Repetition code:

Noise model: Physical qubits are flipped (Pauli X) w. probability p.

We want to protect a quantum state $\alpha|0\rangle + \beta|1\rangle$.

Encode:

codespace := span$\{|000\rangle, |111\rangle\}$

$|0\rangle \longmapsto |0_L\rangle := |000\rangle$

$|1\rangle \longmapsto |1_L\rangle := |111\rangle$

So $\alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha|000\rangle + \beta|111\rangle$

Imagine a single bitflip occurs

e.g. $\alpha|000\rangle + \beta|111\rangle \longrightarrow \alpha|100\rangle + \beta|011\rangle$

How can we correct this error?

Measuring each qubit in Z destroys coherence.

$\alpha|000\rangle + \beta|111\rangle \longrightarrow |000\rangle$ or $|111\rangle$

Instead perform measurements that compare 2 qubits.

e.g. $Z_1 Z_2$

$$Z_1 Z_2 |00\rangle = |00\rangle$$
$$Z_1 Z_2 |11\rangle = |11\rangle$$
$$Z_1 Z_2 |01\rangle = -|01\rangle$$
$$Z_1 Z_2 |10\rangle = -|10\rangle$$

$M_1 = Z_1 Z_2$, $M_2 = Z_2 Z_3$ called 'checks'

	M_1	M_2	Correction
No Error	+1	+1	Do nothing
X error on qubit 1	-1	+1	Apply X to qubit 1
" 2	-1	-1	" 2
" 3	+1	-1	" 3

Each single qubit X error has a unique 'syndrome'.

Choosing a correction based on check-measurement outcomes is called 'decoding'.

Single errors can be corrected, but not 2 qubit errors.

Probability of logical error is $\sim 3p^2$ (improvement)

Unfortunately does not correct against phase-flip (Pauli Z) errors

$$\alpha|000\rangle + \beta|111\rangle \xrightarrow{\text{phase flip on any qubit}} \alpha|000\rangle - \beta|111\rangle$$

Note: Codespace = simultaneous +1 eigenspace of M_1, M_2 = span$\{|000\rangle, |111\rangle\}$.