

Testing Gravity: Connecting Theoretical developments to forthcoming Observations

[Testing Gravity: Th \times Obs]

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Our review paper is now accessible!

PTEP

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Cosmological gravity probes: Connecting recent theoretical developments to forthcoming observations

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Atsushi Taruya^{2,4}, and Junsei Tokuda^{19,20}

- Prog. Theor. Exp. Phys. (2023)7, 072E01
- arXiv:2212.09094

A collective dictionary of gravity theory

1	Introduction				
2	Theories of gravity				
2.1	Scalar-tensor theories	8		2.1	Summary of the status and schedule of ongoing and future CMB experiments
2.1.1	Horndeski theory	8		3.2.2	CMB Polyspectra and Inflation
2.1.2	Degenerate higher-order scalar-tensor theories	9		3.2.3	CMB Lensing
2.1.3	$f(R)$ gravity	11		3.3	Large-scale structure
2.2	Massive gravity and bigravity	12		3.3.1	Summary of the status and schedule of ongoing and future CMB experiments
2.2.1	dRGT massive gravity	12		3.3.2	CMB Polyspectra and Inflation
2.2.2	Extensions of dRGT massive gravity	14		3.3.3	CMB Lensing
2.2.3	Translation breaking theories	15		3.3.4	Large-scale structure
2.2.4	Lorentz-violating massive gravity	16		3.3.5	Summary of the status and schedule of ongoing and future CMB experiments
2.2.5	Massive bigravity theory	16		3.3.6	CMB Polyspectra and Inflation
2.3	Vector-tensor theories	16		3.3.7	CMB Lensing
2.4	Metric-affine gravity	19		3.3.8	Large-scale structure
2.5	Cuscuton and minimally modified gravity	22	4	4	Linear perturbations in modified gravity
2.6	Evading solar-system tests	24	4.1	4.1	Perturbations in scalar-tensor theories
2.6.1	Vainshtein screening	24	4.2	4.2	Perturbations in massive gravity theories
2.6.2	Chameleon and symmetron	27	4.3	4.3	Perturbations in vector-tensor theories
2.7	Positivity bound	29	4.4	4.4	Perturbations in metric-affine gravity
2.7.1	Non-gravitational positivity bound	29	4.5	4.5	Perturbations in cuscuton and minimally modified gravity
2.7.2	Gravitational positivity bound	31	5	5	Numerical tools for theoretical predictions
2.7.3	Implications of gravitational positivity bound	32	5.1	5.1	Boltzmann code
3	Observables for testing gravity	33	5.1.1	5.1.1	Overview
3.1	Basic equations for testing modified gravity against Λ CDM model	34	5.1.2	5.1.2	Formalism
3.2	Cosmic Microwave Background	38	5.1.3	5.1.3	Demonstrations
			5.2	5.2	Predicting non-linear structure
			5.2.1	5.2.1	Overview
			5.2.2	5.2.2	Emulation
			5.2.3	5.2.3	DarkEmulator: halo model meets emulation
			5.2.4	5.2.4	Extensions
6	Outlook				
7	Summary				

Concrete predictions from theories by analytic computations

Numerical tools

How the effect of gravity are captured in observations

Our high-priority subjects

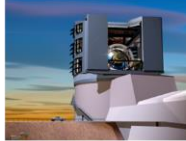
1. Current status

Golden Age of Observational Cosmology

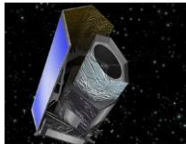
2022 2024 2026 2028 2030



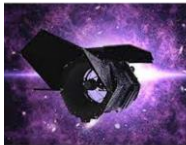
Subaru
(Optical • IR)



Vera C. Rubin
(Optical)



Euclid
(Optical • NIR)



Roman
(NIR)



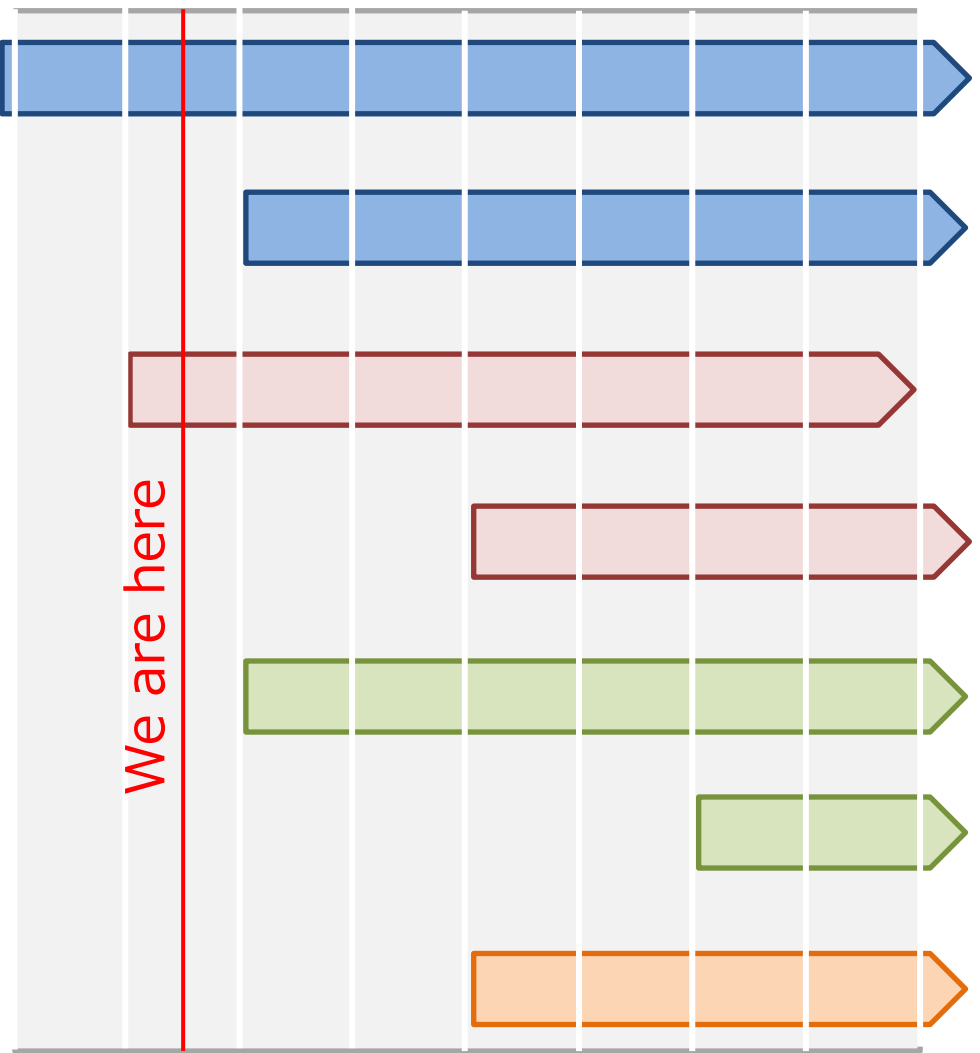
Simons Observatory
(CMB)



LiteBIRD
(CMB)

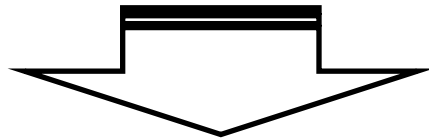


Square Kilometre
Array (radio)



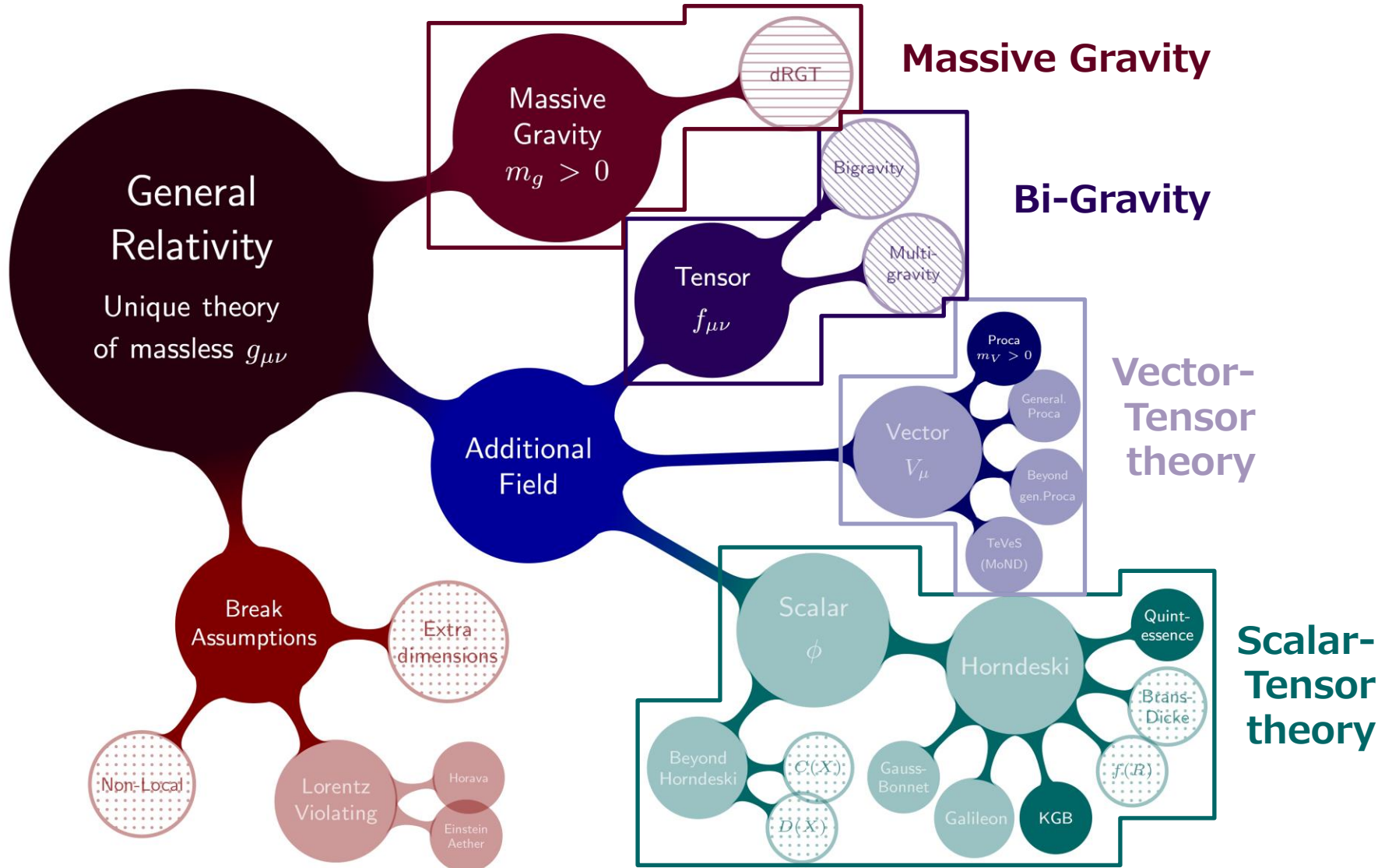
Golden Age of Observational Cosmology

Near-future observations will provide vast high-quality data suitable for proving gravity theory on large scales.

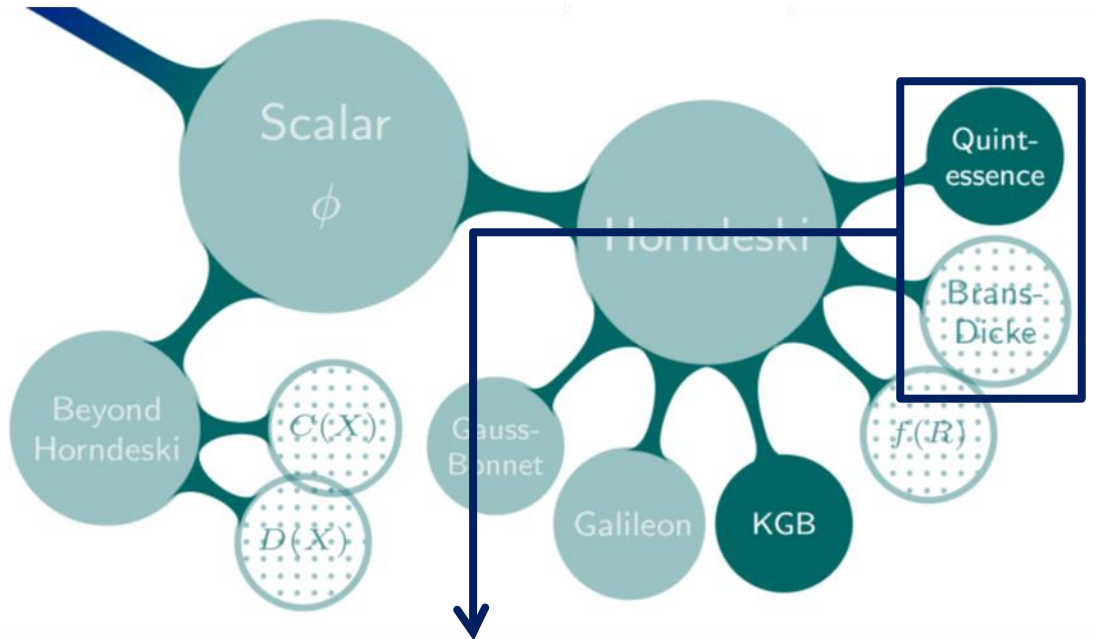


We now need to prepare well-motivated theory and appropriate observables that can indicate any signs beyond GR!

Landscape of Gravity Theory



Example: Scalar-Tensor Theories



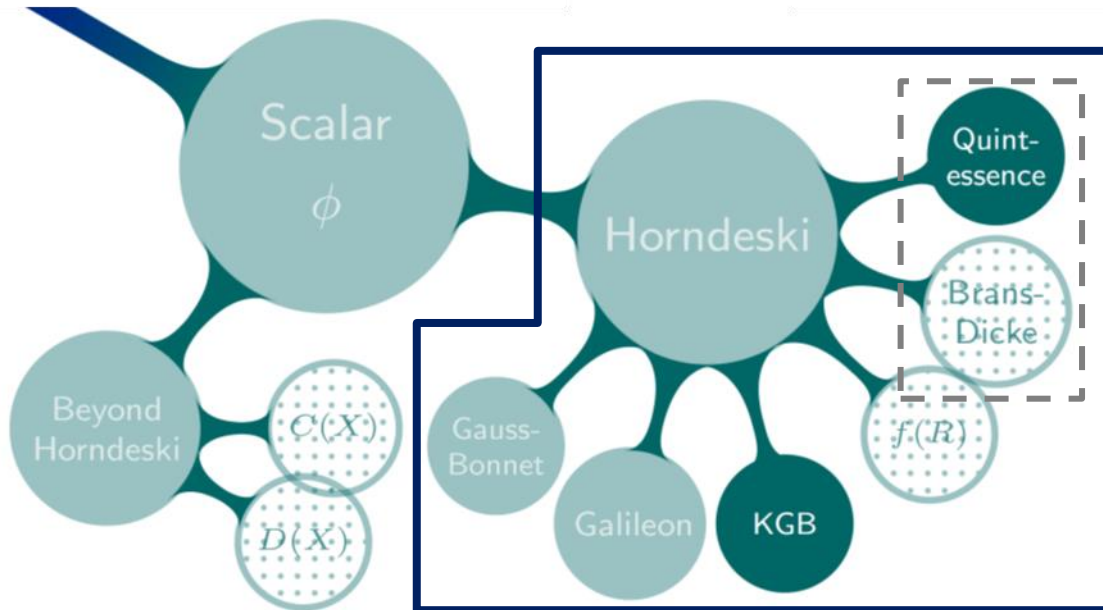
- ◆ (Old) well-known theories: Only one parameter

$$\mathcal{L} = \frac{1}{2} \left[\phi R - \frac{\omega}{\phi} (\partial\phi)^2 \right]$$

[Brans+Dicke, Phys.Rev.124,925(1961),...]

The feasibility can be discussed on a **model-by-model** basis.

Example: Scalar-Tensor Theories



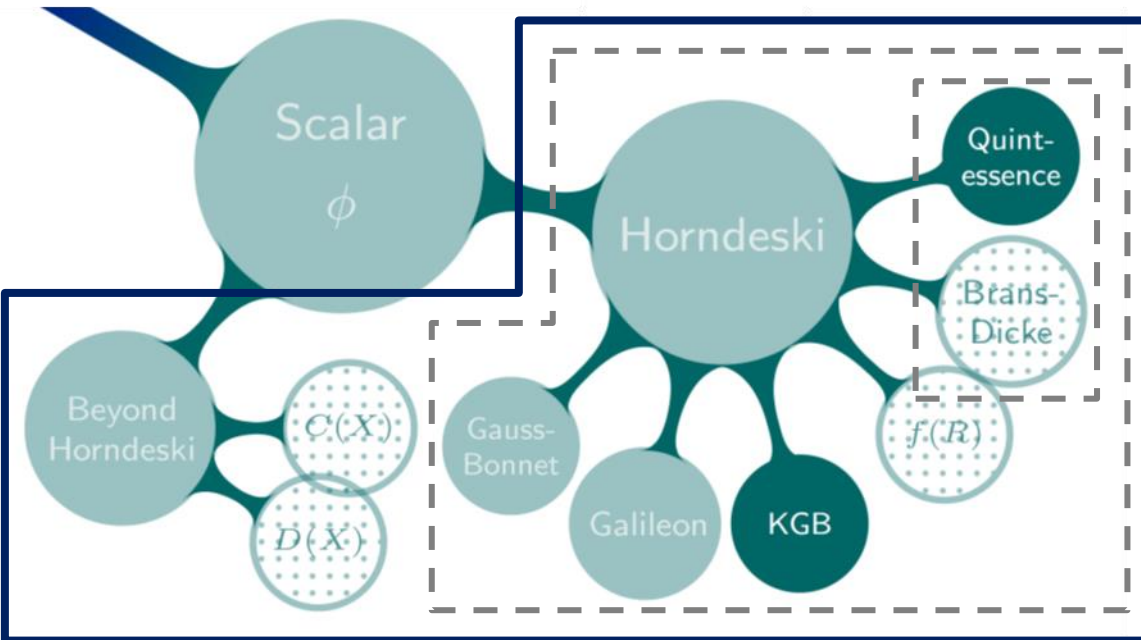
This provides
a bird's eye view
understanding.

◆ Horndeski theory: 4 arbitrary functional DoF

$$\mathcal{L} = P(\phi, X) - Q(\phi, X)\square\phi + G_4(\phi, X)R - \frac{\partial G_4}{\partial X} (\nabla_\mu \nabla_\nu \phi)^2 + \dots$$

[Horndeski, Int. J. Theor. Phys. 10, 363 (1974),
Deffayet+, PRD84, 063039 (2011),
Kobayashi+Yamaguchi+Yokoyama, PTP126, 511 (2011)]

Example: Scalar-Tensor Theories



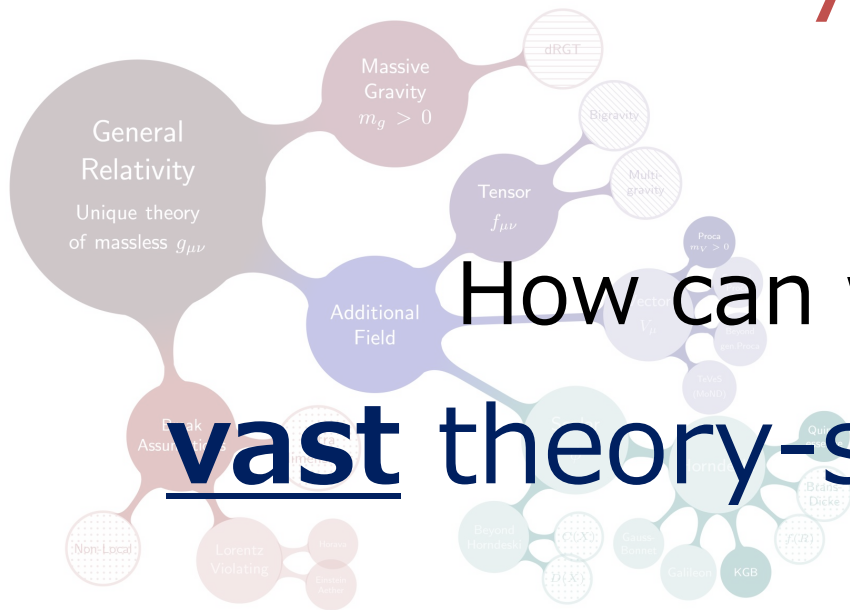
This gives a deeper understanding of its **stability**.

◆ **DHOST** (Degenerate Higher-Order Scalar-Tensor theory) : **15 DoF!**

$$\mathcal{L} = \dots + f(\phi, X)R + A_1(\phi, X) (\nabla_\mu \nabla_\nu \phi)^2 + \dots$$

[Langlois+Noui, JCAP02,034(2015),
Crisostomi+, JCAP04,044(2016),
Ben Achour+, PRD93,124005(2016)]

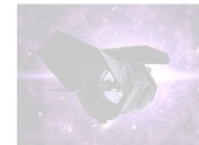
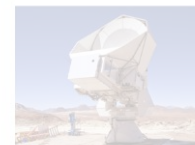
Today's topic



How can we compare
vast theory-space of gravity

to

forthcoming cosmological
observations ?



Take-Home Message

Theoretical

(Th1) Theory

(Th2) Effective Theory

(Obs3) Pheno. model

(Obs2) Pheno. parameter

(Obs1) Observable

Observational

- ◆ Main message: A **hierarchical structure** exists in the cosmological test of gravity.



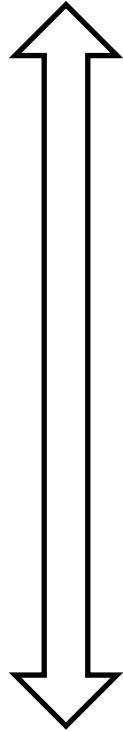
It is essential to link each hierarchy appropriately:

The key is to connect **Theoretical studies with** **Observational ones!**

2. Hierarchical structure from theory to observation (or vice versa)

(Th1) Theory

F(R) , Horndeski, ...

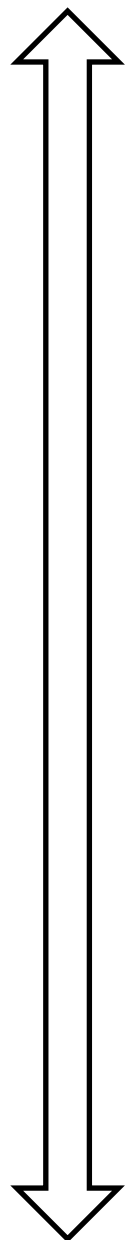


Can we compare
(full-)theory of gravity
to
observational data
directly ?

(Obs1) Observable

CMB, LSS, GW, ...

Theoretical



(Th1) **Theory**

**Unknown
Theory**

(Th2) **Effective
Theory**

modifying gravity
theory at level of
perturbed action

(Obs3) **Pheno.
model**

modifying gravity
model at level of **EoM**

(Obs2) **Pheno.
parameter**

to explore gravity
**w/o changing
 Λ CDM observables**

(Obs1) **Observable**

CMB, LSS, GW, ...

Observational

(Obs2) Pheno. parameter

: w/o changing Λ CDM observables



- ◆ Background level: Hubble expansion rate

$$H^2(a) = H_0^2 \left[\frac{\Omega_{m,0}}{a^3} + \Omega_{DE,0} \exp \left(-3 \int_1^a [1 + w_{DE}(a')] d \ln a' \right) \right]$$

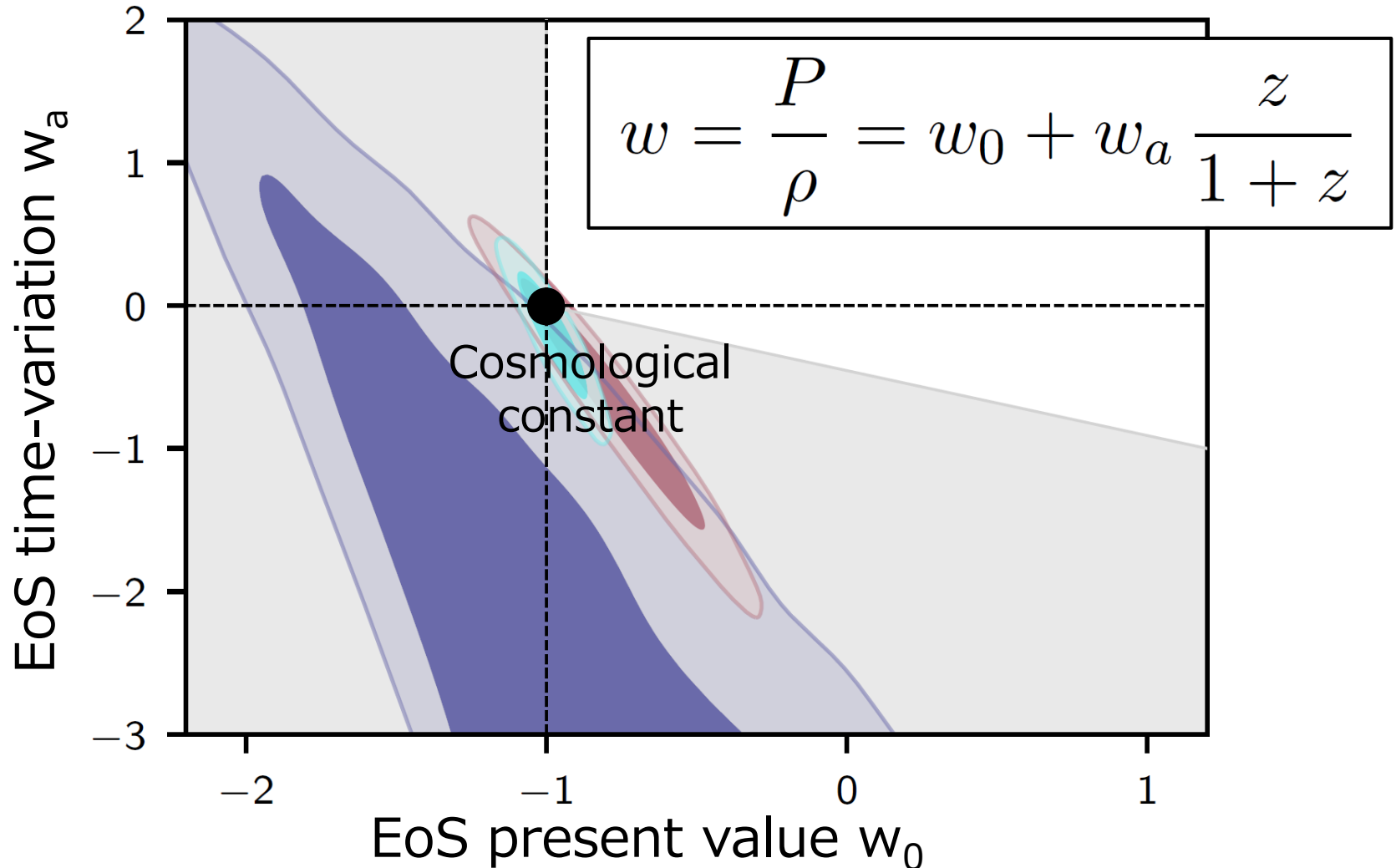
Dark Energy Equation-of-State

- ◆ Perturbed level: Growth rate of density fluct.

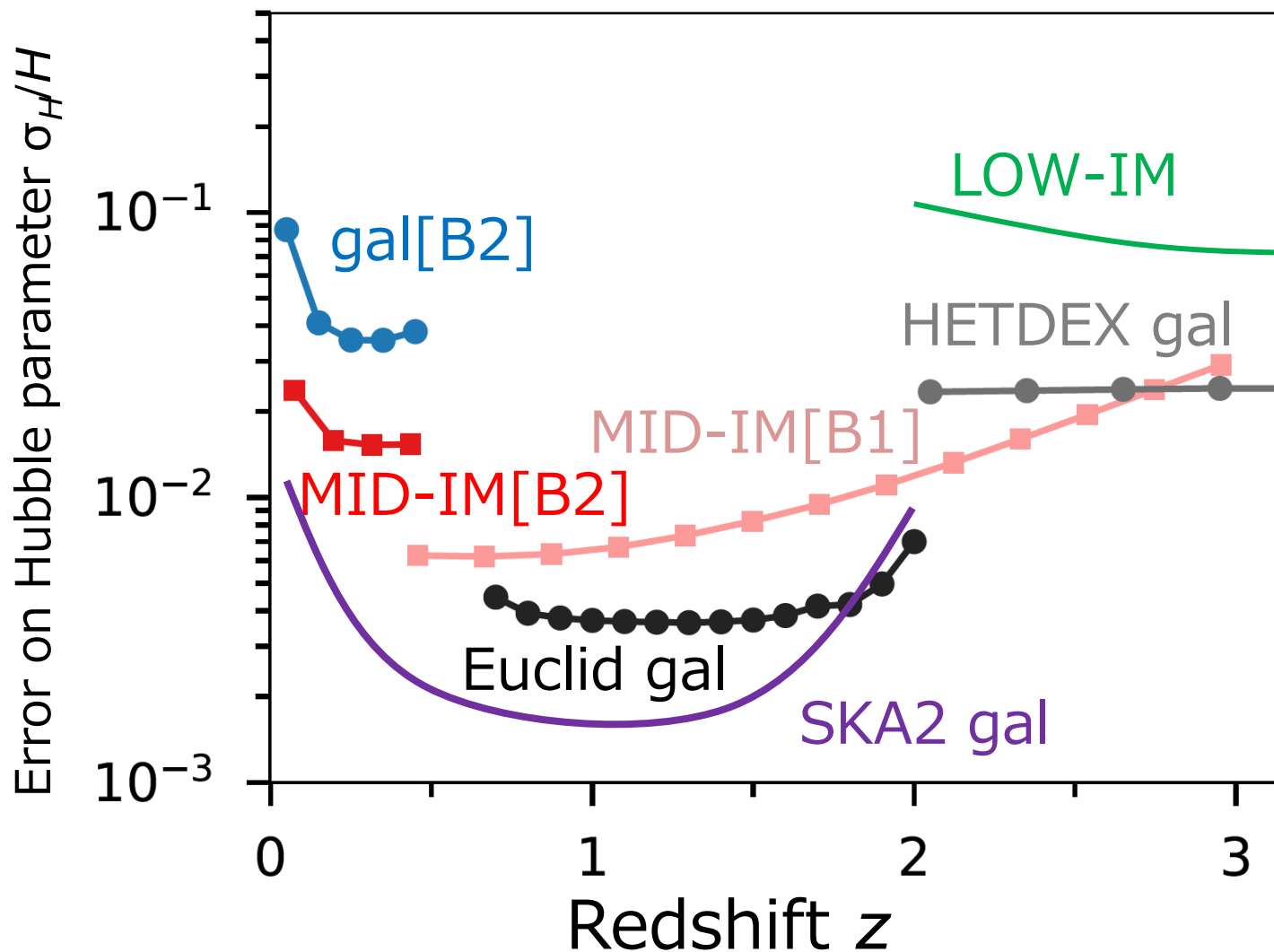
$$\delta(a, \mathbf{k}) = \exp \left(\int_0^a f(a') d \ln a' \right) \delta_*(\mathbf{k})$$

Linear growth rate

Equation-of-State parameter w_{DE}



Cosmic expansion rate



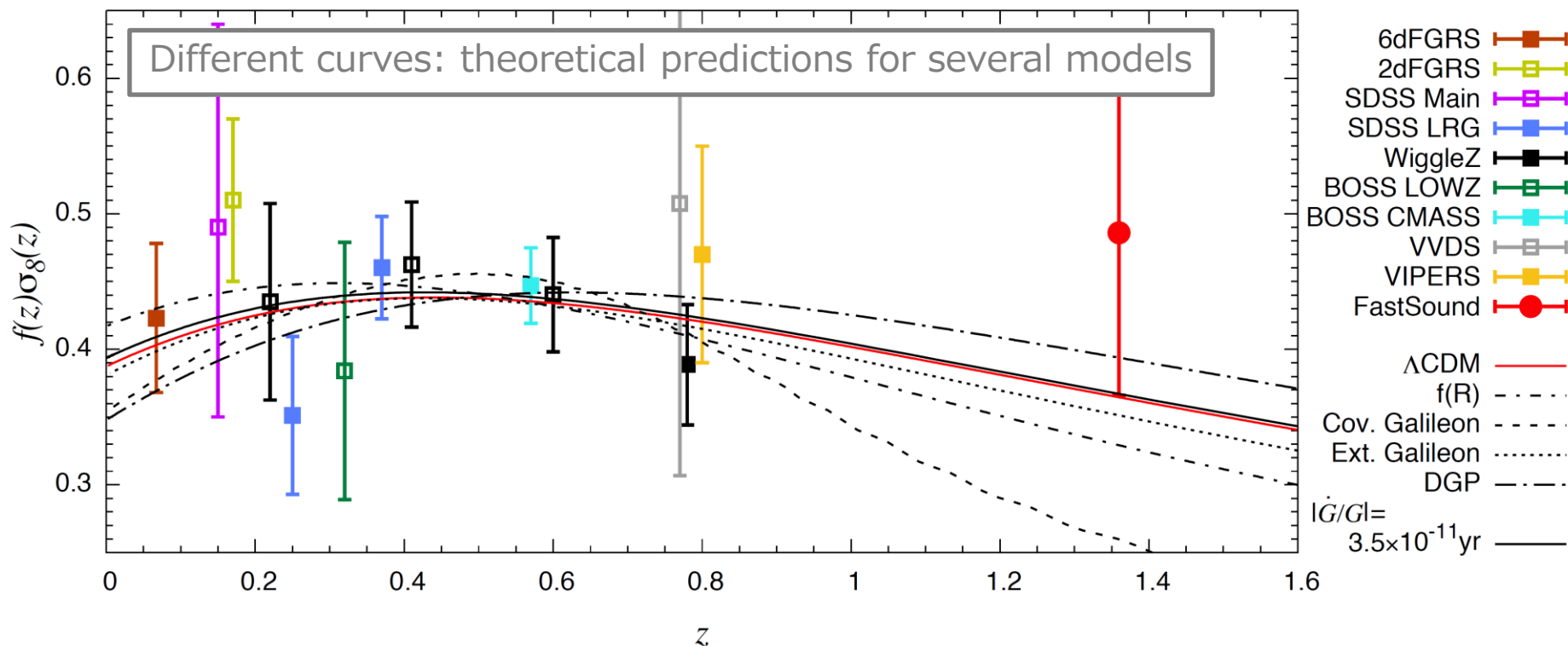
(Obs2) Pheno. parameter

: w/o changing Λ CDM observables



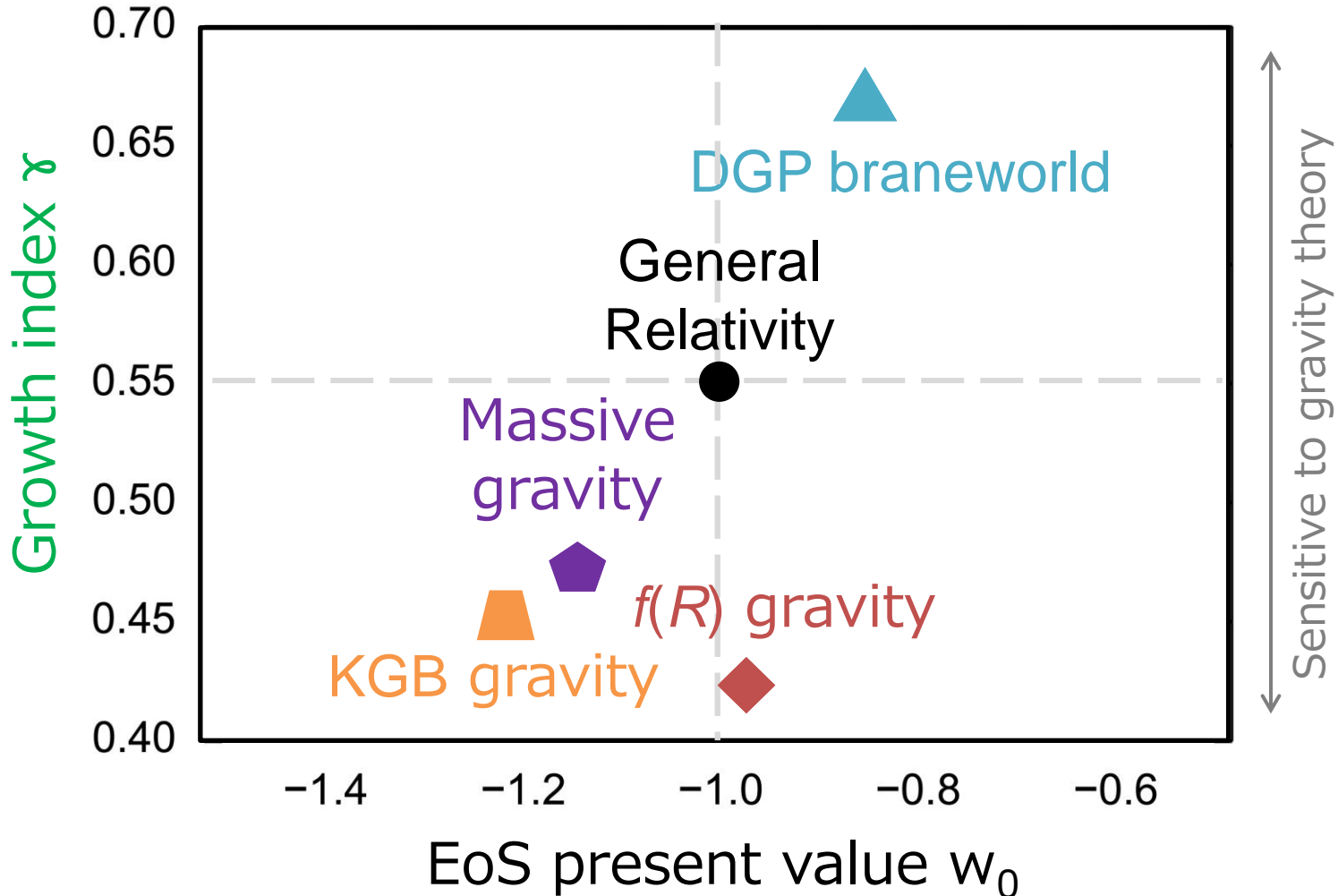
- ◆ Measuring f from RSD is frequently used for test of gravity responsible for current acceleration.

FastSound [Okumura+] PASJ68,3,38(2016)



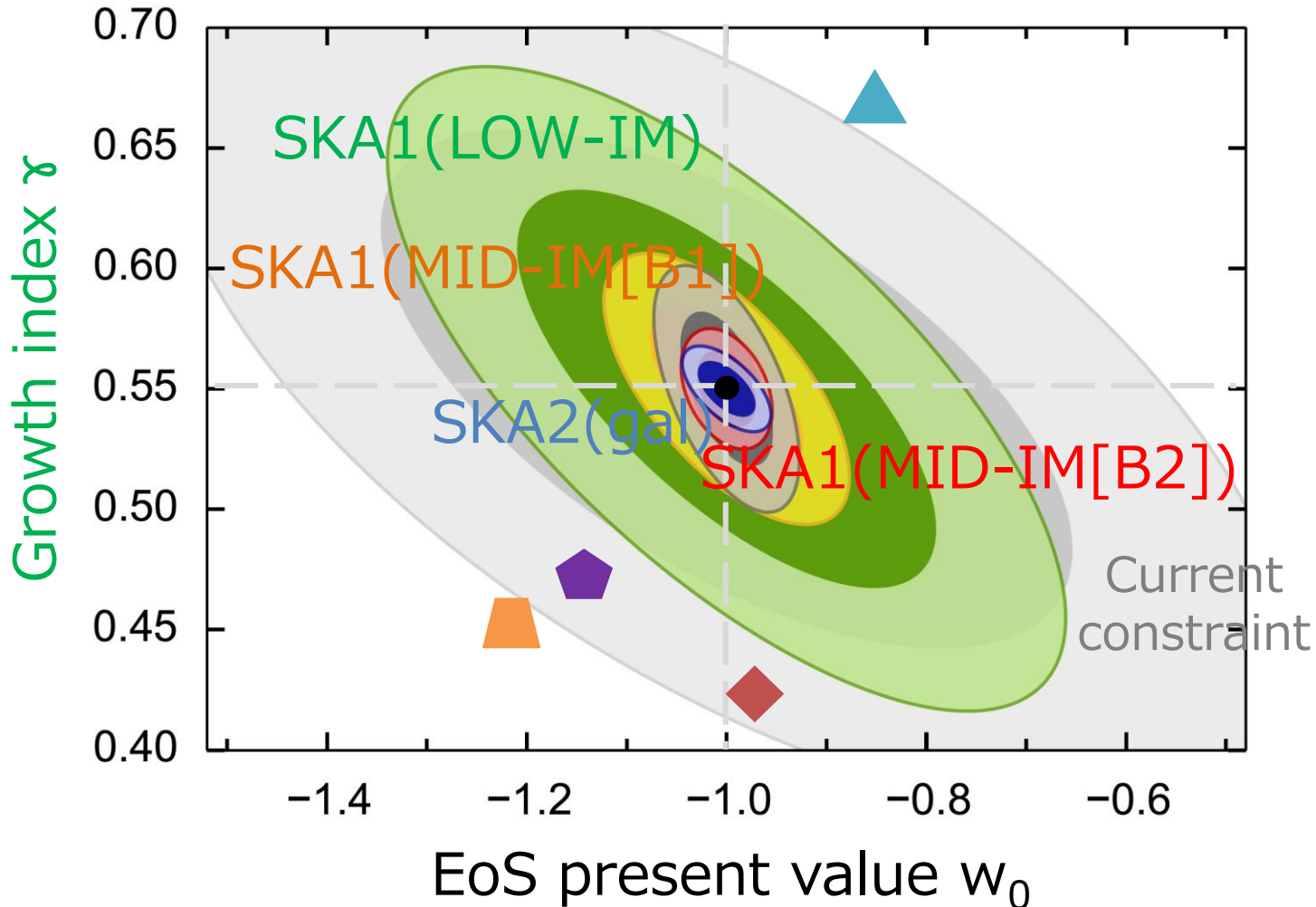
Growth index

$$\frac{d \ln \delta_L}{d \ln a} = f(a) \approx \Omega_m(a)^\gamma$$



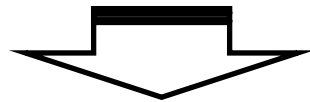
Growth index

$$\frac{d \ln \delta_L}{d \ln a} = f(a) \approx \Omega_m(a)^\gamma$$



Q. Are w_{DE} & f enough to test gravity?

A. **NOT** enough. Even if $w_{\text{DE}} = -1$, $f = f_{\text{GR}}$, it is **NOT** necessary that our Universe is described by ΛCDM with GR.



Nonlinear growth can carry new information that is not included in linear-order.

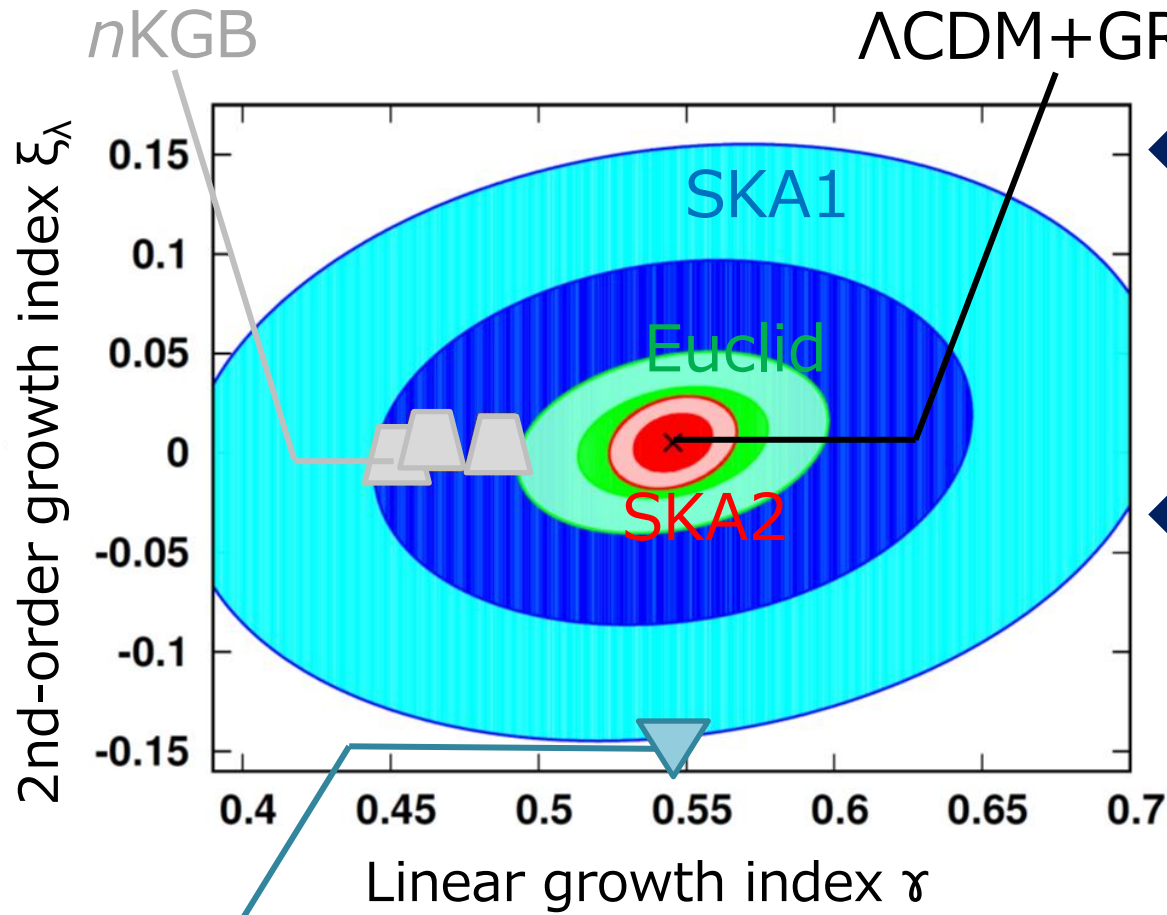
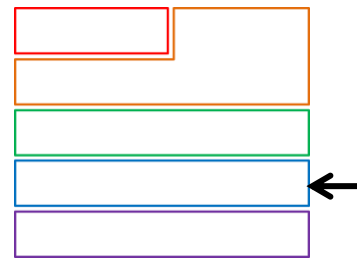
$$\delta(a, \mathbf{k}) = \delta_1(a, \mathbf{k}) + \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \boxed{F_2(a, \mathbf{p}, \mathbf{k} - \mathbf{p})} \delta_1(a, \mathbf{p}) \delta_1(a, \mathbf{k} - \mathbf{p}) + \dots$$

(quasi-)nonlinear growth

[Takushima+(2014,2015),**DY**+(2017),Namikawa+(2018),
Hirano+(2018),Hirano+**DY**+(2020),**DY**+Sugiyama(2022),
DY+(2023),Sugiyama+**DY**+(2023a,b),Yamashita+**DY**+(in prep.)]

(Obs2) Nonlinear growth

: to extract higher-order contributions



- ◆ Nonlinear growth index (ξ_λ) can be used to distinguish various models!
- ◆ Note: Mapping of these parameters to specific theories is **not** fully understood.

A model from Horndeski theory
(specific model with $w_{DE} = -1$, $f = f_{GR}$)

DY+, PRD96, 123516 (2017)
Namikawa+, PRD98, 043530 (2018)

(Obs3) Pheno. model

: modifying gravity at level of EoM



◆ Non-relativistic matter feels

$$\ddot{\delta} + 2H\dot{\delta} + \frac{1}{a^2} \nabla^2 \Phi = 0$$

Gravitational potential (δg_{00})

This term depends on gravity model via **Poisson equation:**

$$\nabla^2 \Phi = 4\pi G a^2 \rho \delta$$

We add

phenomenological functional DoF $\mu(a, k)$

(Obs3) Pheno. model

: modifying gravity at level of EoM



◆ Relativistic matter feels:

$$\nabla^2(\Phi + \Psi) = 8\pi G a^2 \rho \delta$$

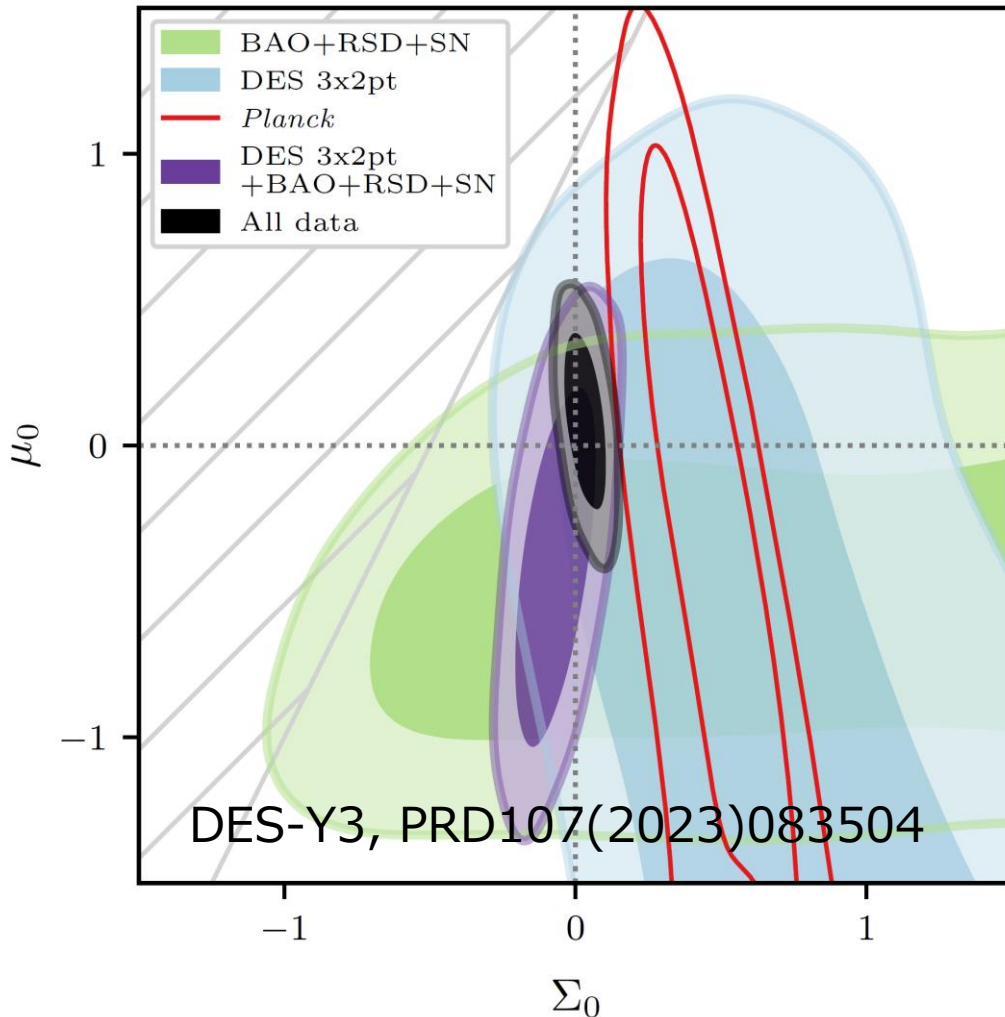
Gravitational potential (δg_{ij})

Phenomenological functional DoF $\Sigma(a, k)$

- ✓ Note: To practically obtain the constraints, a functional form of μ & Σ should be specified.

(Obs3) Pheno. model

: modifying gravity at level of EoM



- ◆ A specific choice of functional forms:

$$\mu(a) = 1 + \mu_0 \frac{\Omega_{\text{DE}}(a)}{\Omega_{\text{DE},0}}$$

$$\Sigma(a) = 1 + \Sigma_0 \frac{\Omega_{\text{DE}}(a)}{\Omega_{\text{DE},0}}$$

- ◆ Pheno. model is useful to investigate how signals deviate from standard ones, although the mapping is not fully understood.

(Th2) Effective Theory

: modifying at level of perturbed action



- ◆ We consider the perturbed action so that the physical meaning of pert. is obvious:

[Example] Effective 2nd-order Lagrangian

$$\mathcal{L} = \frac{1}{16\pi G} \left[(1 + \alpha_T) {}^{(3)}R + \delta K^i_j \delta K^j_i - \delta K^2 + \dots \right]$$

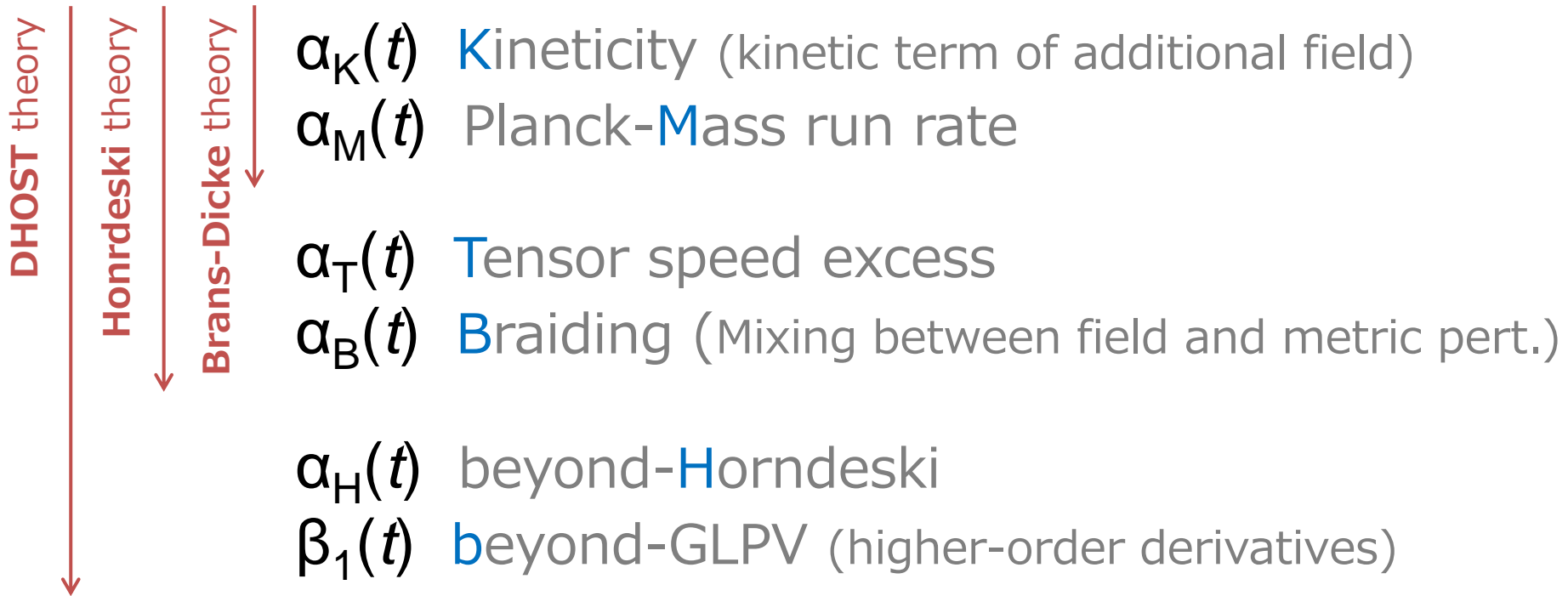
Independently of the details of original theory,
this term always represents
sound speed of GWs !

(Th2) Effective Theory

: modifying at level of perturbed action



- ◆ Even complex full theories can be described by linear theories with **a few** EFT parameters:



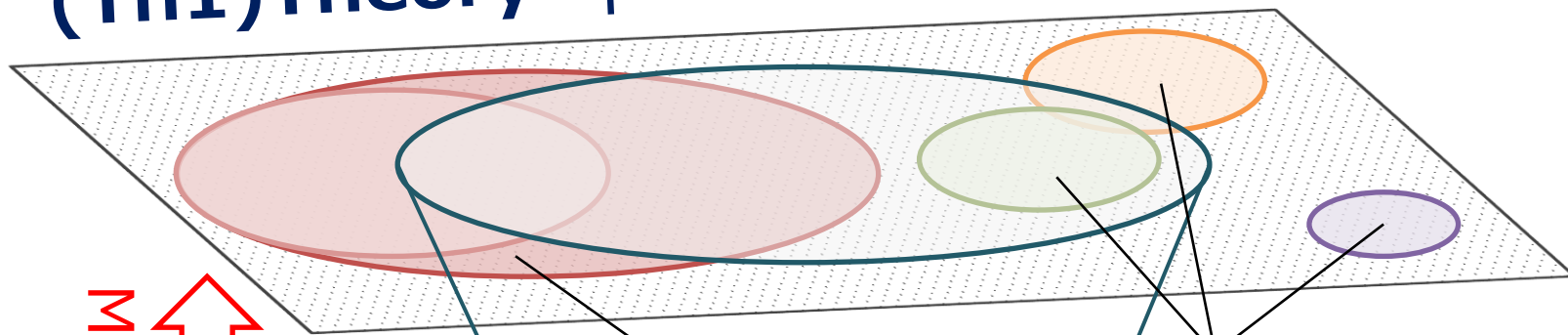
Scalar-Tensor Theories: Bellini+, JCAP07,050(2014), Langlois+, JCAP05,033(2017), ...
Vector-Tensor Theories: Aoki+, JCAP01,056(2022), Fluid: Aoki+, JCAP08,072(2022)

(Th2) Effective Theory

: modifying at level of perturbed action



“(Th1)Theory” parameter space

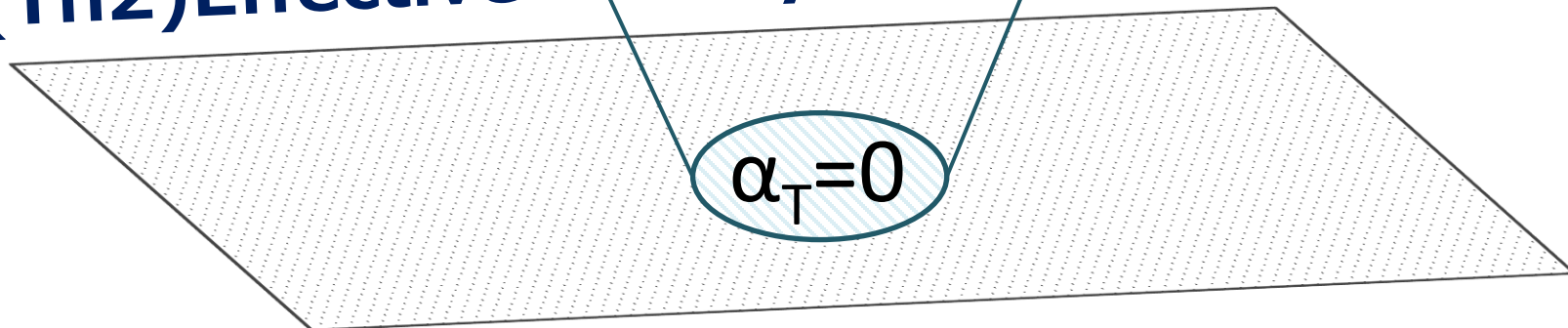


Mapping

A red double-headed vertical arrow indicating a mapping between the two parameter spaces.

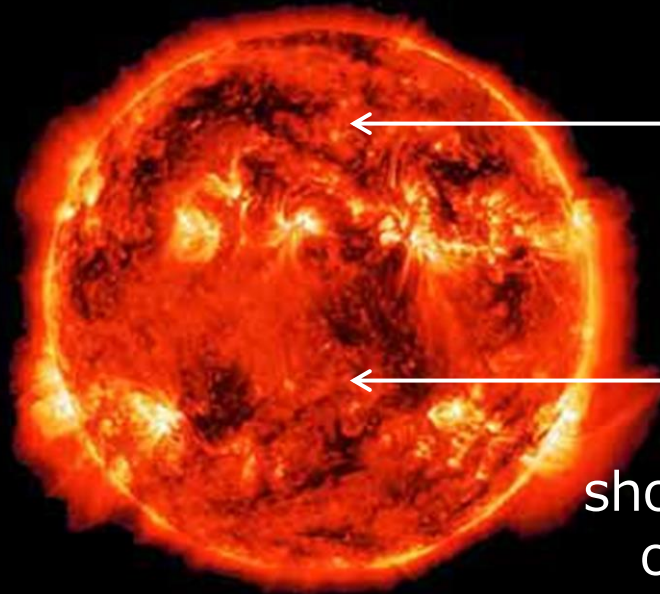
Known models

“(Th2)Effective Theory” parameter space



Other direction: Screening mechanism

- ◆ A new DoF mediate additional a long-range gravitational (fifth-)force at all scales.
- ◆ We need screening mechanism that suppresses new interactions to evade Solar-system tests.

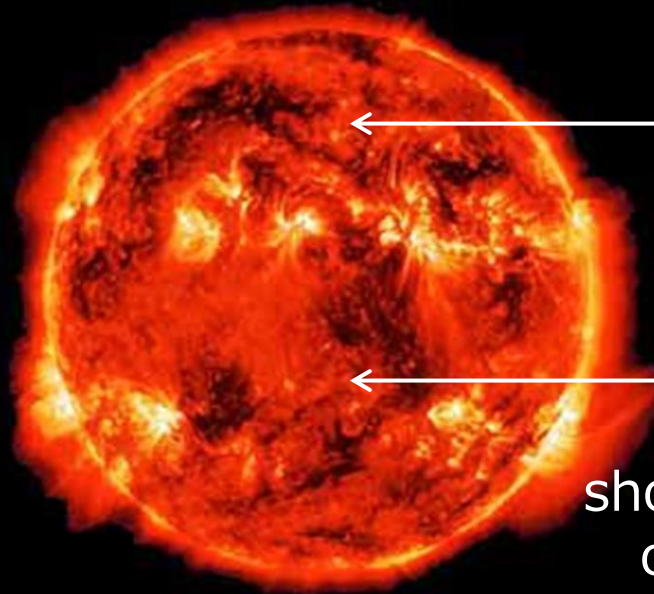


A new DoF is responsible
for cosmic acceleration.

A new DoF
should be screened
on small scales.

Other direction: Screening mechanism

- ◆ **Horndeski theory** exhibits successful screening.
[Narikawa+DY+,PRD87(2013)124006, Kimura+PRD85(2012)024023]
- ◆ **Beyond-Horndeski theories** such as DHOST lead to a deviation from standard Newton law **INSIDE** matter.
[Langlois+DY+,PRD97(2018)061501, Kobayashi+DY+,PRD91(2015)064013]



A new DoF is responsible
for cosmic acceleration.

A new DoF
should be screened
on small scales.

3. Connecting Theoretical studies with Observations (Th × Obs)

General Relativity
of massless $g_{\mu\nu}$

**Vast theory-space
of gravity (Th1)**

A mind map centered on 'Gravity' with branches including: 'General Relativity of massless $g_{\mu\nu}$ ', 'Massive Gravity $m_g > 0$ ' (with sub-nodes 'dRGT' and 'Bigravity'), 'Tensor-Vector-Scalar (TeVeS)', 'Non Local', 'Break Assumptions' (with sub-nodes 'Extra dimensions' and 'Lorentz Violating'), 'Scalar', and 'Feynman-Hellmann'.

Many challenges
lie between
Theory & Observation...

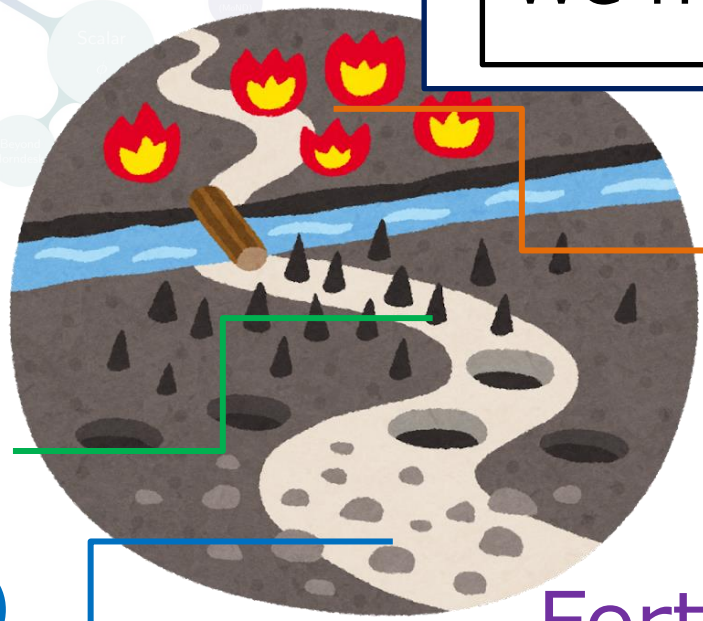
We need a **highway!**

**(Obs3)
Pheno. model**

**(Obs2)
Pheno. parameter**

**(Th2)
Effective Theory**

**Forthcoming
cosmological
observations (Obs1)**



(Th2) Effective Theory → (Obs3) Pheno. model



We have already created a **dictionary** to connect (Th2) and (Obs3) in specific gravity theories.

- Ex) Interaction between metric and scalar-field pert.

$$\mathcal{L}_{\text{int}} = -2M_{\text{Pl}}^2 a H \alpha_B \Phi \nabla^2 \varphi$$



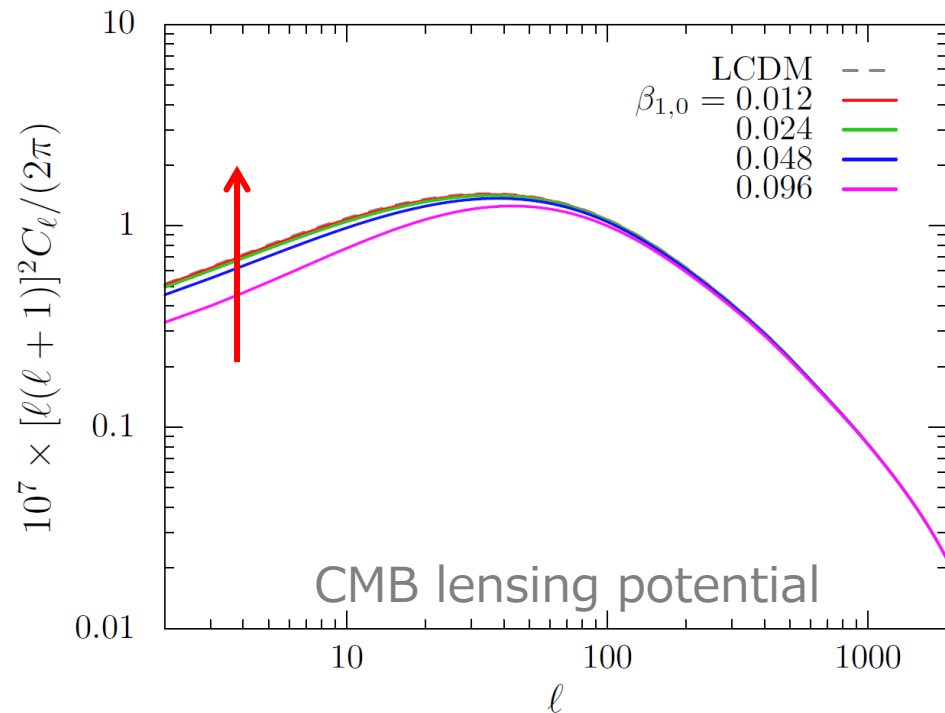
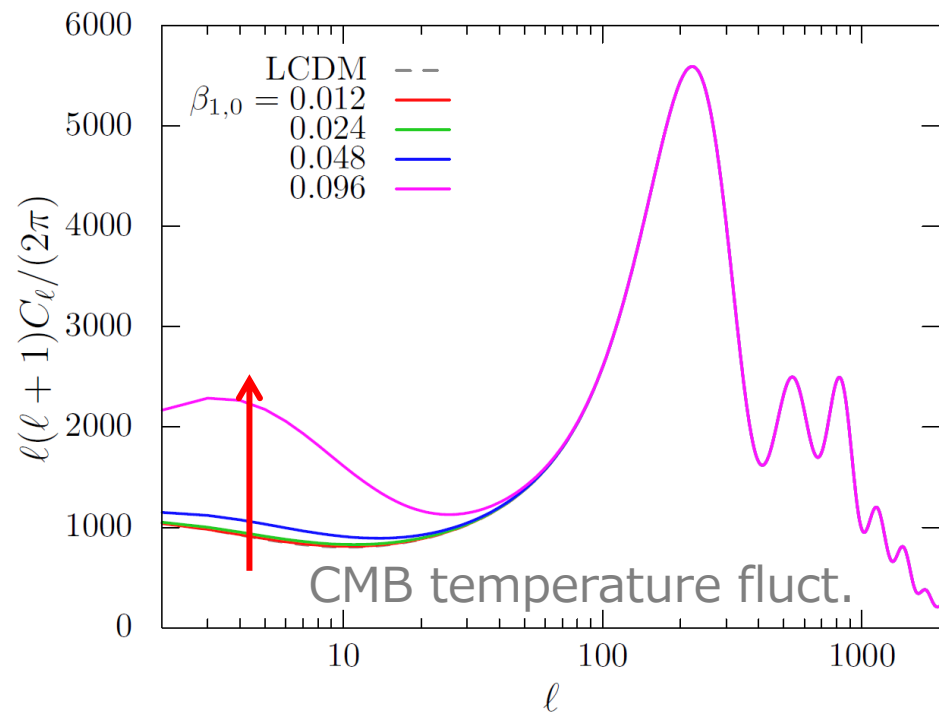
$$\mu = \Sigma = 1 - \frac{\alpha_B^2}{\frac{\dot{H}}{H^2} + \frac{3}{2}\Omega_m + \alpha_B(1 + \alpha_B) + \frac{(H\alpha_B)'}{H^2}}$$

Horndeski theory : Pogosian+, PRD94,104014(2016), Gleyzes+, JCAP02,056(2016)
DHOST theory : Hirano+DY+, PRD99,104051(2019)/PRD102,103505(2020)
Vector-Tensor theory : Aoki+, JCAP01,056(2022), ...

(Th2) Eff. Theo./ (Th1) Theory
 \rightarrow (Obs1) Observation[CMB]



- ◆ T. Hiramatsu (Rikyo U) developed the novel Boltzmann code “**CMB2nd**” that is utilized for computing CMB in the context of **DHOST**.



Hiramatsu+**DY**, PRD102,083525(2020), Hiramatsu, JCAP10(2022)035
 Hiramatsu+Kobayashi, JCAP07, 040 (2022),...

We have started
new Japanese Working Group:
“Testing Gravity: **Th** × **Obs**”

- Start : Aug. 2020
- Aim : Several multi-wavelength wide-field cosmological surveys have been conducted and planned, hence immediate validation system of gravity model needs to be established. For this reason, the aim of this working group is **to strongly connect theoretical and observational studies in Japan.**
- Chairs : Miyatake, Yokoyama, Arai(Nagoya), **DY(OUS)**
- Members : >50

Our review paper is now accessible!

PTEP

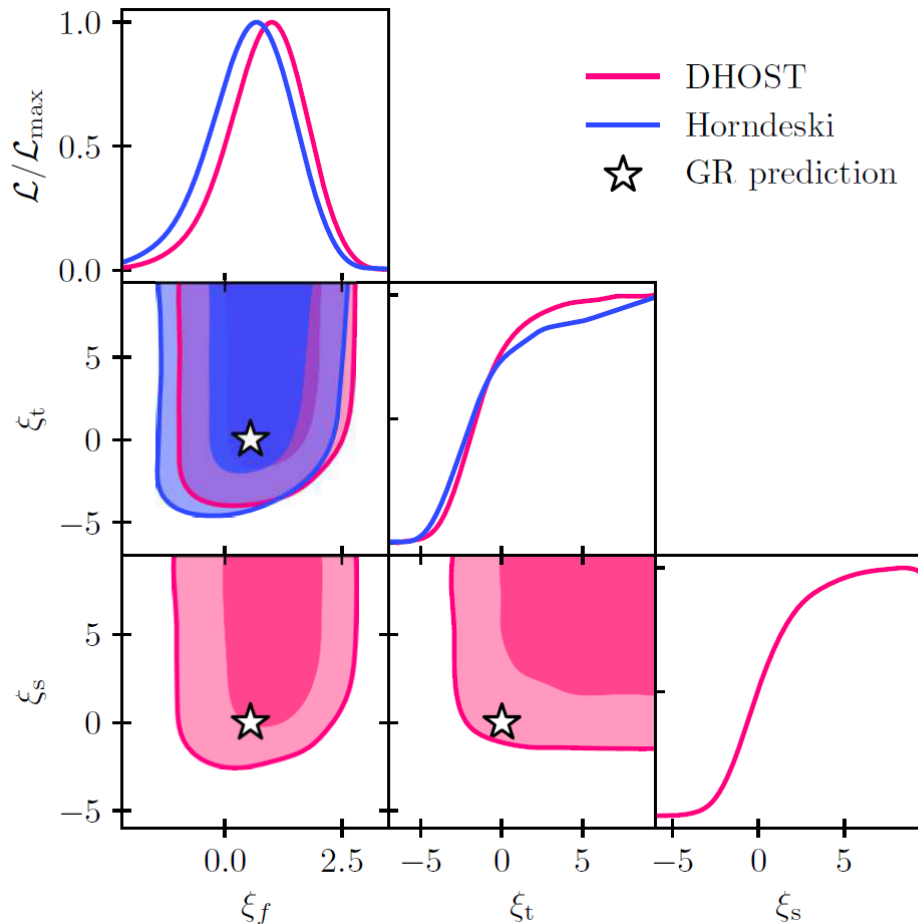
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New Th×Obs project has started!



◆ Requests from **Obs** side :
Novel technique
“anisotropic galaxy 3PCF”

+

◆ Requests from **Theo** side :
Theoretical prediction of
nonlinear growth



New **Th**×**Obs** project
“Test of gravity from
anisotropic galaxy 3PCF”

Sugiyama+**DY**+, MNRAS523, 2, 3133

Sugiyama+**DY**+, MNRAS524, 2, 1651

[Analysis based on **DY**+Sugiyama,PRD105,063515(2022)]

What we have to do are...

□ Coupling between matter and gravity

- ✓ Most calculations are based on minimal coupling.
- ✓ **Nonminimal (e.g. disformal) coupling** may lead to strange phenomena in observables
[Kimura+DY+(2018),Chibana+DY+(2019)]

□ Nonlinearity

- ✓ **Screening mechanism** should be considered. Beyond-Horndeski class such as DHOST leads to partial breaking of screening mechanism
[Kobayashi+DY+(2015),Langlois+DY+(2018)]
- ✓ Deeper understanding of **nonlinear growth** of structure is also needed.

Summary

Theoretical

(Th1) Theory

(Th2) Effective Theory

(Obs3) Pheno. model

(Obs2) Pheno. parameter

(Obs1) Observable

Observational

- ◆ Main message: A **hierarchical structure** exists in the cosmological test of gravity.

The key is to connect **Theoretical studies with** **Observational ones!**

- ◆ Japanese working group "Testing Gravity: Th×Obs"
- ◆ Our review paper is now accessible!