Workshop “Recent developments and challenges in topological phases” @ YITP

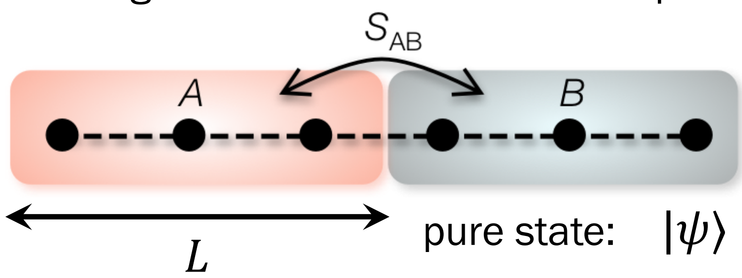
Universality and phase transitions in system-environment entanglement

Yuto Ashida (University of Tokyo)

Refs:
YA, Furukawa, Oshikawa, arXiv:2311.16343
Yokomizo & YA, arXiv:2405.19768.
Masuki, Sudo, Oshikawa, YA, PRL 129, 087001 (2022)
Yokota, Masuki, YA PRA 107, 043709 (2023)
Fuji & YA, PRB 102, 054302 (2020)

Overview: Quantum entanglement in many-body systems

Entanglement between A and B is quantified by a nonlinear function of $\rho_A = \mathrm{tr}_B[|\psi\rangle\langle\psi|]$.



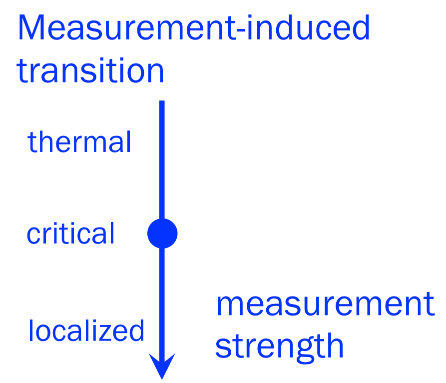
n th Renyi entropy:

$n \to 1$: von Neumann

$n = 2$: purity

$$S_A = \frac{1}{1-n} \ln \mathrm{tr}_A \left[\rho_A^n\right]$$

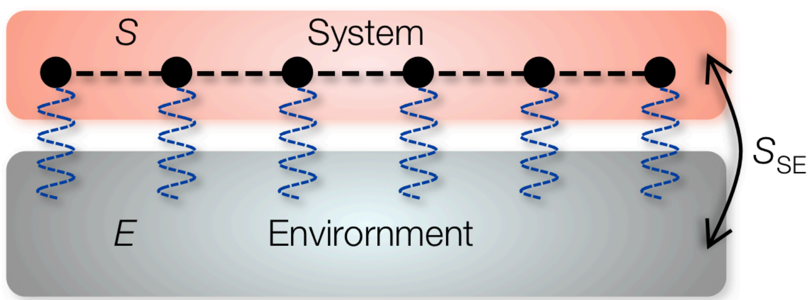
- Volume law $S_A \propto L$ e.g., thermal states
- Logarithmic scaling $S_A \propto \ln L$ e.g., gapless critical states
- Area law $S_A \propto \mathrm{const.}$ e.g., gapped ground states



Fisher et al.,
Ann. Rev. Cond. Matt. 14, 335 (2023)

→ Entanglement *within* a system of interest

What would be entanglement properties unique to open systems?



Universality and transitions in entanglement between system (S) and environment (E)

Collaborators

System-environment entanglement phase transitions

YA, Furukawa, Oshikawa, arXiv:2311.16343

Measurement-induced criticality and transitions under continuous measurement

Fuji and YA, PRB 102, 054302 (2020); Yokomizo and YA, arXiv:2405.19768.

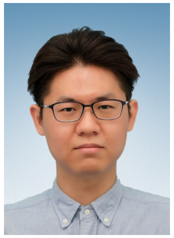
Dissipative QPT in Josephson junctions

Masuki, Sudo, Oshikawa, YA, PRL 129, 087001 (2022); Yokota, Masuki, YA, PRA 107, 043709 (2023)

M. Oshikawa (UT, ISSP)

S. Furukawa (Keio)

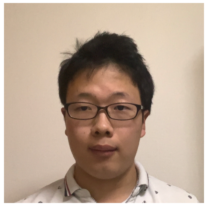
Y. Fuji (UT)



K. Yokomizo (UT)

T. Yokota (RIKEN)

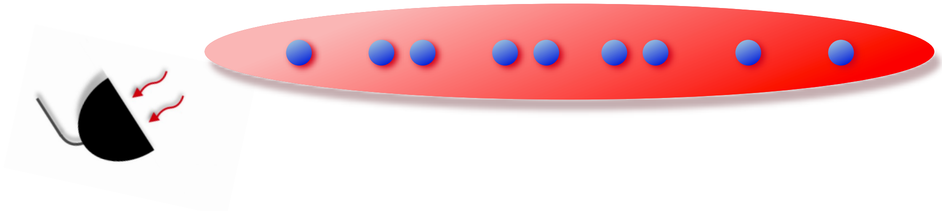
K. Masuki (UT)



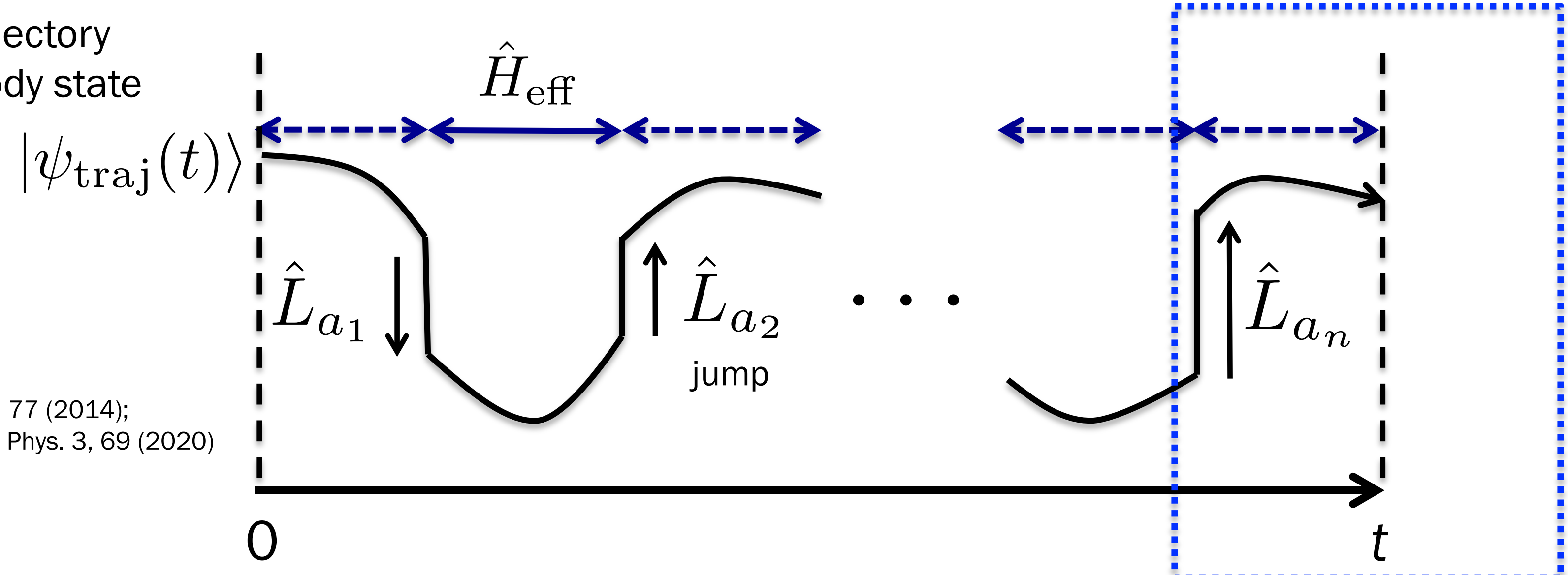
H. Sudo (UT, ISSP)

Introduction: entanglement transition induced by measurement

continuous measurement



quantum trajectory
of a many-body state



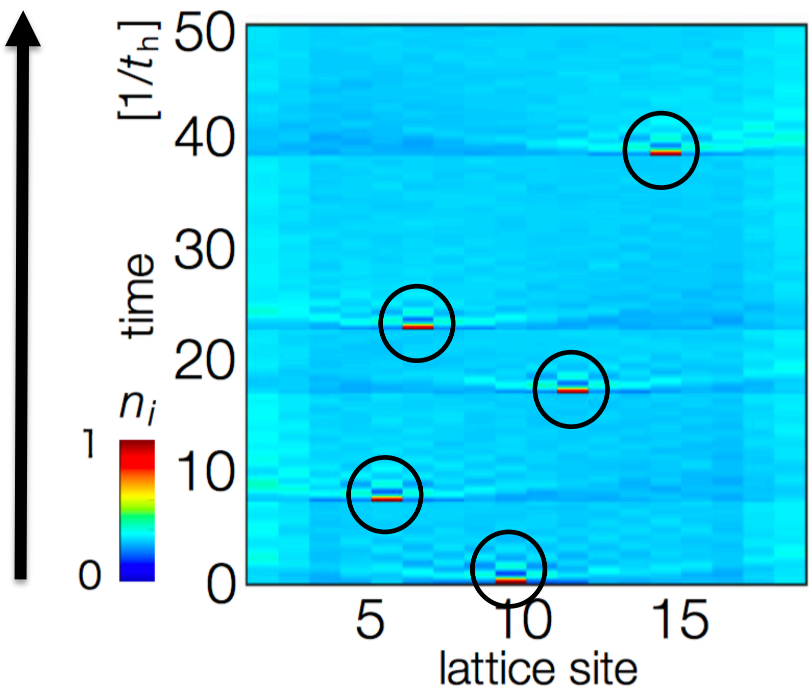
cf. Daley, Adv. Phys. 63, 77 (2014);
YA, Gong, Ueda, Adv. Phys. 3, 69 (2020)
...

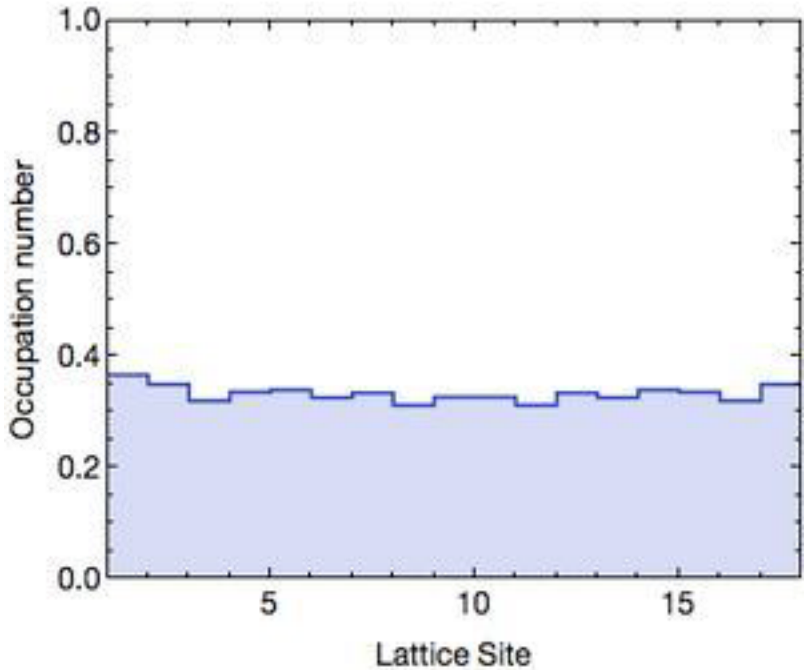
After transient dynamics, the system is expected to reach a **pure *steady* state $|\psi_{\rm traj}\rangle$**.

Illustrative example: nonintegrable hard-core bosons under weak measurement

Nonintegrable hard-core bosons under local density measurement $L_i \propto n_i$.

Quantum jump = Wavefunction localization due to atom detection





- *Weak* measurement: detection rate is low such that the system thermalizes after each jump
- In the long-time limit, the state reaches to an infinite-temperature state → **Volume-law** entanglement in $|\psi_{\mathrm{traj}}\rangle$

What would happen when the measurement strength is increased ?

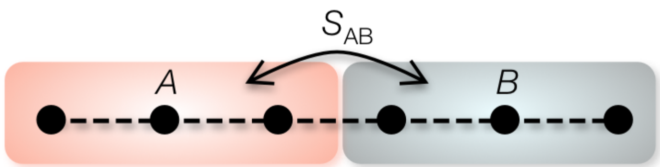
YA, Saito and Ueda, PRL 121, 170402 (2018)

Measurement-induced transition and criticality

Entanglement entropy of $|\psi_{\mathrm{traj}}\rangle$

averaged over trajectories

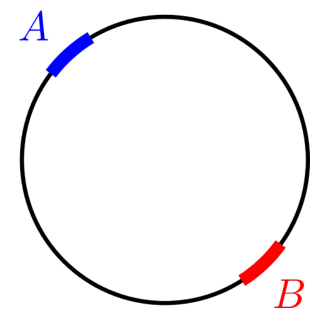
$$S_{AB} = \mathrm{E}[-\mathrm{tr}[\rho_A \ln \rho_A]]$$

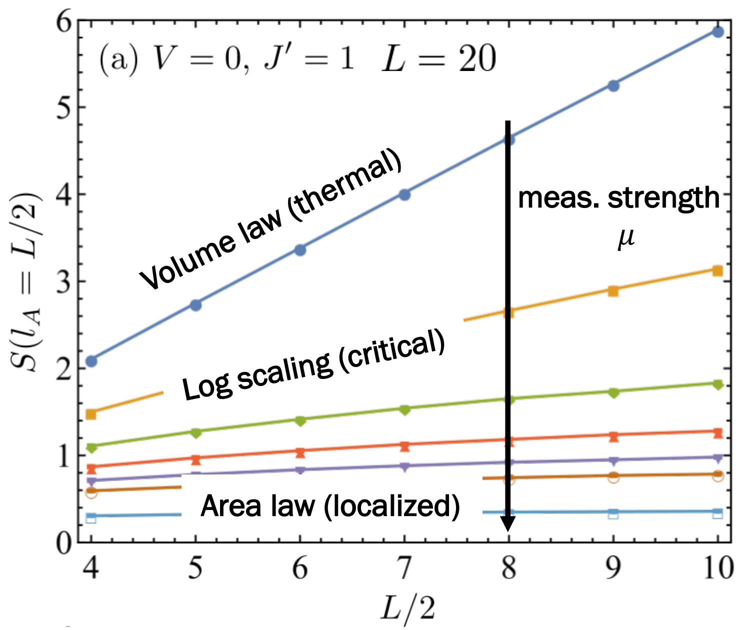


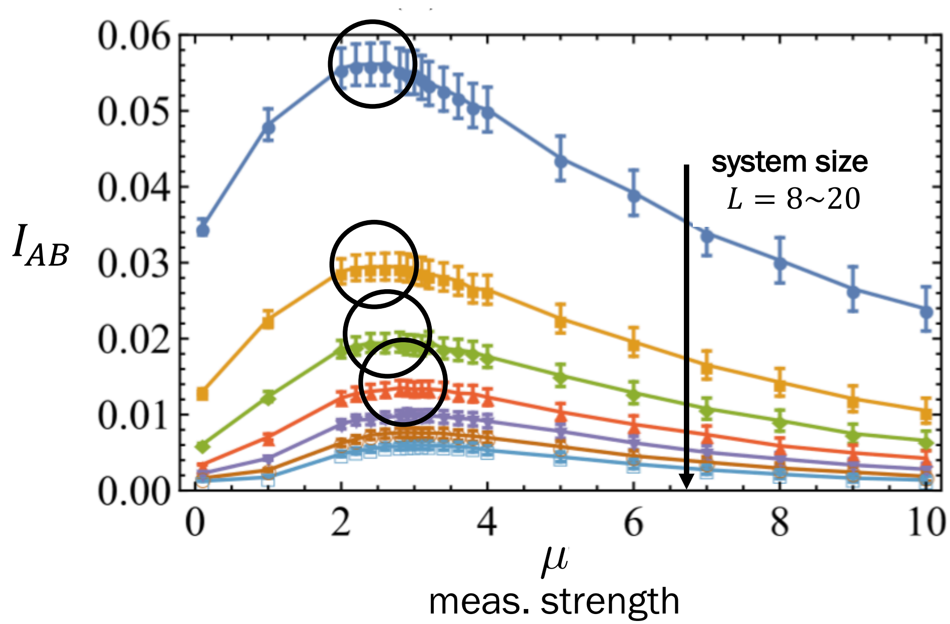
Averaged mutual information

$$I_{AB} = S_A + S_B - S_{A\cup B}$$

$$I_{AB} \geq \frac{|\langle O_A O_B \rangle_c|^2}{2\|O_A\|^2\|O_B\|^2}$$







Measurement-induced criticality:
peak structure at critical point

Y. Fuji and YA, PRB 102, 054302 (2020)

Recent Developments and Challenges

June 3 - June 14 2024

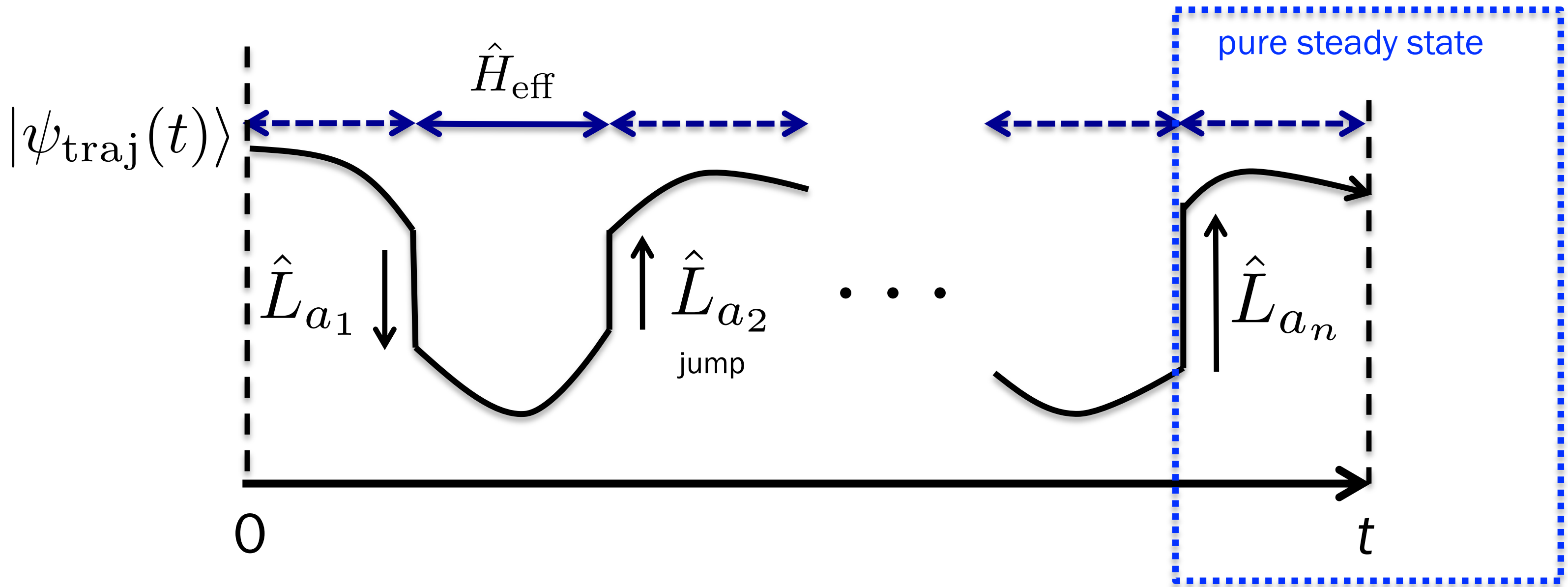
Yukawa Institute for Theoretical Physics, Kyoto University

Postselection problem

Common difficulty:

the necessity of postselections to probe a measurement-induced transition
→ success probability becomes exponentially small as the system size is increased.

Quantum trajectory = a pure state conditioned on measurement outcomes



Entanglement transitions in unconditioned evolution?
(= measurement outcomes are averaged over, and no postselection is required)

System-environment entanglement in open many-body systems

Unconditioned evolution ⇔ CPTP map $\mathcal{E}$ ⇔ Stinespring representation

$$\hat{\rho}_{\mathcal{E}} = \mathcal{E}(\hat{\rho}_S) = \mathrm{tr}_E[\hat{U}(\hat{\rho}_S \otimes \underbrace{\hat{\rho}_E}_{\text{pure}})\hat{U}^\dagger]$$

→ $\rho_{\mathcal{E}}$ is a reduced density matrix on S obtained from a pure state on $S + E$.

Entanglement *between* S and *E* is quantified by Renyi entropy of $\rho_{\mathcal{E}}$:

$$S_{SE}^{(n)} = \frac{1}{1-n} \ln \mathrm{tr}_S\left[\rho_{\mathcal{E}}^n\right]$$

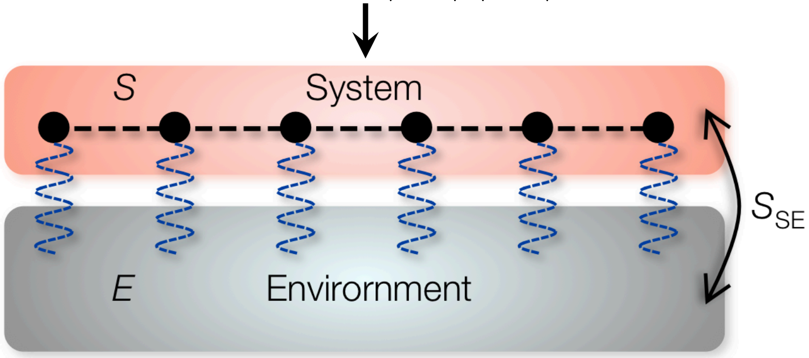
For concreteness, we focus on the case of $n = 2$ (purity):

$$S_{SE} = -\log \mathrm{tr}\left[\hat{\rho}_{\mathcal{E}}^2\right]$$

Universality and transition in S_{SE} ?

Tomonaga-Luttinger liquid (TLL)

$$\hat{\rho}_S = |\Psi_0\rangle\langle\Psi_0|$$



Universality and transition in system-environment entanglement

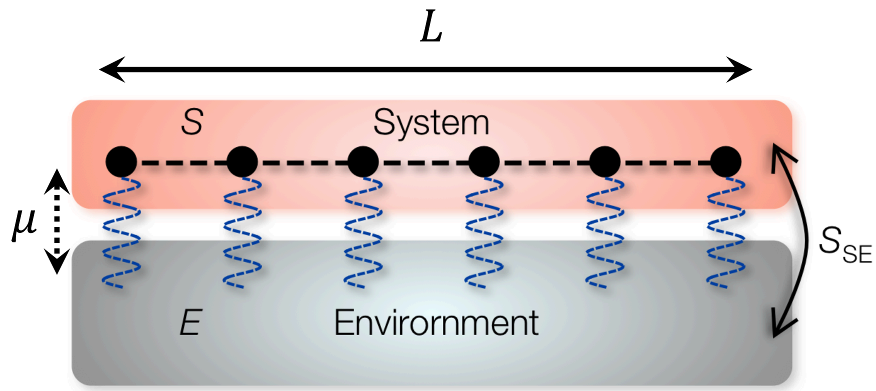
Scaling of system-environment entanglement S_{SE} :

$$S_{SE} = s_1 L - s_0 + \mathrm{o}(1)$$

s_1 : nonuniversal coefficient (depending on UV cutoff)

s_0 : universal term (characterized by K)

$s_0 \neq 0 \Leftrightarrow$ CPTP map $\mathcal{E}$ is relevant to low-energy properties.

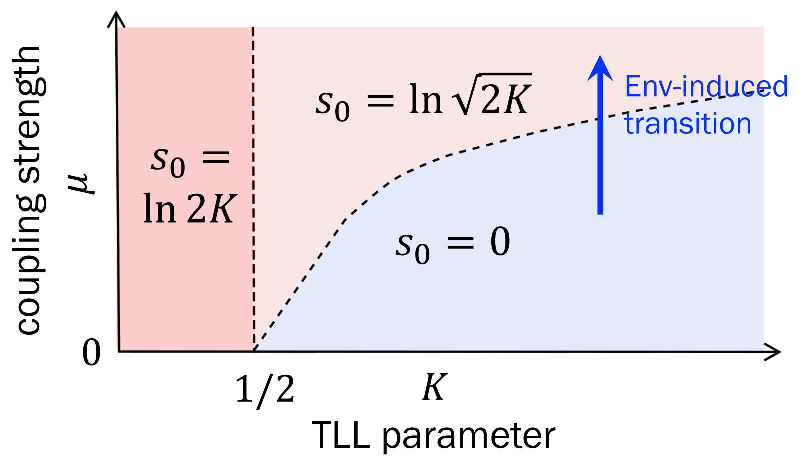


✓ Entanglement transition induced by environment
= Singular change of s_0 as a function of the coupling strength μ

*This transition requires no postselection, but can be detected only by a nonlinear function of $\rho_{\mathcal{E}}$.

→ Makes contrast to conventional dissipative transition, which is probed by a linear function of $\rho_{\mathcal{E}}$.

Phase diagram
(local density measurement)



see also: Stephan, Furukawa, Misguich, Pasquier, PRB 80, 184421 (2009); Zou, Sang, Hsieh, PRL 130, 250403 (2023)

Overview of theoretical framework and derivation

Unconditioned density matrix $\rho_{\mathcal{E}} = \mathcal{E}(\rho_S)$

- Doubled Hilbert space formalism: represent $\rho_{\mathcal{E}}$ as a vector
- Path-integral representation

- cf. talks by Masaya Nakagawa and Shenghan Jiang yesterday

*also used in SPT phases of mixed states: cf. Ma and Turzillo, arXiv:2403.13280 …

Effective field theory in a doubled Hilbert space, where CPTP map $\mathcal{E}$ is represented as a boundary interaction

- (Nonperturbative) RG analysis: identify boundary conditions in the IR limit.
- Boundary CFT : construct conformal boundary states consistent with the b.c.'s.

Goal: determine a value of the universal term s_0 in sys-env entanglement S_{SE}

see also: Lee, Jian, Xu, PRX Quantum 4, 030317 (2023), Bao, Fan, Vishwanath, Altman, arXiv:2301.05687 (2023), …

Effective field theory in a doubled Hilbert space

CPTP map $\mathcal{E}$ ⇔ Kraus representation

$$\hat{\rho}_{\mathcal{E}} = \prod_j \sum_m \hat{K}_{m,j}\hat{\rho}_S\hat{K}^\dagger_{m,j}$$

$K_{m,j}$ acts locally on lattice site j

Doubled Hilbert space formalism:

density matrix (unnormalized) pure state

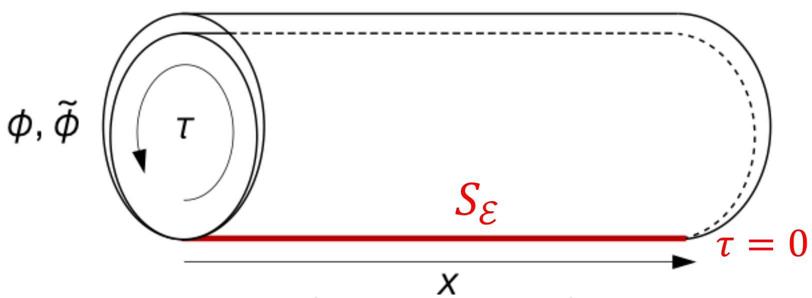
$$\hat{\rho}_{\mathcal{E}} \longrightarrow |\rho_{\mathcal{E}}) = \prod_j \Big(\sum_m \hat{K}_{m,j} \otimes \hat{K}^*_{m,j}\Big)|\rho_S) = \exp\Big(\underbrace{-\mu\sum_j \hat{k}_j \otimes \hat{\tilde{k}}_j}_{S_{\mathcal{E}}}\Big)|\rho_S)$$

Path-integral representation: (1+1)D two-component scalar fields $(\phi, \tilde{\phi})$

$$S_{SE} = -\log \mathrm{tr}\,[|\rho_{\mathcal{E}})(\rho_{\mathcal{E}}|] = -\log\frac{Z_{\mathcal{E}}}{Z_{\mathcal{I}}}$$

$$Z_{\mathcal{E}} = \int \mathcal{D}\phi\mathcal{D}\tilde{\phi}\, e^{-\mathcal{S}^{\mathcal{E}}_{\mathrm{tot}}[\phi,\tilde{\phi}]}$$

$$\mathcal{S}^{\mathcal{E}}_{\mathrm{tot}}[\phi,\tilde{\phi}] \equiv \mathcal{S}_0[\phi] + \mathcal{S}_0[\tilde{\phi}] + \mathcal{S}_{\mathcal{E}}[\phi,\tilde{\phi}]$$



$\mathcal{S}_0$: bulk CFT of initial critical state $|\Psi_0\rangle$

$\mathcal{S}_{\mathcal{E}}$: boundary interaction induced by CPTP map $\mathcal{E}$

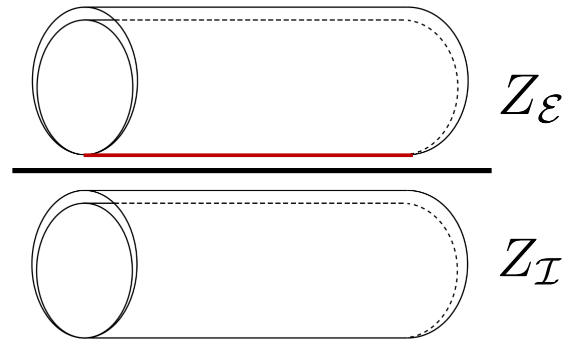
$Z_{\mathcal{I}}$: partition function without boundary interaction

see also: Lee, Jian, Xu, PRX Quantum 4, 030317 (2023); Bao, Fan, Vishwanath, Altman, arXiv:2301.05687 (2023)

Effective field theory in a doubled Hilbert space

Path-integral representation of S_{SE}:

$S_{SE} = -\log$



Each partition function can be evaluated by boundary CFT:

$$\log Z_\xi = b_\xi L + \log g_\xi + \mathrm{o}(1), \quad \xi \in \{\mathcal{I}, \mathcal{E}\}$$

g function ("ground-state degeneracy")

Affleck and Ludwig, PRL 67, 161 (1991)

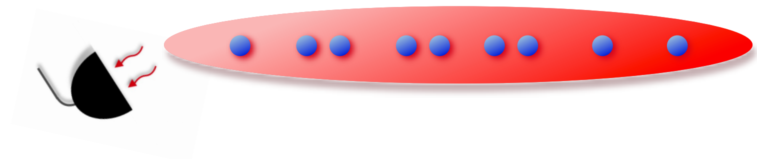
$$S_{SE} = s_1 L - s_0 + \mathrm{o}(1) \rightarrow \; e^{s_0} = \frac{g_{\mathcal{E}}}{g_{\mathcal{I}}}$$

Universality of S_{SE} ⇔ Universality of g function

see also: Stephan, Furukawa, Misguich, Pasquier, PRB 80, 184421 (2009); Zou, Sang, Hsieh, PRL 130, 250403 (2023)

Application to Tomonaga-Luttinger liquid under local measurement

Consider TLL under local density measurement



Effective field theory of unconditioned density matrix $\rho_{\mathcal{E}}$:

$$\mathcal{S}_{\rm tot}[\phi_+,\phi_-] = \mathcal{S}_0[\phi_+] + \mathcal{S}_0[\phi_-] + \mathcal{S}_{\mathcal{E}}[\phi_+,\phi_-]$$

$$\phi_+ = 2\left(\phi + \tilde{\phi}\right)$$

$$\phi_- = 2\left(\phi - \tilde{\phi}\right)$$

bulk action (c = 1 CFT): $$\mathcal{S}_0[\phi_\pm] = \int dx d\tau \frac{1}{16\pi K}\left[(\partial_x\phi_\pm)^2 + (\partial_\tau\phi_\pm)^2\right]$$

boundary interaction: $$\mathcal{S}_{\mathcal{E}} \simeq \int dx d\tau\, \delta(\tau)\left[\boxed{\frac{\gamma}{\Lambda_0}(\partial_x\phi_-)^2} - u_-\Lambda_0 \cos(\phi_-)\right]$$

$\gamma, u_- \propto \mu$ at UV scale Λ_0: UV cutoff

Careful treatment necessary for γ :

it can be relevant in nonperturbative regimes despite being perturbatively irrelevant (i.e., it is dangerously irrelevant)

cf. Masuki, Sudo, Oshikawa, YA., PRL 129, 087001 (2022);
Daviet and Dupuis, PRB 108, 184514 (2023)

Call for nonperturbative RG analysis

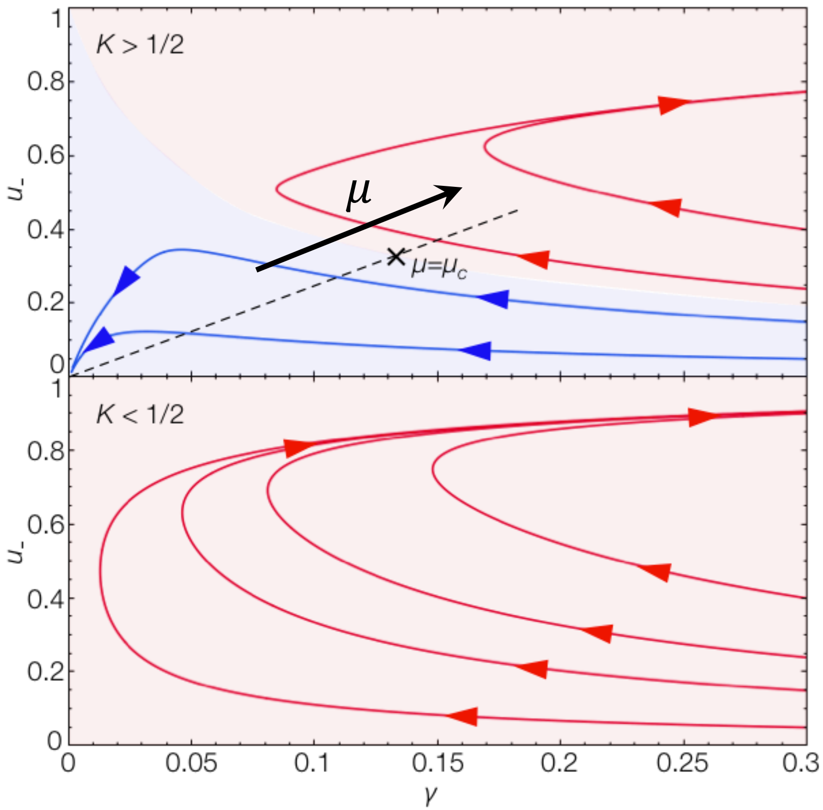
see also: Garratt, Weinstein, Altman, PRX 13, 021026 (2023) for conditioned evolution of TLL under local meas.

Tomonaga-Luttinger liquid under local measurement

$$\mathcal{S}_{\mathcal{E}} \simeq \int dx d\tau\, \delta(\tau)\left[\frac{\gamma}{\Lambda_0}\left(\partial_x\phi_-\right)^2 - u_-\Lambda_0 \cos\left(\phi_-\right)\right]$$

$\gamma, u_- \propto \mu$ at UV scale

Functional RG results



$K > 1/2$

$\mu > \mu_c$: u_- diverges in IR limit $\rightarrow$ ϕ_- obeys Dirichlet b.c.

$\mu < \mu_c$: u_- is irrelevant $\rightarrow$ ϕ_- obeys Neumann b.c.

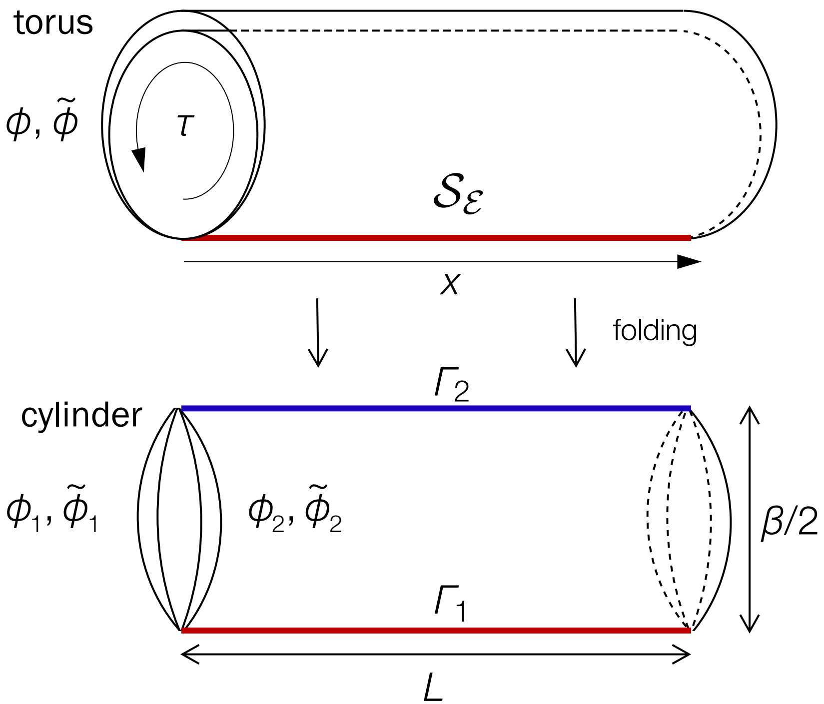
Entanglement transition induced by environment

$K < 1/2$

$u_\pm$ diverge at any $\mu > 0 \rightarrow \phi_\pm$ obey Dirichlet b.c.'s

Arbitrarily weak coupling can affect entanglement in IR limit

Boundary CFT (BCFT) calculation of the g function



Folding → 4-component theory on the cylinder.

$$Z_{\Gamma_1\Gamma_2} = \langle \Gamma_1 | e^{-\frac{\beta}{2}\hat{H}_{\mathrm{CFT}}} | \Gamma_2 \rangle \overset{\beta \gg L}{\simeq} \underline{g_{\Gamma_1} g_{\Gamma_2}} e^{\frac{\pi\beta}{3L}}$$

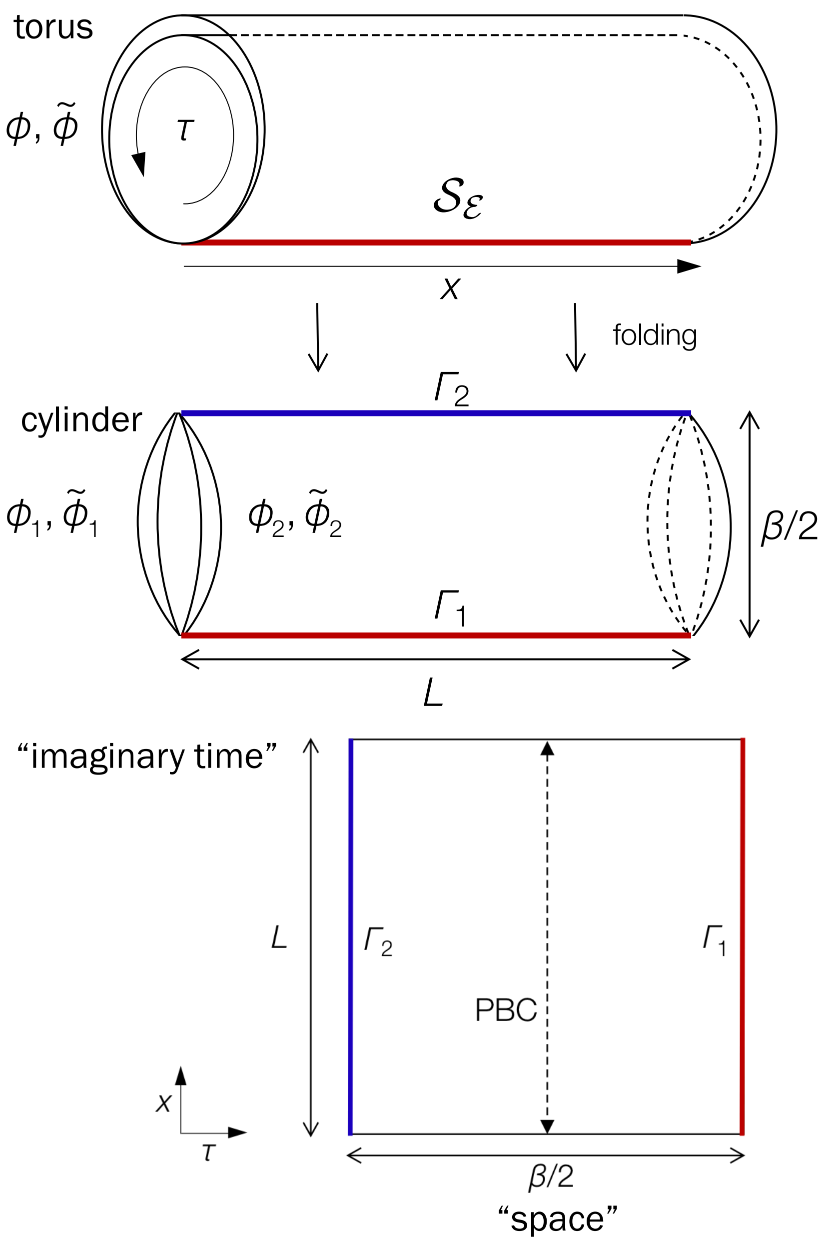
Construct a conformal boundary state $|\Gamma_i\rangle$ that satisfies

- conformal invariance
- invariance under space-time interchange (Cardy condition)
- b.c.'s identified by the RG analysis

→ g function: $\underline{g_{\Gamma_i}} = \langle \Gamma_i | \mathrm{GS} \rangle$

see also: Oshikawa, Chamon, Affleck J. Stac. Mech. P02008 (2006); Furukawa and Kim, PRB 83, 085112 (2011)

Boundary CFT (BCFT) calculation of the g function



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g function as "Ground-state degeneracy":

After interchanging space and time:

$$Z_{\Gamma_1\Gamma_2} = \mathrm{tr}\, e^{-L \hat H_{\mathrm{CFT}}^{\Gamma_1\Gamma_2}}$$

"infinite-size" limit $\beta \gg L$

$$\simeq \underline{g_{\Gamma_1} g_{\Gamma_2}} e^{\frac{\pi\beta}{3L}}$$

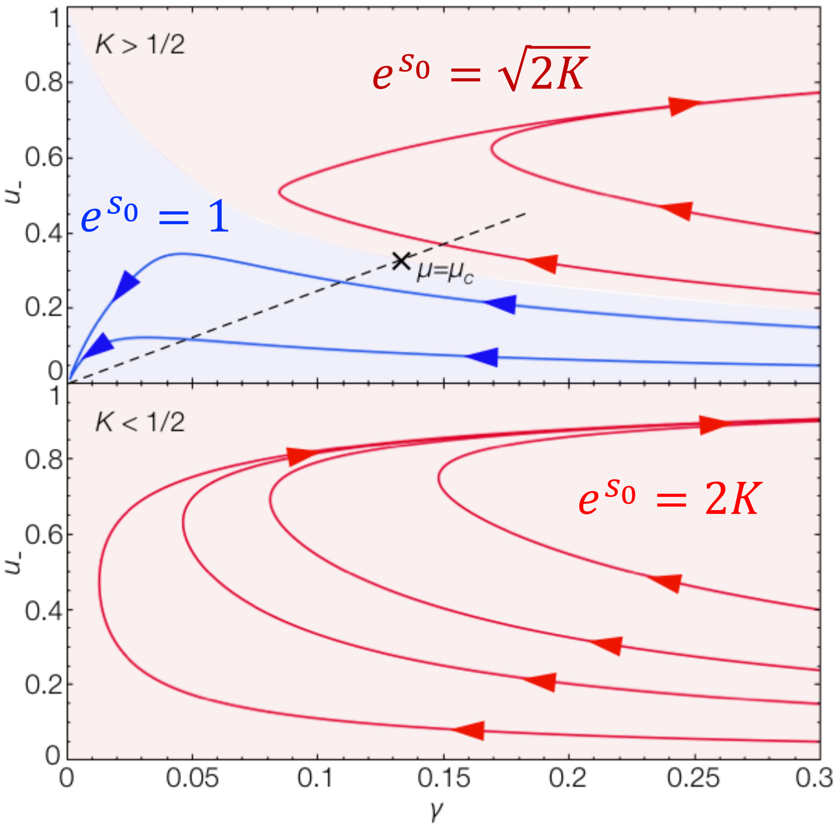
Constant thermal entropy independent of "temperature" $1/L$

g_Γ = "noninteger ground-state degeneracy" of $\hat H_{\mathrm{CFT}}^{\Gamma_1\Gamma_2}$

Affleck and Ludwig, PRL 67, 161 (1991)

Boundary CFT (BCFT) calculation of the g function

RG phase diagram



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BCFT results of TLL under density measurement:

$$e^{s_0} = \frac{Z_{\mathcal{E}}}{Z_{\mathcal{I}}} \approx \frac{g_{\mathcal{E}}}{g_{\mathcal{I}}} = \begin{cases} \sqrt{2K} & \mu > \mu_c,\ K > 1/2 \\ 1 & \mu < \mu_c,\ K > 1/2 \\ 2K & \forall \mu > 0,\ K < 1/2 \end{cases}$$

technical remarks:

- $|\Gamma_i\rangle$: superposition of Ishibashi states
- Mixed Dirichlet-Neumann boundary conditions
- $g_{\mathcal{E}}$ cannot be obtained from a mere product of g functions for single-component theory.
- Additional factor determined from the unit-cell volume of the compactification lattice.

YA, Furukawa, Oshikawa, arXiv:2311.16343 (2023)

Case study of the XXZ chain

Hamiltonian: $$\hat{H}_{\mathrm{XXZ}} = J\sum_{i=1}^{L}\left(\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y\hat{\sigma}_{i+1}^y + \Delta\hat{\sigma}_i^z\hat{\sigma}_{i+1}^z\right) \qquad J>0$$

$K\to\infty$ $\qquad K=1/2$

Initial state ρ_S : the gapless GS of H_{XXZ} with $-1<\Delta\le 1$

Effective Hamiltonian (TLL): $$\hat{H}_{\mathrm{eff}} = \int dx \frac{\hbar v}{2\pi}\left[\frac{1}{K}\left(\partial_x\hat{\phi}\right)^2 + K\left(\partial_x\hat{\theta}\right)^2\right],$$

$$\hat{\sigma}_j^z \simeq \frac{2a}{\pi}\partial_x\hat{\phi} + c_1(-1)^j\cos(2\hat{\phi}),$$

$$\hat{\sigma}_j^+ \simeq e^{i\hat{\theta}}\left[c_2(-1)^j + c_3\cos(2\hat{\phi})\right],$$

*compactification conditions: $\phi\sim\phi+\pi n,\ \ \theta\sim\theta+2\pi m,\ \ n,m\in\mathbb{Z}.$

Decoherence along z axis (= unconditioned evolution under imperfect projection measurements):

$$\hat{\rho}_{\mathcal{E}} = \prod_j\sum_m \hat{K}_{m,j}\hat{\rho}_S\hat{K}_{m,j}^\dagger \quad \text{with} \quad \hat{K}_{0,i} = \cos\zeta\,\hat{I},\ \ \hat{K}_{\pm,i} = \sin\zeta\,\frac{1\pm\hat{\sigma}_i^z}{2}$$

$$\Leftrightarrow \quad |\rho_{\mathcal{E}}) = \exp\left\{-\mu\left[\sum_j(1-\hat{\sigma}_j^z\otimes\hat{\sigma}_j^z)\right]\right\}|\mathrm{GS}\rangle\otimes|\mathrm{GS}\rangle$$

~ density measurement of TLL

$$\Leftrightarrow \quad \rho_{\mathcal{E}} = e^{\mathcal{L}t}\rho_S \qquad \mathcal{L}(\hat{\rho}) = -\frac{1}{2}\sum_j(\hat{L}_j^\dagger\hat{L}_j\hat{\rho} + \hat{\rho}\hat{L}_j^\dagger\hat{L}_j - 2\hat{L}_j\hat{\rho}\hat{L}_j^\dagger) \qquad L_j = \sqrt{\gamma}\sigma_j^z$$

Coupling strength : $\mu = -\ln\cos\zeta = \gamma t$

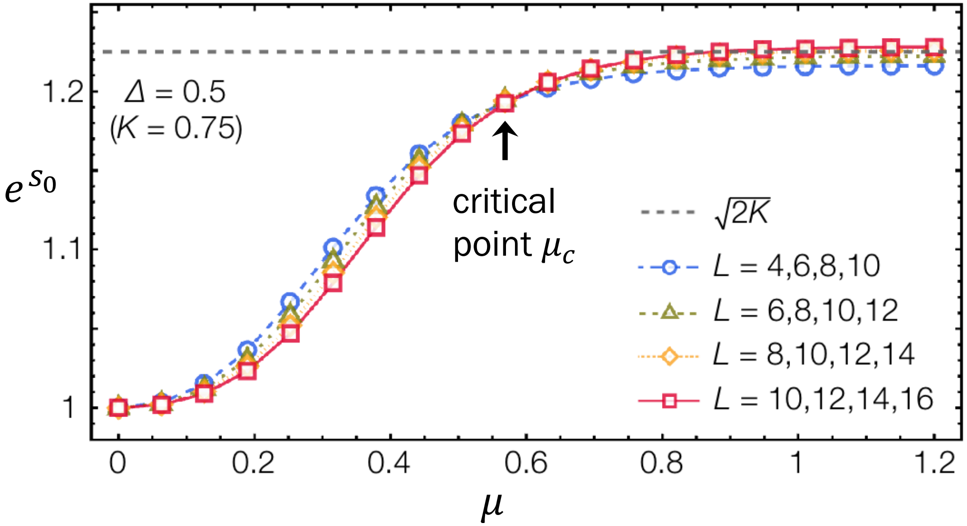
Strong coupling limit : $\mu\to\infty \leftrightarrow \zeta\to\frac{\pi}{2}$ (projection meas.) $\leftrightarrow t\to\infty$ (long-time limit in the Lindblad evolution)

Numerical check in the XXZ chain

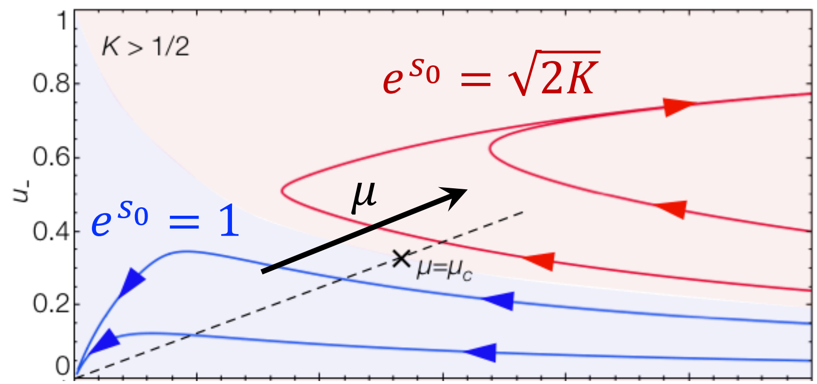
Exact diagonalization of $\hat{H}_{\mathrm{XXZ}} = J\sum_{i=1}^{L}\left(\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y\hat{\sigma}_{i+1}^y + \Delta\hat{\sigma}_i^z\hat{\sigma}_{i+1}^z\right)$ $\quad -1 < \Delta \leq 1$ ($K \to \infty$, $K = \frac{1}{2}$)

→ directly calculate $|\rho_{\mathcal{E}}) = \exp\left\{-\mu\left[\sum_j (1-\hat{\sigma}_j^z \otimes \hat{\sigma}_j^z)\right]\right\}|\mathrm{GS}\rangle\otimes|\mathrm{GS}\rangle$ and $S_{SE} = -\ln(\rho_{\mathcal{E}}|\rho_{\mathcal{E}})$

→ determine s_0 numerically by fitting S_{SE} to $S_{SE} = s_1 L - s_0 + \frac{s_{-1}}{L}$



cf. RG + BCFT analysis:



✓ transition at critical point $\mu = \mu_c$

✓ convergence to $\sqrt{2K}$ at $\mu > \mu_c$

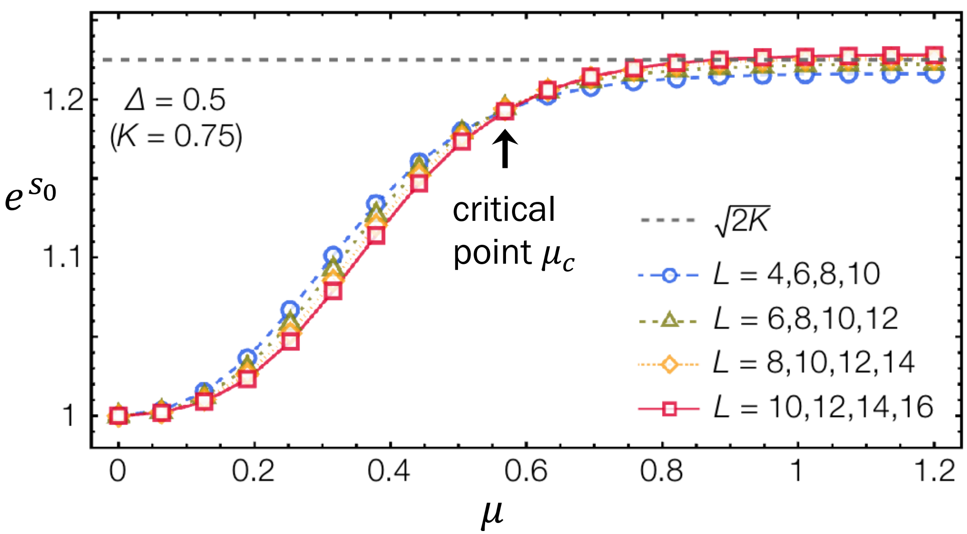
Consistent with RG and BCFT results

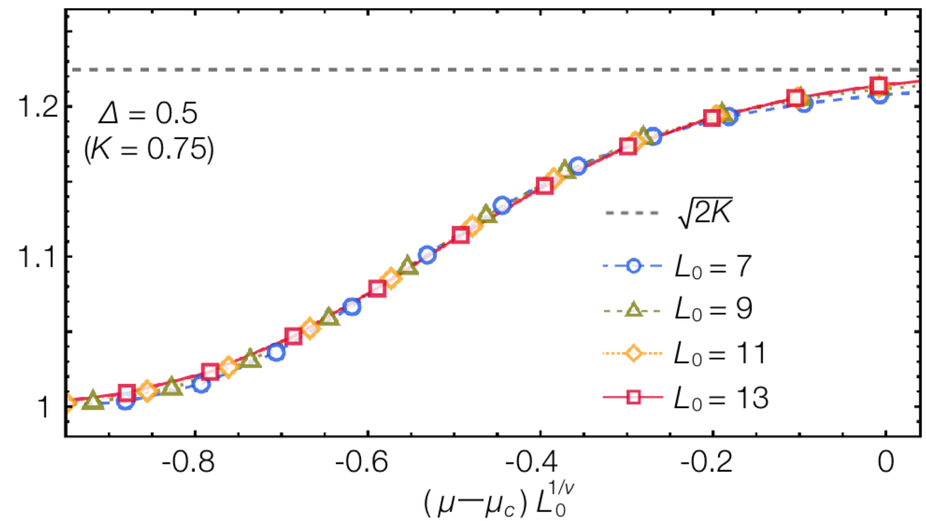
Numerical check in the XXZ chain

Exact diagonalization of $\hat{H}_{\mathrm{XXZ}} = J\sum_{i=1}^{L}\left(\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y\hat{\sigma}_{i+1}^y + \Delta\hat{\sigma}_i^z\hat{\sigma}_{i+1}^z\right)$ $-1 < \Delta \leq 1$ ($K \to \infty$ … $K = \frac{1}{2}$)

→ directly calculate $|\rho_{\mathcal{E}}) = \exp\left\{-\mu\left[\sum_j (1 - \hat{\sigma}_j^z \otimes \hat{\sigma}_j^z)\right]\right\}|\mathrm{GS}\rangle\otimes|\mathrm{GS}\rangle$ and $S_{SE} = -\ln(\rho_{\mathcal{E}}|\rho_{\mathcal{E}})$

→ determine s_0 numerically by fitting S_{SE} to $S_{SE} = s_1 L - s_0 + \frac{s_{-1}}{L}$





✓ transition at critical point $\mu = \mu_c$

✓ convergence to $\sqrt{2K}$ at $\mu > \mu_c$

✓ data collapse with a universal form:

$$\frac{g_{\mathcal{E}}}{g_{\mathcal{I}}} = f\left((\mu - \mu_c)\, L_0^{1/\nu}\right) \qquad \nu \sim 6.0$$

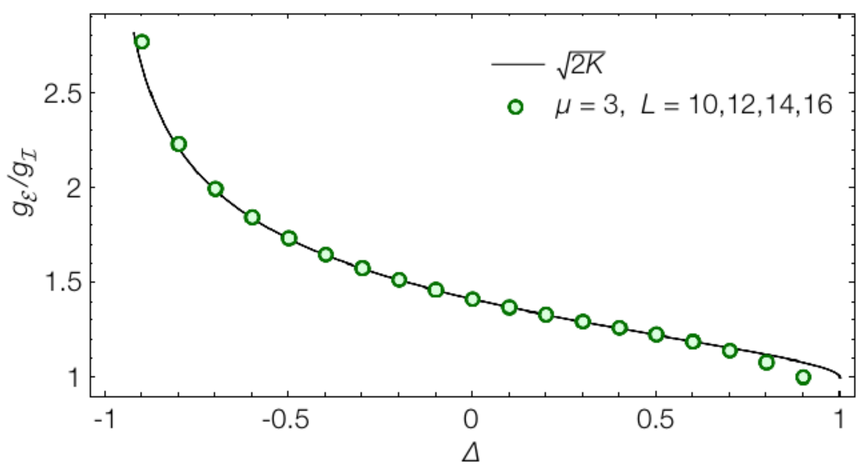
Consistent with RG and BCFT results

Numerical check in the XXZ chain

Exact diagonalization of $\hat{H}_{\mathrm{XXZ}} = J \sum_{i=1}^{L} \left(\hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + \Delta \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z \right)$ $\quad -1 < \Delta \leq 1$

$K \to \infty \qquad K = \frac{1}{2}$

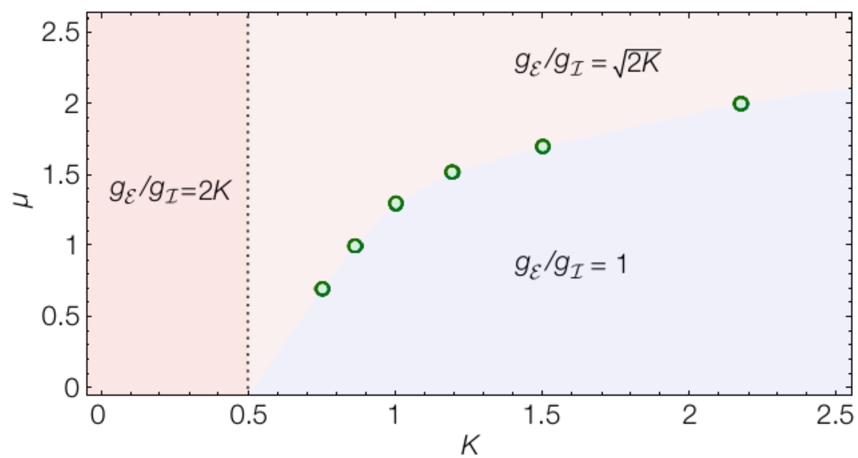
Converged values of e^{s_0} with a varying Δ



- ✓ consistent with Bethe-ansatz result:

$$K = \frac{\pi}{2\left(\pi - \cos^{-1}\Delta\right)}$$

Numerical phase diagram



- ✓ qualitatively consistent with fRG analysis

Numerical results agree with the field-theoretical analysis despite small system sizes.

Possible experimental test in ultracold gases

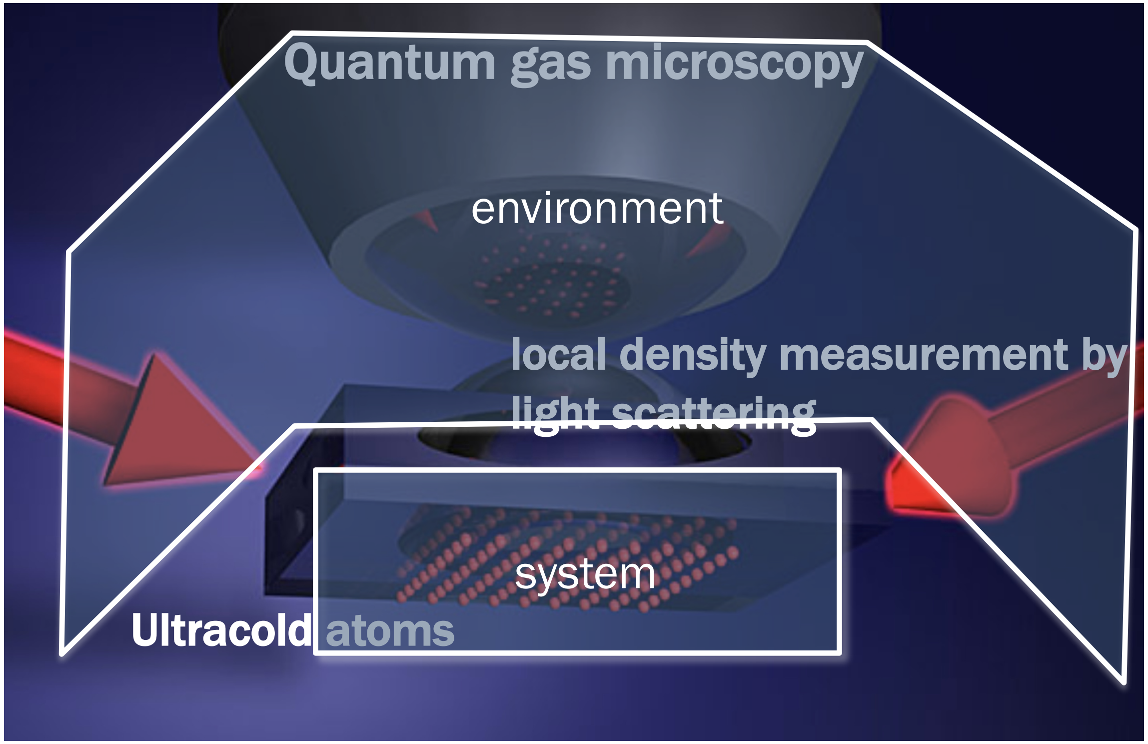
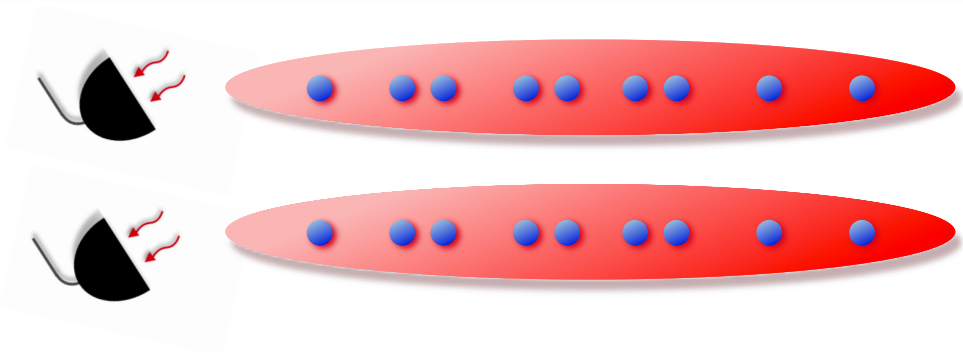


image: Greiner group at Harvard

Possible experimental test in ultracold gases



1) Prepare the two identical copies of a 1D critical Bose gas described by the TLL.

2) Perform a weak density measurement by light scattering while discarding the outcomes.

*Measurement strength μ can be controlled by changing exposure time or intensity of probe light.

3) Perform a beam-splitter operation between the two copies and measure the site-resolved occupation number $\{n_{j,\alpha}\}$ in each copy $\alpha \in \{1,2\}$.

cf. Daley et al., PRL 109, 020505 (2012)

4) Repeat 1) - 3), and evaluate the expectation value of swap operator: $S_{SE} = \mathrm{E}[(-1)^{\sum_j n_{j,2}}]$

*no postselection is required. small size L ~15 would be enough according to our numerical results.

*each step has already been realized in experiments:
e.g., Islam et al., Nature 528, 77 (2015); Lueschen et al., PRX 7, 011034 (2017) …

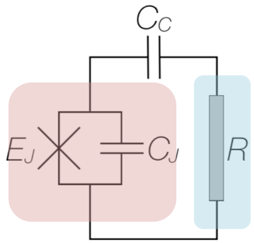
Dissipative QPT in Josephson junctions

Masuki, Sudo, Oshikawa, YA,	PRL 129, 087001 (2022)
Yokota, Masuki, YA,	PRA 107, 043709 (2023)

Dissipative quantum phase transition in Josephson junctions

Resistively shunted Josephson junction (RSJ):

Artificial atom + Resistance



Prototypical model to study quantum dissipation [Caldeira & Leggett, PRL 46, 211 (1981)].

RSJ Hamiltonian

$$\hat{H} = E_C \left(\hat{N} - \hat{n}_r\right)^2 - E_J \cos(\varphi) + \sum_{0<k\leq K} \hbar\omega_k \hat{a}_k^\dagger \hat{a}_k,$$

JJ + interaction — Environment

$$\hat{n}_r = \frac{\sqrt{\alpha}}{2\pi} \sum_{0<k\leq K} \sqrt{\frac{2\pi}{kL}} \left(\hat{a}_k^\dagger + \hat{a}_k\right),$$

$$\hat{N} = -i\frac{\partial}{\partial\varphi}, \alpha = \frac{R_Q}{R} = \frac{h}{4e^2 R}, E_C = \frac{2e^2}{C_J},$$

$$\omega_k = kv,\ k = \frac{\pi}{L}, \frac{2\pi}{L}, \ldots, K.$$

$\alpha = R_Q/R$: dissipation strength

$W = vK$: frequency cutoff in environment

wideband condition ($E_C, E_J \ll \hbar W$)

Controversy regarding quantum phase transition (QPT):
Perturbative RG suggests QPT at α=1

Schmid, PRL 51, 1506 (1983)
Fisher & Zwerger, PRB 32, 6190 (1985)
Kane & Fisher, PRB 46, 15233 (1992)
Furusaki & Nagaosa, PRB 47, 4631 (1993)
...

yet, no concrete experimental evidence...

see e.g., Murani et al., PRX 10, 021003 (2020)

Dissipative quantum phase transition in Josephson junctions

We reexamine the problem by nonperturbative analysis:

Imaginary-time boundary action (the same one as in TLL under density measurement):

$$S[\varphi] = \frac{1}{2}\int_{-W}^{W}\frac{dk}{2\pi}\left(\frac{|k|}{4\pi K} + \frac{\gamma k^2}{W}\right)|\varphi_k|^2 - uW\int_{-\infty}^{\infty} dx\, \cos\left(\varphi(x)\right),$$

$$K \leftrightarrow \frac{R}{2R_Q} \qquad \gamma \leftrightarrow \frac{W}{E_C} \qquad u \leftrightarrow \frac{E_J}{W}$$

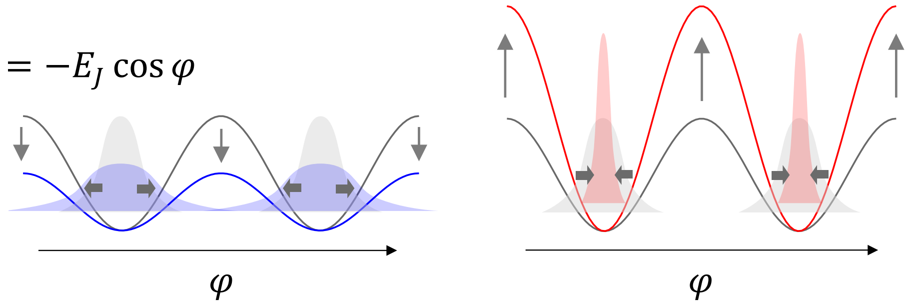
Previous (conventional) understanding:

- Scaling dimension implies that $\gamma = \frac{W}{E_C}$ is irrelevant.
- Assume the validity of $\gamma \to 0$ limit.
- Consider the simplified, boundary sine-Gordon model.

Dissipative quantum phase transition in Josephson junctions

Phase potential: E_J : irrelevant E_J : relevant

$V(\varphi) = -E_J \cos\varphi$



Phase : delocalized ⟺ localized

Insulator ⟺ Superconductor

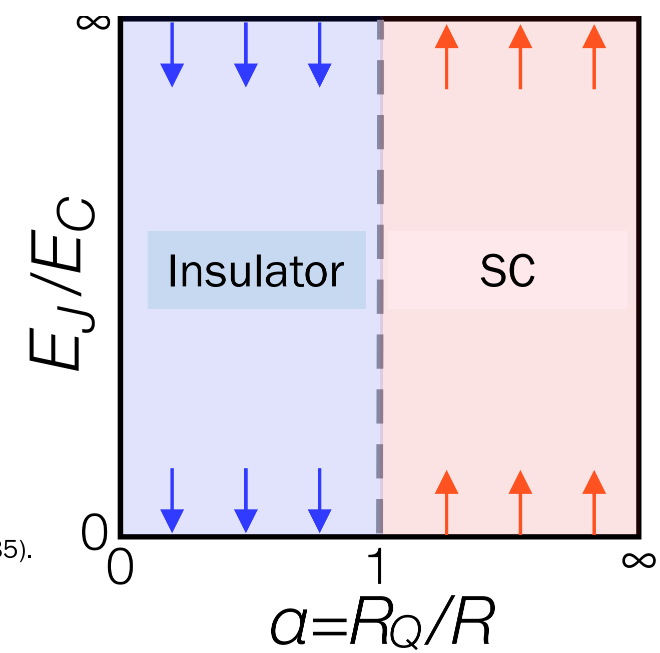
Previous (conventional) understanding:

- Scaling dimension implies that $\gamma = \frac{W}{E_C}$ is irrelevant.
- Assume the validity of $\gamma \to 0$ limit.
- Consider the simplified, boundary sine-Gordon model.

Perturbative RG + duality argument ⟶

$$\begin{cases} \alpha < 1 \to E_J : \text{irrelevant} \\ \alpha > 1 \to E_J : \text{relevant} \end{cases}$$

Fisher and Zwerger, Phys. Rev. B **32**, 6190 (1985).
Kane and Fisher, PRB 46, 15233 (1992).

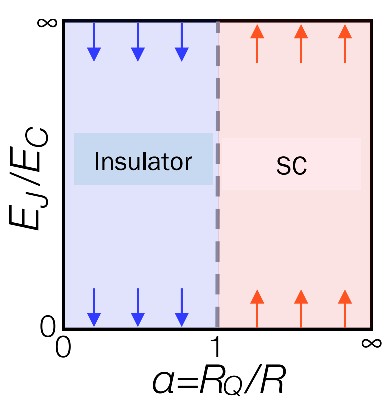


Dissipative quantum phase transition in Josephson junctions

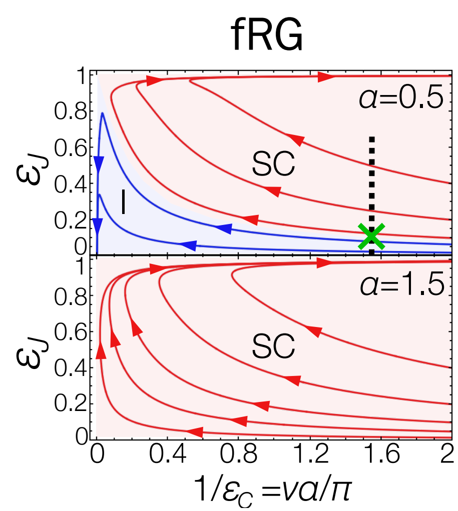
- ✓ Under wideband condition $E_C, E_J \ll \hbar W$,
  $\gamma = \frac{W}{E_C}$ is large at the initial stage of RG flow (i.e., at UV scale).
- ✓ We need a careful analysis of the original action keeping the γ term:

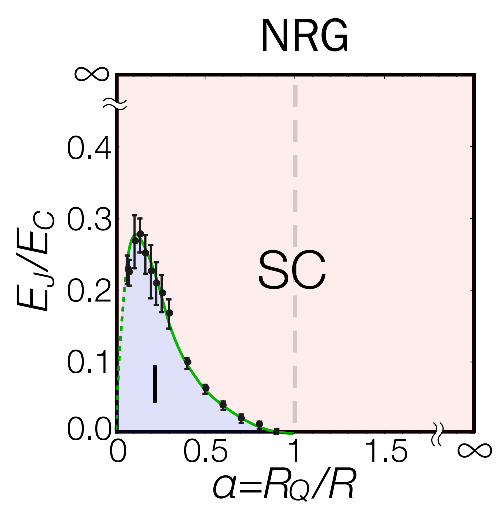
$$S[\varphi] = \frac{1}{2}\int_{-W}^{W} \frac{dk}{2\pi}\left(\frac{|k|}{4\pi K} + \boxed{\frac{\gamma k^2}{W}}\right)|\varphi_k|^2 - uW\int_{-\infty}^{\infty} dx\,\cos(\varphi(x)),$$

Conventional analysis
$\gamma = 0$



Our nonperturbative analysis (γ finite at UV scale)





Nonperturbative effect due to γ can qualitatively modify the phase diagram!

Masuki, Sudo, Oshikawa, YA., PRL 129, 087001 (2022)

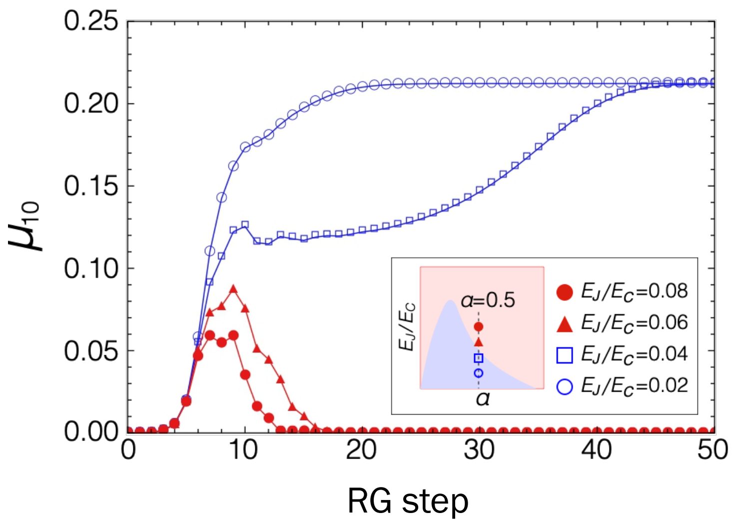
Physical origin: dangerously irrelevant RG flows

dc phase mobility (order parameter)： $\mu = \frac{\alpha}{2\pi} \lim_{\omega \to 0} \omega \langle \varphi\varphi \rangle_\omega$ A. Schmid, Phys. Rev. Lett. 51, 1506 (1983).

$\mu > 0 \;\to$ Insulator (phase delocalized)

$\mu = 0 \;\to$ Superconducting (phase localized)

NRG result

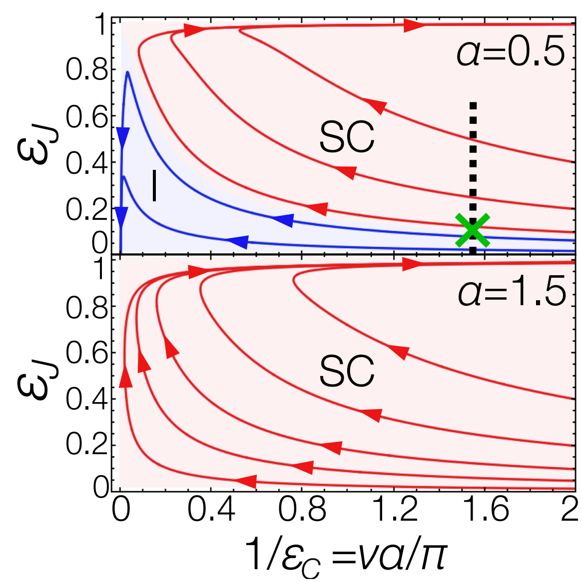


(1) Insulator phase is initially favored.

(2) { small $E_J/E_C \to$ flow to insulator fixed point
large $E_J/E_C \to$ nonmonotonic flow to SC fixed point

FRG result

$(\epsilon_{C(J)} = E_{C(J)}/W)$



*More detailed FRG analyses give the qualitatively same results:

Yokota, Masuki, and YA, PRA 107, 043709 (2023);
Daviet and Dupuis, PRB 108, 184514 (2023).

(1) At the initial RG flows, $d_l(E_J/E_C) < 0$ (insulator favored)

(2) { small $E_J/E_C \to$ flow to insulator fixed point
large $E_J/E_C \to$ nonmonotonic flow to SC fixed point

Both of NRG and FRG results indicate that γ is **dangerously irrelevant.**

Masuki, Sudo, Oshikawa, YA., PRL 129, 087001 (2022)

Experimental relevance to long-high impedance waveguides

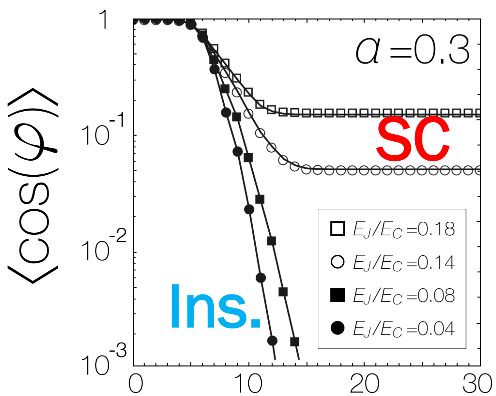
Phase coherence : $\langle \cos(\varphi) \rangle$

Admittance at low frequency: $Y(\omega) = \frac{\delta I(\omega)}{\delta V(\omega)} \propto \frac{\langle \cos(\varphi) \rangle}{\omega}$

Joyez, PRL 110, 217003 (2013)

$\langle \cos(\varphi) \rangle \neq 0 \rightarrow$ superconducting $\langle \cos(\varphi) \rangle = 0 \rightarrow$ insulating

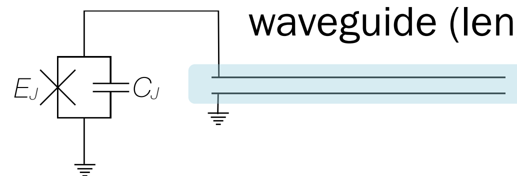
NRG flow of $\langle \cos(\varphi) \rangle$



RG steps : N
(energy scale ~ 2^{-N})

- To identify DQPT in NRG, we need many RG steps = deep IR regime.
- The lowest frequency $\omega_{\min}$ of waveguide or finite temperature determines IR cutoff frequency.

waveguide (length L, velocity v)

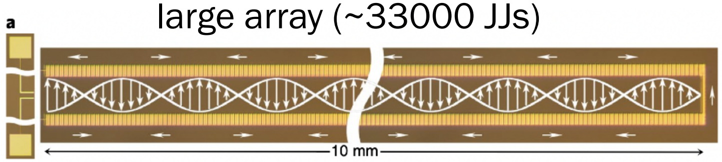


IR cutoff ~ v/L

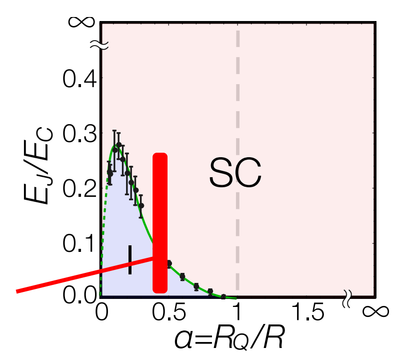
→ Signature of insulating phase ($\langle \cos(\varphi) \rangle \simeq 0$) can be observed at sufficiently low temperature & sufficiently long waveguide.

One possible example: long high-impedance superconducting waveguide

large array (~33000 JJs)



Kuzmin et al.,
PRL **126**, 197701 (2021).



Tendency to insulator phase, $\langle \cos(\varphi) \rangle \lesssim 0.01$, could be experimentally observed.
($L \sim 10$ mm, $E_C = 5.4$ GHz, $\omega_{\min}/2\pi = 63$ MHz, $W/2\pi = 20$ GHz)

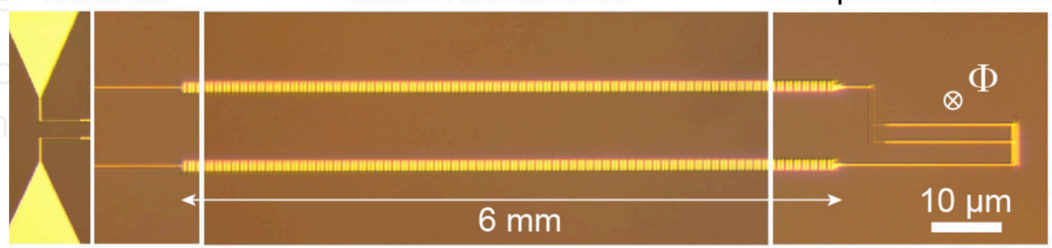
Experimental relevance to long-high impedance waveguides

Observation of the Schmid-Bulgadaev dissipative quantum phase transition

R. Kuzmin,[1,2] N. Mehta,[2] N. Grabon,[2] R. A. Mencia,[2,3] A. Burshtein,[4] M. Goldstein,[4] and V. E. Manucharyan[2,3]

[1]*Department of Physics, University of Wisconsin-Madison, Madison, WI 53706, USA.*[*]
[2]*Department of Physics, University of Maryland, College Park, MD 20742, USA.*
[3]*École Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland.*
[4]*Raymond and Beverly Sackler School of Physics and Astronomy, Tel Aviv University, Tel Aviv 6997801, Israel.*
(Dated: April 13, 2023)

arXiv:2304.05806



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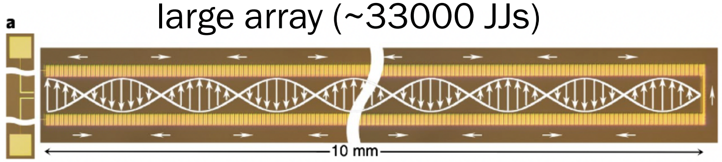
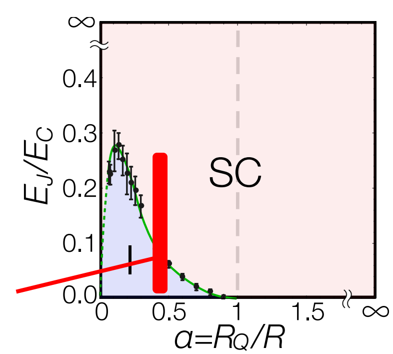


Kuzmin et al.,
PRL **126**, 197701 (2021).



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($L \sim 10$ mm, $E_C = 5.4$ GHz, $\omega_{\rm min}/2\pi = 63$ MHz, $W/2\pi = 20$ GHz)

(short summary)
Measurement-induced phase transition in free bosons

Yokomizo and YA, arXiv:2405.19768.

Previous studies on measurement-induced phase transition (MIPT)

Measurement-induced phase transition (MIPT) =

Transition in the size scaling of the entanglement entropy S_A, which occurs as the measurement strength is increased.

average over trajectories

$$S_A = \mathrm{E}[-\mathrm{tr}[\rho_A \ln \rho_A]]$$

reduced density matrix for each trajectory $\rho_A = \mathrm{Tr}_{\bar{A}}[|\psi_{\mathrm{traj}}\rangle\langle\psi_{\mathrm{traj}}|]$

Common scenario: volume-to-area law MIPT

Volume law	$S_A \propto L$	
Logarithmic scaling	$S_A \propto \ln L$	
Area law	$S_A \propto \mathrm{const.}$	measurement strength ↓

Examples:

- random unitary circuits + random projection measurements — Fisher et al., Ann. Rev. Cond. Matt. 14, 335 (2023)
- interacting many-body systems + local continuous measurements — Fuji and YA, PRB 102, 054302 (2020); Gopalakrishnan and Gullans, PRL 126, 170503 (2021); Turkeshi et al., PRB 103, 224210 ...

Previous studies on measurement-induced phase transition (MIPT)

Measurement-induced phase transition (MIPT) =

Transition in the size scaling of the entanglement entropy S_A, which occurs as the measurement strength is increased.

Situation in free-particle systems is somewhat exceptional.

Two proposed scenarios in free fermions:

(I) subvolume-to-area law MIPT

- quantum-jump trajectories under local continuous measurements
  Alberton et al., Phys. Rev. Lett. 126, 170602 (2021)
  Buchhold et al., Phys. Rev. X 11, 041004 (2021)
- diffusive quantum trajectories in long-ranged systems
  Minato et al., PRL 128, 010603 (2022)
  Mueller et al., PRL 128, 010605 (2022)

(II) absence of MIPT (i.e., always area law)

- Hamiltonian evolution + random projection measurements and its field-theoretical analysis
  Cao et al., SciPost Phys. 7, 024 (2019)
  Poboiko et al., Phys. Rex. X 13, 041046 (2023)
  Poboiko et al., Phys. Rex. Lett. 132, 110403 (2024)

What would happen in free bosons ?

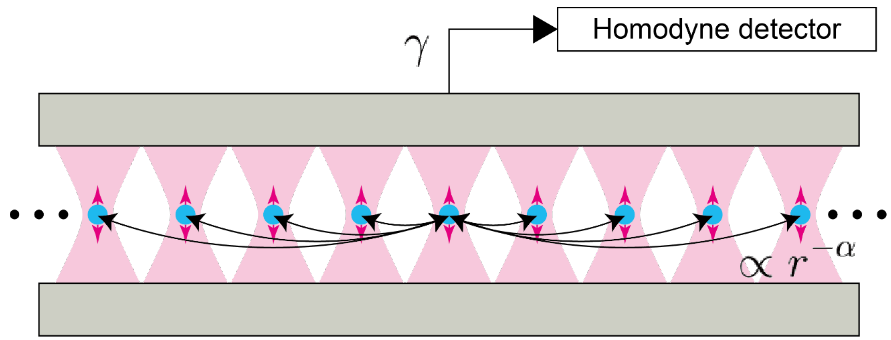
Measurement-induced phase transition (MIPT) in free-bosons

Array of particles in harmonic traps with long-range couplings:

$$\hat{H} = \sum_{j=1}^{L} \frac{\Omega}{2}\left(\hat{p}_j^2 + \hat{x}_j^2\right) + \sum_{j=1}^{L-1}\sum_{r=1}^{L-j} \frac{K}{2r^{\alpha}}\left(\hat{x}_j - \hat{x}_{j+k}\right)^2$$

$\alpha \to \infty$: short-range coupling

$\alpha \to 0$: all-to-all coupling



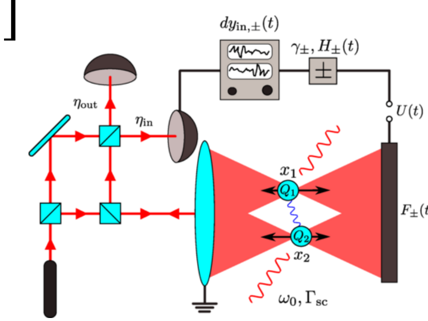
e.g., levitated nanoparticle array [optical binding force]
[cf. Rieser et al., Science 377, 987 (2022)]

Continuous position measurement:

$$E[dW_n] = 0, dW_m dW_n = \delta_{mn} dt$$

$$d|\psi\rangle = \left[-i\hat{H}dt - \frac{1}{2}\sum_n \left(\hat{O}_n - \langle \hat{O}_n \rangle\right)^2 dt + \sum_n \left(\hat{O}_n - \langle \hat{O}_n \rangle\right) dW_n \right] |\psi\rangle$$

$$\hat{O}_n = \begin{cases} \sqrt{\gamma}\,\hat{x}_n & \text{Local} \longrightarrow \text{No MIPT} \\ \sqrt{\dfrac{\gamma}{r^{\alpha}}}\left(\hat{x}_j \pm \hat{x}_{j+r}\right) & \text{Nonlocal} \longrightarrow \text{MIPT} \end{cases}$$

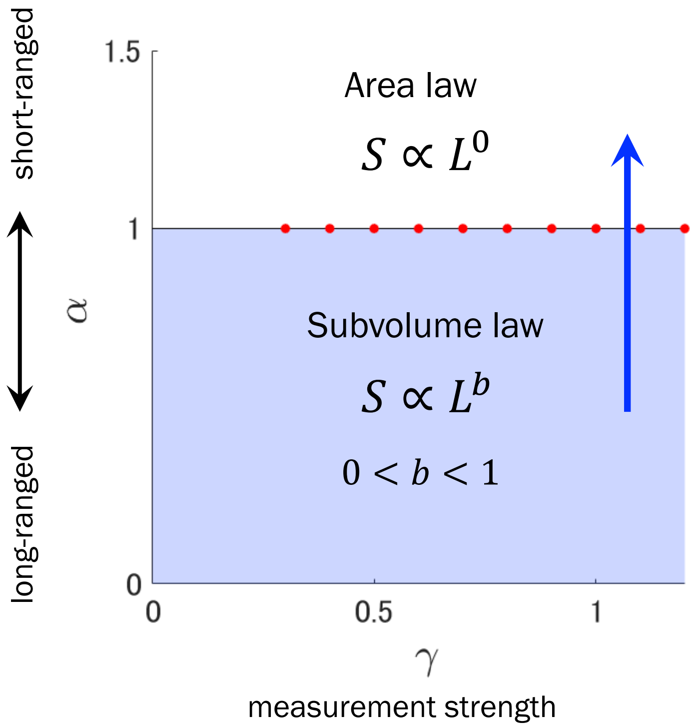


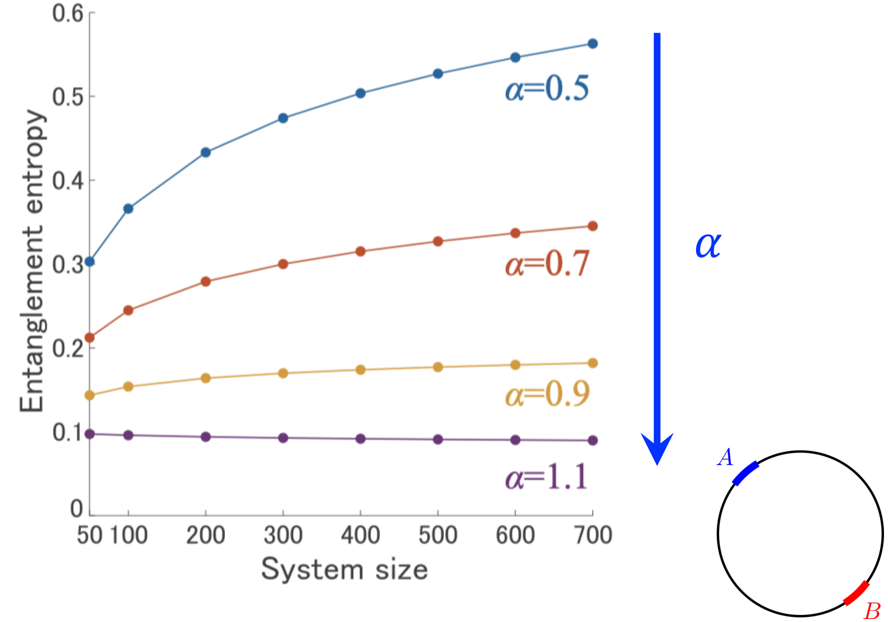
e.g., homodyne detection of scattered light
[cf. Rudolph et al., PRL 129, 193602 (2022)]

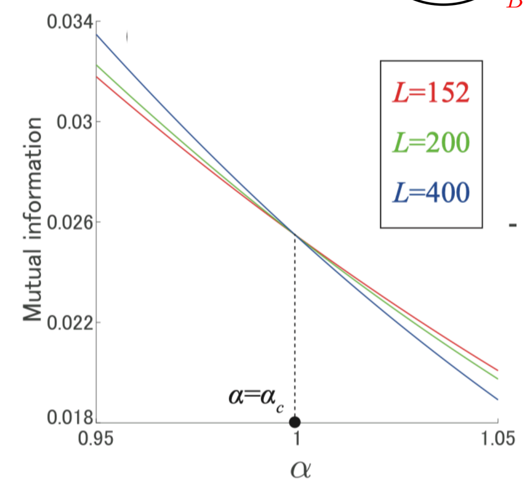
$|\psi\rangle$ remains Gaussian during the time evolution $\longrightarrow$
entanglement can be efficiently obtained from its covariance matrix

Measurement-induced phase transition (MIPT) in free-bosons

Local measurement: $\hat{\mathcal{O}}_n = \sqrt{\gamma}\hat{x}_n$







No MIPT
(transition as a function of the measurement strength γ)

*reproduces the previous result in the short-ranged system ($\alpha \to \infty$).
Minoguchi et al., SciPost 12, 9 (2022)

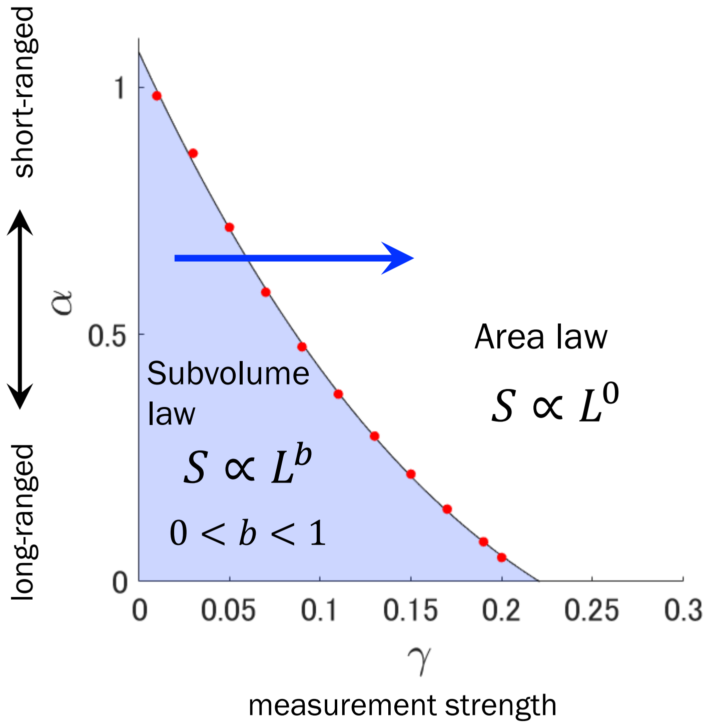
Transition at $\alpha_c = 1$ is induced by long-range couplings:

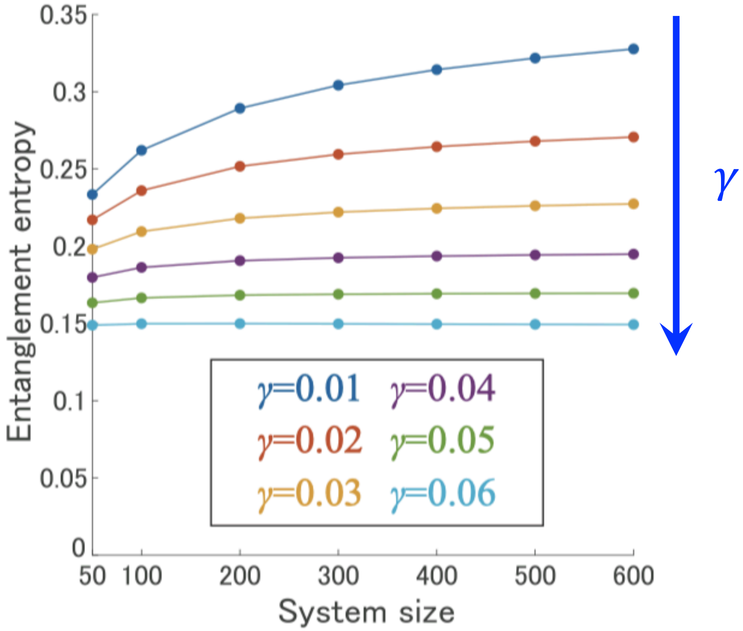
$$\dot{S} \propto \left\| \hat{H}_{\mathrm{AB}} \right\| \propto L^{1-\alpha}$$

Minato, Sugimoto, Kuwahara, Saito, PRL 128, 010603 (2022)

Measurement-induced phase transition (MIPT) in free-bosons

Nonocal measurement: $\hat{O}_n = \sqrt{\frac{\gamma}{r^\alpha}}(\hat{x}_j \pm \hat{x}_{j+r})$



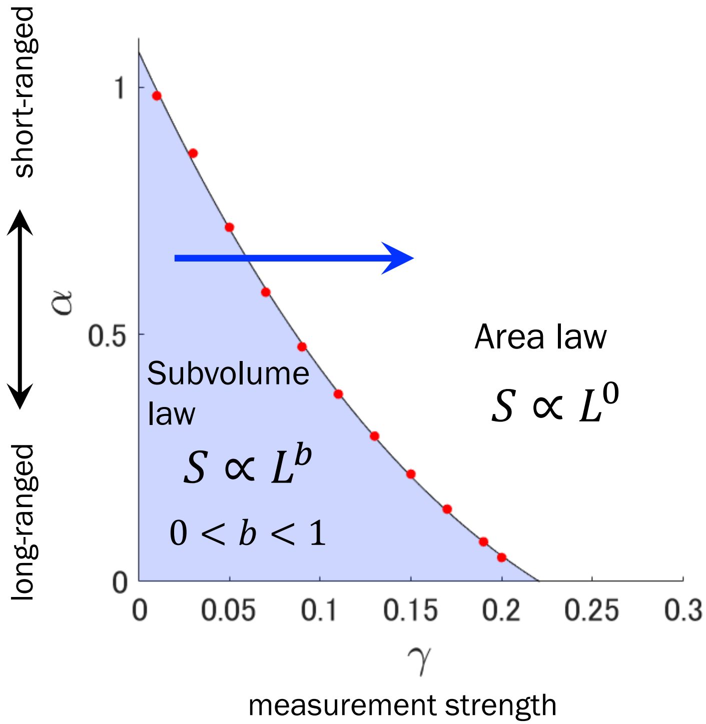


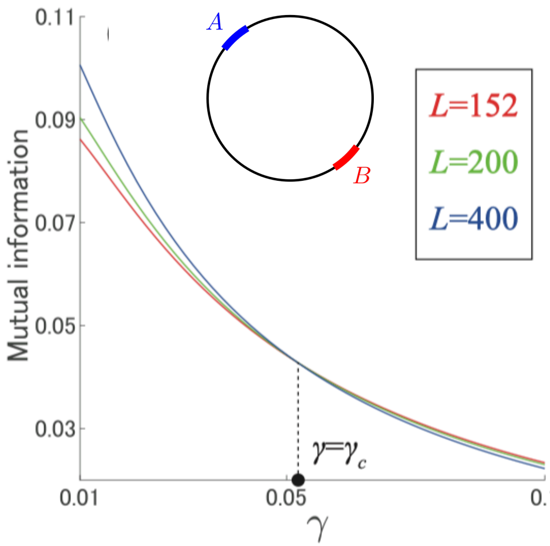
Presence of subvolume-to-area law MIPT
as the measurement strength γ is increased.

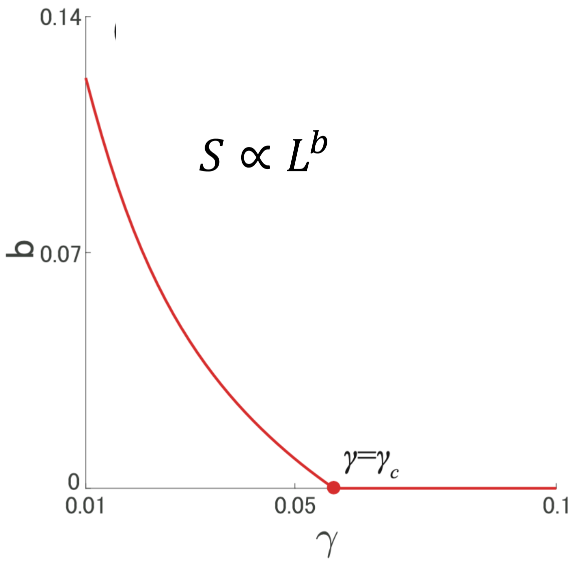
The measurement can now suppress the rapid entanglement growth due to the long-range couplings, and the competition leads to the MIPT.

Measurement-induced phase transition (MIPT) in free-bosons

Nonocal measurement: $\hat{O}_n = \sqrt{\frac{\gamma}{r^\alpha}}(\hat{x}_j \pm \hat{x}_{j+r})$





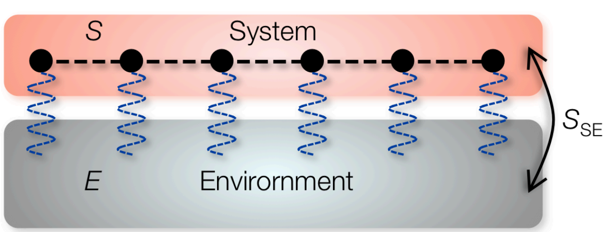


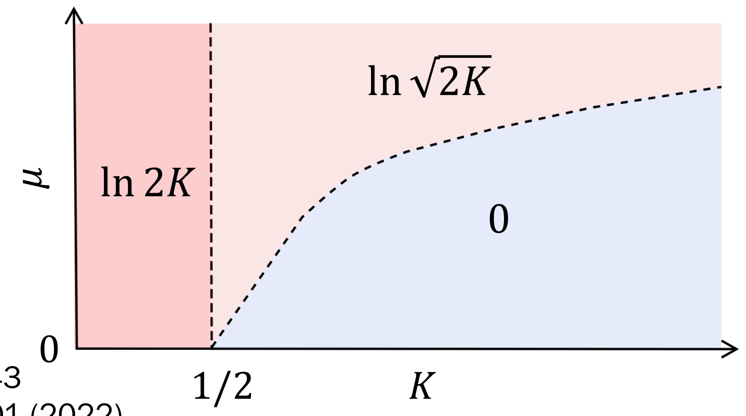
Presence of subvolume-to-area law MIPT as the measurement strength γ is increased.

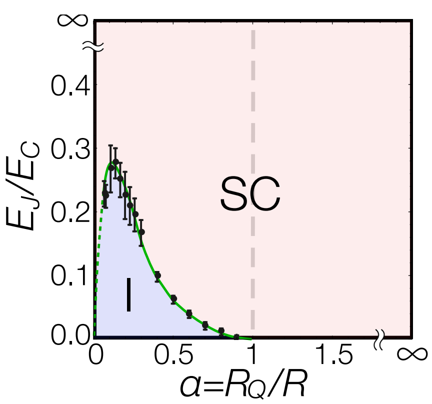
The measurement can now suppress the rapid entanglement growth due to the long-range couplings, and the competition leads to the MIPT.

Summary

- Entanglement between a many-body system and environment can exhibit phase transition, which can be detected by a size-independent universal contribution s_0.
- We have determined s_0 for TLL under local measurement by fRG + BCFT analysis and confirmed it numerically.
- Our analysis suggests that the dissipative QPT in Josephson junction and the entanglement transition should belong to the same universality class.
- MIPT in free bosons can occur when both particle couplings and measurements are long-ranged.







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