Protocol for detecting quantum anomalies in synthetic quantum systems

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Recent Developments and Challenges in Topological Phases @ YITP

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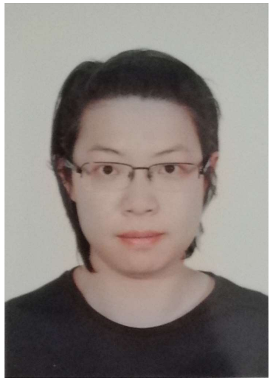
Collaborators



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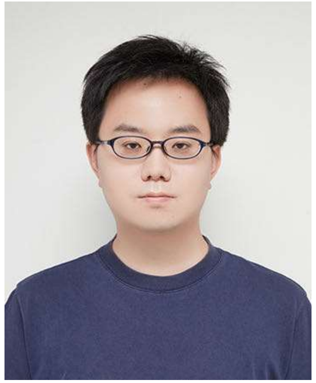

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Outline

- Introduction
  - Quantum anomaly in condensed matter physics
  - Quantum phases in synthetic quantum systems
  - Signature and protocol
- Detecting anomalies in open LSM system
  - Intuition
  - Spectral flow of transfer matrix
  - Numerics
- Detecting anomalies in correlated TI
  - Results for free fermion TI
  - Issues and results on correlated TI
- Summary and Outlook

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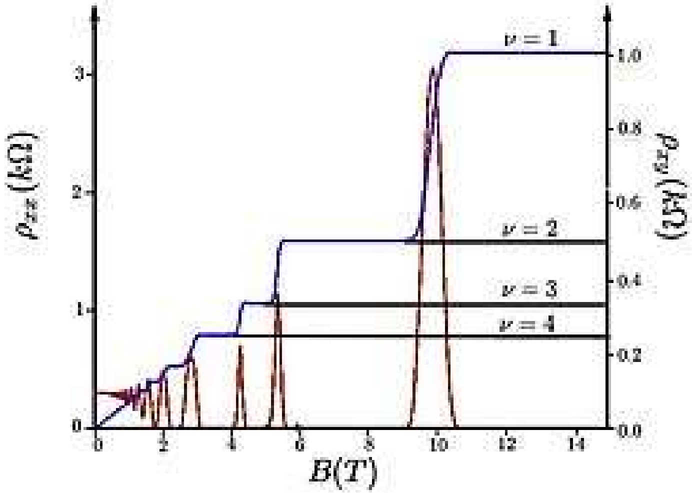
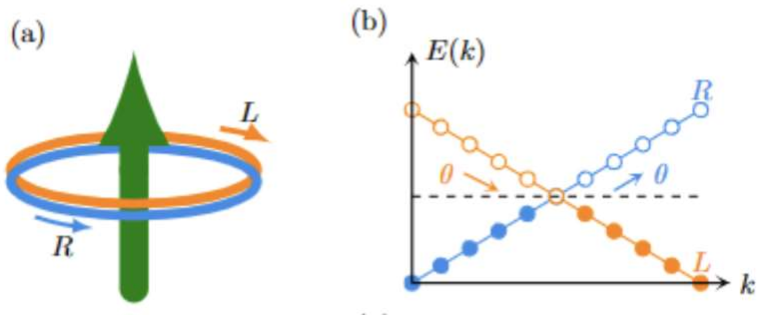
Quantum anomalies in CMP

- Chiral fermion @ edge of IQHE

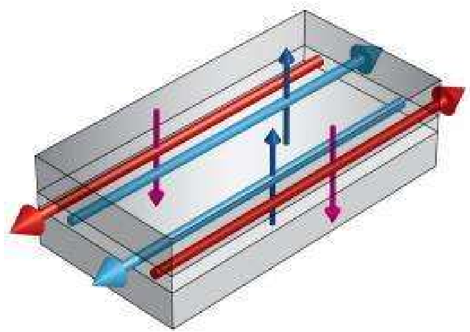


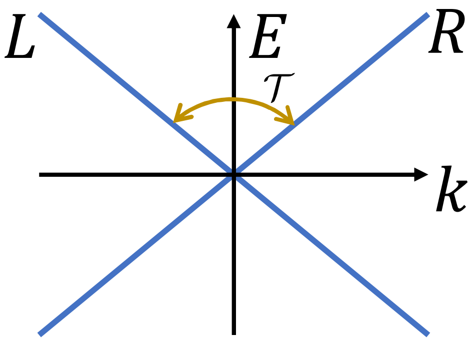


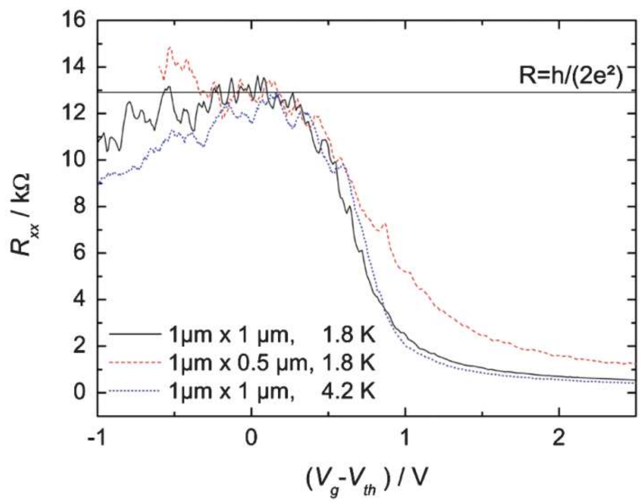
Laughlin's argument, spectral flow and quantum anomaly

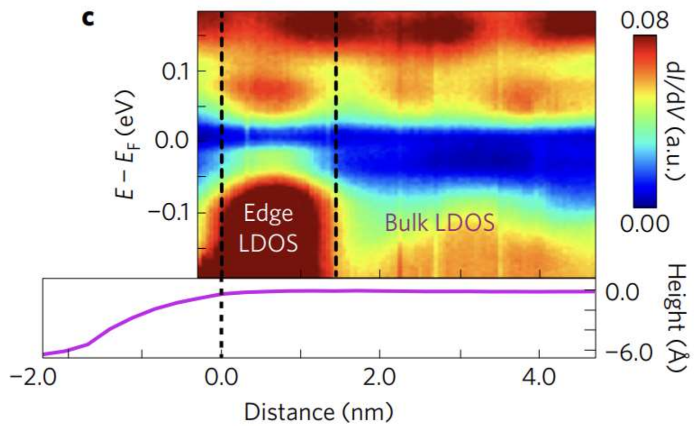
Quantum anomalies in CMP

- Helical fermion @ edge of QSHE (Kane, Mele, Zhang, Monlenkamp, …)









Detecting metalic edge states: quantized R_{xx} (Konig et al.) or STM (Tang et al.)

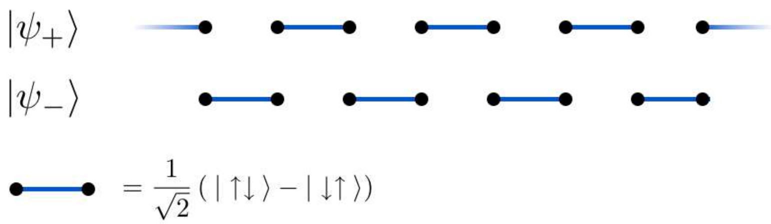
Generalization beyond free fermion: SPT phases, “anomalous” edge

Quantum anomalies in CMP

- Lieb-Schultz-Mattis theorem: Ground states of spin-$\frac{1}{2}$ chains with translational symmetry must either be gapless or spontaneously break the translation symmetry.
- Haldane conjecture: N.N. Heisenberg model, Spin-$\frac{1}{2}$ gapless v.s. Spin-1 gapped

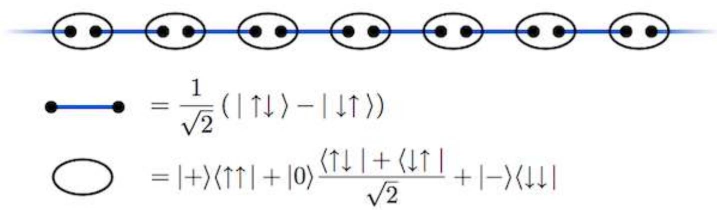
Majumdar–Ghosh model

$$\hat{H} = J\sum_{j=1}^{N} \vec{S}_j \cdot \vec{S}_{j+1} + \frac{J}{2}\sum_{j=1}^{N} \vec{S}_j \cdot \vec{S}_{j+2}$$



AKLT model

$$\hat{H} = \sum_{\langle ij\rangle} P^{(2)}_{\langle ij\rangle} \sim \sum_{j} \vec{S}_j \cdot \vec{S}_{j+1} + \frac{1}{3}(\vec{S}_j \cdot \vec{S}_{j+1})^2$$



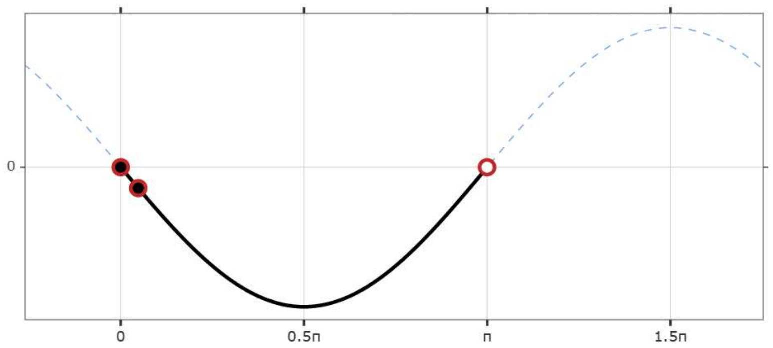
Still works if break symmetry to $U(1) \rtimes Z_2$ (Heisenberg → XXZ), more possible phases:

Unique gapped, Gapless, VBS, (Ising) AFM, Ising FM, ...

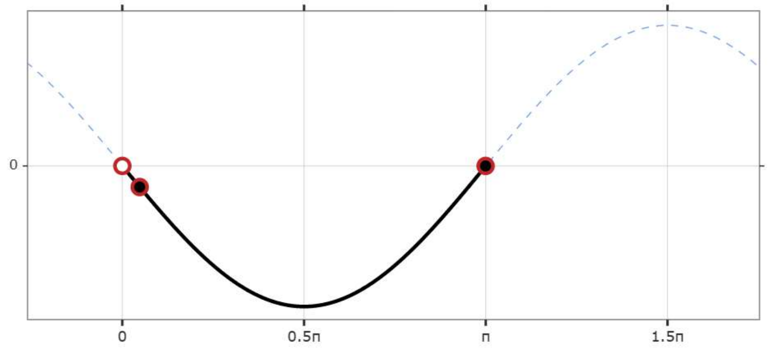
Higher dimensional generalization by Oshikawa & Hastings

LSM theorem and spectral flow

$$H = \sum_j \left(\frac{J}{2} S_j^+ S_{j+1}^- + \mathrm{h.c.} \right) \xrightarrow[n_f = \frac{1}{2} - S^z]{\text{Jordan-Wigner}} H = \sum_j \left(\frac{J}{2} f_j^\dagger f_{j+1} + \mathrm{h.c.} \right)$$



$$\xrightarrow[J(\theta) = J \cdot \exp\left(\mathrm{i}\frac{\theta}{L}\right)]{H \to H(\theta), \theta \in [0, 2\pi)}$$



Inserting 2π flux, spectra back? spectral flow!

2π flux of $U_z(1)$ carries π momentum

LSM theorem and spectral flow

$$H(\theta)=\frac{J}{2}\left[\left(\sum_{j=1}^{L-1}S_j^+S_{j+1}^- + e^{i\theta}S_L^+S_1^- + \right) + \mathrm{h.c.}\right] + \cdots$$

$$T_x(\theta) \equiv e^{i\theta S_1^z}T_x: \begin{cases} S_j \to S_{j+1} \ \ (j<L) \\ S_L \to e^{i\theta}S_1 \end{cases}$$

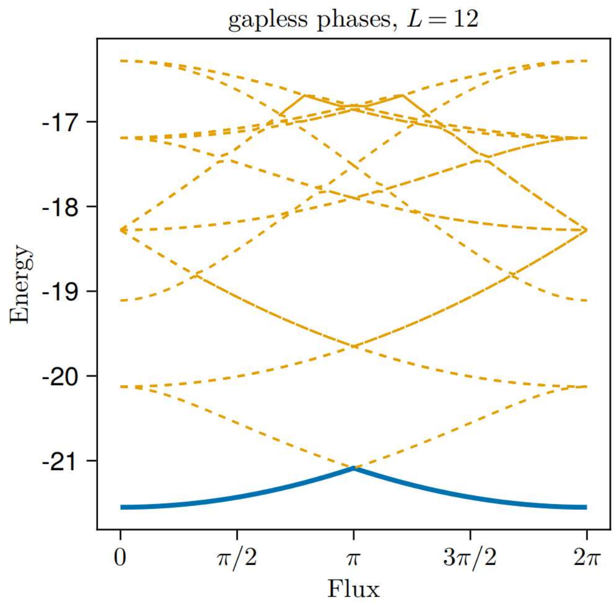
$$[T_x(\theta)]^L = e^{i\theta S_{\mathrm{tot}}^z} \ (= 1 \text{ for singlet})$$

Therefore, many-body momentum $k=\frac{2\pi n}{L}$.

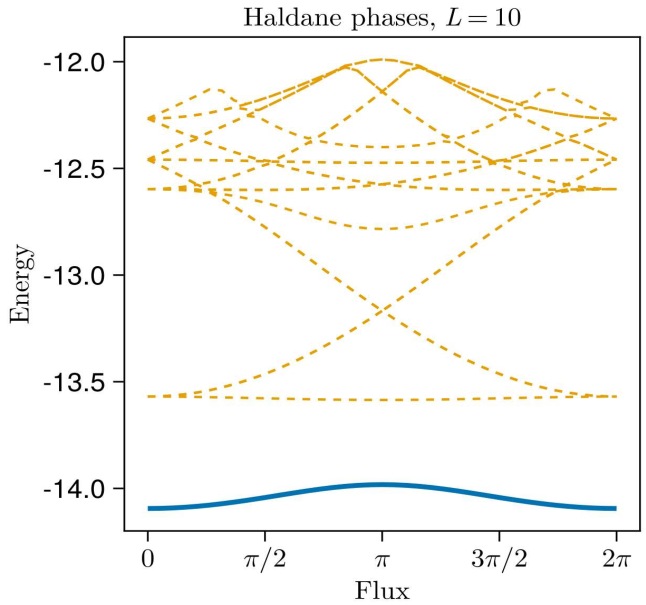
Tracing flow of $|\psi_0(\theta)\rangle, E_0(\theta), k_0(\theta)$

$k_0(\theta)=k_0,$ while $T_x(2\pi)=-T_x$

$H(2\pi)=H(0)$, but $|\psi_0(2\pi)\rangle$ is different from $|\psi_0(0)\rangle$



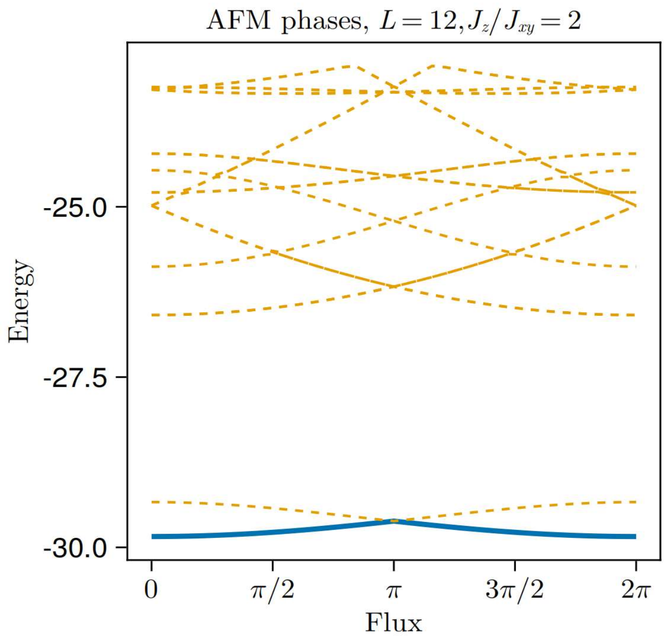
Spin-$\frac{1}{2}$ Heisenberg



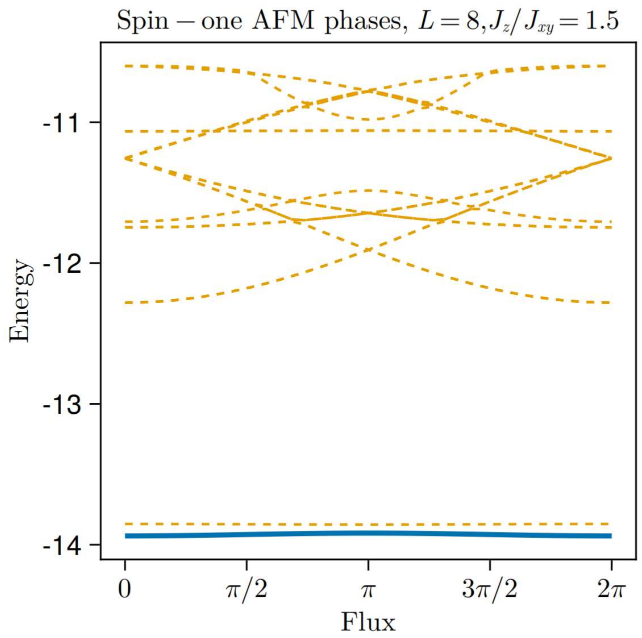
Spin-1 Heisenberg

LSM theorem and spectral flow

Flux insertion for AFM phase on spin-$\frac{1}{2}$ and spin-1 chains



Spin-$\frac{1}{2}$ XXZ



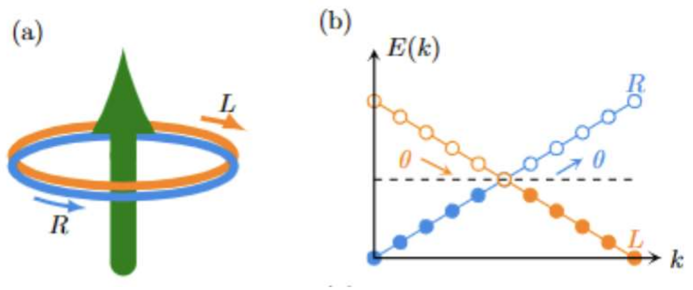
Spin-1 XXZ

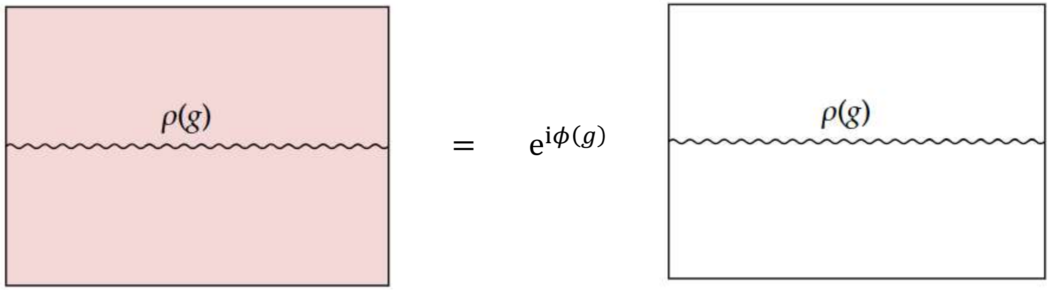
LSM theorem

- 2π flux carries π momentum
- Energy of 2π flux scales to 0 when $L \to \infty$ due to locality

Spectral flow and (mixed) anomaly

- Many faces: classical symmetry broken in quantum level, obstruction to gauge, UV incomplete… definition based on observables?
- Modern view: the (Euclidean) partition function with background gauge fields transforms with additional phase factors under gauge transformations (Cheng & Seiberg)
- Example: mixed anomaly for $U(1) \times G$ in $1+1$D





- Symmetries of left and right edges of quantum Hall effect:

$$U_L(1) \times U_R(1) = U_{tot}(1) \times U_{diff}(1)$$

- Flux insertion and **spectral flow** ~ Mixed anomaly between $U_{tot}(1)$ and $U_{diff}(1)$

$$\boldsymbol{Z(2\pi, g)} \equiv \mathrm{Tr}\big(\rho(g) \cdot \mathrm{e}^{-\beta \quad (2\pi)}\big) = \mathrm{Tr}\big(\rho(g) \cdot U\mathrm{e}^{-\beta \quad} U^\dagger\ \big) = \mathbf{e}^{\mathbf{i}\boldsymbol{\phi}(\boldsymbol{g})}\boldsymbol{Z}(\boldsymbol{g})$$

$\Longrightarrow \rho(g) \cdot U \cdot [\rho(g)]^\dagger = \mathrm{e}^{\mathrm{i}\phi(g)} U,$ **2π flux U carries G quantum number**

(Interacting) quantum phases in synthetic systems

Topological Bands for Ultracold Atoms

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Chiral ground-state currents of interacting photons in a synthetic magnetic field

P. Roushan[1*†], C. Neill[2†], A. Megrant[1†], Y. Chen[1], R. Babbush[3], R. Barends[1], B. Campbell[2], Z. Chen[2], B. Chiaro[2], A. Dunsworth[2], A. Fowler[1], E. Jeffrey[1], J. Kelly[1], E. Lucero[1], J. Mutus[1], P. J. J. O'Malley[2], M. Neeley[1], C. Quintana[2], D. Sank[1], A. Vainsencher[2], J. Wenner[2], T. White[1], E. Kapit[4,5], H. Neven[3] and J. Martinis[1,2]

Probing topological spin liquids on a programmable quantum simulator

G. SEMEGHINI, H. LEVINE, A. KEESLING, S. EBADI, T. T. WANG, D. BLUVSTEIN, R. VERRESEN, H. PICHLER, M. KALINOWSKI, R. SAMAJDAR, A. OMRAN, S. SACHDEV, A. VISHWANATH, M. GREINER, V. VULETIĆ, AND M. D. LUKIN fewer Authors Info & Affiliations

On-chip quantum simulation with superconducting circuits

Andrew A. Houck[1*], Hakan E. Türeci[1] and Jens Koch[2]

Issues in synthetic quantum systems

- Quantum simulation(Hamiltonian): finite temperature
- Quantum circuit (LOCC): decoherence, flux insertion?
- Our goal:
  - Pure state → mixed state topology/anomaly
  - Free fermion → many-body phases
  - Edge states → bulk properties
  - Transport/STM → one-shot measurement on a single state
- Previous works:

Many-body Chern number from statistical correlations of randomized measurements

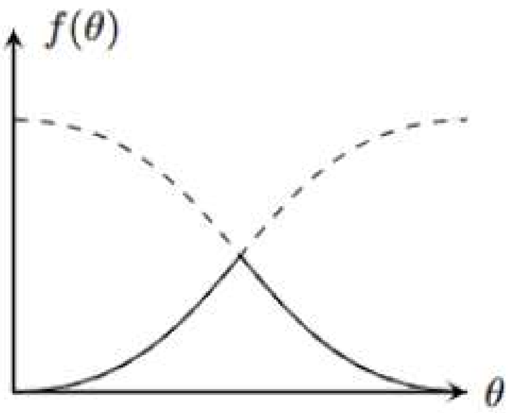
Ze-Pei Cian,[1,2] Hossein Dehghani,[1,2] Andreas Elben,[3,4] Benoît Vermersch,[3,4,5]
Guanyu Zhu,[6] Maissam Barkeshli,[1,7] Peter Zoller,[3,4] and Mohammad Hafezi[1,2]

Signature and protocol

$$f(\theta) = -\lim_{L\to\infty}\frac{1}{L}\ln\left|\mathrm{tr}\left(\rho\, \mathrm{e}^{\mathrm{i}\theta\hat{Q}}\right)\right|, \qquad \theta \in [0,2\pi)$$

For anomalous ρ, unavoid singularity of $f(\theta)$

(See also K.-L. Cai, M. Cheng)



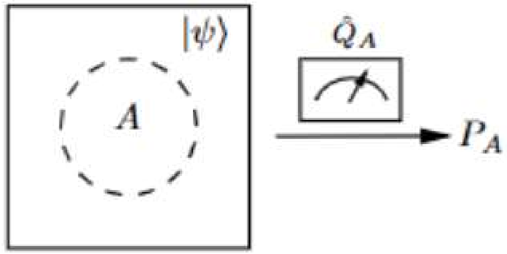
Examples of anomalous ρ?

- Finite temp spin-$\frac{1}{2}$ chain
- Reduced density matrix of 2D SPT with $U(1)$ symmetry (e.g. QSH)

Protocol to get $f(\theta)$?

- One-shot measurement + data post-processing

$$\mathrm{tr}\left(\rho\, \mathrm{e}^{\mathrm{i}\theta\hat{Q}}\right) = \sum_q \langle q|\rho|q\rangle\, \mathrm{e}^{\mathrm{i}\theta q}$$



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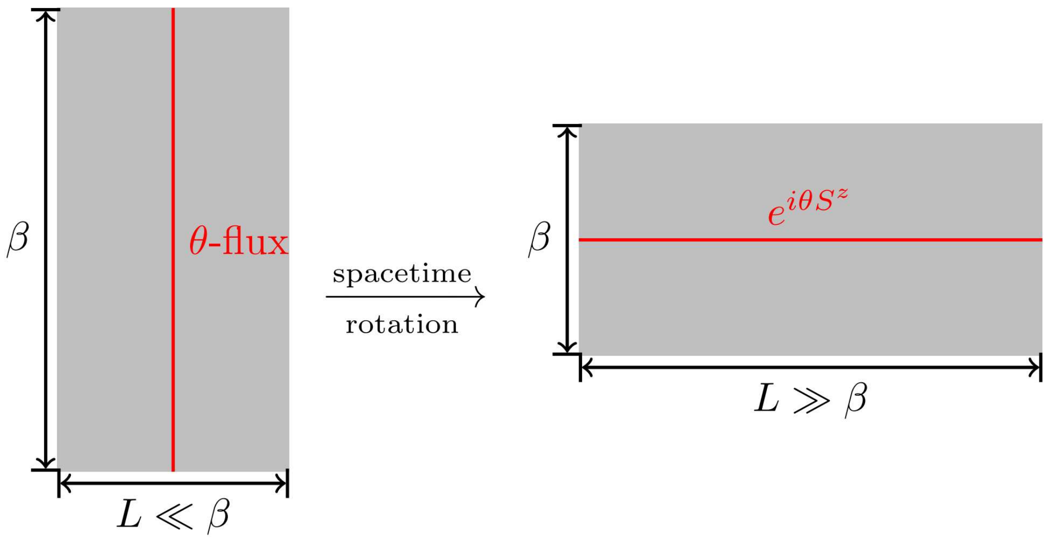
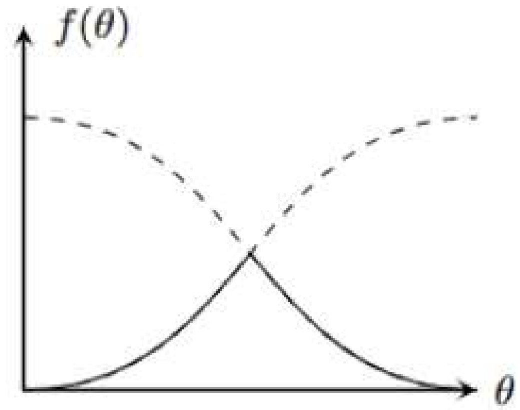
physical intuition from finite temp LSM

$$f(\theta) = -\lim_{L\to\infty}\frac{1}{L}\ln\left|\left\langle e^{i\theta S^z_{\mathrm{tot}}}\right\rangle\right| \quad \text{v.s.} \quad E_{GS}(\theta) = -\lim_{\beta\to\infty}\frac{1}{\beta}\ln Z_\beta(\theta)$$

For half-integer spin chains

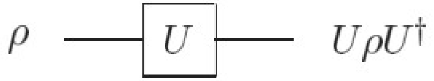
unavoid singularity of $f(\theta)$ v.s. spectral flow of GS energy

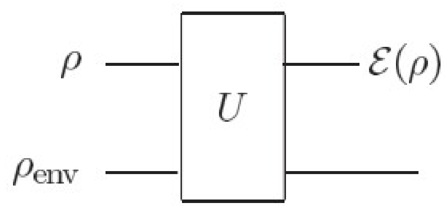




Beyond CFT? Beyond thermal states?

Local quantum channel and MPDO



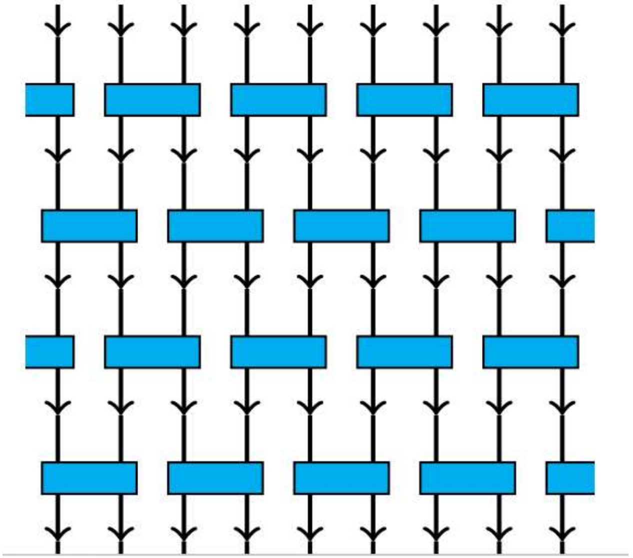


Quantum channel (or quantum operation):

$$\rho \to \mathcal{E}(\rho) = \sum_a K_a \cdot \rho \cdot K_a^\dagger, \qquad \sum_a K_a^\dagger \cdot K_a \leq \hat{1}$$

If assuming Markovian, Master equation:

$$\dot{\rho} = \mathcal{L}(\rho)$$



- For a 1D open system, consider short-range correlated states:

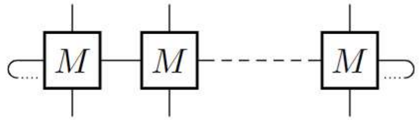
$$\langle O_i O_j \rangle - \langle O \rangle^2 \sim \exp\left(-\frac{|i-j|}{\xi}\right)$$

- Examples include thermal states $\exp(-\beta H)$, or finite depth local quantum operation on ρ_0
- Symmetries of ρ derived from symmetries of the whole system (weak symmetry)

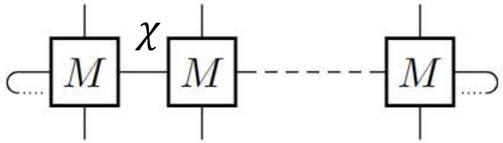
$$\mathbb{U}(g)\,\rho\,\mathbb{U}^\dagger(g) = \rho$$

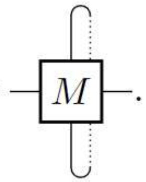
- Represented ρ as MPDO:

$$\rho = \sum_{\{s\},\{s'\}} \mathrm{tr}\left[\cdots \hat{M}^{s_j s_j'} \hat{M}^{s_{j+1} s_{j+1}'} \cdots\right] |\ldots s_j s_{j+1} \ldots\rangle\langle \ldots s_j' s_{j+1}' \ldots| =$$



Transfer matrix spectrum



Transfer matrix $\hat{T} \equiv \sum_s \widehat{M}^{ss} =$ 

- Eigen-decomposition: $\hat{T} = \sum_{\alpha=1}^{\chi} \lambda_\alpha |r_\alpha\rangle \langle l_\alpha|$. Physical meaning of λ's?
- Assuming $L \to \infty$ and $|\lambda_j| \geq |\lambda_{j+1}|$. Set $\lambda_1 = 1$ due to normalization, therefore

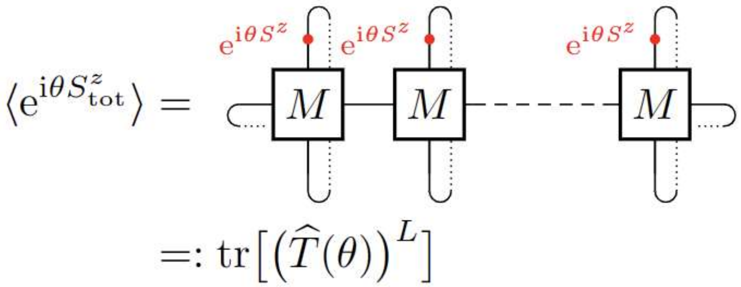
$$\langle \hat{O}_1 \hat{O}_l \rangle \equiv \sum_\alpha \langle l_1|\hat{T}(O)|r_\alpha\rangle \cdot \langle l_\alpha|\hat{T}(O)|r_1\rangle \cdot \lambda_\alpha^{l-2} = \sum_\alpha C_\alpha \mathrm{e}^{\mathrm{i}l\phi_\alpha} \cdot \mathrm{e}^{-\frac{l}{\xi_\alpha}}, \quad \hat{T}(O) \equiv \sum_{\alpha\beta} \sum_{ss'} \left(M_{\alpha\beta}^{ss'} O_{s's} \right) |\alpha)(\beta|.$$

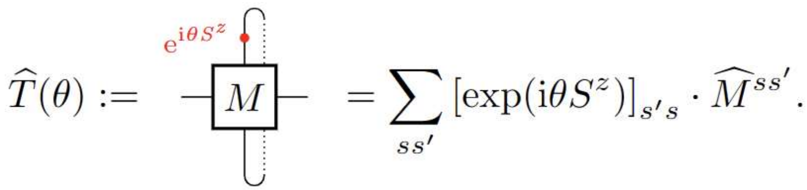
where $\lambda_\alpha = \mathrm{e}^{\mathrm{i}\phi_\alpha} \cdot \mathrm{e}^{-\xi_\alpha^{-1}}$ ϕ_α denotes oscillation wavevector, (dominant) ξ_α correlation length.

- Compare with time correlator for arbitrary state $|\phi_0\rangle$: $\langle \phi_0 | O(t) O(0) | \phi_0 \rangle = \sum_j D_j \mathrm{e}^{-\mathrm{i}E_j t}$
- Short-range correlation $\Leftrightarrow 1 = \lambda_1 > |\lambda_2| \geq \cdots$, non-degeneracy

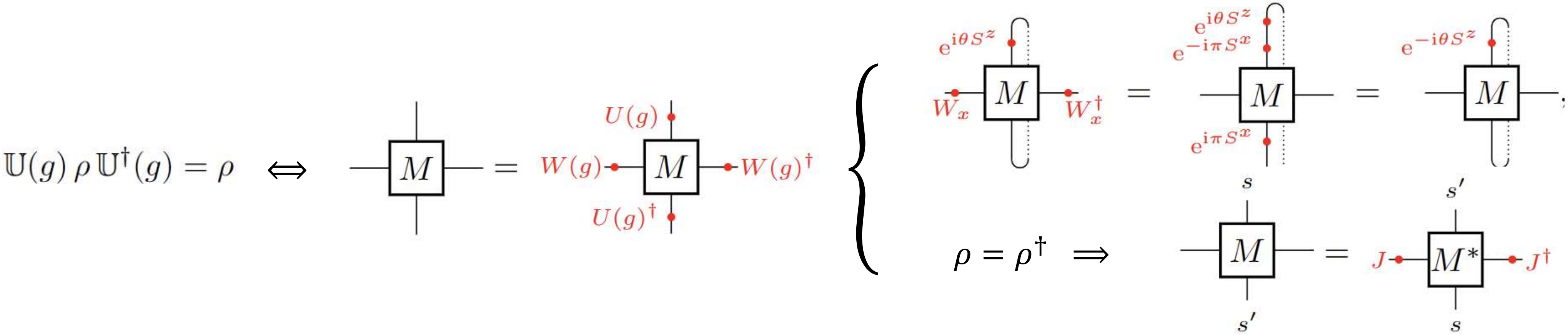
If $1 = |\lambda_2| > |\lambda_3|$, $\exists\, \hat{O}$, s.t. $\langle \hat{O}_1 \hat{O}_l \rangle \xrightarrow{l\to\infty} \mathrm{const} \cdot \mathrm{e}^{\mathrm{i}l\phi_2}$, violates short-range correlation

Twisted transfer matrix and its symmetry





$$f(\theta) = -\lim_{L\to\infty} \frac{1}{L} \ln\left|\langle e^{i\theta S^z_{tot}} \rangle\right| = -\ln|\lambda_{max}(\theta)|$$



Flow of transfer matrix spectrum

$$\hat{\boldsymbol{T}}(-\boldsymbol{\theta}) = \boldsymbol{W}_x \cdot \hat{\boldsymbol{T}}(\boldsymbol{\theta}) \cdot \boldsymbol{W}_x^\dagger = \boldsymbol{J} \cdot \hat{\boldsymbol{T}}^*(\boldsymbol{\theta}) \cdot \boldsymbol{J}^\dagger = \pm\hat{\boldsymbol{T}}(2\boldsymbol{\pi} - \boldsymbol{\theta})$$

where + for spin-1 and − for spin-$\frac{1}{2}$.

When focusing on spin-$\frac{1}{2}$

- $W_x \cdot \hat{T}(\theta) \cdot W_x^\dagger = J \cdot \hat{T}^*(\theta) \cdot J^\dagger \Rightarrow \lambda(\theta)$ is real or comes in conjugate pairs
- $W_x \cdot \hat{T}(\theta) \cdot W_x^\dagger = -\hat{T}(2\pi - \theta) \Rightarrow -\lambda(\theta)$ is an eigenvalue of $\hat{T}(2\pi - \theta)$

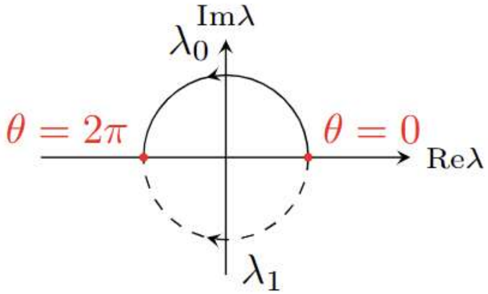
$\lambda(\theta)$'s are reflection symmetric under real and imaginary axis

$$f(\theta) = -\ln|\lambda_{\max}(\theta)|$$

$\lambda_0(\theta)$ labels flow starting at 1

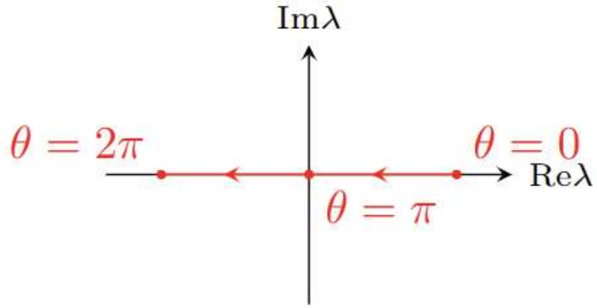
Case 1:

$\lambda_0(0)$ flows to $\lambda_0(2\pi) = -\lambda_0(0)$



degenerate dominant λ's

violates short-range correlation

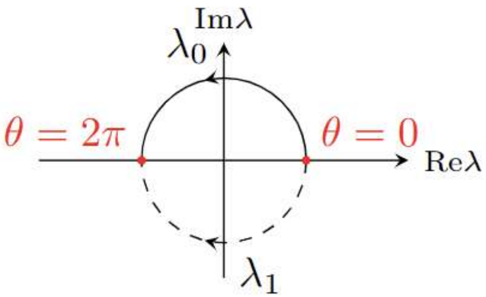


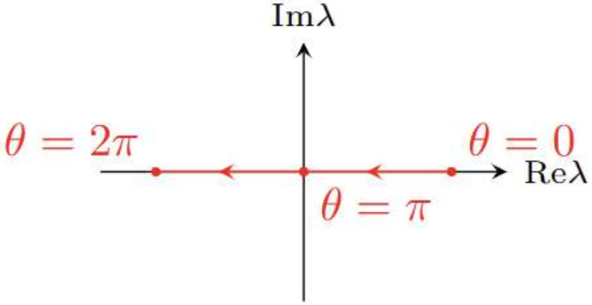
$f(\theta) \to \infty$ at $\theta = \pi$

Symmetries and transfer matrix spectrum

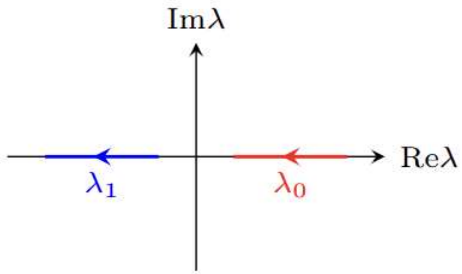
$\lambda(\theta)$ is reflection symmetric under real and imaginary axis

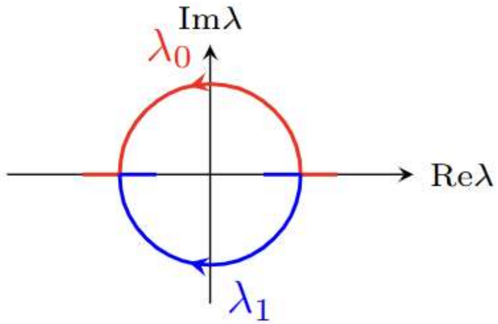
Case 1: $\lambda_0(0)$ flows to $\lambda_0(2\pi) = -\lambda_0(0)$





Case 2: $\lambda_0(0)$ ends at $\lambda_0(2\pi) \neq -\lambda_0(0)$

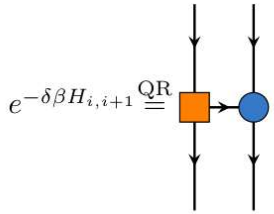


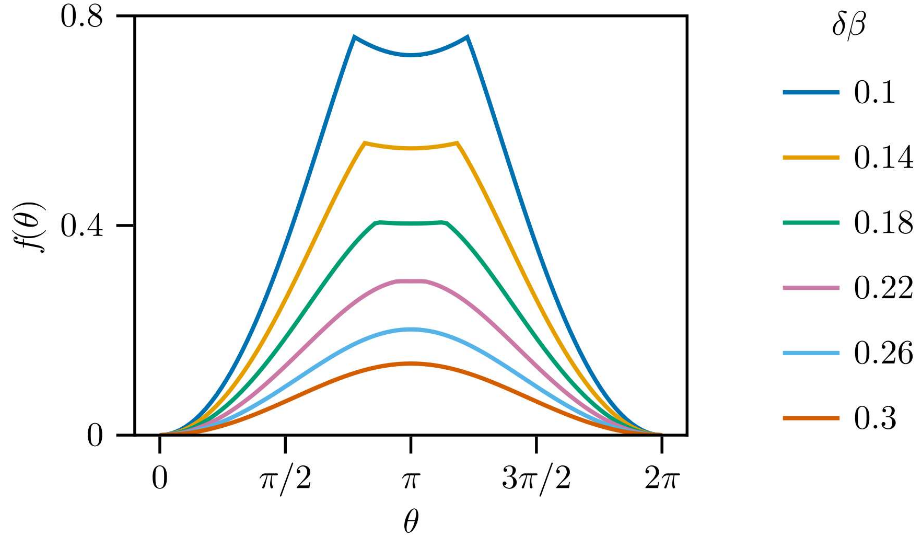


Numerical results

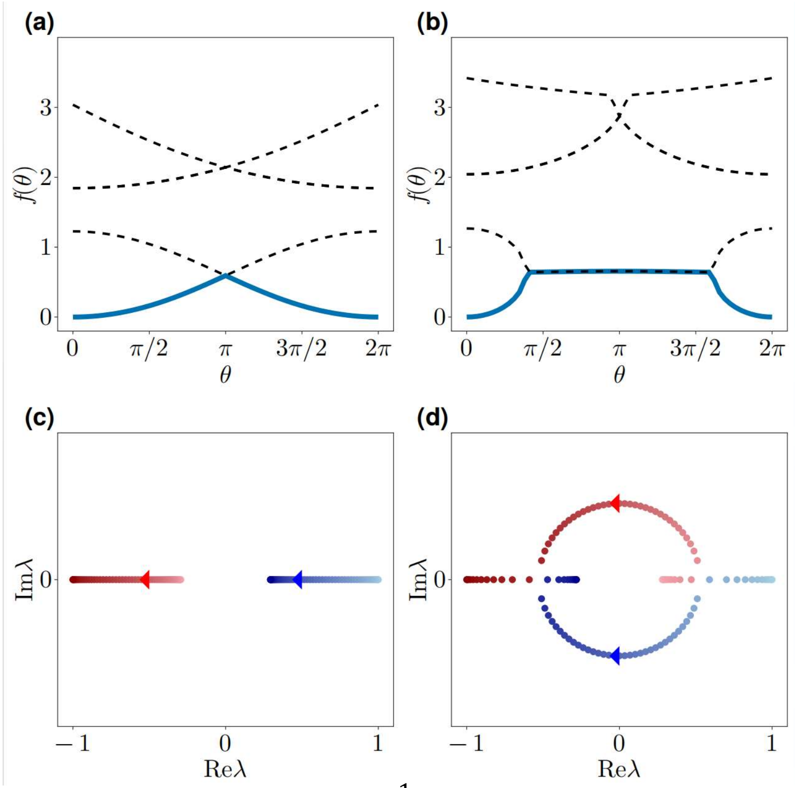
$$\widehat{H} = \sum_{j=1}^{L} \widehat{S}_j^x \widehat{S}_{j+1}^x + \widehat{S}_j^y \widehat{S}_{j+1}^y + \Delta \widehat{S}_j^z \widehat{S}_{j+1}^z$$

$$f(\theta) = -\ln|\lambda_{\max}(\theta)|$$





$f(\theta)$ for spin-1 open chains
"cusp disappear"



$f(\theta)$ & spectral flow for spin-$\frac{1}{2}$ AFM & FM phase at finite β

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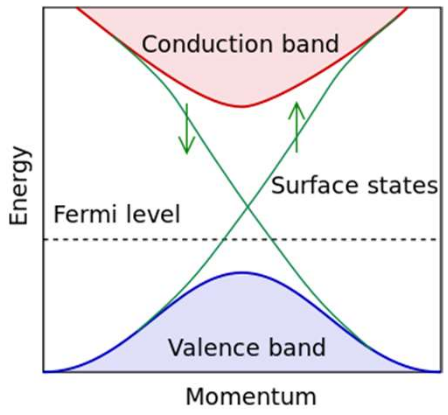
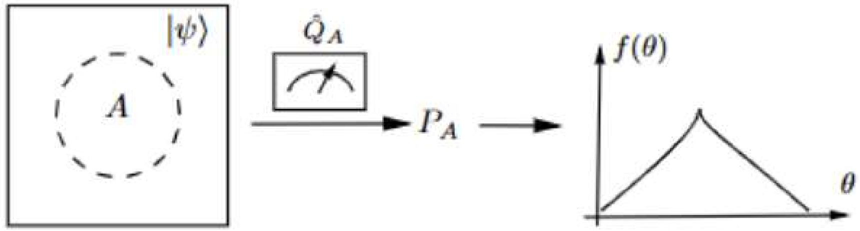
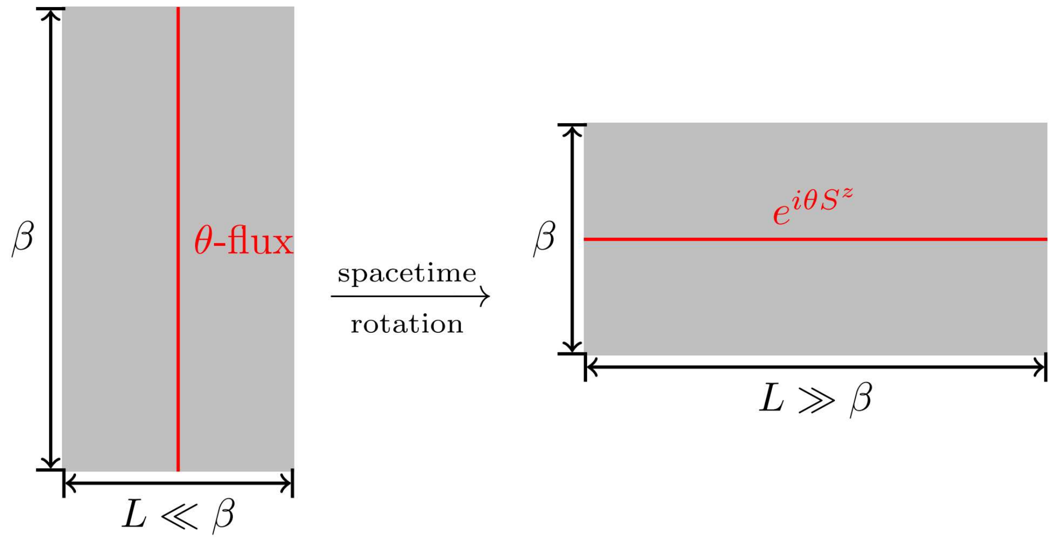
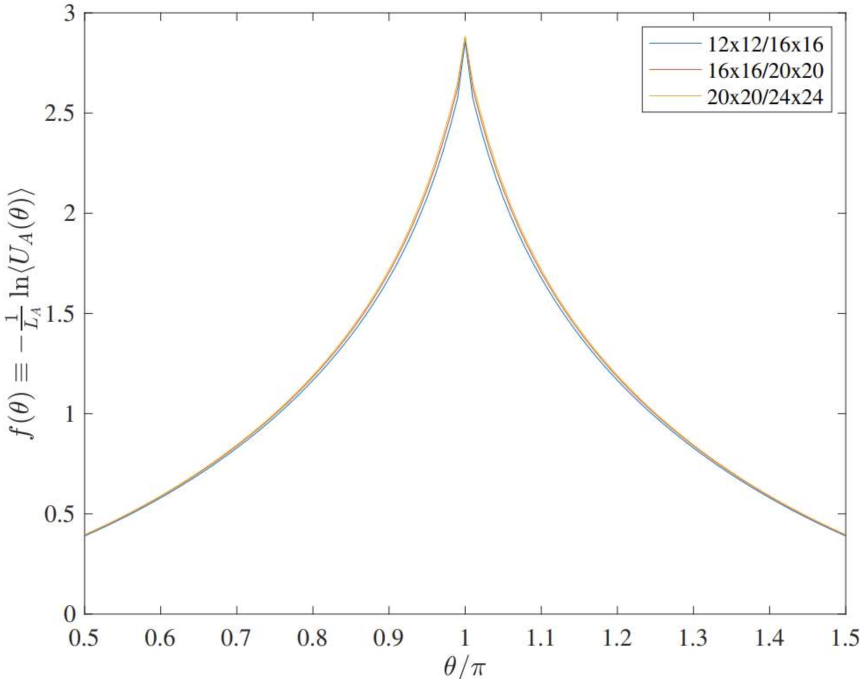
Results from free fermion QSH

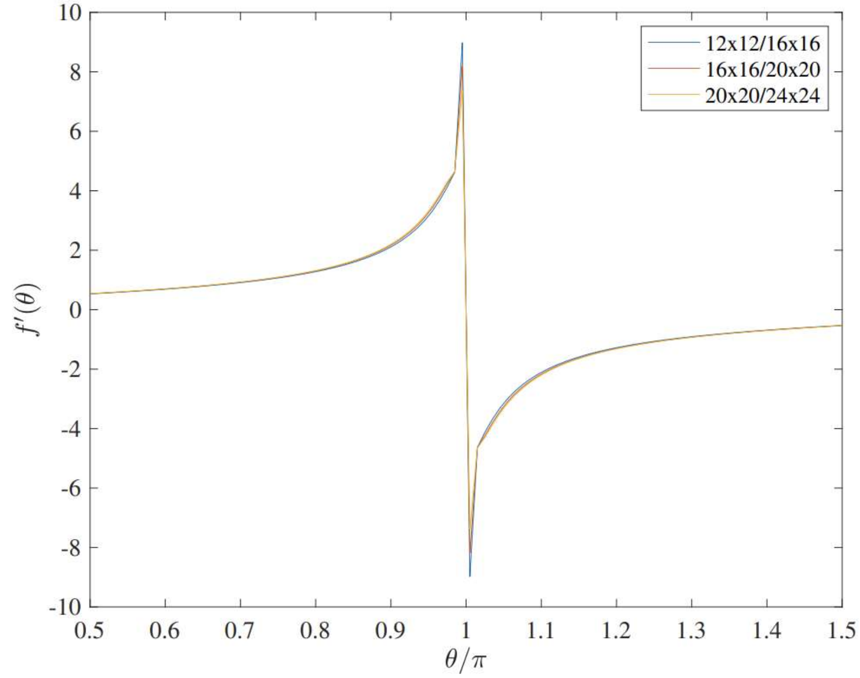






Results from free fermion QSH

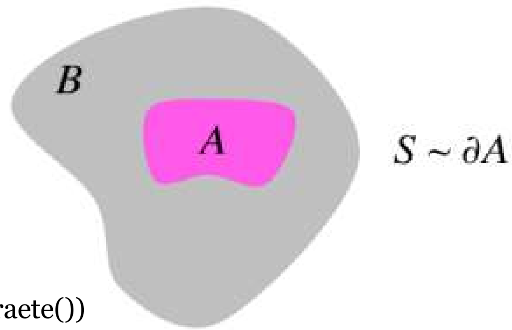


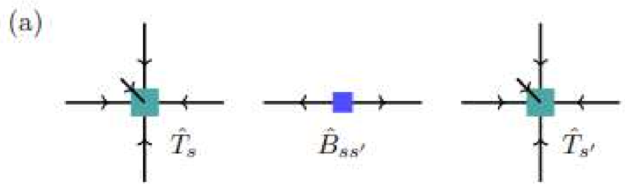


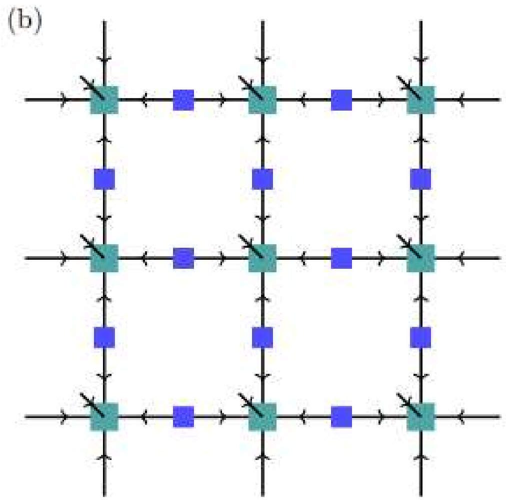
Ansatz: Kane-Mele model on honeycomb lattice

Issues on correlated 2D TI

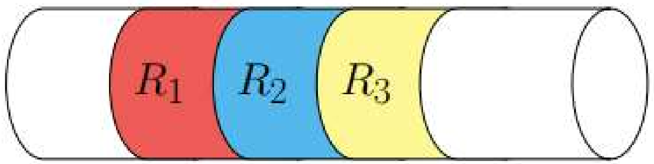
- Issues:
  - How symmetry acts on ρ_A if ρ_A a MPDO?
  - Variational wavefunction for correlated TI?
- Our answer: using (fermionic) PEPS (see our paper PRL 132, 126504)
  - Express reduced density matrix on edges (see Cirac, Poilblanc, Schuch, Verstraete())
  - tensor equations ~ symmetry action on edges
  - variational tensor network wavefunction







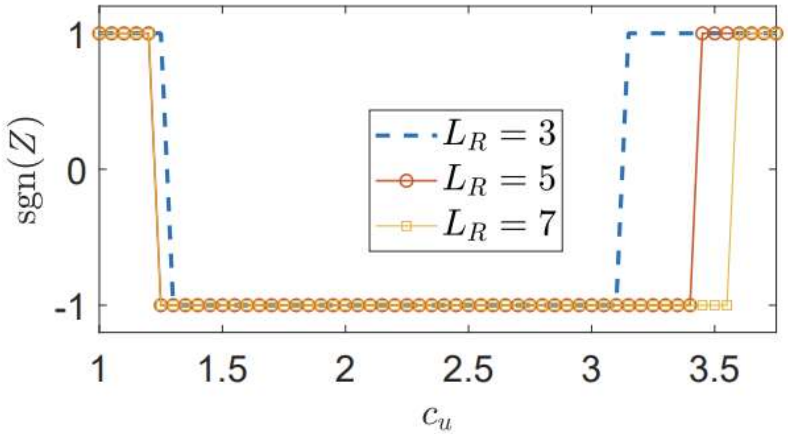
Evidence for correlated TI



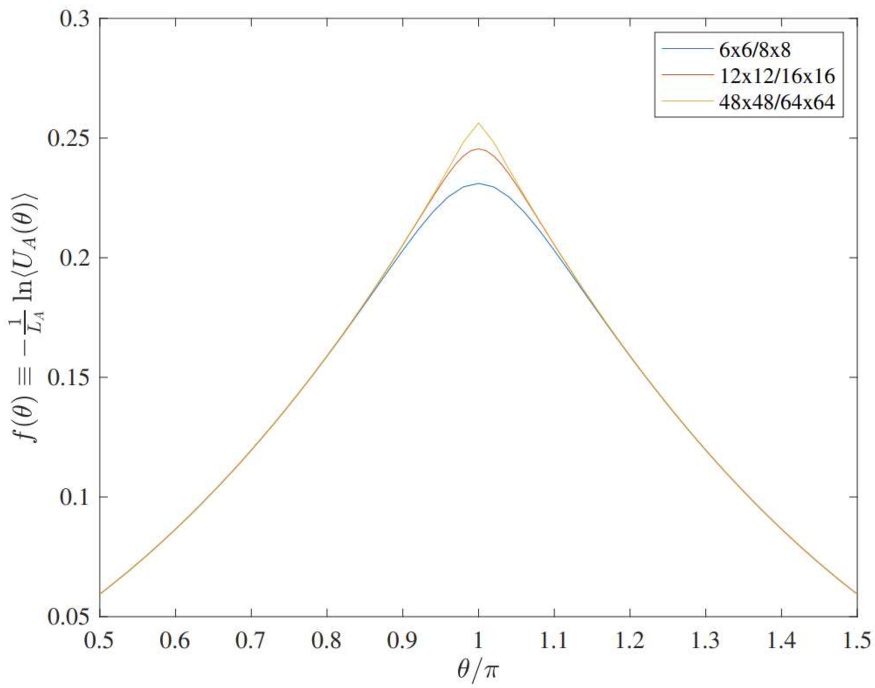
$$Z = \mathrm{Tr}\left[\rho^{+}_{R_1\cup R_3} C_T^{R_1} (\rho^{-}_{R_1\cup R_3})^{\mathsf{T}_1} [C_T^{R_1}]^{\dagger}\right]$$

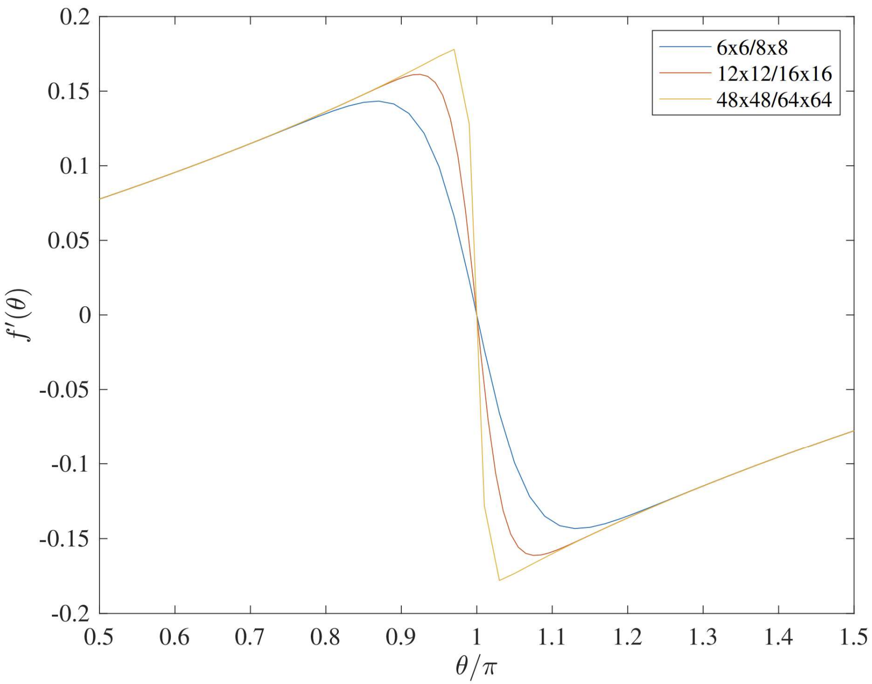
$$\rho^{\pm}_{R_1\cup R_3} = \mathrm{Tr}_{\overline{R_1\cup R_3}}\left[\exp \frac{\pm 2\pi i y \sum_{\mathrm{r}\in R_2} n(\mathrm{r})}{L_y} |\Phi\rangle\langle\Phi|\right]$$

Shiozaki, Shapourian, Gomi, Ryu (2018)



Results on correlated 2D (bosonic) TI





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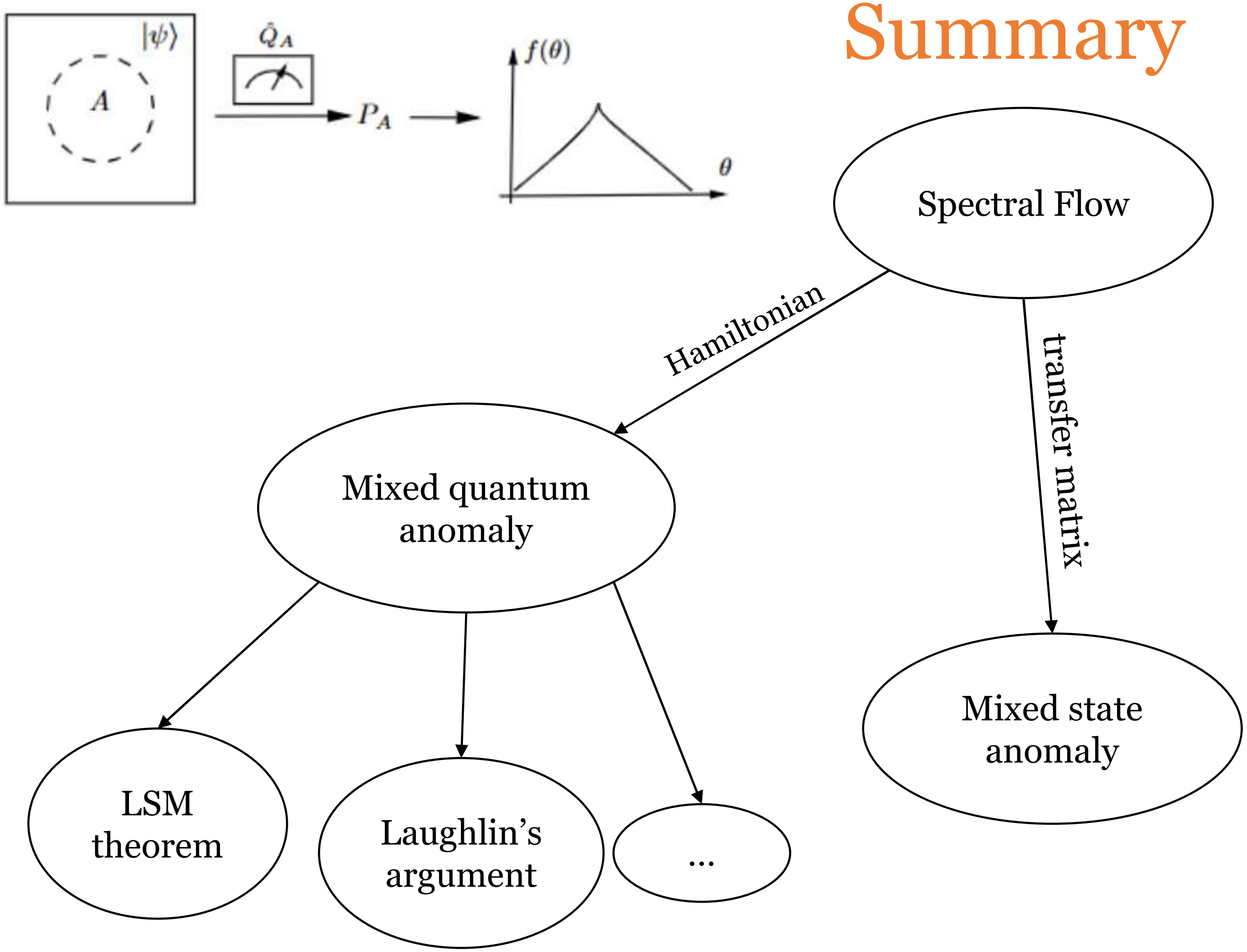
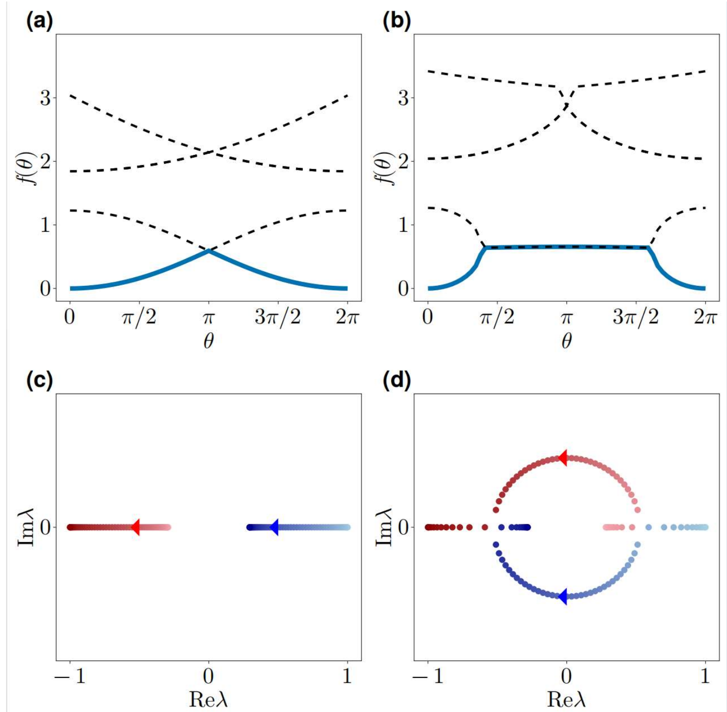
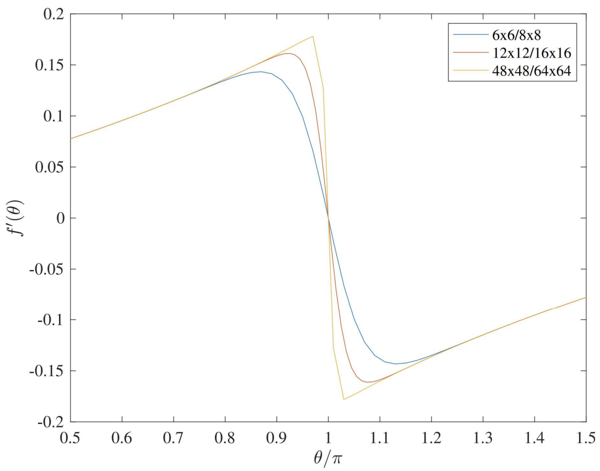
Summary



|ψ⟩
A
$\hat{Q}_A$
P_A
$f(\theta)$
θ
Spectral Flow
Hamiltonian
transfer matrix
Mixed quantum anomaly
Mixed state anomaly
LSM theorem
Laughlin's argument
...



(a)
(b)
(c)
(d)
$f(\theta)$
θ
Imλ
Reλ



6x6/8x8
12x12/16x16
48x48/64x64
$f'(\theta)$
θ/π

Outlook

- Experimental issues?
  - Robust to noises and impurities? Numerical simulation to mimic real experiment, error analysis
  - System size: small v.s. large
  - Preparation: quantum simulator v.s. circuit preparation, isometric PEPS + tensor equation?
- Generalization to detection of anomaly of discrete symmetries? To other anomalies (chiral anomaly, "global"/gravitational anomaly)? To higher dimension? To long-range entangled phases?
- Anomalous symmetry as special case of categorical symmetry; (New) observables from generalized/categorical symmetries?

Thank you!