Solving Naturalness Problem in Matrix Model

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We will discuss the naturalness problem in the context of the IIB matrix model.

Contents

1. IIB matrix model

IIB Matrix Model

$$S = -\frac{1}{g^2} Tr\left(\frac{1}{4}[A^\mu, A^\nu]^2 + \frac{1}{2}\overline{\Psi}\gamma^\mu[A^\mu, \Psi]\right)$$

A candidate of the constructive definition of string theory.

Evidences

(1) World sheet regularization

Green-Schwartz action in the Schild Gauge

$$S = \int d^2\xi \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\overline{\Psi}\gamma^\mu\{X^\mu, \Psi\}\right)$$

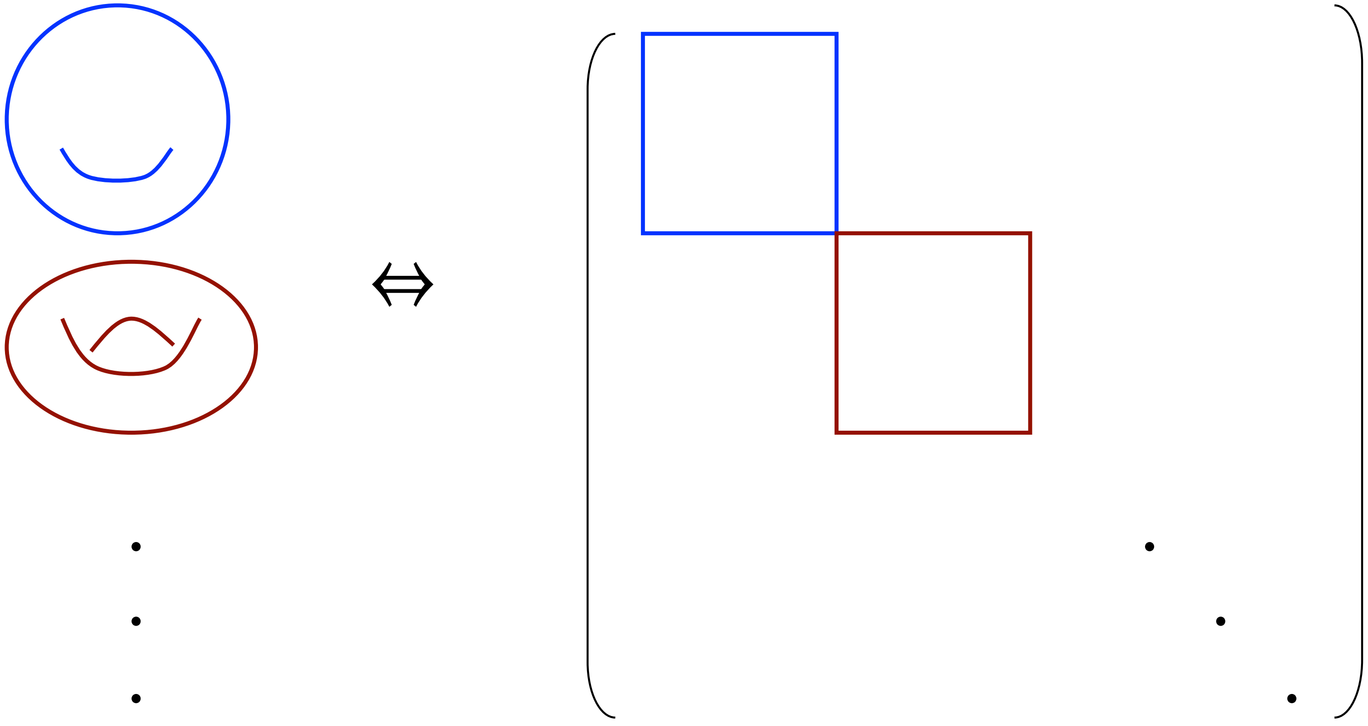
Regularization by matrix

$$\{\ ,\ \} \to [\ ,\ ]$$

$$\int \to Tr$$

$$S = -\frac{1}{g^2} Tr\left(\frac{1}{4}[A^\mu, A^\nu]^2 + \frac{1}{2}\overline{\Psi}\gamma^\mu[A^\mu, \Psi]\right)$$

Multi string states are naturally described in the large-N limit.



(2) Loop equation and string field

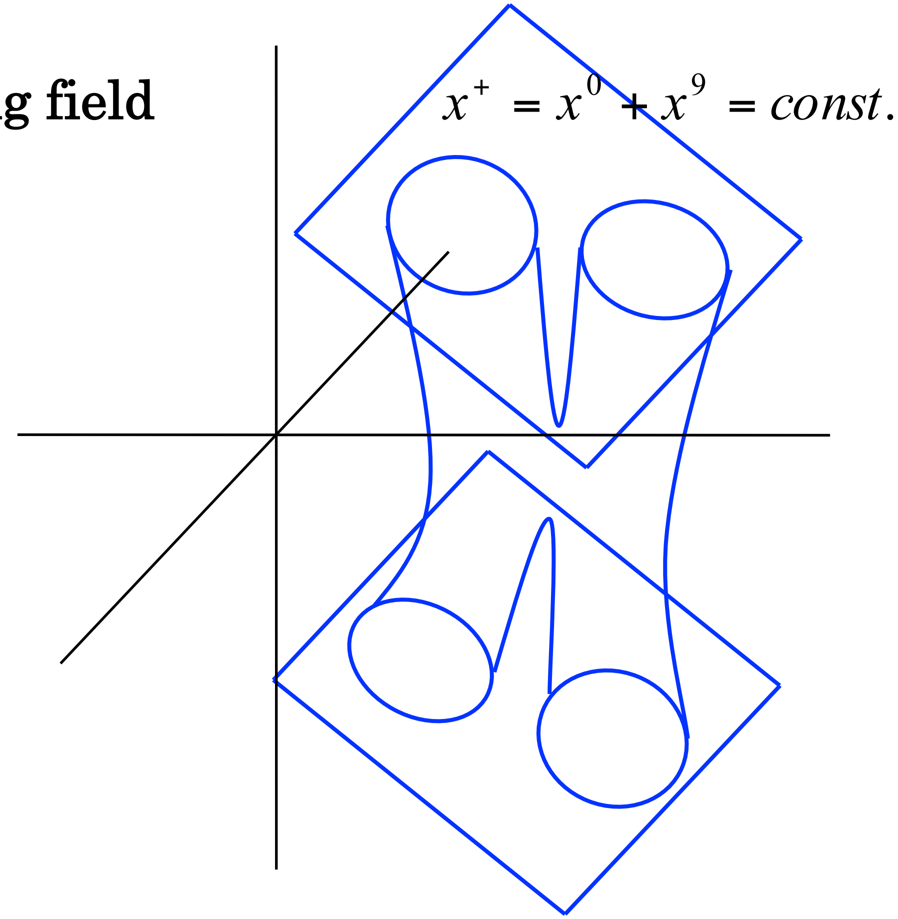
Wilson loop = string field

$$w(k_\mu(\cdot\cdot)) = Tr(P\exp(i\oint d\sigma\, k_\mu(\sigma)\, A^\mu + fermion))$$

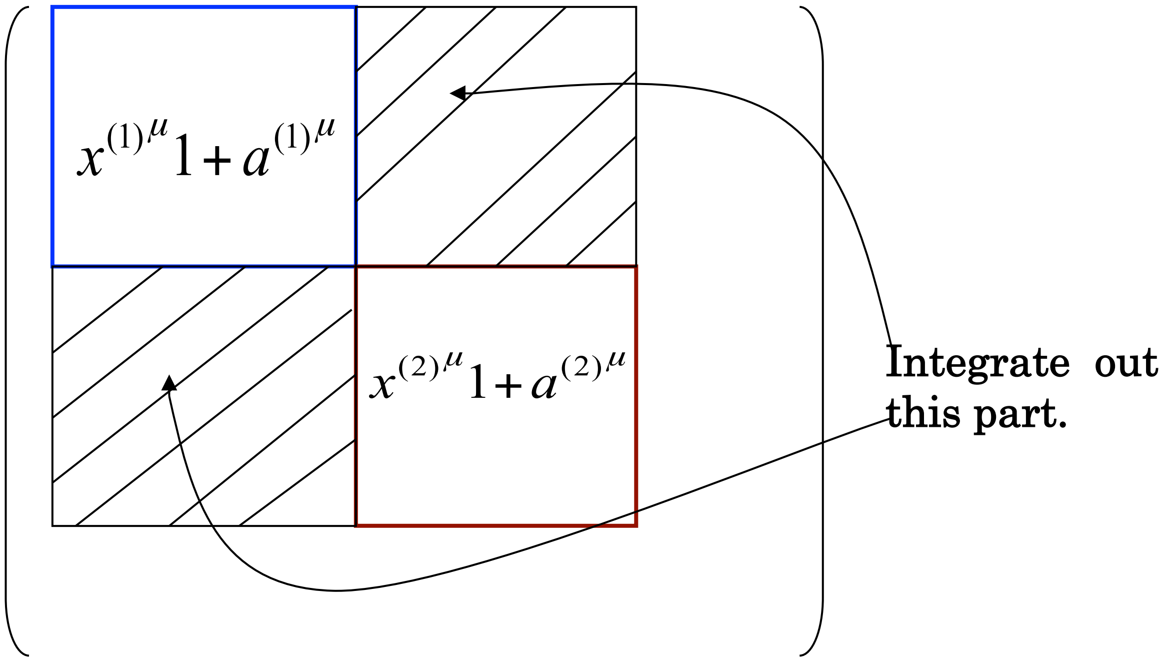
$\Leftrightarrow$ creation annihilation operator of $|k_\mu(\cdot\cdot)>$

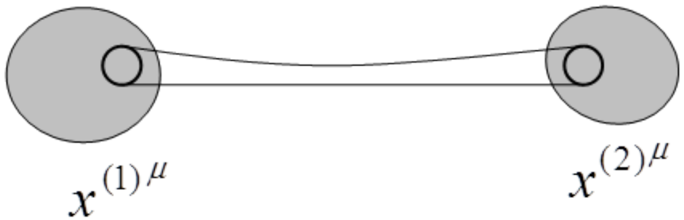
loop equation $\rightarrow$ light-cone string field

This can be shown with some assumptions .



(3) effective Lagrangian and gravity





The loop integral gives the exchange of graviton and dilaton.

$$S_{eff} = -\frac{1}{(x^{(1)} - x^{(2)})^8}\{const \cdot tr(f^{(1)}{}_{\mu\lambda} f^{(1)}{}_{\nu\lambda})\, tr(f^{(2)}{}_{\mu\lambda} f^{(2)}{}_{\nu\lambda})$$

$$-const \cdot tr(f^{(1)}{}_{\mu\nu} f^{(1)}{}_{\mu\nu})\, tr(f^{(2)}{}_{\lambda\rho} f^{(2)}{}_{\lambda\rho}) + \cdots\}$$

2. Emergence of space-time

Various possibilities for the emergence of space-time

$$S = -\frac{1}{g^2} Tr\left(\frac{1}{4}[A^\mu, A^\nu]^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu [A^\mu, \Psi]\right)$$

(1) A_μ as the space-time coordinates

mutually commuting A_μ $\Rightarrow$ space-time

ex. flat space

$A^\mu = x^\mu$ uniformly distributed eigenvalues

$\Rightarrow$ vacuum (flat space-time)

non-commutative lumps

$\Rightarrow$ excitations around the vacuum

(2) A_μ as non-commutative space-time

non-commutative A_μ $\Rightarrow$ NC space-time

fluctuations $\Rightarrow$ local fields in NC space

ex. flat non-commutative space

$$A^\mu = x^\mu, \quad [x^\mu, x^\nu] = i\,C^{\mu\nu}$$

$\Rightarrow$ flat non-commutative space

In non-commutative space, A_μ can be regarded as both coordinates and momenta.

$$p_\mu = C_{\mu\nu} x^\nu$$

(3) A_μ as momenta

A_μ can be regarded as a covariant derivative on any manifold with less than ten dimensions.

$$(A_a\varphi)_\alpha = C_{(a)\alpha}{}^{b,\beta}\,\nabla_b\varphi_\beta$$

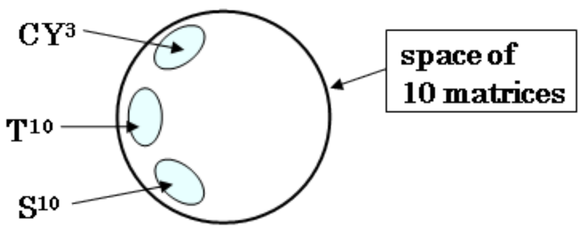
φ_α : regular representation field on manifold M

$C_{(a)\alpha}{}^{b,\beta}$, $(a = 1,..,D)$: the Clebsh-Gordan coefficients

$V_{\text{vector}} \otimes V_r \cong V_r \oplus \cdots \oplus V_r$ $\quad r$: regular representation

ex. derivative on flat space

$$(A_a)_\alpha{}^\beta = C_{(a)\alpha}{}^{b,\beta}\,\partial_b$$



3. Low energy effective theory

Low energy effective theory is obtained by taking the fluctuations into account.

Because of the symmetry, it should be

$$S_{\mathrm{eff}} = \int d^D x \sqrt{g} \left(\kappa R + \Lambda + \text{gauge} + \text{matter} + \ldots \right)$$

Is that all?

Usually, action is additive.

$$S = S_0 + S_{\mathrm{int}},$$

$$S_0 = \int d^4 x \left(\frac{-1}{4} F_{\mu\nu}{}^2 + \overline{\psi} \gamma^\mu \partial_\mu \psi \right),$$

$$S_{\mathrm{int}} = \int d^4 x \left(e A_\mu \overline{\psi} \gamma^\mu \psi \right).$$

why not

$$S = S_0 \, S_{\mathrm{int}}.$$ (Sugawara ~1980)

If it is true, the coupling constant is determuned by the history of the universe:

$$S_{\text{eff}} = \langle S_{\text{int}} \rangle S_0 + \langle S_0 \rangle S_{\text{int}}$$

Actually, in quantum gravity or matrix model, there are some mechanisms that the low energy effective theory becomes

$$S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

$O_i(x)$: local operators

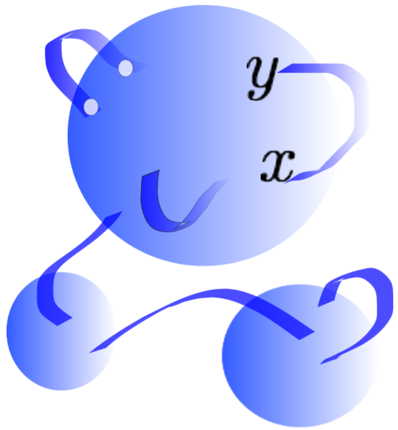
Then the path integral is given by

$$Z = \int d\lambda \, \rho(\lambda) \exp\left(i \sum_i \lambda_i S_i \right).$$

(1) Space-time wormhole and baby universe

action (Euclidean)

$$\int \mathcal{D}g_{\mu\nu} \exp(-S_E)$$



A wormhole induces
a local operator
at each end point

$$\int \mathcal{D}g \, \frac{1}{2} \, c_{ij} e^{-2S_{\rm wh}} \int d^4x d^4y \sqrt{g(x)}\sqrt{g(y)} O^i(x) O^j(y) \exp(-S_E)$$

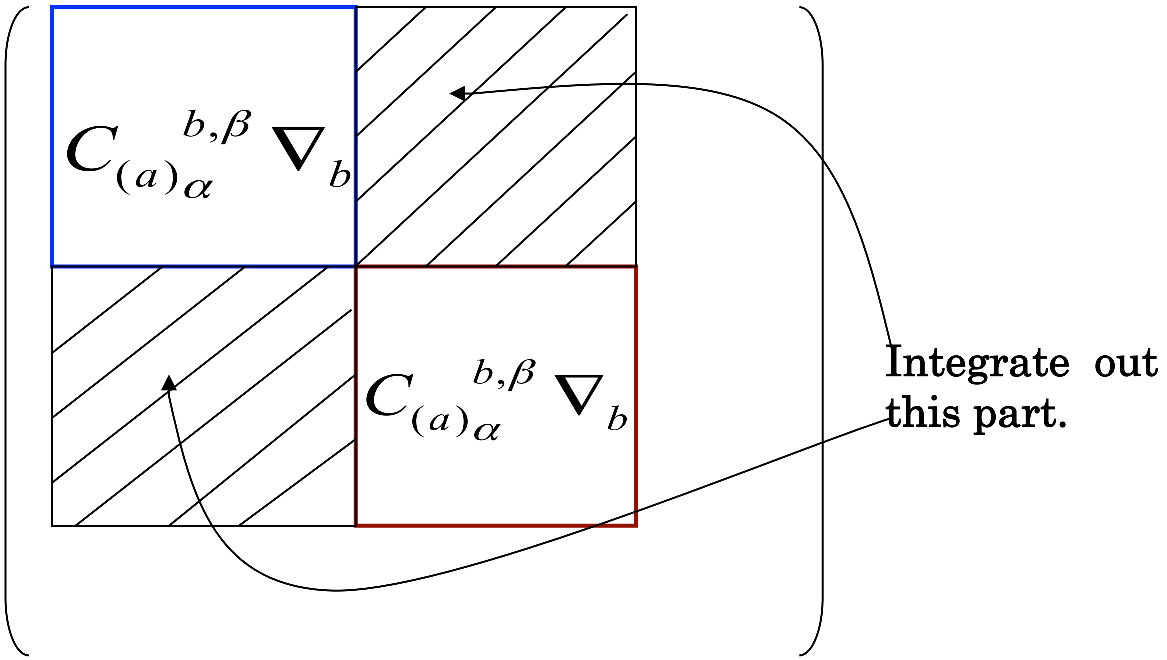
sum over wormholes

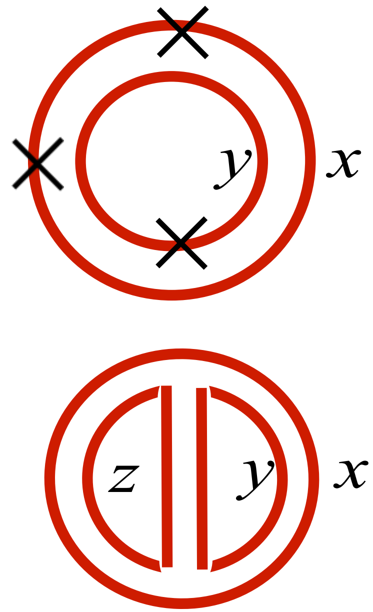
$$\exp\left(\frac{1}{2} c_{ij} e^{-2S_{wh}} \int d^4x d^4y \sqrt{g(x)}\sqrt{g(y)} O^i(x) O^j(y)\right)$$

In matrix model, wormhole-like fluctuations of string scale are expected to exist They need not be classical solutions.

bifurcated wormholes $\Rightarrow$ cubic terms, quartic terms, …

(2) integration of (off) diagonal blocks





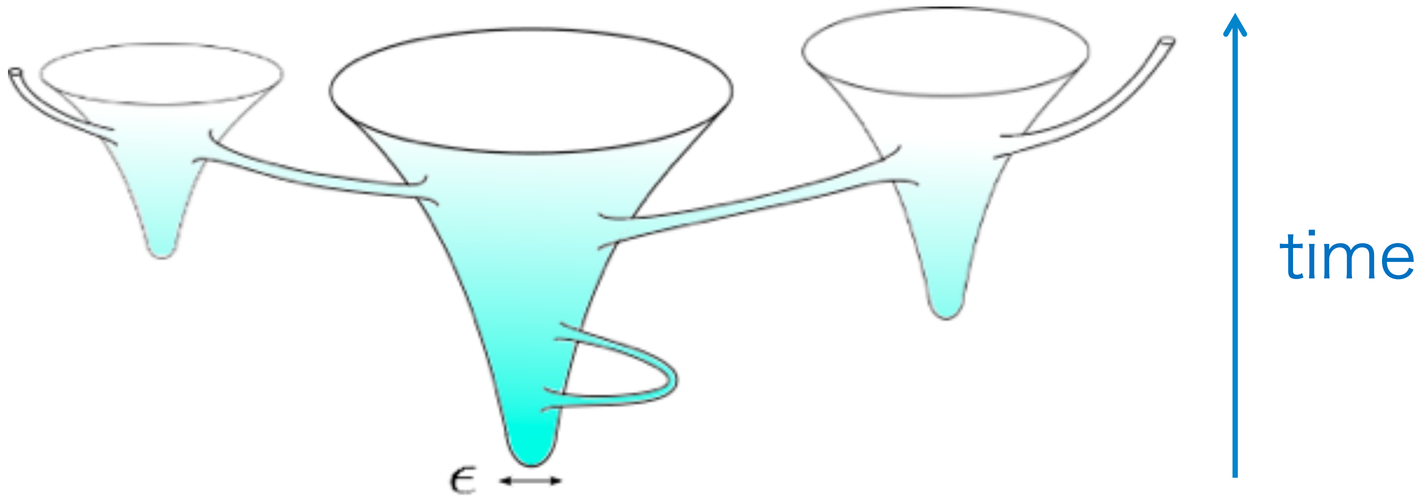
The path integral gives

$$S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

4. Wave function of the multiverse

When we consider the time evolution (the path integral) of the matrix model, a number of universes emerge from fluctuations, and then evolve as almost classical universes,



where the path integral is approximated by

$$Z = \int d\lambda\, \rho(\lambda) \exp\left(i \sum_i \lambda_i S_i \right),$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

[infrared cutoff (1)]

There appears an effective infrared cutoff for the size of universes.

At present, for the IIB matrix model, it is not clear whether we need to introduce an infrared cutoff by hand or not.

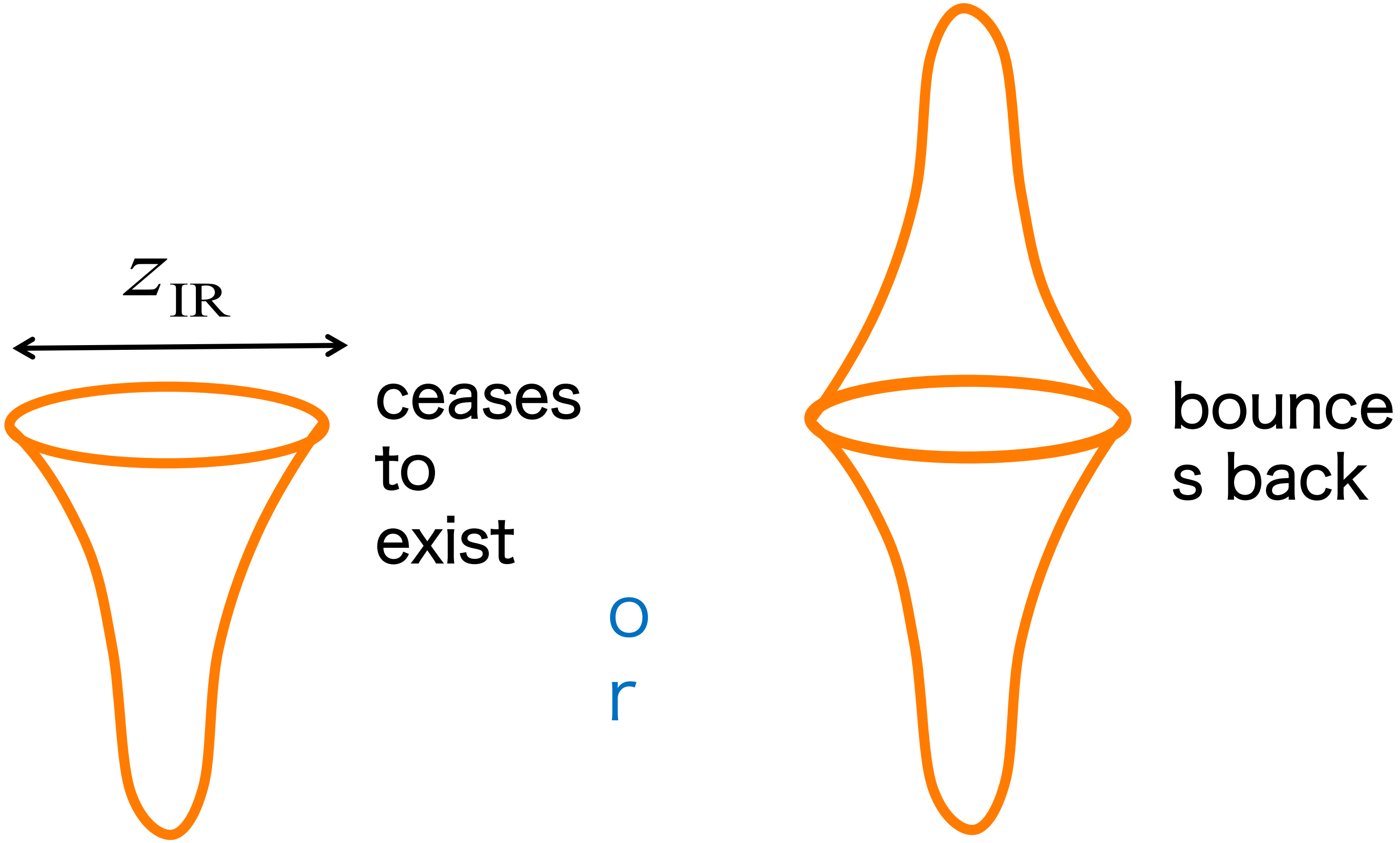
Naively, attractive forces among the eigenvalues of the matrices are canceled by SUSY, and we need to introduce an cutoff by hand.

However, if we take the fermion zero modes into account, then there appears a week attraction among the eigenvalues, and we can show the path integral converges in the Euclidean case.

For Lorentzian case, no definite answer is known.

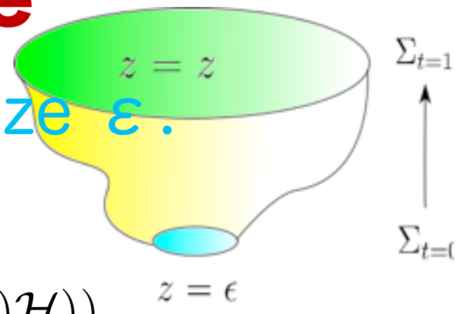
infrared cutoff (2)

We assume that there is an infrared cutoff for the size of universes.



wave function of a universe

Assume that a universe starts from a small size ε .



$$\langle z| e^{-i\hat{\mathrm{H}}}|\ \epsilon\rangle = \int_{z(0)=\epsilon,\ z(1)=z} [dp_z][dz][dN]\exp(i\int_{t=0}^{t=1} dt\ (p_z\dot{z} - N(t)\mathcal{H})).$$

$$N(t) := T$$

$$= \int_{-\infty}^{\infty} d\mathrm{T} \int_{z(0)=\epsilon,\ z(1)=z} [dp_z][dz]\exp\Big(i\int_{t=0}^{t=1} dt\ (p_z\dot{z} - \mathrm{T}\mathcal{H})\Big)$$

$$= \mathcal{C}\times\int_{-\infty}^{\infty} d\mathrm{T}\langle z|e^{-i\mathrm{T}\mathcal{H}}|\ \epsilon\rangle$$

$$= \mathcal{C}\times\langle z|\delta(\mathcal{H})|\epsilon\rangle$$

$$= \mathcal{C}\times\langle z|\delta(\mathcal{H})\Big(\int_{-\infty}^{\infty} dE|\phi_E\rangle\langle\phi_E|\Big)|\epsilon\rangle.$$

$$= \mathcal{C}\times\phi^*_{E=0}(\epsilon)\phi_{E=0}(z)$$

$$\langle\phi'_E|\phi_E\rangle = \delta(E-E'),$$

$$\mathcal{H}|\phi_E\rangle = E|\phi_E\rangle$$

The wave function of the universe that emerges from size ε is

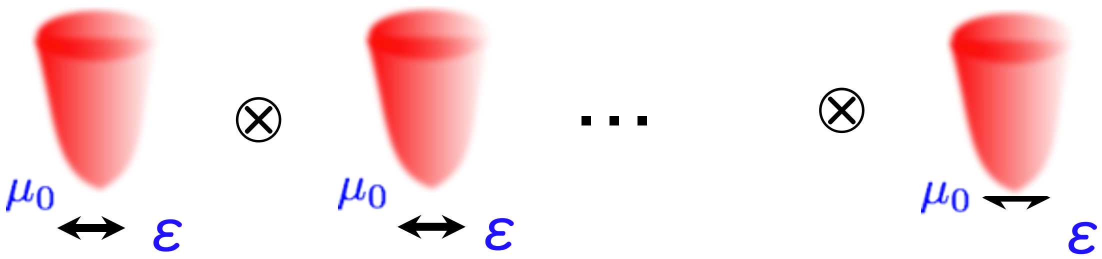
$$\mathcal{C}\times\phi^*_{E=0}(\epsilon)|\phi_{E=0}\rangle$$

Wave Function of N-verse

・The state of each universe is $\mathcal{C} \times \phi^*_{E=0}(\epsilon)|\phi_{E=0}\rangle$.

・Let μ_0 be the prob. amp. of a universe emerging from nothing
to the size ε .

・Then, the N-verse wave fn is given by the tensor product of N universes,



$$|\Phi_N\rangle = \left(\mu_0 \mathcal{C} \times \phi^*_{E=0}(\epsilon)\right)^N |\phi_{E=0}\rangle \cdots |\phi_{E=0}\rangle := \mu^N |\phi_{E=0}\rangle \cdots |\phi_{E=0}\rangle$$

Wave Function of the multiverse

The multiverse: the sate with indefinite number of universes

$$|\phi_{multi}\rangle = \sum_{N=0}^{\infty} |\Phi_N\rangle$$

$$\Phi_N(z_1, \cdots, z_N) = \int d\vec{\lambda}\ \mu^N \times \phi(z_1)\phi(z_2)\cdots\phi(z_N)\ |\vec{\lambda}\rangle$$

where $\qquad \phi(z) \equiv \langle z|\phi_{E=0}\rangle$

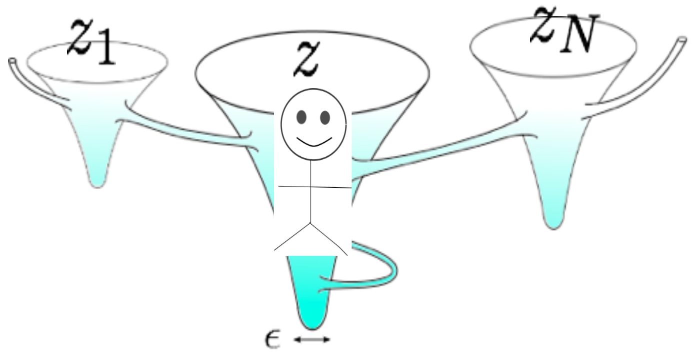
$\vec{\lambda}$: the set of the coupling constants

$$d\vec{\lambda} \equiv \prod_i d\lambda_i.$$

5. Naturalness and the big fix

Density Matrix

- Our universe : a subsystem of the multiverse.
- The density matrix of our is obtained by integrating out the other universes.



ours

$$
\begin{aligned}
\rho(z', z) &= \sum_{N=0}^{\infty} \int \frac{dz_i^N}{N!} \Phi^*_{N+1}(z', z_1, \cdots, z_N)\Phi_{N+1}(z, z_1, \cdots, z_N) \\
&= \sum_{N=0}^{\infty} \frac{1}{N!} \int_{-\infty}^{\infty} d\vec{\lambda}\ |\mu|^2 \phi(z')^* \phi(z) \times \left(\int dz'' |\mu\phi(z'')|^2 \right)^N \\
&= \int_{-\infty}^{\infty} d\vec{\lambda}\ |\mu|^2\ \phi(z')^* \phi(z) \times \exp\left(\int dz'' |\mu\phi(z'')|^2 \right),
\end{aligned}
$$

Recall: $\Phi_N(z_1, \cdots, z_N) = \int d\vec{\lambda}\ \mu^N \times \phi(z_1)\phi(z_2)\cdots\phi(z_N)\ |\vec{\lambda}\rangle$

The Big Fix

$$\rho(z', z) = \int_{-\infty}^{\infty} d\vec{\lambda}\, |\mu|^2\, \phi(z')^* \phi(z) \times \exp\left(\int dz''\, |\mu \phi(z'')|^2 \right)$$

For a given $\vec{\lambda}$, $\phi(z'')$ is in principle determined;

$(\frac{1}{2}\frac{d^2}{dz^2} - U(z))\sqrt{z}\phi(z) = 0$ with $U(z) = \frac{1}{z^{2/3}} - \Lambda - \frac{C_{matt}}{z} - \frac{C_{rad}}{z^{4/3}}$

Note: Λ, C_{matt}, C_{rad} depend on $\vec{\lambda}$

BIG FIX

$\vec{\lambda}$ **are dynamically fixed such that**
the exponent becomes maximum.

Which value of the CC is selected?

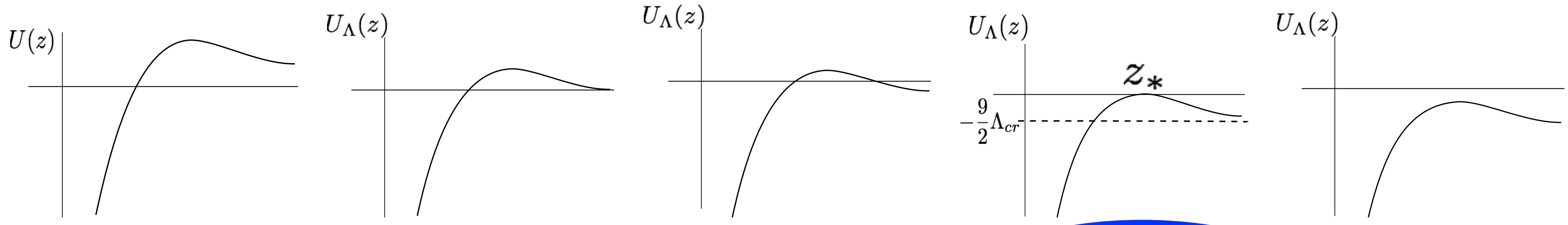
We want to know which Λ maximizes the exponent $\int_0^\infty dz'' |\mu\phi(z'')|^2$.

WKB sol $\phi(z) \sim \frac{1}{\sqrt{zk(z)}}$ with $k = \sqrt{-2U}$

$$U(z) = \frac{1}{z^{2/3}} - \Lambda - \cancel{\frac{C_{matt}}{z}} - \frac{C_{rad}}{z^{4/3}}$$

→ shallower potential is favored as long as no tunneling

As we vary Λ, the potential changes…



(a) $\Lambda < 0$ (b) $\Lambda = 0$ (c) $0 < \Lambda < \Lambda_{cr}$ (d) $\Lambda = \Lambda_{cr}$ (e) $\Lambda > \Lambda_{cr}$

tunneling suppressed

curv.=energy density

Λ_{cr} is given by solving $\begin{cases} U'(z_*, \Lambda_{cr}) = 0 \\ U(z_*, \Lambda_{cr}) = 0 \end{cases}$ → $\Lambda_{cr} \sim 1/C_{rad}$ (extremely small)

The other couplings (Big Fix)

$$\rho(z', z) = \int_{-\infty}^{\infty} d\vec{\lambda}\, |\mu|^2\, \phi(z')^* \phi(z) \times \exp\left(\int dz'' |\mu\phi(z'')|^2\right)$$

The exponent is divergent, and regulated by the IR cutoff :

$$\int_0^{\infty} dz'' |\mu\phi(z'')|^2 \sim \int_0^{\infty \to z_{IR}} dz \frac{1}{z\sqrt{\Lambda_{cr}}} \sim \frac{1}{\sqrt{\Lambda_{cr}}} \log z_{IR}$$

$$\Lambda_{cr} \sim 1/C_{rad}$$

$$\sim \sqrt{C_{rad}} \log z_{IR}$$

BIG FIX 2

$\vec{\lambda}$ is determined in such a way that $C_{rad}(\vec{\lambda})$ is maximized

and the CC is given by $\Lambda = \Lambda_{cr} \simeq 1/C_{rad}(\vec{\lambda})$.

Meaning of the Enhancement

$$\rho(z', z) \;=\; \int_{-\infty}^{\infty} d\vec{\lambda}\; |\mu|^2 \;\phi(z')^*\phi(z) \times \exp\left(\int dz'' |\mu\phi(z'')|^2\right)$$

From the WKB sol., the exponent can be written as

$$\int dz\; |\phi(z)|^2 = \int_{\epsilon}^{z_{IR}} dz\; \frac{1}{zk(z)}$$

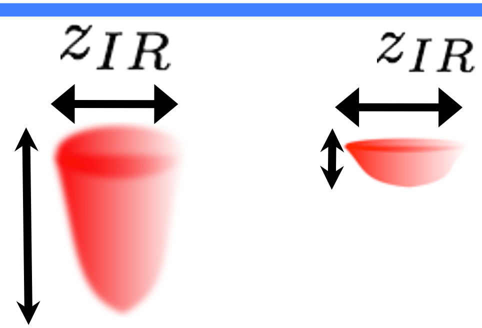
$$H \sim z(-\frac{1}{2}p_z^2 + \cdots)$$

$$\dot{z} = \frac{\partial H}{\partial p_z} \sim -zp_z$$

$$= \underline{\int_{\epsilon}^{z_{IR}} \frac{dz}{\dot{z}}}$$

Equal to lifetime of universe

small Λ
long lifetime



large Λ
short lifetime

→the probability finding small Λ is enhanced.

Example of the big fix

Which couplings are dynamical?

$$\rho(z', z) = \int_{-\infty}^{\infty} d\vec{\lambda} |\mu|^2 \phi(z')^* \phi(z) \times \exp\left(\int dz'' |\mu\phi(z'')|^2 \right)$$

We focus on the Higgs potential.

$$V = -\mu^2 |\Phi_h|^2 + \frac{\lambda_h}{4} |\Phi_h|^4 \qquad m_h = \sqrt{2}\mu = \frac{\lambda_h}{2} v_h$$

We treat the quartic coupling λ_h as dynamical,
assuming that the other parameters in the SM are fixed at the experimentally observed values.

In particular, v_h is fixed at 246 GeV.

We consider what value of λ_h makes the radiation maximum

Higgs mass

If proton decays, the radiation energy in the future is dominated by the radiation produced by its decay.

Thus, λ_h is determined in such a way that the proton number N_P in the universe is maximized.

assuming the leptogenesis

$\lambda_h \searrow$ ⇒ the symmetric phase lasts longer ⇒ $N_P \nearrow$

Most baryons are produced in the symmetric phase in the leptogenesis.

λ_h has a lower bound from the stability of pot. $\lambda_h|_{planck} = 0$

⇒ $\boxed{m_{\text{Higgs}} \sim 140 \pm 20\,\text{GeV}.}$

6. Summary

Summary

In the quantum gravity or matrix model, the multiverse naturally appears, and it becomes a superposition of satates with various values of the coupling constants.

The coupling constants are fixed is such a way that the lifetime of the universe is maximized.

For example the cosmological constant in the far futre is predicted to be very small: $\Lambda(z \to \infty) \sim 0$.

The Higgs mass is predicted at its lower bound provided by the stability of the potential.

Future work

Other operators?
Comparison of different dimensional space-time?
Generalization to the landscape?