Solving Naturalness Problem in Matrix Model

Hikaru KAWAI

2012/ 02/ 20
at Rikkyo Univ.

In collaboration with
G. Ishiki, Y. Asano and T. Okada.
We will discuss the naturalness problem in the context of the IIB matrix model.

Contents

1. IIB matrix model
2. Emergence of space-time
3. Low energy effective theory
4. Wave function of the multiverse
5. Naturalness and the big fix
1. IIB matrix model
IIB Matrix Model

\[ S = - \frac{1}{g^2} \, Tr \left( \frac{1}{4} [A^\mu, A^\nu]^2 + \frac{1}{2} \bar{\Psi} \gamma^\mu [A^\mu, \Psi] \right) \]

A candidate of the constructive definition of string theory.

Evidences

(1) World sheet regularization

Green-Schwartz action in the Schild Gauge

\[ S = \int d^2 \xi \left( \frac{1}{4} \{X^\mu, X^\nu\}^2 + \frac{1}{2} \bar{\Psi} \gamma^\mu \{X^\mu, \Psi\} \right) \]

Regularization by matrix \( \{ , \} \rightarrow [ , ] \)

\[ S = - \frac{1}{g^2} \, Tr \left( \frac{1}{4} [A^\mu, A^\nu]^2 + \frac{1}{2} \bar{\Psi} \gamma^\mu [A^\mu, \Psi] \right) \]
Multi string states are naturally described in the large-N limit.
(2) Loop equation and string field

Wilson loop = string field
\[ w(k_\mu (\cdot)) = Tr(P \exp(i \int d\sigma k_\mu (\sigma) A^\mu + \text{fermion})) \]
\[ \Leftrightarrow \text{creation annihilation operator of } k_\mu (\cdot) > \]

loop equation \rightarrow light-cone string field

This can be shown with some assumptions.

\[ x^+ = x^0 + x^9 = \text{const.} \]
The loop integral gives the exchange of graviton and dilaton.

\[
S_{\text{eff}} = -\frac{1}{(x^{(1)} - x^{(2)})^8} \left\{ \text{const} \cdot \text{tr} \left( f^{(1)}_{\mu \lambda} f^{(1)}_{\nu \lambda} \right) \text{tr} \left( f^{(2)}_{\mu \lambda} f^{(2)}_{\nu \lambda} \right) \\
- \text{const} \cdot \text{tr} \left( f^{(1)}_{\mu \nu} f^{(1)}_{\mu \nu} \right) \text{tr} \left( f^{(2)}_{\lambda \rho} f^{(2)}_{\lambda \rho} \right) + \cdots \right\}
\]
2. Emergence of space-time
Various possibilities for the emergence of space-time

\[ S = -\frac{1}{g^2} \text{Tr} \left( \frac{1}{4} [A^\mu, A^\nu]^2 + \frac{1}{2} \bar{\Psi} \gamma^\mu [A^\mu, \Psi] \right) \]

(1) \( A_\mu \) as the space-time coordinates

mutually commuting \( A_\mu \) \( \Rightarrow \) space-time

ex. flat space

\( A^{\mu} = x^{\mu} \) uniformly distributed eigenvalues

\( \Rightarrow \) vacuum (flat space-time)

non-commutative lumps

\( \Rightarrow \) excitations around the vacuum
(2) $A_\mu$ as non-commutative space-time

non-commutative $A_\mu \Rightarrow$ NC space-time
fluctuations $\Rightarrow$ local fields in NC space

ex. flat non-commutative space

$$A^\mu = x'^\mu, \quad [x'^\mu, x'^\nu] = i C'^{\mu\nu}$$

$\Rightarrow$ flat non-commutative space

In non-commutative space, $A_\mu$ can be regarded as both coordinates and momenta.

$$P_\mu = C_{\mu\nu} x^\nu$$
(3) $A_\mu$ as momenta

$A_\mu$ can be regarded as a covariant derivative on any manifold with less than ten dimensions.

$$\left(A_a \varphi\right)_\alpha = C_{(a)\alpha}^{b,\beta} \nabla_b \varphi_\beta$$

$\varphi_\alpha$: regular representation field on manifold $M$

$C_{(a)\alpha}^{b,\beta}$, $(a = 1, \ldots, D)$: the Clebsh-Gordan coefficients

$V_{\text{vector}} \otimes V_r \cong V_r \oplus L \oplus V_r$  $r$: regular representation

**ex. derivative on flat space**

$$\left(A_a\right)_\alpha^\beta = C_{(a)\alpha}^{b,\beta} \partial_b$$
3. Low energy effective theory
Low energy effective theory is obtained by taking the fluctuations into account.

Because of the symmetry, it should be

$$S_{\text{eff}} = \int d^D x \sqrt{g} (\kappa R + \Lambda + \text{gauge} + \text{matter} + \ldots)$$

Is that all?

Usually, action is additive.

$$S = S_0 + S_{\text{int}},$$

$$S_0 = \int d^4 x \left( \frac{-1}{4} F_{\mu \nu}^2 + \bar{\psi} \gamma^\mu \partial_\mu \psi \right),$$

$$S_{\text{int}} = \int d^4 x \left( e A_\mu \bar{\psi} \gamma^\mu \psi \right).$$

why not

$$S = S_0 S_{\text{int}}.$$  \text{(Sugawara \sim 1980)}
If it is true, the coupling constant is determined by the history of the universe:

\[ S_{\text{eff}} = \langle S_{\text{int}} \rangle S_0 + \langle S_0 \rangle S_{\text{int}} \]

Actually, in quantum gravity or matrix model, there are some mechanisms that the low energy effective theory becomes

\[ S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots, \]

\[ S_i = \int d^D x \sqrt{g(x)} O_i(x). \]

\[ O_i(x) : \text{local operators} \]

Then the path integral is given by

\[ Z = \int d\lambda \rho(\lambda) \exp \left( i \sum_i \lambda_i S_i \right). \]
(1) Space-time wormhole and baby universe

action (Euclidean)

\[ \int \mathcal{D}g_{\mu\nu} \exp(-S_E) \]

A wormhole induces a local operator at each end point

\[ \int \mathcal{D}g \frac{1}{2} c_{ij} e^{-2S_{wh}} \int d^4x d^4y \sqrt{g(x)} \sqrt{g(y)} O^i(x) O^j(y) \exp(-S_E) \]

sum over wormholes

\[ \exp \left( \frac{1}{2} c_{ij} e^{-2S_{wh}} \int d^4x d^4y \sqrt{g(x)} \sqrt{g(y)} O^i(x) O^j(y) \right) \]

In matrix model, wormhole-like fluctuations of string scale are expected to exist. They need not be classical solutions.

bifurcated wormholes ⇒ cubic terms, quartic terms, …
The path integral gives

\[ S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots, \]

\[ S_i = \int d^D x \sqrt{g(x)} O_i(x). \]
4. Wave function of the multiverse
When we consider the time evolution (the path integral) of the matrix model, a number of universes emerge from fluctuations, and then evolve as almost classical universes,

\[
Z = \int d\lambda \rho(\lambda) \exp\left( i \sum \lambda_i S_i \right),
\]

where the path integral is approximated by

\[
S_i = \int d^D x \sqrt{g(x)} O_i(x).
\]
infrared cutoff
(1)

There appears an effective infrared cutoff for the size of universes.

At present, for the IIB matrix model, it is not clear whether we need to introduce an infrared cutoff by hand or not.

Naively, attractive forces among the eigenvalues of the matrices are canceled by SUSY, and we need to introduce an cutoff by hand.

However, if we take the fermion zero modes into account, then there appears a week attraction among the eigenvalues, and we can show the path integral converges in the Euclidean case.

For Lorentzian case, no definite answer is known.
We assume that there is an infrared cutoff for the size of universes.
wave function of a universe

Assume that a universe starts from a small size $\epsilon$.

$$
\langle z|e^{-i\hat{H}}|\epsilon\rangle = \int_{z(0)=\epsilon, \ z(1)=z} [dp_z][dz][dN] \exp(i \int_{t=0}^{t=1} dt \ (p_z \dot{z} - N(t)\mathcal{H})).$$

$$= \int_{-\infty}^{\infty} dT \int_{z(0)=\epsilon, \ z(1)=z} [dp_z][dz] \exp\left(i \int_{t=0}^{t=1} dt \ (p_z \dot{z} - T\mathcal{H})\right)$$

$$= C \times \int_{-\infty}^{\infty} dT \langle z|e^{-iT\mathcal{H}}|\epsilon\rangle$$

$$= C \times \langle z|\delta(\mathcal{H})|\epsilon\rangle$$

$$= C \times \langle z|\delta(\mathcal{H})\left(\int_{-\infty}^{\infty} dE|\phi_E\rangle\langle\phi_E|\right)|\epsilon\rangle.$$  

$$= C \times \phi_{E=0}^*(\epsilon)\phi_{E=0}(z)$$

The wave function of the universe that emerges from size $\epsilon$ is

$$C \times \phi_{E=0}^*(\epsilon)\phi_{E=0}$$
Wave Function of N-verse

- The state of each universe is \( \phi_{E=0}^{\star}(\epsilon)\ket{\phi_{E=0}} \).
- Let \( \mu_0 \) be the prob. amp. of a universe emerging from nothing to the size \( \epsilon \).
- Then, the N-verse wave fn is given by the tensor product of N universes,

\[
|\Phi_N\rangle = \left( \mu_0 C \times \phi_{E=0}^{\star}(\epsilon) \right)^N |\phi_{E=0}\rangle \cdots |\phi_{E=0}\rangle := \mu^N |\phi_{E=0}\rangle \cdots |\phi_{E=0}\rangle
\]
Wave Function of the multiverse

The multiverse: the state with **indefinite** number of universes

\[
|\phi_{\text{multi}}\rangle = \sum_{N=0}^{\infty} |\Phi_N\rangle
\]

\[
\Phi_N(z_1, \cdots, z_N) = \int d\vec{\lambda} \mu^N \times \phi(z_1)\phi(z_2) \cdots \phi(z_N) |\vec{\lambda}\rangle
\]

where \( \phi(z) \equiv \langle z |\phi_{E=0}\rangle \)

\(\vec{\lambda} \): the set of the coupling constants

\(d\vec{\lambda} \equiv \prod_i d\lambda_i.\)
5. Naturalness and the big fix
Density Matrix

- Our universe: a subsystem of the multiverse.
- The density matrix of our is obtained by integrating out the other universes.

\[
\rho(z', z) = \sum_{N=0}^{\infty} \left( \frac{dz_i^N}{N!} \right) \Phi_{N+1}^*(z', z_1, \ldots, z_N) \Phi_{N+1}(z, z_1, \ldots, z_N)
\]

\[
= \sum_{N=0}^{\infty} \frac{1}{N!} \int_{-\infty}^{\infty} d\tilde{\lambda} \left( |\mu|^2 \phi(z')^* \phi(z) \times \left( \int dz'' |\mu\phi(z'')|^2 \right)^N \right)
\]

\[
= \int_{-\infty}^{\infty} d\tilde{\lambda} \left( |\mu|^2 \phi(z')^* \phi(z) \times \exp\left( \int dz'' |\mu\phi(z'')|^2 \right) \right)
\]

Recall: \[\Phi_N(z_1, \ldots, z_N) = \int d\tilde{\lambda} \mu^N \times \phi(z_1)\phi(z_2) \cdots \phi(z_N) |\tilde{\lambda}\rangle\]
The Big Fix

\[
\rho(z', z) = \int_{-\infty}^{\infty} d\vec{\lambda} \ |\mu|^2 \ \phi(z')^* \phi(z) \times \exp \left( \int dz'' |\mu \phi(z'')|^2 \right)
\]

For a given \( \vec{\lambda} \), \( \phi(z'') \) is in principle determined;

\[
\left( \frac{1}{2} \frac{d^2}{dz^2} - U(z) \right) \sqrt{z} \phi(z) = 0 \quad \text{with} \quad U(z) = \frac{1}{z^{2/3}} - \Lambda - \frac{C_{\text{mat}}}{z} - \frac{C_{\text{rad}}}{z^{4/3}}
\]

Note: \( \Lambda, C_{\text{mat}}, C_{\text{rad}} \) depend on \( \vec{\lambda} \)

**BIG FIX**

\( \vec{\lambda} \) are dynamically fixed such that the exponent becomes maximum.
Which value of the CC is selected?

We want to know which $\Lambda$ maximizes the exponent $\int_0^\infty dz'' |\mu \phi(z'')|^2$.

WKB sol $\phi(z) \sim \frac{1}{\sqrt{zk(z)}}$: with $k = \sqrt{-2U}$

$U(z) = \frac{1}{z^{2/3}} - \Lambda - \frac{C_{\text{ matt}}}{z} - \frac{C_{\text{ rad}}}{z^{4/3}}$

shallower potential is favored as long as no tunneling

As we vary $\Lambda$, the potential changes···

![Potential changes](image)

(a) $\Lambda < 0$  (b) $\Lambda = 0$  (c) $0 < \Lambda < \Lambda_{cr}$

(d) $\Lambda = \Lambda_{cr}$  (e) $\Lambda > \Lambda_{cr}$

tunneling suppressed  curv.=energy density

$\Lambda_{cr}$ is given by solving $\begin{align*}
U'(z_*, \Lambda_{cr}) &= 0 \\
U(z_*, \Lambda_{cr}) &= 0
\end{align*}$

$\Lambda_{cr} \sim \frac{1}{C_{\text{ rad}}}$ (extremely small)
The other couplings (Big Fix)

\[ \rho(z', z) = \int_{-\infty}^{\infty} d\lambda |\mu|^2 \phi(z')^* \phi(z) \times \exp\left(\int dz'' |\mu\phi(z'')|^2\right) \]

The exponent is divergent, and regulated by the IR cutoff:

\[ \int_{0}^{\infty} dz'' |\mu\phi(z'')|^2 \sim \int_{0}^{\infty} dz \frac{1}{z\sqrt{\Lambda_{cr}}} \sim \frac{1}{\sqrt{\Lambda_{cr}}} \log z_{IR} \]

\[ \Lambda_{cr} \sim 1/C_{rad} \]

\[ \sim \sqrt{C_{rad} \log z_{IR}} \]

**BIG FIX 2**

\[ \vec{\lambda} \text{ is determined in such a way that } C_{rad}(\vec{\lambda}) \text{ is maximized and the CC is given by } \Lambda = \Lambda_{cr} \sim 1/C_{rad}(\vec{\lambda}) \].
Meaning of the Enhancement

\[ \rho(z', z) = \int_{-\infty}^{\infty} d\lambda |\mu|^2 \phi(z')^* \phi(z) \times \exp\left( \int dz'' |\mu\phi(z'')|^2 \right) \]

From the WKB sol., the exponent can be written as

\[ \int dz |\phi(z)|^2 = \int_{\epsilon}^{z_{IR}} dz \frac{1}{zk(z)} = \int_{\epsilon}^{z_{IR}} dz \frac{1}{\dot{z}} \]

\[ H \sim z(-\frac{1}{2}p_z^2 + \cdots) \]

\[ \dot{z} = \frac{\partial H}{\partial p_z} \sim -zp_z \]

Equal to lifetime of universe

\[ z_{IR} \]

small \( \Lambda \) long lifetime

large \( \Lambda \) short lifetime

→ the probability finding small \( \Lambda \) is enhanced.
Example of the big fix

Which couplings are dynamical?

\[
\rho(z', z) = \int_{-\infty}^{\infty} d\lambda |\mu|^2 \phi(z')^* \phi(z) \times \exp\left(\int dz'' |\mu \phi(z'')|^2\right)
\]

We focus on the Higgs potential.

\[
V = -\mu^2 |\Phi_h|^2 + \frac{\lambda_h}{4} |\Phi_h|^4 \quad m_h = \sqrt{2}\mu = \frac{\lambda_h}{2} v_h
\]

We treat the quartic coupling \( \lambda_h \) as dynamical, assuming that the other parameters in the SM are fixed at the experimentally observed values.

In particular, \( v_h \) is fixed at 246 GeV.

We consider what value of \( \lambda_h \) makes the radiation maximum
Higgs mass

If proton decays, the radiation energy in the future is dominated by the radiation produced by its decay.

Thus, $\lambda_h$ is determined in such a way that the proton number $N_P$ in the universe is maximized, assuming the leptogenesis:

$\lambda_h \downarrow$ the symmetric phase lasts longer $\rightarrow N_P \uparrow$

Most baryons are produced in the symmetric phase in the leptogenesis.

$\lambda_h$ has a lower bound from the stability of $\partial \lambda_h |_{\text{planck}} = 0$

$m_{\text{Higgs}} \sim 140 \pm 20 \text{GeV}$. 
6. Summary
Summary

In the quantum gravity or matrix model, the multiverse naturally appears, and it becomes a superposition of states with various values of the coupling constants.

The coupling constants are fixed is such a way that the lifetime of the universe is maximized.

For example the cosmological constant in the far future is predicted to be very small $\sim 0$.

The Higgs mass is predicted at its lower bound provided by the stability of the potential.

Future work

Other operators?
Comparison of different dimensional space-time?
Generalization to the landscape?