Wilson loops and D-branes in AdS/CFT correspondence

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Based on
hep-th/0601089,hep-th/0603208

Overview

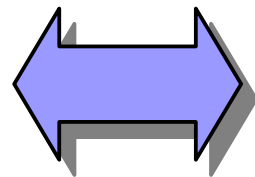AdS$_5$ / CFT$_4$

$=$

AdS5 x S5
IIB SUGRA
or String

$\cup$

Strings,
Branes,
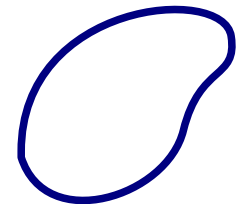Geometries...

$\Longleftrightarrow$
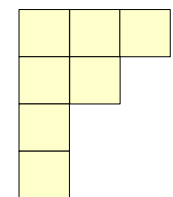
$=$

4dim N=4
Super YM theory

$\cup$

Wilson loop operators

test particle

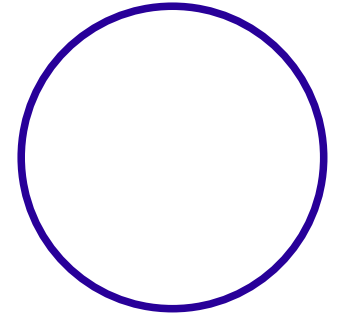1. trajectory

2. coupling

Main result
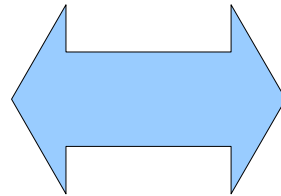
Anti-symmetric representation

Circular loop

AdS5

AdS2 x S4 D5-brane
Electric flux
On-shell action

CFT4

VEV of Circular Wilson loop
of anti-symmetric rep

Calculated using
 the Gaussian matrix model

Completely agree

Field Theory Side

N=4 Super Yang-Mills Theory

Field Contents

- Vector
- Spinors
- Scalars

$$A_\mu, \mu = 0, 1, 2, 3$$

ψ 16 real components

$$\phi_i, i = 4, \cdots, 9$$

Each field is an N x N Hermitian Matrix

Action

$$S_{YM} = \frac{N}{\lambda} \int d^4 x \, \mathrm{tr}\left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \cdots \right]$$
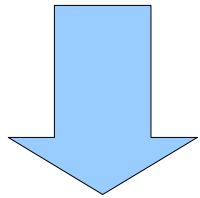
$$\lambda = g_{YM}^2 N \qquad : \text{'t Hooft coupling}$$

½ BPS Wilson loop

Straight Wilson loop

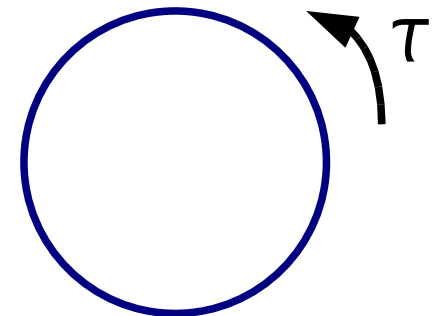$$Tr_R \left[P \exp \int dx^0 \, i \left[A_0 + \phi_4 \right] \right]$$

↑ time

Wick rotation, conformal transformation

Circular Wilson loop

$$W_R := Tr_R \left[P \exp \oint d\tau \left[i A_\mu \dot{x}^\mu(\tau) + \phi_4 |\dot{x}(\tau)| \right] \right]$$

τ

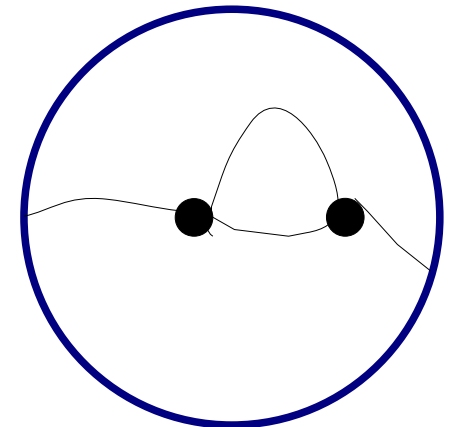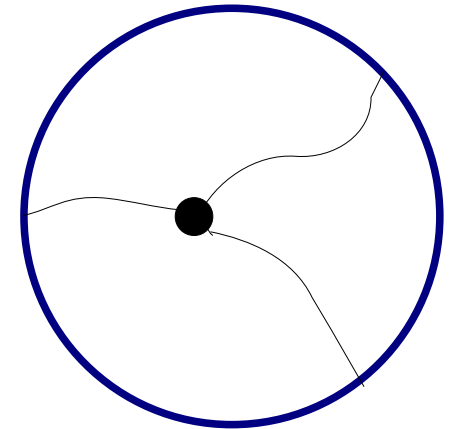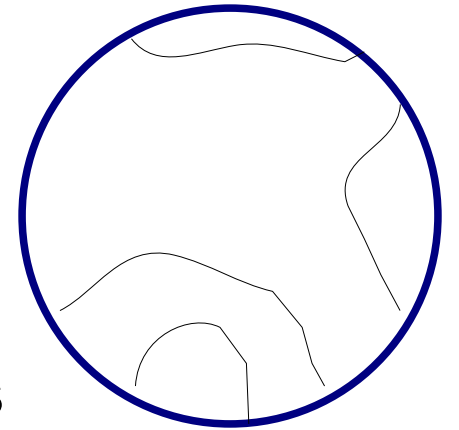# Calculation by perturbation $\langle W_R \rangle$

[Erickson, Semenoff, Zarembo]   [Drukker, Gross]

Conjecture: The diagrams with internal vertices do not contribute (cancel to each other).

"Gaussian"

Fact: The free propagator from edge to edge is independent of the position of the end point.

"Reduce to 0 dimensional theory"

Gaussian matrix model

M : N x N Hermitian matrix

$f(M)$:gauge invariant function of M

$$\langle f(M)\rangle_{mm} := \frac{1}{Z}\int dM\ f(M)\exp\left(\frac{-2N}{\lambda}\operatorname{tr}M^2\right)$$

Consequence of the conjecture

$$\boxed{\langle W_R\rangle_{YM} = \langle \operatorname{Tr}_R e^M\rangle_{mm}}$$
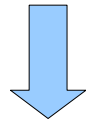
Diagonalize

Measure

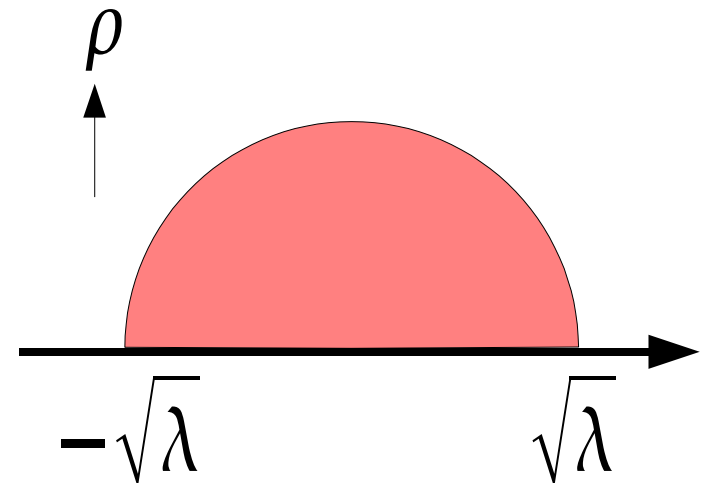$$M = \begin{pmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_N \end{pmatrix}$$

Repulsive force

Maximize

$$Z = \int \prod dm_j \exp\left(\frac{-2N}{\lambda}\sum m_j^2 + 2\sum_{i<j}\log|m_i - m_j|\right)$$

Eigen value density, Large N
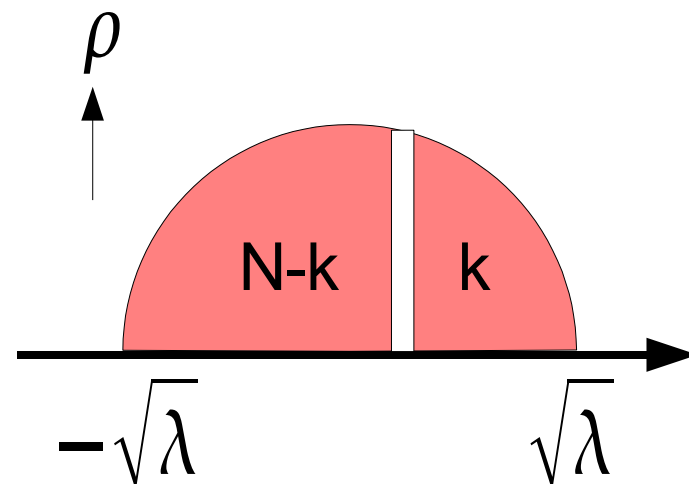
$$\rho(x) = \frac{2N}{\pi\lambda}\sqrt{\lambda - x^2}$$

Anti-symmetric representation

$$\left\langle W_{\substack{ \\ }}^{}\right\rangle_k {}_{YM} = \left\langle \mathrm{Tr}_{\substack{ \\ }}^{}{}_k e^M \right\rangle_{mm}$$

$$= \int \prod dm_j \exp\left(\frac{-2\mathrm{N}}{\lambda}\sum m_j^2 + 2\sum_{i<j}\log|m_i - m_j| + \sum_{j=1}^{k} m_j\right)$$

External force

$$\simeq \exp\left(\sum_{j=1}^{k} m_k(classical)\right)$$

ρ

N-k   k

$-\sqrt{\lambda}$ $\sqrt{\lambda}$

Final result in the field theory side

$$k = \frac{2N}{\pi}\left(\frac{1}{2}\theta_k - \frac{1}{4}\sin 2\theta_k\right)$$

$$\left\langle W \right\}_k \right\rangle_{YM} = \exp\left(\sqrt{\lambda}\,\frac{2N}{3\pi}\sin^3\theta_k\right)$$

$$= \exp\left(k\sqrt{\lambda}\left(1 - \frac{3}{10}\left(\frac{3\pi k}{2N}\right)^{2/3} - \cdots\right)\right)$$

AdS Side

Fundamental strings
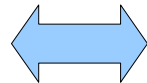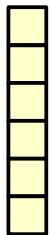
[Rey,Yee],[Maldacena]

$$\langle W_R \rangle_{AdS} \simeq \exp(-S_{on-shell})$$
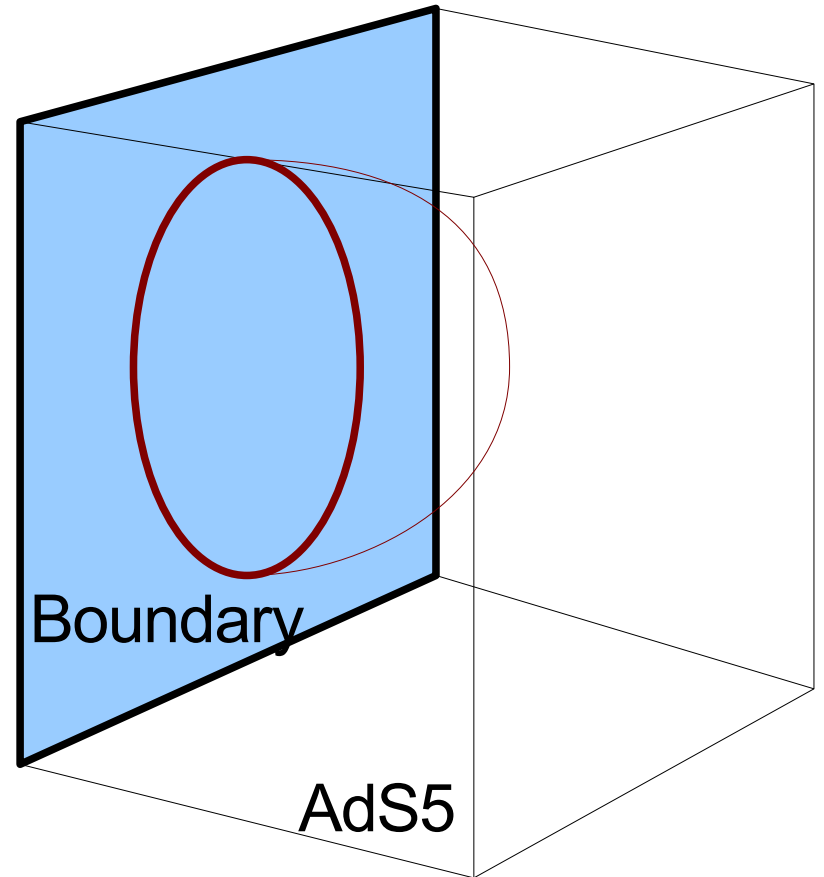
$$\simeq \exp(k\sqrt{\lambda})$$

Number of boxes

We cannot see the detail shape
of the Young diagram

Boundary

AdS5

We propose

D5-brane with electric flux

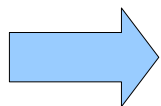# Why we guess D5 = ▯ ? : Bubbling geometry
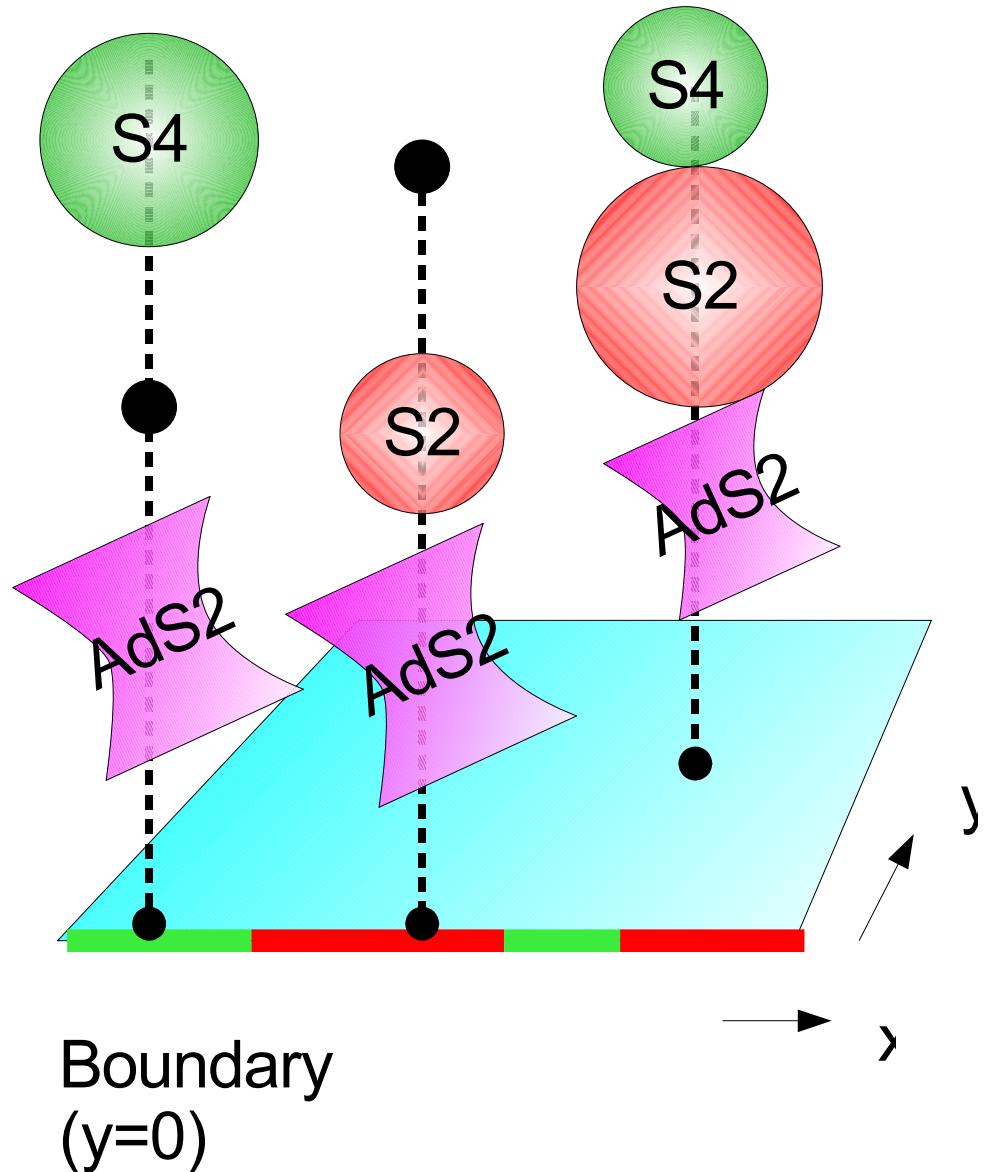
Supergravity picture
for Wilson loop
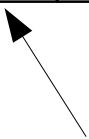
[Yamaguchi], [Lunin]

AdS5 x S5

AdS2 x S4 shaped Object

D5-brane

S4

S4

S2

S2

AdS2

AdS2

AdS2

AdS2

Boundary
(y=0)

Why we guess D5 = ▯ ? : Brane picture

N D3-branes, 1 D5-branes, k F-strings
in flat spacetime

\	0	1	2	3	4	5	6	7	8	9
D3	O	O	O	O						
D5	O					O	O	O	O	O
F1	O			O						

NSR formulation of F1

8 Dirichlet-Neumann directions

F-string ground state is a fermion
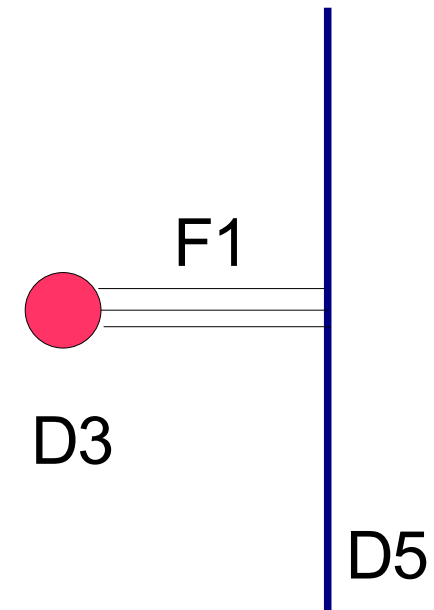
Level	R	NS
...	...	...
2	O	O
1	O	X
0	X	X

The only degeneracy of the ground states
is the Chern-Paton indeces at the D3 end.

Chern-Paton indeces at the D3 ends
are anti-symmetrized.

k-th anti-symmetric
representation

F1

D3

D5
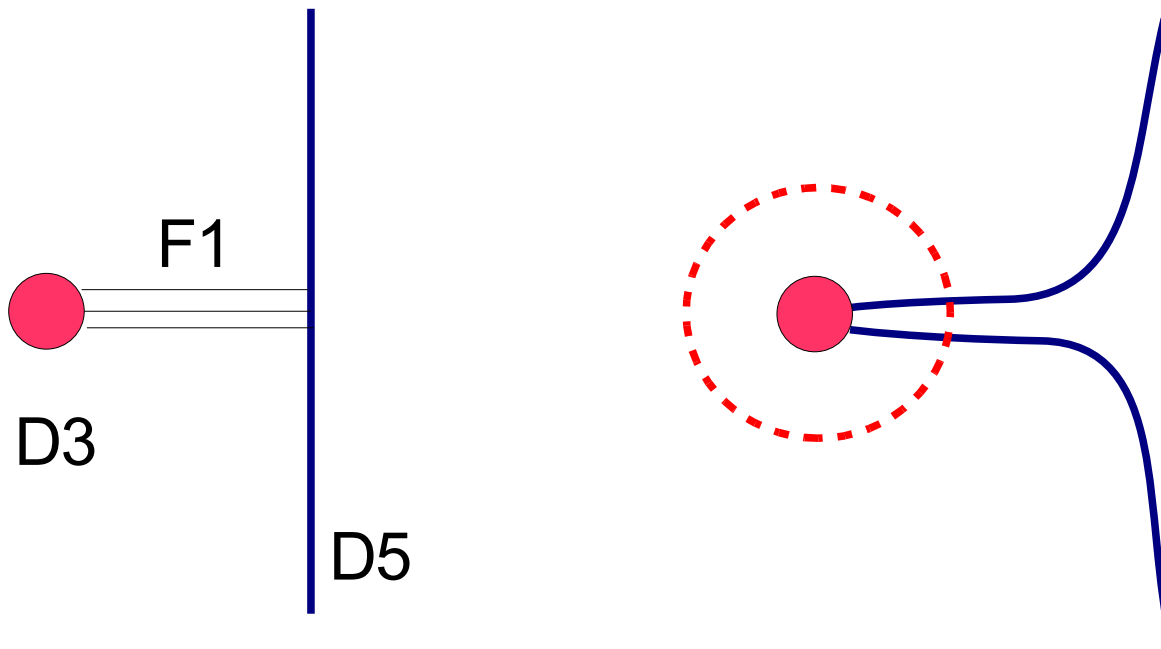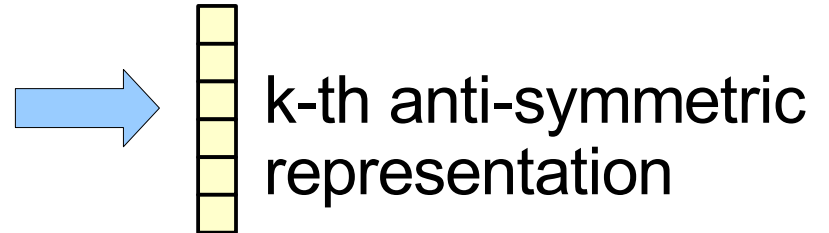
N D3-branes   sugra solution, near horizon   AdS5 x S5
F1+D5-branes   spike solution of DBI   AdS2 x S4

Calculation using D5-brane

Action of D5-brane

$$S_{D5} = T_5 \int d^6\xi \sqrt{det(G+F)} - iT_5 \int F \wedge C_4$$

Electric flux

$$F = \cos\theta_k \, vol(AdS2)$$

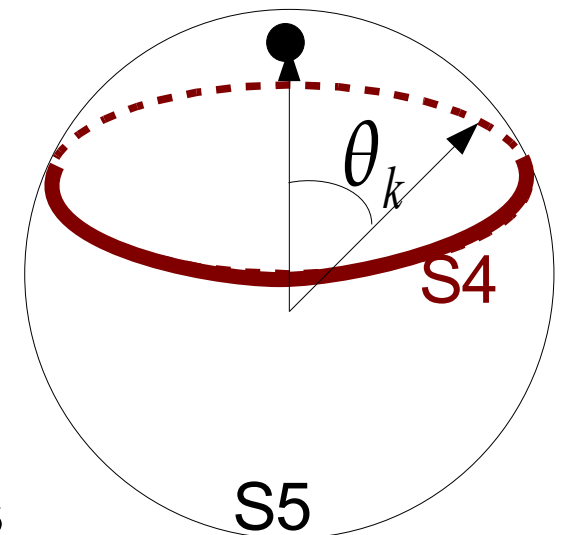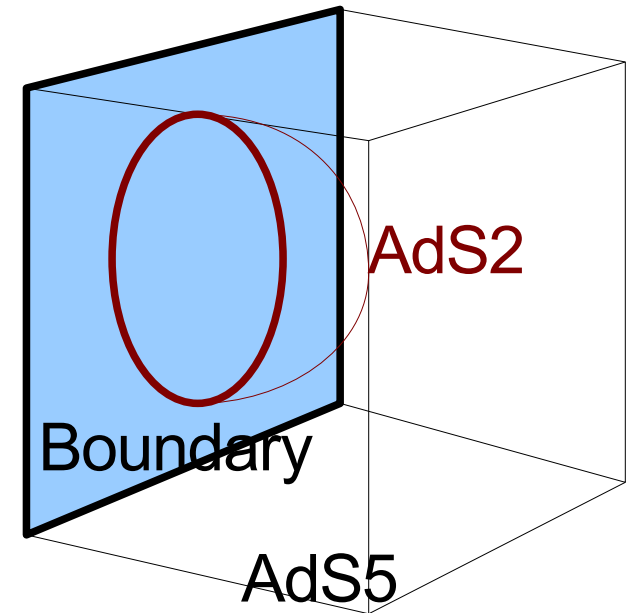String charge

$$k = \frac{\delta S_{D5}}{\delta B_{01}} = \frac{2N}{\pi}\left(\frac{1}{2}\theta_k - \frac{1}{4}\sin 2\theta_k\right)$$

VEV of Wilson loop

$$\left\langle W \left.\begin{matrix}\\\\\\\\\end{matrix}\right\}_k \right\rangle_{AdS} = \exp\left(-S_{on-shell}\right)$$

Need to include some boundary terms

AdS2

Boundary

AdS5

θ_k

S4

S5

Final result in the AdS side

$$\langle W_{\substack{\Big\} \, k}} \rangle_{AdS} = \exp\left(\sqrt{\lambda}\,\frac{2N}{3\pi}\sin^3\theta_k\right)$$

$$k = \frac{2N}{\pi}\left(\frac{1}{2}\theta_k - \frac{1}{4}\sin 2\theta_k\right)$$

Summary

We calculated the expectation value of the Wilson loop of k-th anti-symmetric tensor representation in the field theory and in the AdS2 x S4 D5-brane picture. They agree including some non-trivial 1/N corrections.

$$k = \frac{2N}{\pi} \left(\frac{1}{2} \theta_k - \frac{1}{4} \sin 2\theta_k \right)$$

$$\langle W_{\left. \begin{array}{c} \\ \\ \\ \end{array} \right\} k} \rangle = \exp \left(\sqrt{\lambda} \frac{2N}{3\pi} \sin^3 \theta_k \right)$$

$$= \exp \left(k \sqrt{\lambda} \left(1 - \frac{3}{10} \left(\frac{3\pi k}{2N} \right)^{2/3} - \cdots \right) \right)$$

Comments and Discussions

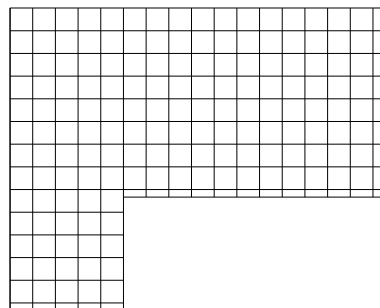🔵 **D3-brane**   [Rey, Yee], [Drukker, Fiol]

AdS2 x S2 D3-brane   ⟷   k-th wound (or symmetric ?) Wilson loop

🔵 **Expectation value from supergravity?**

Young diagrams with big block

c.f.   [Yamaguchi], [Lunin]