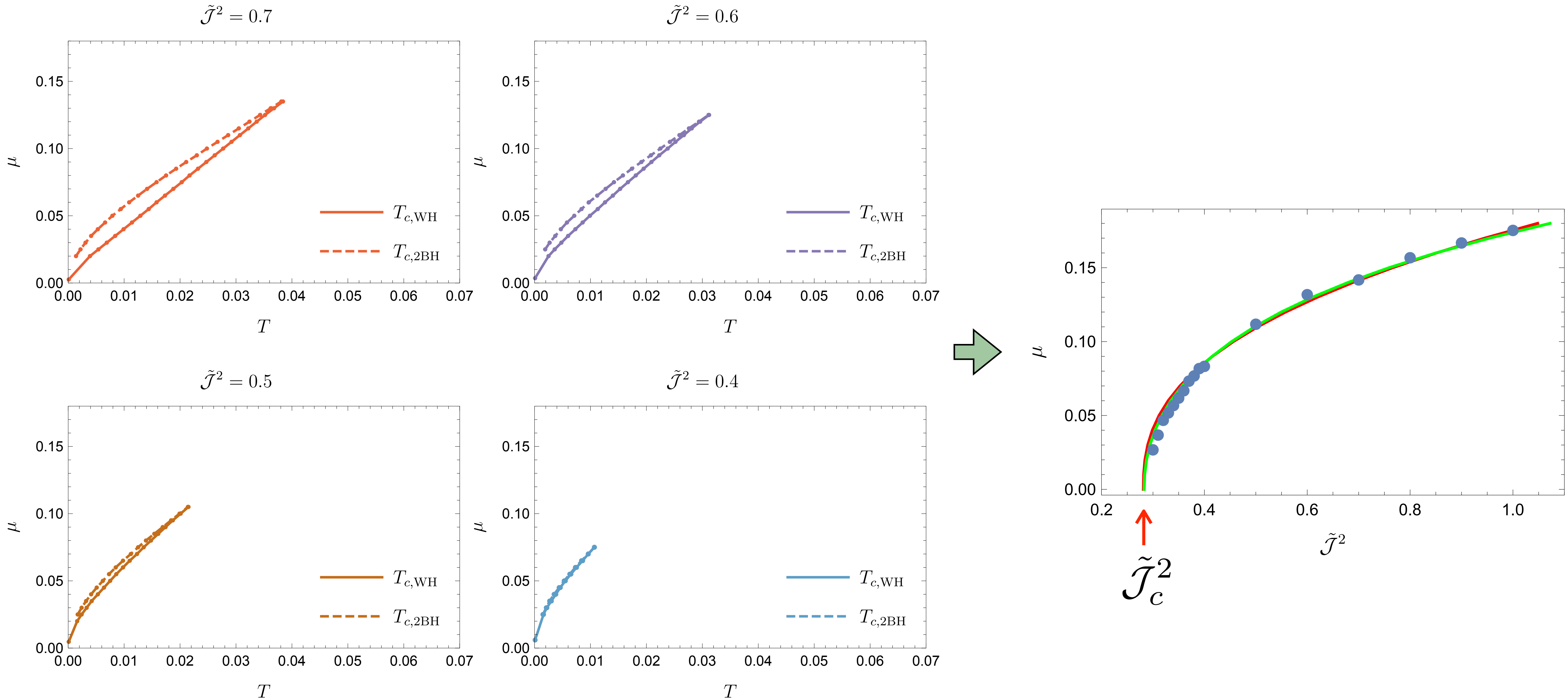
(also for $\tilde{\mathcal{J}}^2 = 0.6 \to 0.4$ in previous slide)

μ_c and $T_c(\mu_c)$ also decreases in $\tilde{\mathcal{J}}^2$ ⇨ phase transition happens in smaller regime

Let us "define" critical correlation $\tilde{\mathcal{J}}_c^2$ for revival as $\mu_c(\tilde{\mathcal{J}}_c^2) = 0$

(i.e. phase transition completely disappears)

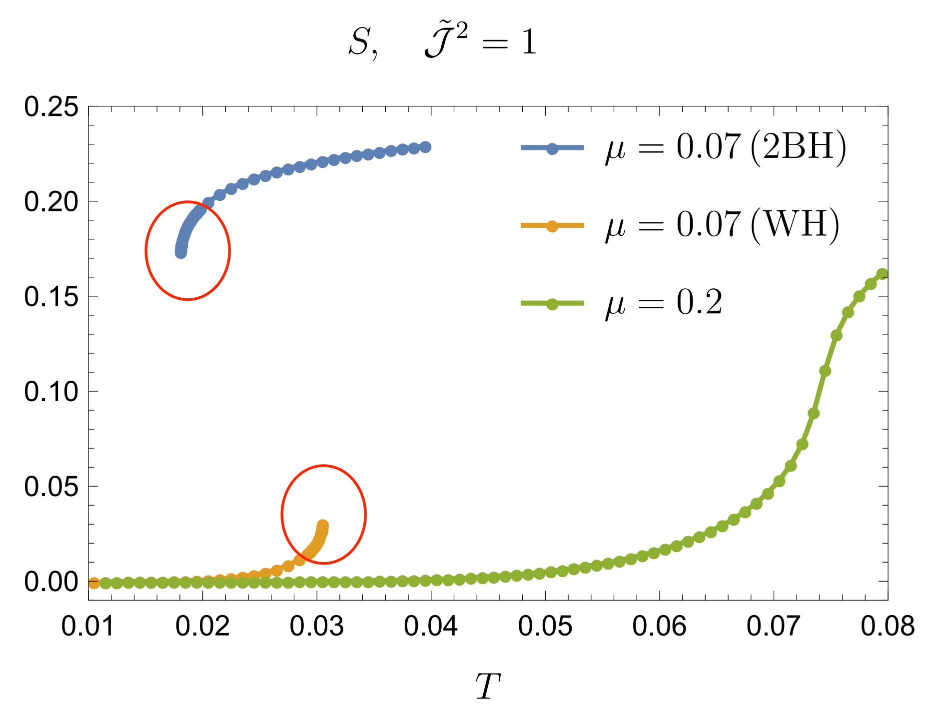
Critical value of J^L_{ijkl}-J^R_{ijkl} correlation

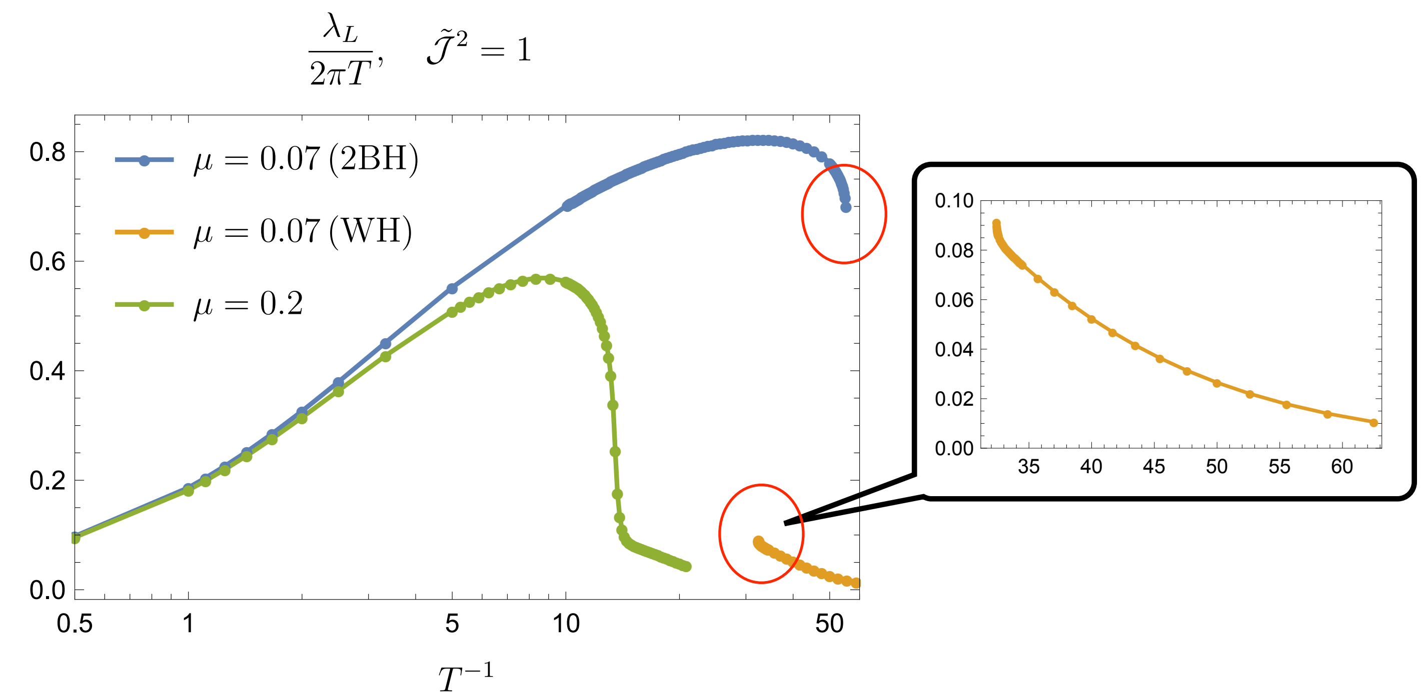


No phase transition for $\tilde{\mathcal{J}}^2 < \tilde{\mathcal{J}}_c^2 \approx 0.25$!

We can also see this (semi-)analytically in large *q* limit of SYK$_q$.

Analytic evidence in large *q* SYK



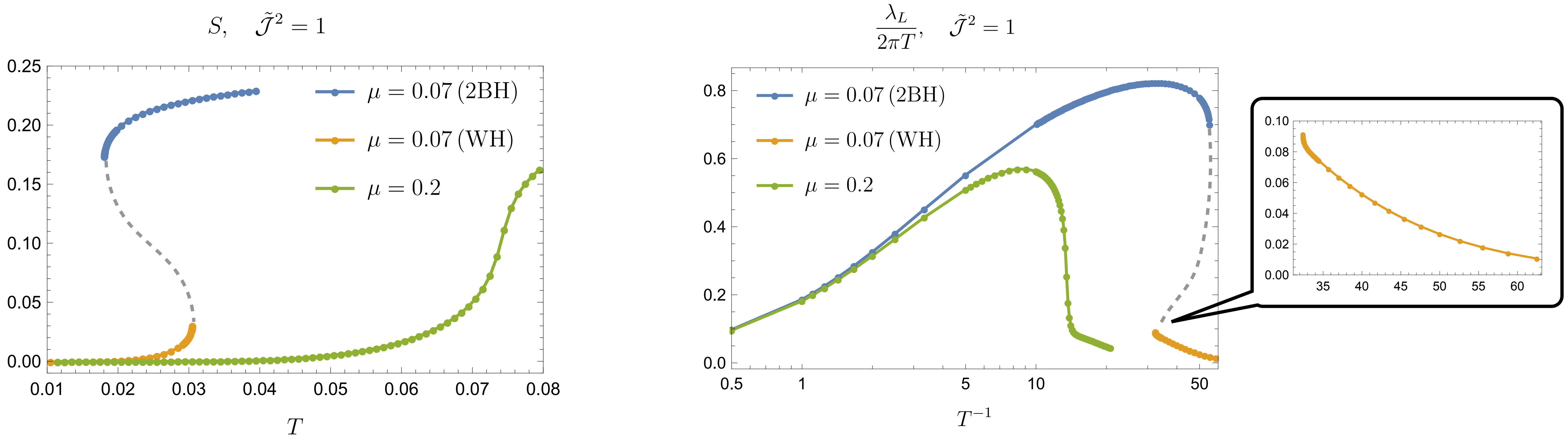


Divergent slope at $T \to T_{c,\mathrm{2BH}}, T_{c,\mathrm{WH}}$ suggests two phases are smoothly connected by an unstable phase

(c.f. smooth in microcanonical ensemble)

[Maldacena,Qi,'18][Maldacena,Milekhin,'19]

Analytic evidence in large *q* SYK



Divergent slope at $T \to T_{c,\mathrm{2BH}}, T_{c,\mathrm{WH}}$ suggests two phases are smoothly connected by an unstable phase

(c.f. smooth in microcanonical ensemble)

[Maldacena,Qi,'18][Maldacena,Milekhin,'19]

Phase transition ⟺ non-monotonicity of $T(S)$

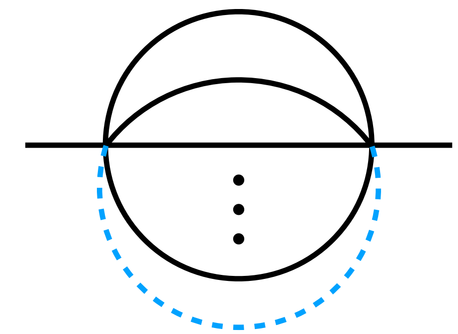
$$\mu_c \Longleftrightarrow \min_S \frac{dT(\mu_c, S)}{dS} = 0$$

We can obtain μ_c analytically for *q*-body generalization of SYK with *q* >> 1

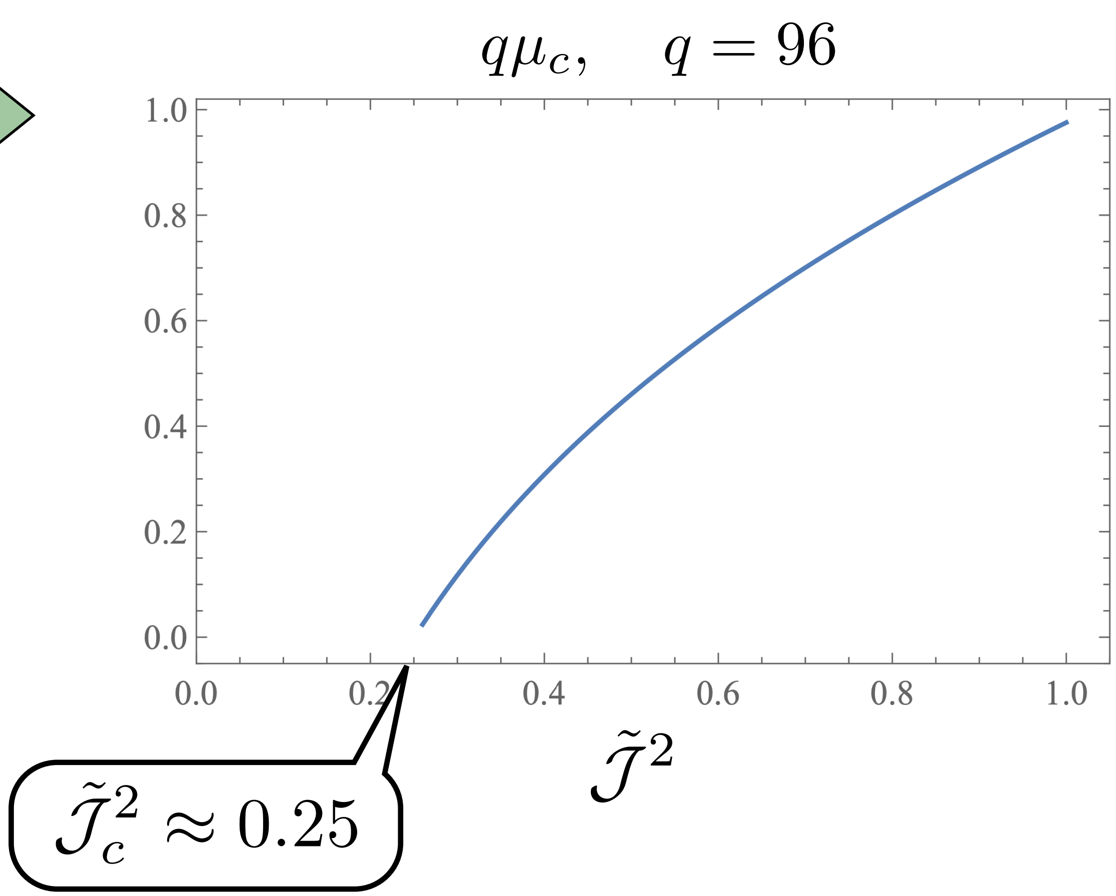
Critical $\tilde{\mathcal{J}}_c^2$ for 2-coupled large *q* SYK

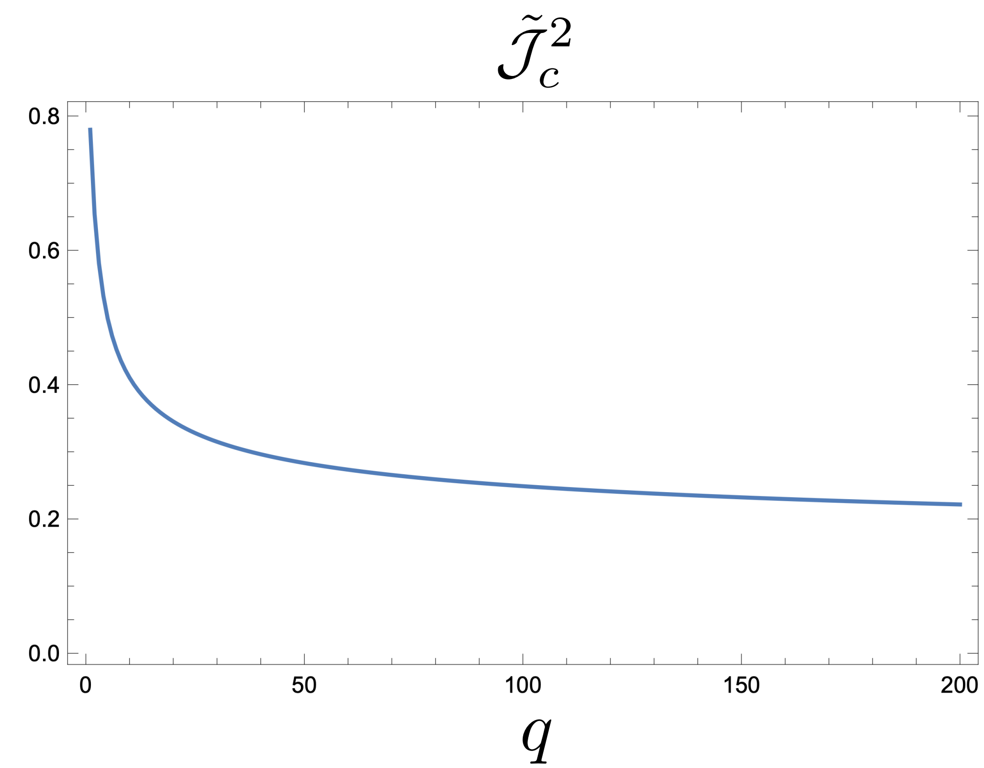
$$H = \sum_{i_1<\cdots<i_q} J^L_{i_1\cdots i_q}\psi^L_{i_1}\cdots\psi^L_{i_q} + \sum_{i_1<\cdots<i_q} J^R_{i_1\cdots i_q}\psi^R_{i_1}\cdots\psi^R_{i_q} + i\mu\sum_i \psi^L_i\psi^R_i$$

$$\langle (J^L_{i_1\cdots i_q})^2\rangle = \frac{2^{q-1}(q-1)!}{qN^{q-1}} \qquad \langle J^L_{i_1\cdots i_q} J^R_{i_1\cdots i_q}\rangle = \tilde{\mathcal{J}}^2\langle (J^L_{i_1\cdots i_q})^2\rangle$$



Qualitatively similar to finite *q* model in $q\to\infty$ with $\mu\sim q^{-1}$, $\beta\sim q\log q$





$\tilde{\mathcal{J}}_c^2 \sim \dfrac{1}{2\log q} \to 0$ at $q\to\infty$, but $\tilde{\mathcal{J}}_c^2 \gtrsim 0.2$ for wide range of *q*

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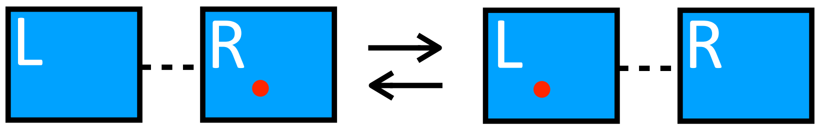
4. Future problems

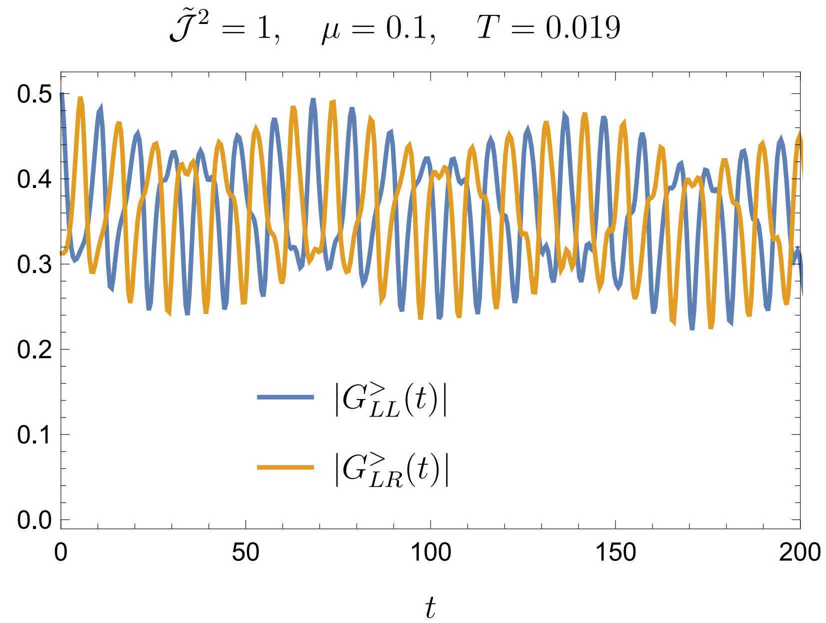
Summary

2-coupled SYK model is useful to study regenesis / revival.



At low temperature phase, two point fcns $G_{LL}(t), G_{LR}(t)$

oscillate **out-of-phase** ⇨ interpreted as revival



Low temperature phase is described well by quasiparticles

1st quasiparticle excitation ⇨ $\Gamma_1, \lambda_L \sim e^{-\frac{\beta E_{\rm gap}}{2}}$

1st + 2nd excitations ⇨ revival

Large correlation of SYK disorders $\tilde{\mathcal{J}}_c^2 = \langle J^L_{ijkl} J^R_{ijkl} \rangle$ is crucial for revival.

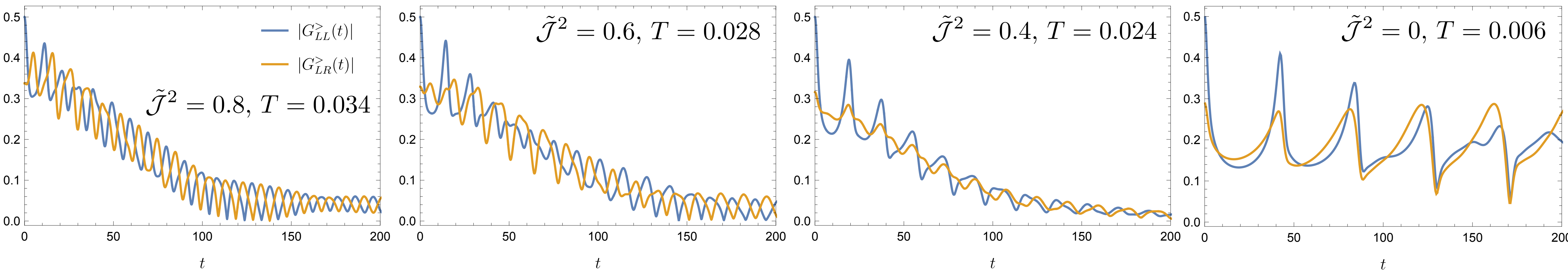
⇨ consistent with requirement for regenesis.

Phase transition disappears at finite $\tilde{\mathcal{J}}_c^2$ (≈ 0.25) ⇨ "no revival" for $\tilde{\mathcal{J}}^2 < \tilde{\mathcal{J}}_c^2$

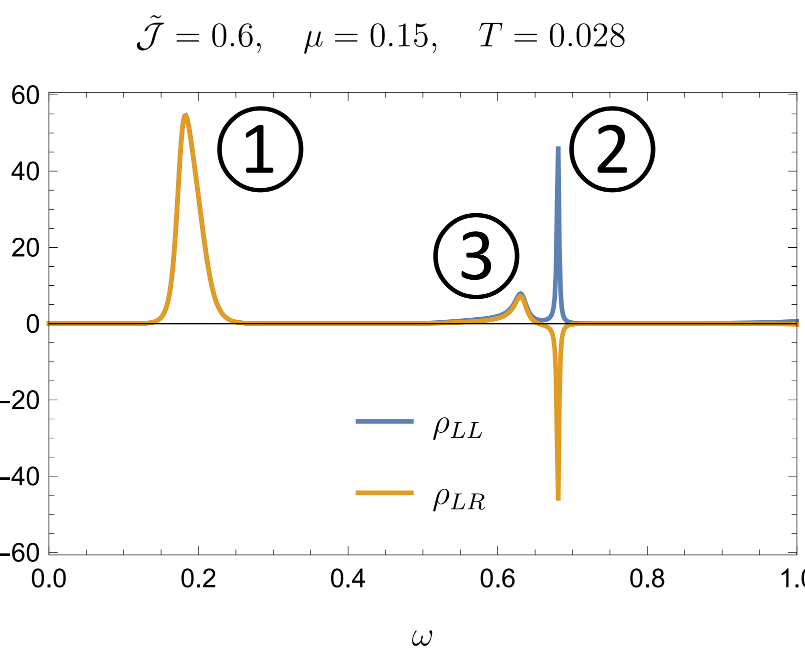
Another criterion for "no revival"?

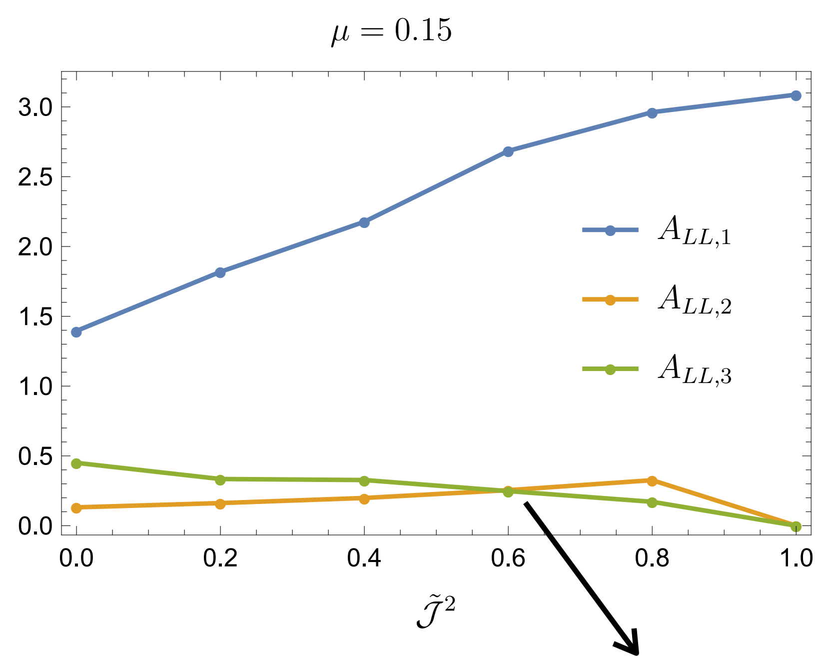
As $\tilde{\mathcal{J}}^2$ decreases, oscillation becomes not only suppressed but also more "in-phase".

$|G^{>}_{ab}(t)|$ at μ=0.15 (*different T's):



In spectral function, new in-phase peak emerges



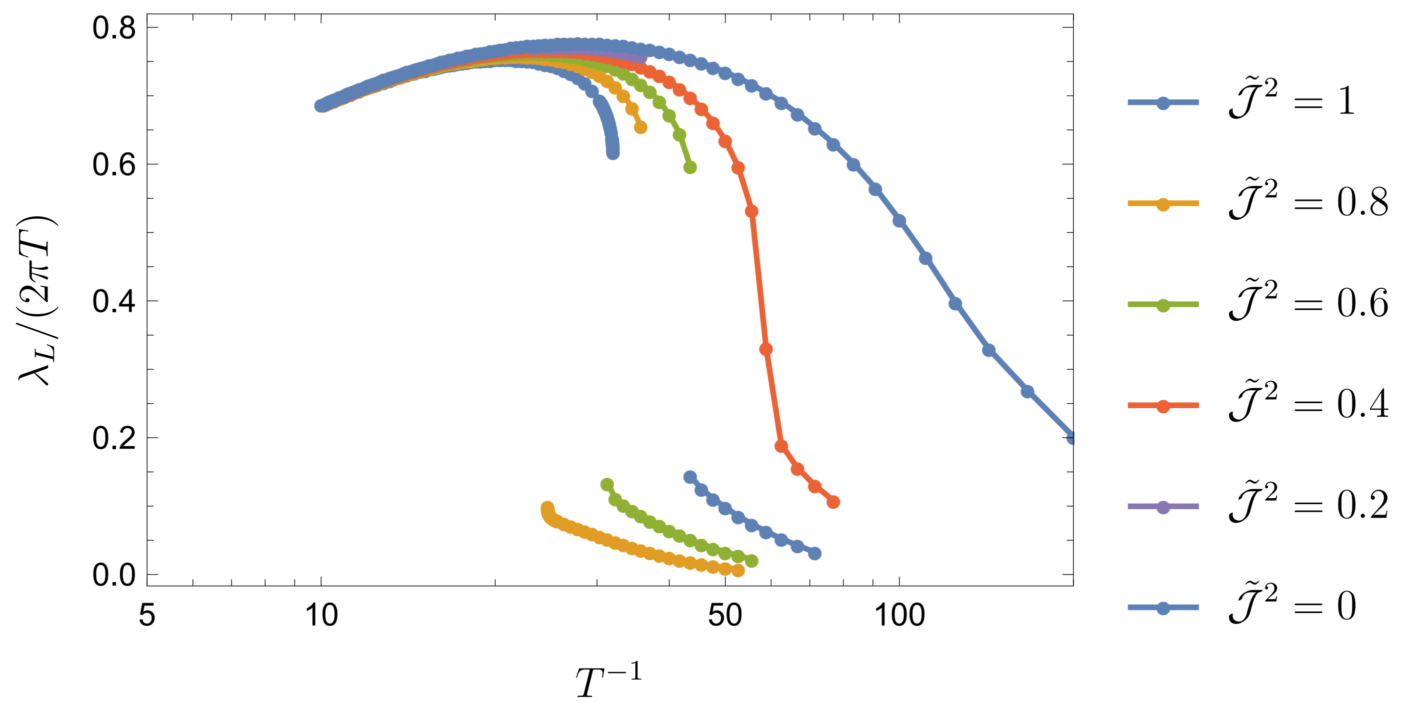


③: $A_{3,LL} = A_{3,LR} > 0$

⇨ ①+③ = in-phase

in-phase /out-of-phase transition??

Physical interpretation of $\lambda_L(\tilde{\mathcal{J}}^2 < 1)$?



low temperature:

As $\tilde{\mathcal{J}}^2$ increase, revival increase and λ_L decrease

Assume λ_L measures mainly operator growth within single side

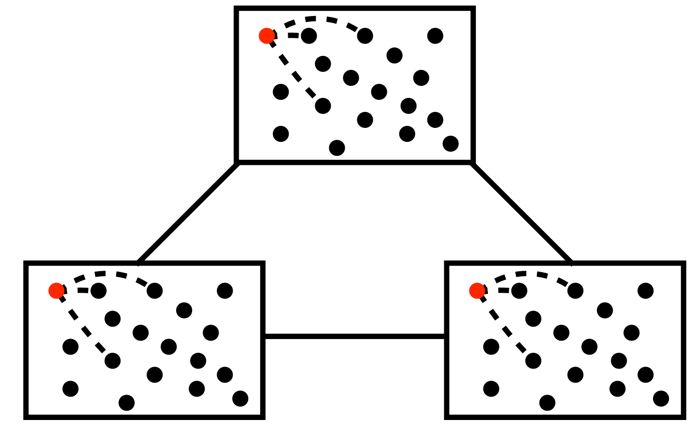
⇨ interpretation: "leakage" by revival prevents operator growth within single side?

high temperature → ?

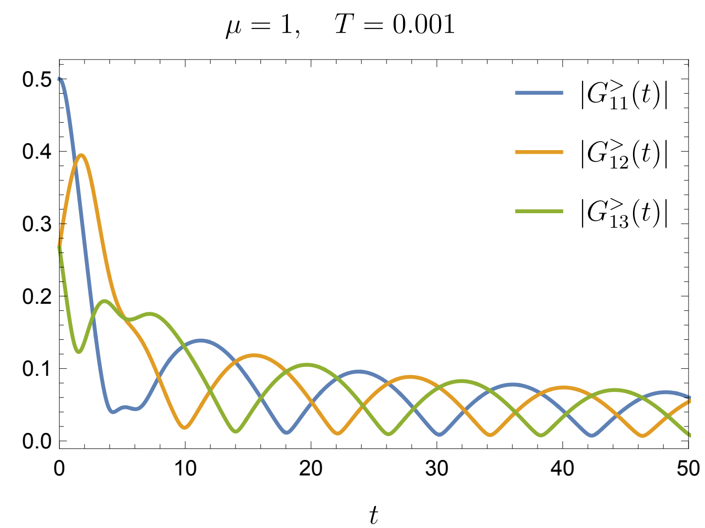
Multi-site generalization

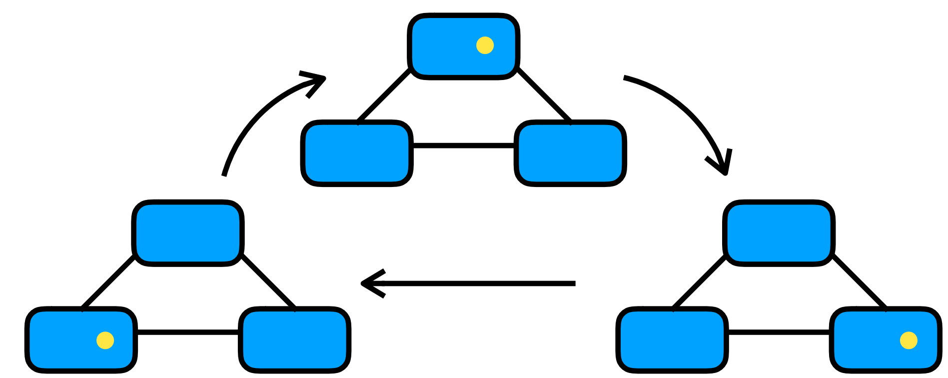
[TN,Numasawa,Peng,Yang, work in progress]

$$H = \sum_{a=1}^{L} H_{\text{SYK}}(J^{(a)}_{ijk\ell}; \psi^{(a)}_i) + H_{int}$$



Multi-site revival oscillation

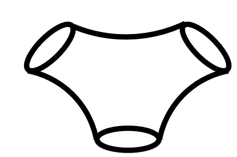




[NNPY, in preparation]

- What is regenesis analog? (*L*-partite entanglement?)

Can we find a solution for multi-boundary wormhole?



- How should we choose H_{int} (right *L*-partite entanglement of the ground state?)

- Complex J_{ijkl} might help? [Garcia-Garcia,Godet,'20]