Physics, Astrophysics, & Simulation of Gravitational Wave Sources

Christian D. Ott
TAPIR, Burke Institute
California Institute of Technology
cott@tapir.caltech.edu

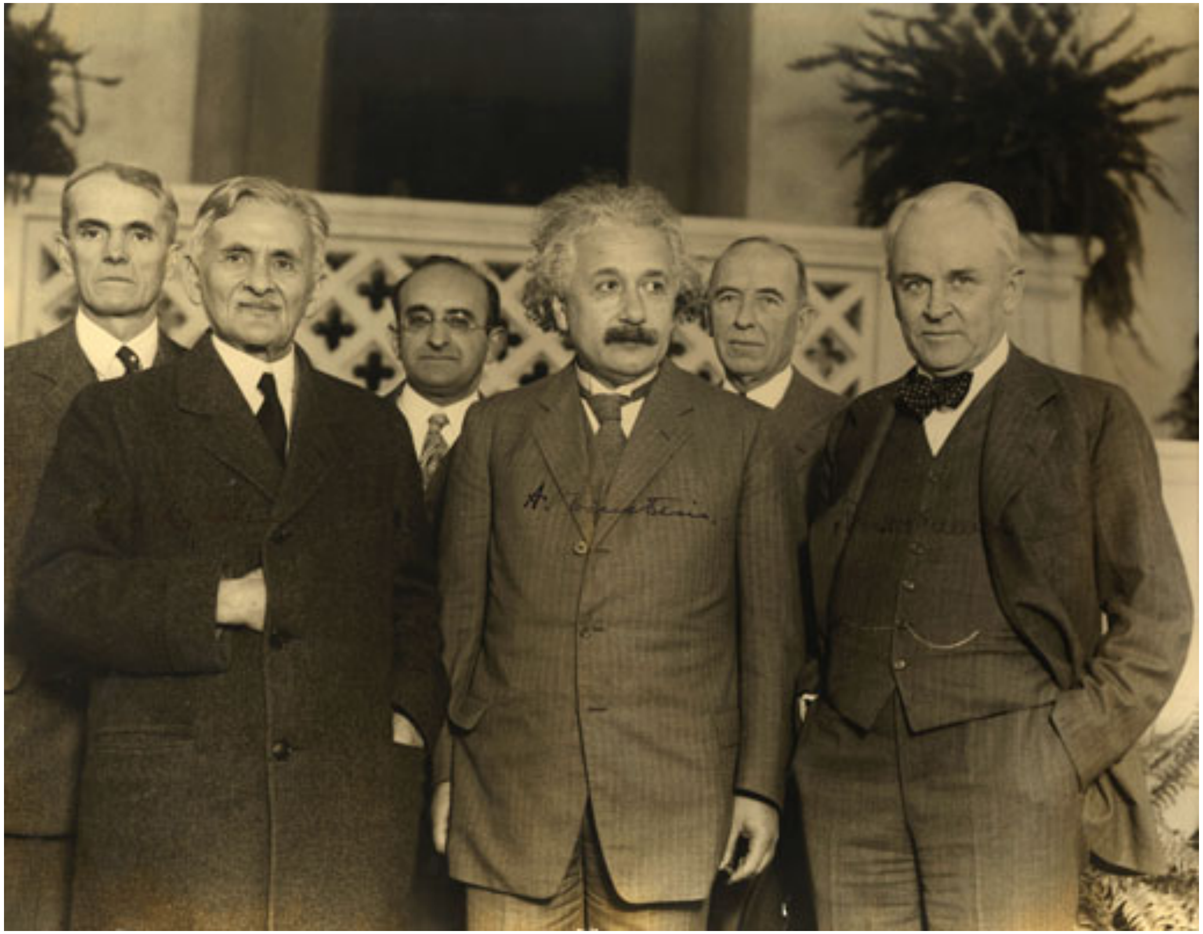
BURKE INSTITUTE

Caltech

Lecture Plan

- Lecture 1 (now!)
  - (a) General Relativity & Gravitational Wave Refresher
  - (b) Overview of GW sources & phenomenology.
  - (c) Numerical relativity and general-relativistic (magneto-)hydrodynamics.
- Lecture 2 (Thursday)
  - (a) Continuation of Lecture 1, Part (c).
  - (b) Microphysics of neutron star mergers and stellar collapse.
  - (c) Neutron star mergers and Nucleosynthesis
- Lecture 3 (Friday)
  - (a) Massive star evolution, stellar collapse.
  - (b) Core-collapse supernovae and long gamma-ray bursts.
  - (c) Neutron star and black hole formation.

General Relativity & Gravitational Waves



Caltech

1931

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

Warning

Goal: Remind you about some key aspects of GR and GWs.

Will skip over many details!

Will not provide proofs or detailed derivations!

Recommended texts:

Carroll, ***Spacetime and Geometry: an Introduction to General Relativity***
Schutz, ***A First Course in General Relativity***
Misner, Thorne, and Wheeler, ***Gravitation***

Sean Carroll's online notes on GR:
http://preposterousuniverse.com/grnotes/

Recall: General Relativity

Einstein, 1915

Curvature of Spacetime: Einstein tensor

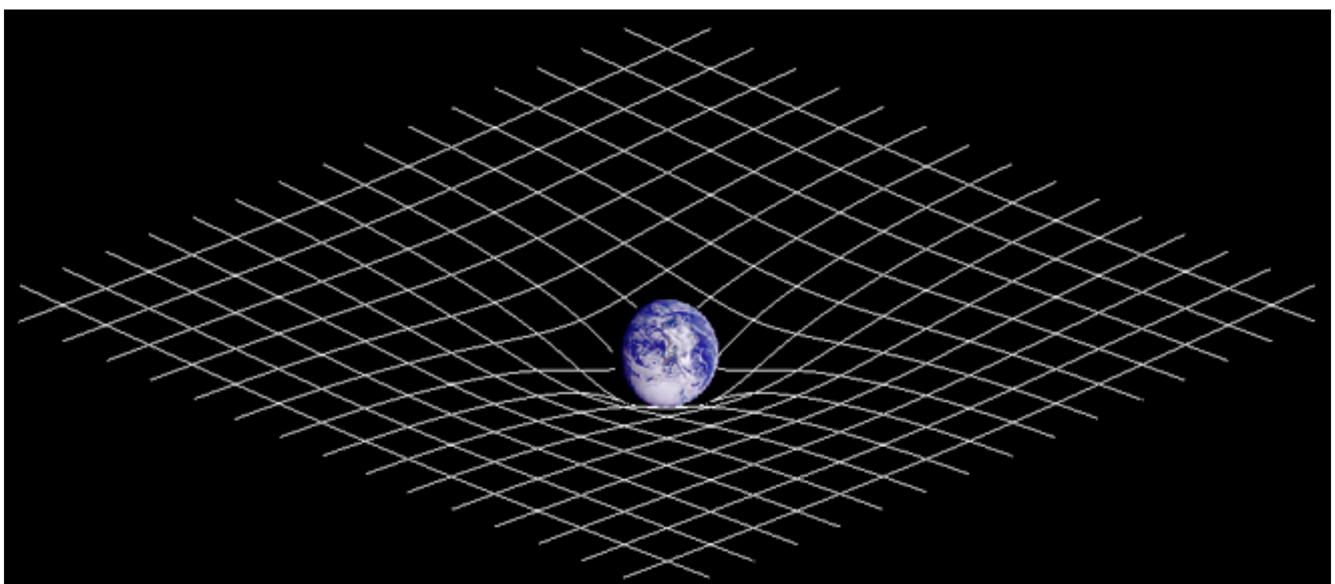
$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

(symmetric; 10 indep. components)

Source of Curvature: Stress-Energy Tensor

"Matter tells space how to curve and space tells matter how to move"

- John Archibald Wheeler



Refresher: Metric & Notation

Units: $G = c = M_\odot = 1$ (most of the time, but not always in this lecture!)

Indices:
$$\text{Latin}: \quad i, j, k, \ldots \quad \to \{1,2,3\}$$
$$\text{Greek}: \quad \alpha, \beta, \gamma, \nu, \mu, \ldots \quad \to \{0,1,2,3\}$$

Metric: Measure distances in spacetime

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \sum_{\mu\nu} g_{\mu\nu} dx^\mu dx^\nu$$

Einstein Sum Convention: Assume sum over identical indices ("dummy indices")

Scalar product:

$$\mathbf{A} \cdot \mathbf{B} = A_\nu B^\nu = g_{\mu\nu} A^\mu B^\nu$$

(-> metric "lowers" and "raises" indices physics independent indices)

Trivial example: length of a coordinate 4-vector in flat Cartesian space:

Minkowski

$$A^2 = -dt^2 + dx^2 + dy^2 + dz^2 = \eta_{\mu\nu} A^\mu A^\nu = \mathrm{diag}(-1,1,1,1) \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} \cdot \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$

"spacelike signature" – sign convention; alternative is

$$A^2 = dt^2 - dx^2 - dy^2 - dz^2$$

Refresher: More GR

Covariant derivative: Derivative in curved spacetime

$$\nabla_\mu V^\mu = \partial_\mu V^\mu + \Gamma^\nu_{\mu\lambda} V^\lambda$$

$$\nabla_\mu V^\mu = \boxed{V^\mu{}_{;\mu}}$$ (shorthand notation)

$$\partial_\mu V^\mu = \frac{\partial}{\partial x^\mu} V^\mu = \boxed{V^\mu{}_{,\mu}}$$

Connection Coefficients
(Christoffel Symbols)

$$\Gamma^\sigma_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} (g_{\nu\rho,\mu} + g_{\rho\mu,\nu} - g_{\mu\nu,\rho})$$

Note: $g_{\mu\nu;\sigma} = 0 \quad g^{\mu\nu}{}_{;\sigma} = 0$

(covariant derivative of the metric is zero)

Partial derivative:
coordiante dependent

Covariant derivative:
coordinate independent

Any law of physics must be independent of coordinates!

-> covariant derivative crucial for formulating the laws of physics in curved spacetime.

Refresher: Yet More GR

Riemann curvature tensor:

$$R^{\sigma}_{\mu\alpha\beta} = \Gamma^{\sigma}_{\mu\beta,\alpha} - \Gamma^{\sigma}_{\mu\alpha,\beta} + \Gamma^{\sigma}_{\alpha\lambda}\Gamma^{\lambda}_{\mu\beta} - \Gamma^{\sigma}_{\beta\lambda}\Gamma^{\lambda}_{\mu\alpha}$$

Encapsulates *physical* curvature of spacetime (= curvature due to gravity).

$R^{\sigma}_{\mu\alpha\beta} = 0$ if and only if spacetime is **flat**.

Flat: there exists a global coordinate system in which the metric components are everywhere constant.

$$R_{\mu\nu\rho\sigma} = -R_{\mu\nu\sigma\rho} = -R_{\nu\mu\rho\sigma} \qquad R_{\mu\nu\rho\sigma} = R_{\rho\sigma\mu\nu}$$

$$R_{\mu[\nu\rho\sigma]} = R_{\mu\nu\rho\sigma} + R_{\mu\rho\sigma\nu} + R_{\mu\sigma\nu\rho} = 0$$

Riemann Tensor has 4^4 = 256 coeff's. Only 20 are independent.

$$R_{\mu\nu[\rho\sigma;\lambda]} = 0 = R_{\mu\nu\rho\sigma;\lambda} + R_{\mu\nu\sigma\lambda;\rho} + R_{\mu\nu\lambda\rho;\sigma}$$

$$\boxed{\begin{array}{ll} R_{\alpha\beta} = R_{\beta\alpha} = R^{\lambda}_{\alpha\lambda\beta} & \text{Ricci Tensor} \\ R = R^{\nu}{}_{\nu} = g^{\mu\nu}R_{\mu\nu} & \text{Ricci Scalar} \end{array}}$$

Refresher: Even More GR

Einstein Tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \qquad \nabla^\mu G_{\mu\nu} = G_{\mu\nu}{}^{;\mu} = 0$$

(due to Bianchi identity)

Einstein Equation:

$T_{\mu\nu}$ Stress-Energy Tensor $\qquad \nabla^\mu T_{\mu\nu} = 0$

(energy conservation)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

curavature

source of curvature

$$\nabla^\mu T_{\mu\nu} = 0$$

equation of motion

Gravitational Waves

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

linearize

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$$

flat space metric

metric perturbation

$$\Box h^{\mu\nu} = \left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) h^{\mu\nu} = -\frac{16\pi G}{c^4} T^{\mu\nu}$$

inhomogeneous wave equation -> gravitational waves (GWs)

Gravitational Waves: A little more detail

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu} \qquad ||h_{\mu\nu}|| \ll 1$$

(linear perturbation; raise/lower indices with Minkowski metric)

Linearized Riemann Tensor:

$$R^{\mu}_{\nu\alpha\beta} = \frac{1}{2}\eta^{\mu\delta}(h_{\delta\beta,\nu\alpha} - h_{\nu\beta,\delta\alpha} - h_{\delta\alpha,\nu\beta} + h_{\nu\alpha,\delta\beta})$$

This is invariant under gauge transformation: $x^{\alpha'} = x^{\alpha} + \xi^{\alpha}$

$$g_{\mu'\nu'} = \eta_{\mu\nu} + h_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu}$$

Further:

$$\overline{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h^{\lambda}{}_{\lambda}$$ ("trace-reverse")

Require Lorentz gauge: $\overline{h}^{\nu\mu}{}_{,\nu} = 0$

(one can show that this is always possible)

Gravitational Waves: A little more detail

Lorentz gauge, construct Ricci, plug into Einstein tensor:

$$G^{\mu\nu} = -\frac{1}{2}\Box\bar{h}^{\mu\nu} = -\frac{1}{2}\bar{h}^{\mu\nu}{}_{,\sigma}{}^{\sigma} = \left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}^{\mu\nu}$$

d'Alembert operator

Linearized Einstein equation:

$$\Box\bar{h}^{\mu\nu} = -16\pi T^{\mu\nu}$$

In vacuum:

$$\Box\bar{h}^{\mu\nu} = 0$$

Plane wave solutions:

$$\bar{h}^{\mu\nu} = A^{\mu\nu}\exp(ik_\alpha x^\alpha)$$

$$k_\nu k^\nu = 0$$

tangent to the worldline of a photon -> GWs travel with the speed of light!

Transverse-Traceless Gauge

There is remaining gauge freedom, since any coordinate change ξ^α with

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\xi^\alpha = 0 \qquad \text{allowed.}$$

$$\bar{h}^{\mu\nu} = A^{\mu\nu}\exp(ik_\alpha x^\alpha)$$

Transverse-traceless (TT) conditions:

$$A^\alpha{}_\alpha = 0 \qquad A_{\alpha\beta}U^\alpha = 0 \qquad \text{with:} \quad U^\nu U_\nu = -1$$

Note: $\bar{h}^{TT}_{\mu\nu} = h^{TT}_{\mu\nu}$

Reduces GW field to two independent components: “Polarizations”

Example: pick $U^\nu = \delta^\nu_0$

& wave traveling in +z

-> wave oscillation transverse to direction of propagation

$$A^{TT}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & A_{xy} & 0 \\ 0 & A_{xy} & -A_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

GW Effect on Test Particles

Free particles travel on geodesics:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0 \qquad U^\mu = \frac{dx^\mu}{d\tau}$$

acceleration → $$\frac{dU^\mu}{d\tau} + \Gamma^\mu_{\nu\delta}U^\nu U^\delta = 0$$

Particle initially at rest, waves into +z comes by:

$$h^{TT}_{\alpha\beta} = A^{tt}_{\alpha\beta}\exp i(\omega t - k_z z) \qquad A^{TT}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & A_{xy} & 0 \\ 0 & A_{xy} & -A_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\frac{dU^\alpha}{d\tau} = -\Gamma^\alpha_{00} = -\frac{1}{2}\eta^{\alpha\beta}(h^{TT}_{\beta 0,0} + h^{TT}_{0\beta,0} - h^{TT}_{00,\beta})$$

$= 0$ (!!!! – so does GW have no effect?!?)

GW Effect on Test Particles

Not so fast! Consider GW effect on separation of 2 test masses:

Mass 1: $x = y = z = 0$ Mass 2: $x = \epsilon, y = z = 0$

Physical ("proper") distance between the test masses:

$$\Delta l = \int |ds^2|^{1/2} = \int |g_{\alpha\beta} dx^\alpha dx^\beta|^{1/2}$$

$$= \int |g_{xx}|^{1/2} dx \approx |g_{xx}(x=0)|^{1/2} \epsilon$$

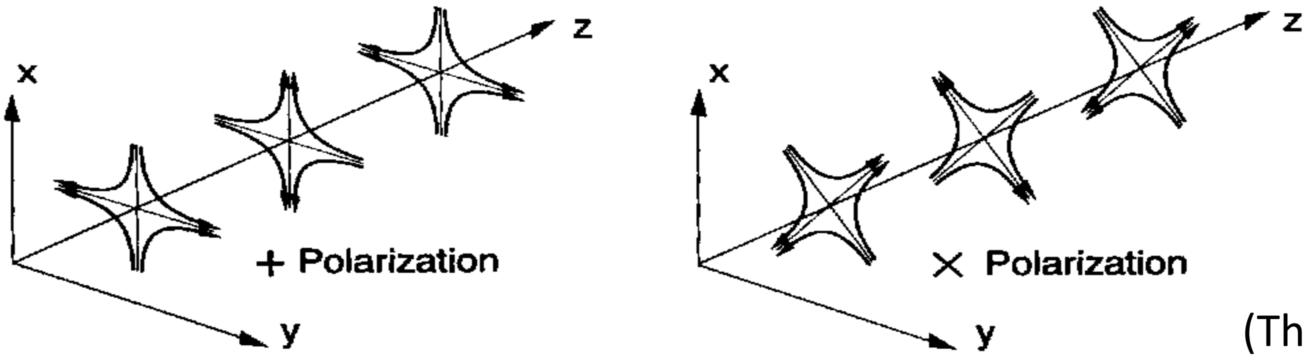
$$\approx \left[1 + \frac{1}{2} h_{xx}^{TT}(x=0)\right] \epsilon$$

-> **TT GWs do not change the coordinate locations, but stretch & squeeze separation between test masses.**

In other gauges: may have coordinate changes, but not physically meaningful!

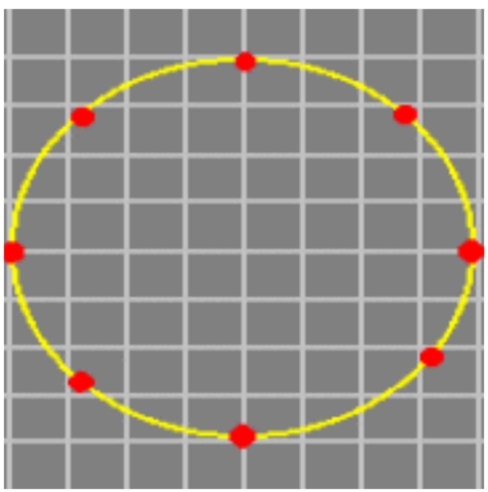
GW Polarizations

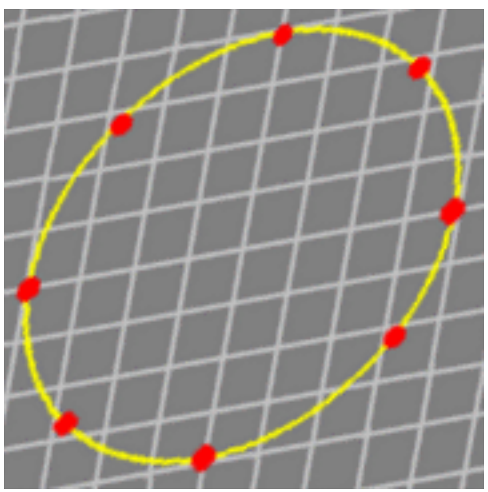
In transverse-traceless gauge (TT) all gauge degrees of freedom fixed:



(Thorne)

GW effect on ring of test masses



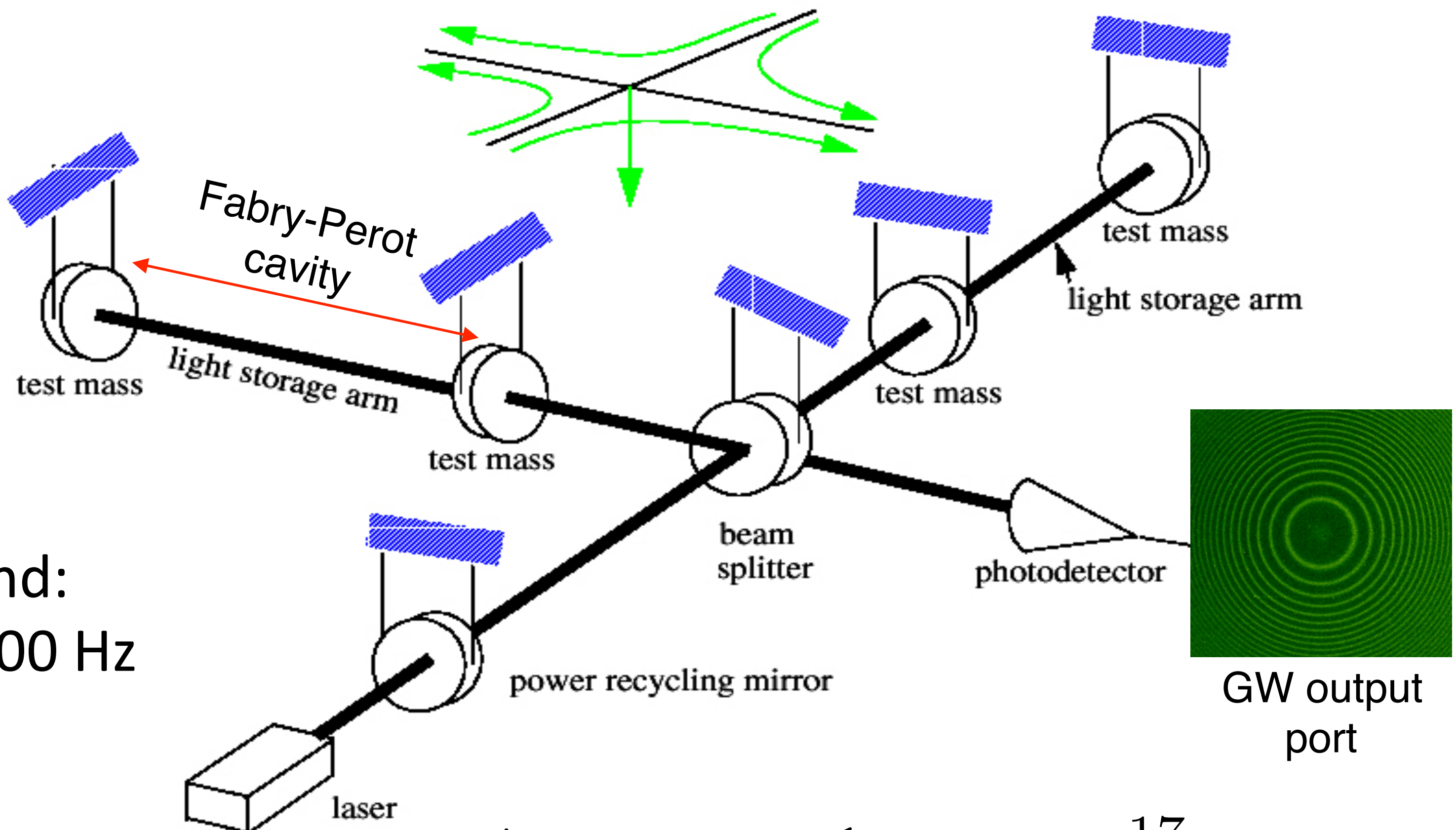


http://www.johnstonsarchive.net/relativity/pictures.html

"+ Polarization" "x Polarization"

GW Detection

(schematic: see P. Brady's lectures for full picture!)



Broadband:
~10 – 2000 Hz

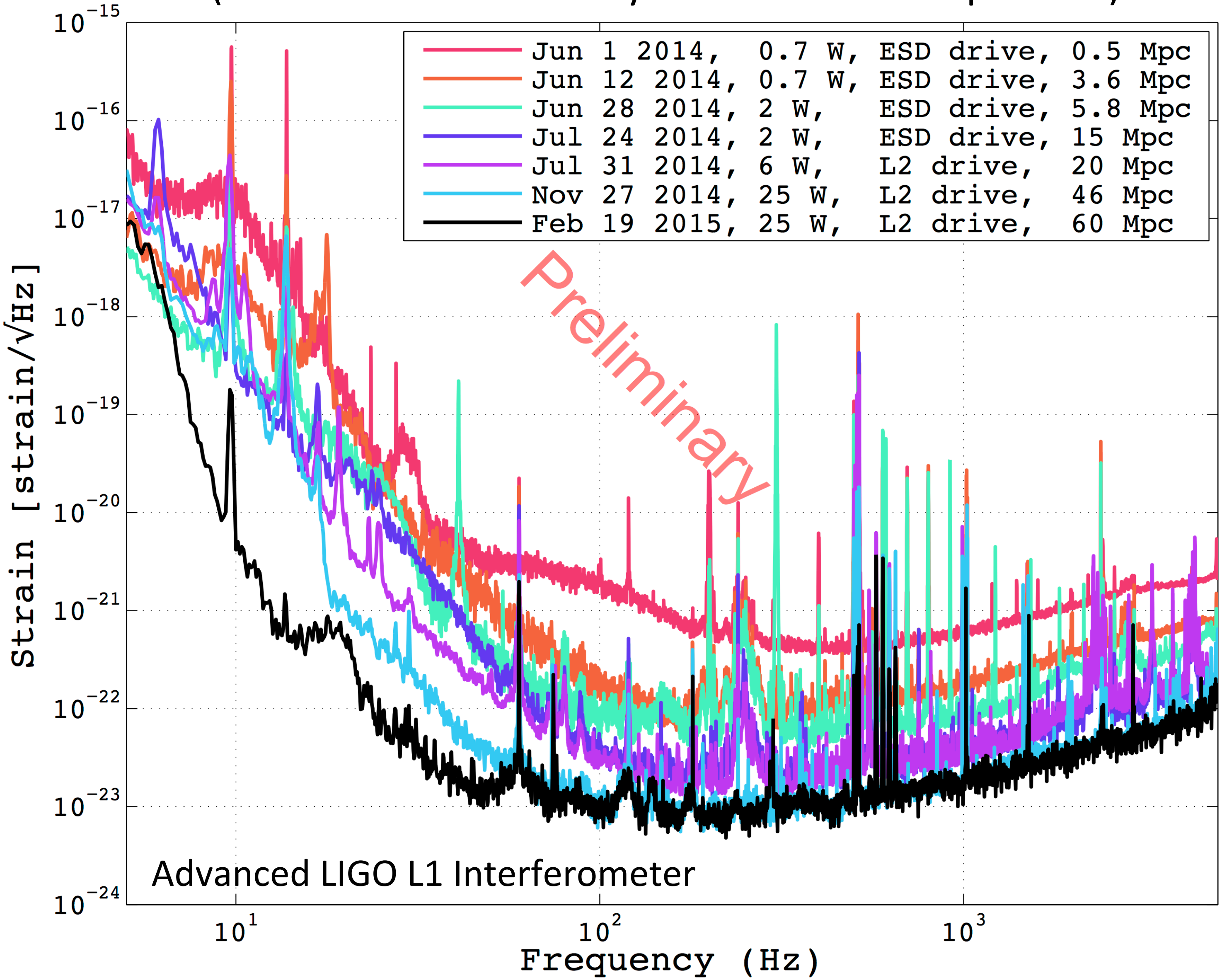
$$\Delta L = hL \lesssim 4 \times 10^{-17}\,\mathrm{cm}$$

Why is h so small?

$$h \lesssim 10^{-22} \qquad L = 4\,\mathrm{km}$$

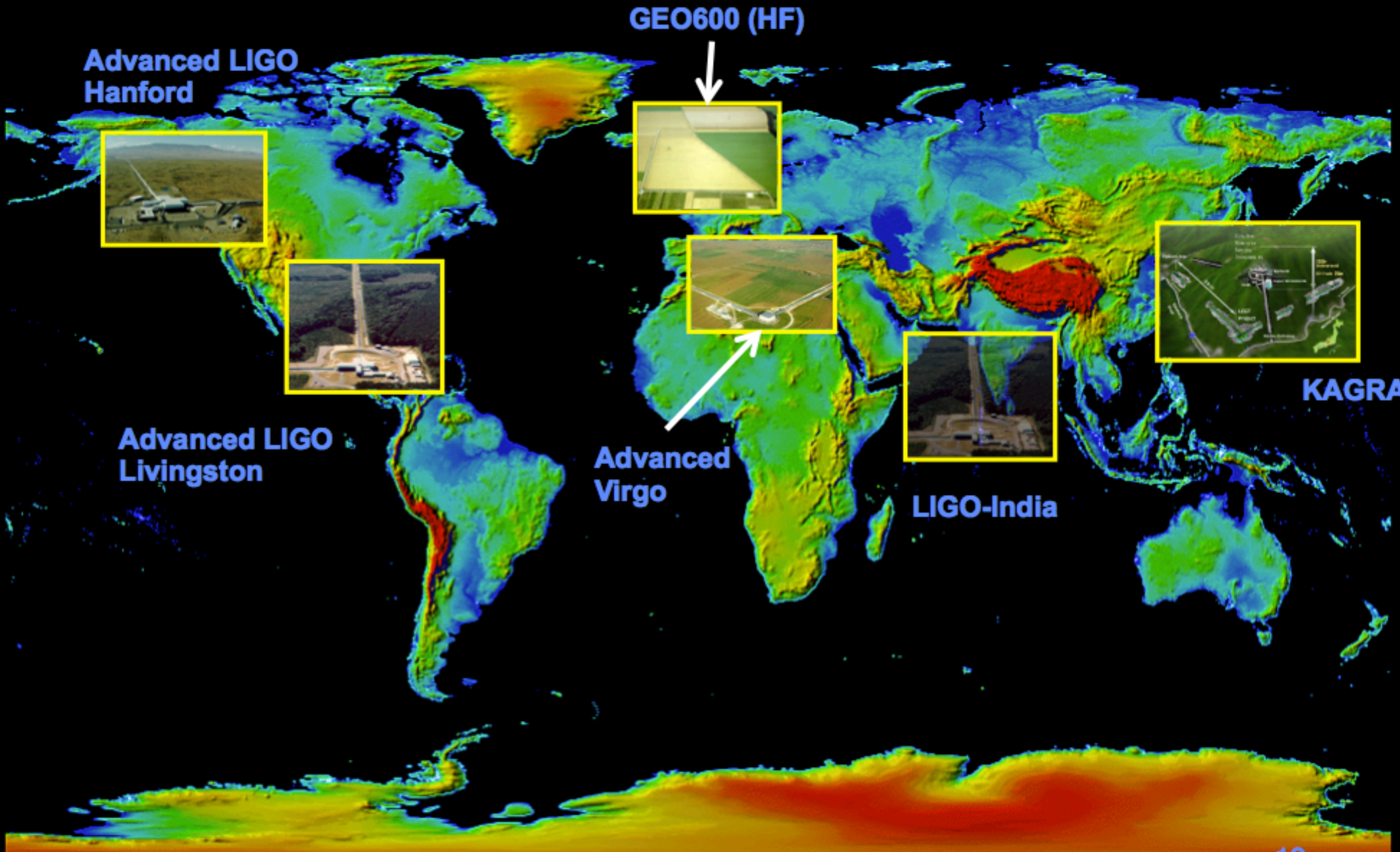
Sensitivity of Laser Interferometers

(schematic: see P. Brady's lectures for full picture!)



https://dcc.ligo.org/LIGO-G1401390/public

The Advanced GW Detector Network



(from Alan Weinstein)

Gravitational Wave Emission

- GWs (in GR!) are to lowest-order **quadrupole waves**.
- Emitted by accelerated aspherical bulk mass-energy motions.
- "Slow-motion" "weak-field" quadrupole approximation:

$$h_{jk}^{TT}(t,\vec{x}) = \left[\frac{2}{c^4}\frac{G}{|\vec{x}|}\ddot{I}_{jk}(t-\frac{|\vec{x}|}{c})\right]^{TT}$$

dimensionless GW "strain" (displacement)

mass quadrupole moment

$$\frac{G}{c^4} \approx 10^{-49}\,\mathrm{s}^2\,\mathrm{g}^{-1}\,\mathrm{cm}^{-1}$$

First Numerical Estimate: $M \equiv$ "aspherical mass"

$$I_{jk} = \int \rho x_j x_k d^3x \quad \frac{d^2}{dt^2}I \sim \mathcal{O}(Mv^2) \quad h \sim \frac{2G}{c^4 D}Mv^2$$

$$M = 1M_\odot \quad v = 0.1c$$
$$D = 10\,\mathrm{kpc}$$
$$\longrightarrow \quad h \sim 10^{-19}$$

GW Emission

- **GWs** are very weak and interact weakly with matter.
  - No human-made sources.

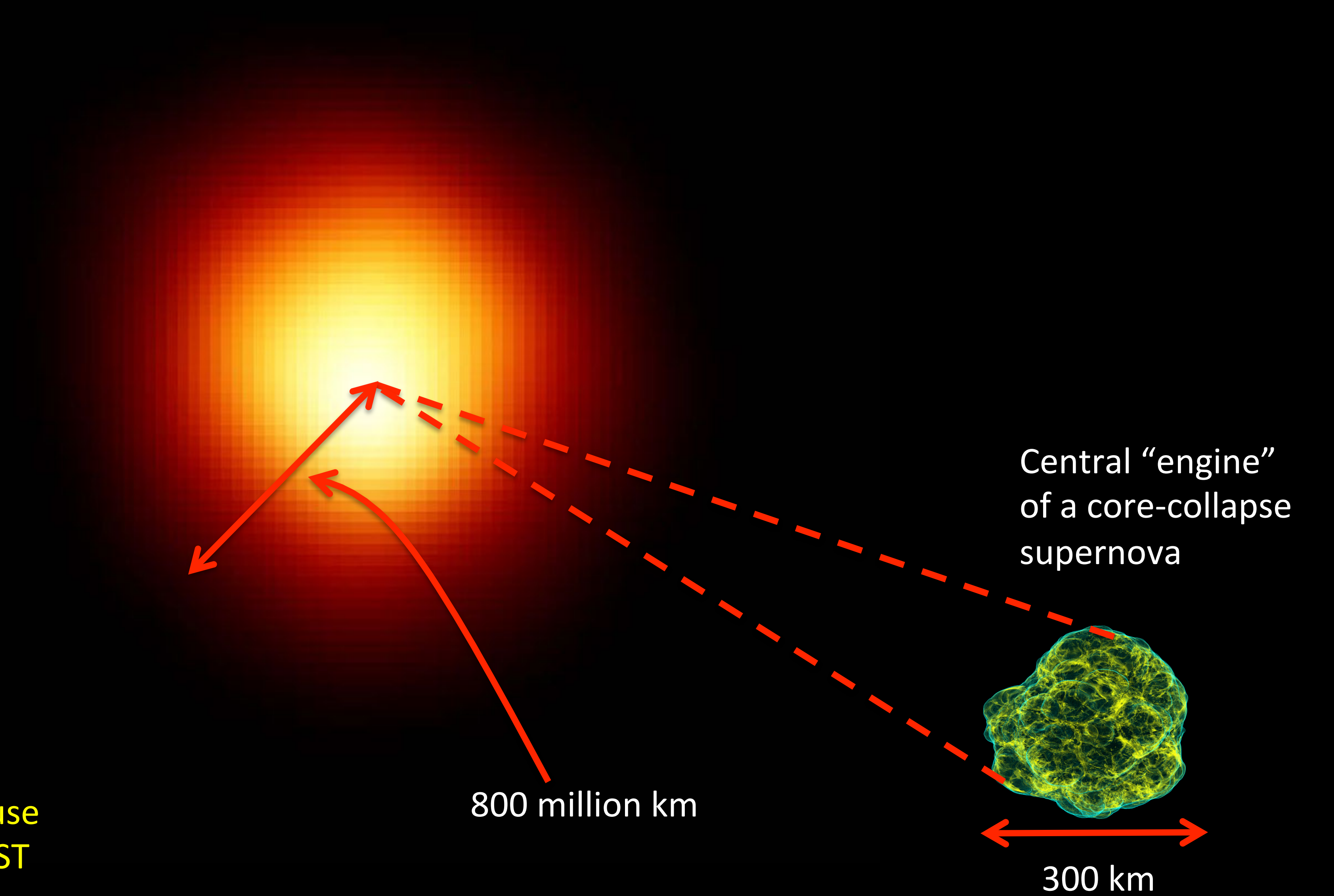
GW generator, TAPIR group, Caltech

GW Emission

- **GWs** are very weak and interact weakly with matter.
  - No human-made sources.
  - Bad: *Very hard to detect*.
  - Good: ***Travel from source to detectors unscathed by intervening material.***

-> **Great opportunity!**

- Study regions of spacetime that opaque to other radiations.



Betelgeuse
NASA/HST

GW Emission

- **GWs** are very weak and interact weakly with matter.
  - No human-made sources.
  - Bad: *Very hard to detect*.
  - Good: ***Travel from source to detectors unscathed by intervening material.***

-> **Great opportunity!**

- Study regions of spacetime that are **opaque to other radiations**.
- Study regions of spacetime that **do not emit other radiations**. (*Kip Thorne: the dark and warped side of the universe*)

GW Emission

- **GWs** are very weak and interact weakly with matter.
  - No human-made sources.
  - Bad: *Very hard to detect*.
  - Good: ***Travel from source to detectors unscathed by intervening material.***

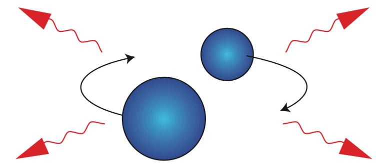
-> **Great opportunity!**

- Study regions of spacetime that are **opaque to other radiations**.
- Study regions of spacetime that **do not emit other radiations**. (*Kip Thorne: the dark and warped side of the universe*)
- Waves linear when they get to us, BUT generated in the strong-field non-linear regime of GR!
  GWs are the only way to probe GR in the non-linear regime!
  (-> see Nico Yunes's lectures)

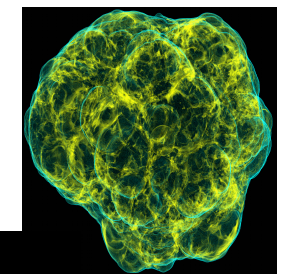
(Expected) Astrophysical Sources of GWs

-> Anything that gives a large time-changing quadrupole moment!

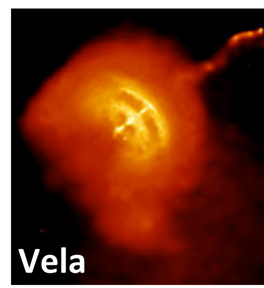
Coalescing binaries of compact stars

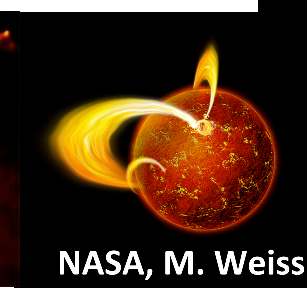


Stellar collapse & core-collapse supernovae



Galactic neutron stars:
mountains, glitches, quakes



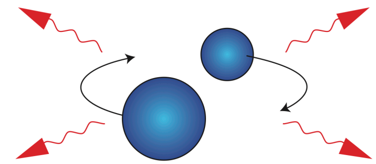


Cosmological and astrophysical stochastic backgrounds

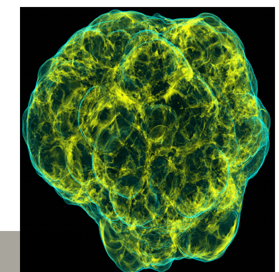
Cosmic strings, fast radio bursts,
+ your favorite crazy source

(Expected) Astrophysical Sources of GWs

-> Anything that gives a large time-changing quadrupole moment!

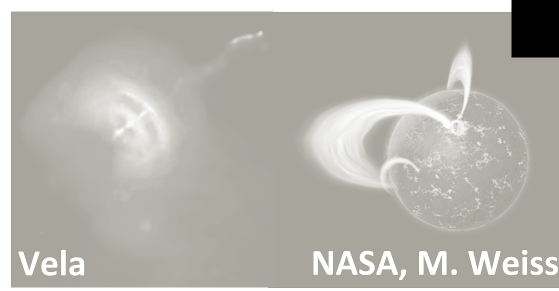


Coalescing binaries of compact stars
(-> this lecture & lecture 2)



Stellar collapse & core-collapse supernovae
(-> lecture 3)

Galactic neutron stars:
mountains, glitches, quakes



Cosmological and astrophysical stochastic backgrounds

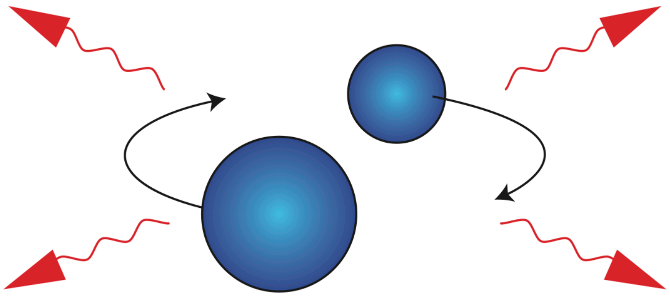
Cosmic strings, fast radio bursts,
+ your favorite crazy source

GWs from Coalescing Binaries

GW Emission guaranteed!

- **Neutron-Star – Neutron-Star (NSNS)**
- Black-Hole – Neutron-Star (BHNS)
- Black-Hole – Black Hole
  (BBH – binary black hole)
- White-Dwarf – White Dwarf (WDWD)

Credit: D. Tsang



Radio Astronomy:

- Discovery of pulsars 1967 (radio emitting NSs)
- Pulses show periodic variation if pulsar in binary system. Pulsars with NS companions & double pulsar!

- Five NSNS systems known to merge within Hubble time.

GWs from NSNS Binaries

PSR Name	P_s (ms)	$\dot{P}_s$ 10^{-18} (ss^{-1})	M_{psr} ($M_\odot$)	M_c ($M_\odot$)	P_{orb} (hr)	e	$f_{b,obs}$	$f_{b,eff}$	τ_{age}^{a} (Gyr)	τ_{mgr} (Gyr)
Tight binaries										
B1913+16	59.	8.63	1.44	1.39	7.75	0.617	5.72	2.26	0.0653	0.301
B1534+12	37.9	2.43	1.33	1.35	10.1	0.274	6.04	1.89	0.200	2.73
J0737-3039A	22.7	1.74	1.34	1.25	2.45	0.088		1.55	0.142	0.086
J0737−3039B	2770.	892.			2.45	0.088		14.	0.0493	
J1756−2251	28.5	1.02	1.4	1.18	7.67	0.181		1.68	0.382	1.65
J1906+0746	144.	20300.	1.25	1.37	3.98	0.085		3.37	0.000112	0.308
Wide binaries										
J1518+4904	40.94	0.028	1.56	1.05	206.4	0.249		1.94	29.2	$>\tau_H$
J1811−1736	104.18	0.901	1.60	1.00	451.2	0.828		2.92	1.75	$>\tau_H$
J1829+2456	41.01	0.053	1.14	1.36	28.3	0.139		1.94	12.3	$>\tau_H$
J1753−2240[c]	95.14	0.97	*1.25*	*1.25*	327.3	0.303		2.80	1.4	$>\tau_H$

O'Shaughnessy & Kim 2010

Many more may be out there, pulsar surveys may miss vast majority.

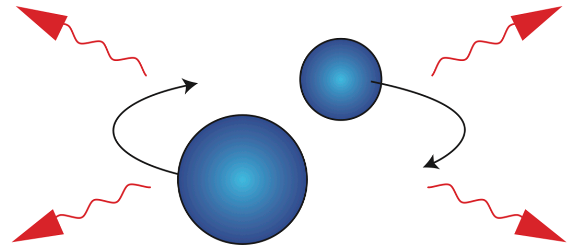
A few questions:

- How strong GWs? How sensitive detector to see how far?
- How estimate time to merger, typical GW frequency near merger?
- How many mergers do we expect to see with a given detector?

A Simplistic Signal Model for Merging Binaries (1)

(see Patrick Brady's lectures for better models!)

Consider a circular binary of point particles in x-y plane.
Equal mass: $m_1 = m_2 = m$



$$r_1^i(t) = \frac{a}{2}\{\cos\theta, \sin\theta, 0\} \qquad a = |r_1| + |r_2| \text{ (semi-major axis)}$$

$$r_2^i(t) = \frac{a}{2}\{-\cos\theta, -\sin\theta, 0\} \qquad M = m_1 + m_2 = 2m$$

$$\theta = \omega t = 2\pi f_{\mathrm{orb}} t = 2\pi \frac{t}{P_{\mathrm{orb}}} \qquad \omega = \sqrt{\frac{GM}{a^3}}$$

Now evaluate:

$$I_{jk} = \int \rho x_j x_k d^3x \qquad h_{jk}^{TT}(t,\vec{x}) = \left[\frac{2}{c^4}\frac{G}{|\vec{x}|}\ddot{I}_{jk}(t - \frac{|\vec{x}|}{c})\right]^{TT}$$

A Simplistic Signal Model for Merging Binaries (2)

$$I_{xx} = \int d^3x(\rho x^2) = 2mx_1^2$$

$$= 2m\frac{a^2}{4}\cos^2\omega t = \frac{ma^2}{2}\cos^2\omega t$$

use: $$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$= \frac{ma^2}{4}(1 + \cos 2\omega t)$$

Now ignore constant term (will drop out anyway).

A Simplistic Signal Model for Merging Binaries (3)

$$I_{ij} = \frac{1}{4} m a^2 \begin{pmatrix} \cos 2\omega t & \sin 2\omega t & 0 \\ \sin 2\omega t & -\cos 2\omega t & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\ddot{I}_{ij} = m a^2 \omega^2 \begin{pmatrix} -\cos 2\omega t & -\sin 2\omega t & 0 \\ -\sin 2\omega t & \cos 2\omega t & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

For observer at distance D along the z axis already in TT gauge:

$$h_{ij}^{TT} = \frac{2G}{c^4} \frac{m a^2 \omega^2}{D} \begin{pmatrix} -\cos 2\omega t & -\sin 2\omega t & 0 \\ -\sin 2\omega t & \cos 2\omega t & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

A Simplistic Signal Model for Merging Binaries (4)

$$h_+ = -\frac{2G}{c^4}\frac{ma^2\omega^2}{D}\cos 2\omega t$$

$$h_\times = -\frac{2G}{c^4}\frac{ma^2\omega^2}{D}\sin 2\omega t$$

GW frequency
= 2 x orbital frequency

$$\frac{dE_{\rm GW}}{dt} = P = \frac{G}{c^5}\langle \dddot{I}_{ij}\dddot{I}_{ij}\rangle$$

time average
over a cycle

Radiated energy must come from orbital energy -> also change of angular momentum. Change of orbital separation:

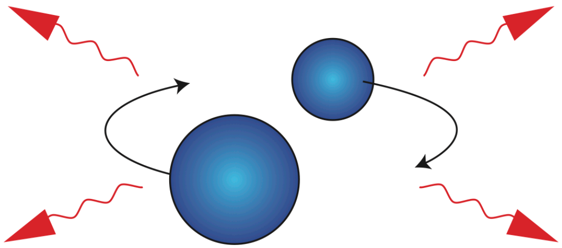
$$\left\langle \frac{da}{dt}\right\rangle = -\frac{64}{5}\frac{G^3}{c^5}\frac{m_1 m_2 M}{a^3} \qquad a(t) = \left(\frac{256}{5}\frac{G^3}{c^5}\mu M^2\right)^{\frac{1}{4}}(t_c - t)^{\frac{1}{4}}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad M = m_1 + m_2$$

time of a = 0
(merger time)

A Simplistic Signal Model for Merging Binaries (5)

Can now make useful estimates:



NSNS: $m \approx 1.4 M_\odot$

At merger: $a \approx 2R \approx 2 \times 12\,\mathrm{km}$

$$f_\mathrm{merge} = \frac{1}{\pi}\sqrt{\frac{2Gm}{a^3}}$$

$$f_\mathrm{merge} \approx 1650\,\mathrm{Hz}$$

$$v_\mathrm{merge} = \omega_\mathrm{merge} a_\mathrm{merge} \approx 0.4c$$

$$h_\mathrm{merge} D \approx 0.7\,\mathrm{km} \qquad 1\,\mathrm{pc} = 3.086 \times 10^{18}\,\mathrm{cm}$$

$$h_\mathrm{merge} \approx 2 \times 10^{-22} (100\,\mathrm{Mpc}/D)$$

A Simplistic Signal Model for Merging Binaries (6)

BBH: Black holes more massive than NSs.
Assume 10 $M_\odot$ + 10 $M_\odot$ BBH coalescence.

$$m = 10\,M_\odot \quad a = 2R_s \approx 2 \times 30\,\mathrm{km} = 60\,\mathrm{km}$$

$$f_\mathrm{merge} \approx 1100\,\mathrm{Hz} \qquad h_\mathrm{merge} \approx 5 \times 10^{-22} (1\,\mathrm{Gpc}/D)$$

WDWD: WDs less massive than NSs. 2 x 0.6 $M_\odot$ typical.

$$a \approx 2R_\mathrm{WD} \approx 2 \times 6000\,\mathrm{km}$$

$$f_\mathrm{merge} = 0.27\,\mathrm{Hz}$$

-> not a good source for ground-based detectors
(dominated by local noise background at these f < 10 Hz)

A Simplistic Signal Model for Merging Binaries (7)

Coalescence/Merger time:

$$\tau_{\mathrm{merge}} = a_0^4 \frac{5}{256} \frac{c^5}{G^3} \frac{1}{\mu M^2}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

current separation

$$M = m_1 + m_2$$

For m1=m2=1.4 $M_\odot$:

a_0 = 10^6 km -> τ_{merge} ~ 120 x 10^6 yrs.

a_0 = 1000 km -> τ_{merge} ~ 3700 s

a_0 = 100 km -> τ_{merge} ~ 370 ms

Frequency Evolution

$$\dot{a} = -\frac{64}{5}\frac{G^3}{c^5}\frac{\mu M^2}{a^3}$$

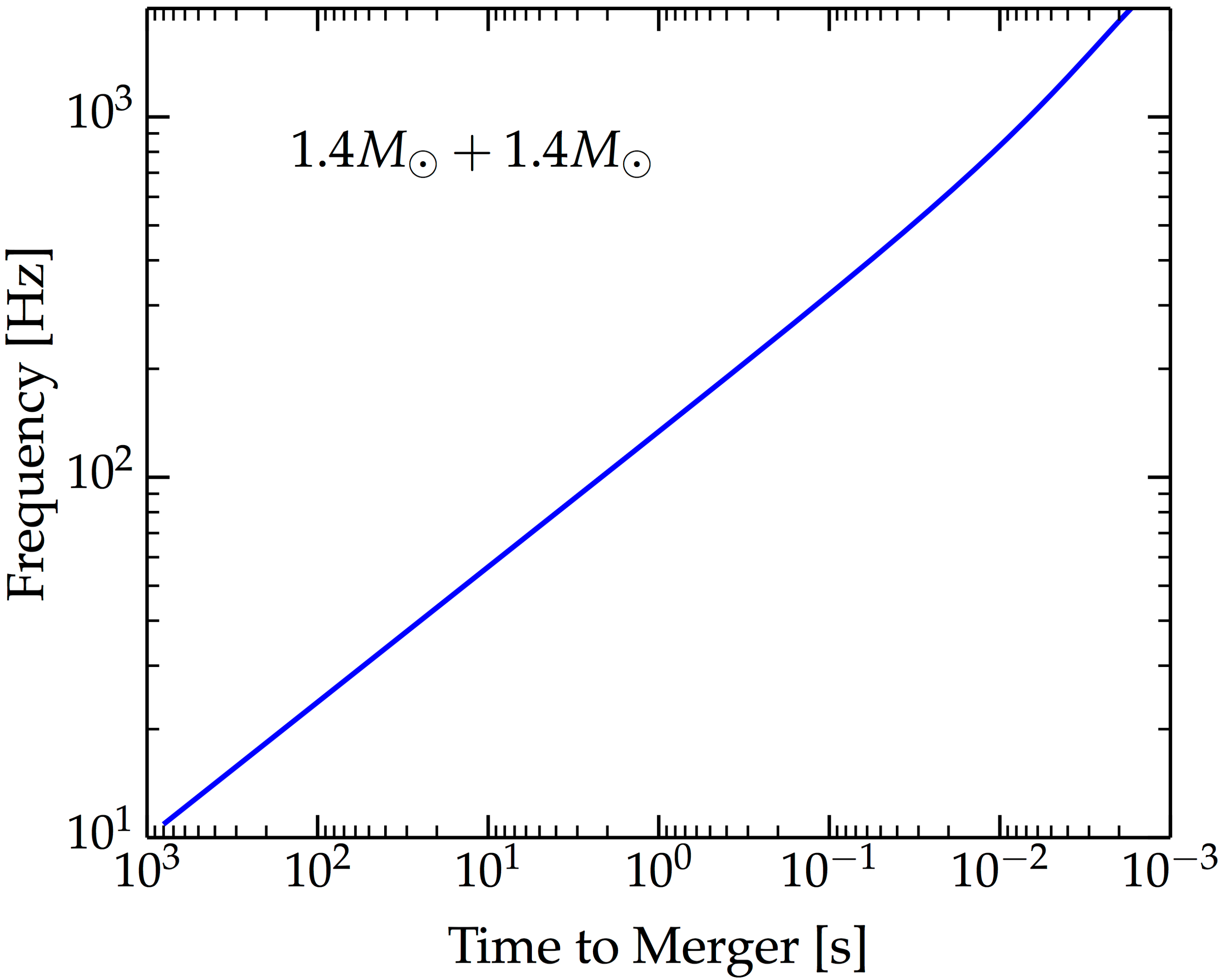
$$f = 2\frac{\omega}{2\pi} = \frac{1}{\pi}(GM)^{\frac{1}{2}}a^{-\frac{3}{2}}$$

$$\dot{f} = \frac{96}{5}\pi^{8/3}\frac{G^{\frac{5}{3}}}{c^5}\mu M^{\frac{2}{3}}f^{\frac{11}{3}}$$

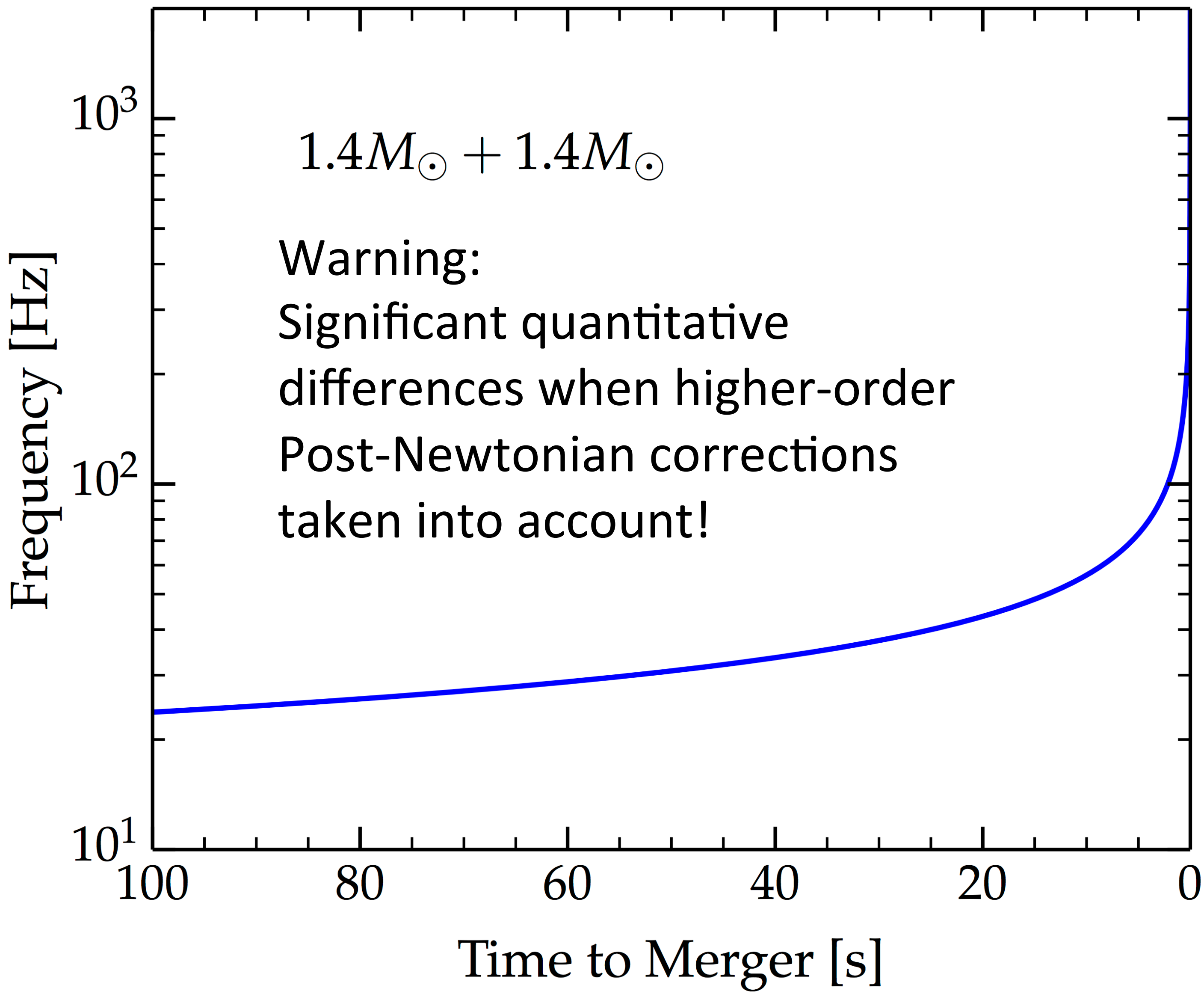
$$\dot{f} = \frac{96}{5}\pi^{\frac{8}{3}}\frac{G^{\frac{5}{3}}}{c^5}\mathcal{M}^{\frac{5}{3}}f^{\frac{11}{3}}$$

$$\mathcal{M} = \mu^{3/5}M^{2/5}$$ "Chirp Mass"

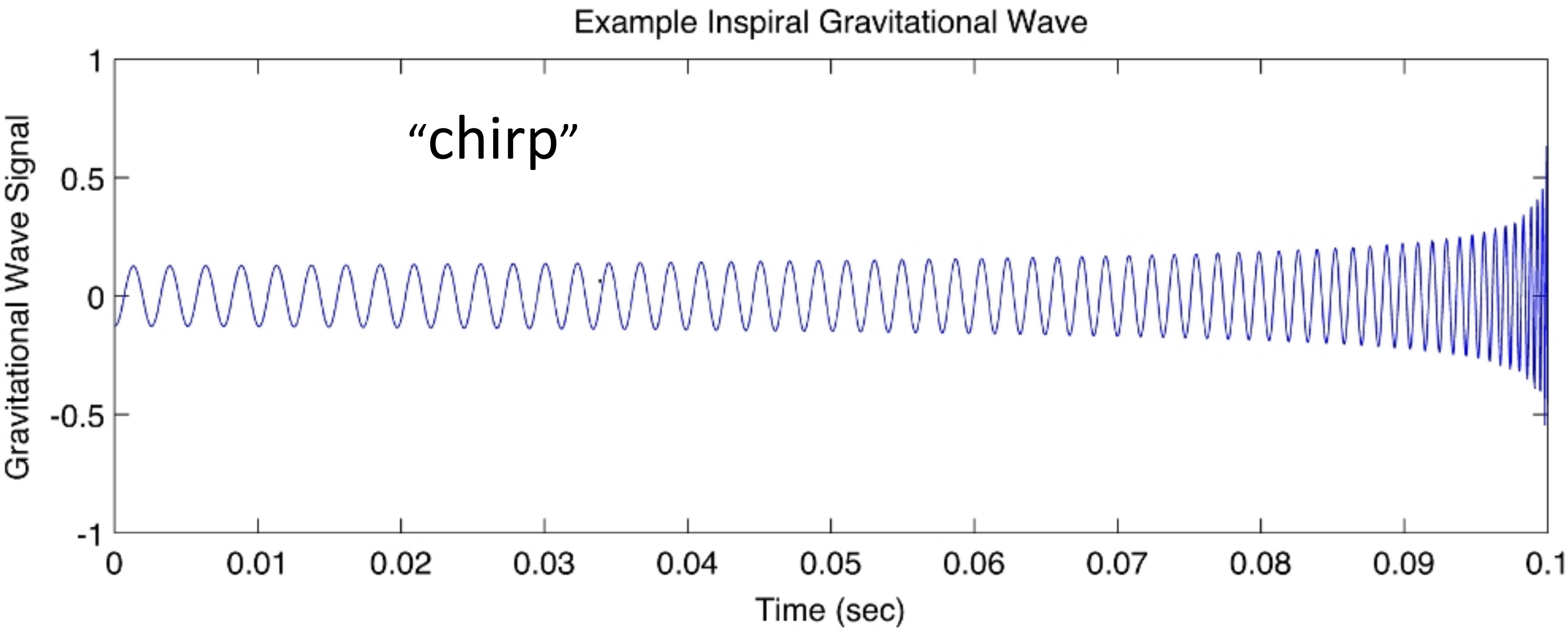
Frequency Evolution



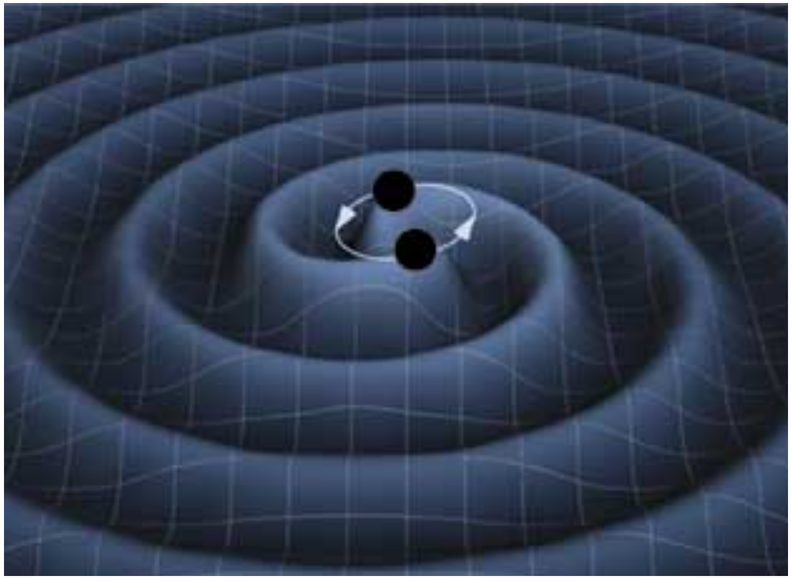
Frequency Evolution



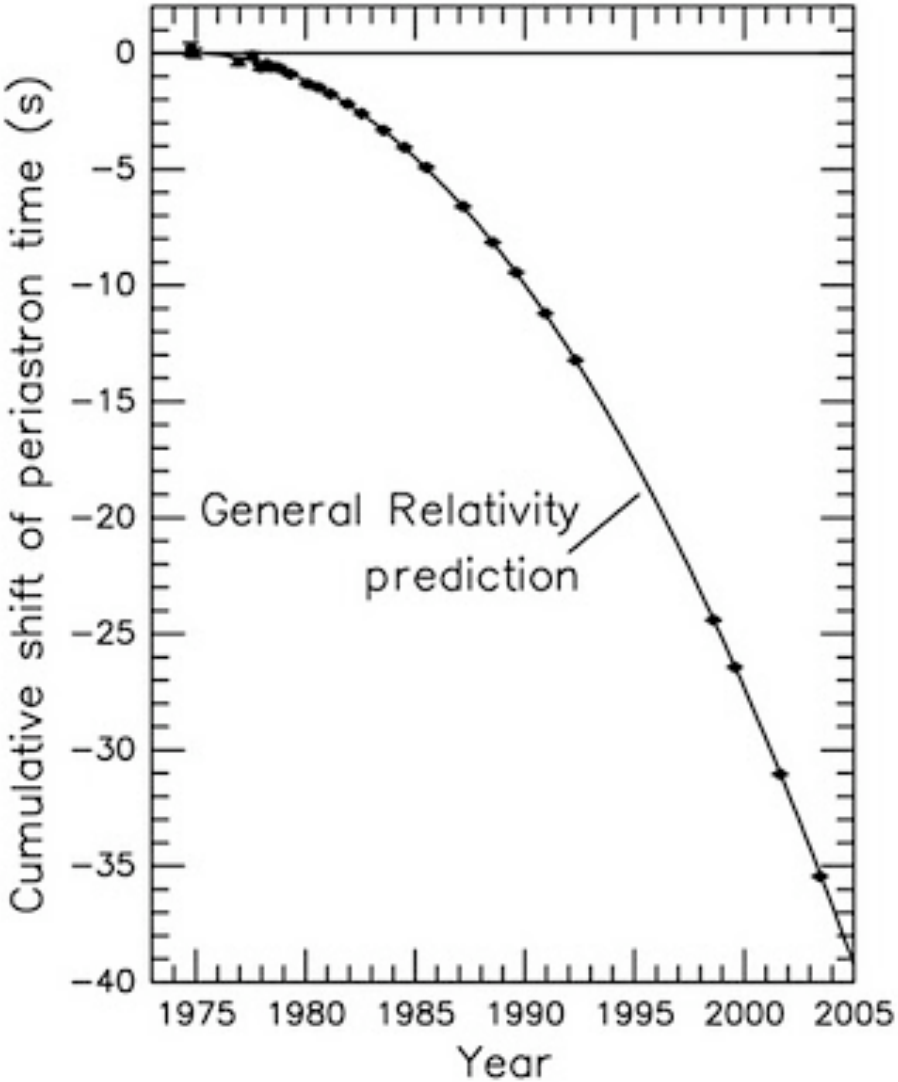
GW Signal: Chirp



Is this real? YES!



- GWs lead to “orbital decay”
  -> binary stars get closer
  to each other.
- Double neutron star systems in the Milky Way.
- PSR 1913+16:
  “Hulse-Taylor Pulsar”
  -> Nobel prize in Physics 1993



What is the merger & detection rate? NSNS

PSR Name	P_s (ms)	$\dot{P}_s$ 10^{-18} (ss^{-1})	M_{psr} ($M_\odot$)	M_c ($M_\odot$)	P_{orb} (hr)	e	$f_{b,obs}$	$f_{b,eff}$	τ_{age}^{a} (Gyr)	τ_{mgr} (Gyr)
Tight binaries										
B1913+16	59.	8.63	1.44	1.39	7.75	0.617	5.72	2.26	0.0653	0.301
B1534+12	37.9	2.43	1.33	1.35	10.1	0.274	6.04	1.89	0.200	2.73
J0737-3039A	22.7	1.74	1.34	1.25	2.45	0.088		1.55	0.142	0.086
J0737−3039B	2770.	892.			2.45	0.088		14.	0.0493	
J1756−2251	28.5	1.02	1.4	1.18	7.67	0.181		1.68	0.382	1.65
J1906+0746	144.	20300.	1.25	1.37	3.98	0.085		3.37	0.000112	0.308
Wide binaries										
J1518+4904	40.94	0.028	1.56	1.05	206.4	0.249		1.94	29.2	$>\tau_H$
J1811−1736	104.18	0.901	1.60	1.00	451.2	0.828		2.92	1.75	$>\tau_H$
J1829+2456	41.01	0.053	1.14	1.36	28.3	0.139		1.94	12.3	$>\tau_H$
J1753−2240[c]	95.14	0.97	*1.25*	*1.25*	327.3	0.303		2.80	1.4	$>\tau_H$

O'Shaughnessy & Kim 2010

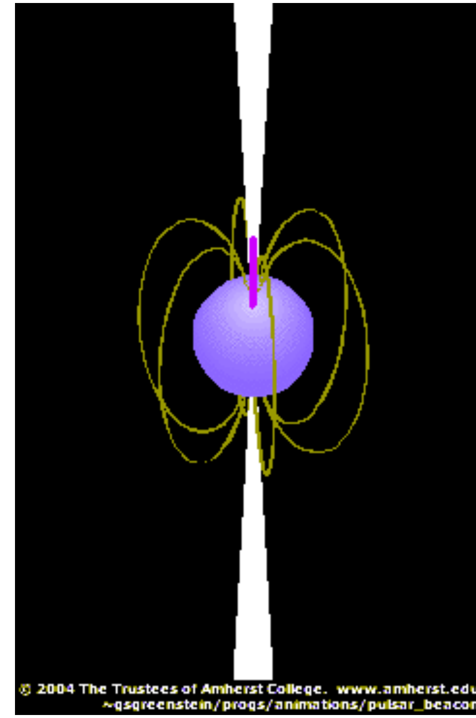
Use observational data to estimate merger rate in the Milky Way:

$R_{\rm MW}$ -> # of mergers / year / MW-equivalent galaxy

Detection rate: $\dot{N} = R_{\rm MW} \times N_{\rm MWG}$

# of MW-equiv. galaxies in observable volume

Milky Way NSNS Merger Rate



- Need to know how many NSNS systems in the Milky Way.
- Need to know the typical lifetime.

$$R_{\mathrm{MW}} = \frac{N_{\mathrm{NSNS}}}{\tau} \qquad \tau = \frac{P}{2\dot{P}} + \tau_{\mathrm{merge}}$$

Characteristic pulsar age.

- Problem: know only 5 NSNS binaries that will merge. How estimate how many we are missing?

MW volume

$$R_{\mathrm{MW}} = \sum_i \frac{V_{\mathrm{MW}}}{V_{\mathrm{max},i}} \frac{1}{\tau_i}$$

Volume out to which binary could have been found.

Further reading, e.g.: Kalogera+01,04, Phinney 1991, Kim+03, O'Shaughnessy&Kim 10

Milky Way NSNS Merger Rate

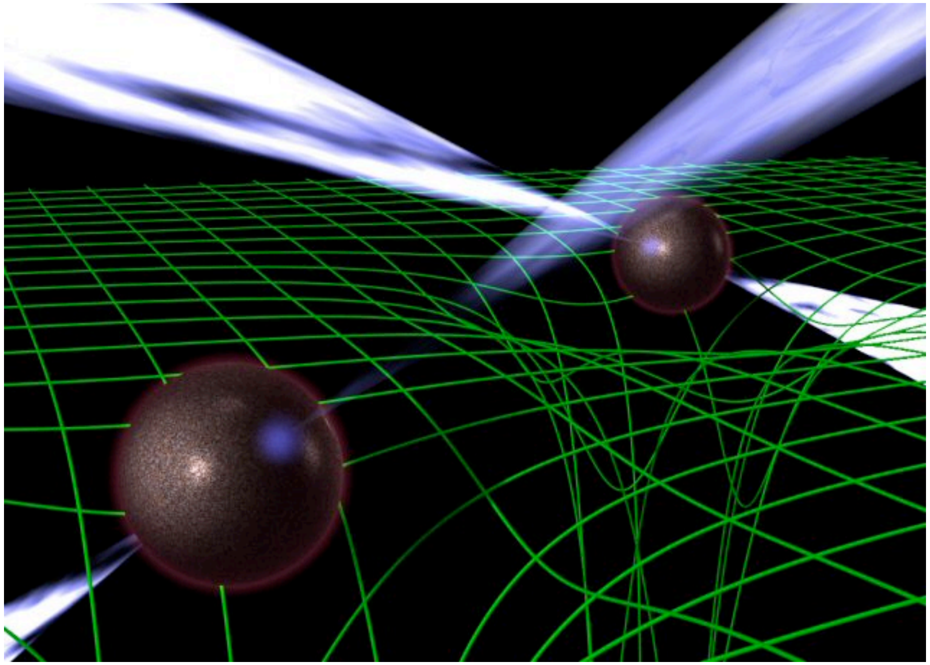
- Merger rate dominated by double-pulsar system J0737–3039.

$$R_{\rm MW} \approx \frac{10^4}{142\,{\rm Myr} + 86\,{\rm Myr}} \approx 44\,{\rm Myr}^{-1}$$

Uncertainty in estimate of existence of ~10^4 similar NSNSs.

- Luminosity distribution of pulsars (can't find faint pulsars).
- Pulsar beaming widths un-certain.
- Distribution of pulsars in MW.

Roughly factor ~10-100 uncertainty.



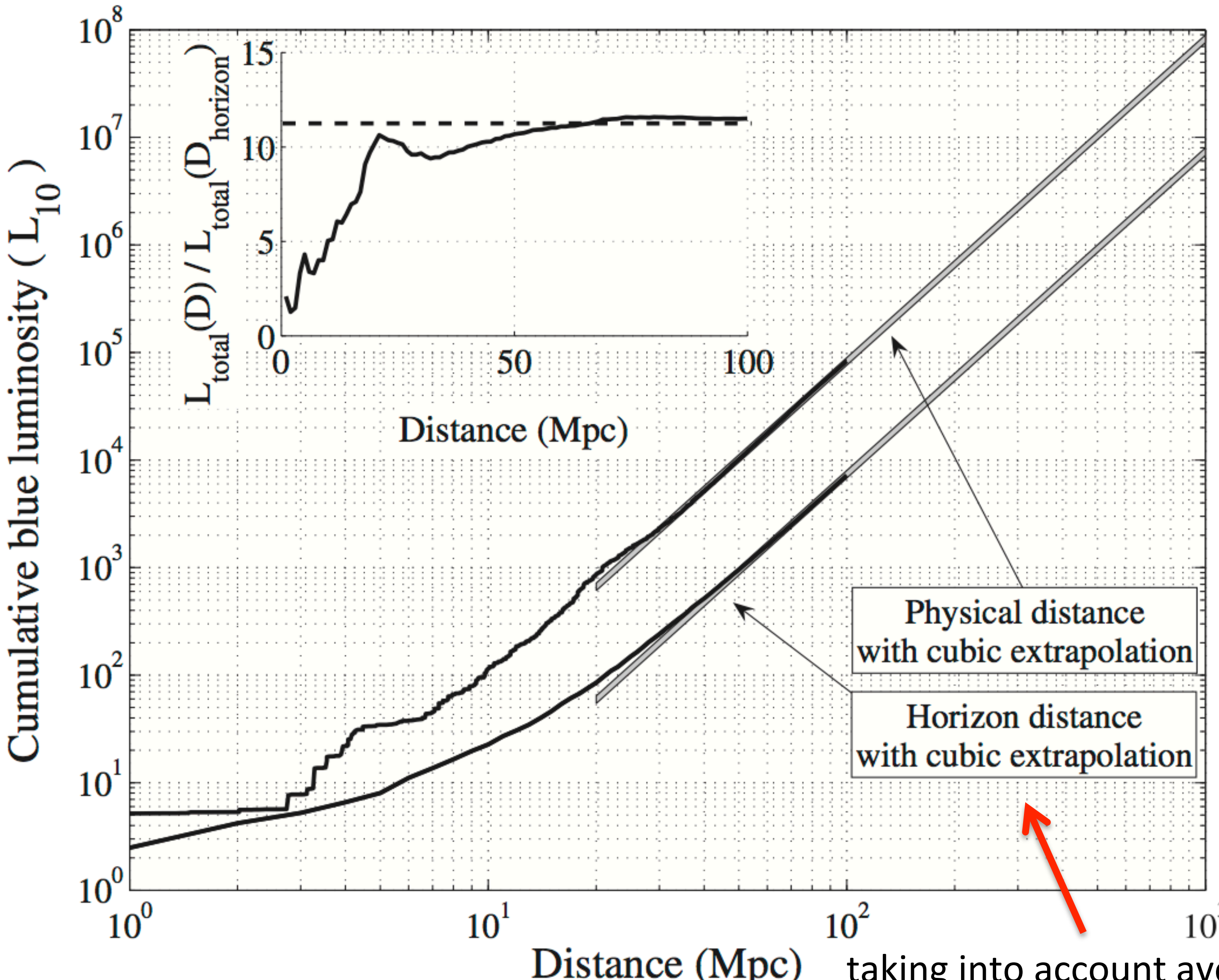
http://www.jb.man.ac.uk/pulsar/doublepulsarcd/

NSNS Detection Rate

- Horizon distance D_H: Distance at which optimally oriented NSNS merger observed by single detector w/ signal-to-noise ratio = 8.
- Advanced LIGO: $D_H \sim 445$ Mpc.
- For estimating detection rate: need N_{MWG} out to 445 Mpc.
- Must take into account that few NSNS optimally oriented.

- "Young" systems will dominate merger rate: young star populations have a high B (blue) luminosity (young, hot & massive stars).
- MW blue luminosity: 1.7 L_{10} (10^{10} $L_{B\odot}$).
- Sum up *accessible* blue luminosity as a function of distance.

NSNS Detection Rate



taking into account averages over sky-location & orientation

Kopparapu+08

NSNS Detection Rate

- Our estimate:

$$N_{\rm MWG} \approx \frac{4\pi}{3} \left(\frac{D_{\rm H}}{\rm Mpc}\right)^3 \frac{0.0116}{(2.26)^3}$$

Good fit for D_H > 30 Mpc.

Abadie et al. 2010, CQG 27, 173001

$$\dot{N} = R_{\rm MW} \times N_{\rm MWG}$$

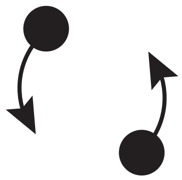
$$R_{\rm MW} \approx 44\,{\rm Myr}^{-1} \quad D_{\rm H} \approx 445\,{\rm Mpc}$$

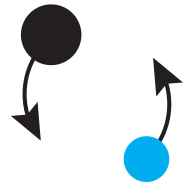
$$N_{\rm MWG} \approx \frac{4\pi}{3} \left(\frac{D_{\rm H}}{\rm Mpc}\right)^3 \frac{0.0116}{(2.26)^3} \approx 3.7 \times 10^5$$

$$\dot{N} \approx 16\,{\rm yr}^{-1}$$

-> so expect of order 10 detected NSNS mergers / year at aLIGO design sensitivity.
But: uncertainties -> could be 1 (or less!), could be 100.

BBH and BHNS Merger Rates





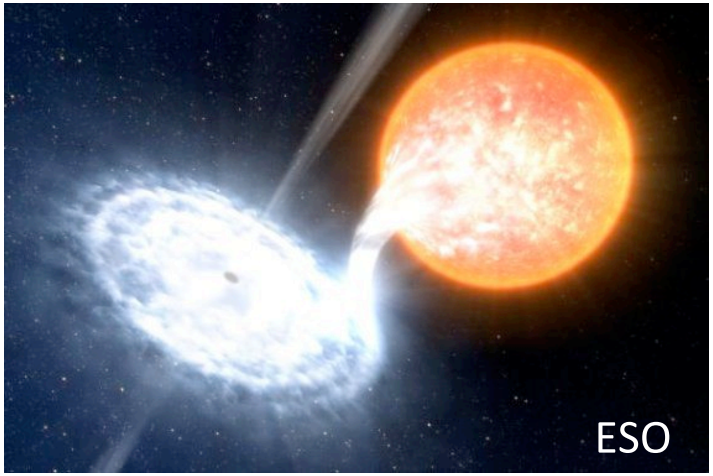
- Problem: No known galactic BBH and BHNS binaries!
- Cannot use same approach as for NSNS.

Answer: **Population Synthesis**

- Monte-Carlo binary evolution model:

  Randomly select initial parameters for large ensemble of massive binary stars, follow stellar evolution.

  Predict theoretical NSBH, BHBH (and also NSNS) occurrence rates.



References for further reading:
Dominik+12,13,14
O'Shaughnessy+12
Postnov & Yungelson 06

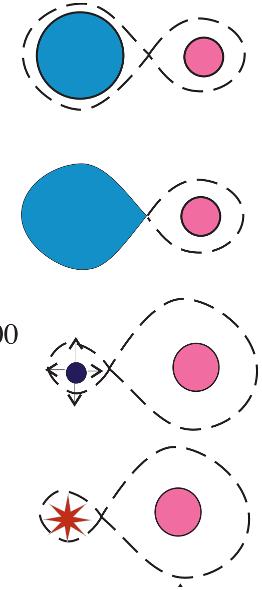
Schematic Massive Binary Evolution

$T \sim 3$ Myr, $N \sim 10^4$ — Two OB main-sequence stars

$T \sim 10^4$ yr, $N \sim 30$ — More massive star (primary) overfills Roche lobe

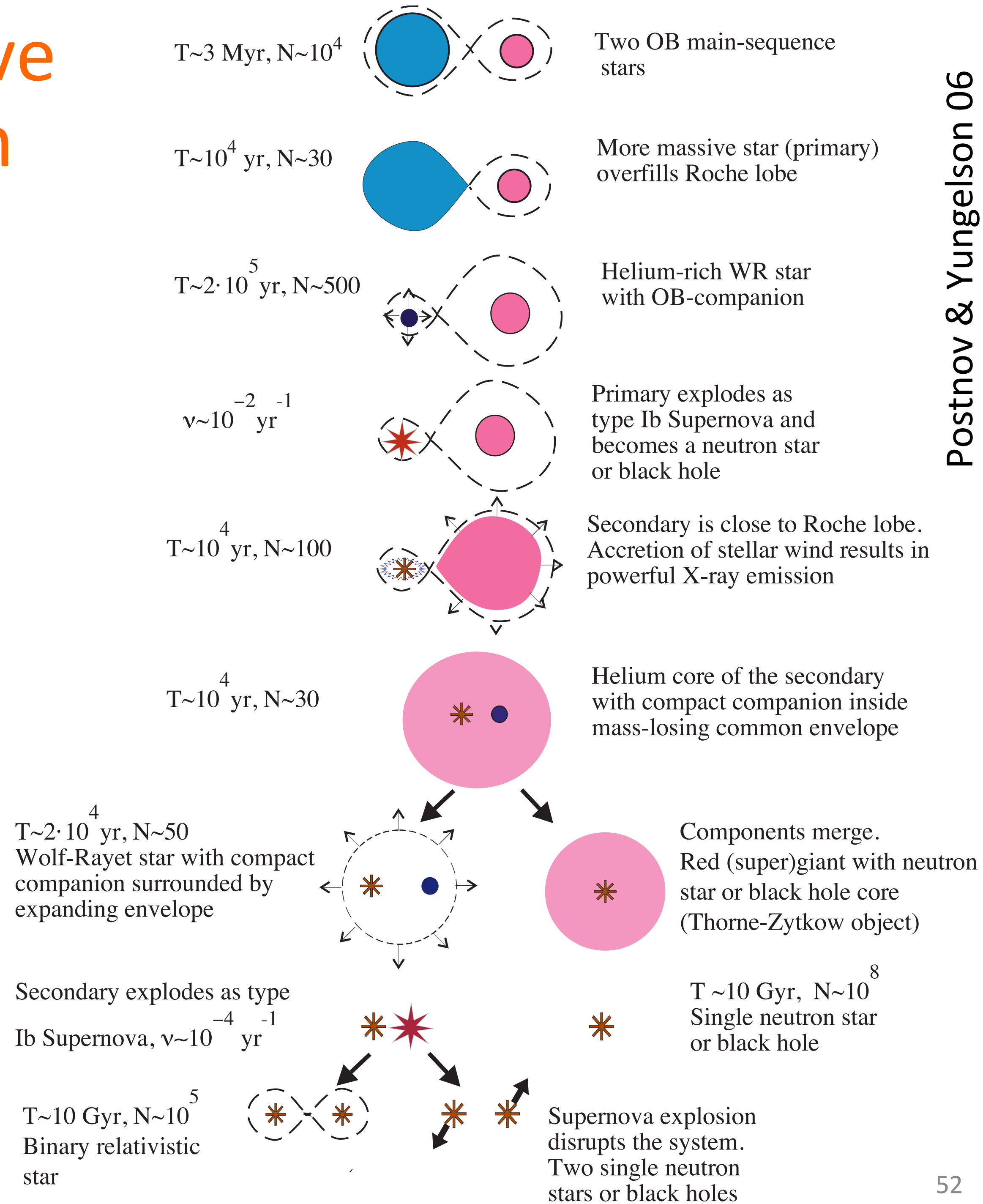
$T \sim 2 \cdot 10^5$ yr, $N \sim 500$ — Helium-rich WR star with OB-companion

$\nu \sim 10^{-2}$ yr^{-1} — Primary explodes as type Ib Supernova and becomes a neutron star or black hole



Postnov & Yungelson 06

Schematic Massive Binary Evolution



Postnov & Yungelson 06

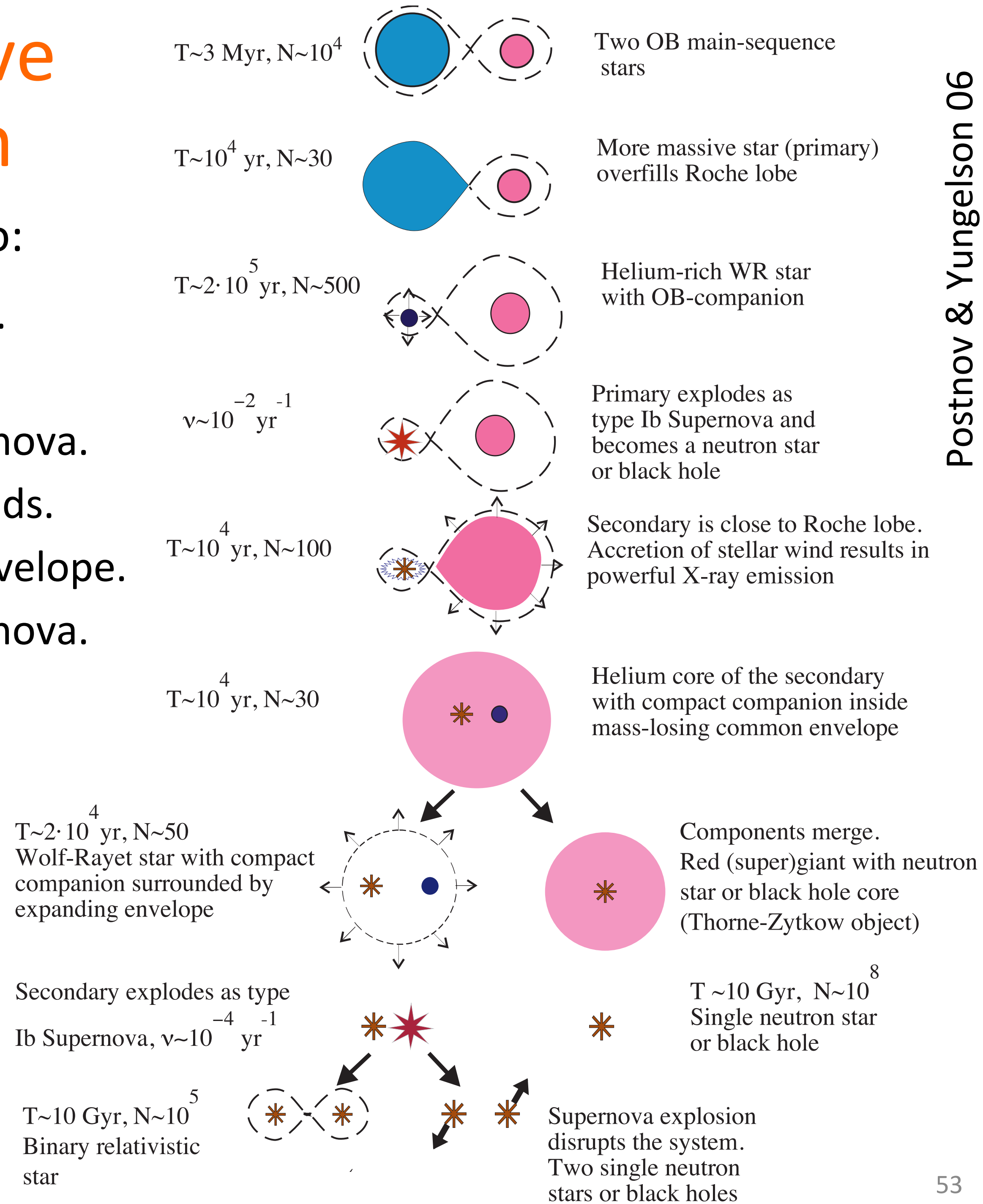
Schematic Massive Binary Evolution

One possible general scenario:

- Primary evolves and expands.
- Mass transfer.
- Collapse #1 & perhaps Supernova.
- Secondary evolves and expands.
- Mass transfer or common envelope.
- Collapse #2 & perhaps Supernova.

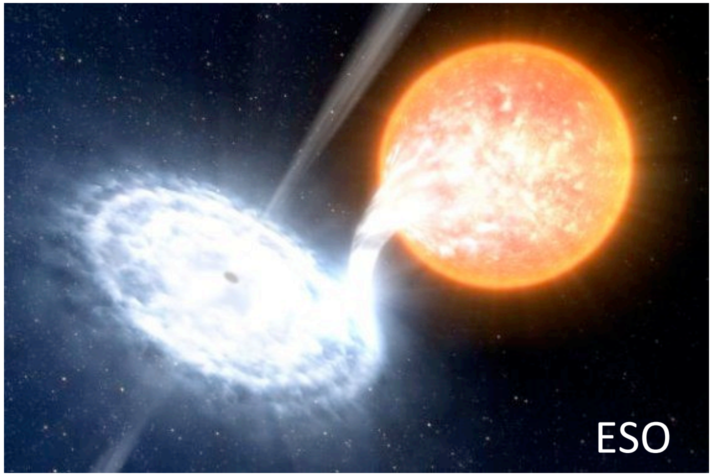
Other considerations:

- Mergers possible.
- SNe disrupt binary due to kick on NS or BH.
- Common envelope crucial to reduce orbital separation.



BBH and BHNS Merger Rates

- Problem: No known galactic BBH and BHNS binaries!
- Cannot use same approach as for NSNS.



Answer: **Population Synthesis**

- Monte-Carlo binary evolution model.
- Parameterizes uncertain astrophysics:
  - binary fraction;
  - mass exchange and "common envelope" evolution;
  - kicks;
  - BH vs. NS formation in supernovae.

HUGE uncertainties!!!

- Saving grace: it appears that binary fraction ~1 for massive stars; 70% will interact. (Sana+12)

References for further reading:
Dominik+12,13,14
O'Shaughnessy+12
Postnov & Yungelson 06

Recent Rate Estimates for Advanced Detectors

Abadie et al. 2010, CQG 27, 173001

("official" LIGO Scientific Collaboration rate estimates)

IFO	Source[a]	$\dot{N}_{\rm low}$ yr^{-1}	$\dot{N}_{\rm re}$ yr^{-1}	$\dot{N}_{\rm high}$ yr^{-1}	$\dot{N}_{\rm max}$ yr^{-1}
	NS–NS	0.4	40	400	1000
	NS–BH	0.2	10	300	
Advanced	BH–BH	0.4	20	1000	

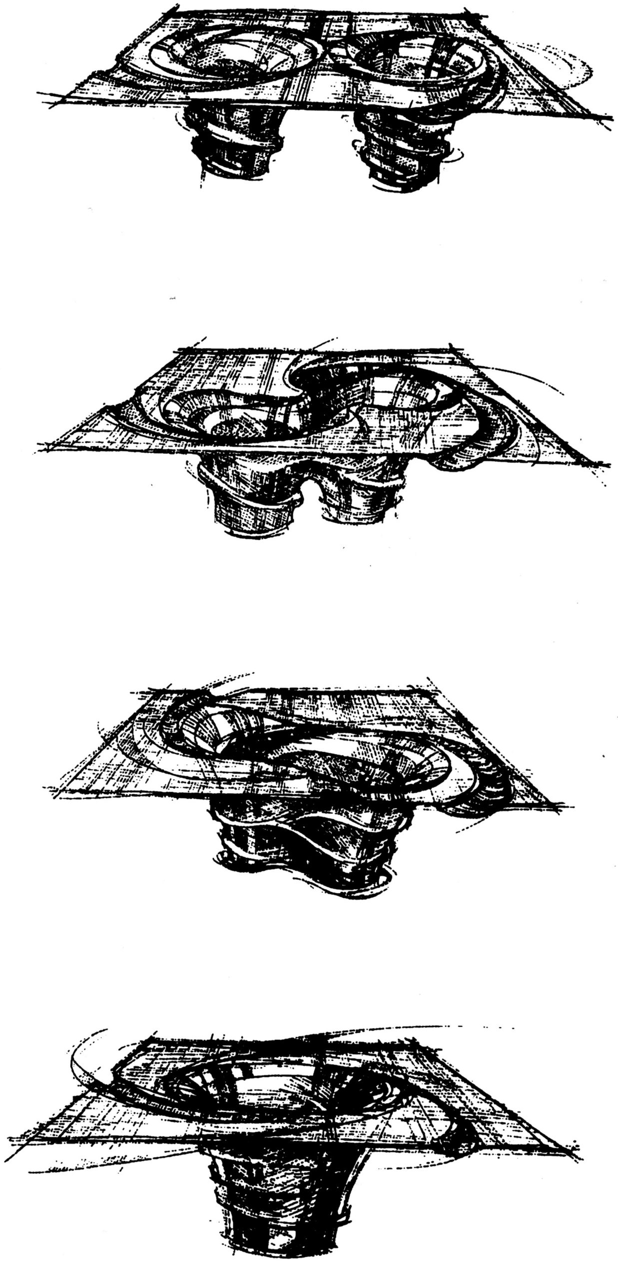
Warning: Population synthesis!

"Realistic" (=best-guess) event rates per year with advanced detectors later this decade

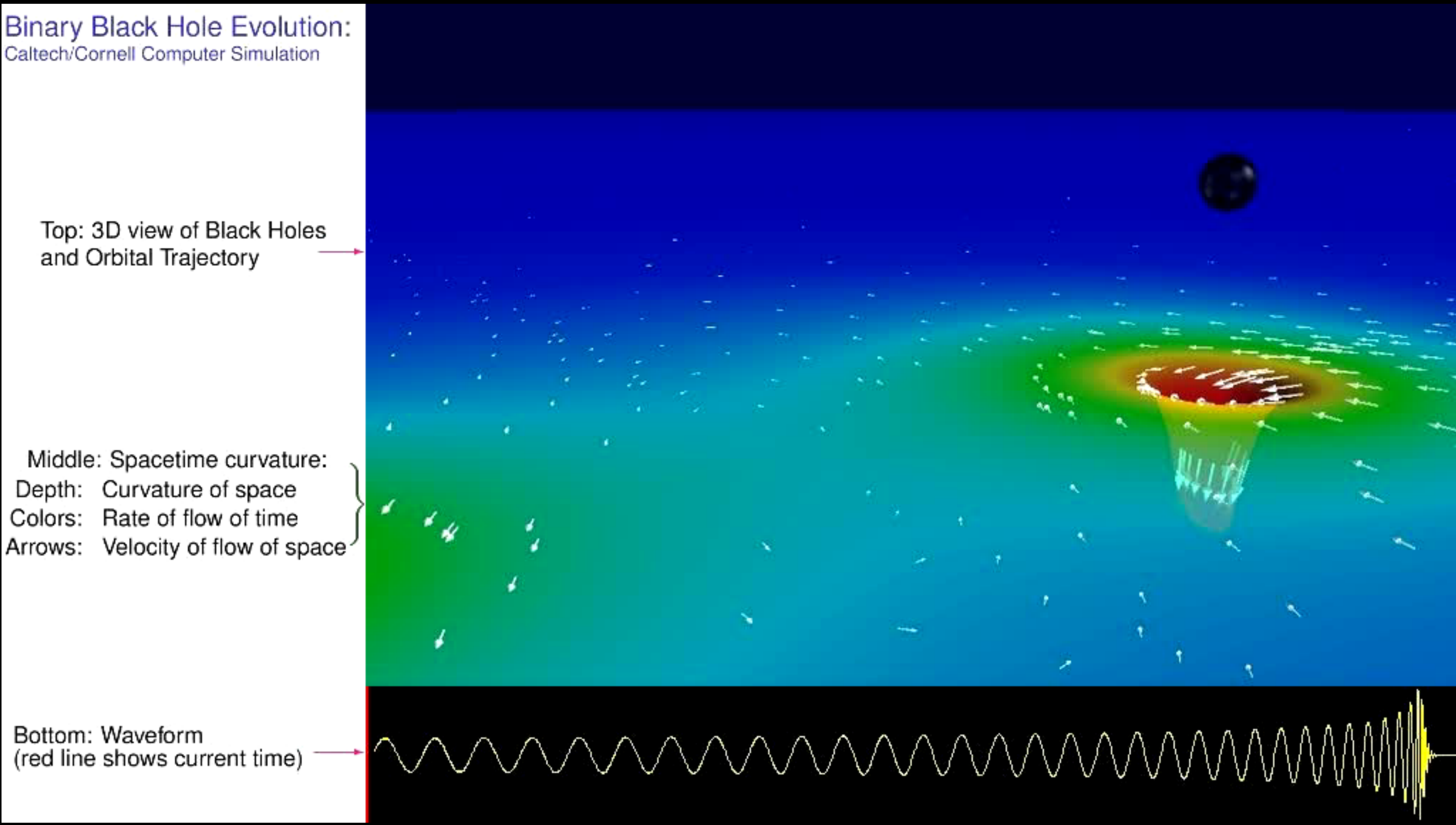
Model	R_D (aLIGO $\rho \geq 8$) yr^{-1}	R_D (3-det network $\rho \geq 10$) yr^{-1}
NS-NS		
Standard	1.3 (1.1)	3.2 (2.7)
Optimistic CE	3.9 (3.3)	9.2 (7.7)
Delayed SN	1.9 (1.7)	4.5 (4.0)
High BH Kick	1.2 (1.1)	3.0 (2.7)
BH-NS		
Standard	1.0 (1.2)	2.4 (2.7)
Optimistic CE	5.7 (6.5)	13.8 (15.4)
Delayed SN	0.5 (0.9)	1.1 (2.3)
High BH Kick	0.01 (0.08)	0.04 (0.2)
BH-BH		
Standard	227 (427)	540 (1017)
Optimistic CE	676 (1585)	1610 (3773)
Delayed SN	232 (394)	552 (938)
High Kick	22 (34)	51 (81)

Dominik+14

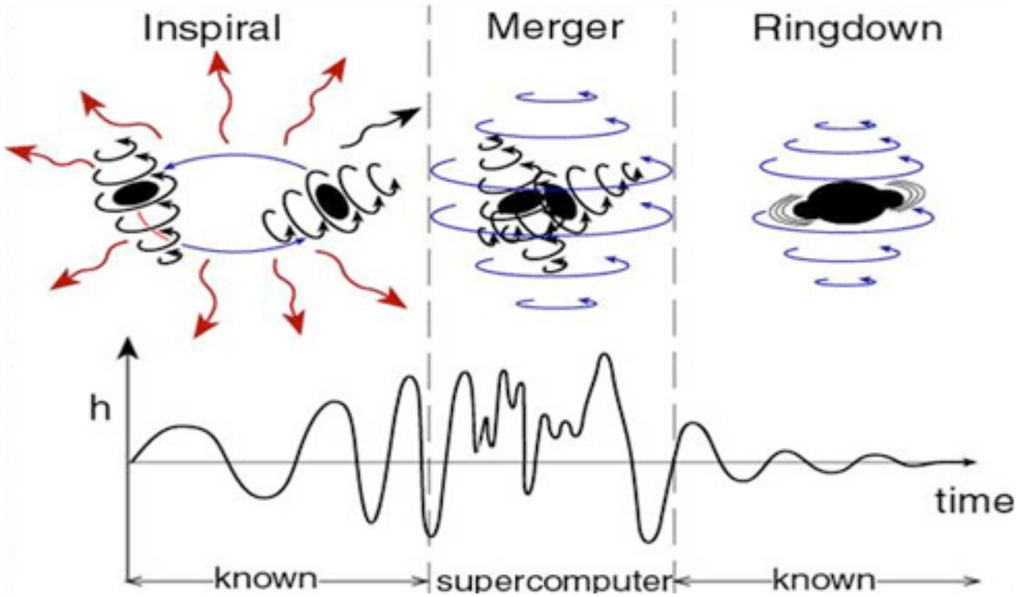
BBH Mergers (The Primary GW Source?)



- "Cleanest" GW source – pure curvature, no messy matter around. But: now EM counterpart!
- Most extreme GW source – GWs near merger probe truely non-linear strong-field, fast-motion GR.
- Strong field limit most likely place for GR to fail.
- Can make exact (approximation free) prediction of GWs using **numerical relativity**.
- Use GW observations of BBH mergers to test general relativity!


Binary Black Hole Evolution:
Caltech/Cornell Computer Simulation
Top: 3D view of Black Holes and Orbital Trajectory
Middle: Spacetime curvature:
Depth: Curvature of space
Colors: Rate of flow of time
Arrows: Velocity of flow of space
Bottom: Waveform
(red line shows current time)

BBH: Stages of a Coalescence



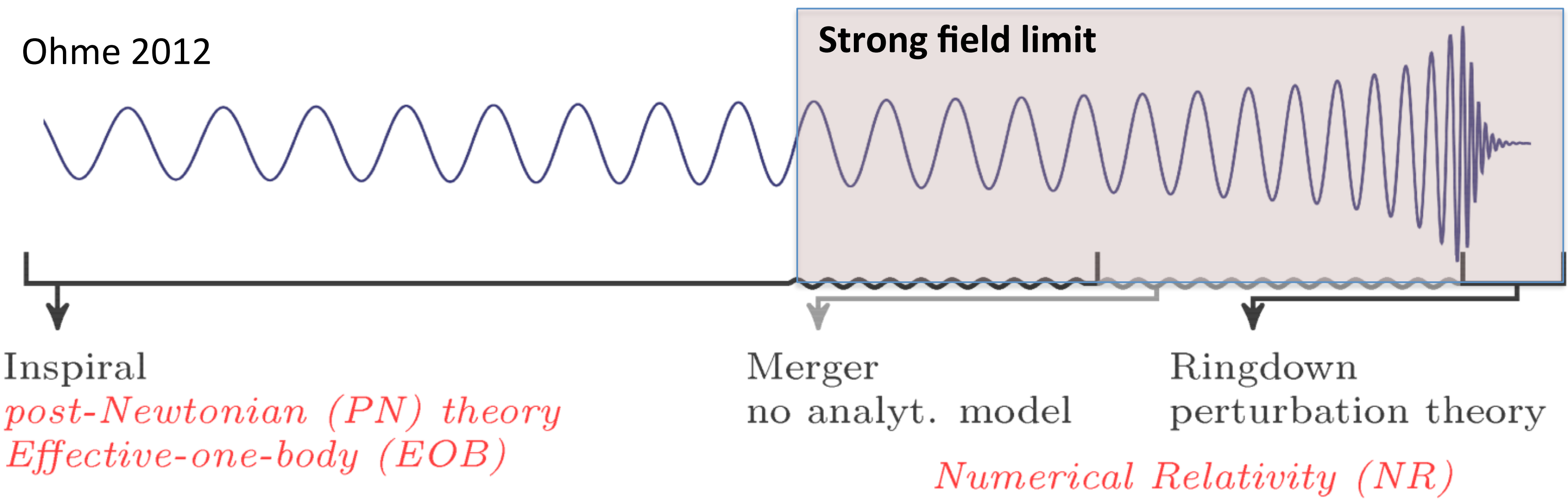
Inspiral

Merger

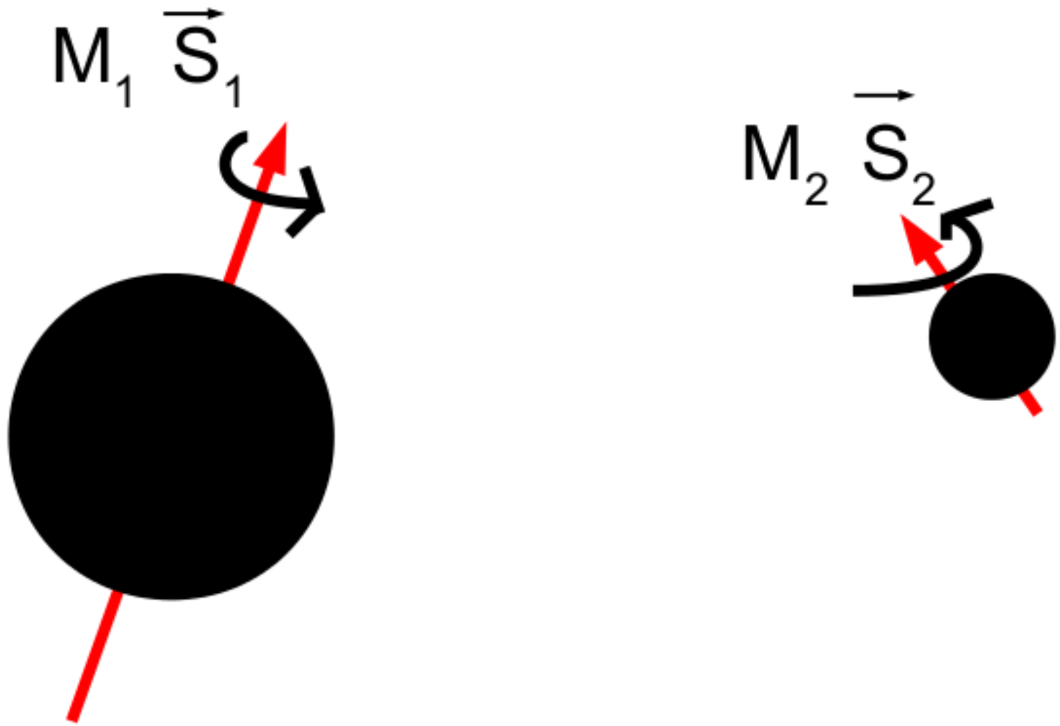
Ringdown

Credit:
Kip Thorne

Ohme 2012



BBH Parameter Space



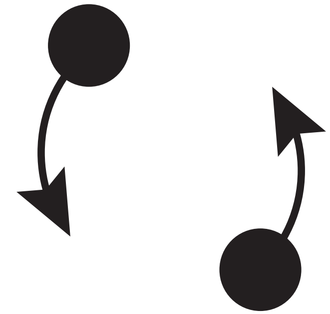
credit:
Jonathan
Blackman

No hair theorem: BHs may have mass, spin, ~~charge~~

BBH parameter space: $q=M_1/M_2$, 6 spin components -> 7 dimensional

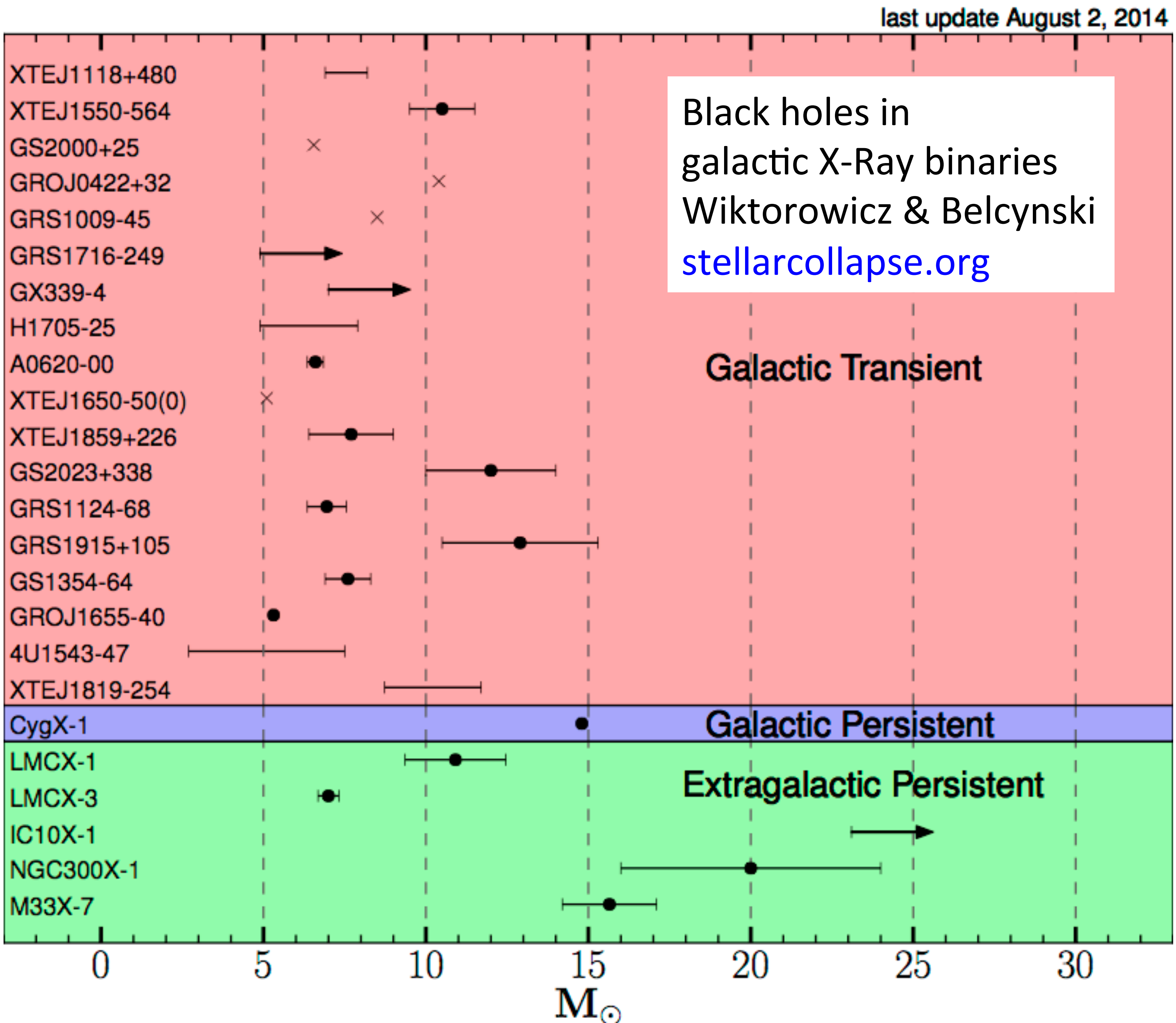
Additional parameter: orbital eccentricity (likely small in most cases).

BBH Parameter Space

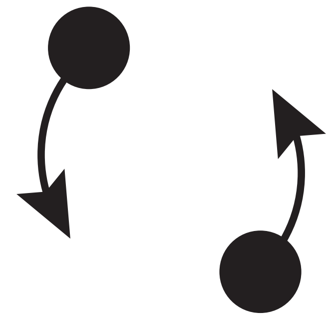


Component Masses: uncertain

- $M > 2.5 - 3\ M_\odot$, probably $> 5\text{-}7\ M_\odot$
- X-ray binaries: $5\text{-}20\ M_\odot$
- Depends on stellar structure & supernova mechanism, fallback.


last update August 2, 2014
Black holes in
galactic X-Ray binaries
Wiktorowicz & Belcynski
stellarcollapse.org
XTEJ1118+480
XTEJ1550-564
GS2000+25
GROJ0422+32
GRS1009-45
GRS1716-249
GX339-4
H1705-25
A0620-00
XTEJ1650-50(0)
XTEJ1859+226
GS2023+338
GRS1124-68
GRS1915+105
GS1354-64
GROJ1655-40
4U1543-47
XTEJ1819-254
Galactic Transient
CygX-1
Galactic Persistent
LMCX-1
LMCX-3
IC10X-1
NGC300X-1
M33X-7
Extragalactic Persistent
0
5
10
15
20
25
30
$M_\odot$

BBH Parameter Space



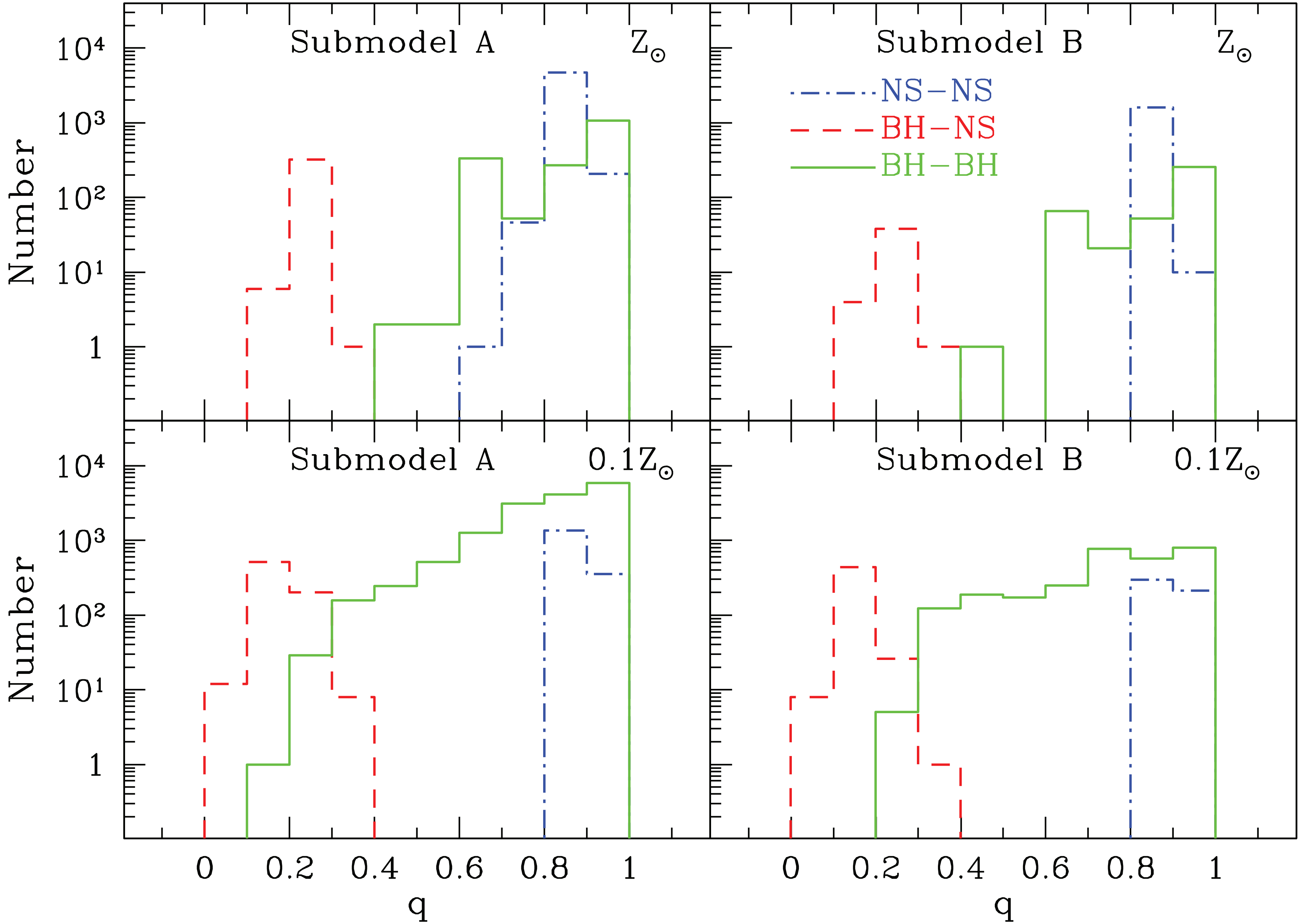
Component Masses: uncertain

- $M > 2.5 - 3\ M_{\odot}$, probably $> 5\text{-}7\ M_{\odot}$
- X-ray binaries: 5-20 $M_{\odot}$
- Depends on stellar structure & supernova mechanism, fallback.

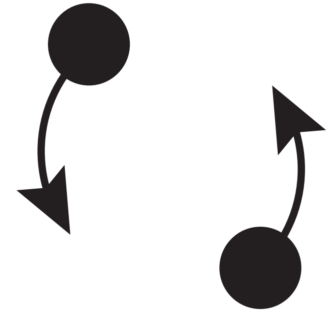
Mass ratio: highly uncertain

- Relies on population synthesis.
- Dominik+12: $q < \sim 5$ in most scenarios.

Dominik+12, showing 1/q



BBH Parameter Space



Component Masses: uncertain

- $M > 2.5 - 3\ M_\odot$, probably $> 5\text{-}7\ M_\odot$
- X-ray binaries: 5-20 $M_\odot$
- Depends on stellar structure & supernova mechanism, fallback.

Mass ratio: highly uncertain

- Relies on population synthesis.
- Dominik+12: $q < \sim 5$ in most scenarios.

Spin magnitude: uncertain

- Dimensionless spin: $a_* = J/M^2 < 1$
- BHs born spinning, accrete J.
- High spin $a_* > 0.8$ could be typical for BBH.

BHs in X-Ray Binaries

Spin measurements results to date for nine stellar-mass BHs using the continuum-fitting method [a]

	Source	MT Type[b]	$P_{\rm orb}$ (days)[b]	Spin a_*	Reference
(*)	GRS 1915+105	RLO	33.9	>0.98	McClintock et al. (2006)
*	Cyg X-1	Wind	5.60	>0.983	Gou et al. (2014)
*	LMC X–1	Wind	3.91	$0.92^{+0.05}_{-0.07}$	Gou et al. (2009)
*	M33 X–7	Wind	3.45	0.84 ± 0.05	Liu et al. (2008, 2010)
	4U 1543–47	RLO	1.12	0.80 ± 0.05	Shafee et al. (2006)
	GRO J1655–40	RLO	2.62	0.70 ± 0.05	Shafee et al. (2006)
*	XTE J1550–564	RLO	1.54	$0.34^{+0.20}_{-0.28}$	Steiner et al. (2011)
*	LMC X–3	RLO	1.70	$0.25^{+0.13}_{-0.16}$	Steiner et al. (2014a)
	A0620–00	RLO	0.32	0.12 ± 0.19	Gou et al. (2010)

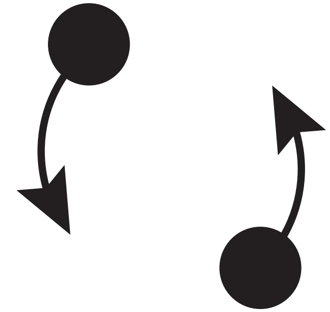
[a] Errors are quoted at the 68% level of confidence.

[b] McClintock & Remillard (2006) and references therein

Table from Fragos & McClintock 2014

* High-mass X-ray binary (HMXB)
-> companion is a massive star

BBH Parameter Space



Component Masses: uncertain

- $M > 2.5 – 3\ M_\odot$, probably $> 5\text{-}7\ M_\odot$
- X-ray binaries: $5\text{-}20\ M_\odot$
- Depends on stellar structure & supernova mechanism, fallback.

Mass ratio: highly uncertain

- Relies on population synthesis.
- Dominik+12: $q < \sim 5$ in most scenarios.

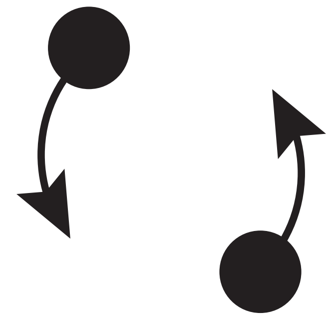
Spin magnitude: uncertain

- Dimensionless spin: $a_* = J/M^2 < 1$
- BHs born spinning, accrete J.
- High spin $a_* > 0.8$ could be typical for BBH.

Spin orientation: highly uncertain

- BHs may form via collapse w/o explosion or w/ explosion.
- If matter ejected aspherically -> momentum kick -> spin misalignment.

BBH Parameter Space



Component Masses: uncertain

- $M > 2.5 – 3\ M_\odot$, probably $> 5\text{-}7\ M_\odot$
- X-ray binaries: $5\text{-}20\ M_\odot$
- Depends on stellar structure & supernova mechanism, fallback.

Mass ratio: highly uncertain

- Relies on population synthesis.
- Dominik+12: $q < \sim 5$ in most scenarios.

Spin magnitude: uncertain

- Dimensionless spin: $a_* = J/M^2 < 1$
- BHs born spinning, accrete J.
- High spin $a_* > 0.8$ could be typical for BBH.

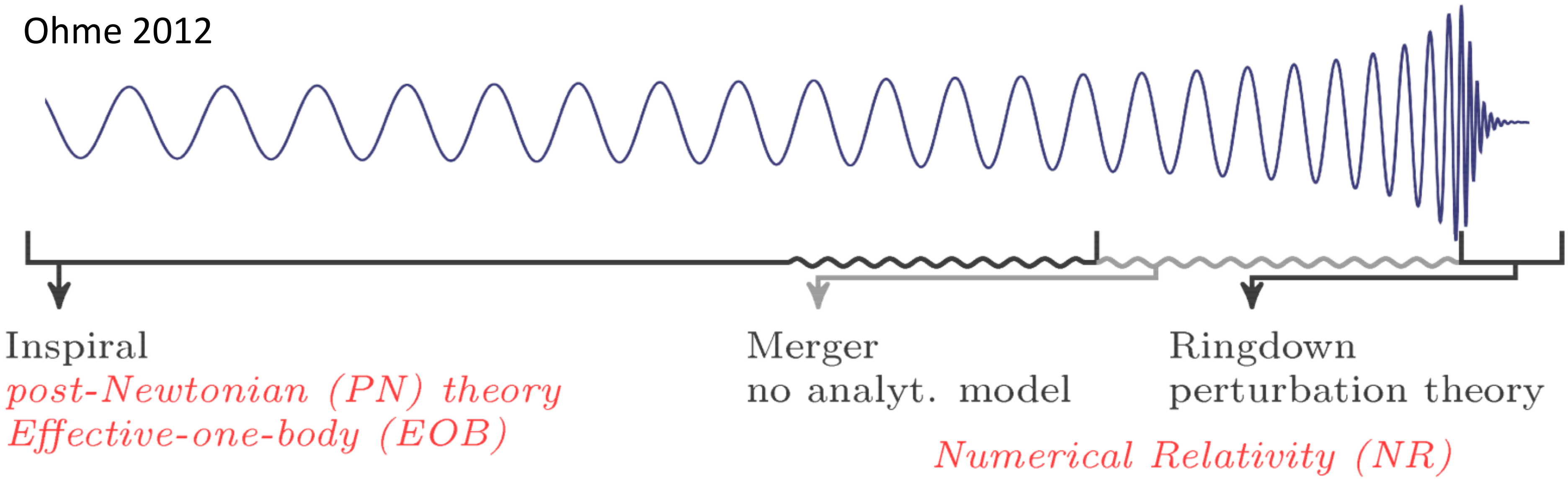
Spin orientation: highly uncertain

- BHs may form via collapse w/o explosion or w/ explosion.
- If matter ejected aspherically -> momentum kick -> spin misalignment.

GW Observations will tell us! Will learn new Astrophysics!

The Need for Numerical Relativity

Ohme 2012



GW data analysis & parameter estimation
Matched-filtering ("templated") -> Patrick Brady's lectures

- Need "template bank" for detection and parameter estimation**. Must cover parameter space densely**. Template generation must be fast.
- Inspiral: post-Newtonian waveforms.
- Late inspiral/merger: **Numerical Relativity** -> tune phenomenological WFs (Ajith+) and/or effective-on-body (EOB) WFs (e.g., Buonanno, Damour+)
- Ring-down: BH ringdown can be treated perturbatively.

The Need for Numerical Relativity

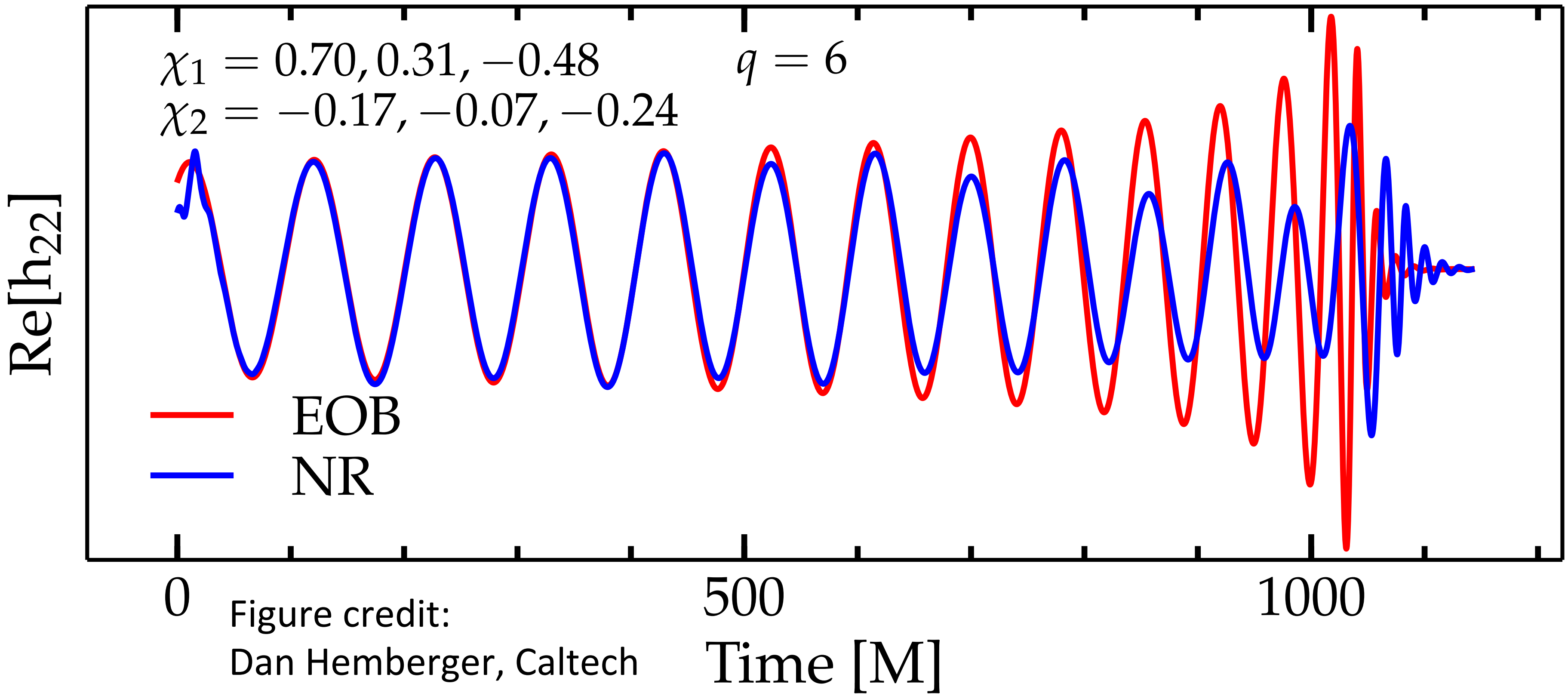
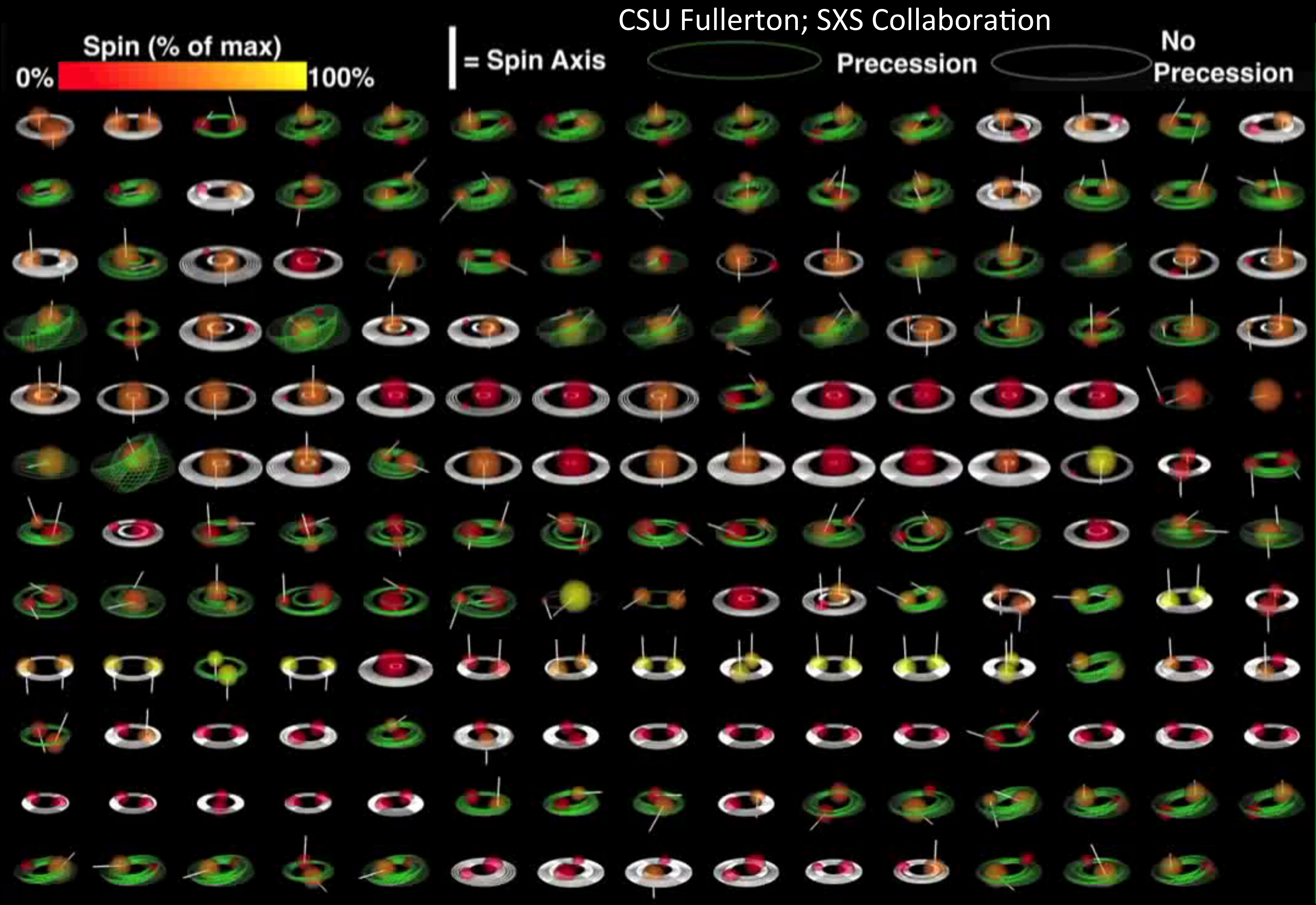


Figure credit:
Dan Hemberger, Caltech

Issue: Phenomenological/EOB waveforms calibrated on finite set of NR WFs. Pick different parameters, calibration fails!

Solution: Build template bank based on numerical relativity WFs. Find efficient way to interpolate in sparse template bank.


CSU Fullerton; SXS Collaboration
Spin (% of max)
0%
100%
= Spin Axis
Precession
No Precession

Numerical Relativity

MISNER summarized the discussion of this session: "First we assume that you have a computing machine better than anything we have now, and many programmers and a lot of money, and you want to look at a nice pretty solution of the Einstein equations. The computer wants to know from you what are the values of $g_{\mu\nu}$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ at some initial surface, say at t = 0. Now, if you don't watch out when you specify these initial conditions, then either the programmer will shoot himself or the machine will blow up. In order to avoid this calamity you must make sure that the initial conditions which you prescribe are in accord with certain differential equations in their dependence on x, y, z at the initial time. These are what are called the "constraints." They are the equations analogous to but much more com-

Proceedings of the GR1 Conference on the role of gravitation in physics
University of North Carolina, Chapel Hill [January 18-23, 1957]
(via Pablo Laguna & Deirdre Shoemaker)

Recommended texts:

Baumgarte & Shapiro, ***Numerical Relativity***

Alcubierre, ***Introduction to 3+1 Numerical Relativity***