Topological
approach to
quantum error correction

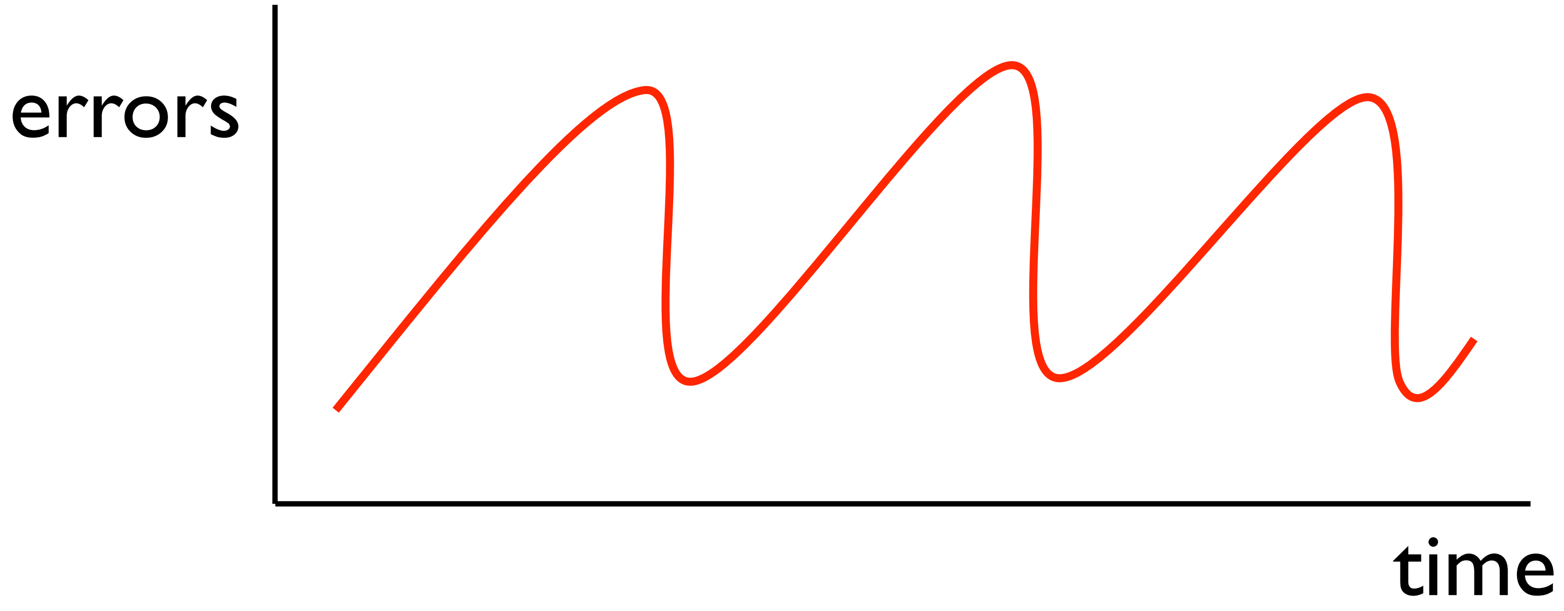
Outline

- Quantum error correction
- Topological codes
- Gauge color codes

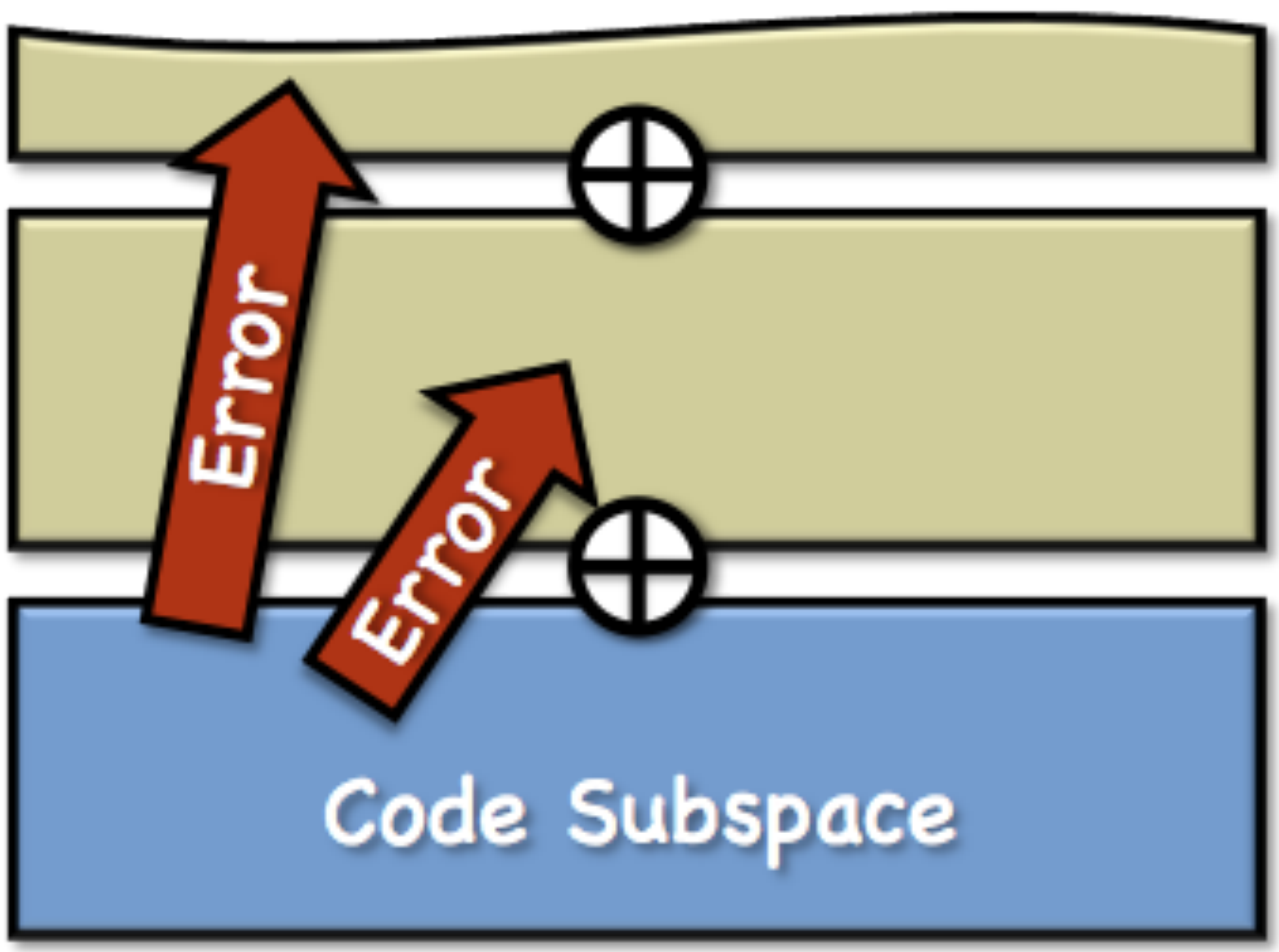
Simulating physics with (quantum) computers

Quantum computers

- Want: isolation + control
- Have: decoherence + imprecision
- Need: error correction



Error correction



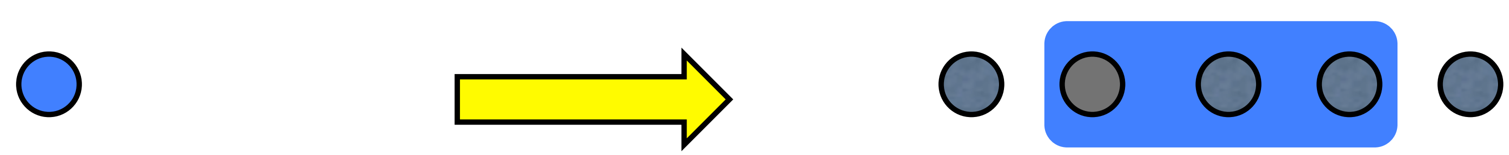
$$\mathcal{H} = \bigoplus_i \mathcal{H}_i$$

Locality of noise

- Errors are local

likely　　　　unlikely

- Encode in collective degrees of freedom

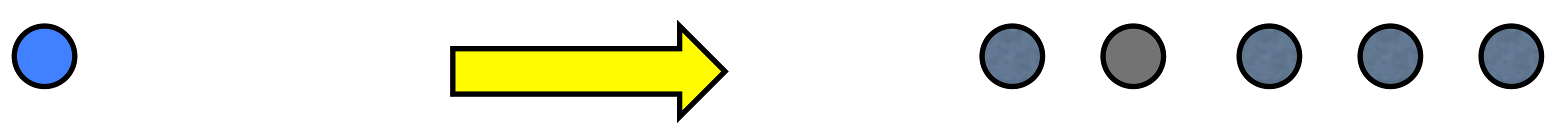


logical qubit　　　　physical qubits

Transversal gates

- To compute we need unitary ‘gates’ on logical qubits
- Aim: preserve locality (do not spread errors)
- How: act on subsystems separately

$$U_{\text{logical}} \qquad\qquad U_1 \otimes U_2 \otimes U_3 \otimes U_4 \otimes U_5$$



Transversal gates

No code admits a universal transversal set of gates

Eastin & Knill '09

Alternatives:

- Magic state distillation Bravyi & Kitaev '03
- Gauge fixing Paetznick & Reichardt '13
- Concatenation Jochym-O'Connor & Laflamme '14

Fault-tolerant quantum computation

Quantum computations with any **size, time and precision** can be performed with a reasonable resource **overhead** if noise level is below a **threshold** value.

Often very high overhead
or very low threshold

Outline

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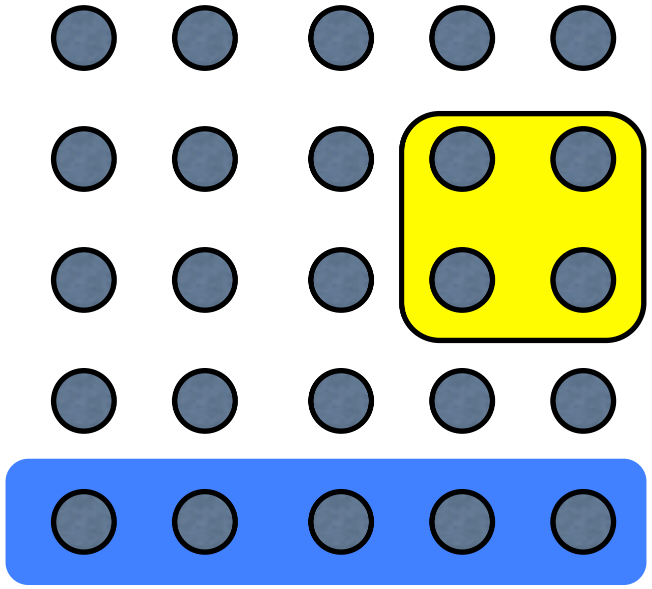
- **Topological codes**
- Gauge color codes

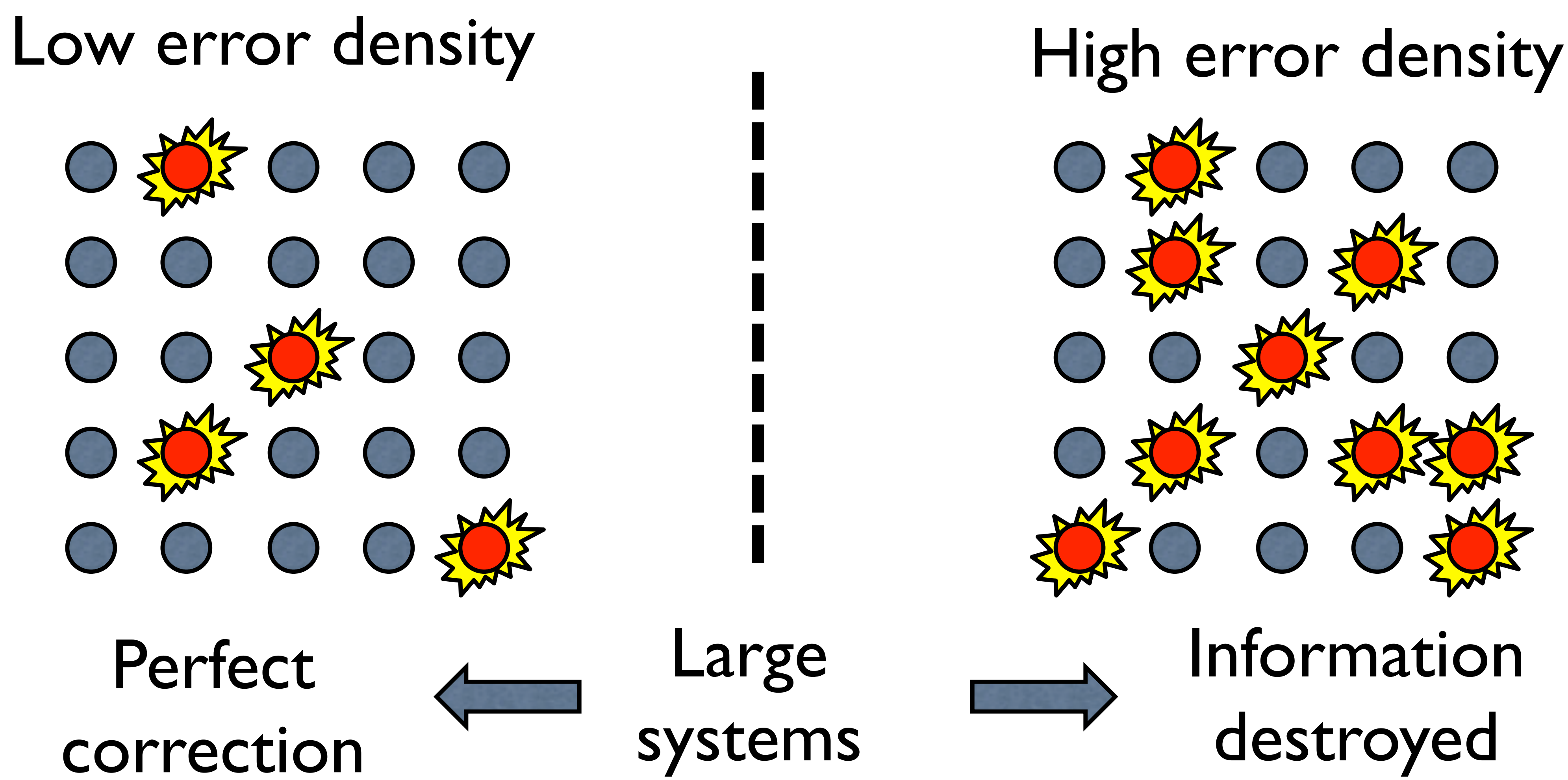
Topological codes

Kitaev '97

- Physical qubits on a lattice
- Local check operators
- Global logical operators



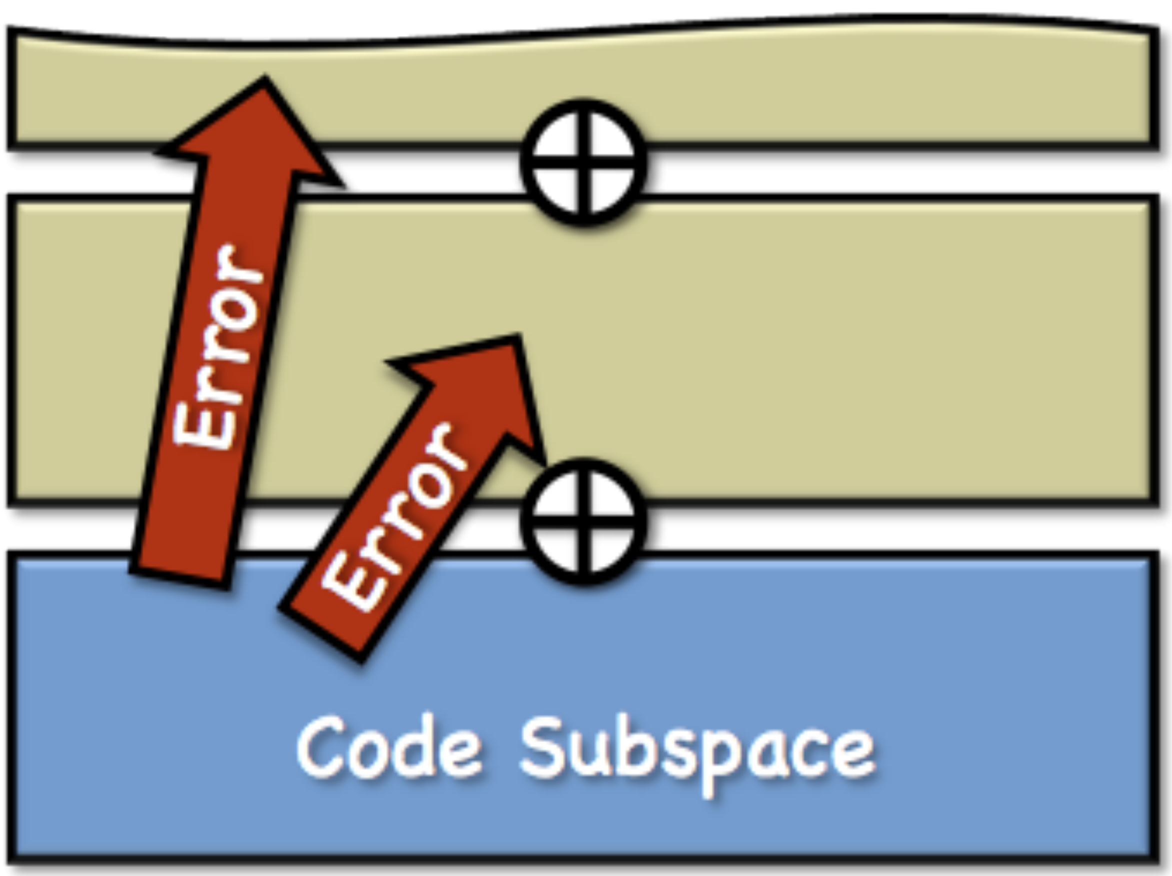
Error threshold



- Phase transition!
- Connects with classical statistical physics

Topological order

- A passive approach to quantum error correction?

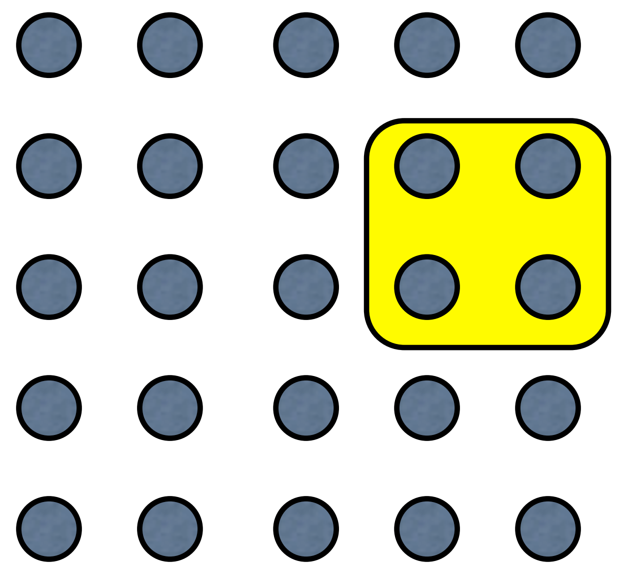


Excited states

Gap

Ground states

Topological order

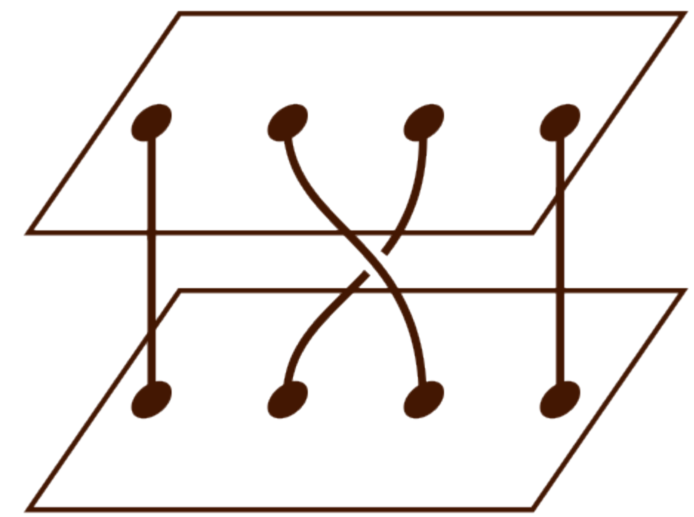


$$H = -J \sum_i P_i$$

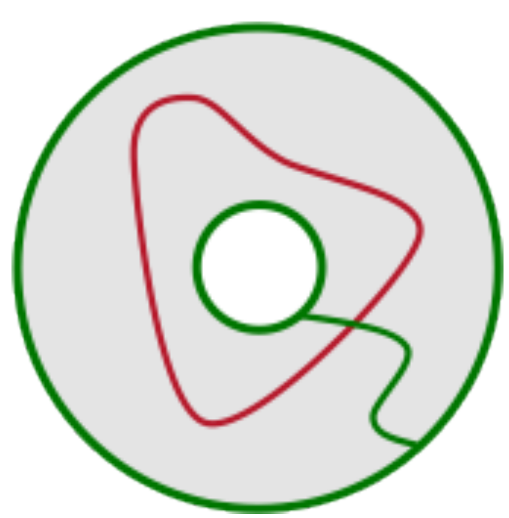
- Gapped (local) quantum Hamiltonian
- Locally undistinguishable ground states
- Robust against deformations

Topological order

- In 2D excitations are **anyons**
- Planar codes can be derived using anyon physics:

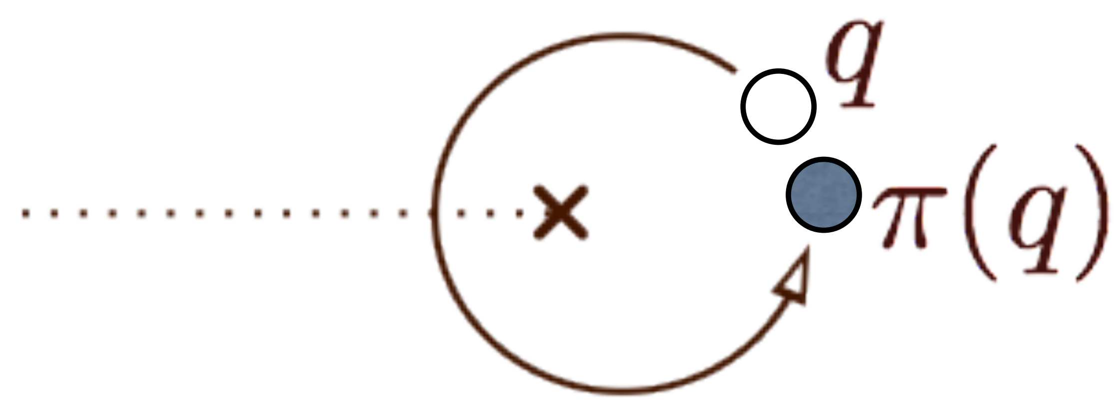


Boundaries / condensation



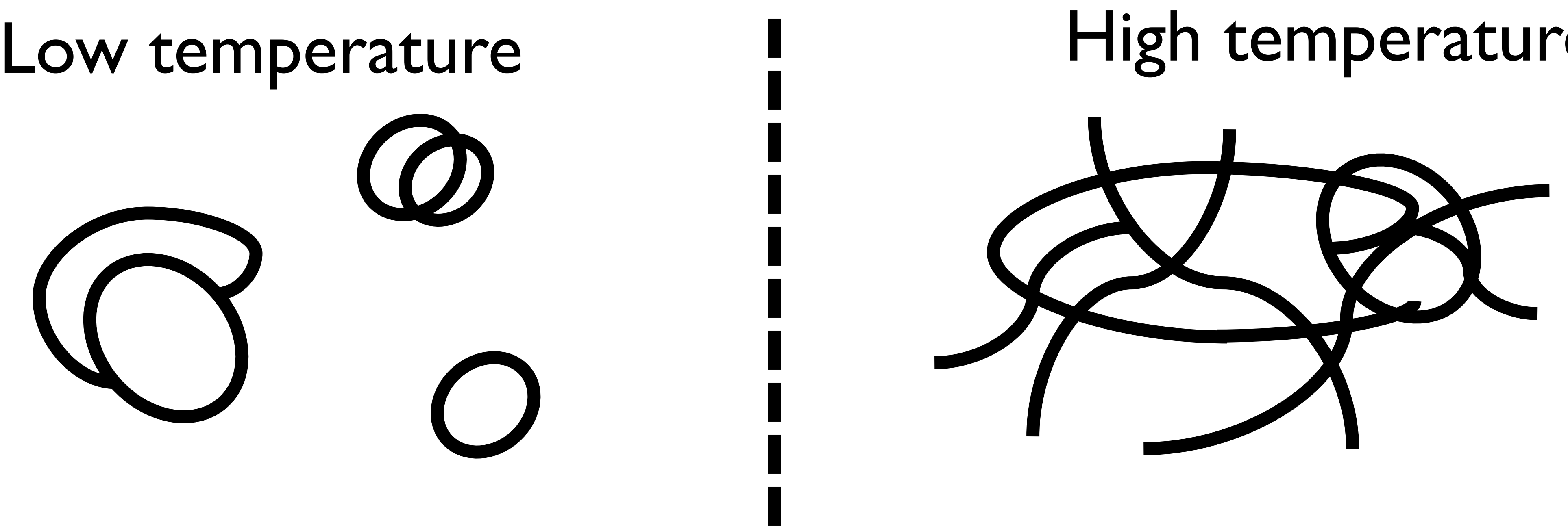
Twists / symmetries

Bombin '10



Self-correction

- For D $\geq$ 4 all excitations can be extended objects



Perfect preservation ← Large systems → Information destroyed

- What about D = 2, 3? Open question

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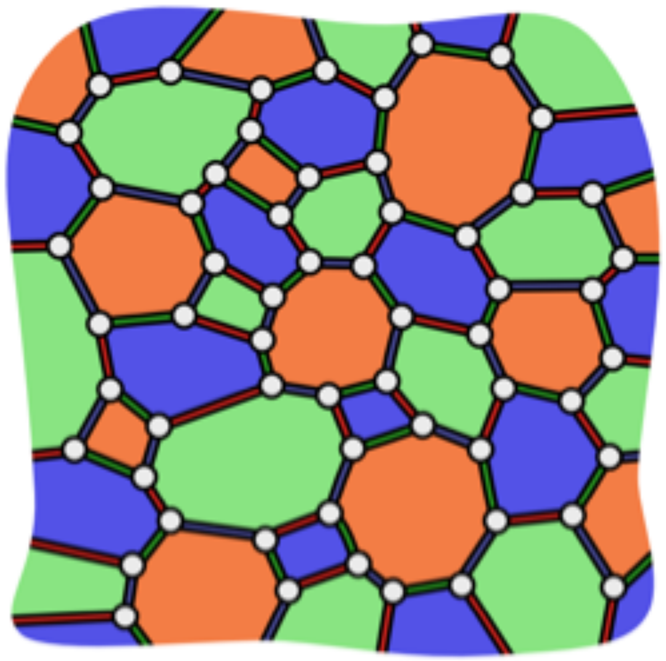
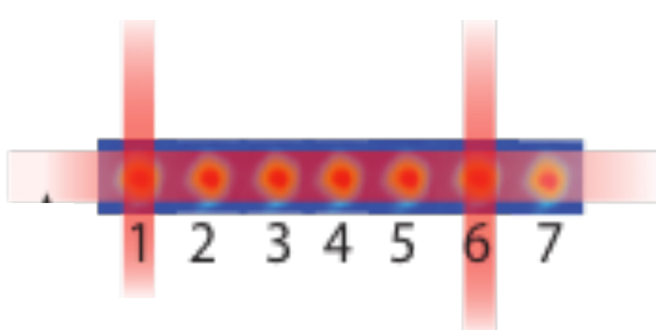
Color codes



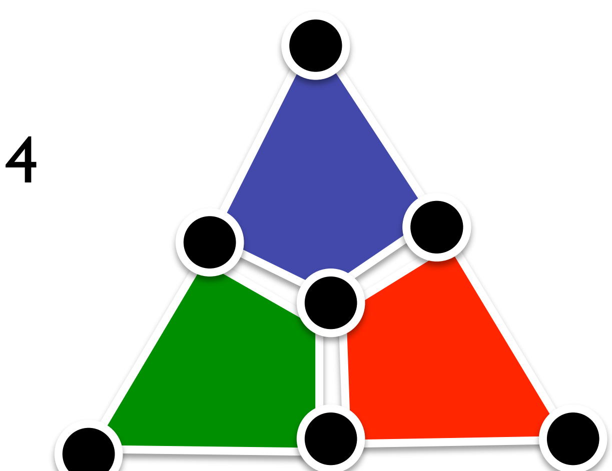
Topology + transversality !

- 2*D*: Clifford group ➔ Distillation
- *D*>2: Cnot + $R_D := \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^D} \end{pmatrix}$
- 3*D* ➔ Hadamard via ancilla |+>

Demonstrated with 7 trapped ions



Nigg et al '14



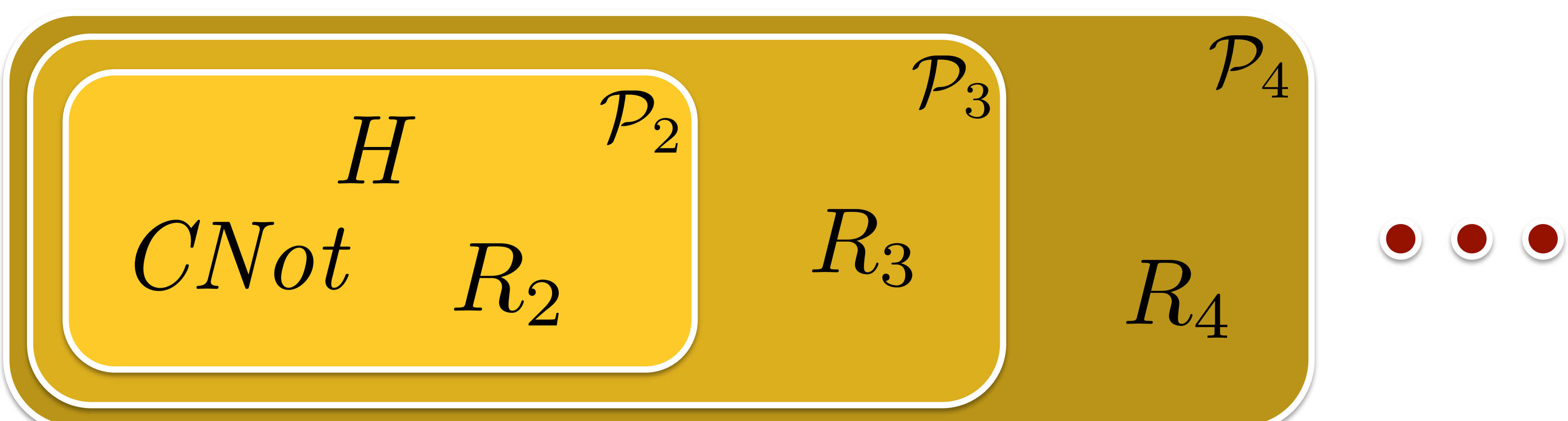
Dimensional restrictions

Bravyi & Koenig '13

- In topological stabilizer codes only some gates can be transversal depending on the dimension

$$\mathcal{P}_D := \{U \,|\, U\mathcal{P}U^\dagger \subseteq \mathcal{P}_{D-1}\}, \qquad \mathcal{P}_1 := \mathcal{P}$$

Gottesman & Chuang '99



$$R_D := \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^D} \end{pmatrix}$$

Color codes

Difficulties of 3D color codes:

- Many-body measurements of 20+ qubits
- Ancilla encoded qubit for *H* gate

Gauge Color codes

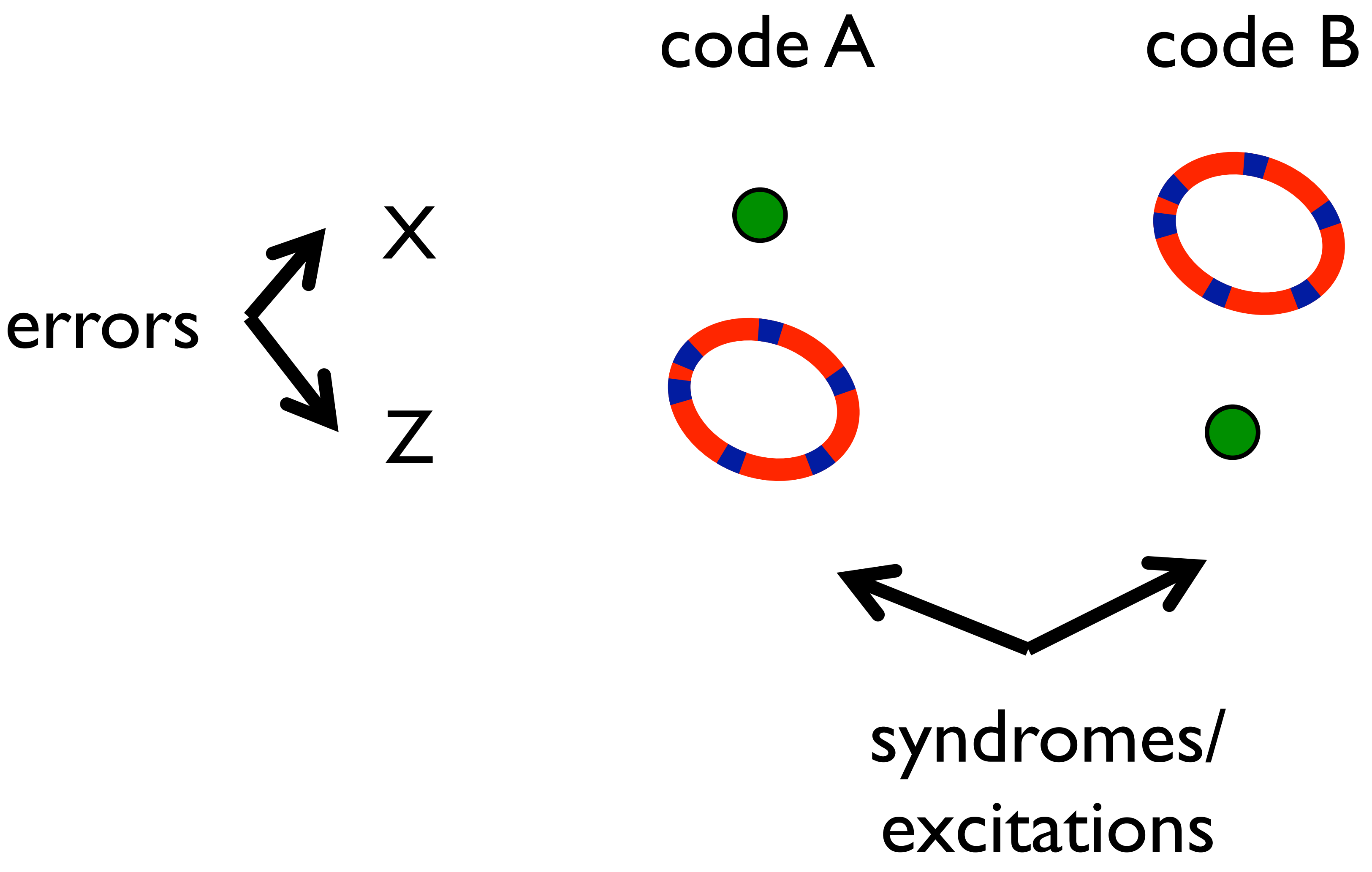
~~Difficulties of~~ 3D color codes:

just 6

- Many-body measurements of ~~20+~~ qubits
- NO Ancilla encoded qubit for *H* gate
- **Single-shot** FT QEC
- Constant time overhead

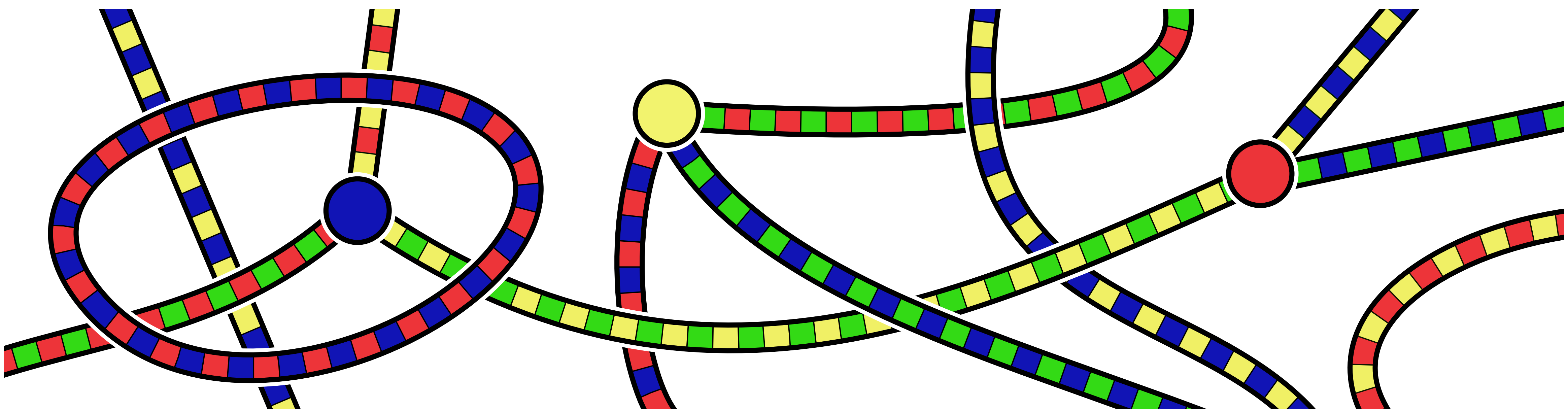
3D color codes

- For a given geometry, two dual codes



3D color codes

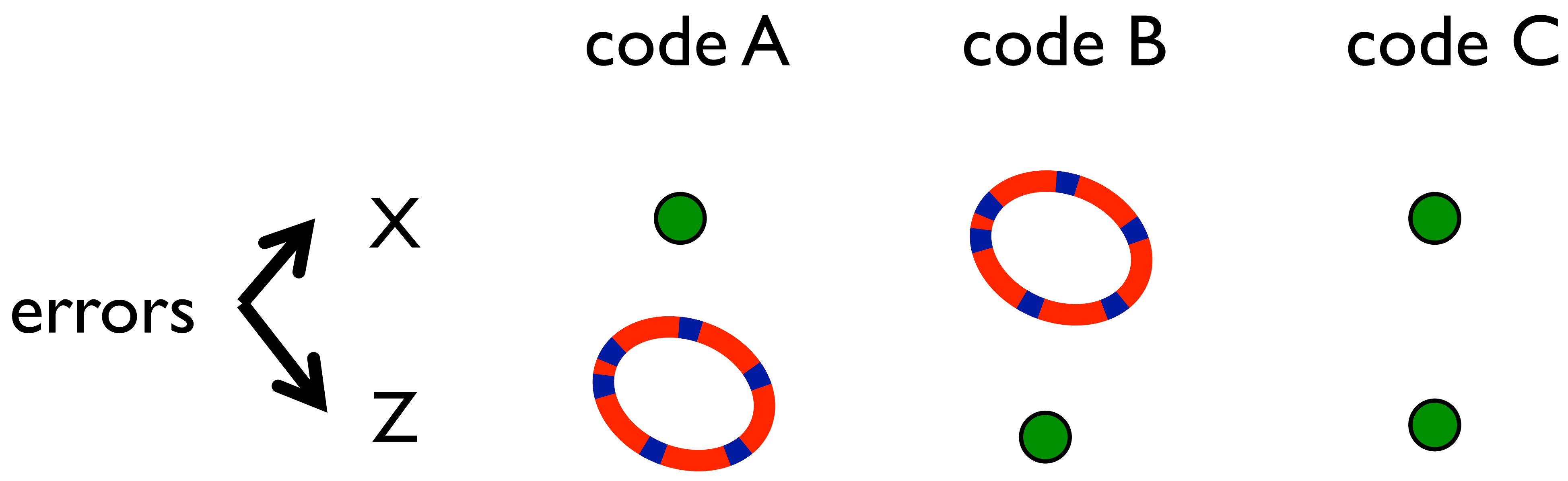
- But fluxes and charges of dual codes are related!



- The branching points of fluxes in a code correspond to charges in the dual code

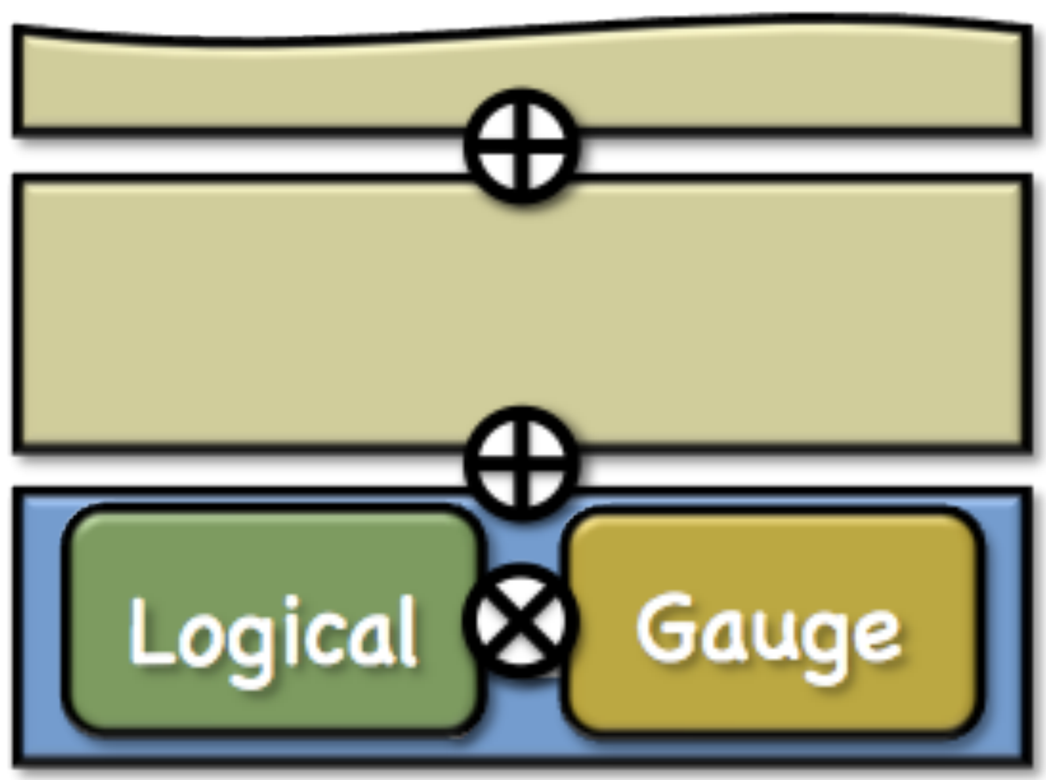
3D color codes

- For some boundary conditions, processes involving only fluxes (without branching points) cannot give rise to errors
- We can consider codes were only charges are measured



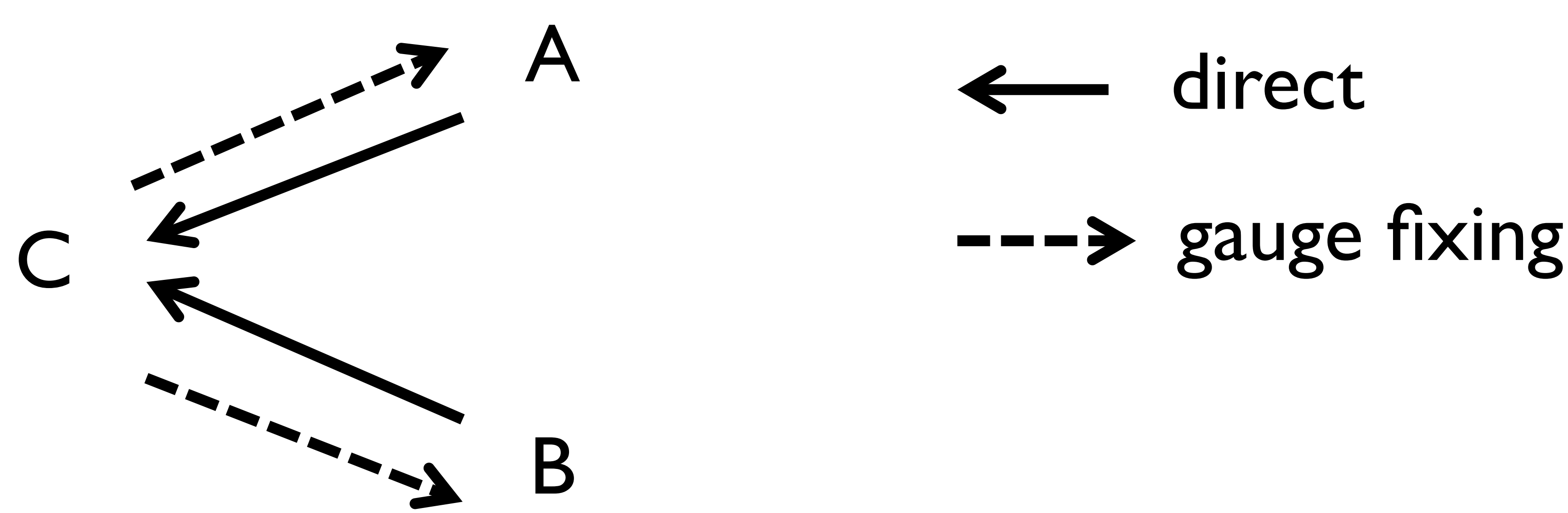
3D gauge color codes

- The result is a subsystem code, with fluxes corresponding to gauge degrees of freedom



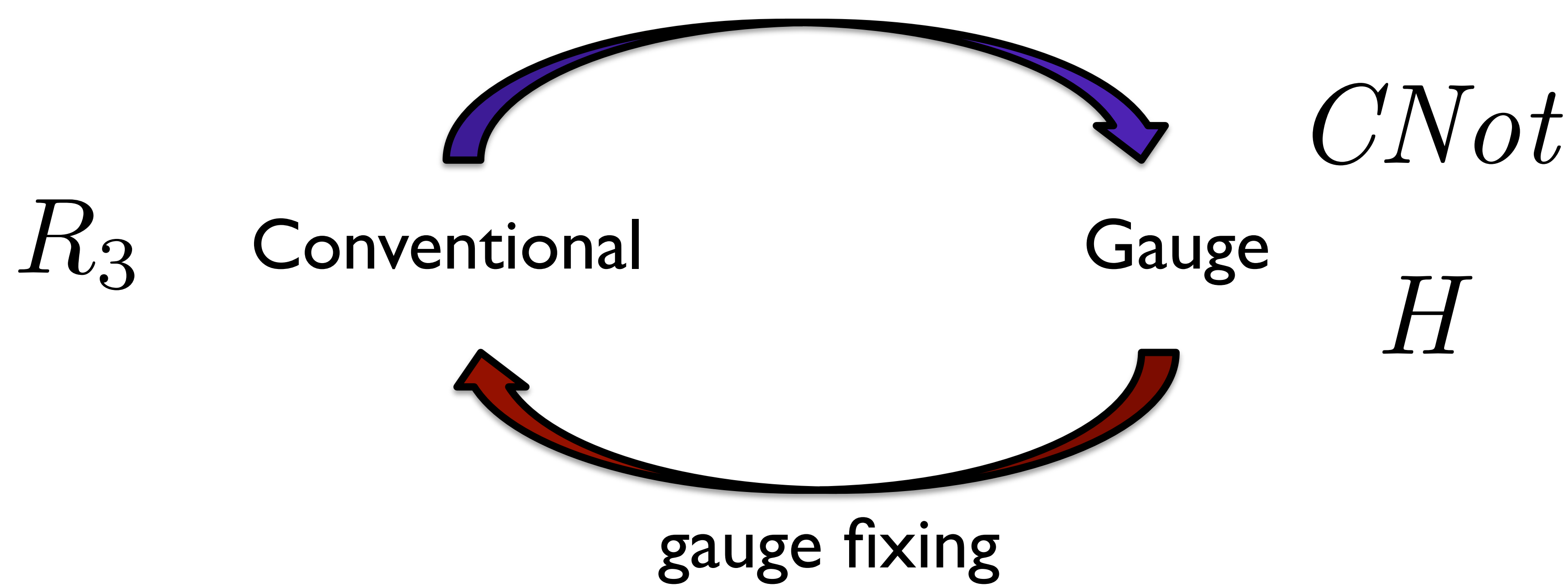
3D gauge color codes

- For suitable boundary conditions:
  - Hadamard is transversal in the gauge code
  - The codes have the same logical operators
- We can jump between them!



3D gauge color codes

- We can now recover a universal set of transversal gates!



- Measurements can be made 6-local by measuring fluxes (gauge degrees of freedom), not charges directly

Single-shot fault tolerant QEC

- Because syndrome measurements are faulty, they have to be repeated or large errors can be introduced when trying to do error correction
- Not for 3D gauge color codes! A small wrong ‘flux syndrome’ gives rise to wrong charge syndromes that are close!
- This is reminiscent of confinement…

Single-shot fault tolerant QEC

- Codes that yield self-correcting topological order also are single-shot!

- For 3D gauge color codes, all operations (initialization, gates, error correction, measurements) can be made in constant time using only local quantum + global classical operations
- Fault-tolerance with constant time overhead

Summary & discussion

- Topological codes are a natural tool to make quantum computation feasible
- 3*D* gauge color codes have **many** interesting practical properties
- Error thresholds?
- What are the limitations in 2D?
- What about non-geometrical locality?
- Could there be related 3*D* self-correcting systems?