Physics, Astrophysics, & Simulation of Gravitational Wave Sources

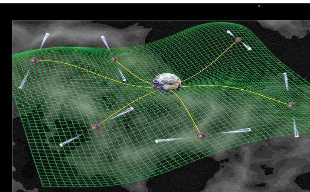
Christian D. Ott
TAPIR, Burke Institute
California Institute of Technology
cott@tapir.caltech.edu

CGWAS 2015

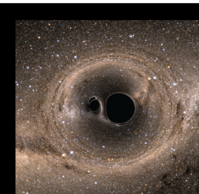
Caltech Gravitational Wave Astrophysics School 2015

A Summer School for Graduate Students and Senior Undergraduates in Physics and Astrophysics

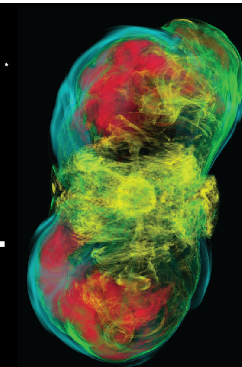
©LIGO



©MPI for Radio Astronomy



©SXS



July 6 - 10, 2015

California Institute of Technology

Topics:

Gravitational Waves and Sources
Gravitational Wave Detection & Data Analysis
Electromagnetic Counterparts & Transient Astronomy
Neutrino Counterparts and Observations
Pulsar Timing Arrays
Numerical Relativity

Confirmed Lecturers:

Duncan Brown (Syracuse)
Sean Carroll (Caltech)
Yanbei Chen (Caltech)
Alessandra Corsi (Texas Tech)
Jessica McIver (UMass)
Brian Metzger (Columbia)
Chiara Mingarelli (Caltech/JPL)
Samaya Nissanke (Radboud)
Erin O'Connor (Duke)
Benjamin Owen (Texas Tech)
Christian Ott (Caltech)
Jocelyn Read (CSUF)
Mark Scheel (Caltech)
Patricia Schmidt (Caltech)
Michele Vallisneri (Caltech/JPL)
Alan Weinstein (Caltech)

Apply now at: http://www.cgwas.org
Deadline: April 15, 2015
Limited participant support available!

For more information and registration scan the QR code or visit our site: **http://www.cgwas.org**
Contact us at: **cgwas@cgwas.org**

Lecture Plan

- Lecture 1 (Wednesday)
  - (a) General Relativity & Gravitational Wave Refresher
  - (b) Overview of GW sources & phenomenology.
  - (c) Numerical relativity and general-relativistic (magneto-)hydrodynamics.
- Lecture 2 (Thursday)
  - (a) Continuation of Lecture 1, Part (c).
- Lecture 3 (now!)
  - (a) Microphysics of neutron star mergers and stellar collapse.
  - (b) Neutron star mergers and Nucleosynthesis
  - (c) Massive stars, stellar collapse, core-collapse supernovae.
  - (d) Neutron star and black hole formation.

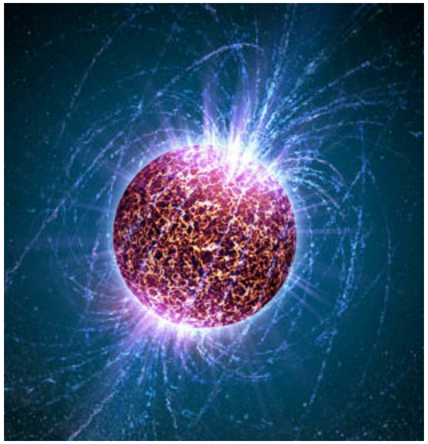
Microphysics for Neutron Star Mergers and Stellar Collapse/Core-Collapse Supernovae (CCSNe)

(aka: messy nuclear astrophysics)

Vote:

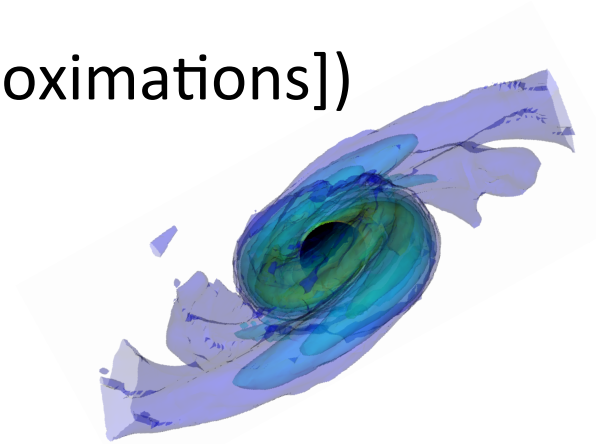
What provides the pressure that stabilizes **neutron stars** against gravity?

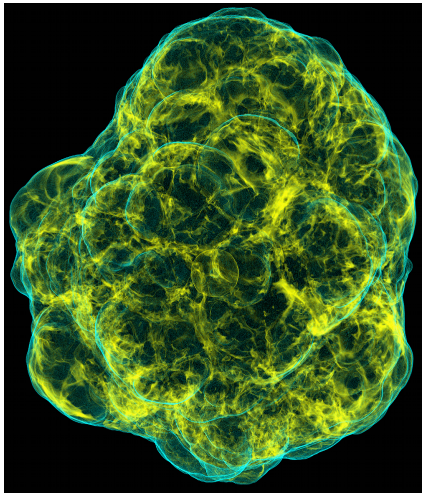
(a) Neutron degeneracy.
(b) Mixture of neutron and proton degeneracy.
(c) None of the above.

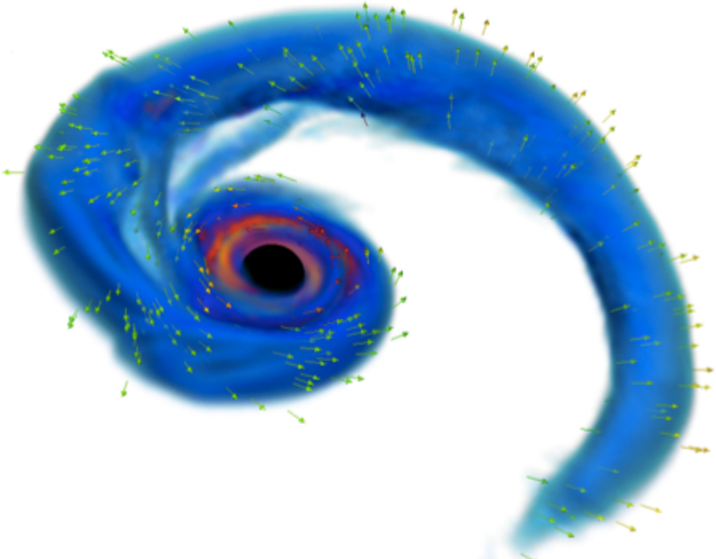


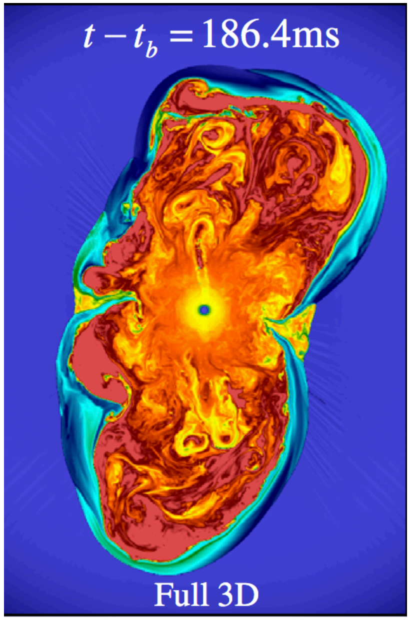
Microphysics?

- Equation of state $P = P(\rho, T, X_i)$
- Neutrino interactions (and transport [approximations])
- Nuclear reactions & nucleosynthesis
- Other: transport properties, elastic moduli (crust), etc.









Microphysics in Mergers

Nuclear Equation of State (EOS)

Crust Physics & Superfluidity (SF)

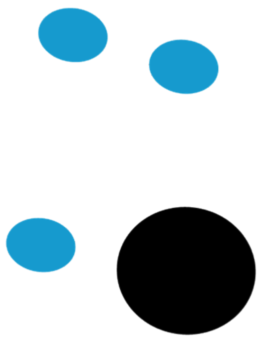
Neutrinos/Neutrino Interactions

Nuclear Reactions & Opacities

Inspiral

Inspiral

Late Inspiral

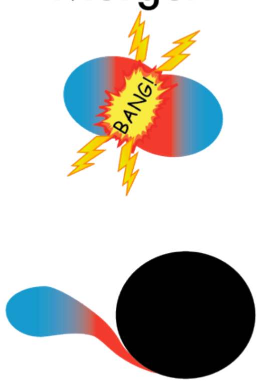


Late Inspiral

Crust/SF

EOS

Merger



Merger/Disruption

hot EOS

Neutrinos

Postmerger

Postmerger

hot EOS

Neutrinos

Nuclear

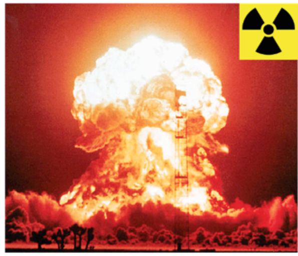
GRB?

hot EOS

Neutrinos

Nuclear

Late Time
(t >~1s)



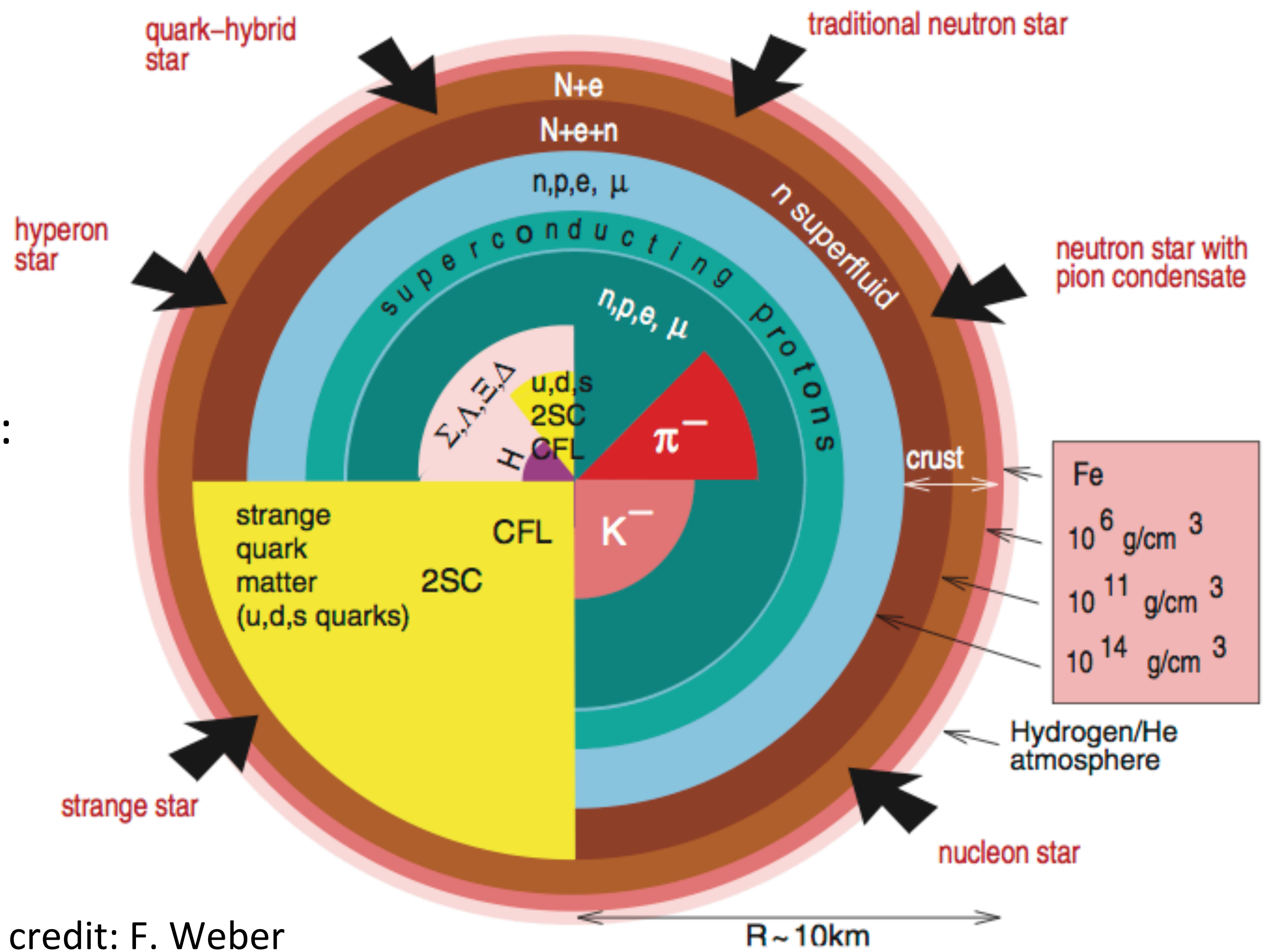
EM afterglow/
counterpart

hot EOS

Neutrinos

Nuclear

Neutron Star Composition



Simplest model:
neutrons +
few protons

credit: F. Weber

Equation of State: Some Thermodynamics

First Law
$$dQ = TdS = dE + PdV - \sum_i \mu_i dN_i$$

$$n = \frac{N}{V}$$

In specific quantities per particle (baryon):

$$d\epsilon = -Pd\left(\frac{1}{n}\right) + Tds + \sum_i \mu_i d\left(\frac{n_i}{n}\right)$$

$$dV = \frac{1}{N} d\left(\frac{1}{n}\right)$$

$$\frac{n_i}{n} = Y_i$$

$$\epsilon = \epsilon(n, s, \{Y_i\})$$

But in n+p neutron star (or in nuclear statistical equilibrium [NSE]):

$$\epsilon = \epsilon(n, s, Y_e)$$

Y_e electron fraction

Prefer to work in $X = X(n, T, Y_e)$

Helmholtz free energy:

$$f = f(n, T, Y_e) = \epsilon - Ts$$

At fixed T, n, and composition, Helmholtz Free Energy is minimized in equilibrium.
At zero temperature: Helmholtz free energy <-> internal energy.

Equation of State: Some Thermodynamics

EOS from the Free Energy:

$$f = f(n, T, Y_e) = \epsilon - Ts$$

$$df = -Pd\left(\frac{1}{n}\right) + sdT + \sum_i \mu_i d\left(\frac{n_i}{n}\right) \qquad \frac{d}{d(\frac{1}{n})} = -n^2 \frac{d}{dn}$$

Obtain thermodynamic quantities via derivatives of free energy f:

$$P = n^2 \frac{\partial f}{\partial n}\bigg|_{T,Y_e} \qquad s = -\frac{\partial f}{\partial T}\bigg|_{n,Y_e} \qquad \mu_i = \frac{\partial f}{\partial n_i}\bigg|_{n,T}$$

Finding the EOS = min(f) for a given n, T, Y_e. This also fixes mass fractions of constituent particles.

Typical constituents: n, p, α, representative nucleus with (A,Z) or NSE ensemble {A_i, Z_i}.
At high densities: exotica such as hyperons, kaons, etc.

Generally: $f = f_{\mathrm{baryon}} + f_e + f_\gamma$ (electrons, photons independent of baryons)

Important Equilibria (1)

- Nuclear Statistical Equilibrium (NSE)
  - At T > 0.5 MeV, nuclear forward and backward reactions go into statistical equilibrium -> chemical equilibrium:

$$Z_i \mu_p + N_i \mu_n = \mu_i$$

← nuclear species i

$$n = \sum_i n_i A_i$$

Mass conservation

$$n Y_e = n_p + 2 n_\alpha + \sum_i Z_i n_i$$

Charge conservation

  - Saha-like equations for abundances $Y_i = \frac{n_i}{n}$

nuclear part. fn.

$$Y_{Z_i,A_i} = \frac{G_{Z_i,A_i}}{2^A (m_u kT/(2\pi\hbar)^{(3/2[A-1])}} (\rho N_A)^{A-1} Y_p^Z Y_n^N \exp\left(\frac{Q}{kT}\right)$$

$$Q = Z m_p + N m_n - M(N,Z) \qquad A = N + Z$$

  - Abundances completely determined by ρ, T, Y_e.

Important Equilibria (2)

- Weak equilibrium (beta equilibrium):
  - At high densities (>10^{12} g/cm^3), weak reactions reach equilibrium.

$$p + e^- \leftrightarrow n + \nu_e \qquad \mu_p + \mu_e = \mu_n + \mu_{\nu_e}$$

  - In hot dense matter: neutrinos trapped, Y_e and Y_ν determined by trapped lepton fraction Y_{lep}.
  - In cold dense matter: neutrinos escape, $\mu_{\nu_e} \approx 0$
    Y_e completely determined by ρ, T.
    "**neutrino-less beta equilibrium**"
    -> condition in old neutron stars & in coalescing NSNS until ~final orbit.

A Simple Neutron Star Equation of State

- T=0, pure nonrelativistic degenerate neutron & proton gas.

$$\epsilon(n_n, n_p) = \frac{3}{5}\frac{p_{F,n}^2}{2m_n}\frac{n_n}{n} + \frac{3}{5}\frac{p_{F,p}^2}{2m_p}\frac{n_p}{n} \quad p_F = (3\pi^2\hbar^3)^{1/3} n^{1/3}$$

$$P = n^2 \frac{\partial \epsilon}{\partial n} \propto n^{5/3} \qquad \Gamma = \left.\frac{d \ln P}{d \ln \rho}\right|_s = \frac{5}{3}$$

- Composition (n_n, n_p) determined by neutrino-less beta equilibrium.
- Write as polytropic EOS for fixed (constant) composition, e.g. pure neutrons:

$$P = K\rho^{5/3}$$

$$K = 5.3802 \times 10^9 \ [\mathrm{cgs}]$$

Equation of State -> Neutron Star Structure

Newtonian: $$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \qquad \frac{dM}{dr} = 4\pi r^2 \rho$$ (no maximum mass!)

GR:

Tolman-Oppenheimer-Volkoff

$$\frac{dP}{dr} = -G(\rho(1+\epsilon/c^2) + P/c^2)\frac{M + 4\pi r^3 p/c^2}{r(r - 2GM/c^2)}$$

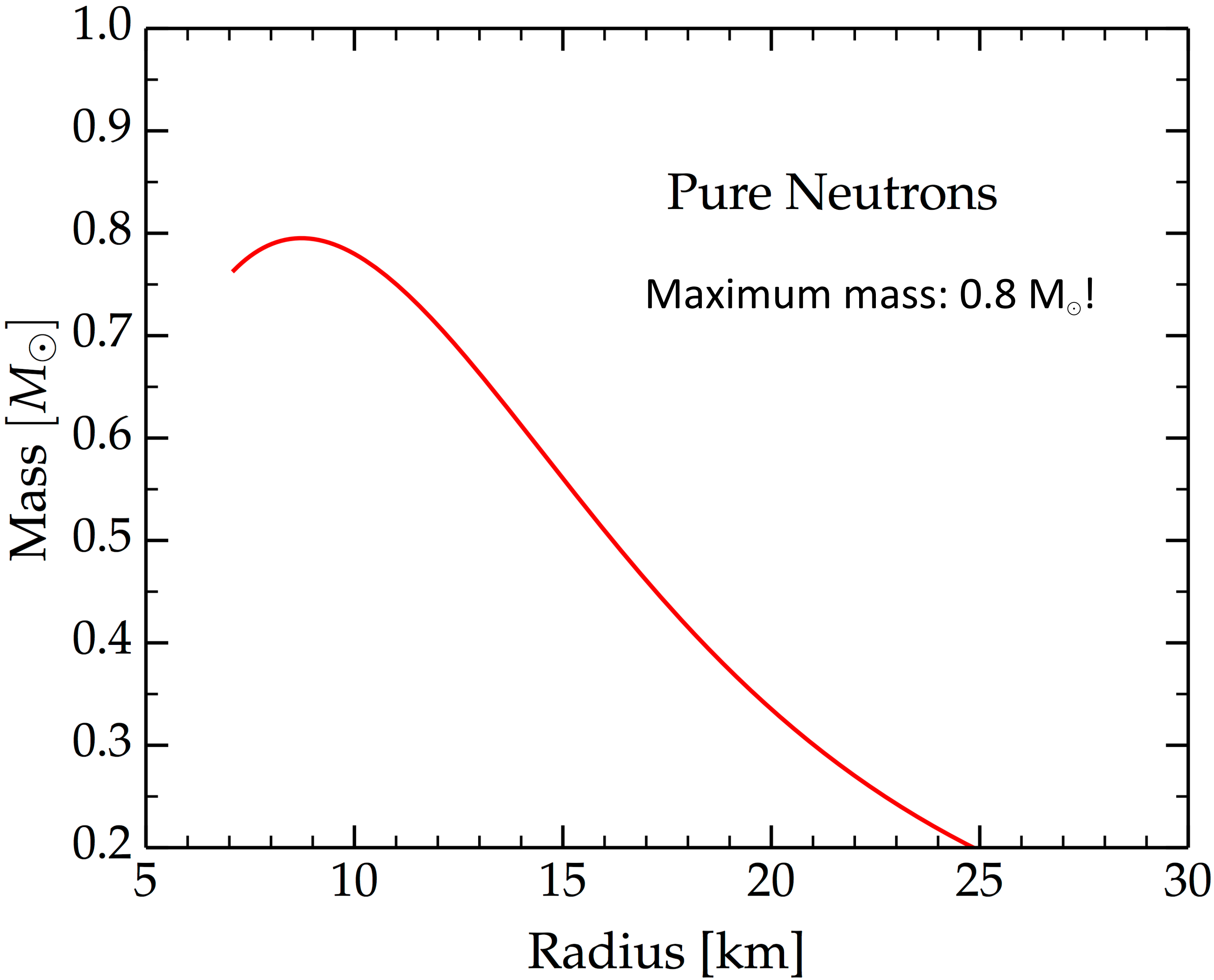
$$\frac{dM_g}{dr} = 4\pi r^2 \rho(1+\epsilon/c^2)$$ gravitational mass

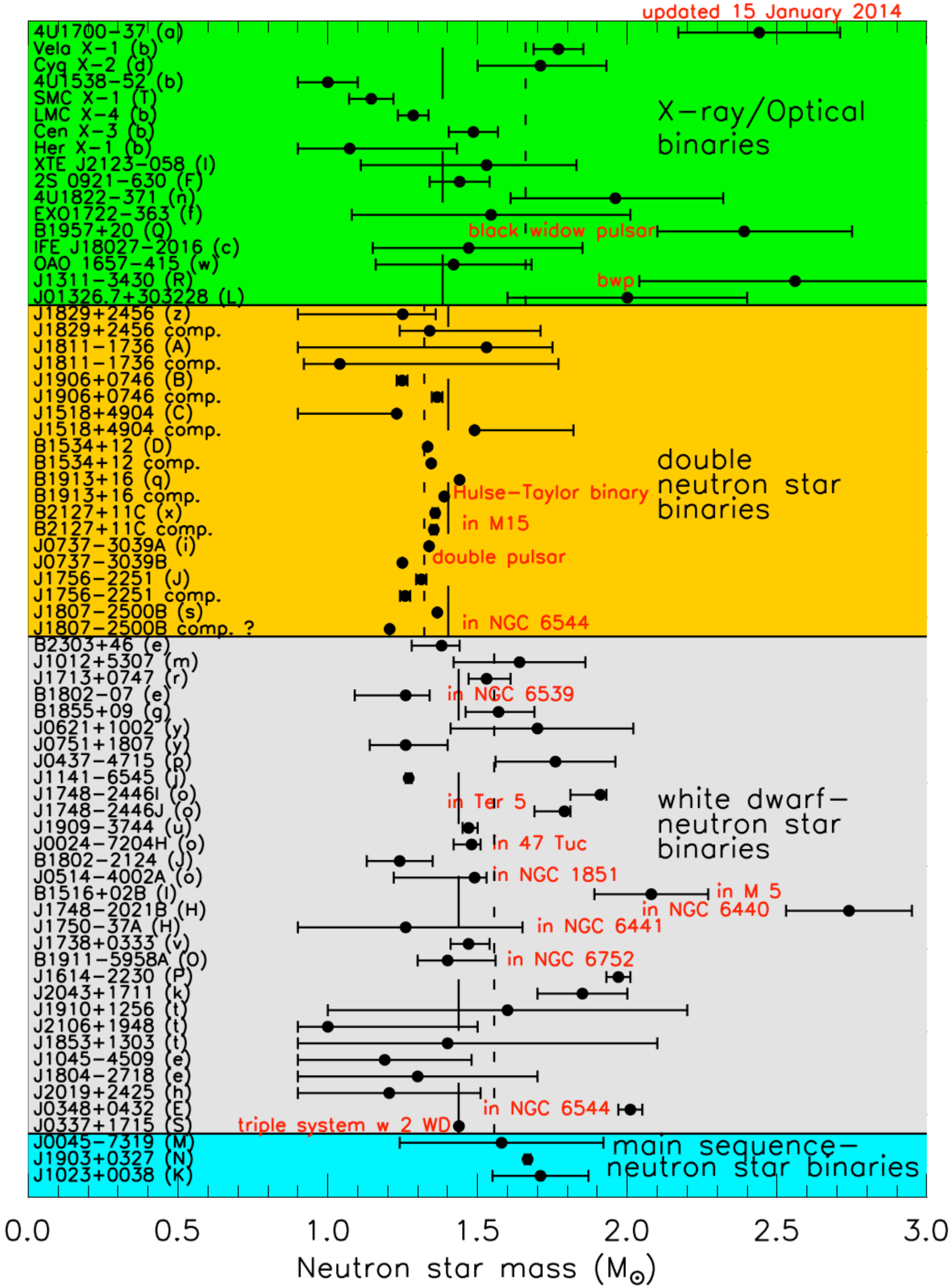
$$\frac{dM_b}{dr} = \frac{4\pi r^2}{\sqrt{1-\frac{2GM}{rc^2}}}\rho$$ baryonic mass

Radius is circumferential radius!

- Solve by ODE integration from r=0, invert P(ρ) at each step to obtain ρ.

Equation of State -> Neutron Star Structure





Lattimer 14,
http://stellarcollapse.org

X-ray binaries

NS+NS

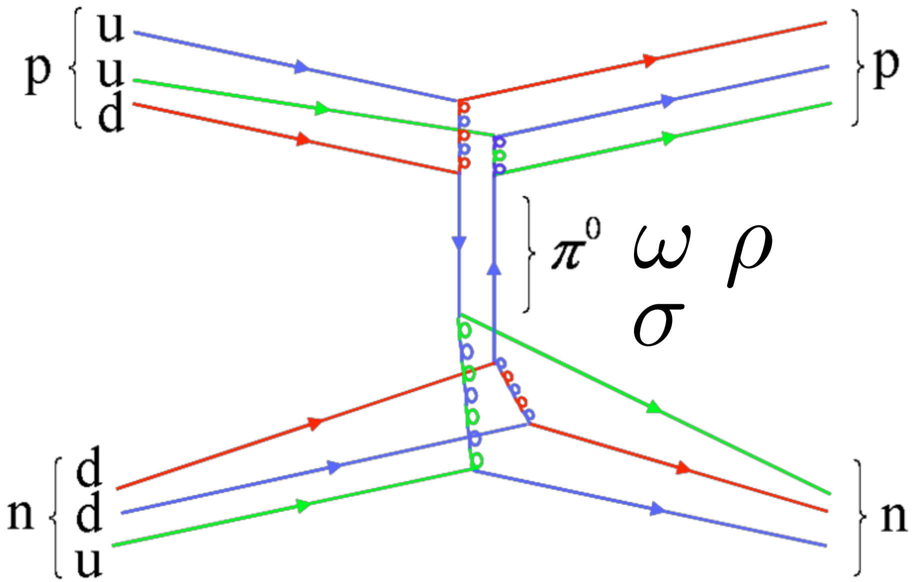
Most massive:
PSR J1614-2230
1.97+-0.04 $M_\odot$
PSR J0348+0432
2.01+-0.04 $M_\odot$

WD+NS

NS + normal star

What is missing?

Nucleon-Nucleon Interaction!



- Nuclear force is NN many-body interaction = "effective" strong force interaction.
- Mediated by mesons:
  π (s=0), σ (s=0), ω (s=1), ρ (s=1)
- Dependent on separation and spin orientation. **Scalar**, **vector**, and **tensor** components.
  Vector component is repulsive.

- T=0, interacting pure neutron & proton gas.

$$\epsilon(n_n, n_p) = \frac{3}{5}\frac{p_{F,n}^2}{2m_n}\frac{n_n}{n} + \frac{3}{5}\frac{p_{F,p}^2}{2m_p}\frac{n_p}{n} + \frac{V_{np}(n_n, n_p)}{n}$$

Nucleon-Nucleon Interaction

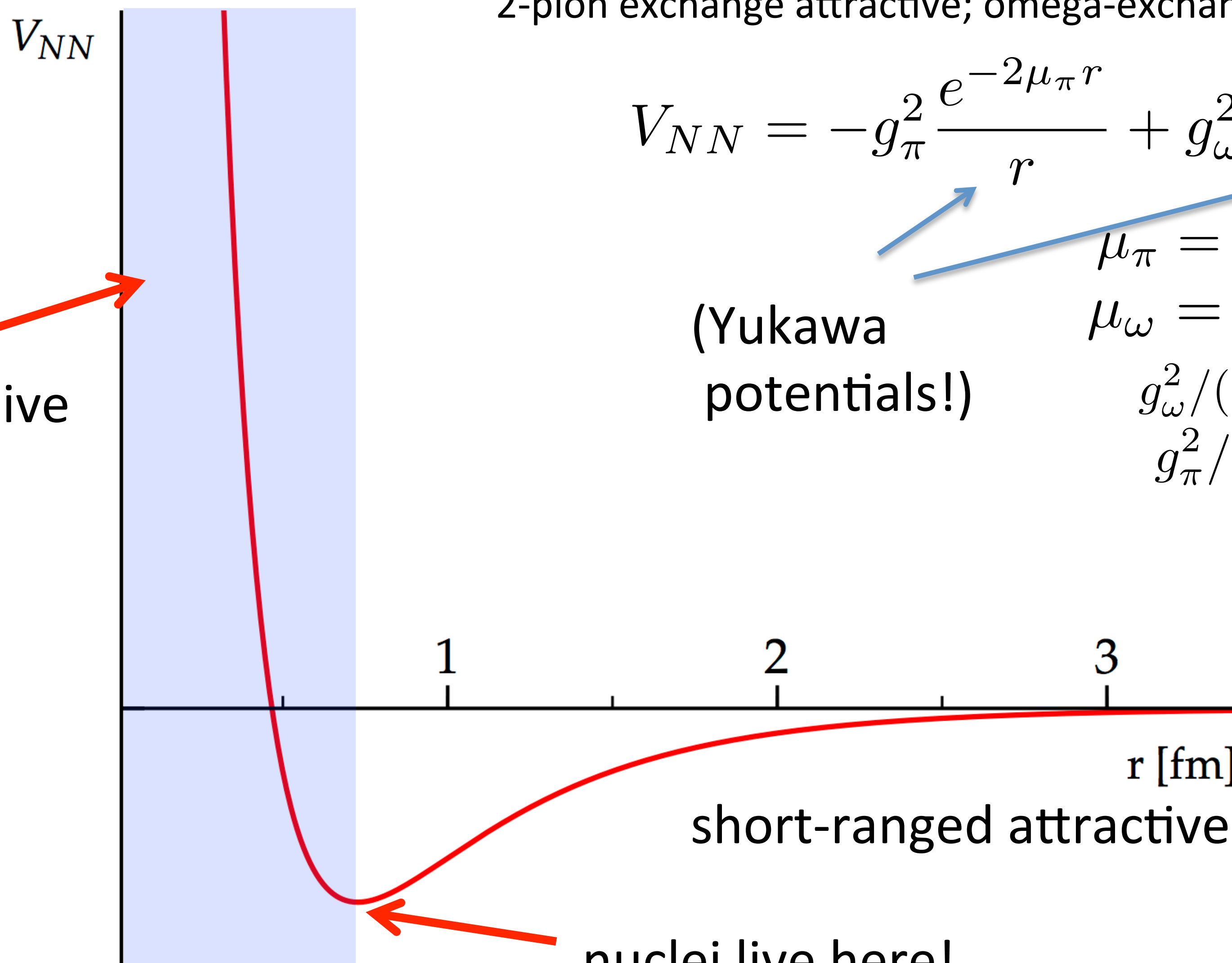
Example: Bethe & Johnson 74

2-pion exchange attractive; omega-exchange repulsive

$$V_{NN} = -g_\pi^2 \frac{e^{-2\mu_\pi r}}{r} + g_\omega^2 \frac{e^{-\mu_\omega r}}{r}$$

(Yukawa potentials!)

$$\mu_\pi = m_\pi/(\hbar c)$$
$$\mu_\omega = m_\omega/(\hbar c)$$
$$g_\omega^2/(\hbar c) = 29.6$$
$$g_\pi^2/(\hbar c) = 10$$

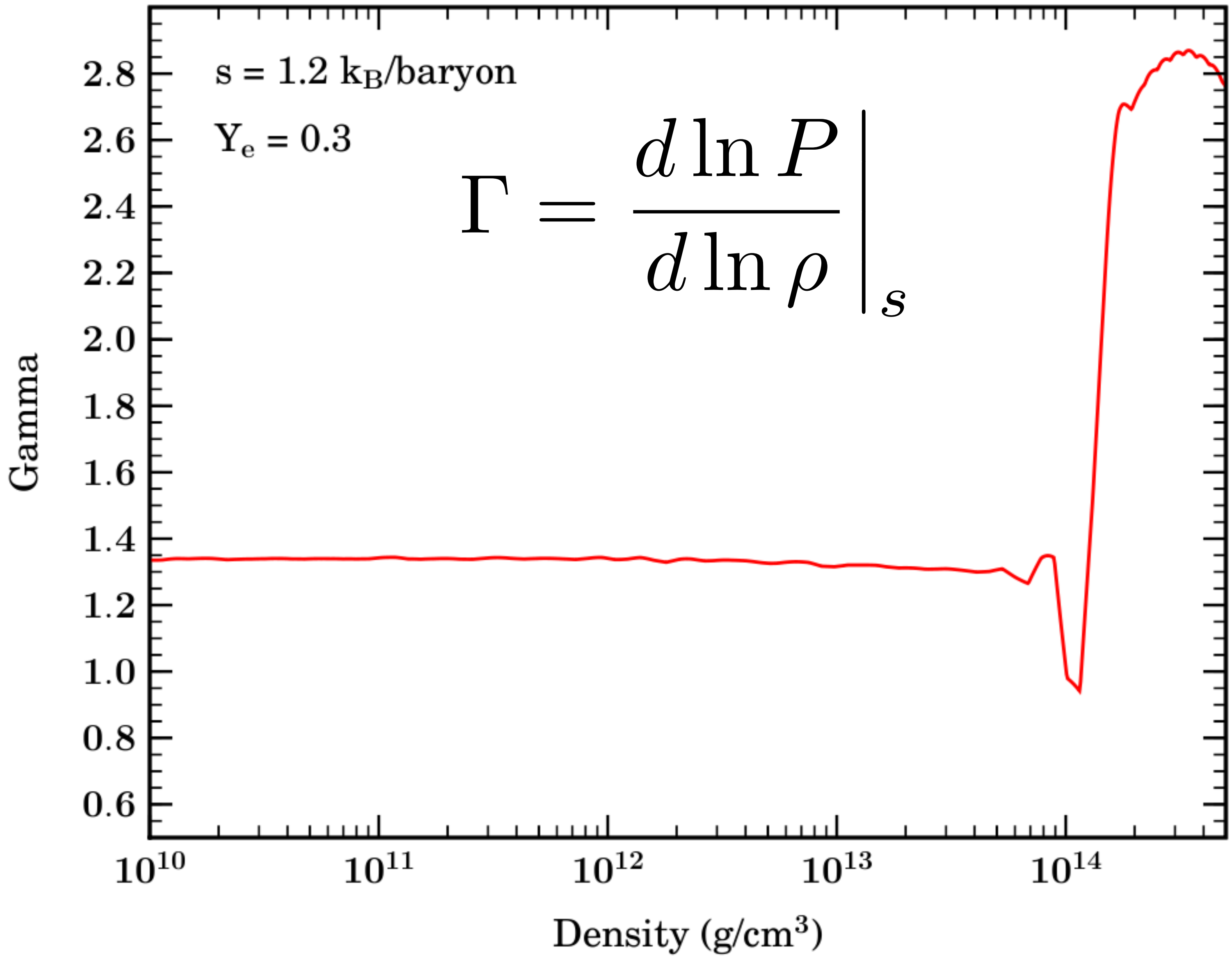


"Repulsive Core"

short-ranged attractive force

nuclei live here!

"Stiffness" of the Nuclear EOS



Obtaining an EOS

- Brute force: Solve quantum many-body interactions with V_{NN} (e.g. via Hartree-Fock approach).

- Mean field approximation (write down Lagrangian for nucleons moving in effective meson fields), introduce parameters to match laboratory nuclei or observations.

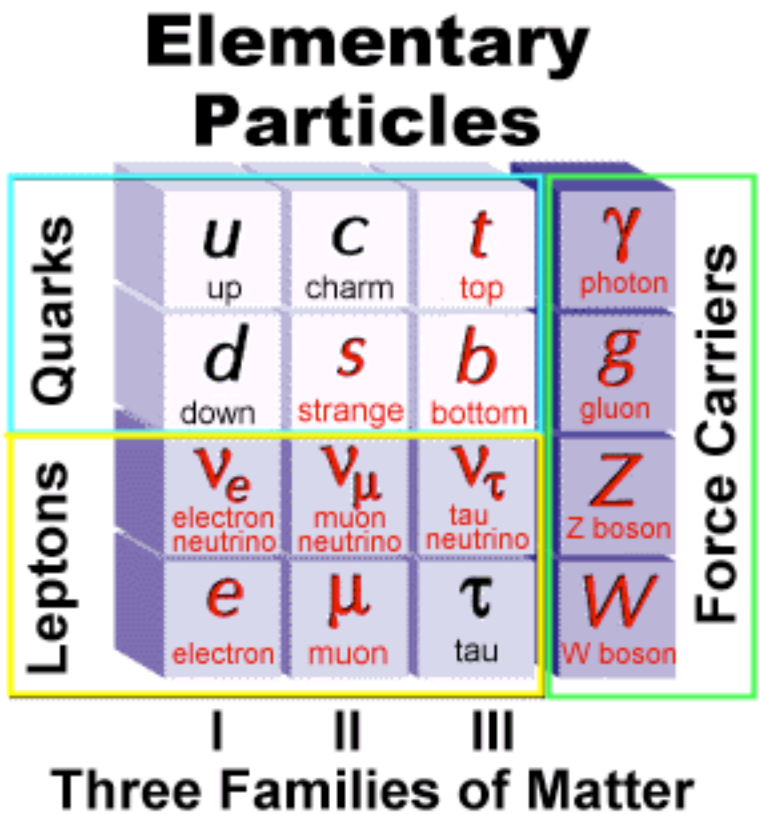
- Phenomenological approach: Liquid drop model with parameters from theory (V_{NN}), experiments, and observations.

Neutrinos

+ anti-particles:

$$\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$$

$$e^+, \mu^+, \tau^+$$



Masses:

$$M_{e^-} = 0.511\,\mathrm{MeV}$$

$$M_{\mu^-} = 105.7\,\mathrm{MeV}$$

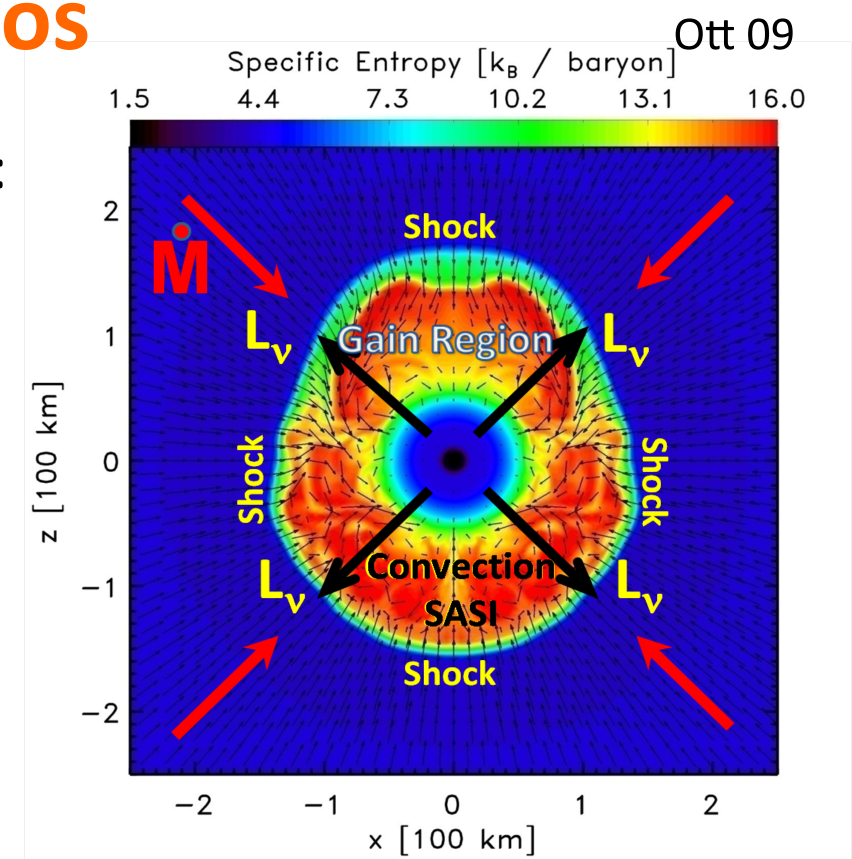
$$M_{\tau^-} = 1777\,\mathrm{MeV}$$

not shown:
anti-neutrinos

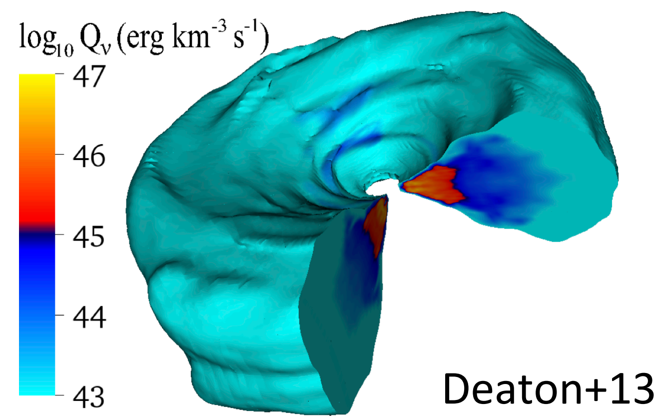
http://particlezoo.net/

Neutrinos

- Stellar collapse & core-collapse SNe:
  -> neutrinos dominate energetics, carry away 99% of energy.
  -> potentially crucial for driving explosion.
  -> **must not be neglected**!



- Neutron star mergers (NSNS/BHNS):
  -> cool hypermassive NS, accretion torus, can drive wind.
  -> affect composition of ejected material – **set Y_e** -> r-process nucleosynthesis
  -> **must not be neglected!**



Neutrino Emission, Scattering & Absorption

- Weak interaction: charged and neutral current processes. $\sigma \propto \epsilon_\nu^2$

- Charged current:

$$p + e^- \longleftrightarrow n + \nu_e$$
$$n + e^+ \longleftrightarrow p + \bar{\nu}_e$$
$$W^+, W^-$$

Only electron-type participate in charged current interactions in CCSN & merger context.

+ neutrino oscillations

- Neutral current: Z^0

$$\nu_i + (n, p, e^{+,-}) \longrightarrow \nu_i + (n, p, e^{+,-})$$ (scattering)

$$e^- e^+ \longleftrightarrow \nu_i \bar{\nu}_i$$ (pair annihilation/creation)

$$NN \longleftrightarrow \nu_i \bar{\nu}_i$$ (nucleon-nucleon bremsstrahlung)

$$\tilde{\gamma} \longleftrightarrow \nu_i \bar{\nu}_i$$ (plasmon decay)

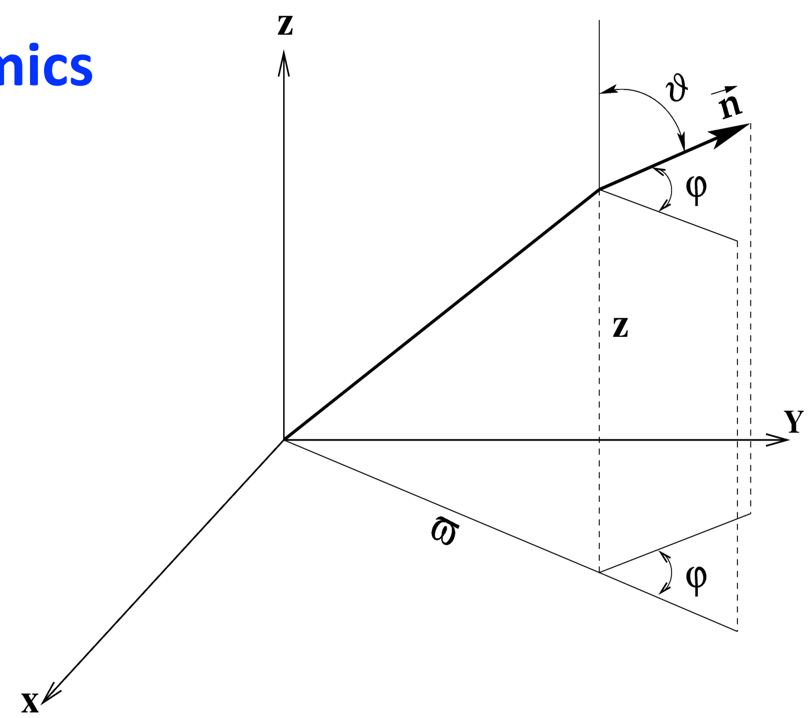
Emission: $\propto T^9$

Neutrino Transport

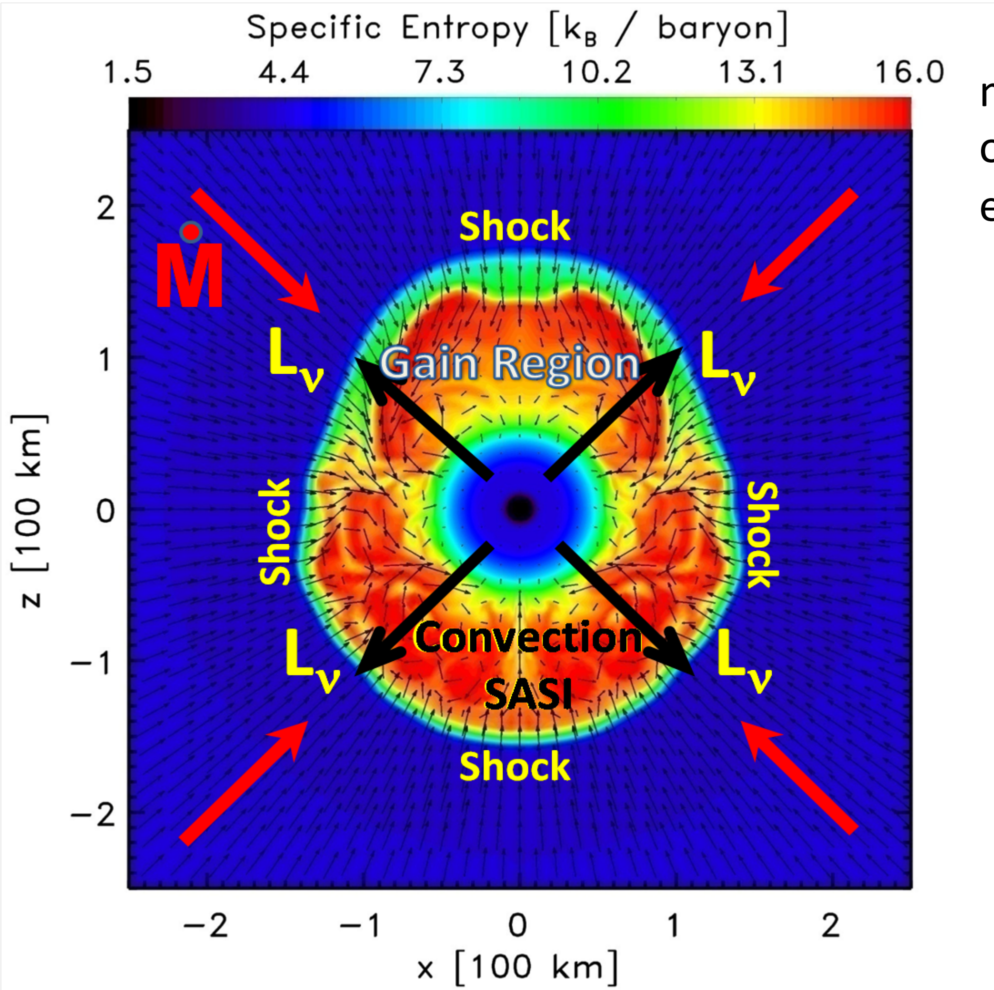
$$\frac{1}{c}\frac{\partial I(\vec{r},\vec{n},\epsilon_\nu)}{\partial t} + \vec{n}\,\vec{\nabla} I(\vec{r},\vec{n},\epsilon_\nu) = \Xi[I(\vec{r},\vec{n},\epsilon_\nu),\rho,T,Y_e]$$

$$J = \frac{1}{4\pi}\oint I d\Omega \qquad \vec{H} = \frac{1}{4\pi}\oint \vec{n} I d\Omega \qquad \mathbf{K} = \frac{1}{4\pi}\oint \vec{n}\cdot\vec{n}\, I d\Omega$$

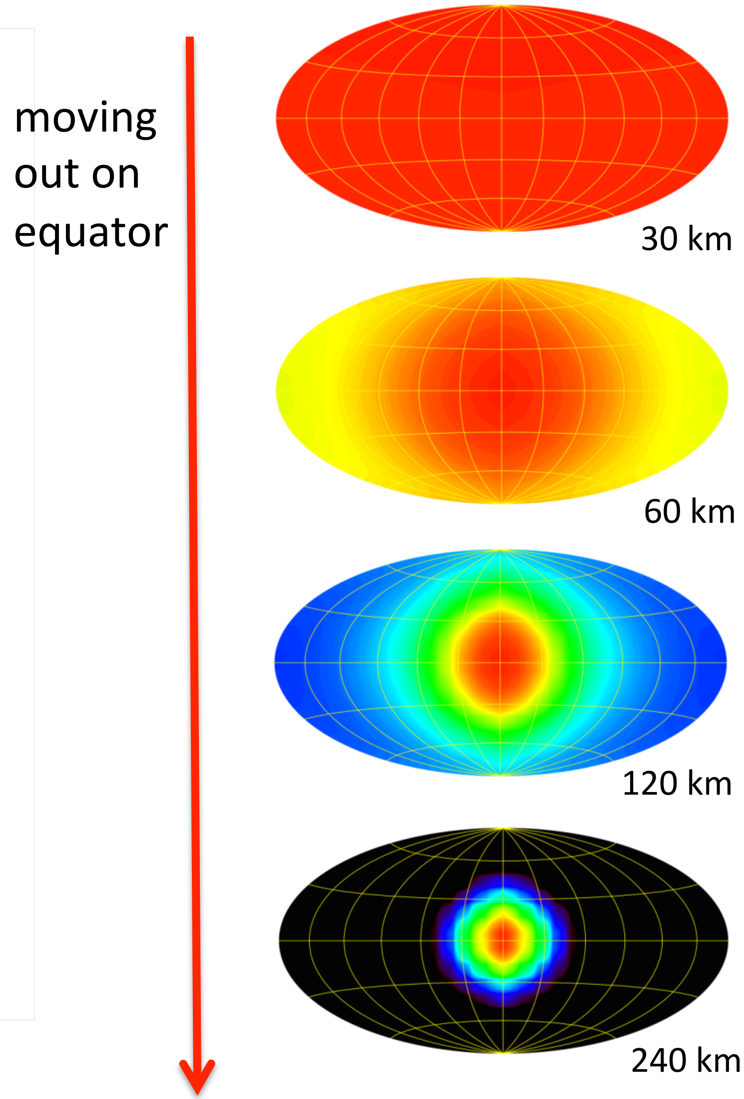
- **Continuum assumption of fluid dynamics fails for neutrinos.**
- Kinetic Theory (Boltzmann transport)
  6+1D problem: 3D space,
  3D (ϵ, θ, ϕ) momentum space.
- Limiting cases – easy to handle:
  (1) Diffusion (isotropic radiation field)
  (2) Free streaming
  (“forward-peaked” radiation field)



Example: Neutrino Radiation Field in a CCSN



specific neutrino intensity



Handling Neutrino Transport in Simulations (1)

Neutrino transport dominates computational complexity of merger and CCSN simulations.
Need efficient & accurate approximation.

(1) Ignore it.

Mergers: okay for inspiral & merger, postmerger evolution wrong.
CCSN: not an option.

(2) Neutrino "leakage" (energy averaged [gray], or energy-dependent)
(e.g., Ruffert+96,Rosswog+03,O'Connor&Ott'10,Sekiguchi+11,Deaton+13,Perego+14)

Estimate neutrino optical depth: $\tau_\nu = \int \kappa_\nu dr$

Estimate diffusion time: $T_\mathrm{diff} \sim \frac{\tau \Delta r}{c}$ $Q_\mathrm{diff} = E_\nu / T_\mathrm{diff}$

Interpolate between diffusion and free emission:
Leakage captures overall energetics & lepton # changes; incorrect in detail.

$$Q^\mathrm{leak} = \frac{Q_\mathrm{free}}{1 + Q_\mathrm{free}/Q_\mathrm{diff}}$$

Handling Neutrino Transport in Simulations (2)

(3) Flux-limited diffusion (gray or energy-dependent)

(e.g., Bruenn+85,Burrows+95,Dessart+09, Burrows+07)

Solve equation for 0th moment (energy density), use ad-hoc "limiter" to interpolate between diffusion and free streaming.

$$\frac{1}{c}\frac{\partial}{\partial t}J_\nu + \frac{1}{4\pi}\nabla \mathbf{F}_\nu = \eta_\nu - \sigma^a_\nu J_\nu \qquad \mathbf{F}_\nu \approx -\frac{4\pi}{3\sigma^t_\nu}\nabla J_\nu$$

Works well in 1D, 2D, in 3D problematic due to need for implicit time integration (inversion of large matrix). Causality issues.

(4) Isotropic diffusion source approximation (energy-dependent)

(Liebendoerfer+09, 1D version available as open source)

Split radiation field into diffusive and free-streaming part; evolved separately. Free-streaming angle dependent, but assumed stationary -> elliptic equation.

Good results, but expensive in full 3D.

"ray-by-ray" approach (e.g., Suwa+10,14,Takiwaki+13,14).

Handling Neutrino Transport in Simulations (3)

(5) Two-Moment approximation (gray or energy-dependent) with analytic or variable Eddington factor.
(e.g., Rampp & Janka 02, Mueller+09, Shibata+11, Kuroda+13,15, Roberts 13, Just+15, Foucart+15, Sekiguchi+15 O'Connor 15: GR1D http://gr1dcode.org -- open source.)
Equations for 0-th and 1-st moment. Analytic closure relation ("M1 approximation") or closure by solution of time-independent Boltzmann equation (MPA Garching).

(6) Monte-Carlo (e.g., Abdikamalov+12)
Direct simulation by Monte-Carlo experiment. Promising, but so far only 1D implementation.

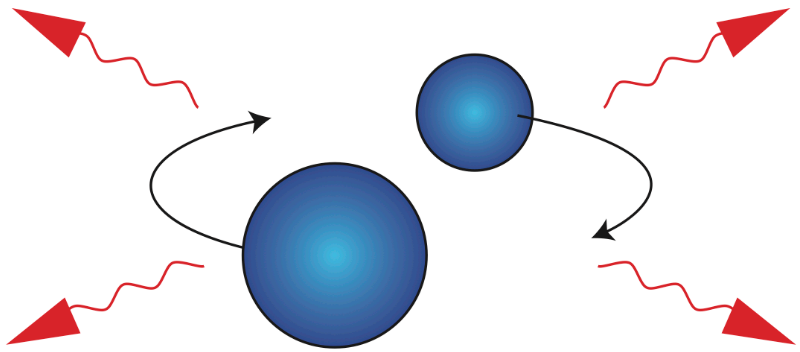
(7) S_N discretization (e.g., Livne+04,Ott+08,Nagakura+14,Sumiyoshi+12,15)
Direct discretization in angle. Extremely computationally intense. Yields fully time-dependent solution of Boltzmann equation. 1D, 2D, 3D, but 2D and 3D expensive.

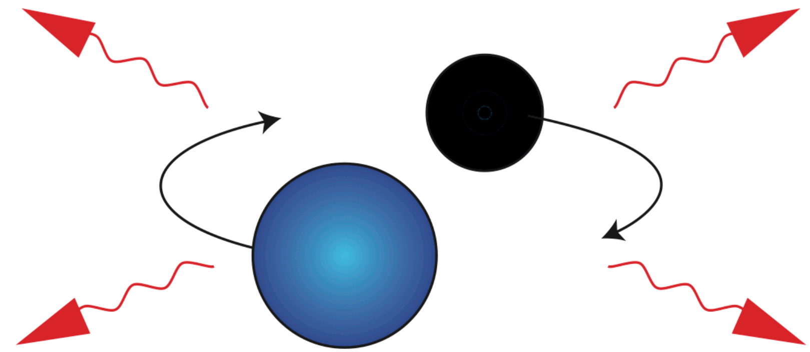
Merger Simulations and Nucleosynthesis

Neutron Star Mergers

- Neutron Star + Neutron Star (NSNS)
- Black Hole + Neutron Star (BHNS)

credit: D. Tsang

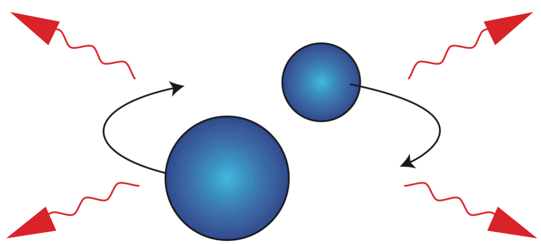




$M_1 \sim M_2 \sim 1.4\ M_{Sun}$
-> galactic NSNS binaries!

$M_{BH} \sim 7\text{-}10 \times M_{NS}$ (Belczynski+'10)
(but no BHNS systems known)

NSNS Merger Scenarios



m_1, m_2

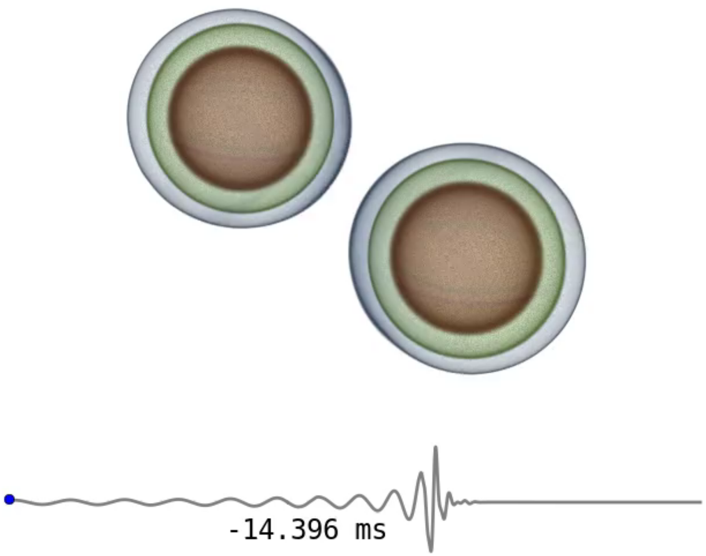
$M = m_1+m_2$

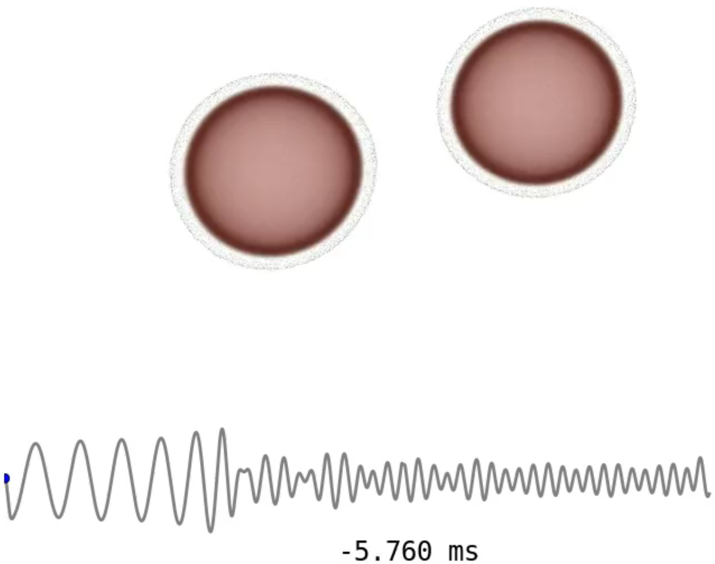
- Prompt BH
- HMNS -> BH
- Stable NS Remnant

$M < M_{max}(EOS)$

Outcome most sensitive to total mass of binary and nuclear EOS.

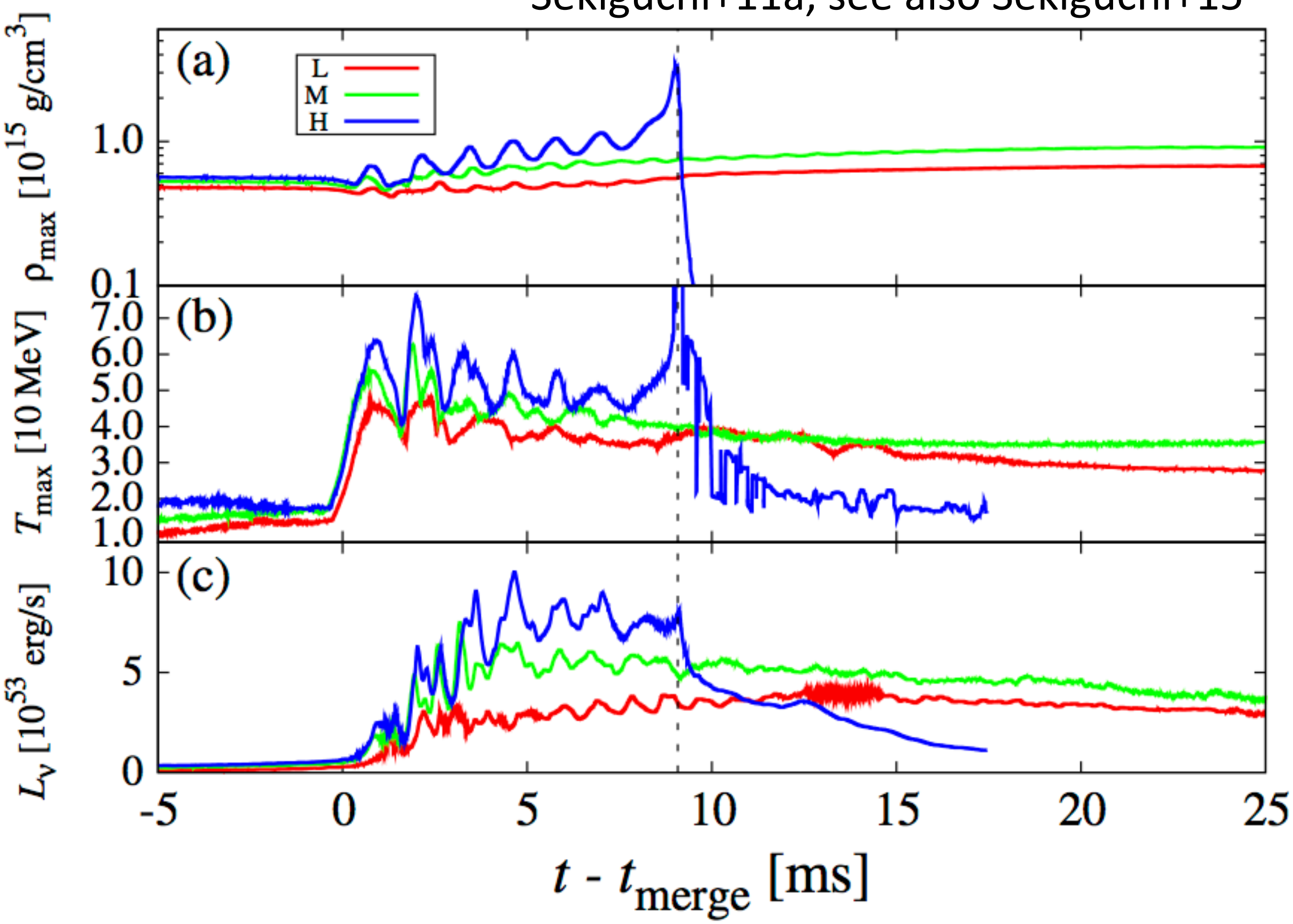
NSNS Merger Scenarios





NSNS Postmerger Evolution

Sekiguchi+11a, see also Sekiguchi+15



H. Shen EOS

L: 2x1.35 $M_\odot$;
M: 2x1.5 $M_\odot$;
H: 2x1.6 $M_\odot$

Total baryonic masses: (2.90, 3.28, 3.54) $M_\odot$

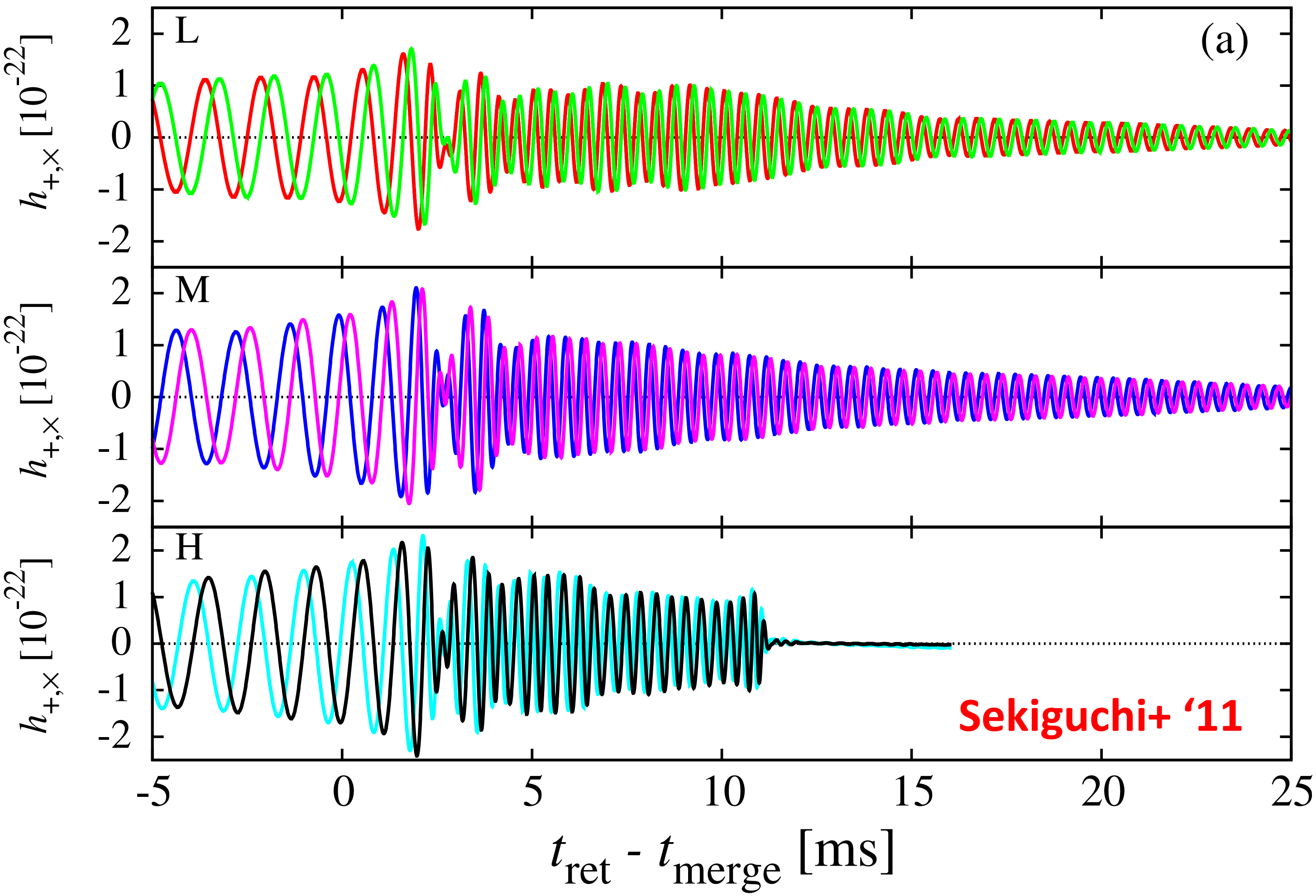
TOV: 2.56 $M_\odot$;
uniform rot.: 3.05 $M_\odot$;
diff. rot: no formal limit

-> see also Bauswein+10,12,14,15

HMNS: support by differential rotation, only small thermal contribution.
Secular evolution: governed by energy loss to GWs, neutrinos, and angular momentum redistribution by 3D torques / magnetorotational instability.

NSNS Postmerger Evolution

Sekiguchi+11a, see also Sekiguchi+15



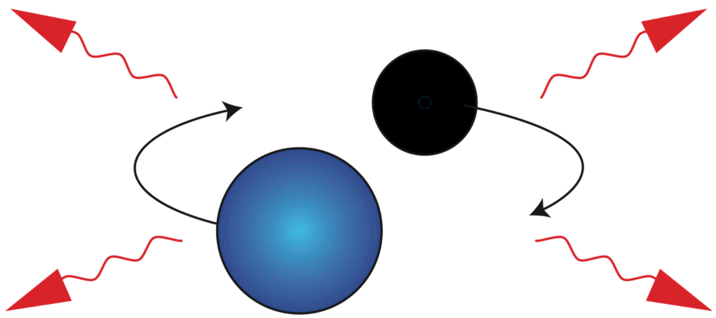
Mass [M_{Sun}]

1.35 + 1.35

1.50 + 1.50

1.60 + 1.60

BHNS Merger Scenarios

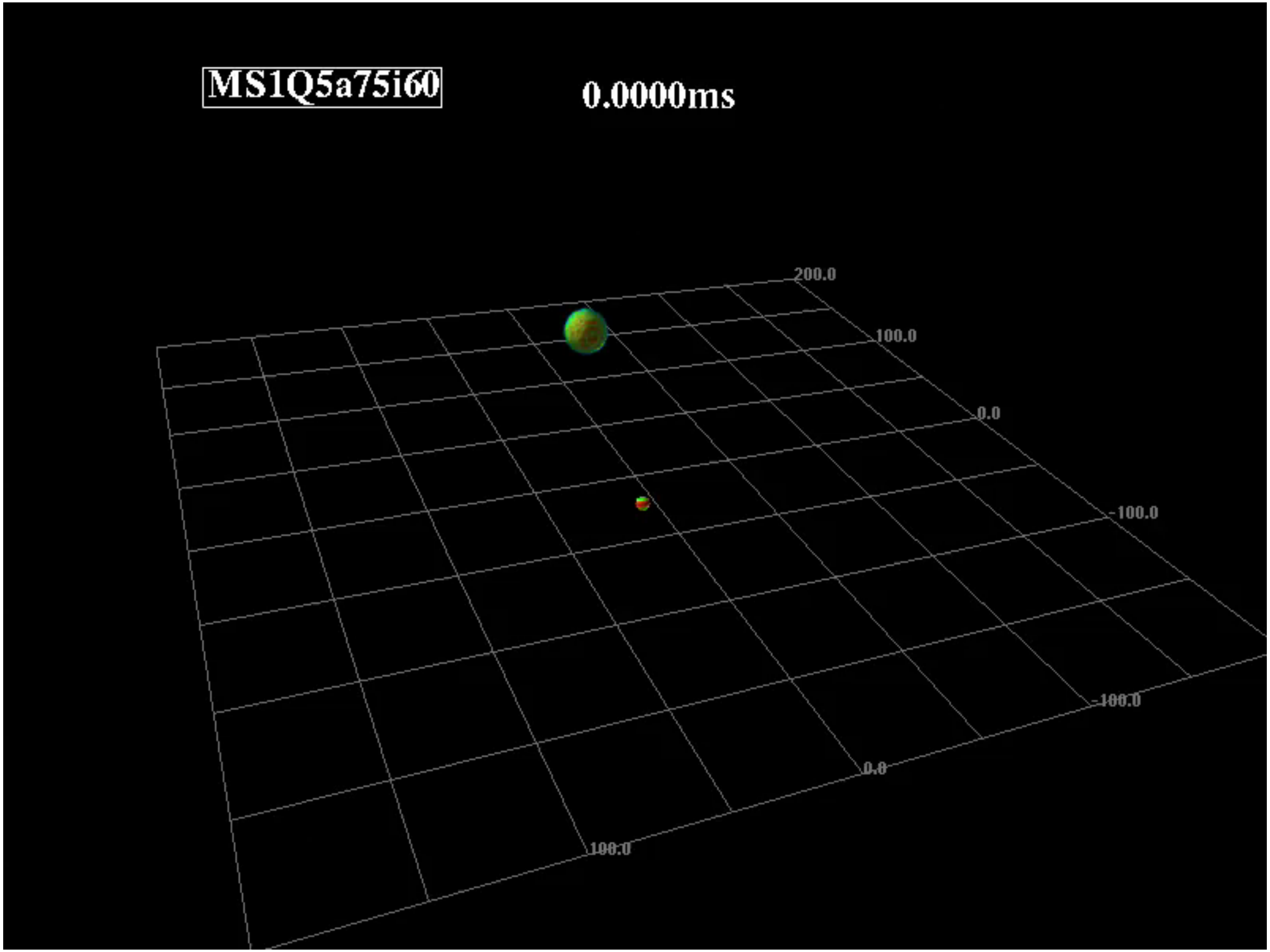


$$a^* = \frac{J}{M^2}$$

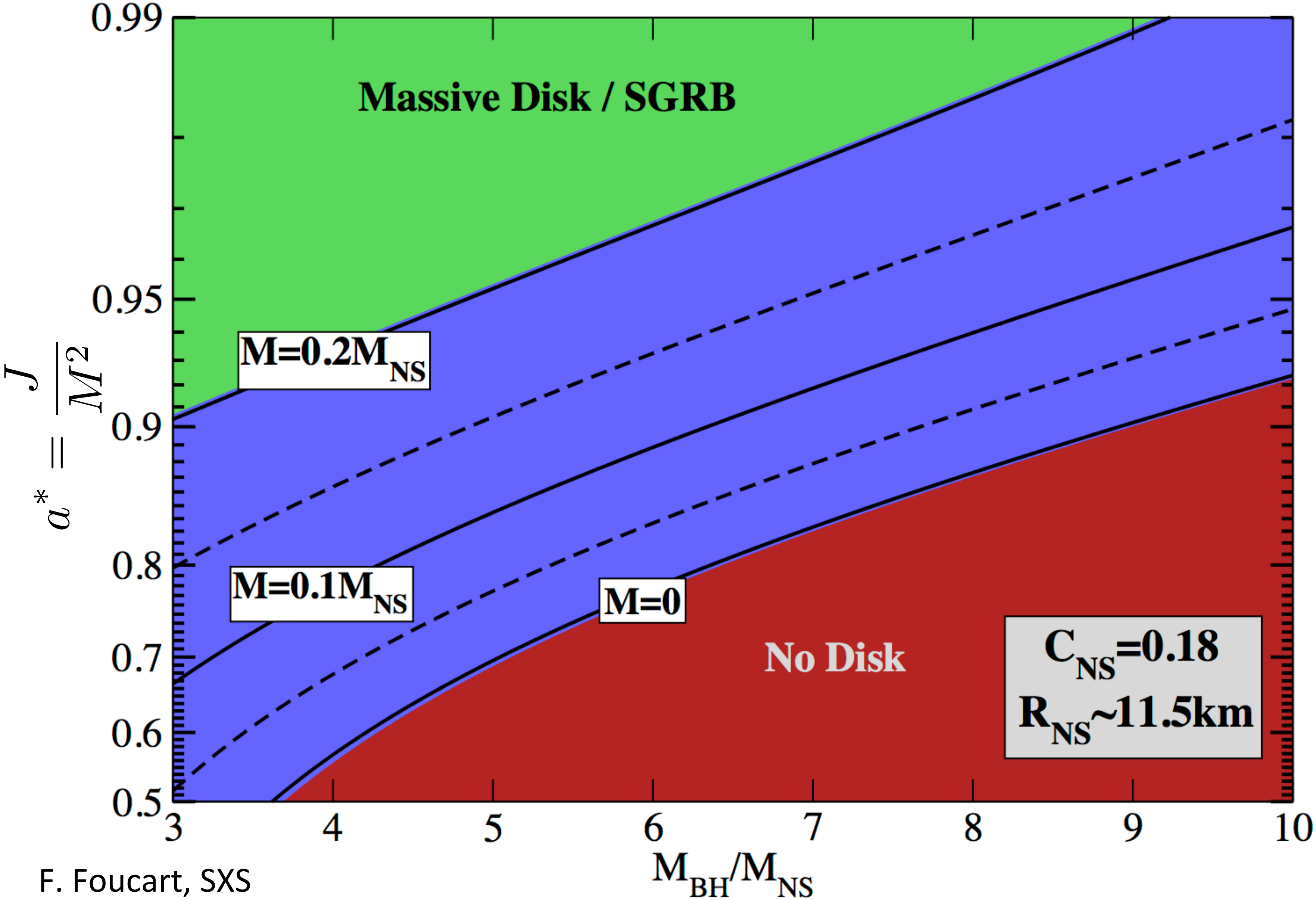
- Tidal disruption or complete "swallow".
- The greater BH spin a*, the stronger disruption.
- The larger M_{BH}, the more spin required for disruption.
- Typical BH/NS mass-ratio uncertain.
  Best guess: 7:1 – 10:1.
- Spins likely misaligned -> precession.

BHNS Merger Scenarios

Kyohei Kawaguchi

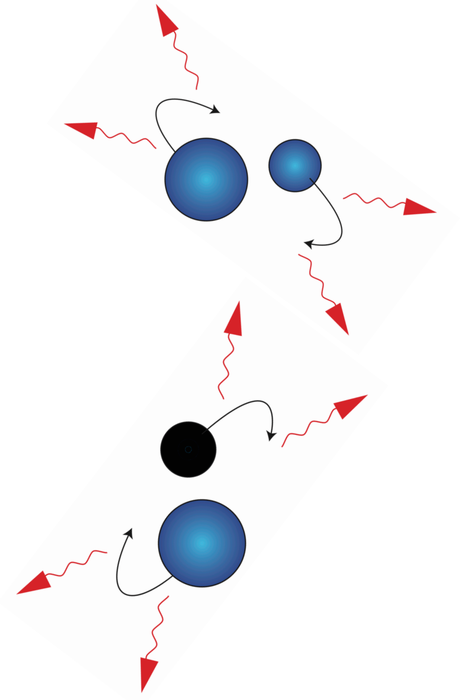


BHNS Merger Scenarios: Remnant



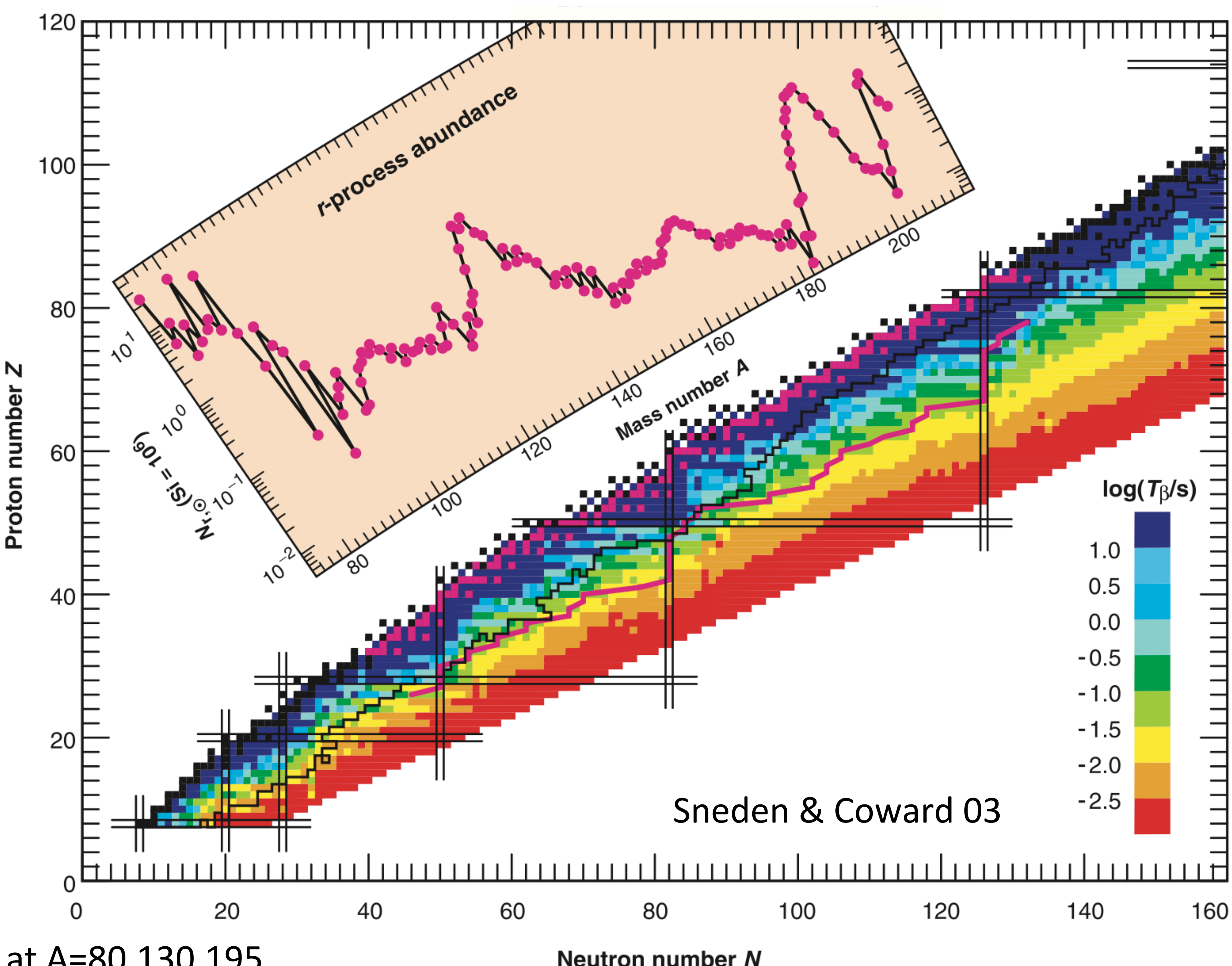
F. Foucart, SXS

Ejecta & Nucleosynthesis



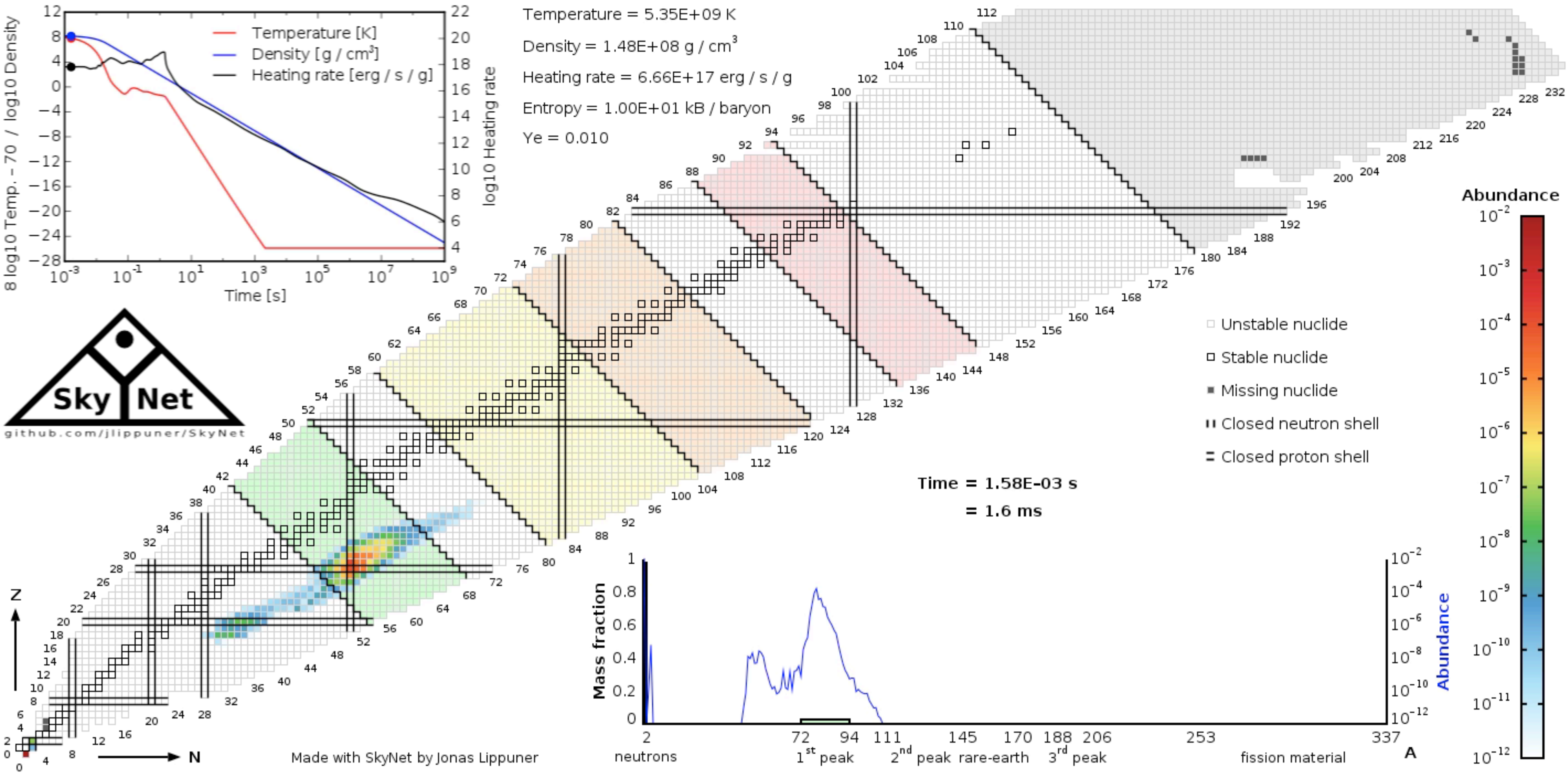
- NSNS & NSBH mergers eject neutron rich material.
- NSNS: (e.g., Sekiguchi+15)
  - Dynamical at merger: tidal tails, shock heating. Outflow mass: $10^{-3} – 10^{-2}\ M_\odot$
  - Postmerger (neutrino-driven) outflows.
  - Typical $Y_e \sim 0.2 – 0.3$.
- BHNS: (e.g., Foucart+15)
  - Dynamical at tidal disruption, $0.05 – 0.1\ M_\odot$, $Y_e < 0.1$.
  - Disk wind.

r-Process Nucleosynthesis: Rapid Neutron Capture

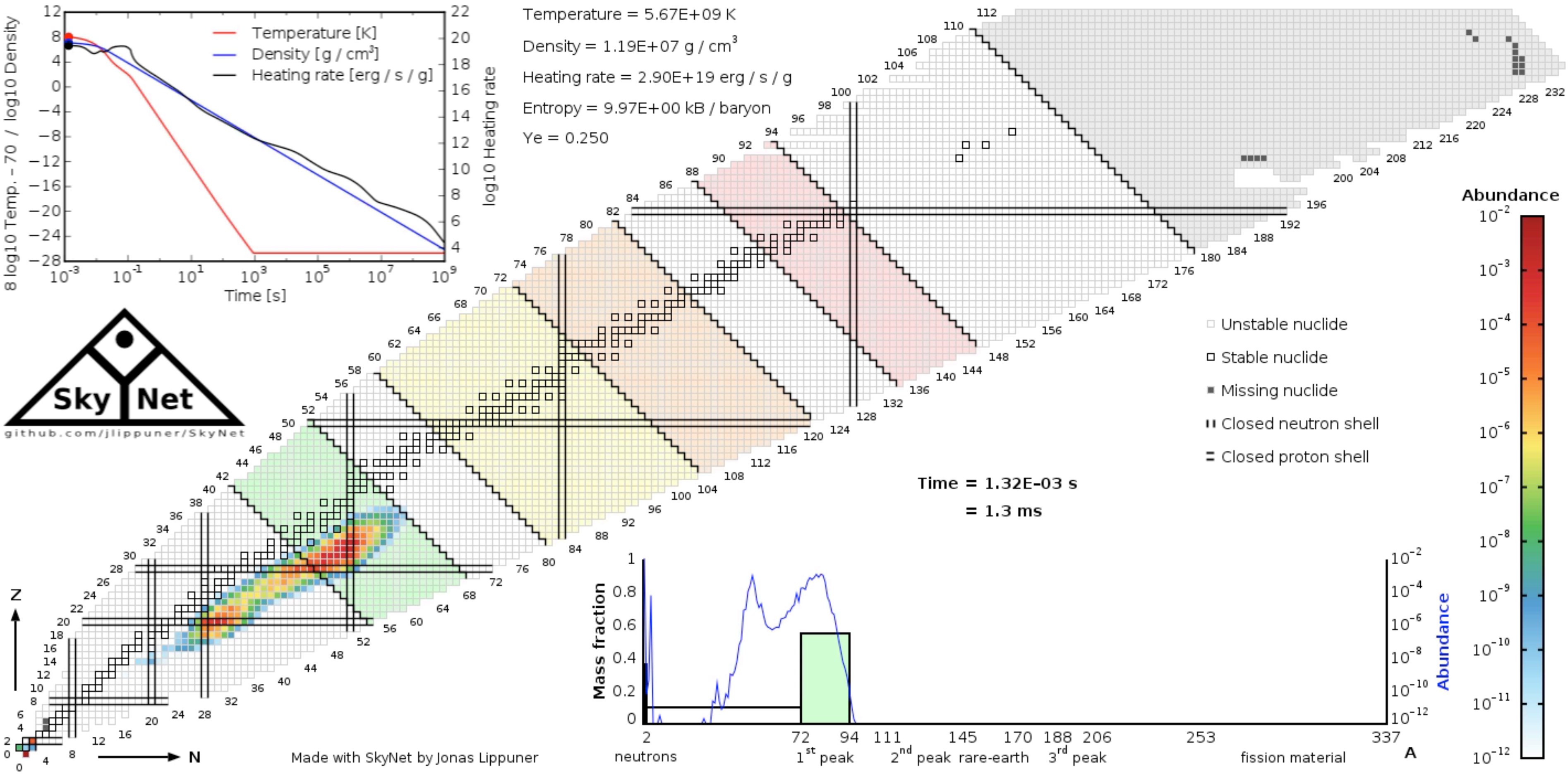


Peaks at A=80,130,195

$Y_e = 0.01$ r-Process Nucleosynthesis



$Y_e = 0.25$ r-Process Nucleosynthesis



Stellar Collapse &
Core-Collapse Supernovae

Massive Star Evolution Recap

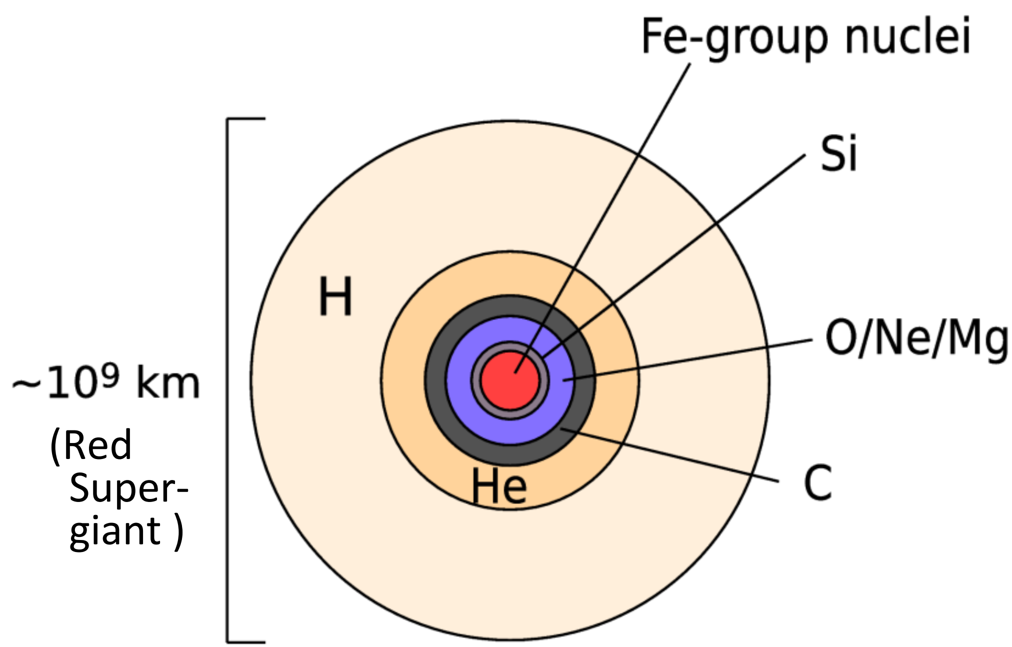
- CCSN Mass: $\sim 7\ M_{SUN} \leq M \leq \sim 100\ M_{SUN}$.
- Nuclear Burning:

H -> He -> C/O -> O/Ne/Mg -> Si -> Fe

$M < \sim 7\ M_{SUN}$ → **Envelope ejection** → C-O White Dwarf

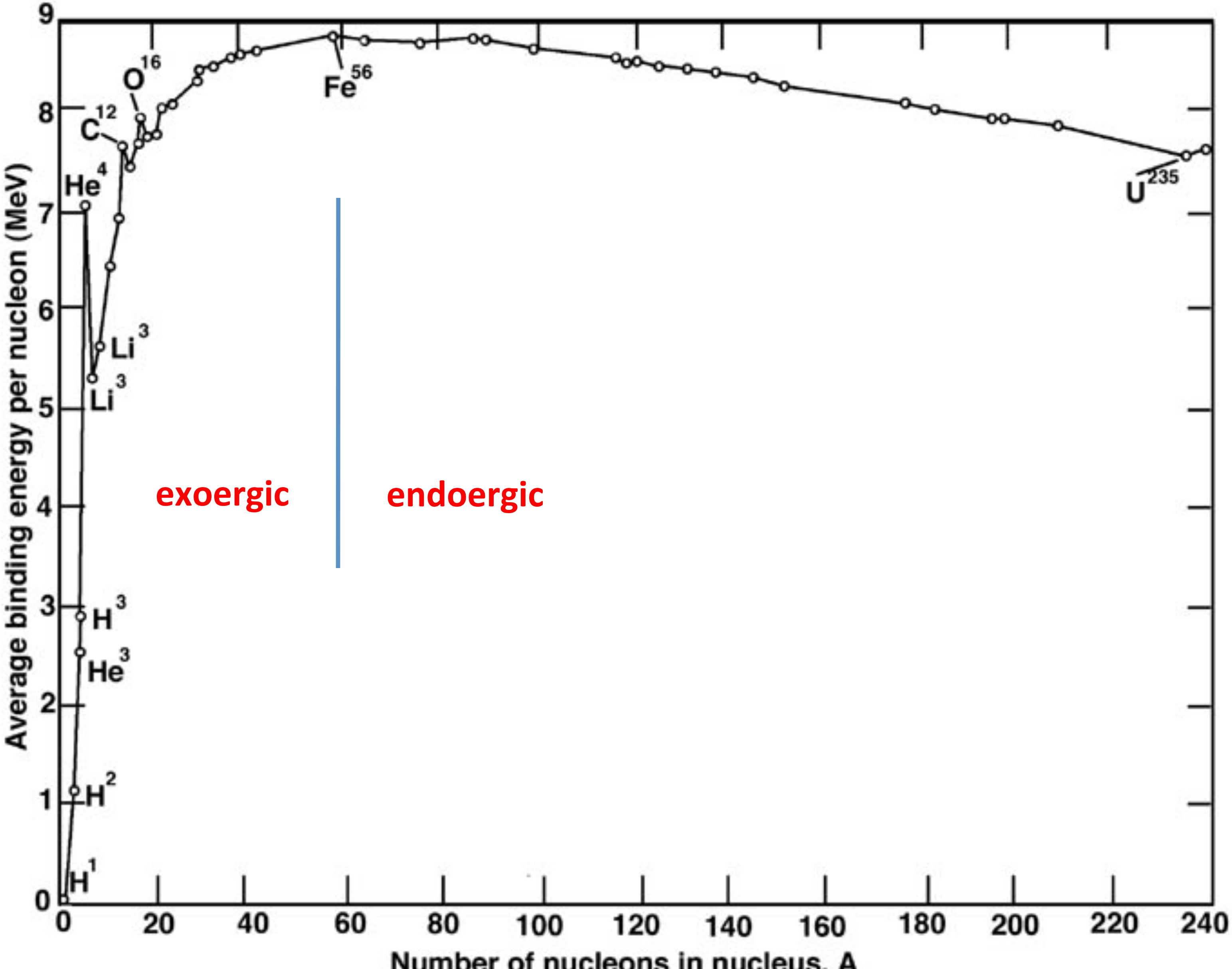
$\sim 7\ M_{SUN} < M < \sim 10\ M_{SUN}$ → **Envelope ejection** → O-Ne White Dwarf

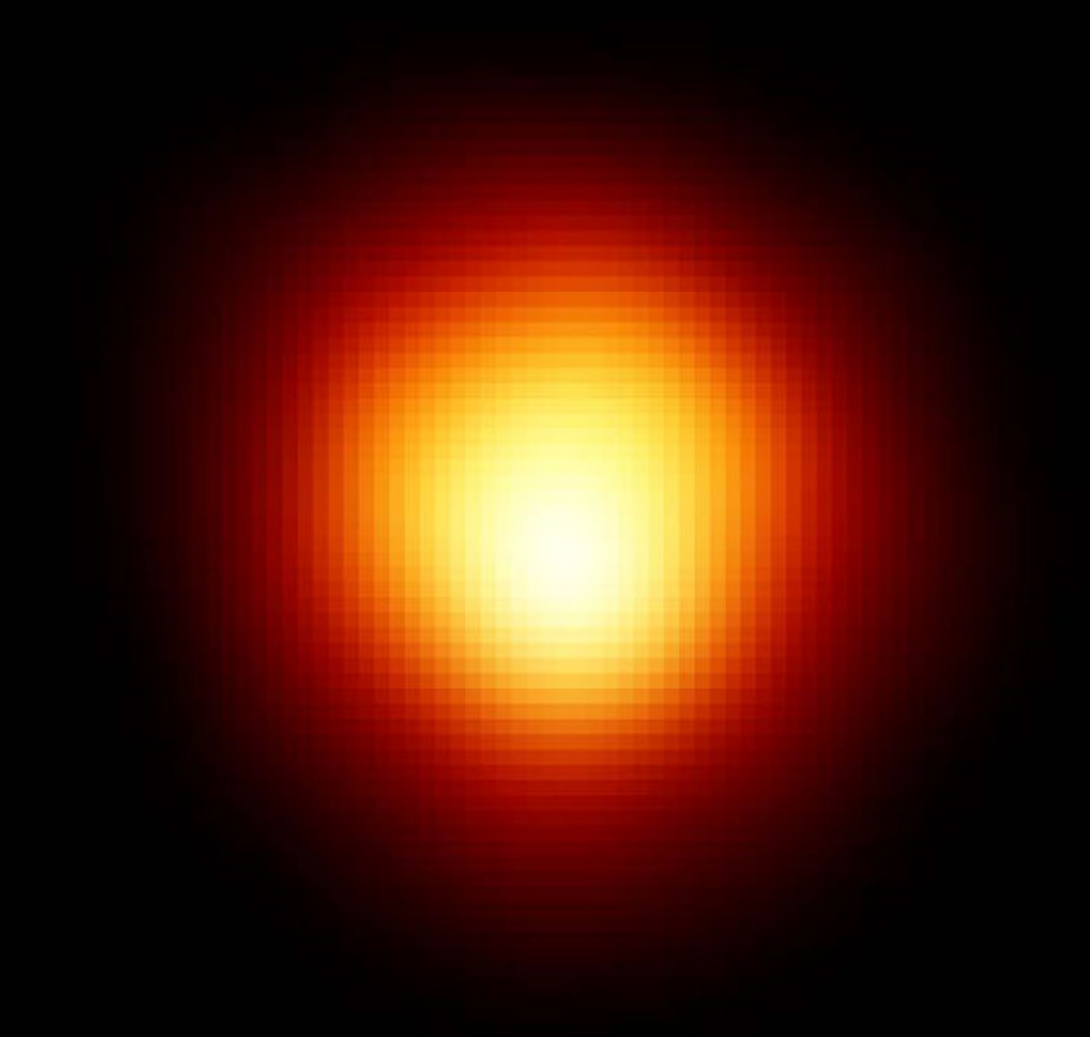
$M > \sim 10\ M_{SUN}$



- Key parameters controlling stellar evolution:
  - Mass
  - Metallicity (mass fraction of elements heavier than H, He)
  - Binary Interactions
  - **Rotation**

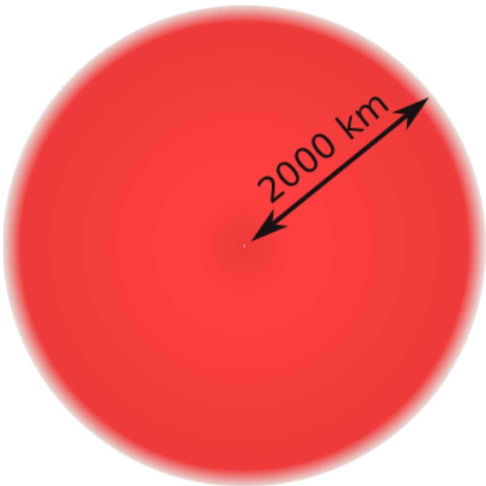
Massive Star Evolution





Betelgeuse, $M \sim 20\ M_{\odot}$, $R \sim 8 \times 10^{13}$ cm $\sim 1000\ R_{\odot}$
(HST)

Hydrostatics of the Iron Core



Iron Core

$\rho_c \approx 10^{10}$ g/cm^3

T ≈ 1 MeV

$Y_e \approx 0.5$

(in reality: T lower and Y_e slightly lower)

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

What produces the pressure?

ions (iron-group nuclei)

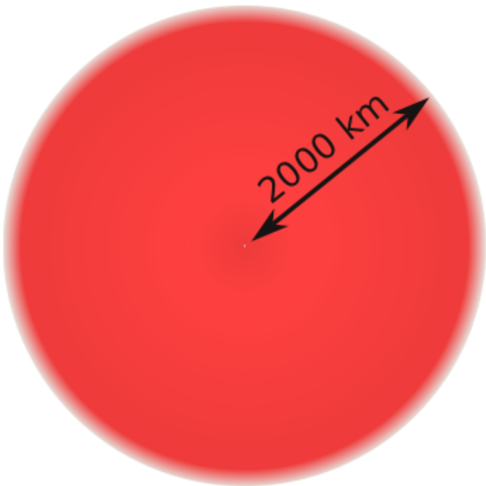
electrons

photons

$$P = P_{\rm ion} + P_{\rm rad} + P_e$$

What dominates?

Equation of state in the Iron Core



Iron Core

$$P = P_{\text{ion}} + P_{\text{rad}} + P_e$$

$$P_{\text{ion}} = 1.7 \times 10^{26} \left(\frac{\rho}{10^{10}\,\text{g}\,\text{cm}^{-3}} \right) \left(\frac{T}{1\,\text{MeV}} \right) \text{dyn}\,\text{cm}^{-2}$$

$$P_\gamma = \frac{1}{3} a T^4 = 4.6 \times 10^{25} \left(\frac{T}{1\,\text{MeV}} \right)^4 \text{dyn}\,\text{cm}^{-2}$$

$$P_e = 10^{28} \left(\frac{Y_e}{0.5} \right)^{4/3} \left(\frac{\rho}{10^{10}\,\text{g}\,\text{cm}^{-3}} \right)^{4/3} \text{dyn}\,\text{cm}^{-2}$$

$\rho_c \approx 10^{10}$ g/cm^3

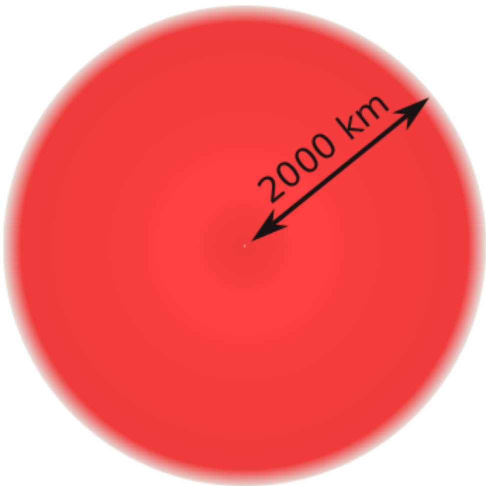
$T \approx 1$ MeV

$Y_e \approx 0.5$

(in reality: T lower and Y_e slightly lower)

$$P_e \gg P_{\text{ion}} \gg P_{\text{rad}}$$

Onset of Collapse



Iron Core

$\rho_c \approx 10^{10}$ g/cm^3

$T \approx 1$ MeV

$Y_e \approx 0.5$

(in reality: T lower and Y_e slightly lower)

- Chandrasekhar:

$$M_{\rm Ch,eff} \approx 1.44 \left(\frac{Y_e}{0.5}\right)^2 M_\odot$$

+ corrections: GR, thermal, surface P etc.

No equilibrium solutions exists for relativistic & degenerate electron gas for

$$M > M_{\rm Ch,eff}$$

-> radial instability -> core collapse!

Two ways to get there:

(1) Silicon shell burning adding mass to the core.

(2) Reduction of Y_e.

-> electron capture $p + e^- \rightarrow n + \nu_e$

Things that happen during collapse:

In collapse, pressure support is reduced by

- **Photodissociation** of heavy nuclei: ~125 MeV/reaction

$$\gamma + {}^{56}_{26}\mathrm{Fe} \rightleftharpoons 13\alpha + 4n$$

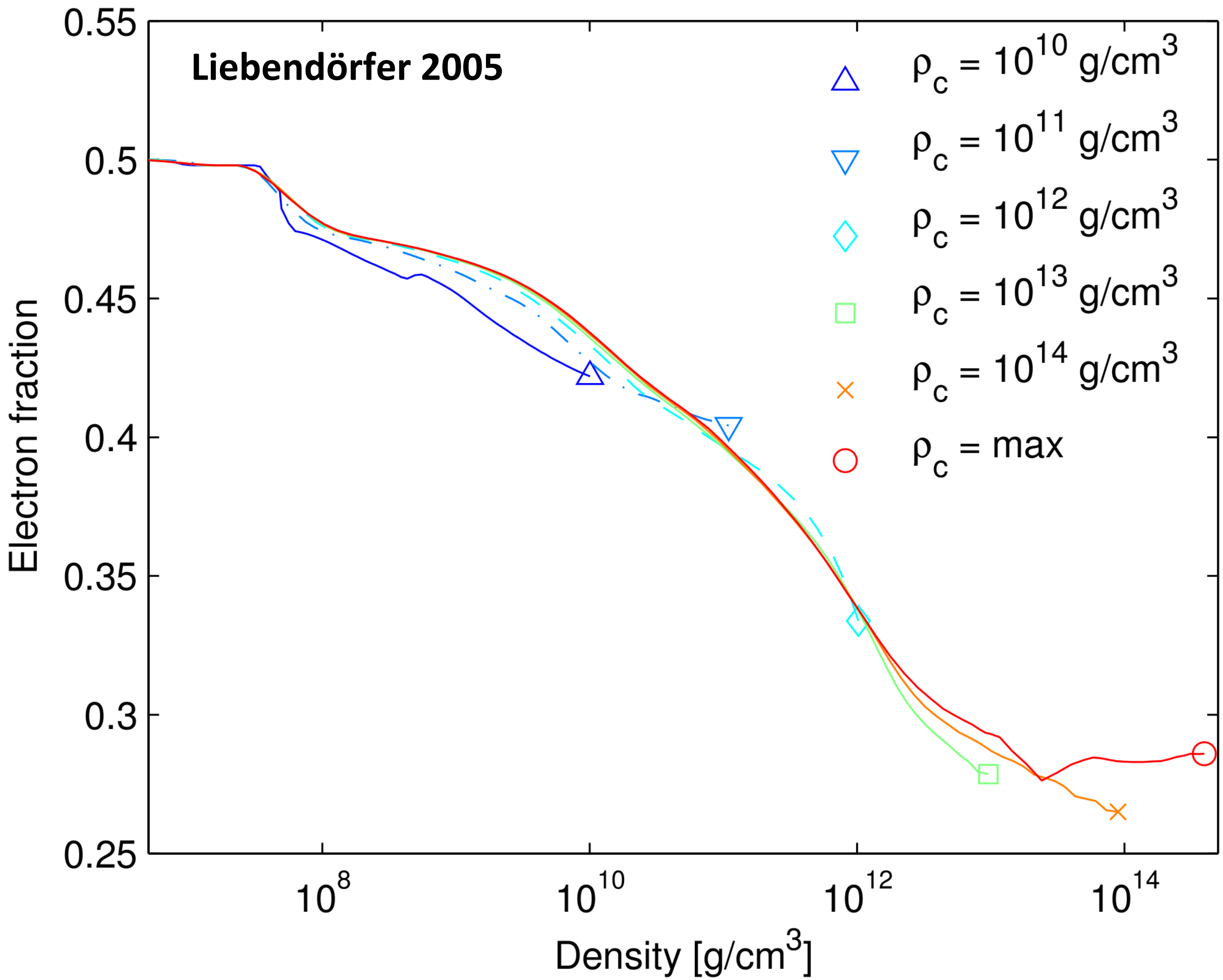
- **Electron Capture** $e^- + (Z, A) \stackrel{(W)}{\longrightarrow} \nu_e + (Z-1, A)$

$$e^- + p \stackrel{(W)}{\longrightarrow} \nu_e + n\ .$$

$$\frac{\partial}{\partial t} Y_e \propto \mu_e^5 \propto \rho^{5/3}$$

- Neutrinos stream off almost freely at densities below ~10^{12} g/cm^3.
  -> core "**deleptonizes**" during collapse.
- Net entropy change is small,
  -> **collapse proceeds practically adiabatically.**

Deleptonization



Neutrino Trapping

- Collapse phase: Neutrino opacity dominated by coherent neutrino-nucleus scattering: $\nu + (A,Z) \longleftrightarrow \nu + (A,Z)$

  Neutrino mean-free path: $\lambda_\nu \approx 10^7\,\mathrm{cm} \left(\frac{10^{12}\,\mathrm{g\,cm^{-3}}}{\rho}\right) \frac{A}{N^2} \left(\frac{10\,\mathrm{MeV}}{\epsilon_\nu^2}\right)$

- For $\rho \geq 3 \times 10^{12}$ g/cm^3, diffusion time τ_{diff} >> collapse time
  -> **neutrinos become dynamically trapped in the collapsing core.**

- **Consequences**:

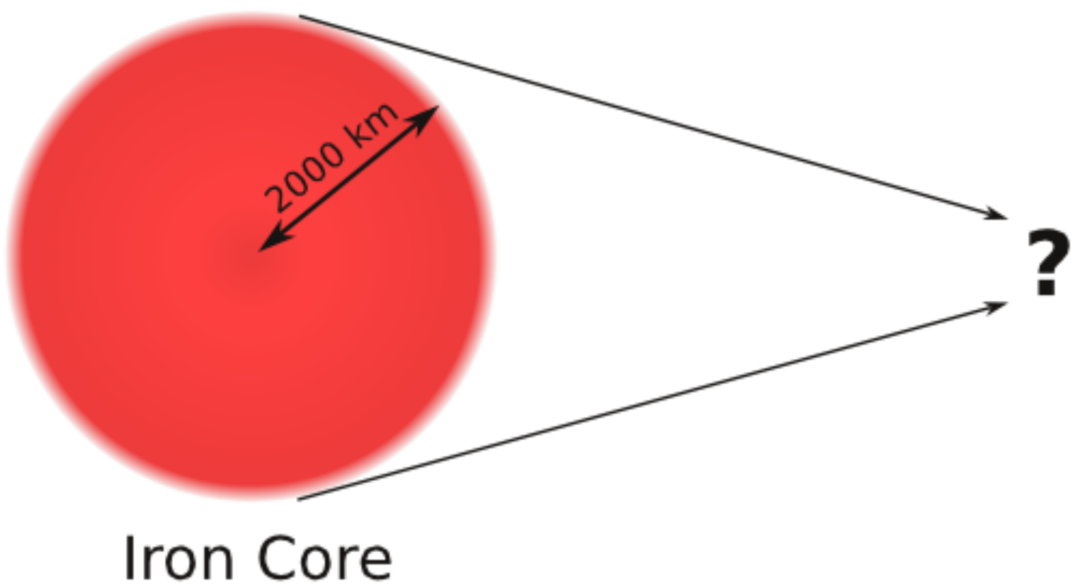
  Deleptonization stopped $Y_{\mathrm{lep}} = Y_e + Y_\nu = \mathrm{const.}$

  Detailed simulations: $Y_{\mathrm{lep}} \approx 0.32$

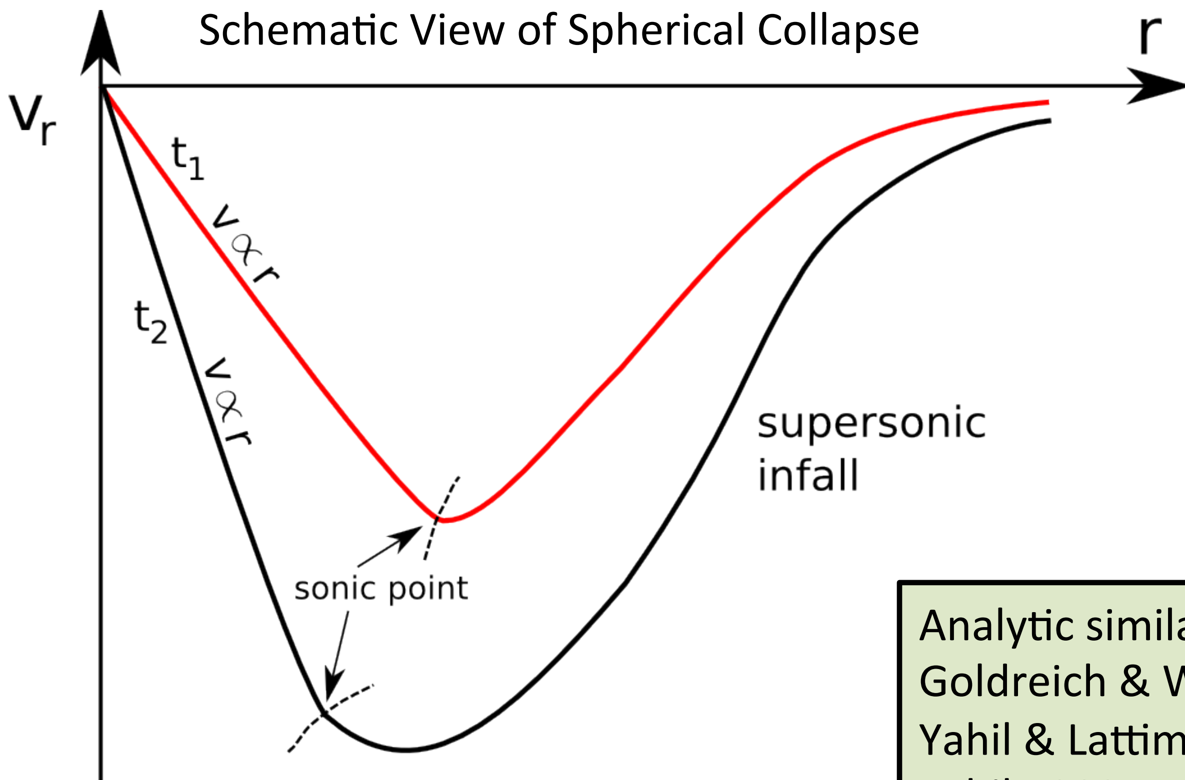
  Beta Equilibrium $e^- + p \longleftrightarrow \nu_e + n$

  $$\mu_e + \mu_p = \mu_\nu + \mu_n$$

still collapsing...



Self-Similarity in Core Collapse



Analytic similarity solutions:
Goldreich & Weber 1980
Yahil & Lattimer 1982
Yahil 1983

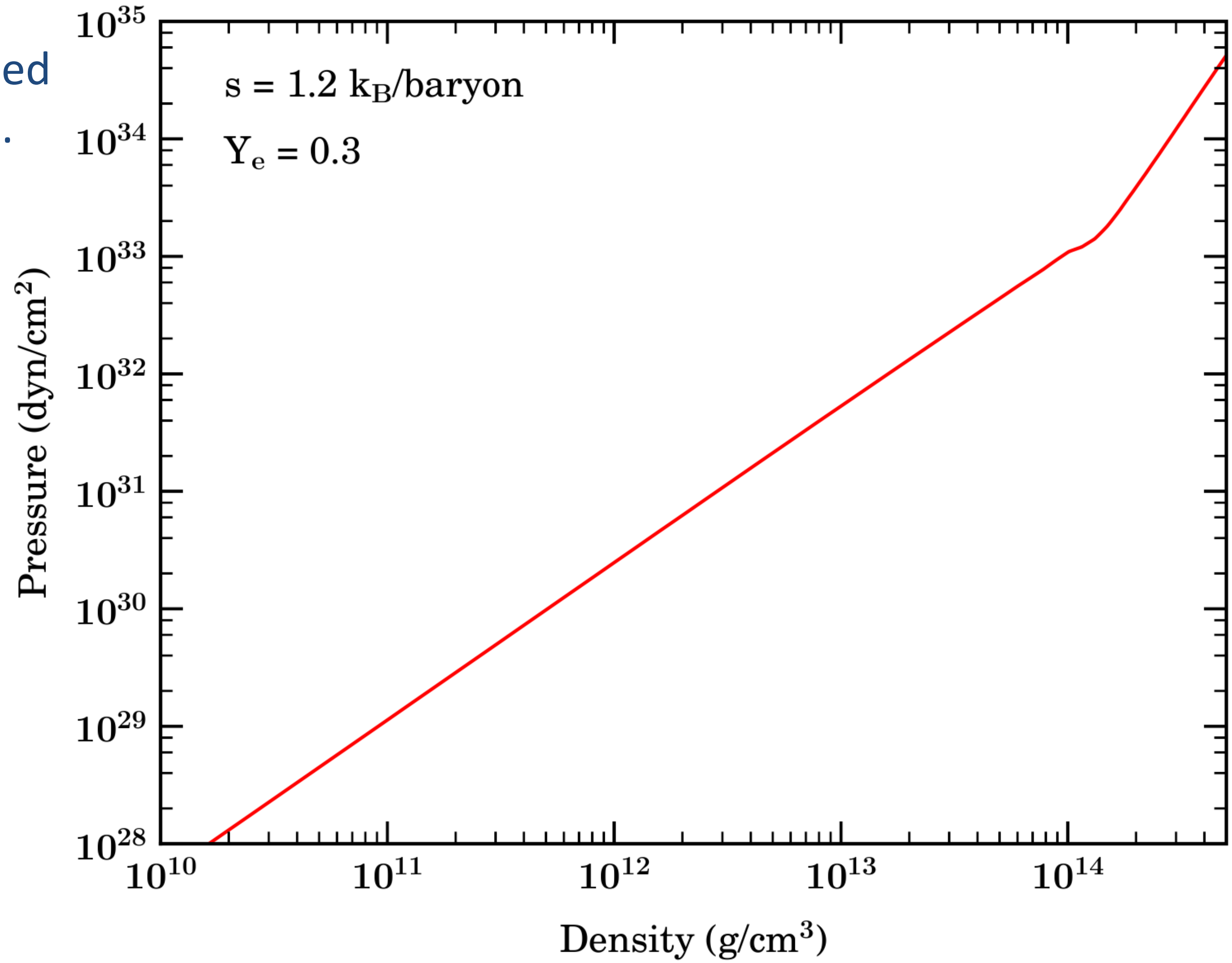
- Separation into **homologously ($v \propto r$) collapsing inner core** and **supersonically collapsing outer core**.

Equation of State in Collapse

Nuclear Statistical Equilibrium ($\rho > 10^7$ g/cm^3, T > 0.5 MeV)

-> $P = P(\rho, T, Y_e)$

Composition determined by Saha-type equation.

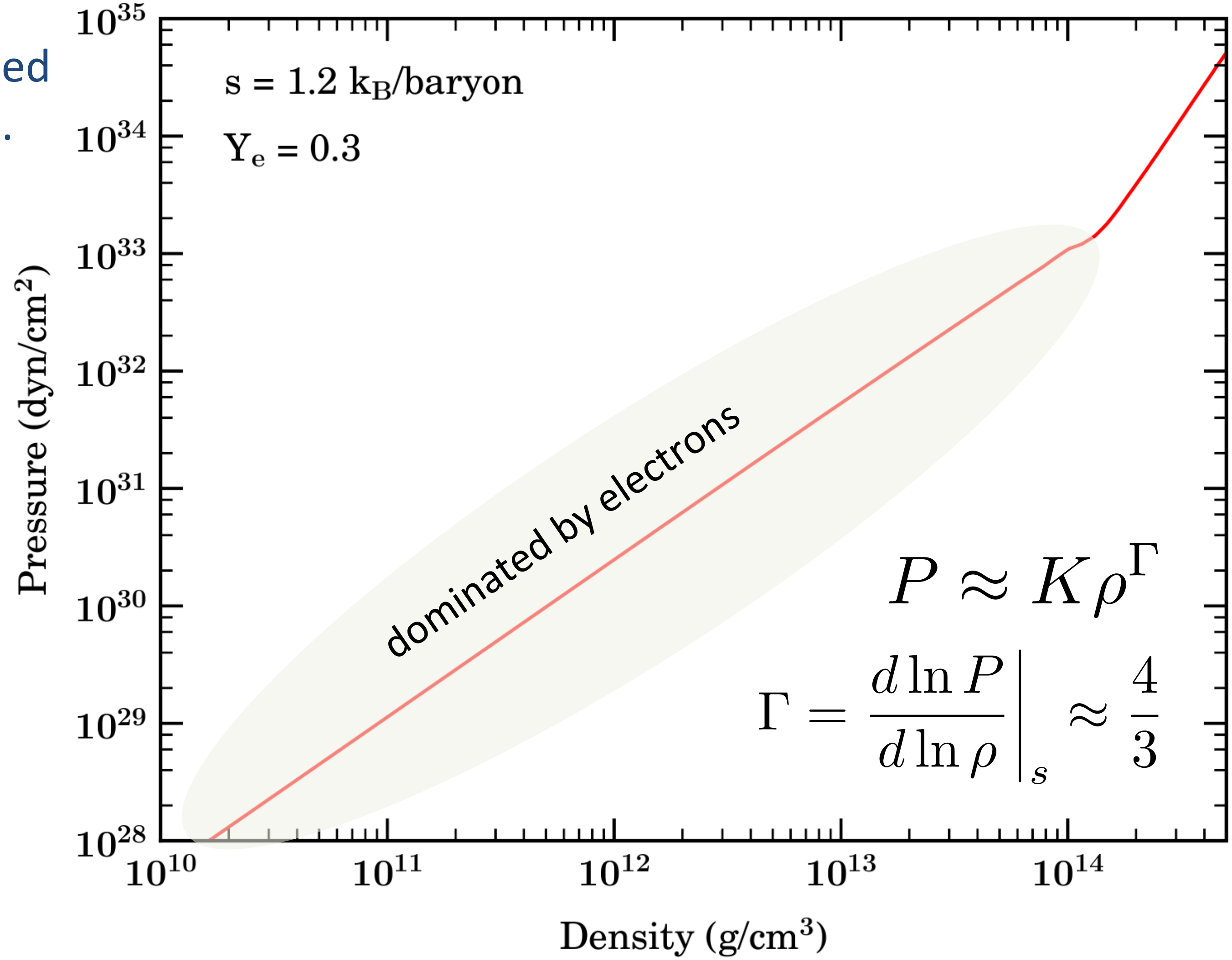


Equation of State in Collapse

Nuclear Statistical Equilibrium ($\rho > 10^7$ g/cm^3, T > 0.5 MeV)

-> $P = P(\rho, T, Y_e)$

Composition determined by Saha-type equation.

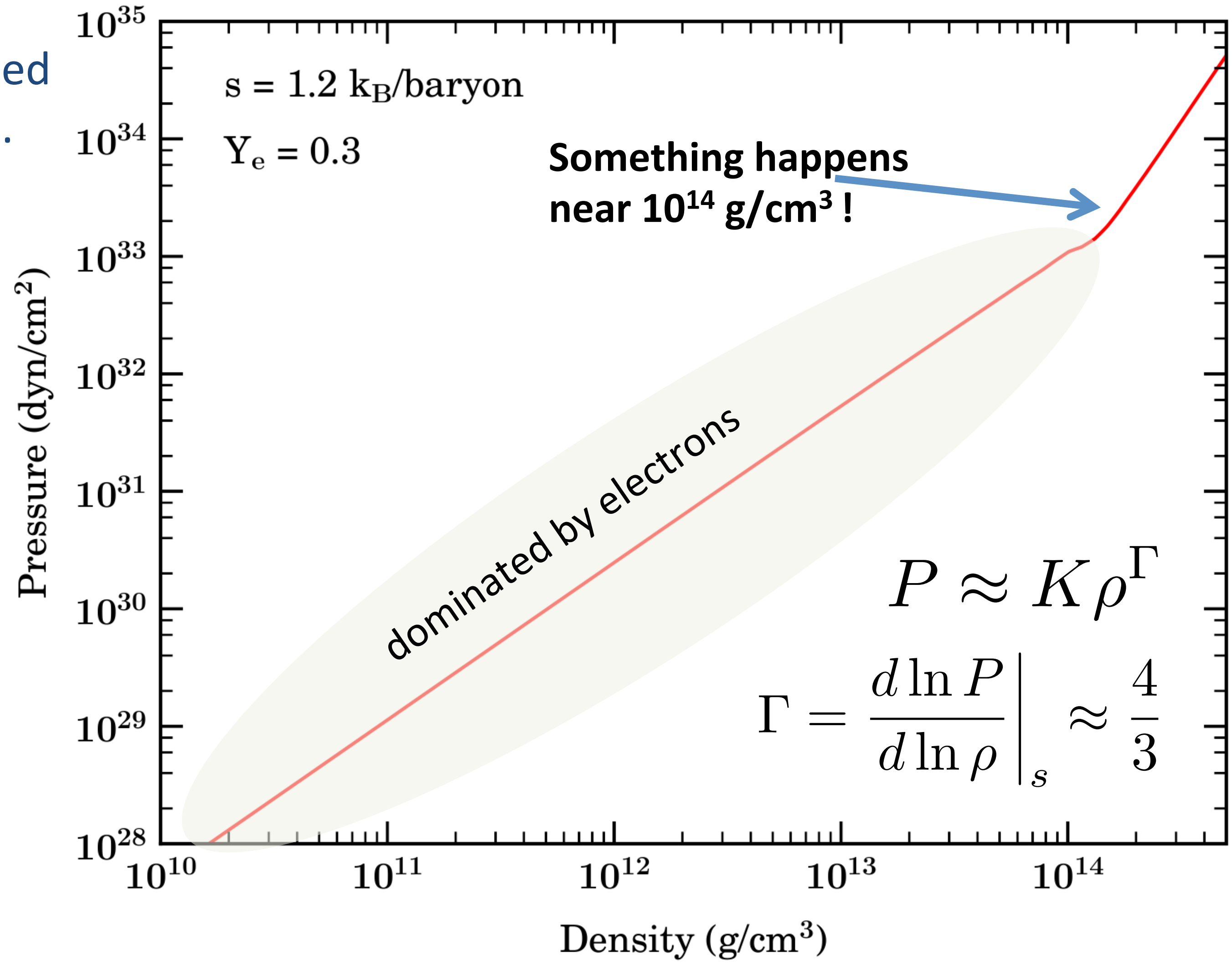


Equation of State in Collapse

Nuclear Statistical Equilibrium ($\rho > 10^7$ g/cm^3, T > 0.5 MeV)

-> $P = P(\rho, T, Y_e)$

Composition determined by Saha-type equation.



Nuclear Equation of State

Nuclear Physics:

$$R_{\rm nuc} = A^{1/3} r_0$$

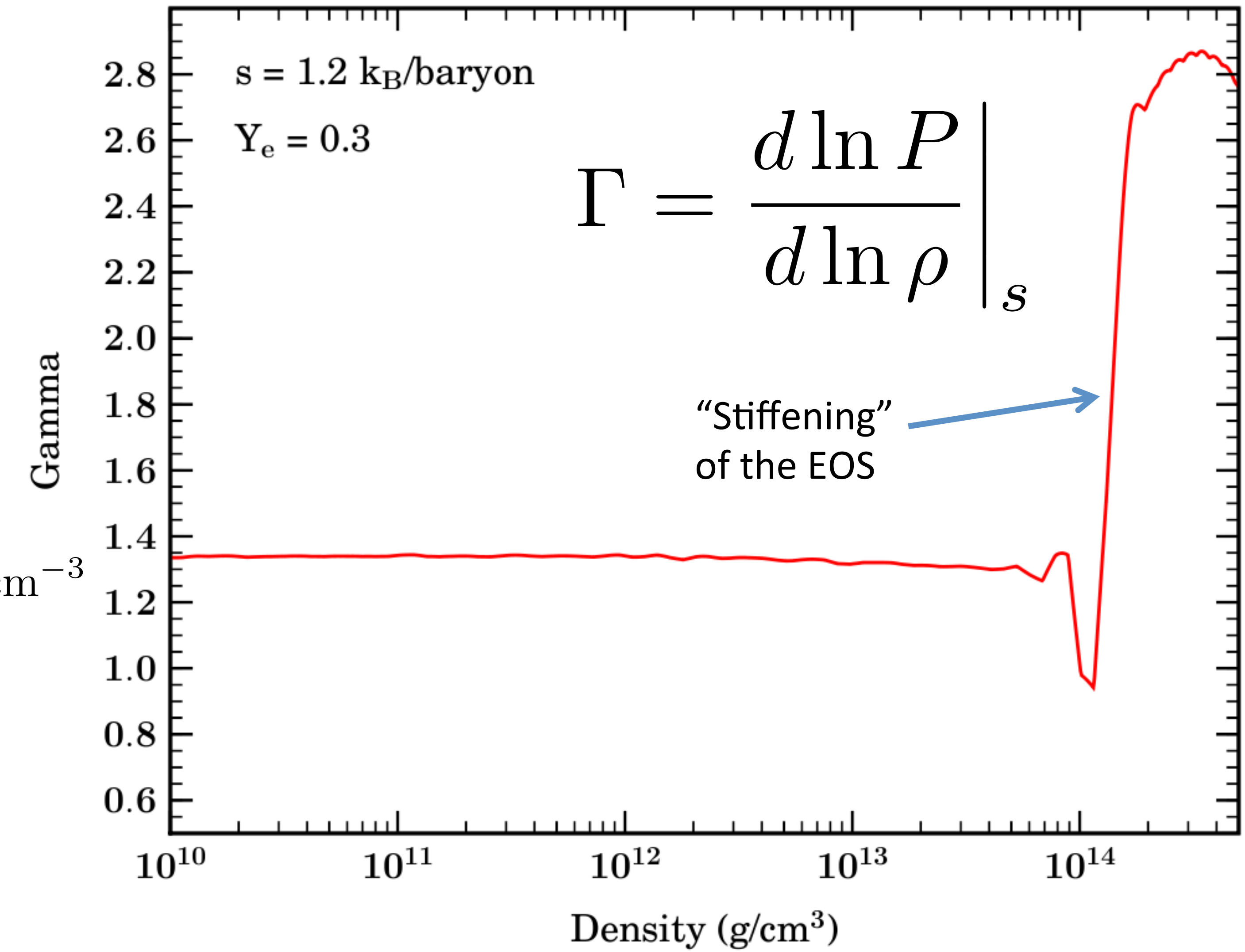
$$r_0 = 1.25\,{\rm fm}$$

Nuclear Density:

$$\bar{\rho}_{\rm nuc} = \frac{A\, m_b}{\frac{4}{3}\pi R_{\rm nuc}^3}$$

$$\rho_{\rm nuc} \sim 2.7 \times 10^{14}\,{\rm g\,cm^{-3}}$$

$$n_{\rm nuc} \sim 0.16\,{\rm fm^{-3}}$$



Nuclear Equation of State

Nuclear Physics:

$$R_{\rm nuc} = A^{1/3} r_0$$

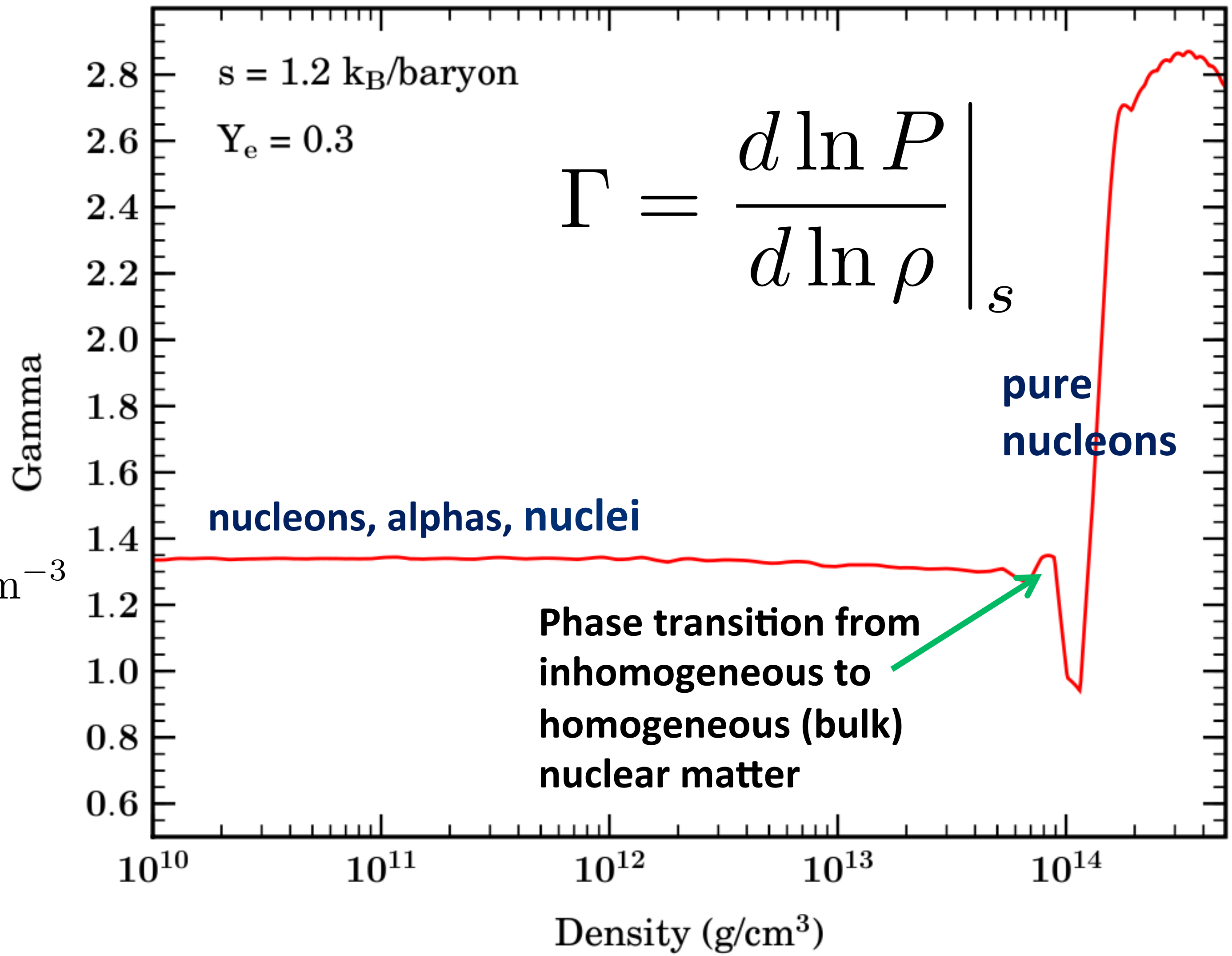
$$r_0 = 1.25\,\mathrm{fm}$$

Nuclear Density:

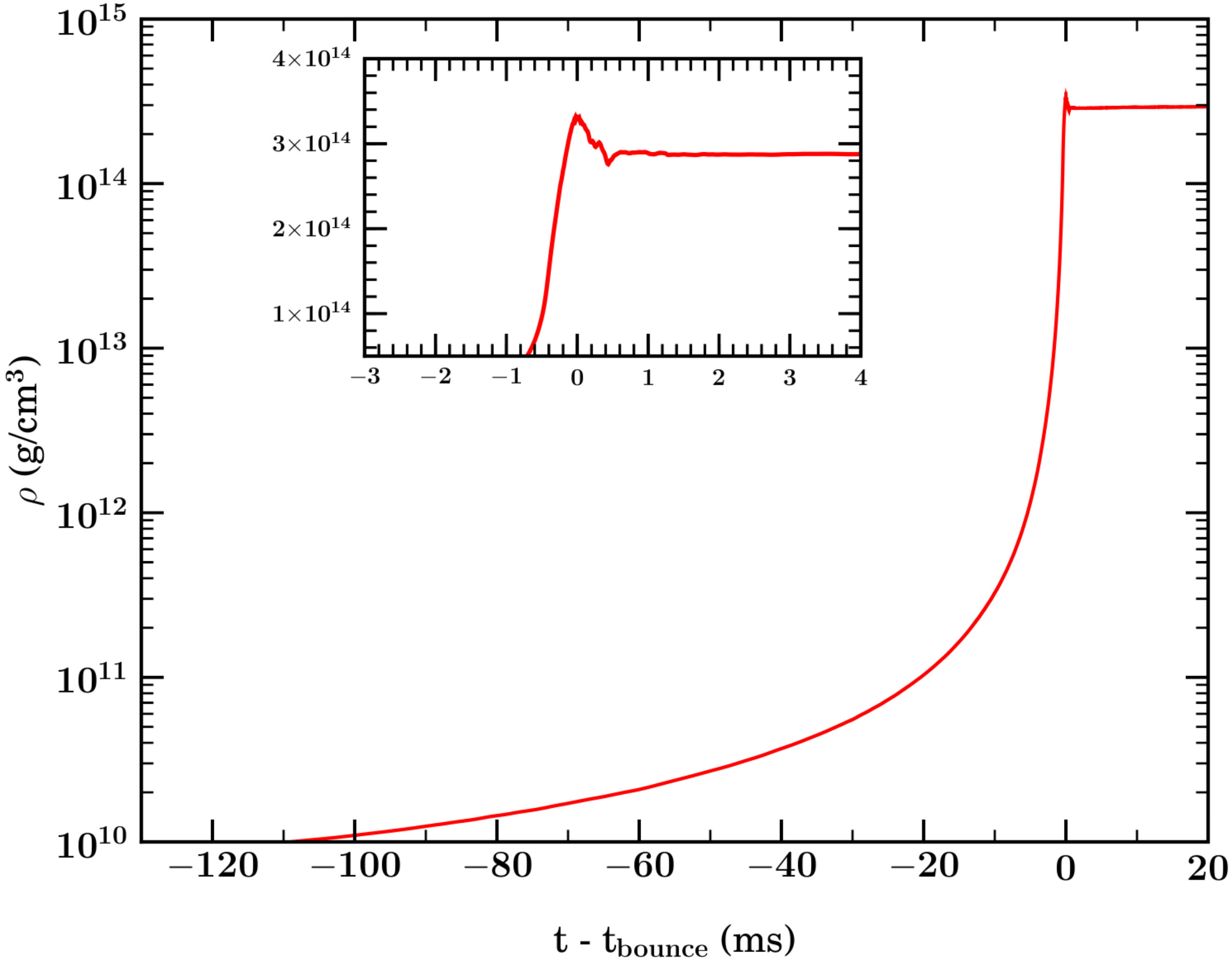
$$\bar{\rho}_{\rm nuc} = \frac{A\,m_b}{\frac{4}{3}\pi R_{\rm nuc}^3}$$

$$\rho_{\rm nuc} \sim 2.7 \times 10^{14}\,\mathrm{g\,cm^{-3}}$$

$$n_{\rm nuc} \sim 0.16\,\mathrm{fm^{-3}}$$

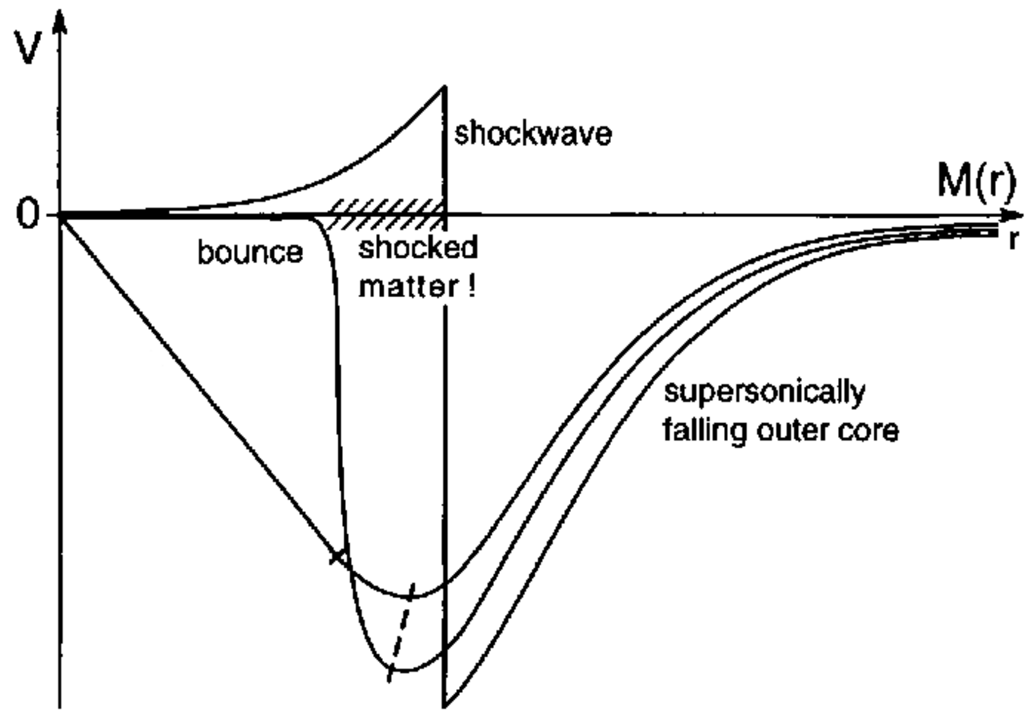


Effect of Stiffening



- **Inner Core** reaches ρ_{nuc}, rebounds ("bounces") **into still infalling outer core**.

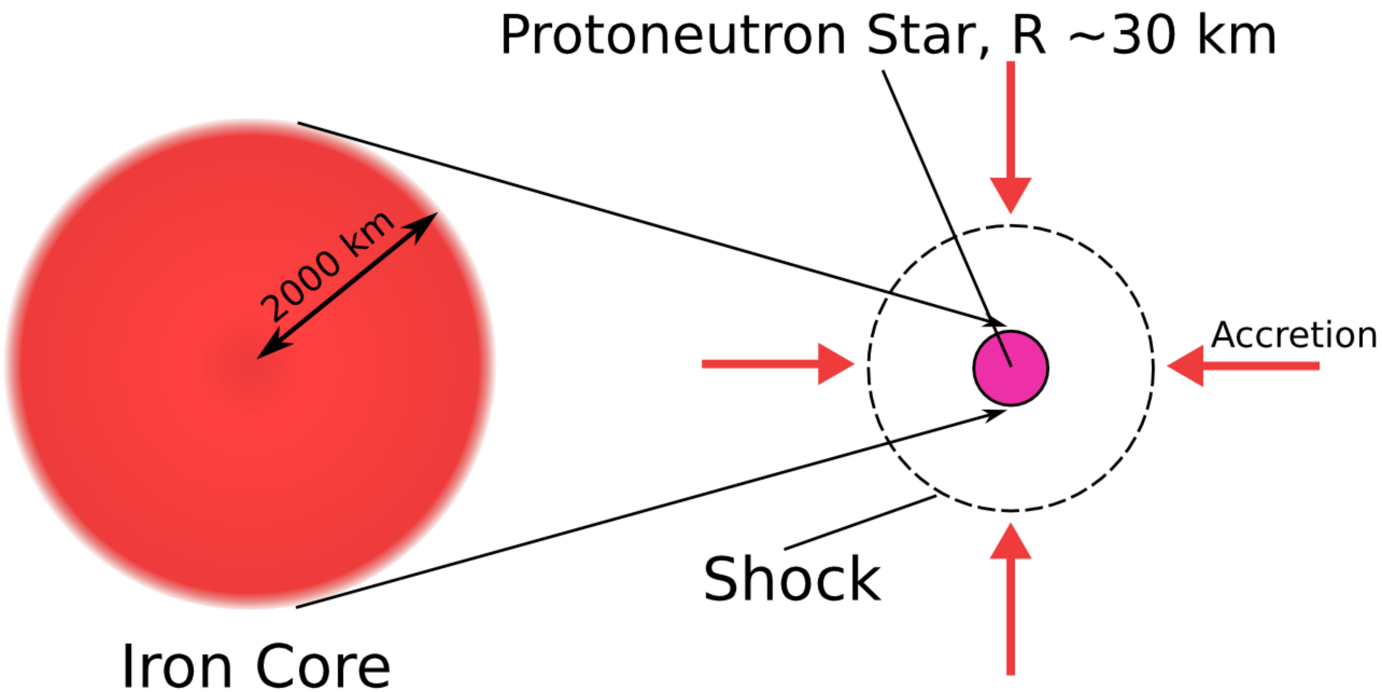
Shock Formation



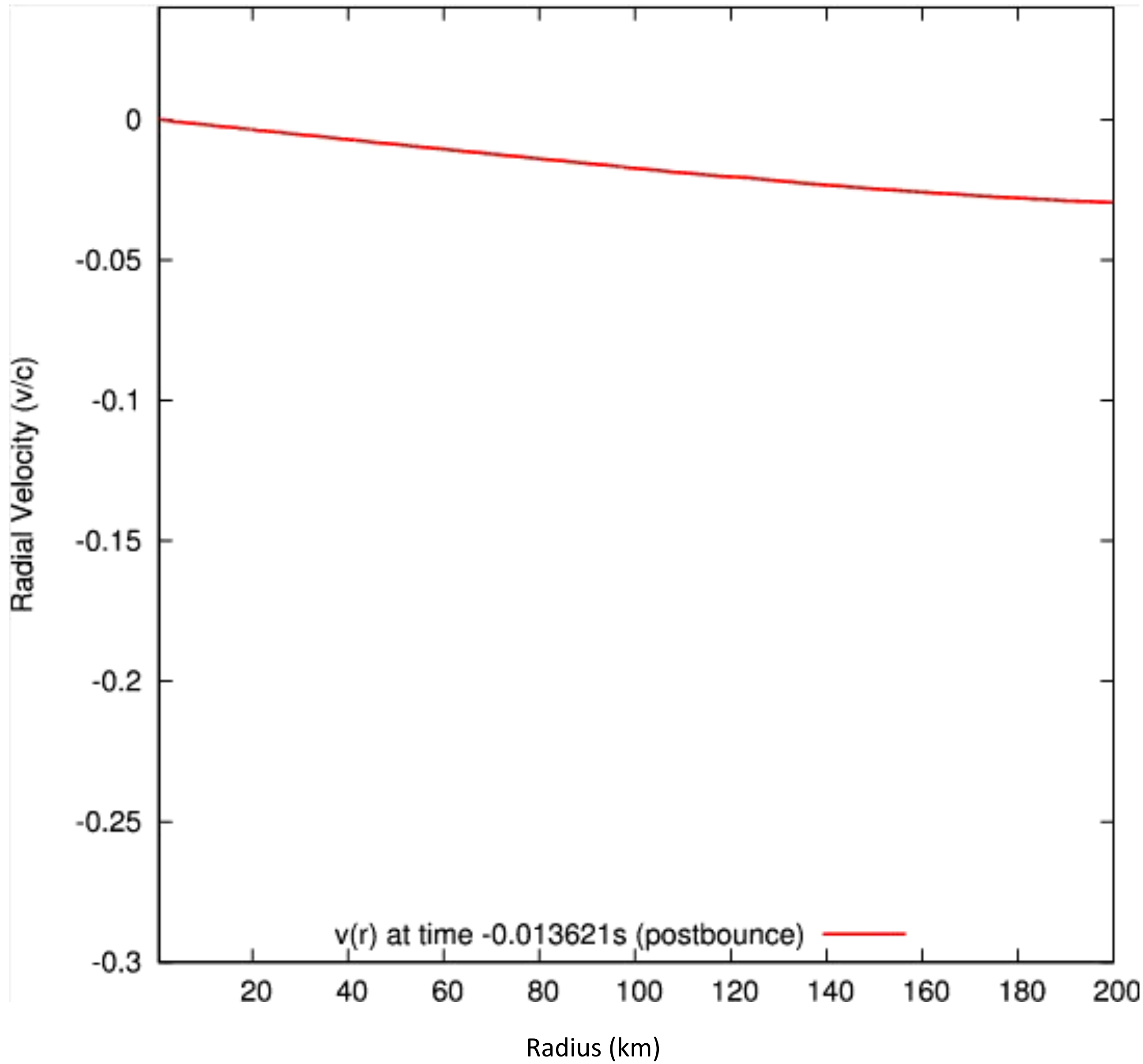
Credit:
E. Müller
Saas-Fee Lectures 1998

- Stiffening of EOS leads to sound wave that propagates through the inner core and steepens to a shock at the sonic point.

Core Bounce and Shock Formation



The Supernova Problem

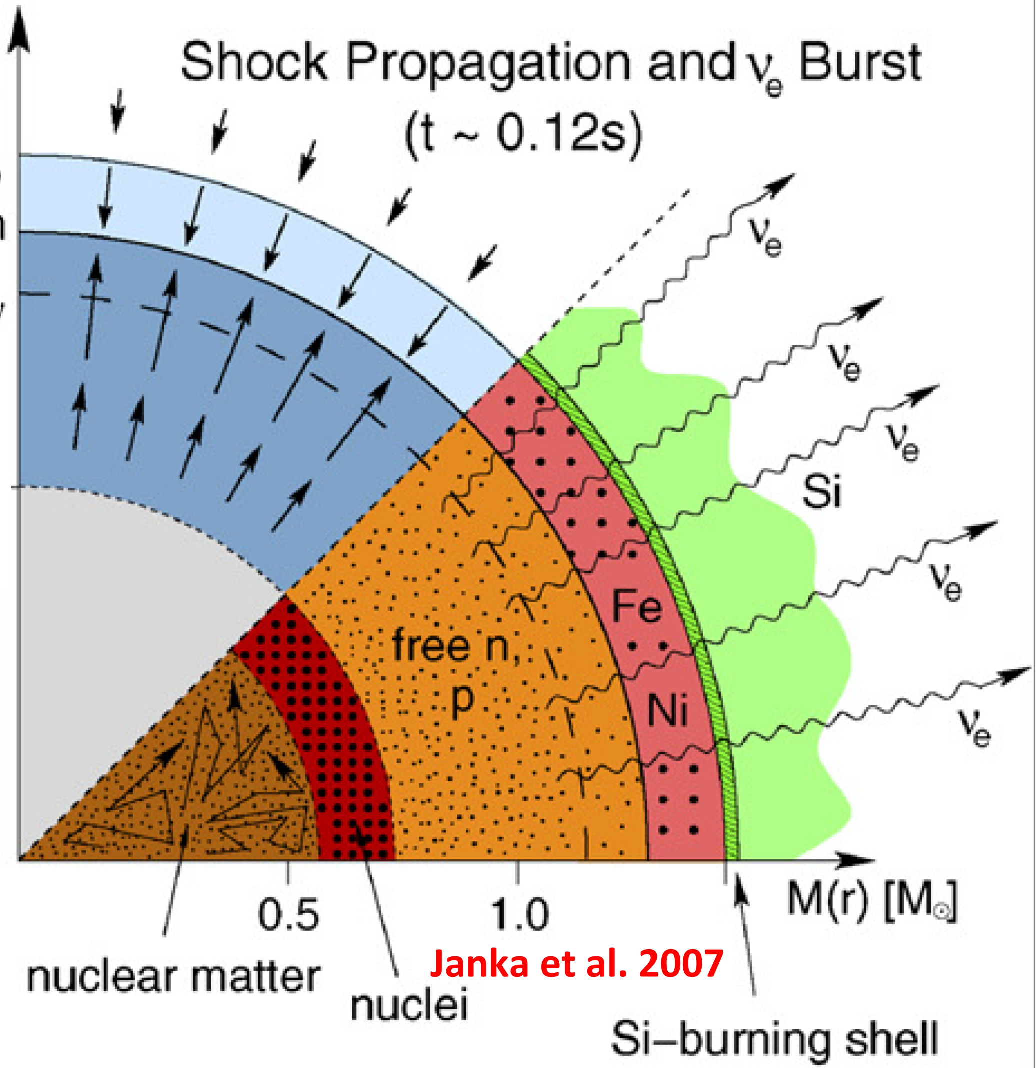


Movie by Evan O'Connor

Why does the shock stall?

- Shock loses energy to:
  - Dissociation of infalling heavy nuclei:
    ~8.8 MeV/baryon
    -> 17 x 10^{51} erg / $M_{\odot}$
  - Neutrinos that stream away from behind the shock.

Inner core -> Core of the proto-neutron star (PNS)



Janka et al. 2007

Neutrino Burst

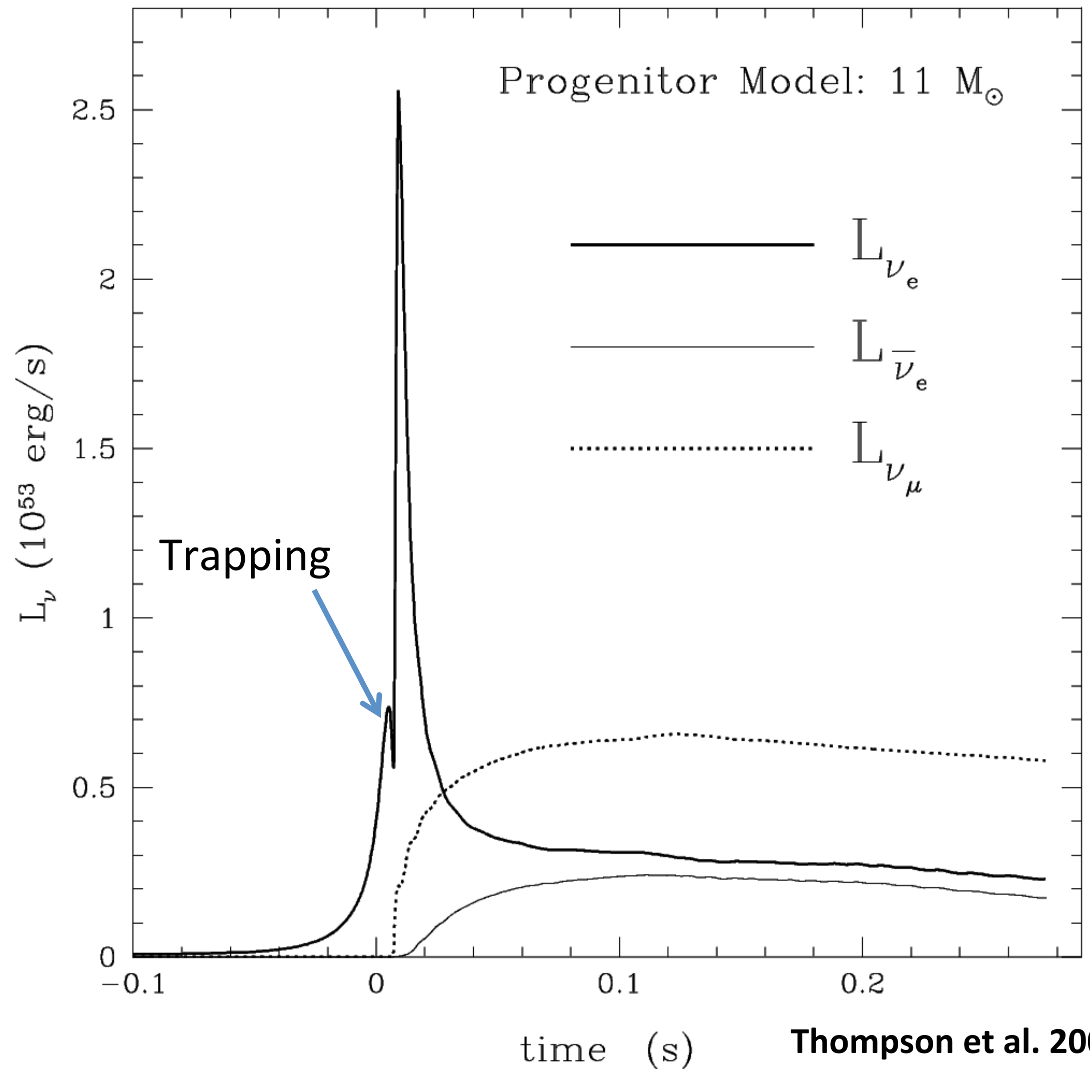
- Optical depth

$$\tau_\nu(r) = \int_\infty^r \frac{1}{\lambda_\nu} dr'$$

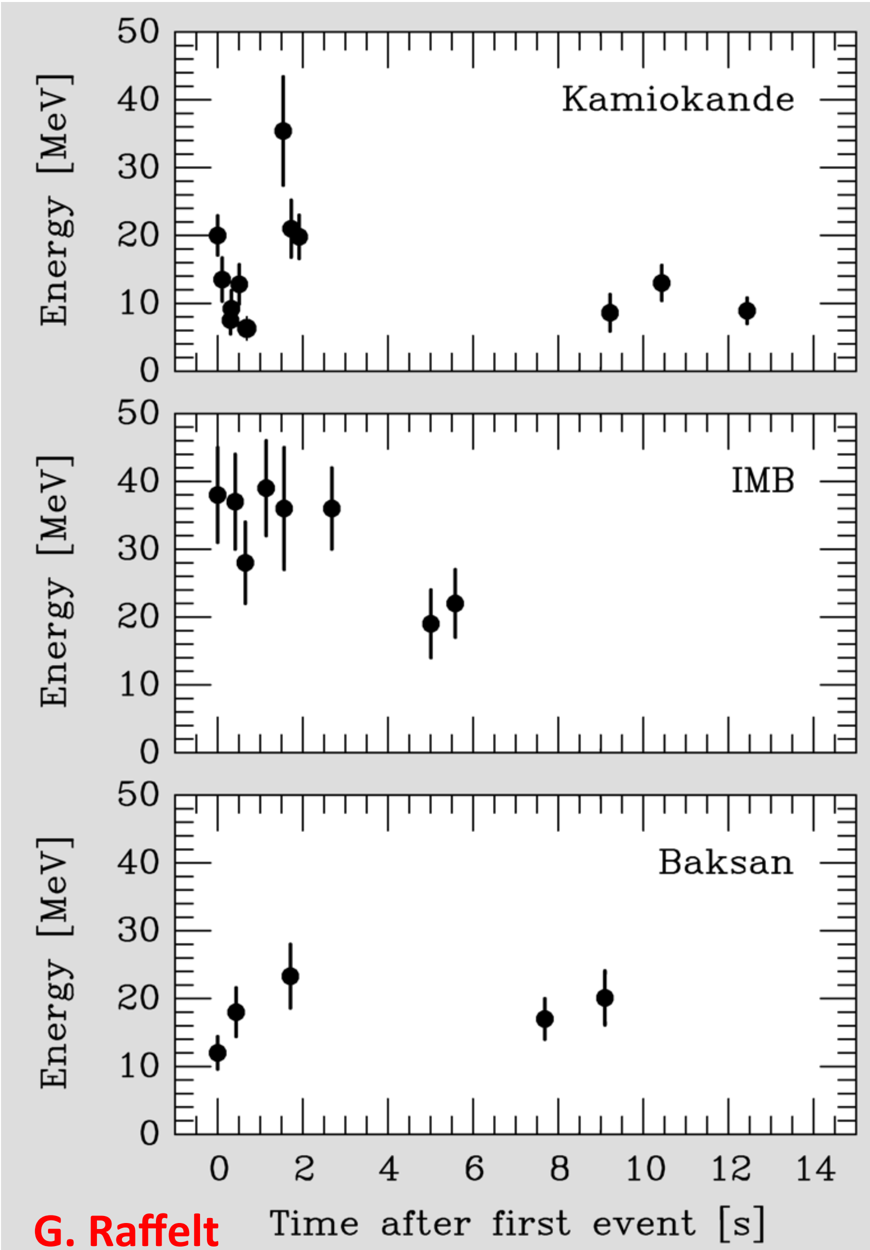
- Neutrinosphere:

$$R_\nu = R\left(\tau_\nu = \frac{2}{3}\right)$$

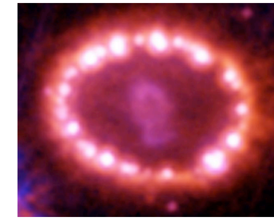
Depends on $(\epsilon_\nu)^2$



Thompson et al. 2003



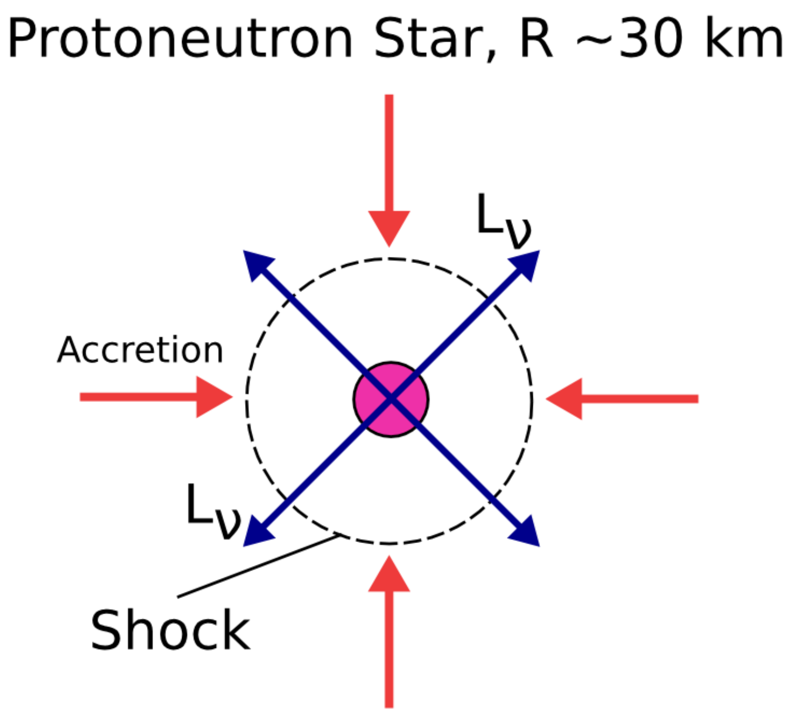
Neutrinos from SN 1987A



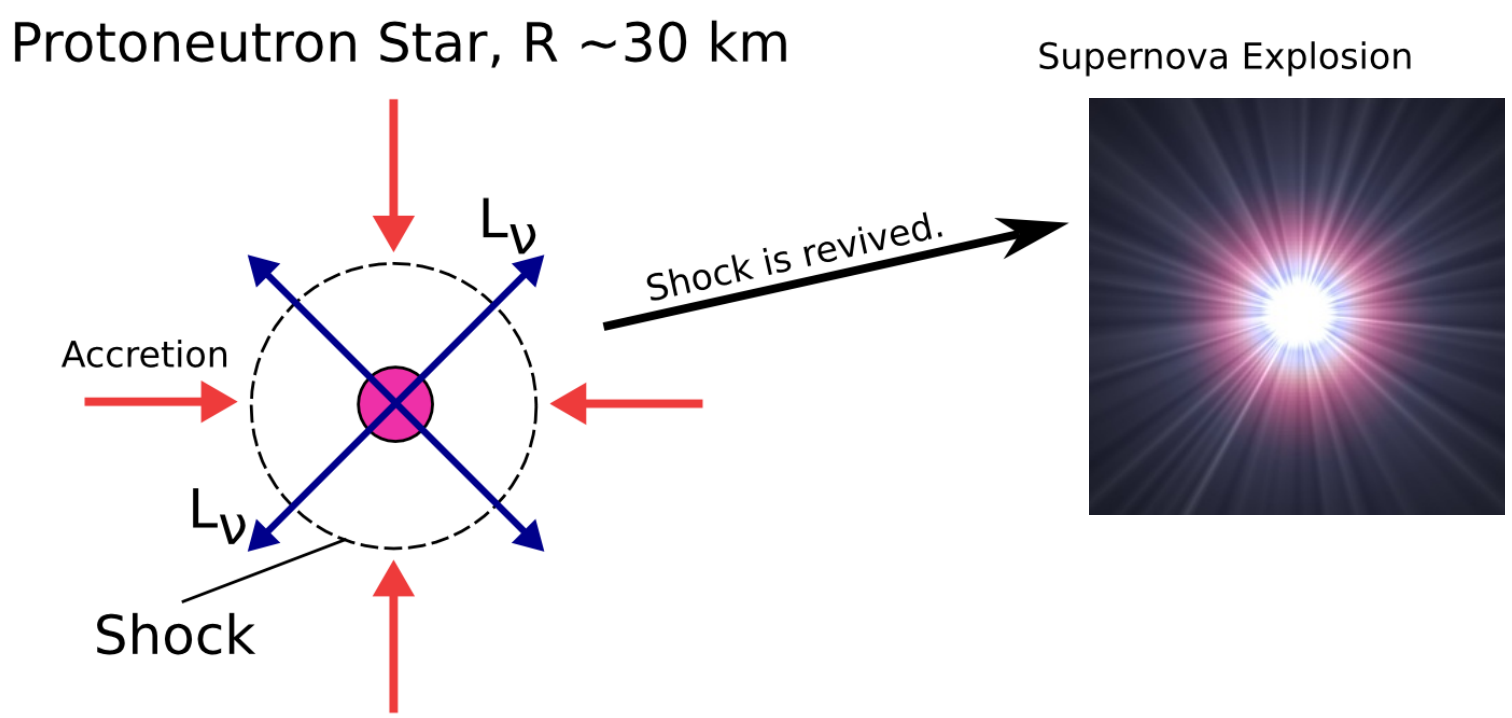
Observed about 20 neutrinos from SN 1987A in the LMC in Kamiokande II (Japan) and IMB (US) experiments.

Confirmation of the basics of core-collapse supernova theory.

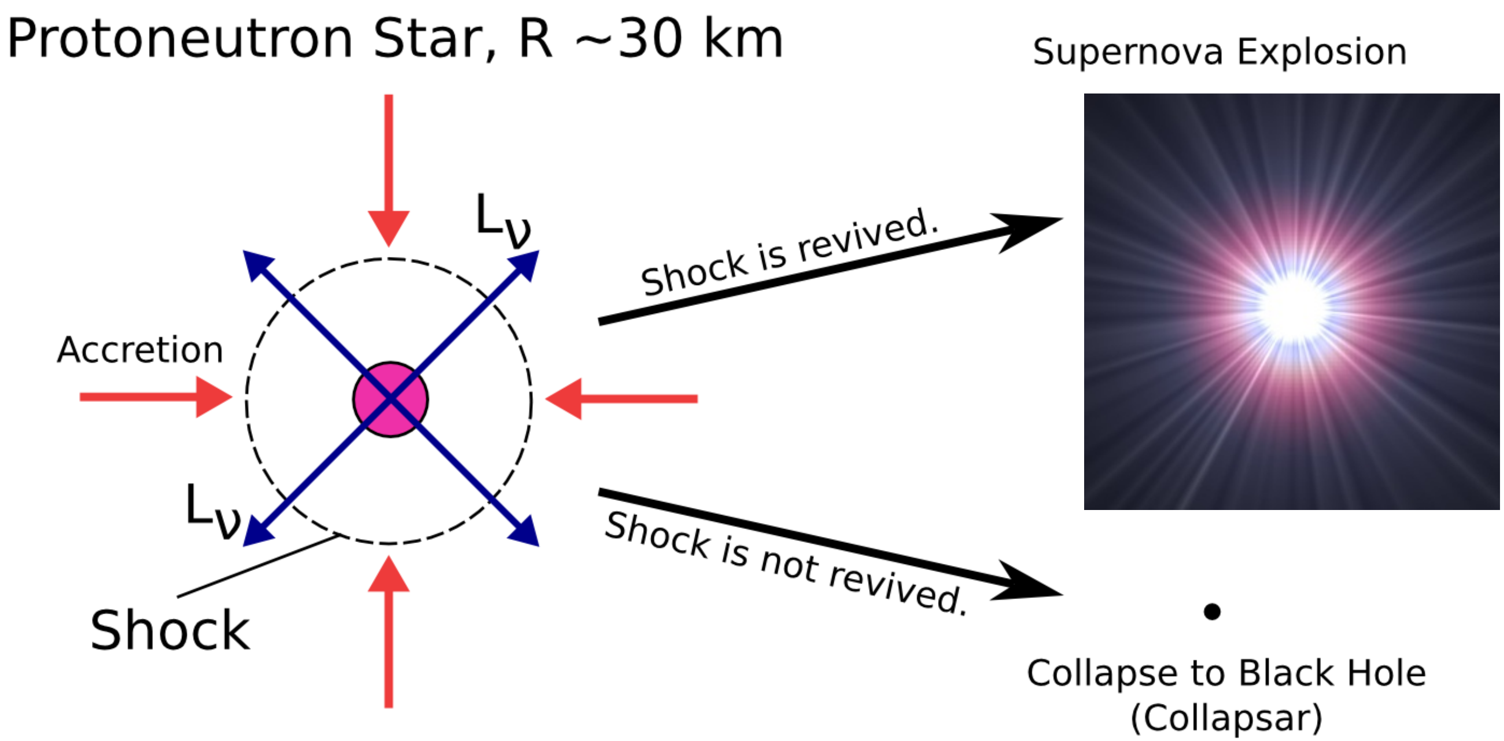
Core-Collapse Supernova – The Big Picture



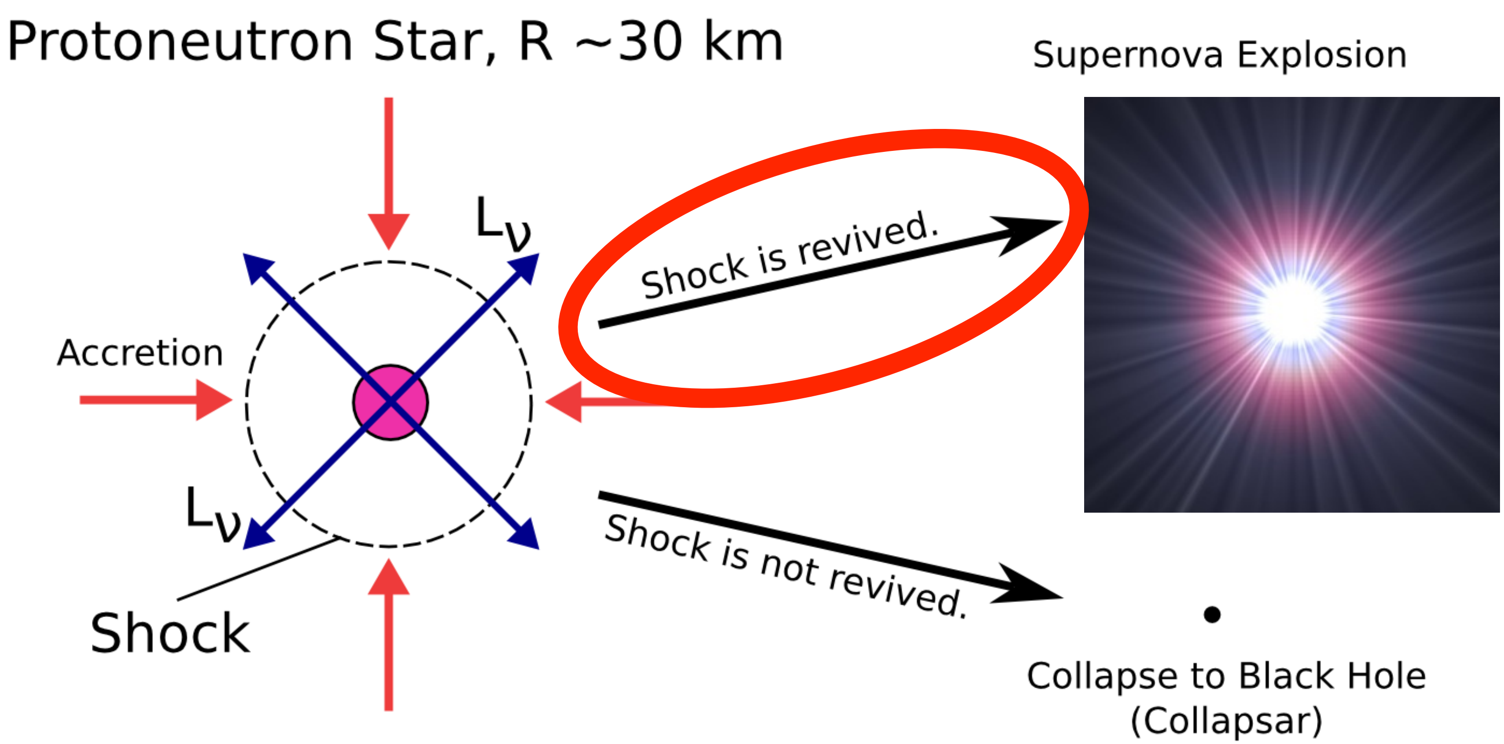
Core-Collapse Supernova – The Big Picture



Core-Collapse Supernova – The Big Picture



The Supernova Problem



What is the Mechanism of shock revival?

Essence of any Explosion Mechanism

- Collapse to neutron star:
  $\sim 3 \times 10^{53}$ erg = 300 Bethe [B] gravitational energy.

- $\sim 10^{51}$ erg = 1 B kinetic and internal energy of the ejecta.
  (Extreme cases: 10^{52} erg; "hypernova")

- 99% of the energy is radiated as neutrinos over hundreds of seconds as the protoneutron star (PNS) cools.

Explosion mechanism must tap the gravitational energy reservoir and convert the necessary fraction into energy of the explosion.

Core-Collapse Supernova Simulations

Fully coupled!

- Magneto-Hydrodynamics / Plasma Physics → Dynamics of the stellar fluid.
- General Relativity → Gravity
- Nuclear and Neutrino Physics → Nuclear EOS, nuclear reactions & ν interactions.
- Transport Theory → **Neutrino transport**.

- Additional Complication: **Supernovae are 3D**
  - Rotation, **fluid instabilities** (convection, turbulence, advective-acoustic, rotational), **MHD**, precollapse multi-D perturbations.
    -> **Need multi-D (ideally 3D) treatment.**
- Route of Attack: **Computational Simulation**
  - First 1D computations in the late 1960's: **Colgate & White, Arnett, Wilson**
  - Best current simulations still 1D.
  - **Good 2D Models (with various approximations [Gravity/Transport]).**
  - **First 3D Models.**

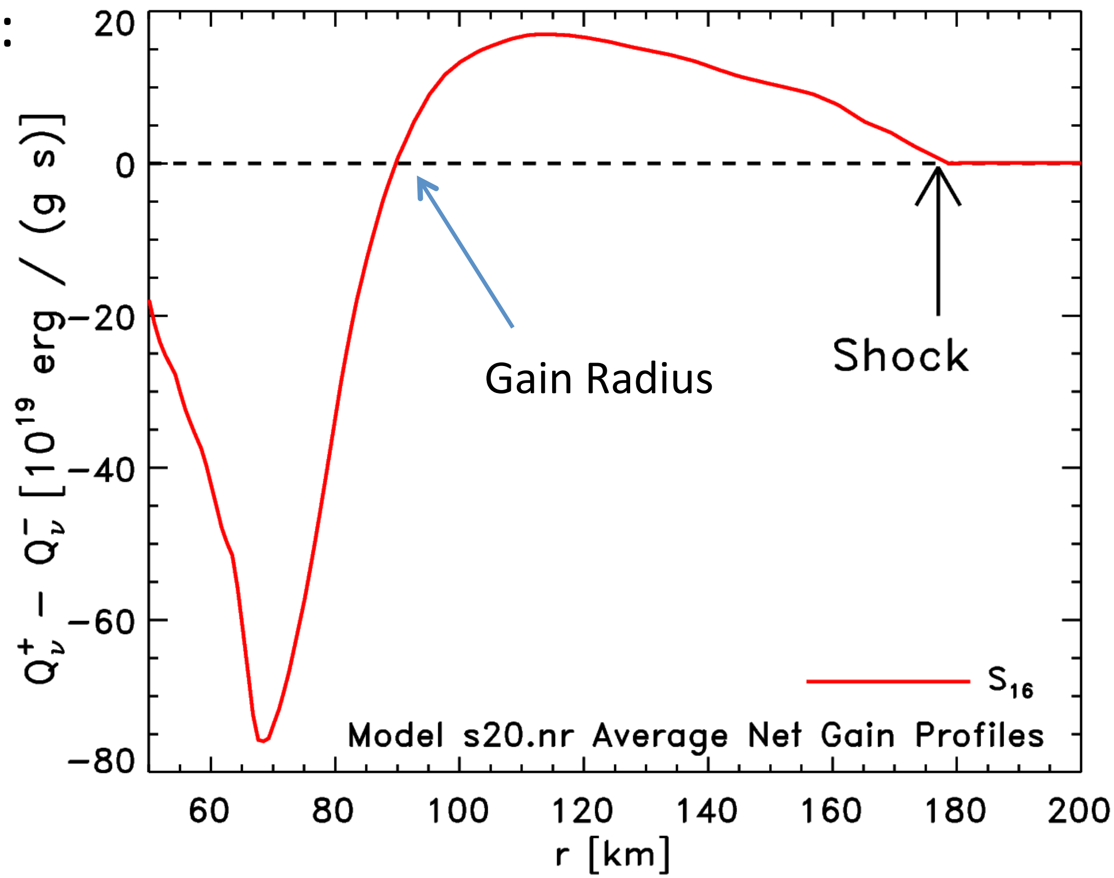
Neutrino Mechanism

Bethe & Wilson 1985

Neutrino cooling: $Q_\nu^- \propto T^6, T^9$ Net heating where:

Neutrino heating: $Q_\nu^+ \propto L_\nu r^{-2} \langle \epsilon_\nu^2 \rangle$ $Q_\nu^+ > Q_\nu^-$

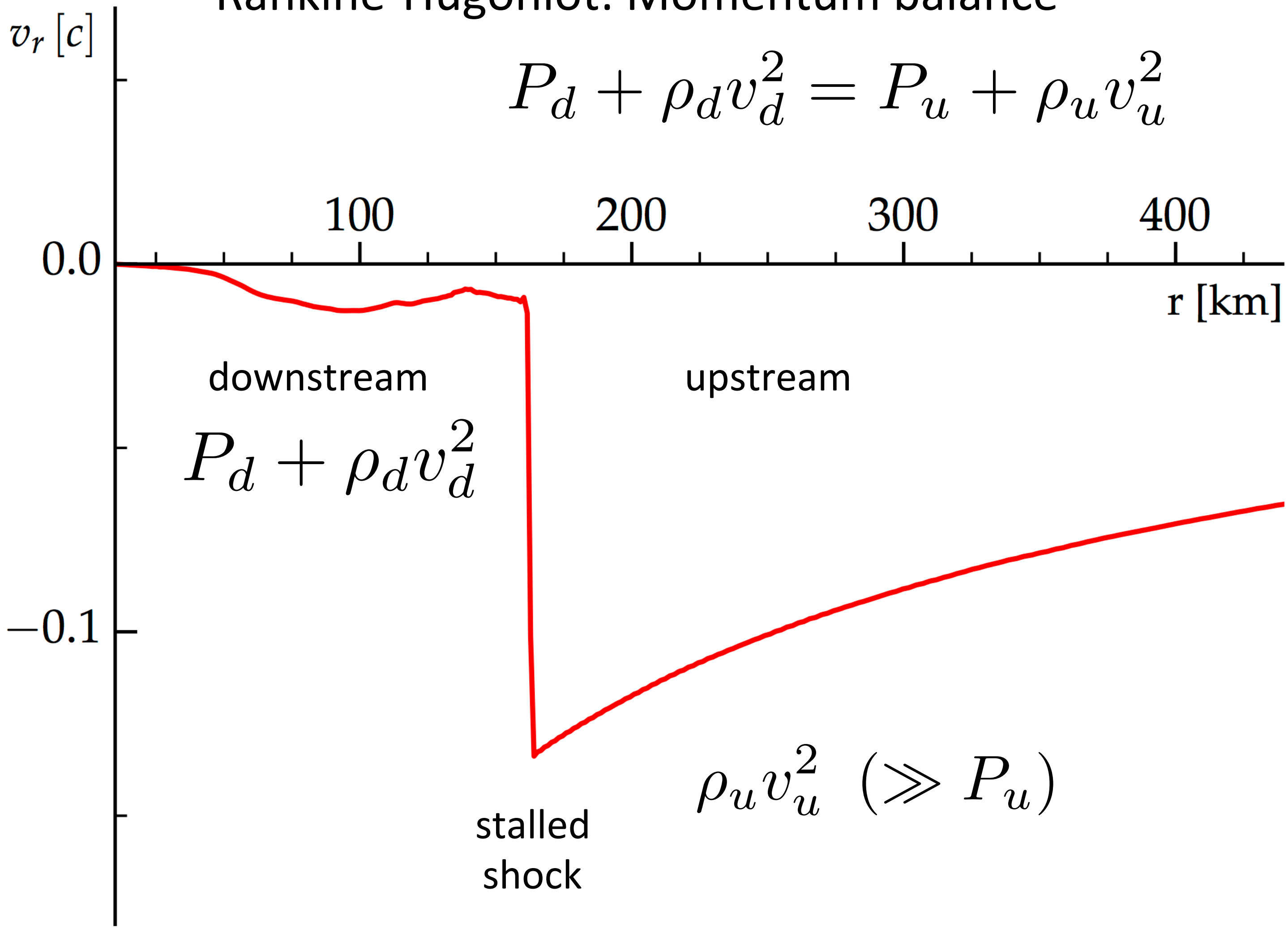
- Neutrino-driven mechanism: Based on subtle imbalance between neutrino heating and cooling in postshock region.



Basic Stalled-Shock Situation

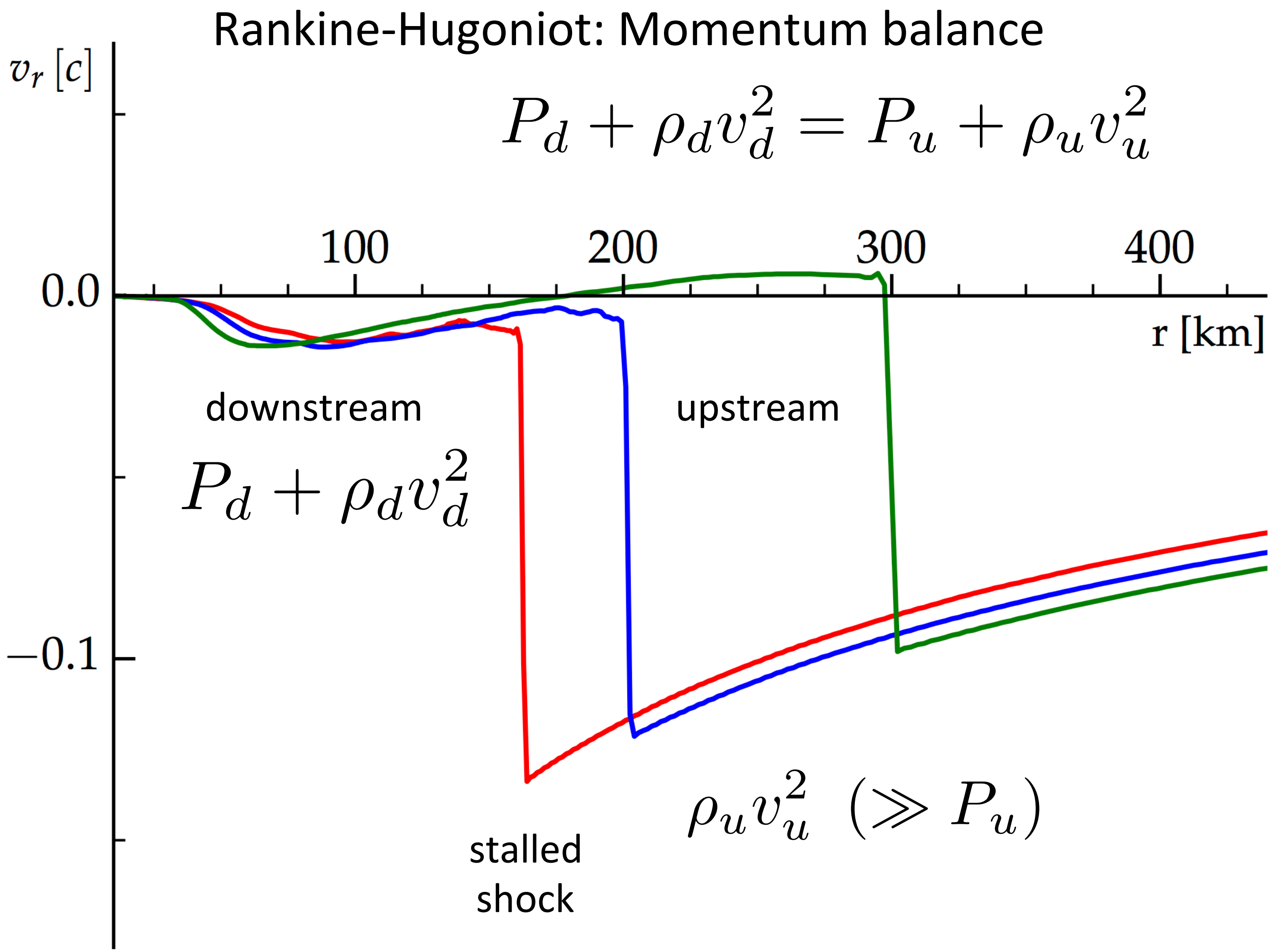
Rankine-Hugoniot: Momentum balance

$$P_d + \rho_d v_d^2 = P_u + \rho_u v_u^2$$



GR1D simulation
http://stellarcollapse.org/

Basic Stalled-Shock Situation

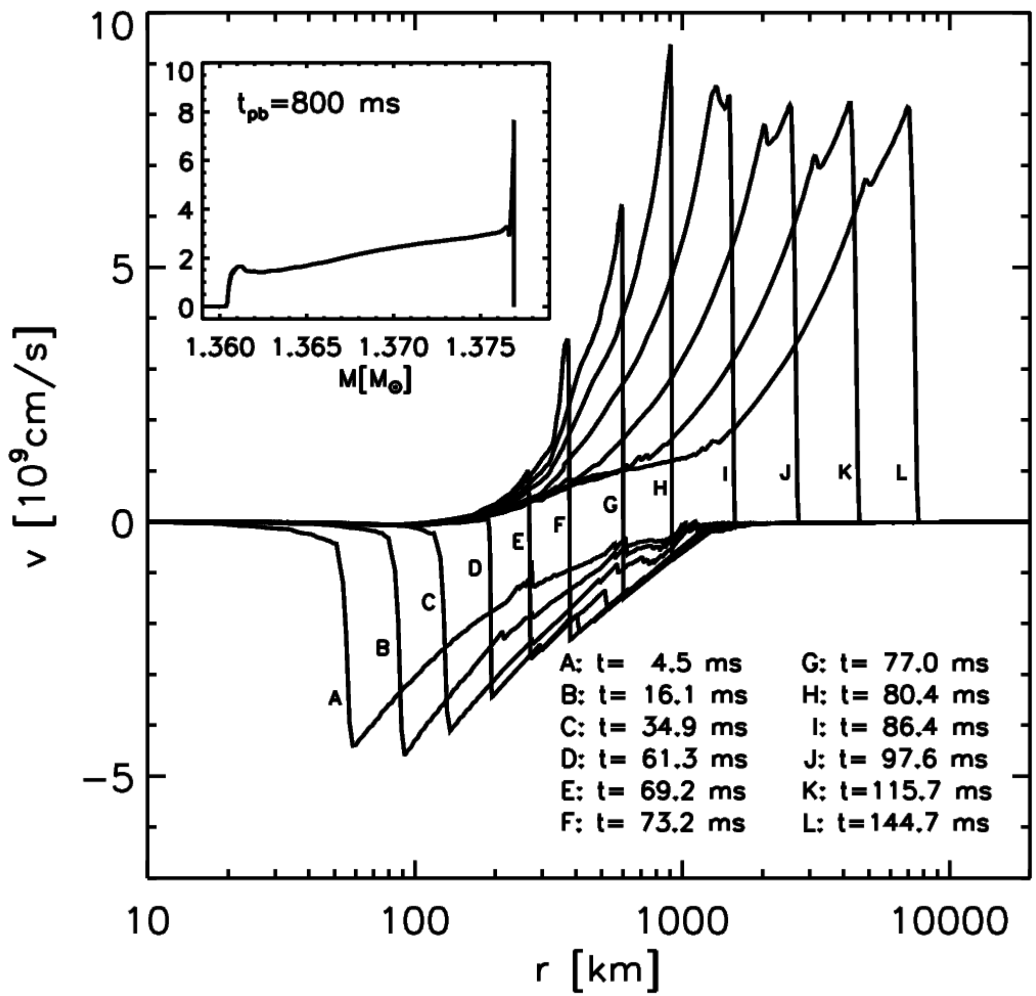


GR1D simulation
http://stellarcollapse.org/

Does this really work?

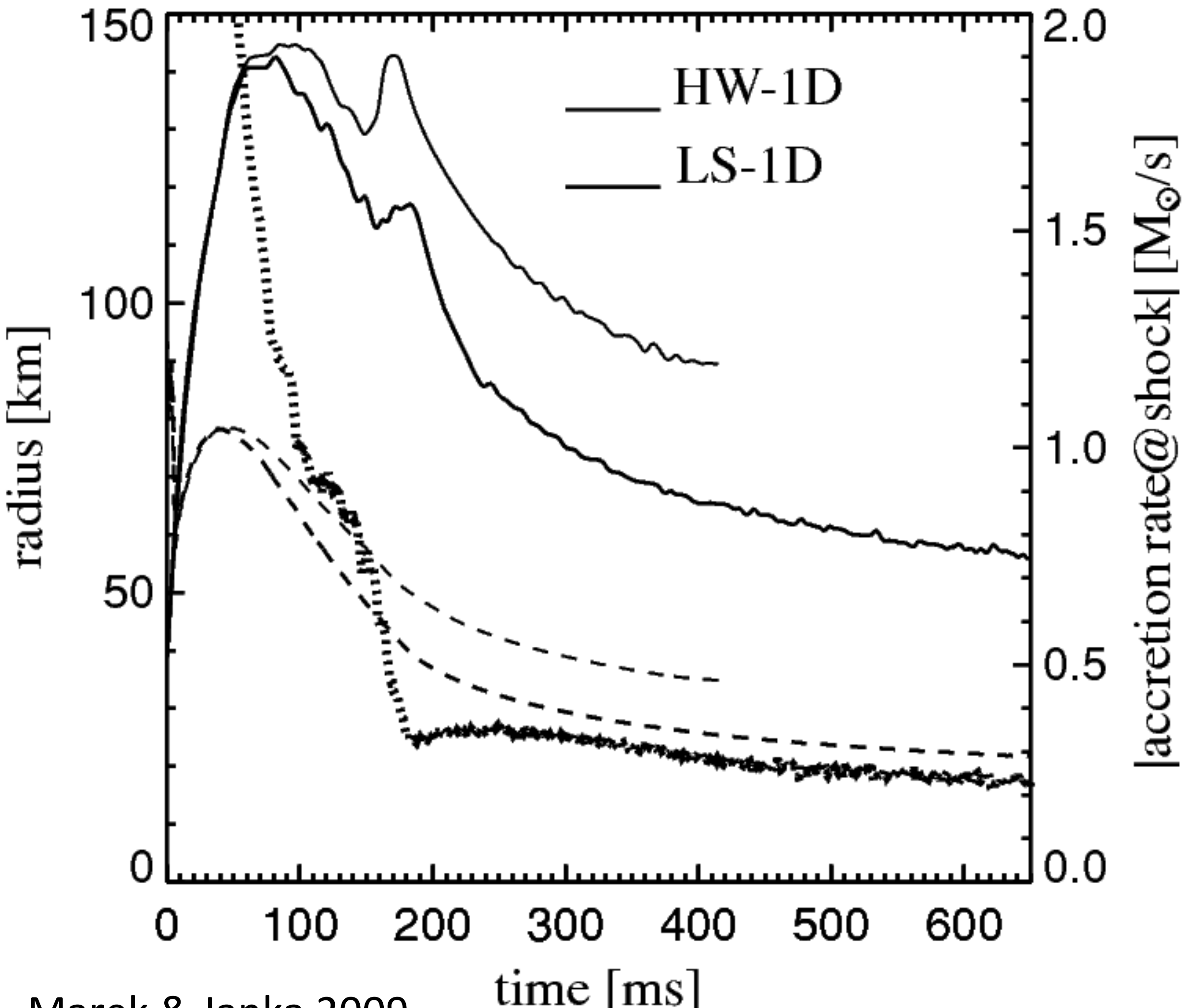
- **Yes!**
  BUT:
  Only for lowest-mass massive stars.

- **FAILS in spherical symmetry (1D) for more massive stars** in simulations with best neutrino physics and neutrino transport



Kitaura+06

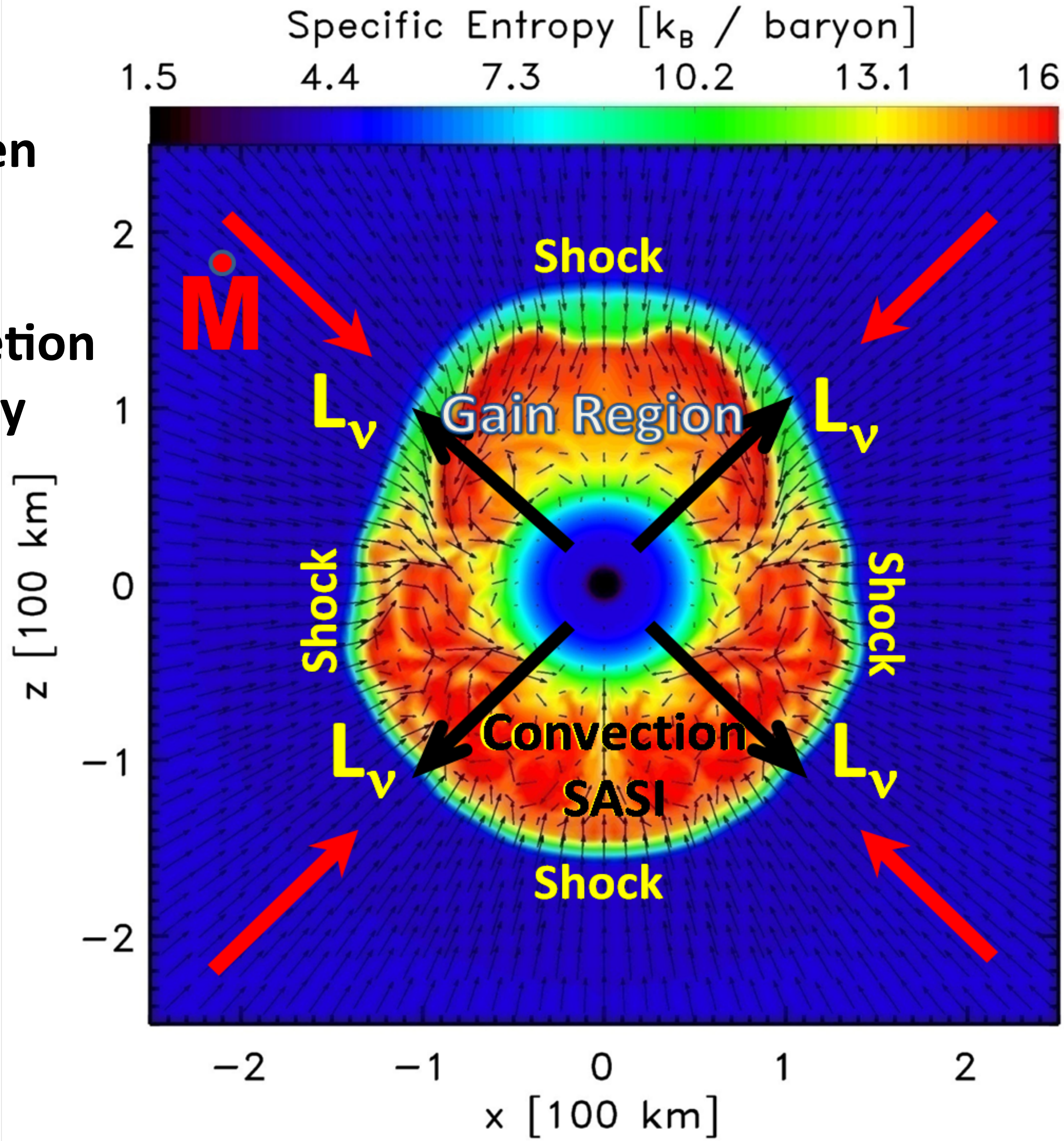
Failure of the Neutrino Mechanism in 1D



Marek & Janka 2009

Neutrino-Driven Convection

Standing Accretion Shock Instabilty (SASI)



[Ott et al. 2008, Ott 2009]

Multi-Dimensional Simulations: Effects

(1) Lateral/azimuthal flow: "Dwell time" in gain region increases.

(2) Turbulent ram pressure (Couch&Ott 15, Murphy+13, Müller&Janka 14)

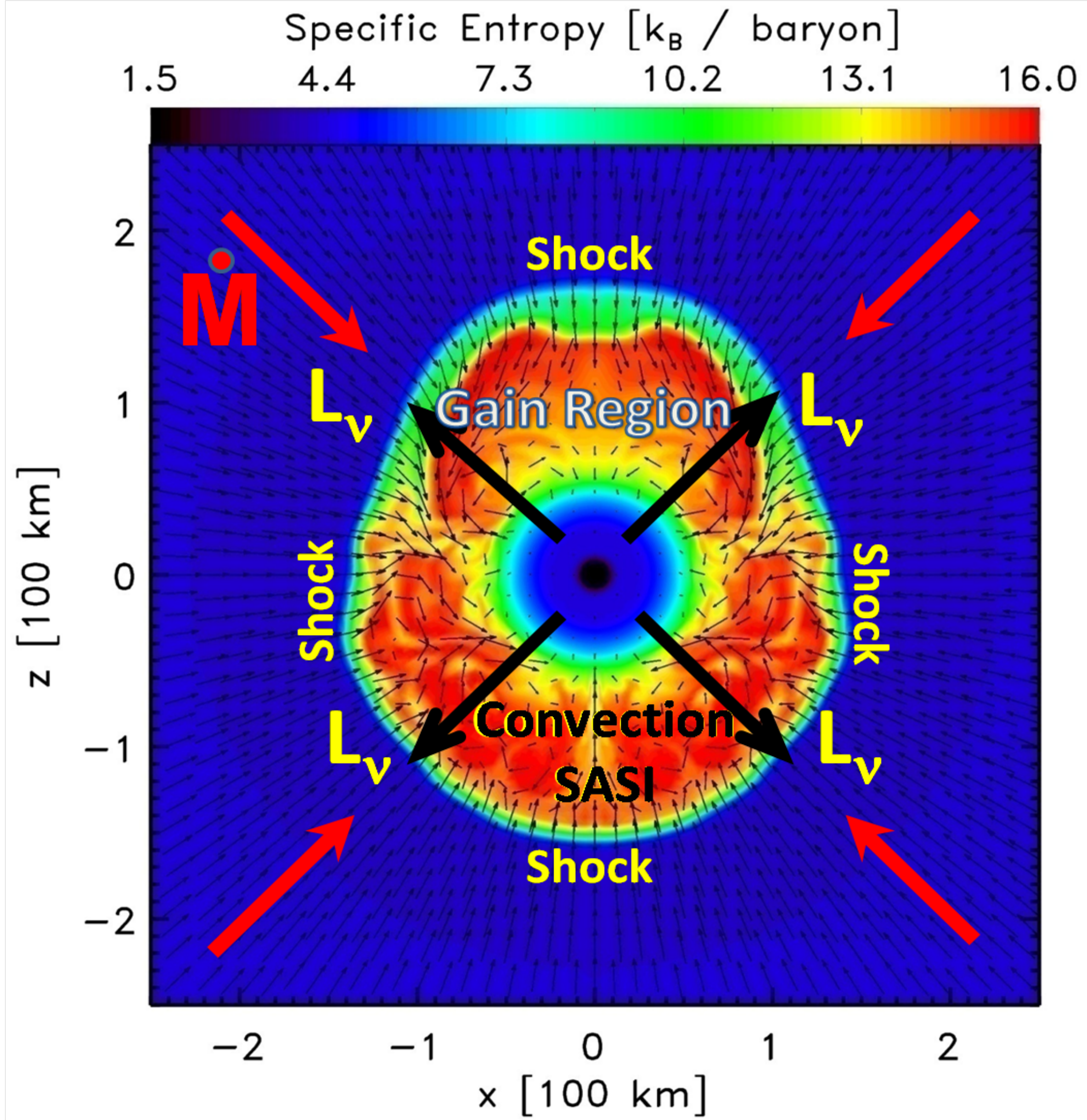
Reynolds stress tensor:

$$R_{ij} = \overline{\delta v_i \delta v_j}$$

$$\delta v_i = v_i - \overline{v_i}$$

Effective ram pressure:

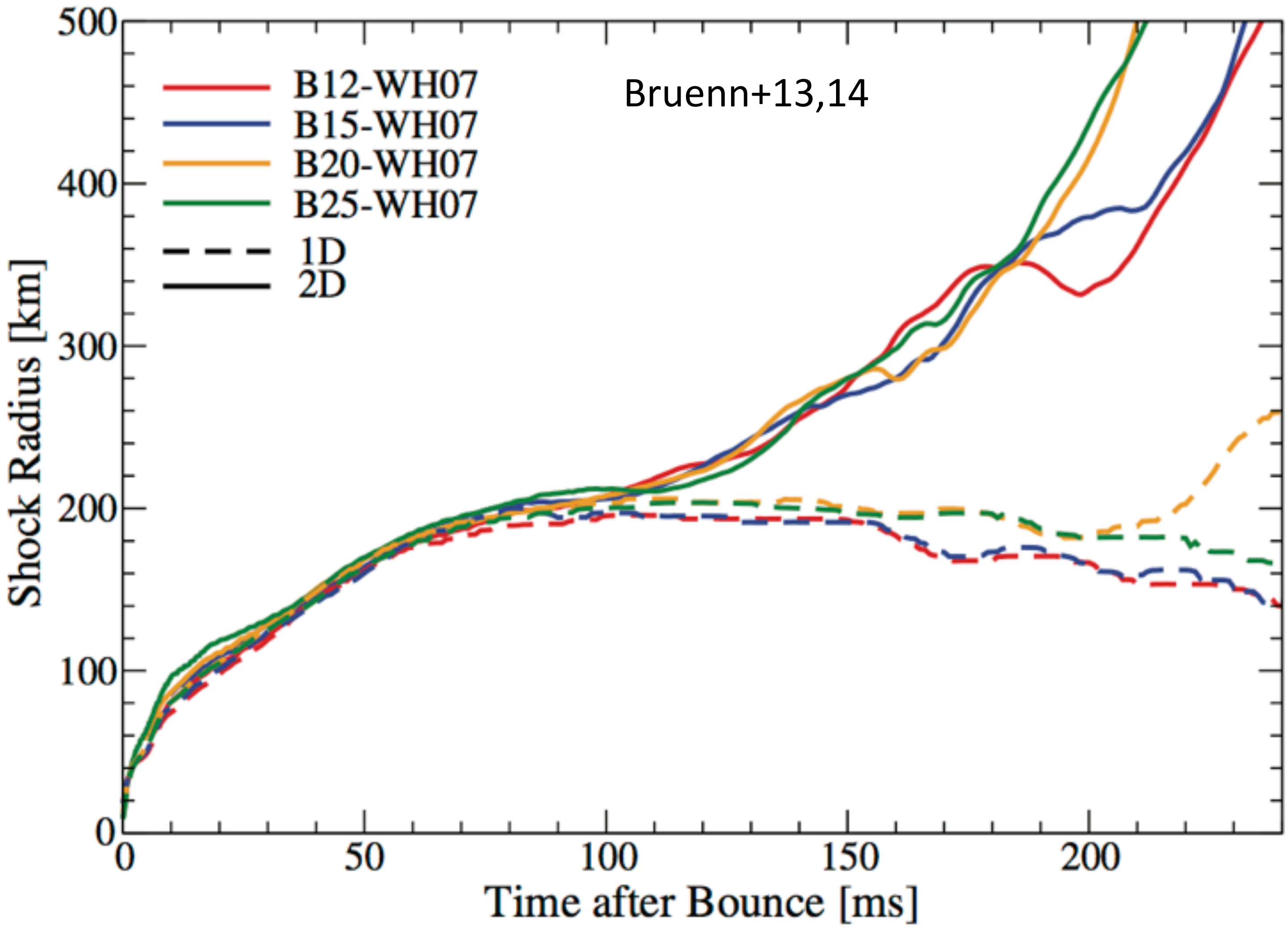
$$P_{\rm turb} = \rho R_{rr}$$



Ott+13, ApJ

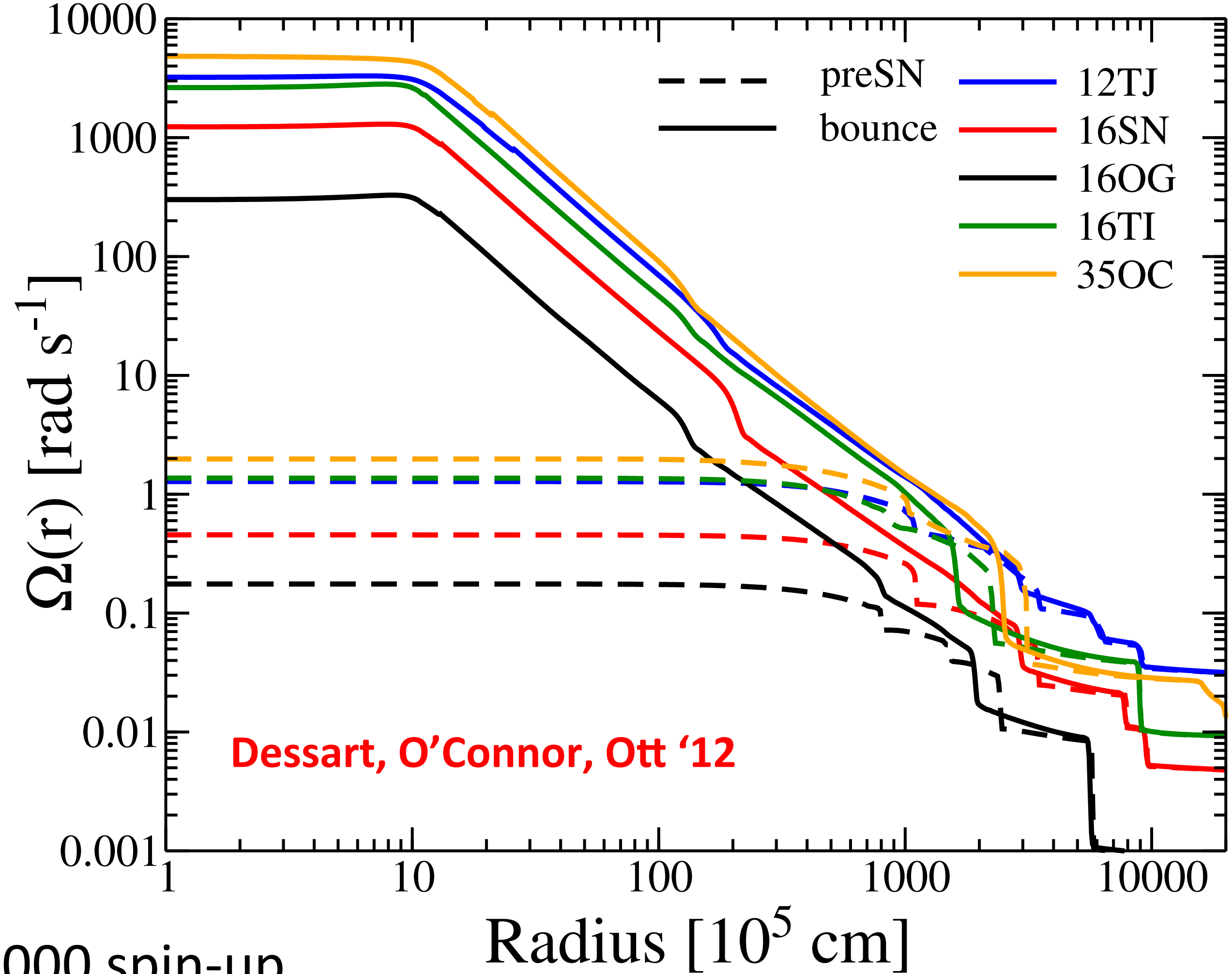
-6.18 ms

Multi-Dimensional Simulations



- 2D (axisymmetric) simulations explode.
- 3D: explosions appear harder (->because turbulent pressure lower?).

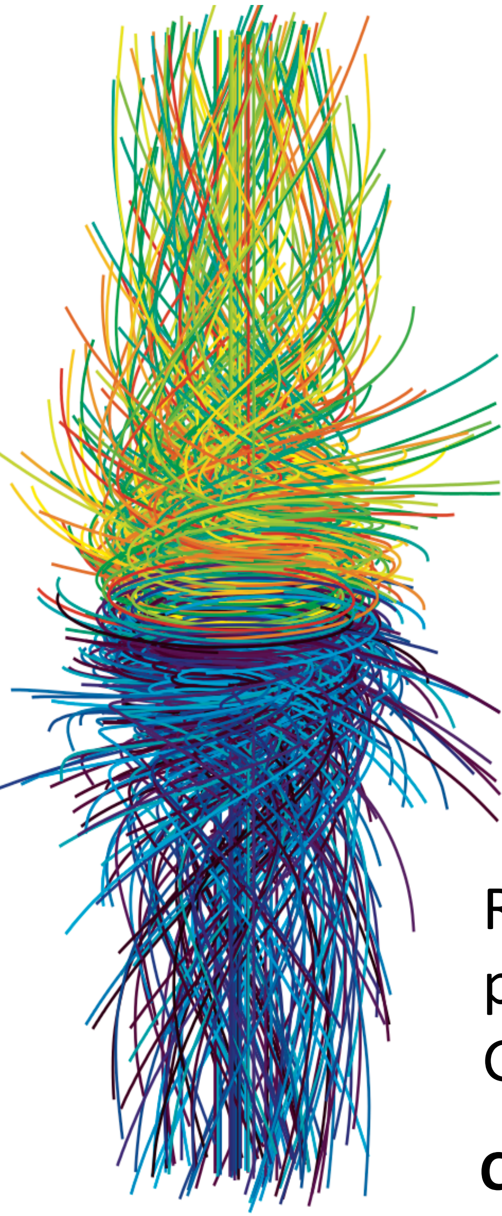
"Magnetorotational Explosions"



- Core: x 1000 spin-up
- Differential rotation -> reservoir of free energy.
- Spin energy tapped by magnetorotational instability (MRI)?

Magnetorotational Mechanism

[LeBlanc & Wilson 70, Bisnovatyi-Kogan 70,
Burrows+ 07, Cerda-Duran+07, Takiwaki & Kotake 11, Winteler+ 12]



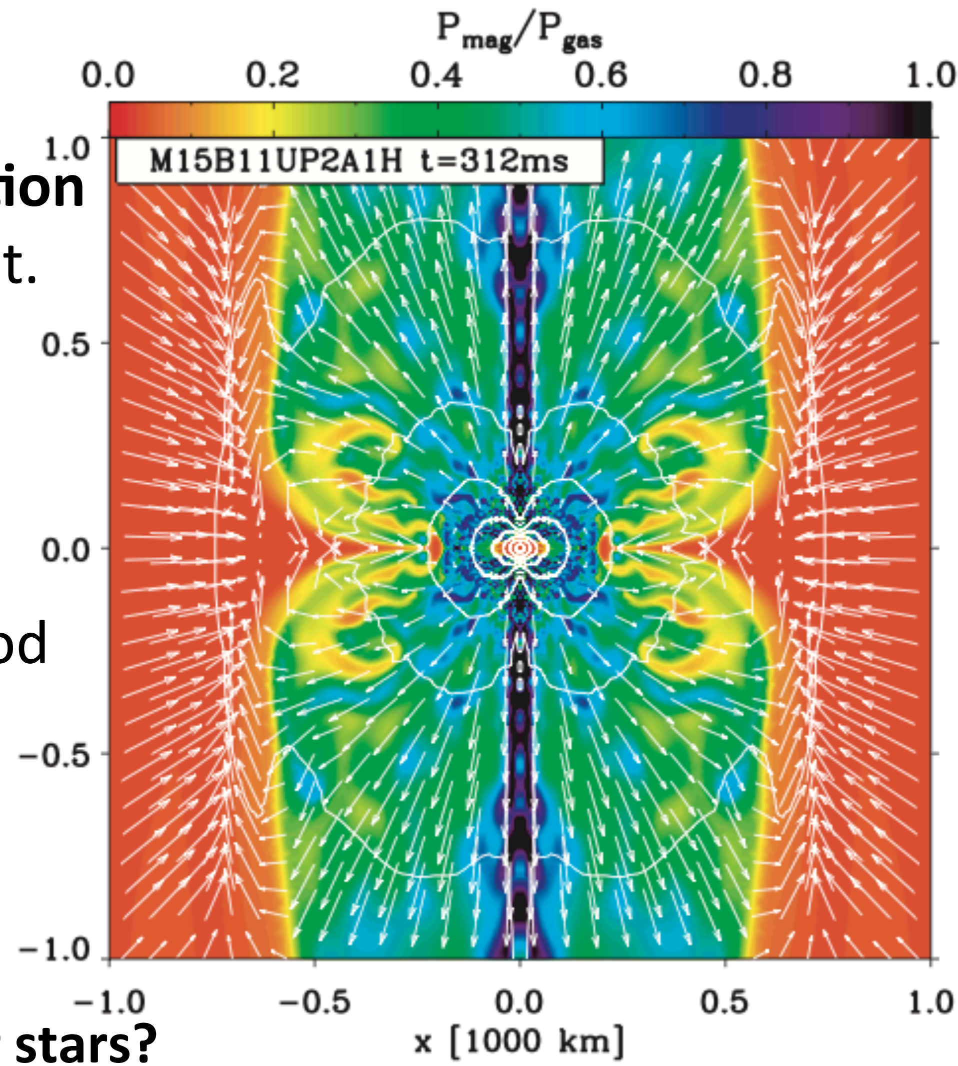
Burrows+07

Rapid Rotation +
B-field amplification
(need magnetorot. Instability [MRI])

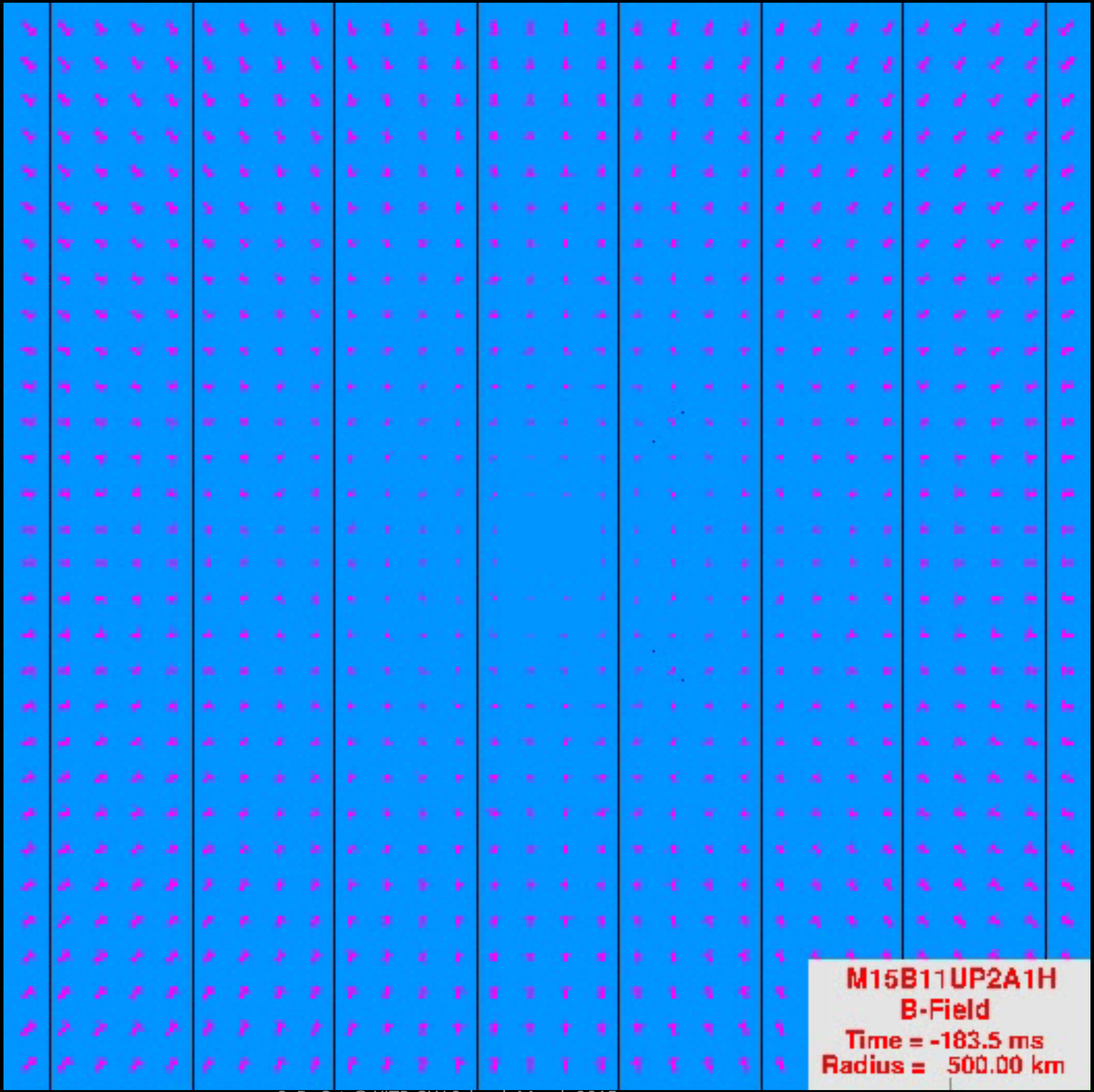
Energetic bipolar explosions.

Results in ms-period proto-magnetar.
GRB connection?

Caveat: Need high core spin; only in very few progenitor stars?



Burrows+07



Burrows+'07

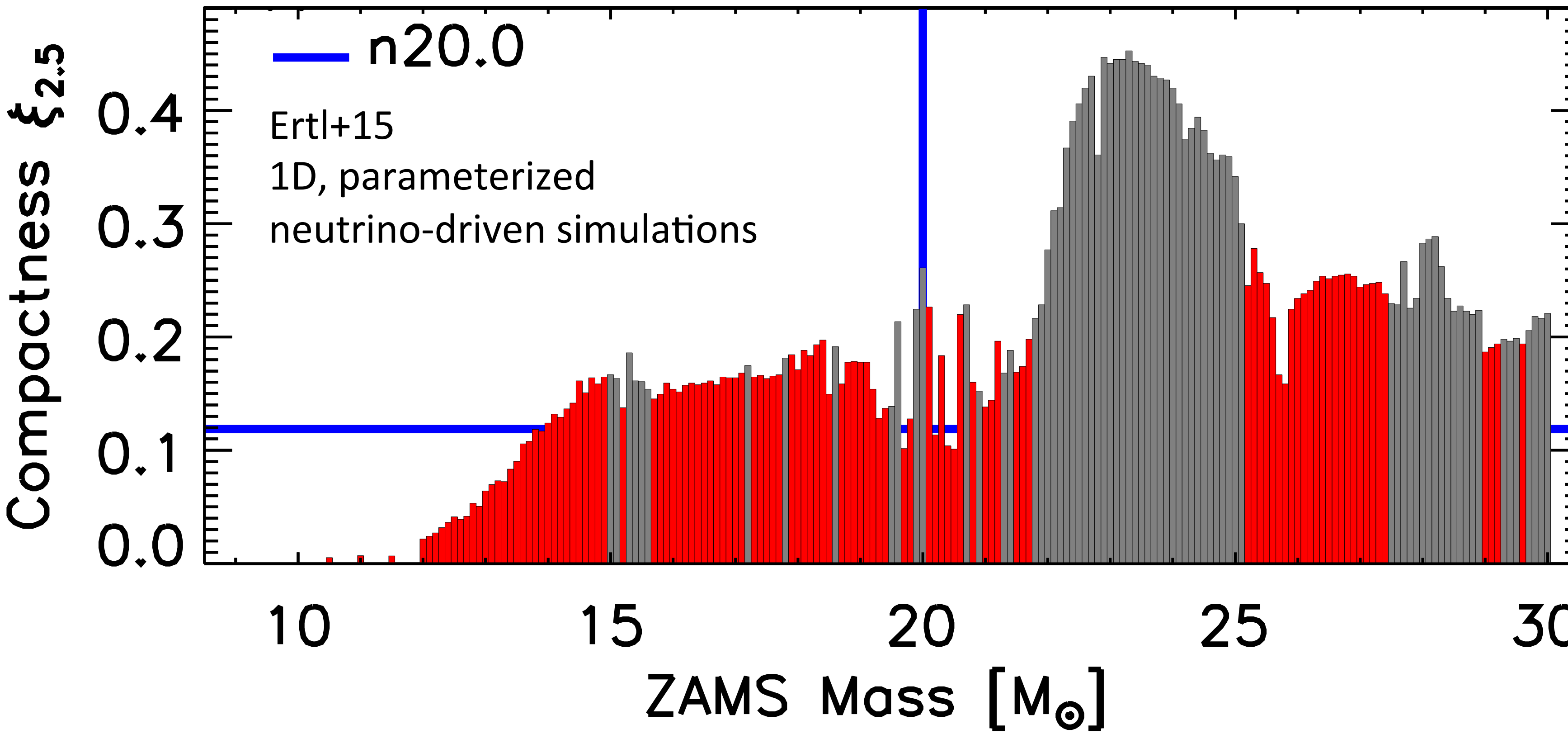
$$\beta = \frac{P_{\rm gas}}{P_{\rm mag}}$$

t = -4.95 ms

Mösta+ 2014
ApJL

What stars make black holes?

- We do not know yet!



$$\xi_M = \frac{M/M_\odot}{R(M_{\rm bary}=M)/1000\,{\rm km}}\Big|_{t=t_{\rm bounce}}$$

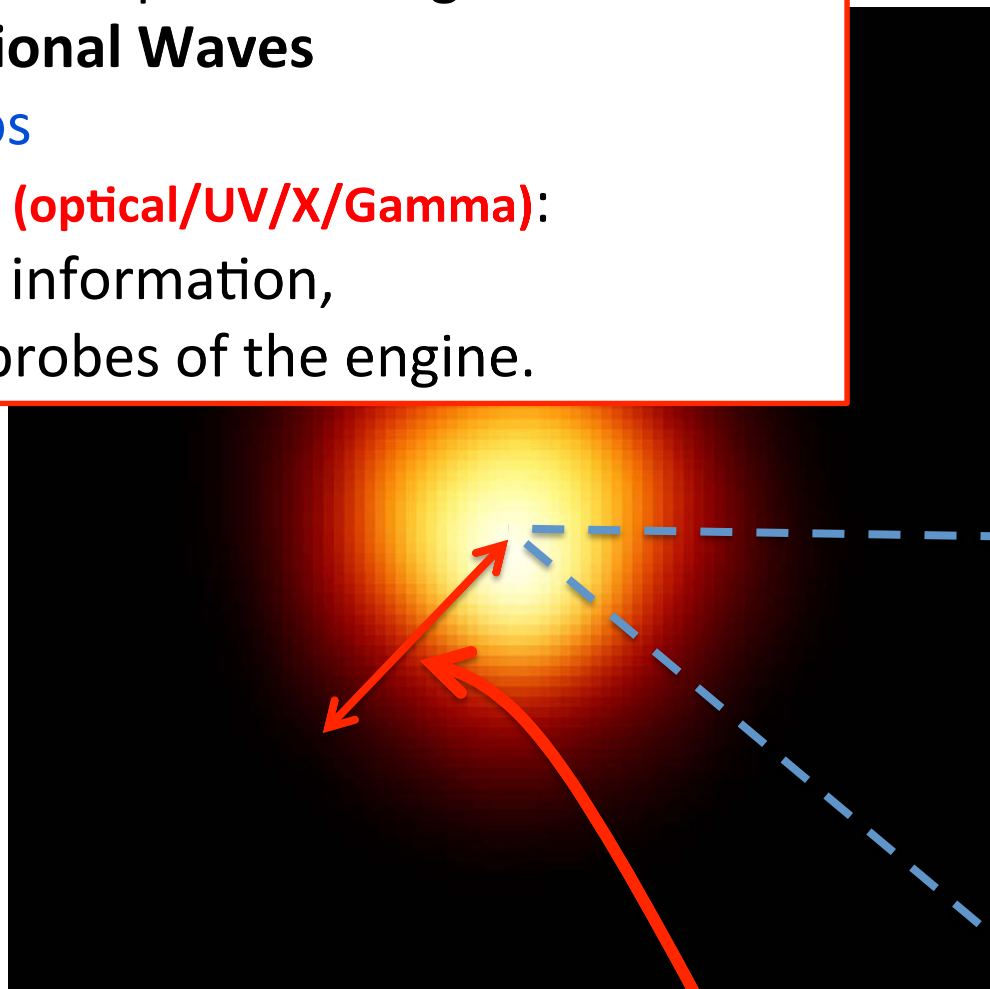
Stars get harder to explode with increasing compactness parameter.

Observing the CCSN Mechanism

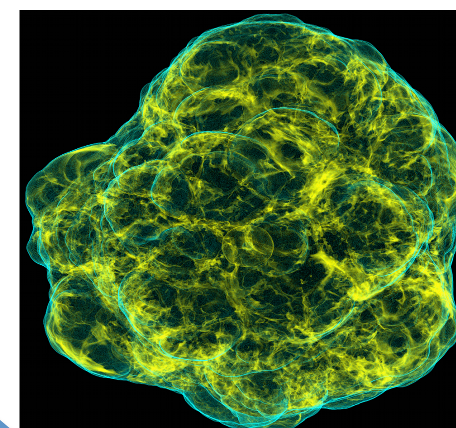
Probing the "Supernova Engine"
- **Gravitational Waves**
- Neutrinos

EM waves (optical/UV/X/Gamma):
secondary information,
late-time probes of the engine.

Red Supergiant
Betelgeuse
D ~200 pc



Supernova "Central Engine"



Gravitational-Waves from Core-Collapse Supernovae

Recent reviews: Ott '09, Kotake '11, Fryer & New '11

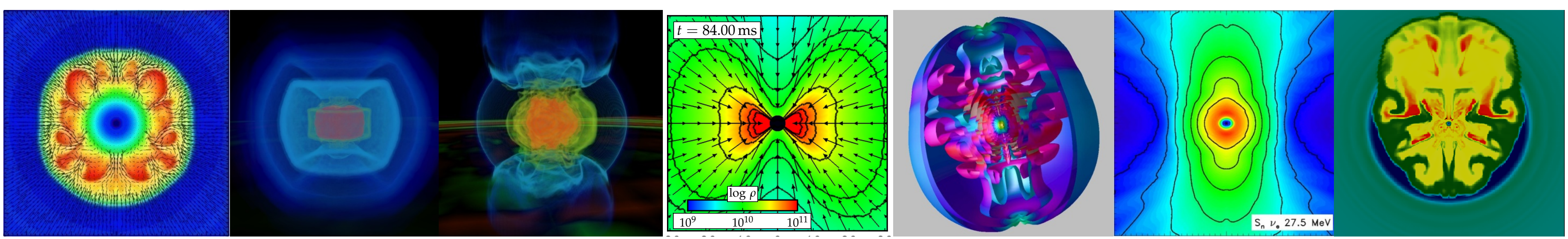
Need:

$$h_{jk}^{TT}(t,\vec{x}) = \left[\frac{2}{c^4}\frac{G}{|\vec{x}|}\ddot{I}_{jk}\left(t - \frac{|\vec{x}|}{c}\right)\right]^{TT}$$

→ accelerated aspherical (quadrupolar) mass-energy motions

Candidate Emission Processes:

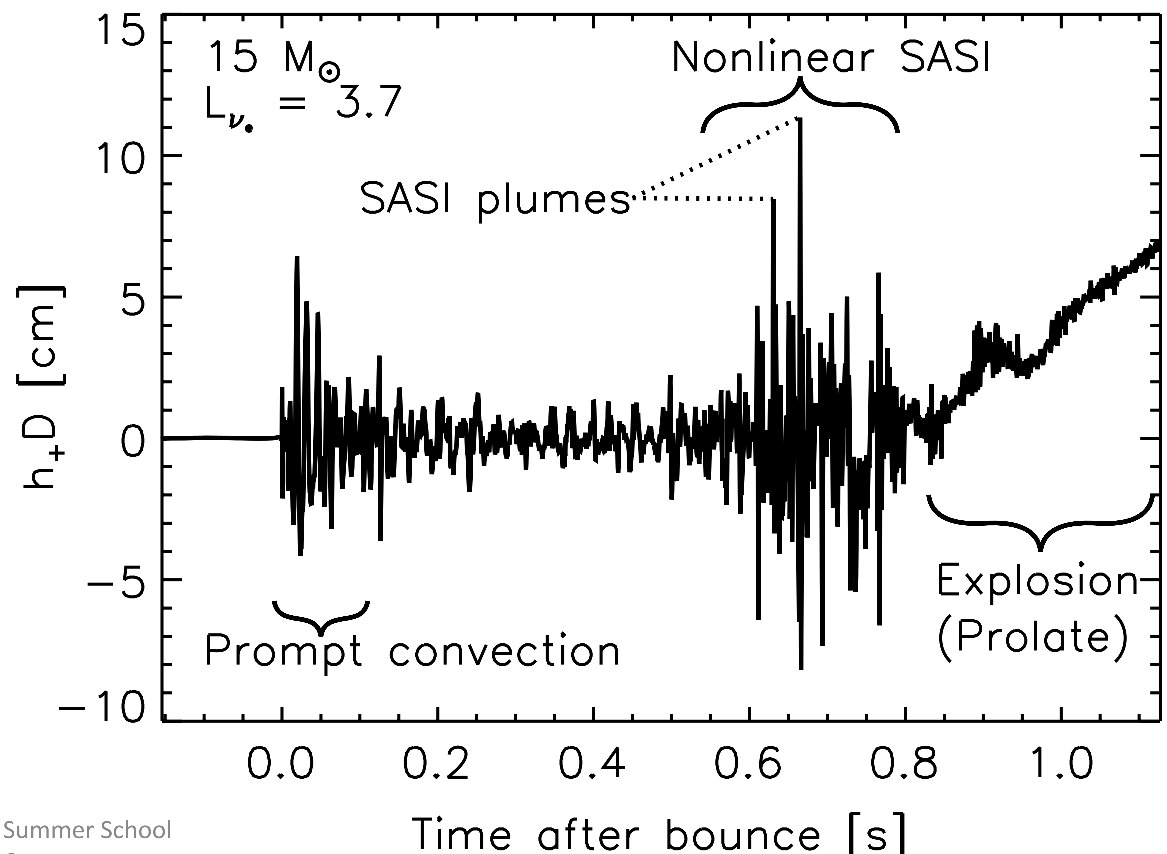
- Convection and SASI
- Rotating collapse & bounce
- Rotational 3D instabilities
- Black hole formation
- Pulsations of the protoneutron star
- Anisotropic neutrino emission
- Aspherical accelerated outflows
- Magnetic stresses



GWs from Convection & SASI

Recent work: Kotake+ '09, '11, Murphy+'09, Yakunin+'10 E. Müller+'12, B.Müller+'13

- Prompt convection soon after bounce (Marek+ '09, Ott '09).
- Neutrino-driven convection & SASI (recent: Murphy+'09, Yakunin+10, Müller+12).
- Protoneutron star convection (e.g., Keil+ '96, Müller+'04)

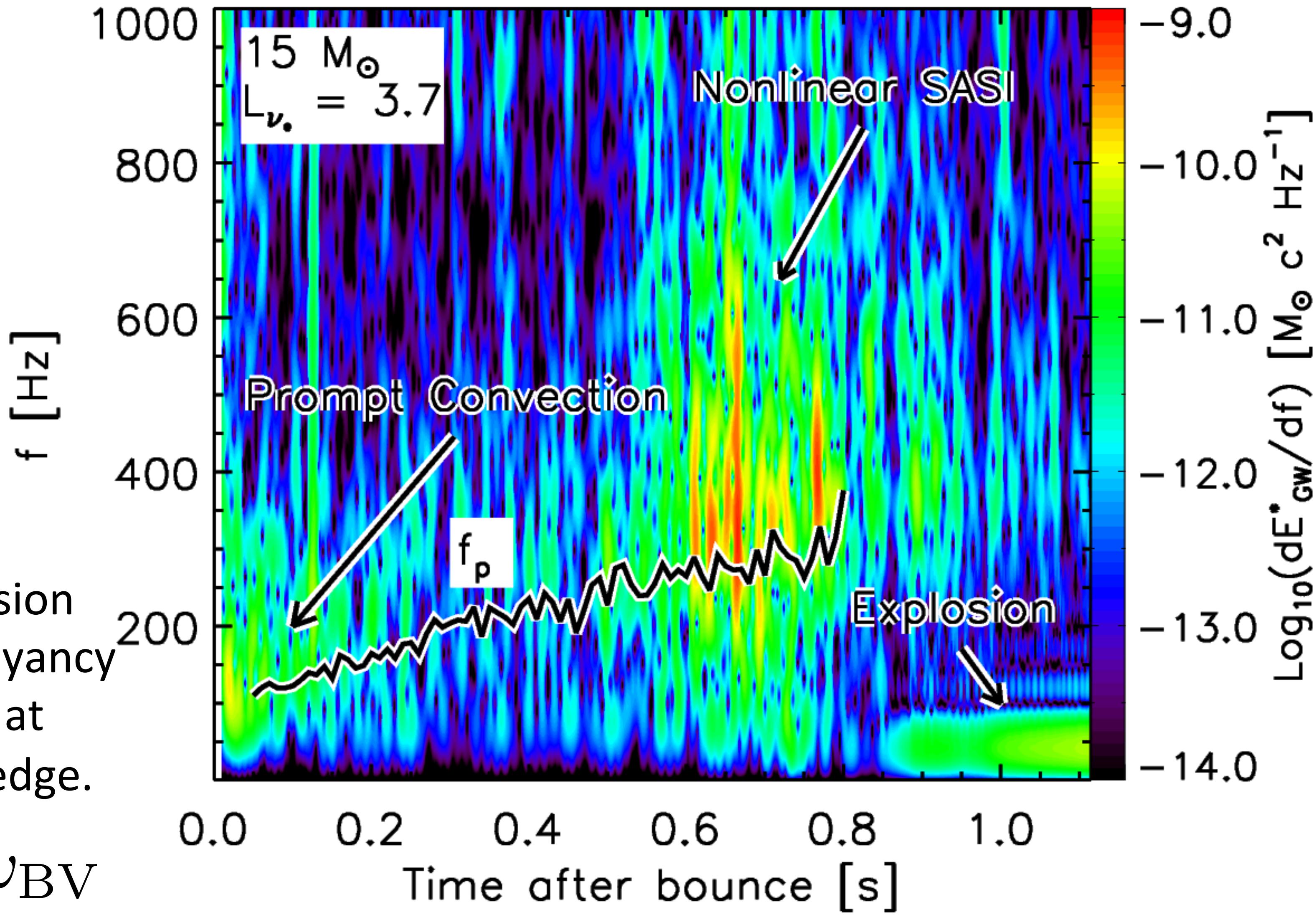


Murphy+ '09, using simplified heating/cooling scheme.

Expect also: Correlations with neutrino signal. Lund+ '10,'12, Marek+'09, Brandt+'11

Time-Frequency Analysis of GWs

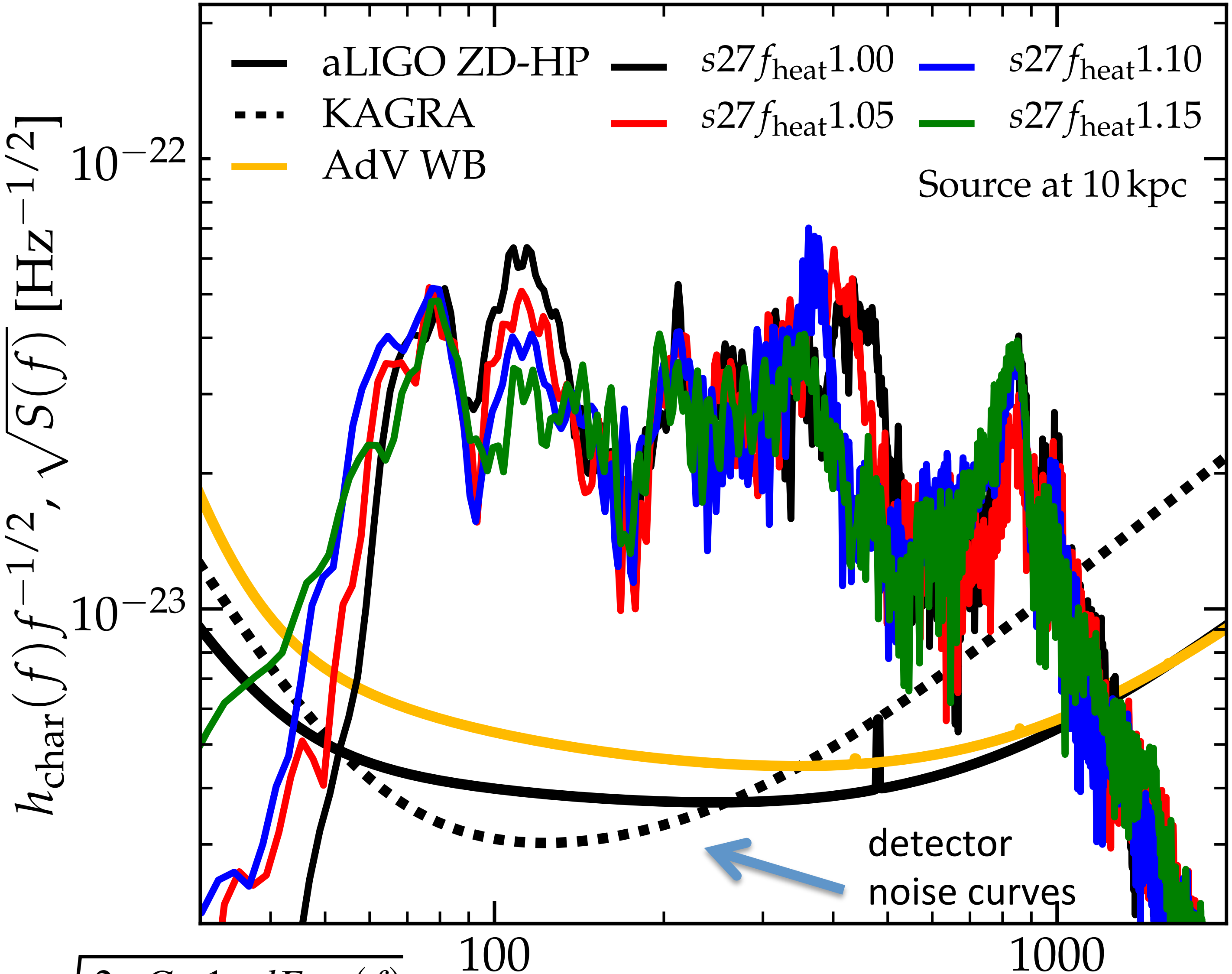
Murphy, Ott, Burrows '09, see also B. Müller+'13



Peak emission traces buoyancy frequency at proto-NS edge.

$$f_p \sim \frac{\omega_{\rm BV}}{2\pi}$$

Can we observe GWs from Core-Collapse Supernovae?



Ott+13, ApJ 768:115

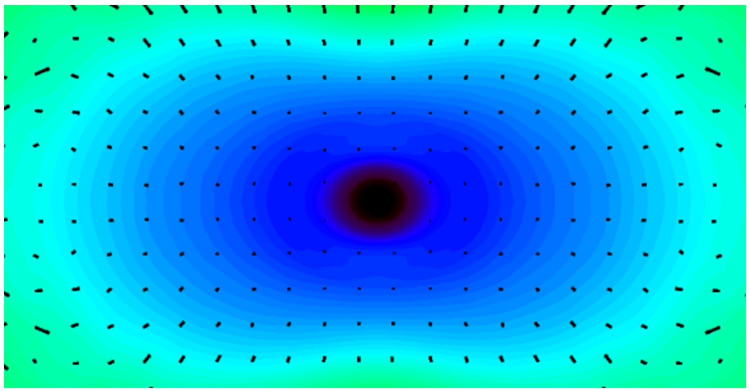
$$h_{\rm char}(f) = \sqrt{\frac{2}{\pi^2} \frac{G}{c^3} \frac{1}{D^2} \frac{dE_{\rm GW}(f)}{df}},$$

$E_{\rm GW} \gtrsim 4 \times 10^{-10} \, M_\odot c^2$

GWs from Rotating Collapse & Bounce

Recent work: Dimmelmeier+ '08, Scheidegger+ '10, Ott+ '12, Kuroda+ '13

Rapid rotation:
Oblate deformation of the inner core

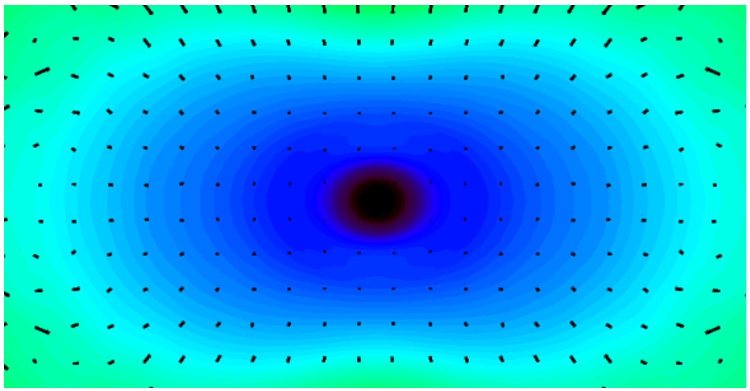


- Most extensively studied GW emission in core collapse
- **Axisymmetric: ONLY h_+**
- Simplest GW emission process: **Rotation** + **Gravity** + **Stiffening of nuclear EOS**.
- Strong signals for rapid rotation (-> millisecond proto-NS).

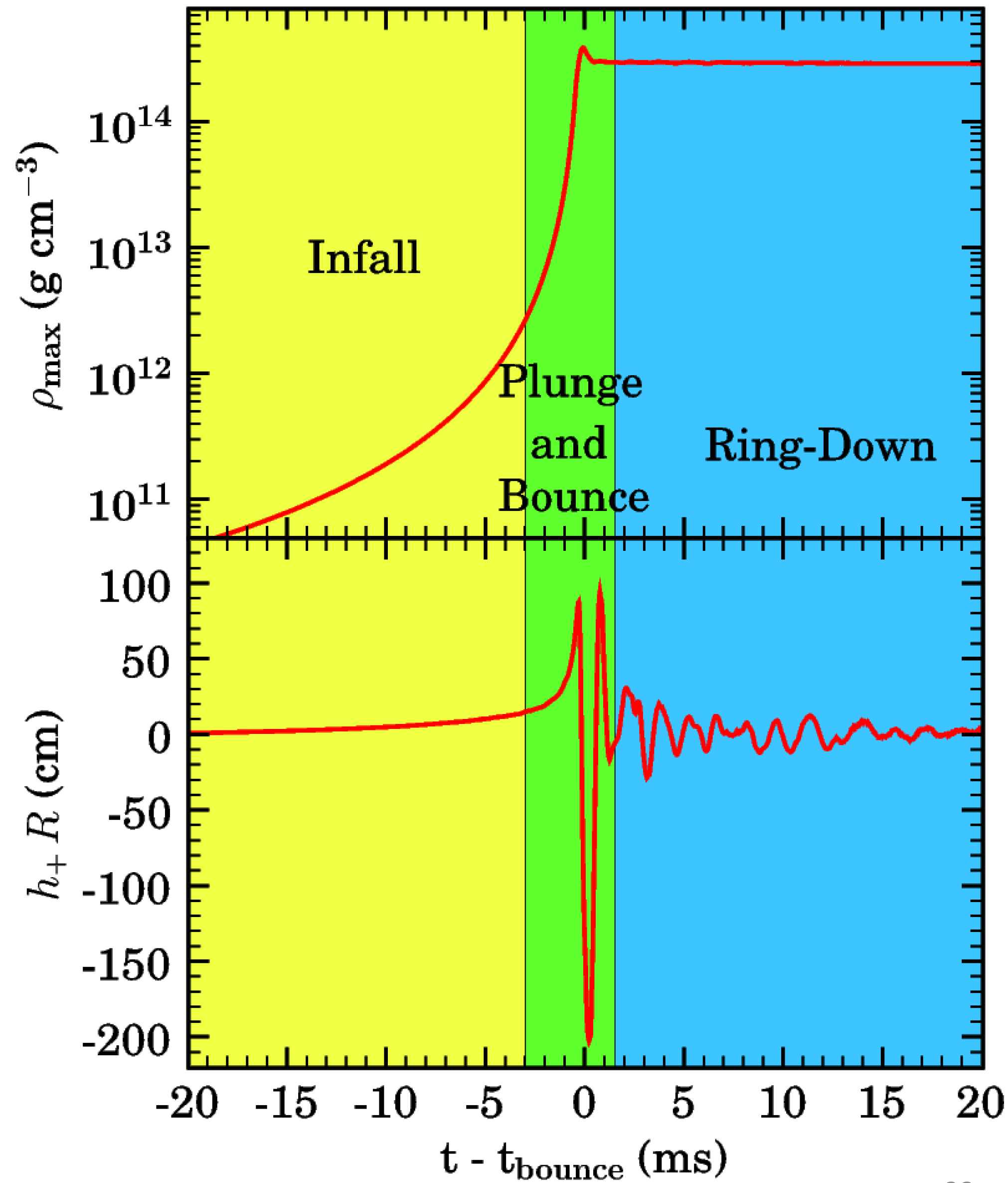
GWs from Rotating Collapse & Bounce

Recent work: Dimmelmeier+ '08, Scheidegger+ '10, Ott+ '12, Kuroda+ '13

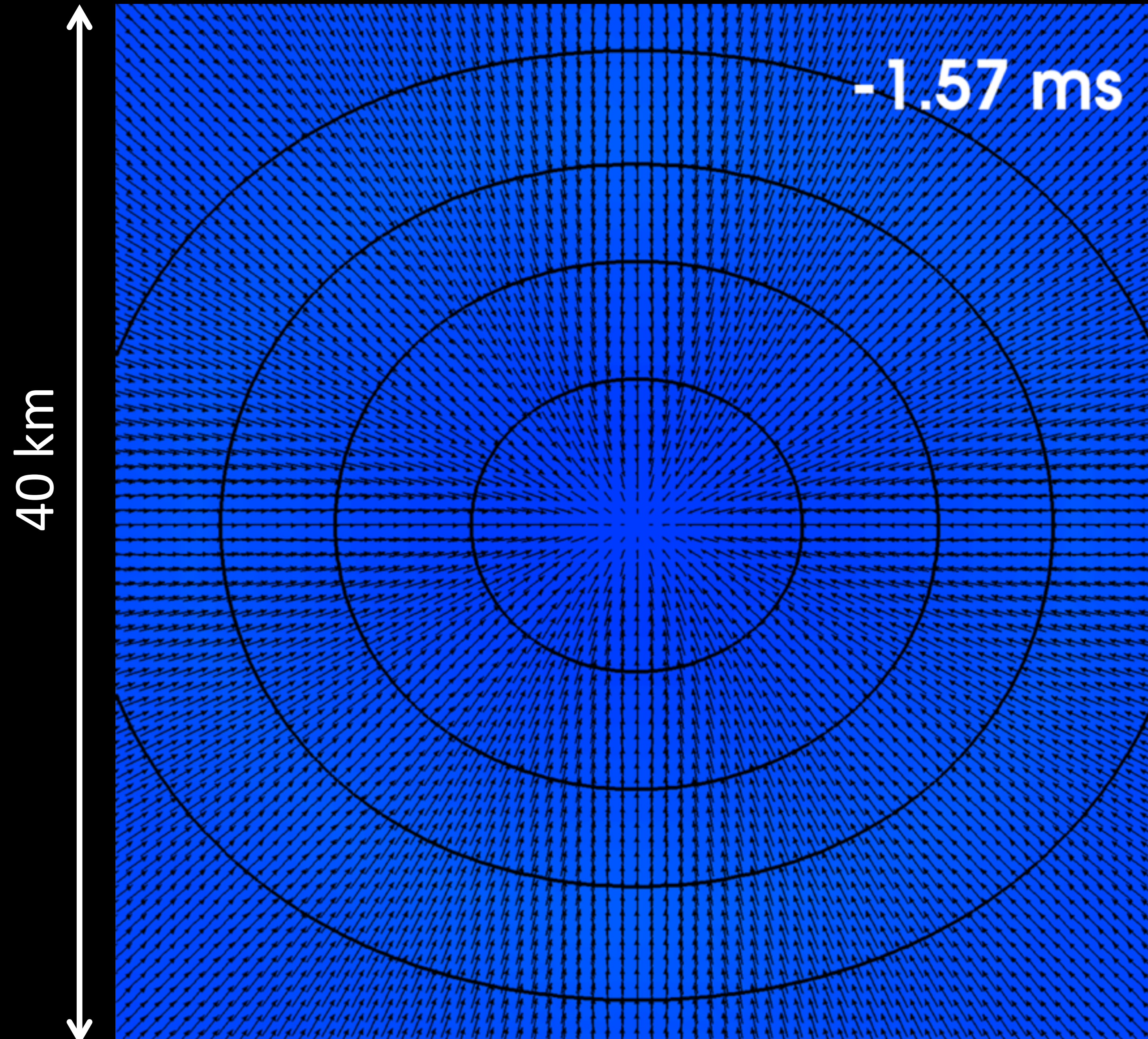
Rapid rotation:
Oblate deformation of the inner core



- Most extensively studied GW emission in core collapse
- **Axisymmetric: ONLY h_+**
- Simplest GW emission process: **Rotation** + **Gravity** + **Stiffening of nuclear EOS**.
- Strong signals for rapid rotation (-> millisecond proto-NS).



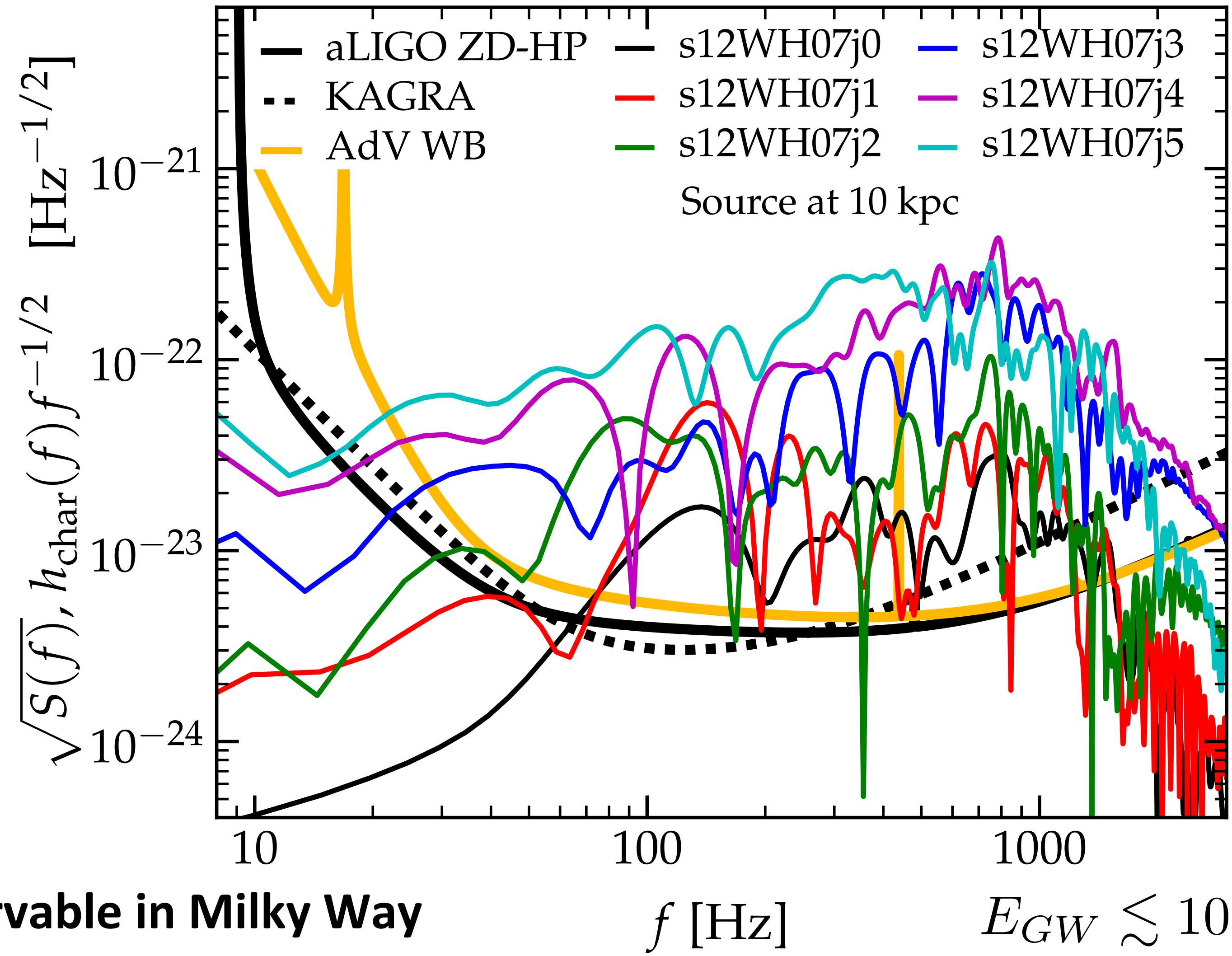
Ott+12



Can we observe this?

Ott+ '12, PRD

Gravitational Waves



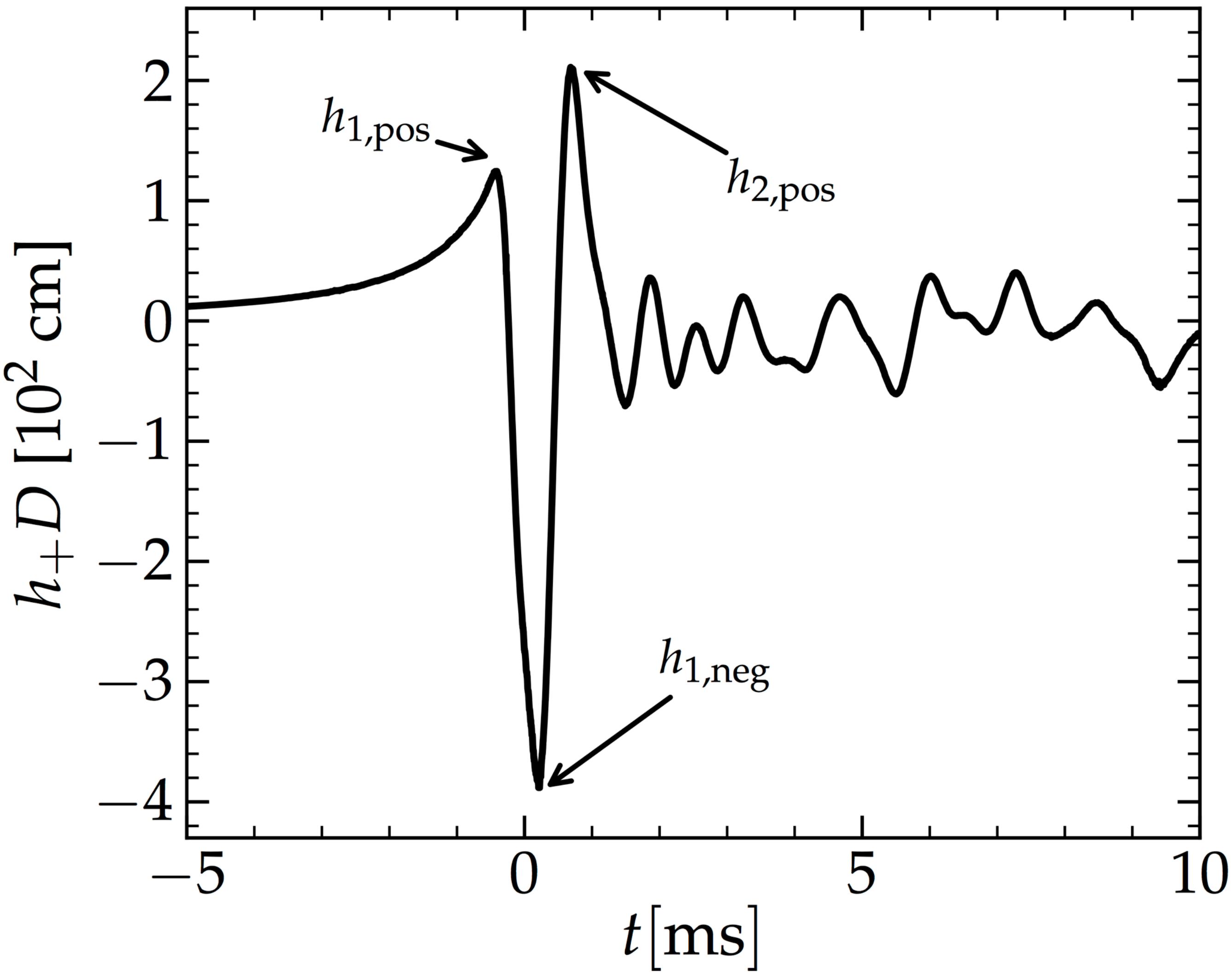
$E_{GW} \lesssim 10^{-8} M_\odot c^2$

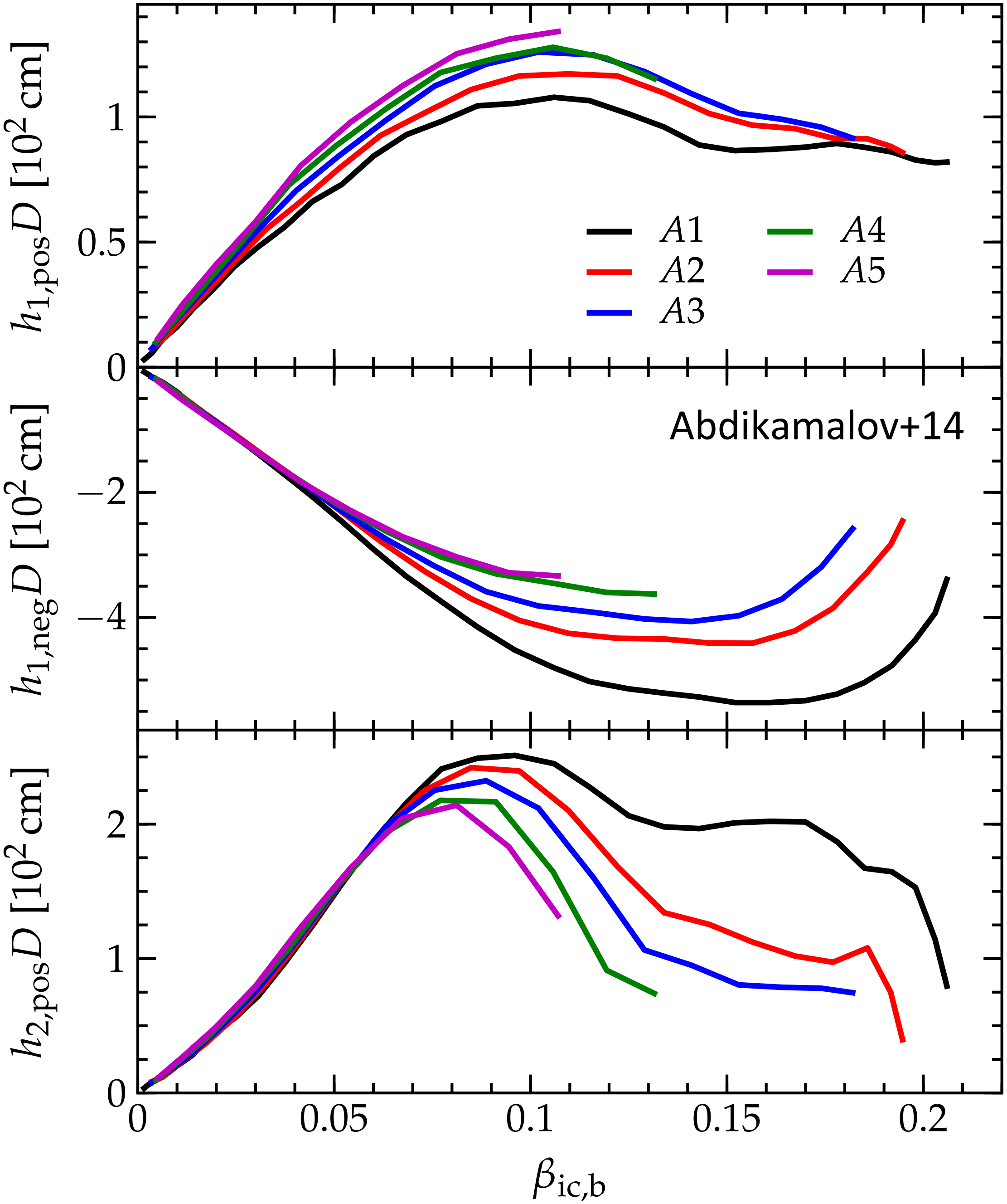
-> Observable in Milky Way with aLIGO

GWs from Rotating Collapse & Bounce

Abdikamalov, Gossan, DeMaio, Ott 2014, PRD 90, 044001

Simple signal features:





Measure for "total rotation" of the inner core:

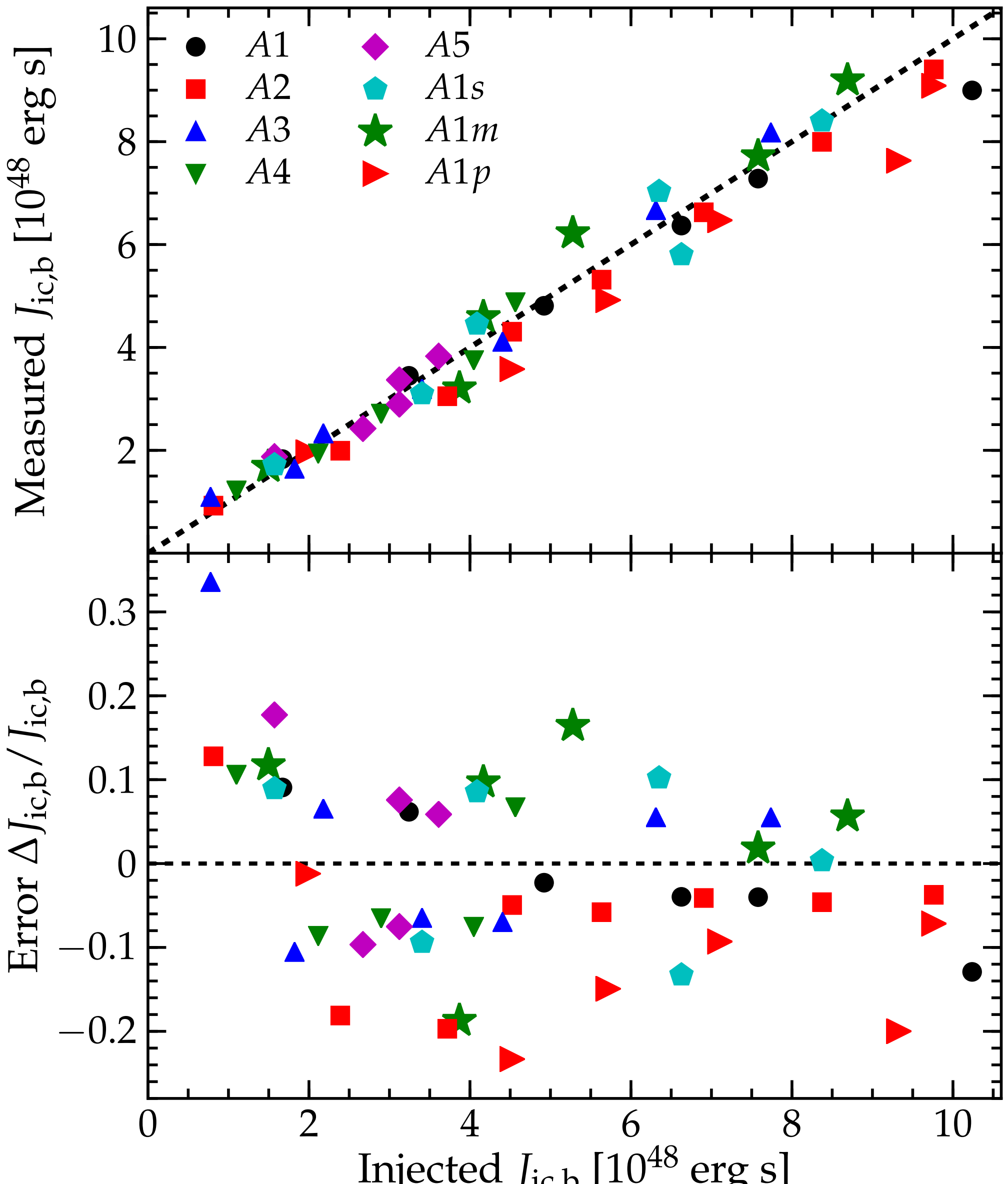
$$\beta = \frac{T}{|W|}$$

Closely related to inner core angular momentum

A1(most) – A5(least) differential rotation.

Measuring Inner Core Angular Momentum

Abdikamalov, Gossan, DeMaio, Ott 2014, PRD 90, 044001



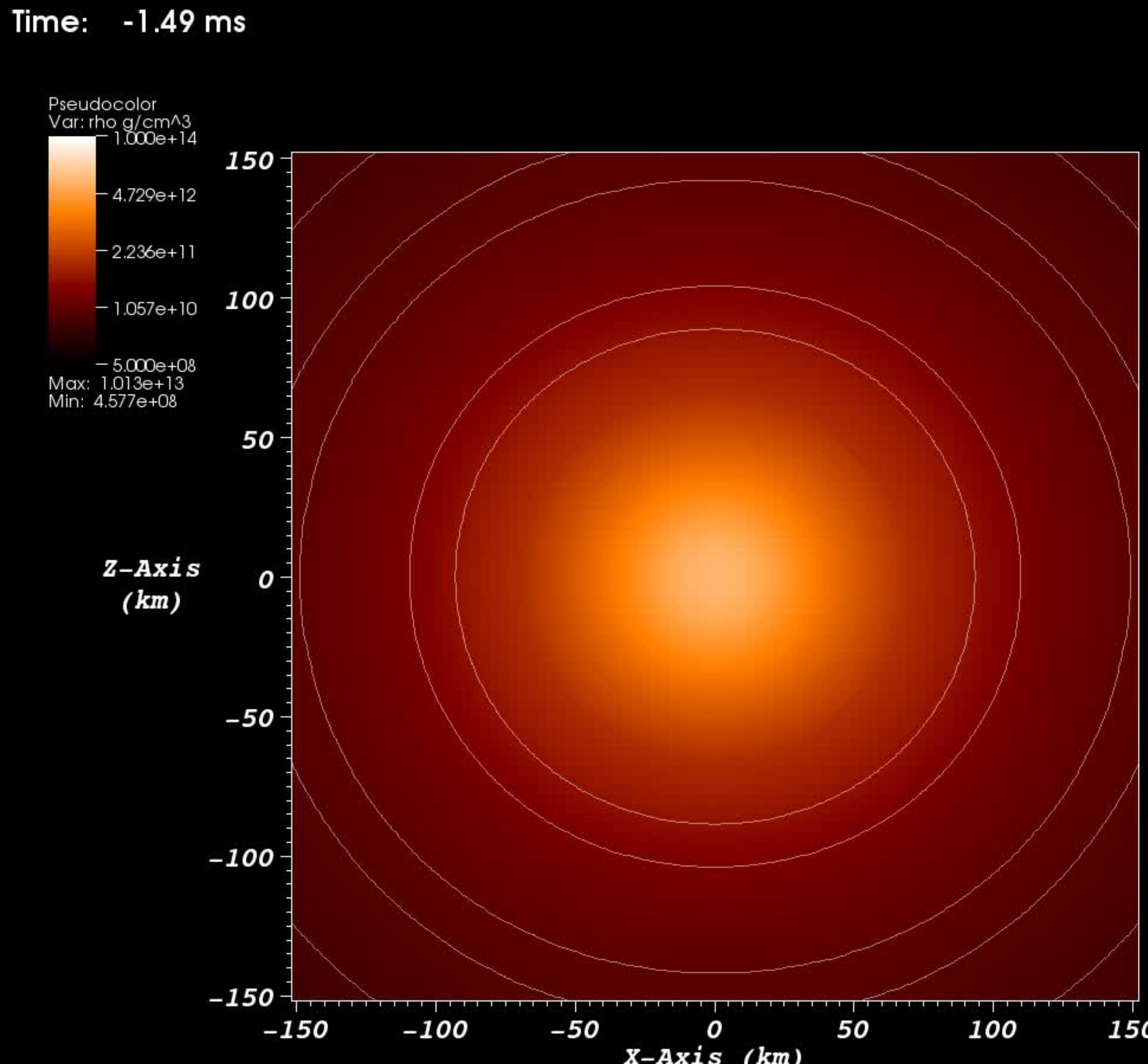
"Matched-filtering" analysis.

Unknown signal injected into simulated detector noise.

Can measure inner core angular momentum with < 30% error!

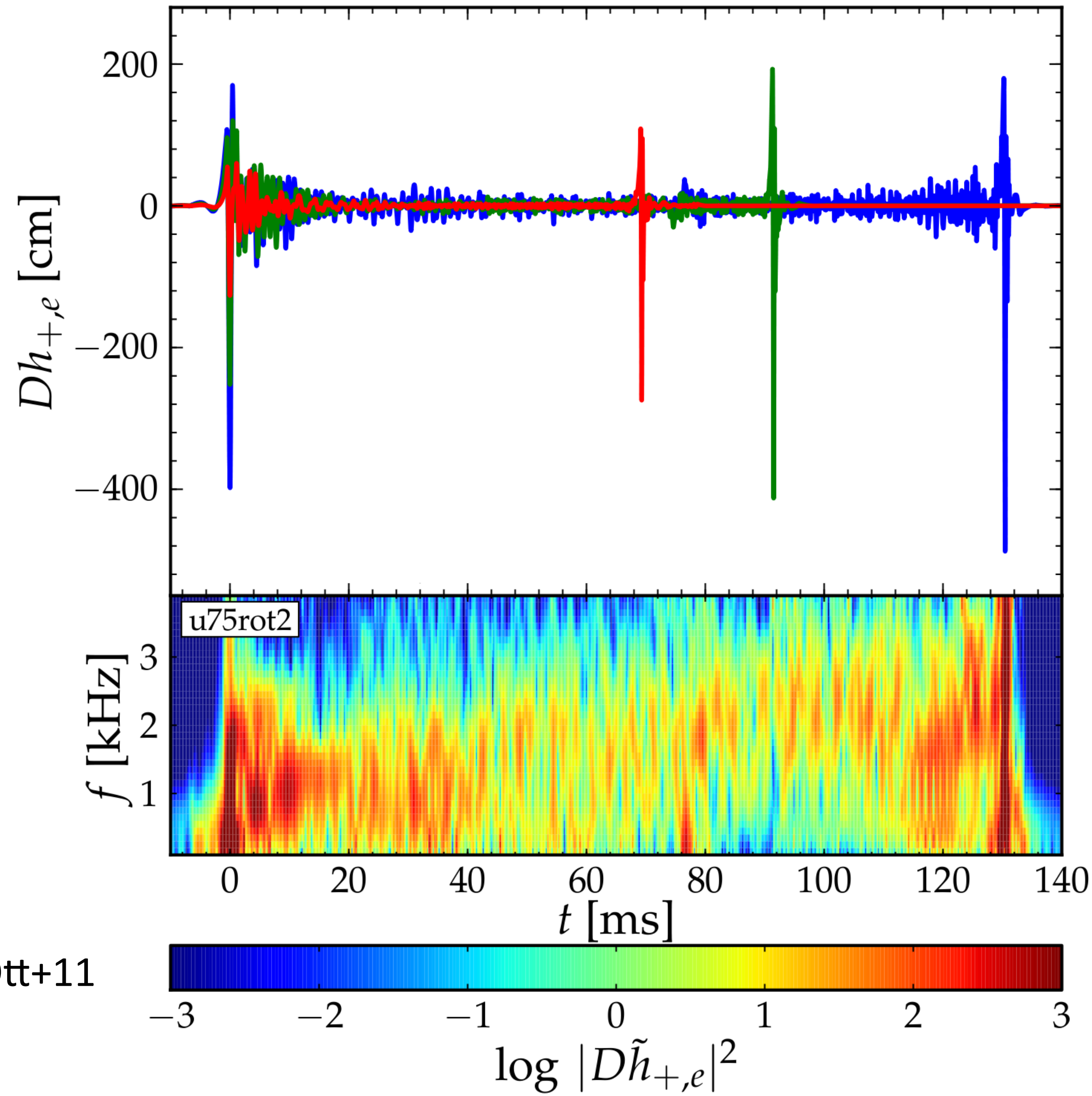
3+1 GR simulation, simplified microphysics

u75 progenitor of Woosley+02



Ott+11

Gravitational Waves from BH Formation



Ott+11

Rotating Black Hole Formation

$$E_{\rm GW} \sim 10^{-7}\,M_\odot\,c^2$$

But: observable only for nearby (galaxy/SMC/ LMC events)

Done!

Supplemental Slides

Ion EOS in the Iron Core

- Ideal Boltzmann gas of non-interacting particles.

$$P_{\rm ion} = n_{\rm ion} kT \quad n = \frac{\rho}{\mu m_u} \quad \mu = \left(\sum_i \frac{X_i}{A_i}\right)^{-1}$$

For pure, say, ^{56}Ni: $\mu = 56$

$$P_{\rm ion} = \frac{\rho N_A}{56} kT = 1.7 \times 10^{26} \left(\frac{\rho}{10^{10}\,{\rm g\,cm^{-3}}}\right) \left(\frac{T}{1\,{\rm MeV}}\right) {\rm dyn\,cm^{-2}}$$

Photon EOS in the Iron Core

- Ideal Bose gas:

$$P_\gamma = \frac{1}{3} a T^4 = 4.6 \times 10^{25} \left(\frac{T}{1\,\mathrm{MeV}} \right)^4 \mathrm{dyn\,cm}^{-2}$$

Electron EOS in the Iron Core

- Ideal Fermi gas, but electrons are *relativistic* and *degenerate*:

$$\eta = \frac{\mu_e}{kT} \gg 1 \qquad \beta = \frac{kT}{m_e c^2} \gg 1$$

degeneracy parameter relativity parameter

In this case:

$$P_e = K\rho^\gamma = 1.2435 \times 10^{15} Y_e^{4/3} \rho^{4/3}$$

$$P_e = 10^{28} \left(\frac{Y_e}{0.5}\right)^{4/3} \left(\frac{\rho}{10^{10}\,\mathrm{g\,cm^{-3}}}\right)^{4/3} \mathrm{dyn\,cm^{-2}}$$

Liquid Drop Model

Bethe & von Weizsäcker 1935/37

Nuclear masses:

$$M(N,Z) = Zm_p + Nm_n - BE$$

$$BE = a_V A - a_\sigma A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{\rm sym} \frac{(N-Z)^2}{A} + \delta(N,Z)$$

Volume term

Surface term

Coloumb term

Symmetry Term

Pairing Term

$$a_V \simeq 16\,\mathrm{MeV} \quad a_\sigma \simeq 18\,\mathrm{MeV} \quad a_C \simeq 0.7\,\mathrm{MeV} \quad a_{\rm sym} \simeq 23\,\mathrm{MeV}$$

$$\delta(N,Z) = \begin{cases} -\delta_0 & Z, N \,\mathrm{even} \\ 0 & Z+N \,\mathrm{odd} \\ \delta_0 & Z, N \,\mathrm{odd} \end{cases} \qquad \delta_0 = \frac{a_P}{A^{1/2}} \qquad a_P \simeq 12\,\mathrm{MeV}$$

Liquid Drop Model -> EOS

(e.g. Lattimer & Swesty 1991, Lattimer & Prakash 2007, Lattimer & Lim 2013)

- Near nuclear saturation density $n_s \sim 0.16\ \mathrm{fm}^{-3}$, expand energy per baryon:

$$\epsilon(n,x) = -16\,\mathrm{MeV} + \frac{1}{18}K\left(1-\frac{n}{n_s}\right)^2 + \frac{K'}{27}\left(1-\frac{n}{n_s}\right)^3 + E_{\mathrm{sym}}(n)(1-2x)^2 + \dots$$

At T=0: $f = \epsilon$ $\qquad x = Y_p = Y_e$

$K \simeq 240\,\mathrm{MeV}$ incompressibility

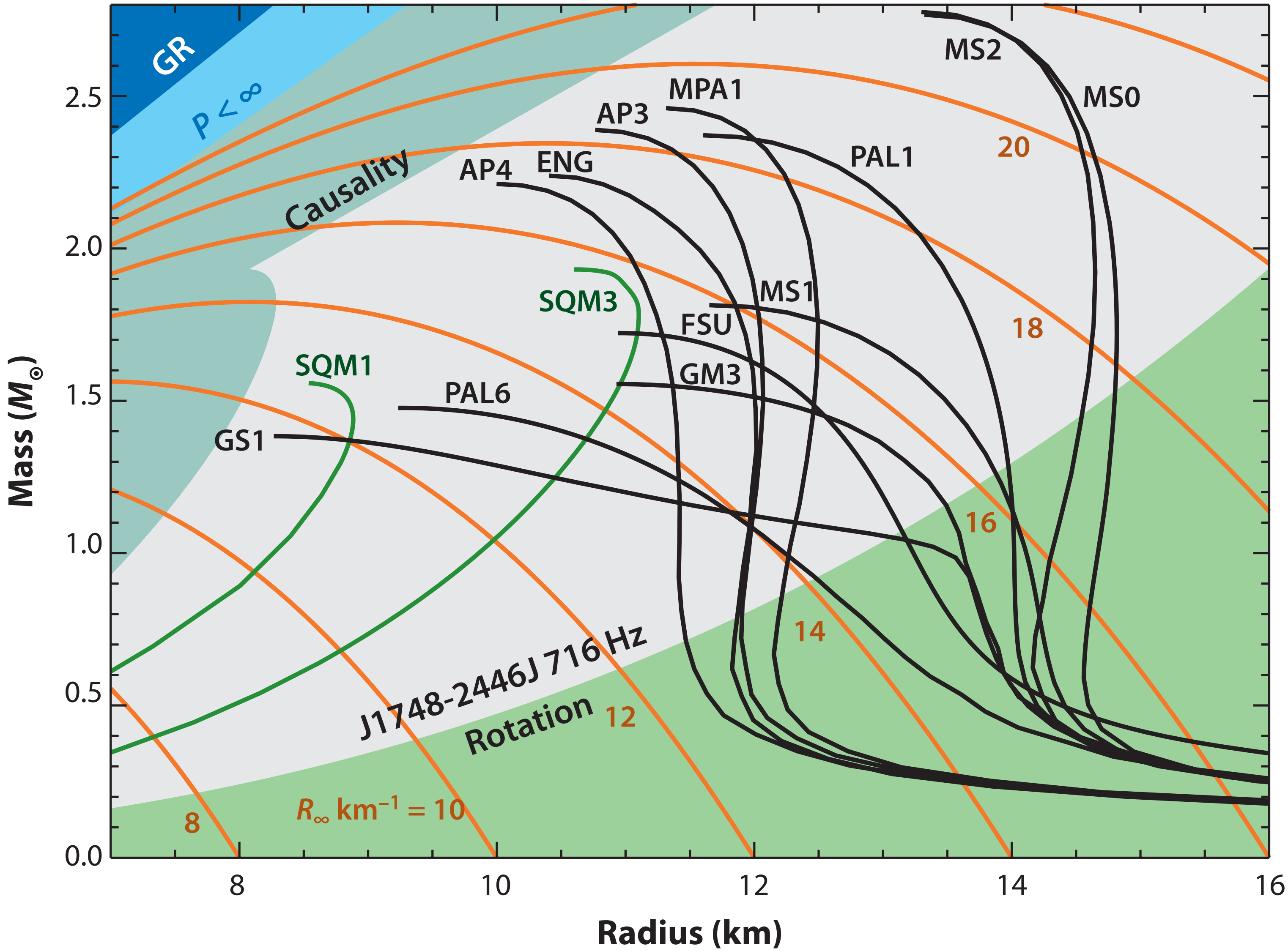
$E_{\mathrm{Sym}}(n_s) = S_v \approx 29.0 - 32.7\,\mathrm{MeV}$ symmetry energy

$K' \approx 1780 - 2380\,\mathrm{MeV}$ skewness

} experimental & astrophysical constraints

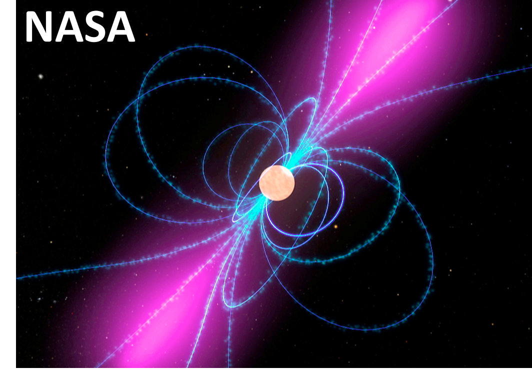
- Write out energy of bulk nuclear matter according to nuclear force model (e.g., Skyrme 1959) and use T=0, n=ns, and above expansion to set parameters of nuclear force.
- Introduce model for nuclei & alpha particles, then minimize f.

EOS & Neutron Star Structure



Knowing masses and radii would really help!!!

Neutron Star Masses



Pulsar with binary companion:
Mass function for pulsar

$$f_p = \left(\frac{2\pi}{P}\right)^2 \frac{(a_p \sin i)^3}{G} = \frac{(M_c \sin i)^3}{M^2}$$

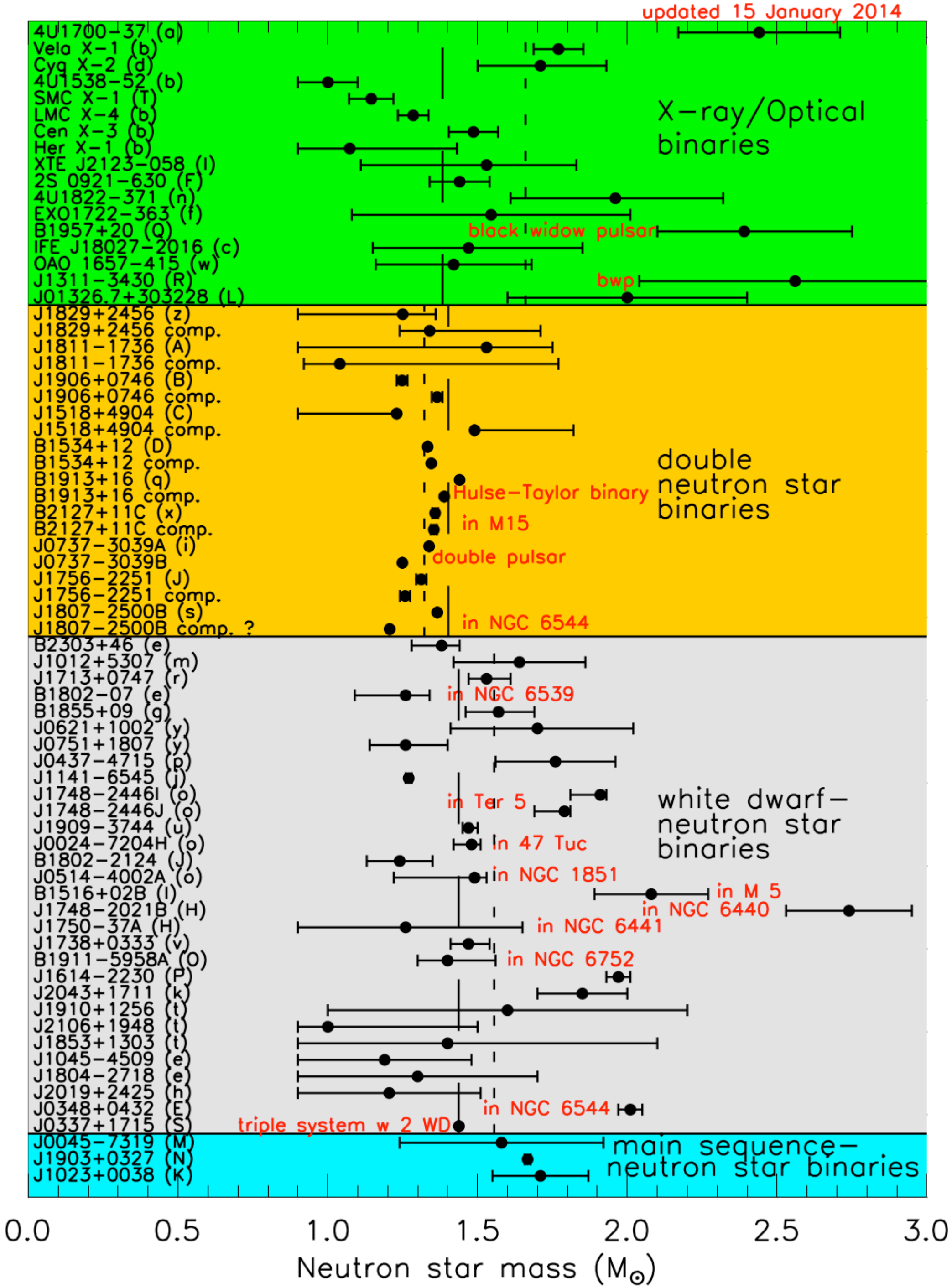
Companion mass

Orbital inclination

Total system mass

- Must know/infer **companion mass** and **inclination** to get M_P.
- Different kinds of binaries:
  X-ray binaries (accreting NSs), double NS binaries, NS–normal-star binaries, NS–WD binaries.
- Companion mass: via stellar models or relativistic effects.
- Inclination: most difficult. In relativistic binaries:
  Shapiro time delay (delay of pulsar pulses by gravity of companion)

Lattimer 14,
http://stellarcollapse.org



X-ray binaries

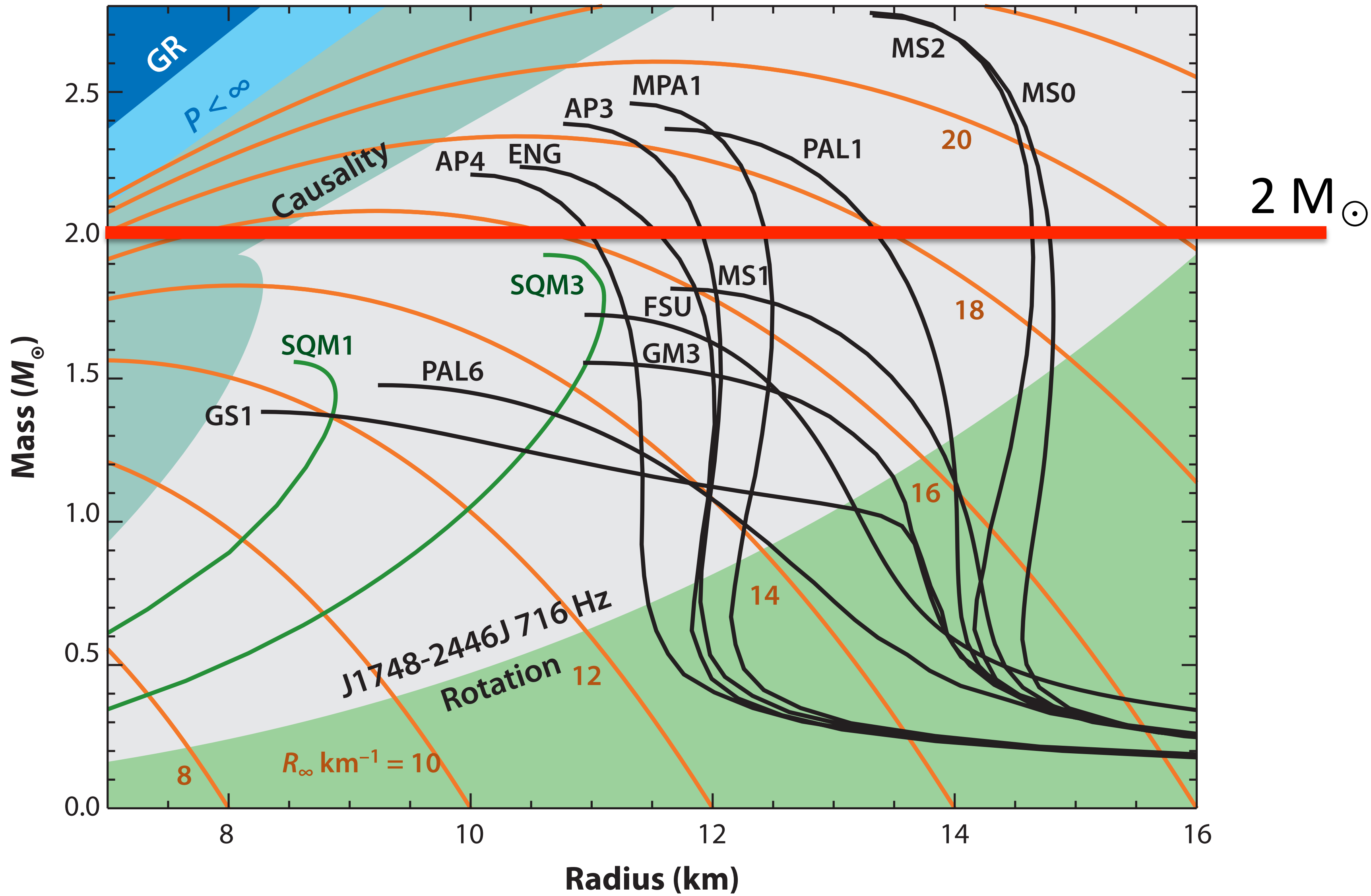
NS+NS

Most massive:
PSR J1614-2230
1.97+-0.04 $M_\odot$
PSR J0348+0432
2.01+-0.04 $M_\odot$

WD+NS

NS + normal star

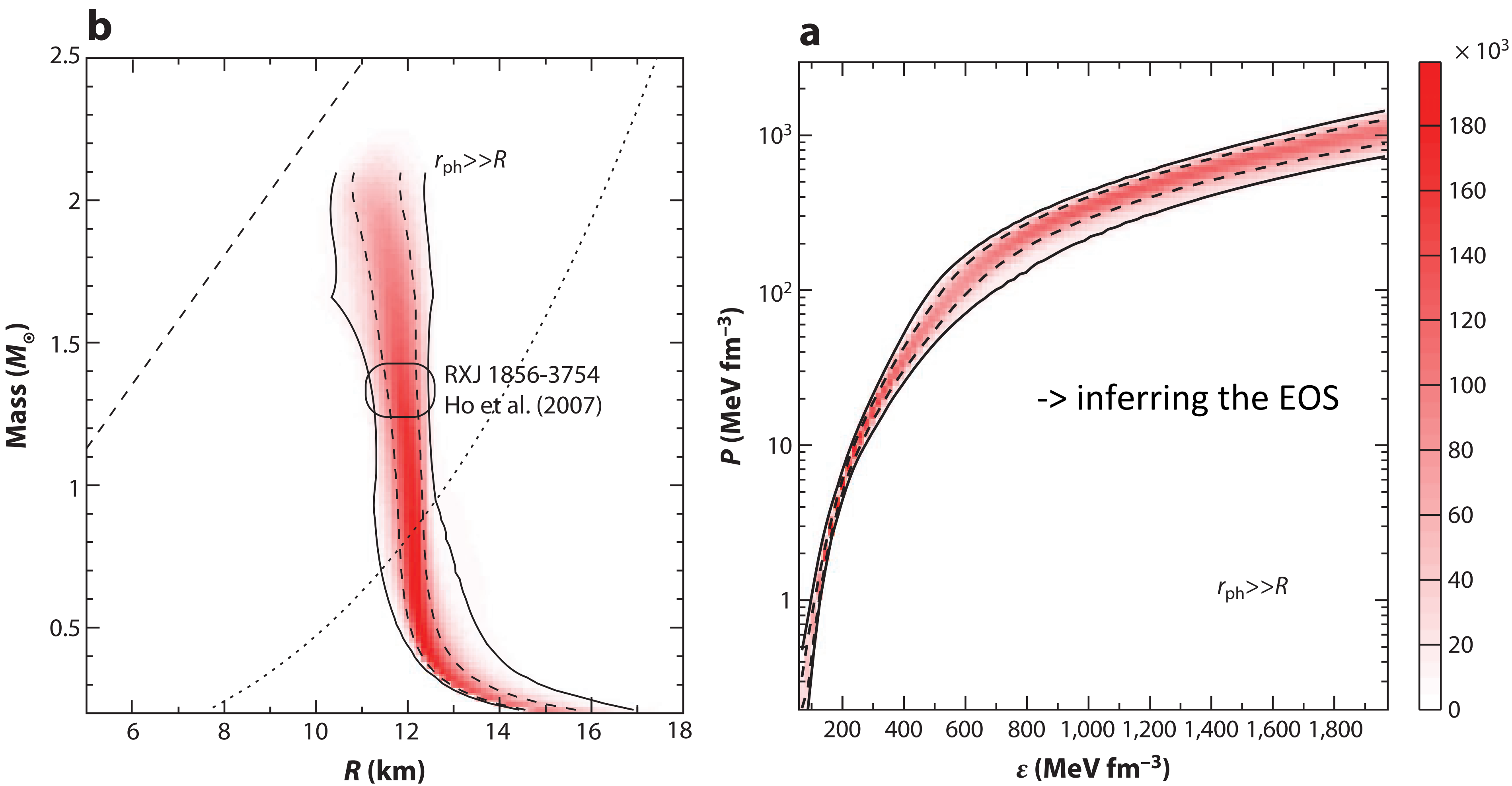
Neutron Star Structure & EOS Constraints



Neutron Star Radii

- So far no robust NS radius (or mass&radius) measurements.
- Main approaches: (see Lattimer 2012)
  - X-ray observation of quiescent and bursting NSs in galactic X-ray binaries.
  - **GW signal from tidal deformation and disruption of NS in BHNS merger.**
  - **GW signal from tidal deformation and postmerger oscillations in NSNS merger.**
  - Neutrino signal from the proto-NS in the next galactic CCSN.

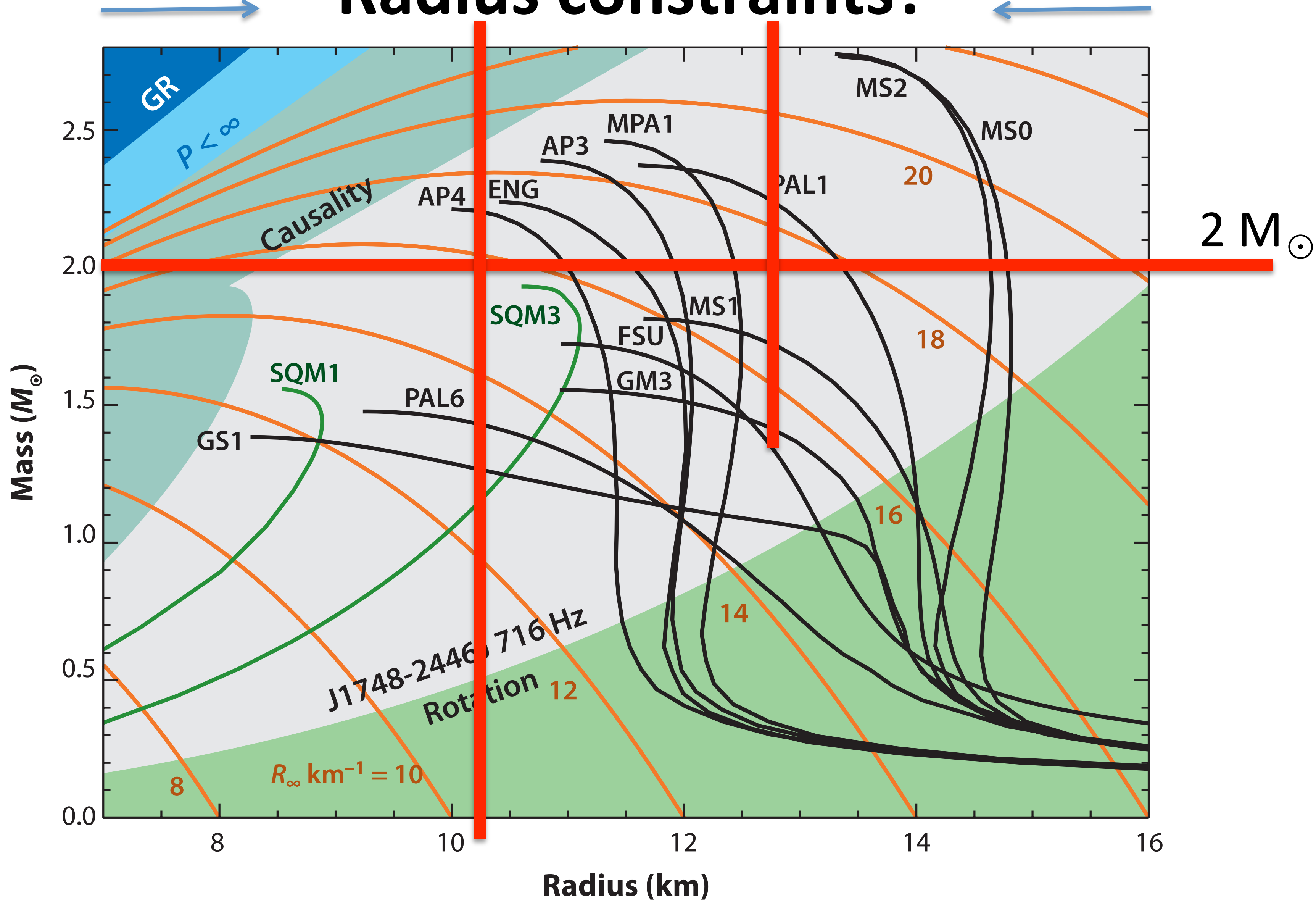
Neutron Star Masses & Radii



Statistical Analysis of observational data: Steiner+10,+12, Lattimer 12

Warning: Does not fix model dependence of M, R estimates!

Radius constraints?

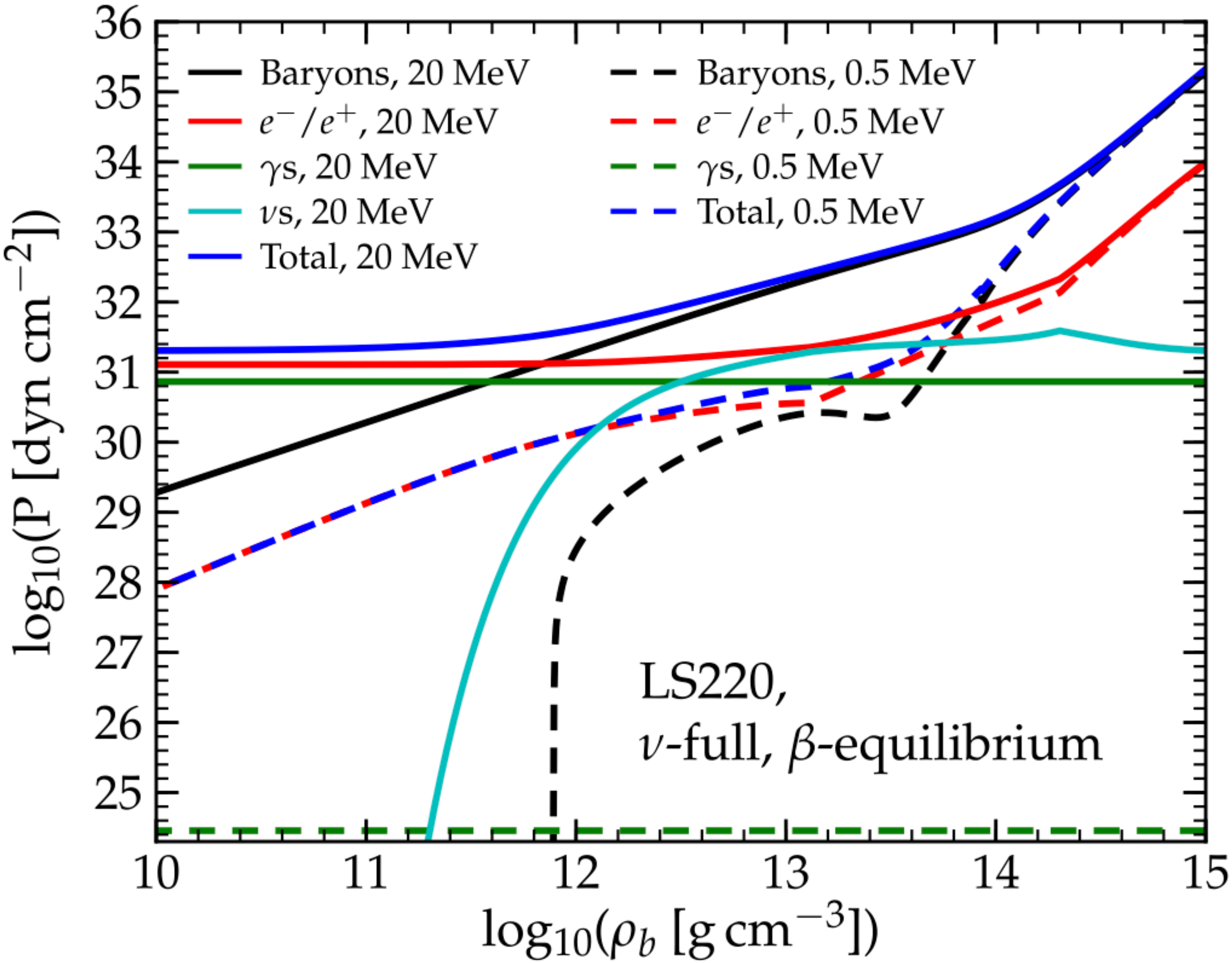


Literature on the Nuclear EOS

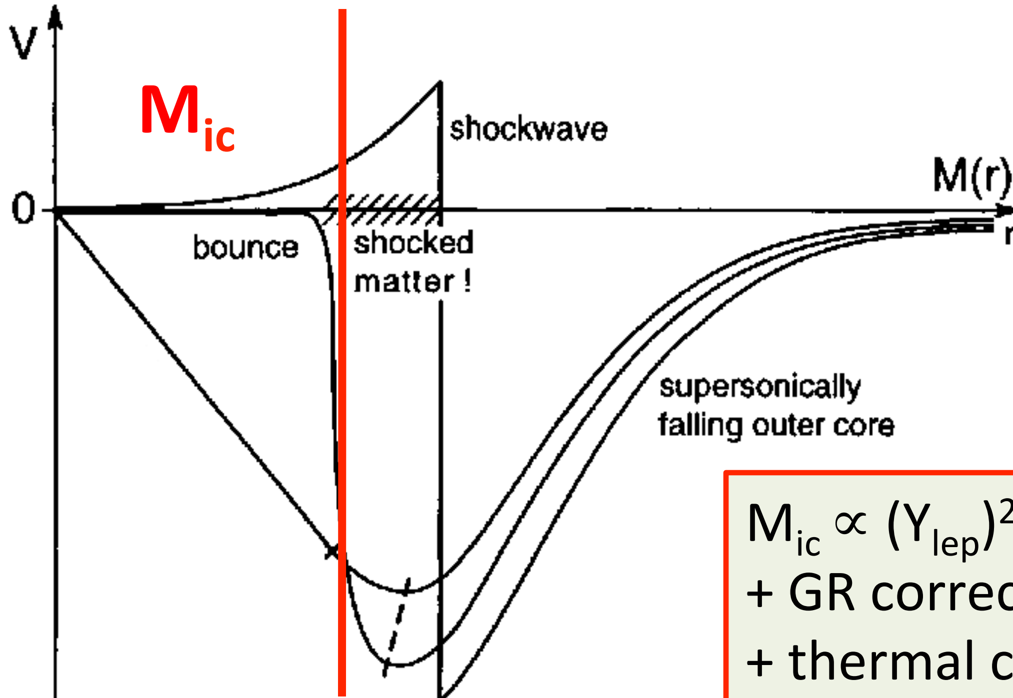
- Shapiro & Teukolsky, Black Holes, White Dwarfs, and Neutron Stars, Wiley-VCH, 1983
- Haensel, Pothekhin, Yakhovlev, Neutron Stars – Equation of State and Structure, Springer, 2007
- Bethe 1990, Rev. Mod. Phys. 62, 801
- Bethe, Brown, Applegate, Lattimer 1979, Nuc. Phys. A 324, 487
- Lattimer, Pethick, Ravenhall 1985, Nuc. Phys. A 432, 646
- Lattimer & Swesty 1991, Nuc. Phys. A 535, 331
- Lattimer & Prakash 2007, Phys. Rep., 442, 109
- Lattimer 2012, Ann. Rev. Nuc. Par. Sc., 62, 485

NSNS Postmerger: Sources of Pressure

Kaplan+14



Universality of Core Collapse



$M_{ic} \propto (Y_{lep})^2$
+ GR correction (-)
+ thermal correction (+)
+ rotation (+)

The Mass M_{ic} of the inner core at bounce is determined by nuclear physics and weak interactions, is ~0.5 M_{SUN}, and is practically independent of progenitor star mass and structure.