QUANTUM SIMULATION OF LATTICE GAUGE THEORIES WITH COLD ATOMS

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In collaboration with E. Zohar (Tel-Aviv) and J. Ignacio Cirac, (MPQ)

YITP workshop on quantum information, August 2th 2014

OUTLINE

- Preliminaries
  - --Quantum Simulations
  - --Ultracold Atoms
  - --Structure of HEP (standard) models
  - --Hamiltonian Formulation of Lattice Gauge Theory

- Simulating Lattice gauge theories
- Local gauge invariance from microscopic physics
- Examples: Abelian (cQED), Non Abelian (YM SU(2)) , .

- outlook.

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

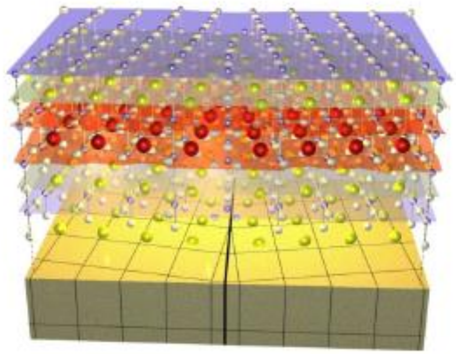
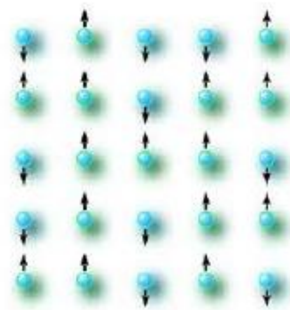
1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have

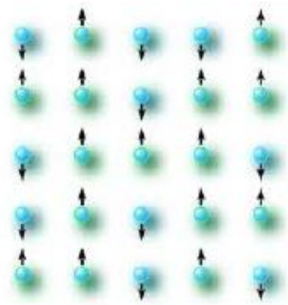
be understood very well in analyzing the situation. And I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. Thank you.

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

QUANTUM ANALOG SIMULATION

PHYSICAL SYSTEM	QUANTUM SIMULATOR
	
(Phenomenological) Hamiltonian	Physical Hamiltonian
$H = \ldots$	$H = \ldots$

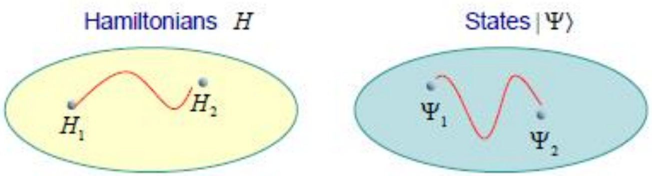
QUANTUM ANALOG SIMULATION



$$H = \ldots$$

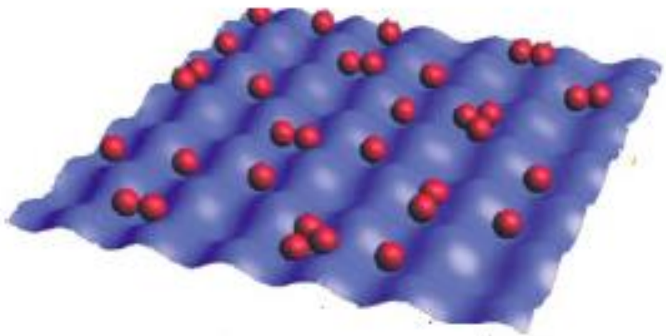
- **Questions:**
  - Dynamics: $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$
  - Ground state: $H|\Psi_0\rangle = E_0|\Psi_0\rangle$

Adiabatic algorithms



SIMULATED PHYSICS

- <u>Condensed matter</u>
  (e.g. for testing model for high TC superconductivity)



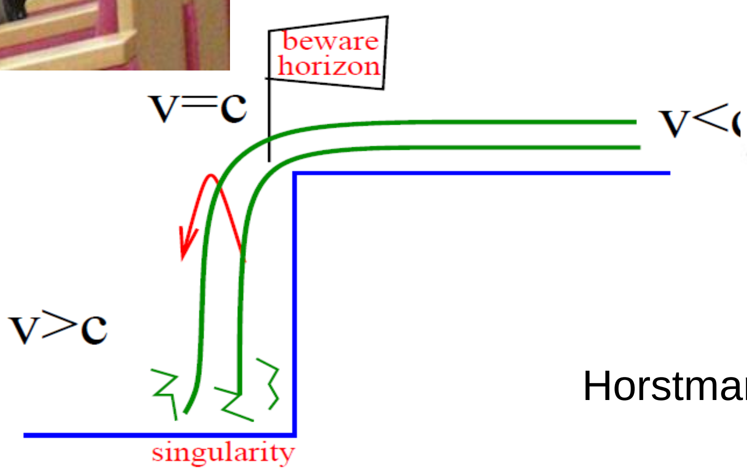
$\Rightarrow$ Hubbard and spin models
$\Rightarrow$ External (classical) artificial gauge potential
Abelian/non-Abelian.

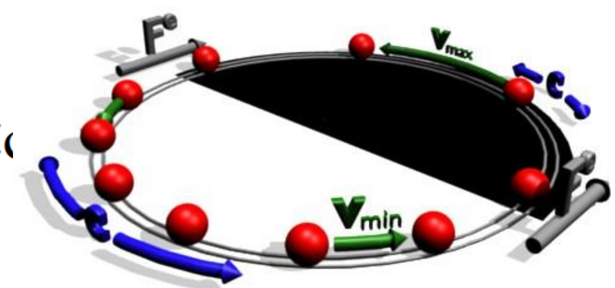
SIMULATED PHYSICS

- **Gravity:** BH, Hawking/Unruh, cosmological effects ..

Sonic (fluid) BH
(Unruh 81,96)

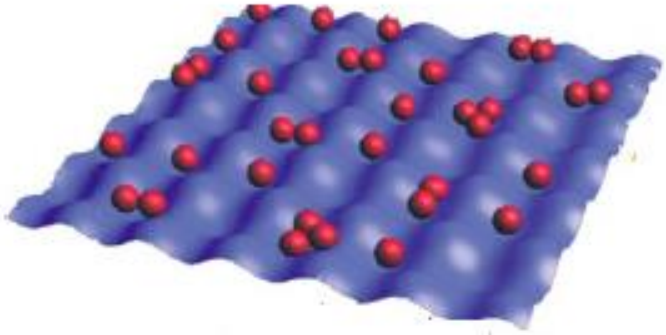
Discrete version of a black hole



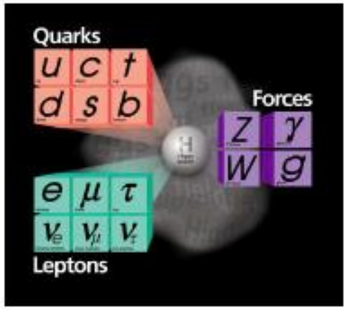


Horstman, BR, Fagnocchi, Cirac, PRL (2010)

SIMULATED PHYSICS



High Energy physics (HEP)?



SIMULATING SYSTEMS

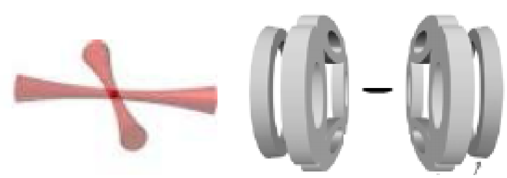
- Bose Eienstein Condensates
- Atoms in optical lattices
- Rydberg Atoms
- Trapped Ions
- Superconducting devices
- …

COLD ATOMS

- Control: External fields

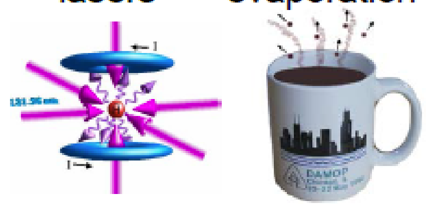
Trapping:

lasers Magnetic fields

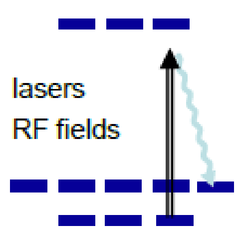


Cooling:

lasers evaporation



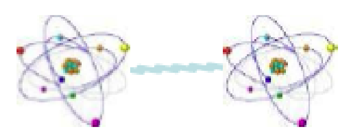
Internal manipulation



purification
coherence
detection

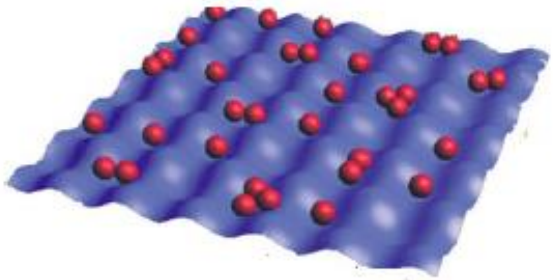
Interactions

tune scattering length

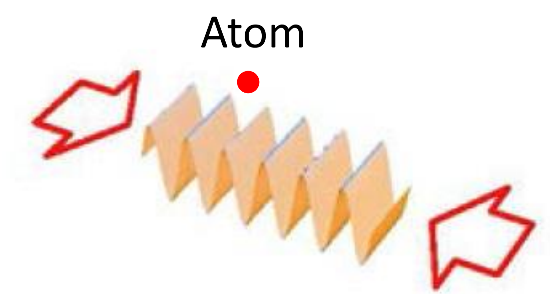


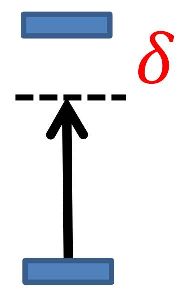
COLD ATOMS
OPTICAL LATTICES

Laser Standing waves: dipole trapping



COLD ATOMS
OPTICAL LATTICES





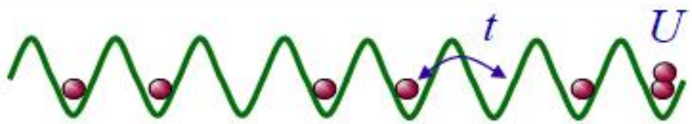
In the presence $\boldsymbol{E}(r,t)$ the atoms has a time dependent dipole moment $d(t) = \alpha(\omega)\, \boldsymbol{E}(r,t)$ of some non resonant excited states.
Stark effect:

$$\mathrm{V}(\mathrm{r}) \equiv \Delta\mathrm{E}(\mathrm{r}) = \alpha(\omega)\langle\, \boldsymbol{E}(r)\, \boldsymbol{E}(r)\rangle/\delta$$

COLD ATOMS
OPTICAL LATTICES

- Laser standing waves: dipole-trapping

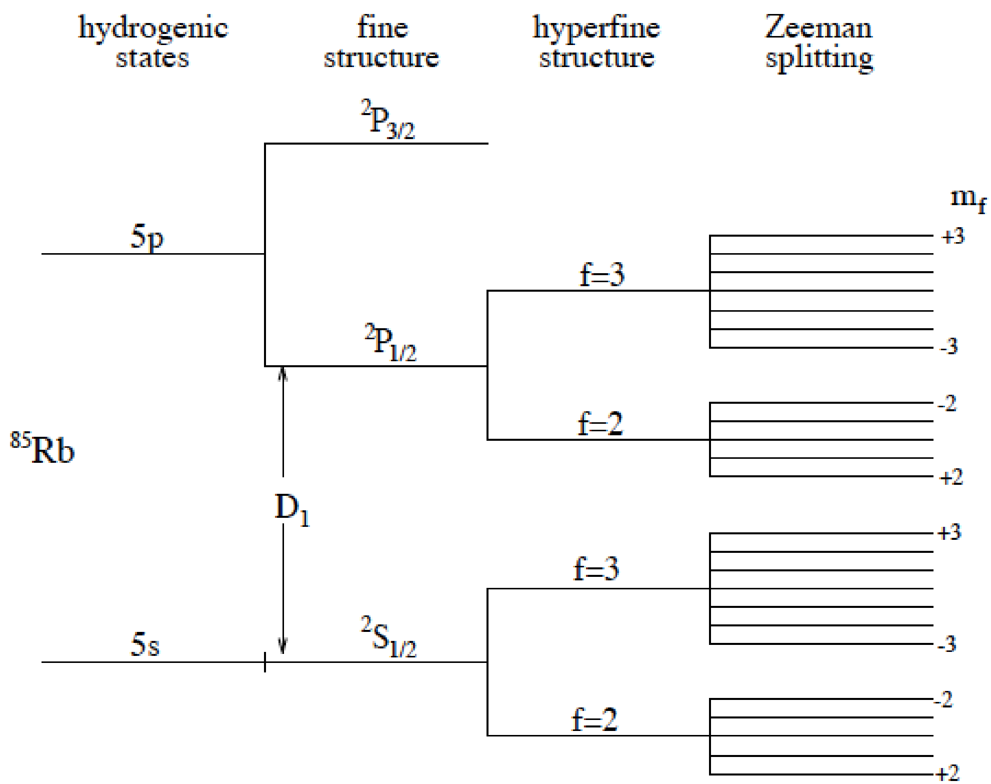
$$H=\int \Psi_{\sigma}^{\dagger}\left(-\nabla^{2}+V(r)\right)\Psi_{\sigma}+u_{\sigma_i}\int \Psi_{\sigma_1}^{\dagger}\Psi_{\sigma_2}^{\dagger}\Psi_{\sigma_3}\Psi_{\sigma_4}$$



Lattice theory: Bose/Fermi-Hubbard model

$$H=-t\sum_{n}\left(a_{n}^{\dagger}a_{n+1}+h.c\right)+U\sum_{n}a_{n}^{\dagger 2}a_{n}^{2}$$

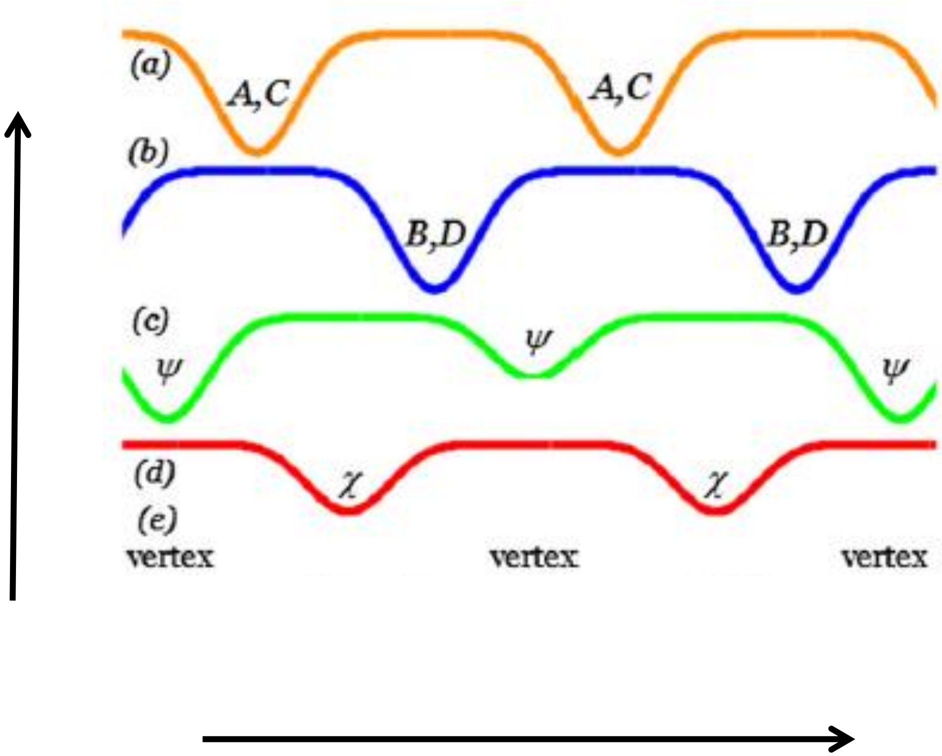
→ Superfluid to Mott insulator, phase transition (I. Bloch)



Level diagram of ^{85}Rb. $\mathbf{I} = 5/2$. The splittings are not to scale.

“Super lattice!”

Resolved (hyperfine levels) potentials



Spatial direction

THE STANDARD MODEL: CONTENTS

Matter Particles: Fermions

Quarks and Leptons:

Mass, Spin, Flavor

Coupled by force Carriers / Gauge bosons,

Massless, chargeless photon (1): Electromagnetic, U(1)

Massive, charged Z, W's (3): Weak interactions, SU(2)

Massless, charged Gluons (8): Strong interactions, SU(3)

GAUGE FIELDS

Abelian Fields **Maxwell theory**	**Non-Abelian fields** **Yang-Mills theory**
Massless	Massless
Long-range forces	**Confinement**
Chargeless	Carry charge
Linear dynamics	Self interacting & NL

QED: THE CONVENIENCE OF BEING ABELIAN

$$\alpha_{QED} \ll 1, \qquad V_{QED}(r) \propto \frac{1}{r}$$

We (ordinarily) don't need second quantization and quantum field theory to understand the structure of atoms:

$$m_e c^2 \gg E_{Rydberg} \simeq \alpha_{QED}^2 \, m_e c^2$$

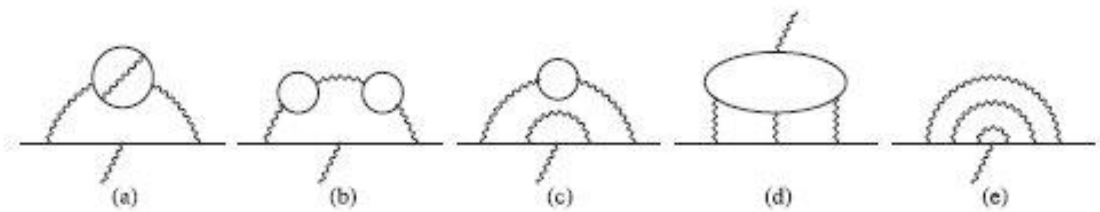
Also in higher energies (scattering, fine structure corrections), where QFT is required, perturbation theory (Feynman diagrams) works well.

CALCULATE!

e.g. , the anomalous electron magnetic moment:

(g-2)/2= ....

...+



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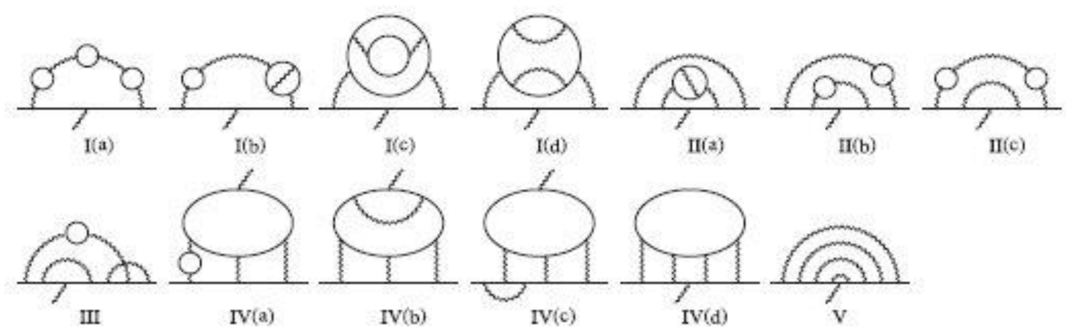
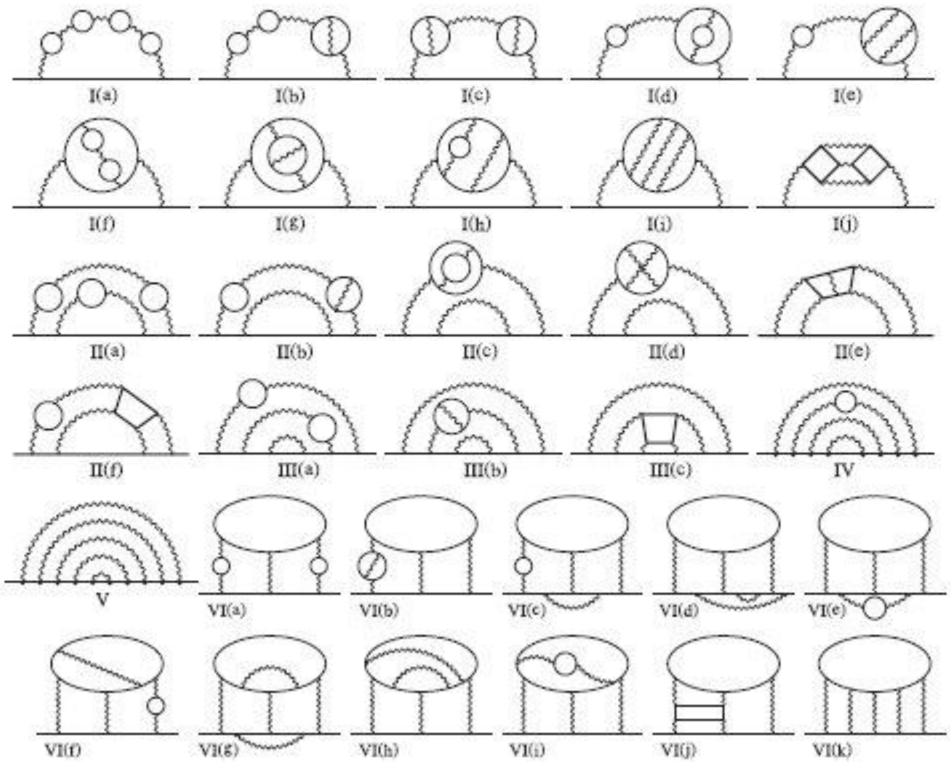


+...

891 vertex diagrams

...+



12672 self energy diagrams

$(g\text{-}2)/2 = 1\ 159\ 652\ 180.73\ (0.28) \times 10^{-12}$ ✓

g − 2 measurement by the Harvard Group using a Penning trap

T. Aoyama et. al. Prog. Theor. Exp. Phys. **2012**, 01A107

THE LOW ENERGY PHYSICS OF HIGH ENERGY PHYSICS, OR THE DARK SIDE OF ASYMPTOTIC FREEDOM

$$\alpha_{QCD} > 1 \, , V_{QCD}(r) \propto r$$

non-perturbative confinement effect!

No free quraks: they construct Hadrons:

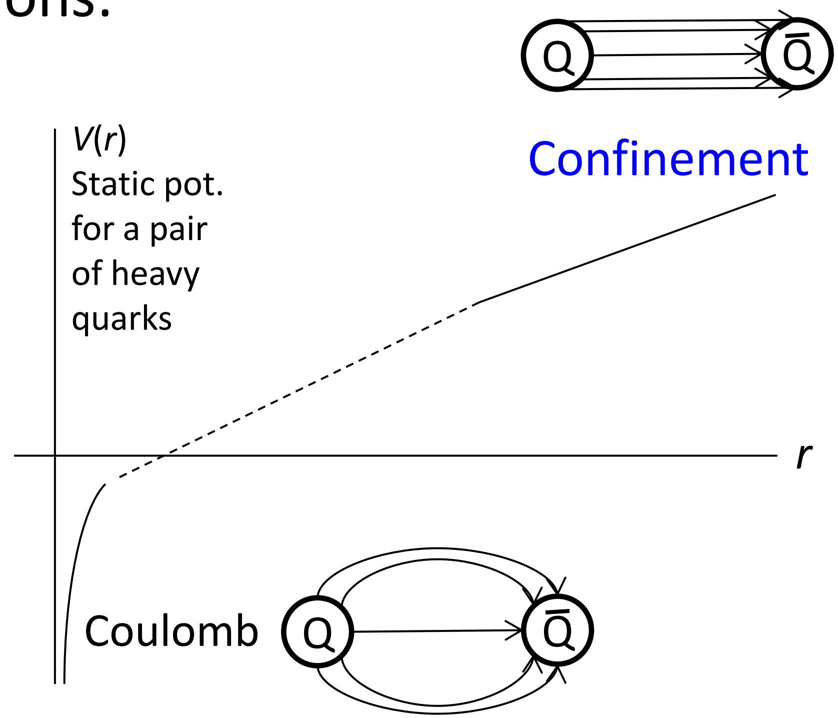
Mesons (two quarks),

Baryons (three quarks),

...

Color Electric flux-tubes:

“a non-abelian Meissner effect”.



ASYMPTOTIC FREEDOM

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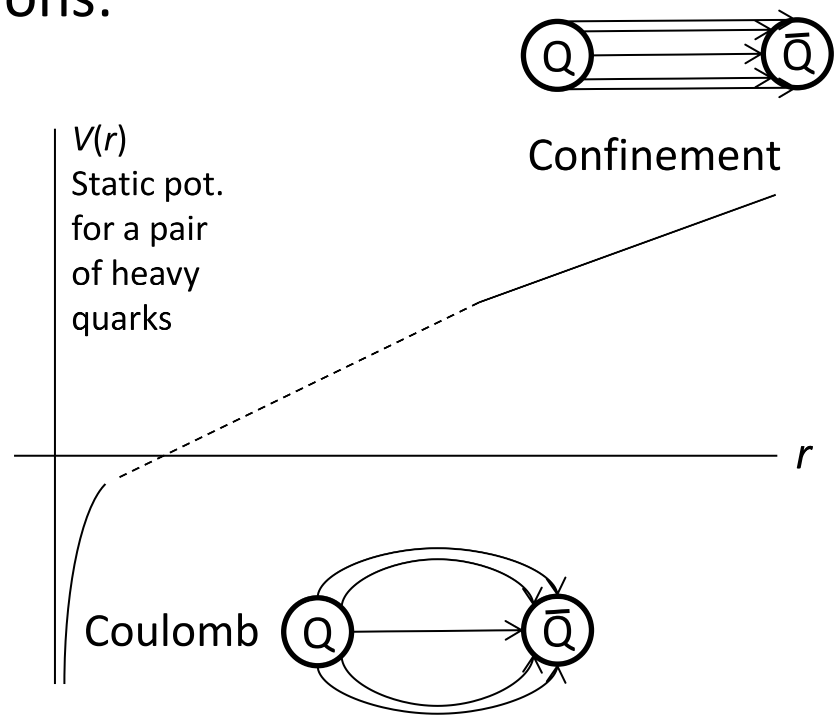


and Calculate!

Compared with CM simulations, several additional requirements when trying to simulate HEP models

REQUIREMENT 1

One needs both bosons and fermions

Fermion fields : = Matter

Bosonic, Gauge fields:= Interaction mediators

Ultracold atoms:
One can have bosonic
and fermionic species

REQUIREMENT 2

The theory has to be relativistic = have a causal structure.

The atomic dynamics (and Hamiltonian) is nonrelativistic.

We can use lattice gauge theory.
The continuum limit will be then relativistic.

REQUIREMENT 3

The theory has to be local gauge invariant.

local gauge invariance = "charge" conservation

Atomic Hamiltonian conserves total number – seem to have *only global symmetry*

It turns out that local gauge invariance can be obtained as either :

I)– a low energy approximate symmetry.

II)– or "fundamentally" *from symmetries of atomic interactions.*

LATTICE GAUGE THEORY

- A very useful nonperturbative approach to gauge theories, especially QCD.
- Lattice partition and correlation functions computed using Monte Carlo methods in a discretized Euclidean spacetime (Wilson).

- However:
  Limited applicability with too many quarks / finite chemical potential (quark-gluon plasma, color superconductivity): Grassman integration → the computationally hard "sign problem"
- Euclidean correlations – No real time dynamics

LATTICE GAUGE THEORIES
HAMILTONIAN FORMULATION

PHYSICAL REVIEW D VOLUME 11, NUMBER 2 15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

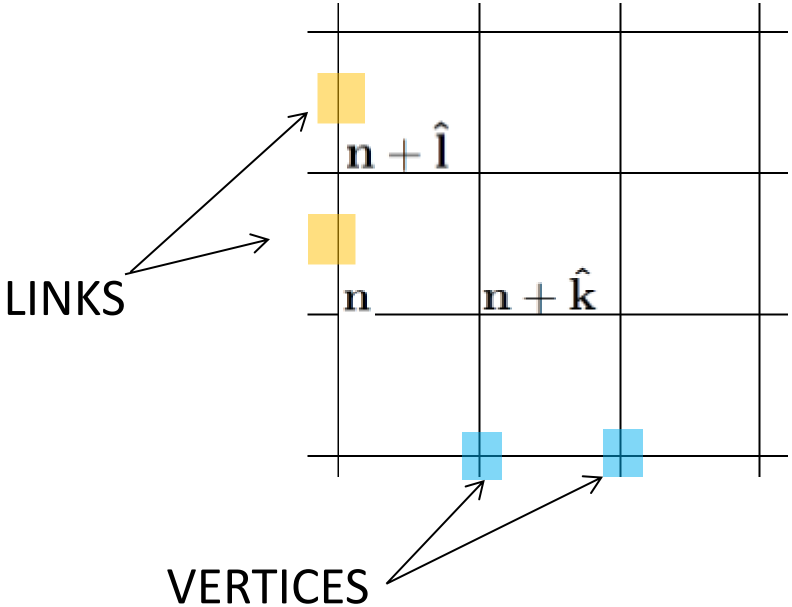
Leonard Susskind†
Belfer Graduate School of Science, Yeshiva University, New York, New York
and Tel Aviv University, Ramat Aviv, Israel
and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received 9 July 1974)

Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.

LATTICE GAUGE THEORIES

DEGREES OF FREEDOM

Gauge field degrees of freedom:
U(1), SU(N), etc, unitary matrices



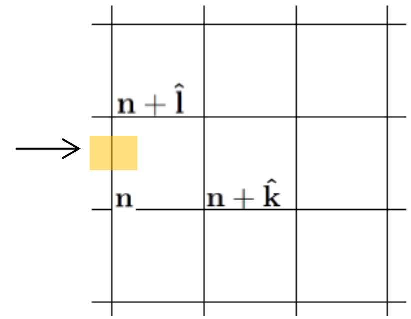
Matter degrees of freedom :
Spinors

$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \dots \end{pmatrix}$$

Gauge fields on the links

Gauge group elements:

U^r is an element of the gauge group (in the representation r), on each link



Left and right generators:

$$[L_a, U^r] = T_a^r U^r \quad ; \quad [R_a, U^r] = U^r T_a^r$$

$$[L_a, L_b] = -i f_{abc} L_c \quad ; \quad [R_a, R_b] = i f_{abc} R_c \quad ; \quad [L_a, R_b] = 0$$

$$\sum_a L_a L_a = \sum_a R_a R_a \equiv \sum_a E_a E_a$$

$\left\{ \; |J, m, m'\rangle \; \right\}$

Dynamical!

Gauge transformation:

Generators: $U^r_{\mathbf{n},k} \rightarrow V^r_{\mathbf{n}} U^r_{\mathbf{n},k} V^{\dagger r}_{\mathbf{n}+\hat{\mathbf{k}}}$

$$(G_{\mathbf{n}})_a = \mathrm{div}_{\mathbf{n}} E_a = \sum_k \left((L_{\mathbf{n},k})_a - \left(R_{\mathbf{n}-\hat{\mathbf{k}},k} \right)_a \right)$$

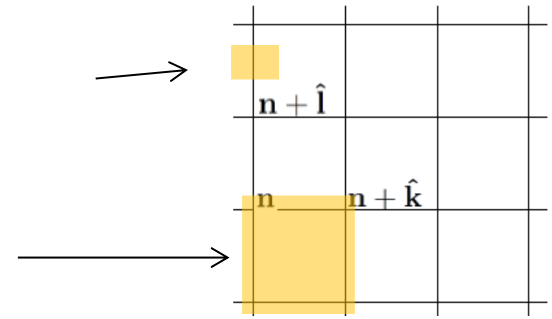
Left and right “electric” fields

LATTICE GAUGE THEORIES
NON-ABELIAN HAMILTONIAN

Gauge field dynamics (Kogut-Susskind Hamiltonian):

$$H_E = \frac{g^2}{2} \sum_{\mathbf{n},k,a} \left(E_{\mathbf{n},k}\right)_a \left(E_{\mathbf{n},k}\right)_a$$

$$H_B = -\frac{1}{g^2} \sum_{\text{plaquettes}} \left(\mathrm{Tr} \left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$



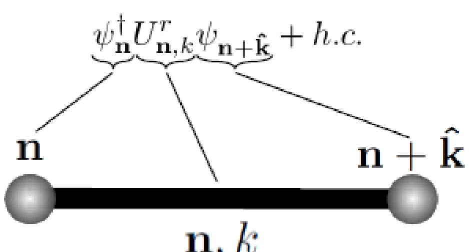
Local gauge invariance: acting on a single vertex

Strong coupling limit: $g >> 1$
Weak coupling limit: $g << 1$

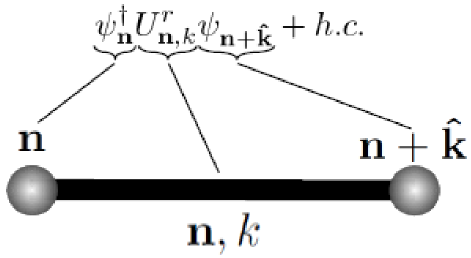
Matter dynamics:

$$H_M = \sum_{\mathbf{n}} M_{\mathbf{n}} \psi_{\mathbf{n}}^\dagger \psi_{\mathbf{n}}$$

$$H_{int} = \epsilon \sum_{\mathbf{n},k} \left(\psi_{\mathbf{n}}^\dagger U_{\mathbf{n},k}^r \psi_{\mathbf{n}+\hat{\mathbf{k}}} + h.c. \right)$$



Local Gauge invariance



$$\psi_{\mathbf{n}} \to V^{r}_{\mathbf{n}} \psi_{\mathbf{n}}$$

$$U^{r}_{\mathbf{n},k} \to V^{r}_{\mathbf{n}} U^{r}_{\mathbf{n},k} V^{\dagger r}_{\mathbf{n}+\hat{\mathbf{k}}}$$

$$H_{int} = \epsilon \sum_{\mathbf{n},k} \left(\psi^{\dagger}_{\mathbf{n}} U^{r}_{\mathbf{n},k} \psi_{\mathbf{n}+\hat{\mathbf{k}}} + h.c. \right)$$

A symmetry that is satisfied for each link separately

Example
compact – QED (cQED)

U(1) gauge theory

Start with a hopping fermionic Hamiltonian, in 1 spatial direction

$$H = \sum_n M_n \psi_n^\dagger \psi_n + \alpha_n \left(\psi_n^\dagger \psi_{n+1} + H.c.\right)$$

This Hamiltonian is invariant to global gauge transformations,

$$\psi_n \longrightarrow e^{-i\Lambda} \psi_n \quad ; \quad \psi_n^\dagger \longrightarrow e^{i\Lambda} \psi_n^\dagger$$

U(1) gauge theory

Promote the gauge transformation to be local:

$$\psi_n \longrightarrow e^{-i\Lambda_n}\psi_n \quad ; \quad \psi_n^\dagger \longrightarrow e^{i\Lambda_n}\psi_n^\dagger$$

Then, in order to make the Hamiltonian gauge invariant, add unitary operators, U_n,

$$H = \sum_n M_n \psi_n^\dagger \psi_n + \alpha_n\left(\psi_n^\dagger U_n \psi_{n+1} + H.c.\right)$$

$$U_n = e^{i\theta_n} \quad ; \quad \theta_n \longrightarrow \theta_n + \Lambda_{n+1} - \Lambda_n$$

Dynamics

Add *dynamics* to the gauge field:

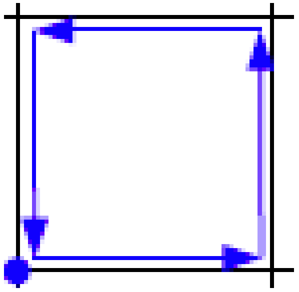
$$H_E = \frac{g^2}{2} \sum_n L_{n,z}^2$$

L_n is the angular momentum operator conjugate to θ_n , representing the (integer) electric field.

Plaquette

In d>1 spatial dimensions, interaction terms along <u>plaquettes</u>

$$-\frac{1}{g^2}\sum_{m,n}\cos\left(\theta^1_{m,n}+\theta^2_{m+1,n}-\theta^1_{m,n+1}-\theta^2_{m,n}\right)$$



In the continuum limit, this corresponds to $(\nabla\times\boldsymbol{A})^2$ - gauge invariant magnetic energy term.

Figure from ref [6]

cQED -> QED

$$U_{\mathbf{n},k} = e^{i\phi_{\mathbf{n},k}}$$

$$[E_{\mathbf{n},k}, \phi_{\mathbf{m},l}] = -i\delta_{\mathbf{nm}}\delta_{kl}$$

E is quantized! = L_z

$$(\nabla \times \boldsymbol{A})^2$$

plaquette

$$\frac{g^2}{2}\sum_{\mathbf{n},k} E^2_{\mathbf{n},k} - \frac{1}{g^2}\sum_{\mathbf{n}} \cos\left(\phi_{\mathbf{n},1} + \phi_{\mathbf{n}+\hat{\mathbf{1}},2} - \phi_{\mathbf{n}+\hat{\mathbf{2}},1} - \phi_{\mathbf{n},2}\right)$$

$$+ \ \epsilon\sum_{\mathbf{n},k}\left(\psi^\dagger_{\mathbf{n}} e^{i\phi_{\mathbf{n},k}}\psi_{\mathbf{n}+\hat{\mathbf{k}}} + \psi^\dagger_{\mathbf{n}+\hat{\mathbf{k}}} e^{-i\phi_{\mathbf{n},k}}\psi_{\mathbf{n}}\right) + M\sum_n (-1)^n \psi^\dagger_n \psi_n$$

$$J \cdot A$$

Gauge-Matter interaction

End

Example (cQED)

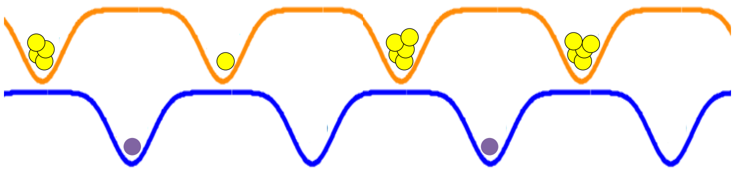
Next: we move on to atomic lattices

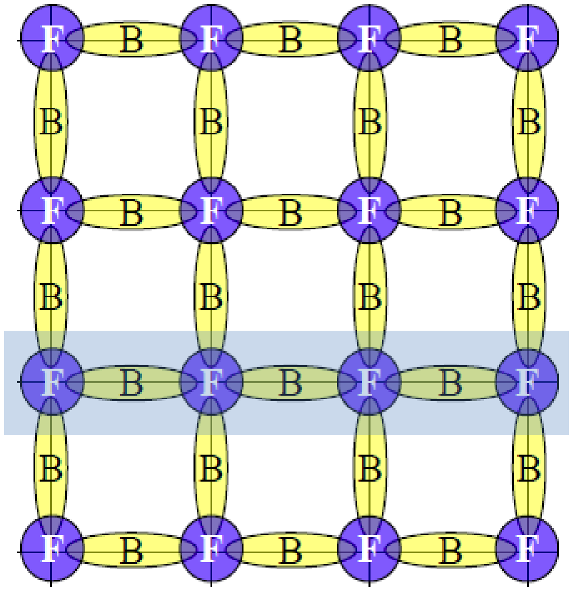
QUANTUM SIMULATION

COLD ATOMS

- Fermion matter fields
- Bosonic gauge fields

Superlattices:





$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \cdots \end{pmatrix} \longrightarrow$$ Atom internal levels

QUANTUM SIMULATION
LOCAL GAUGE INVARIANCE

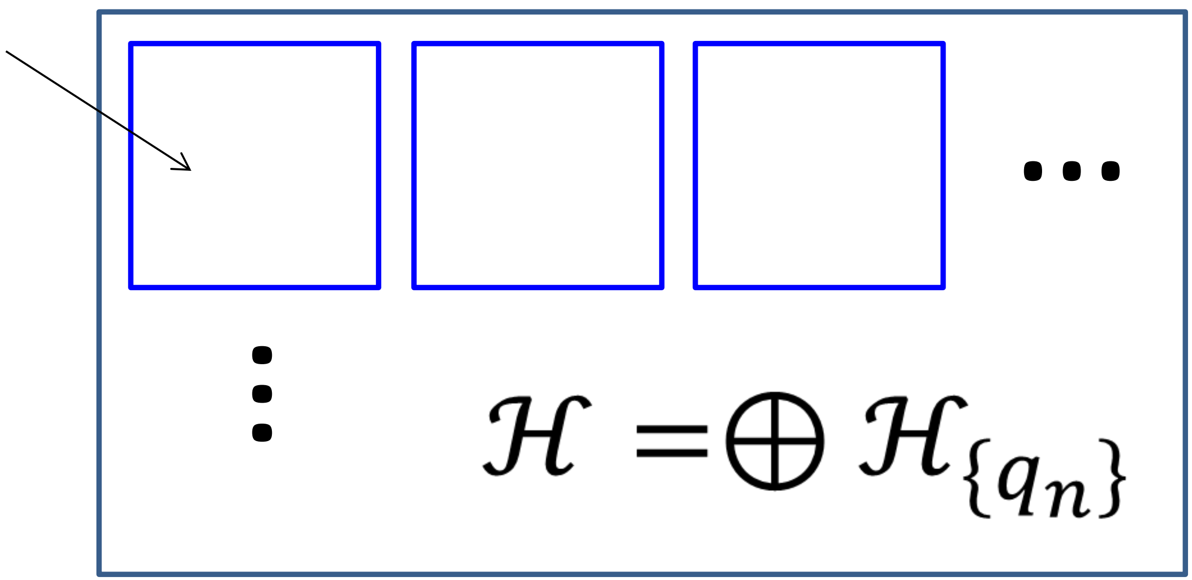
- Generators of gauge transformations:

$$(G_{\mathbf{n}})_a = \mathrm{div}_{\mathbf{n}} E_a - Q_{\mathbf{n}}$$

$$G_{\mathbf{n}} \left| phys \right\rangle = q_{\mathbf{n}} \left| phys \right\rangle$$

$$[G_n, H] = 0$$

Sector w. fixed charge



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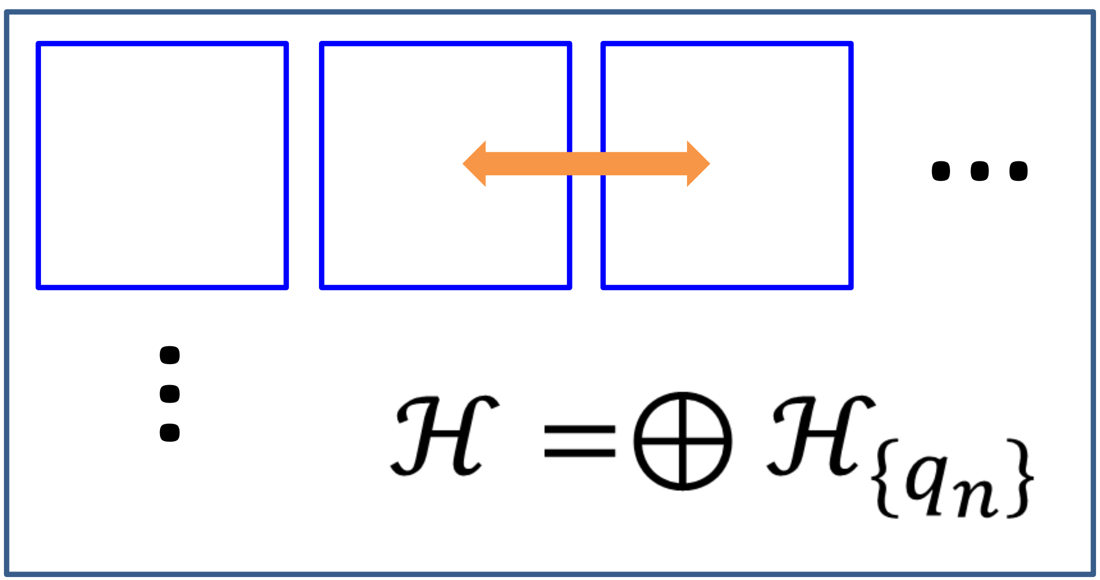

$$\mathcal{H} = \bigoplus \mathcal{H}_{\{q_n\}}$$

QUANTUM SIMULATION
LOCAL GAUGE INVARIANCE

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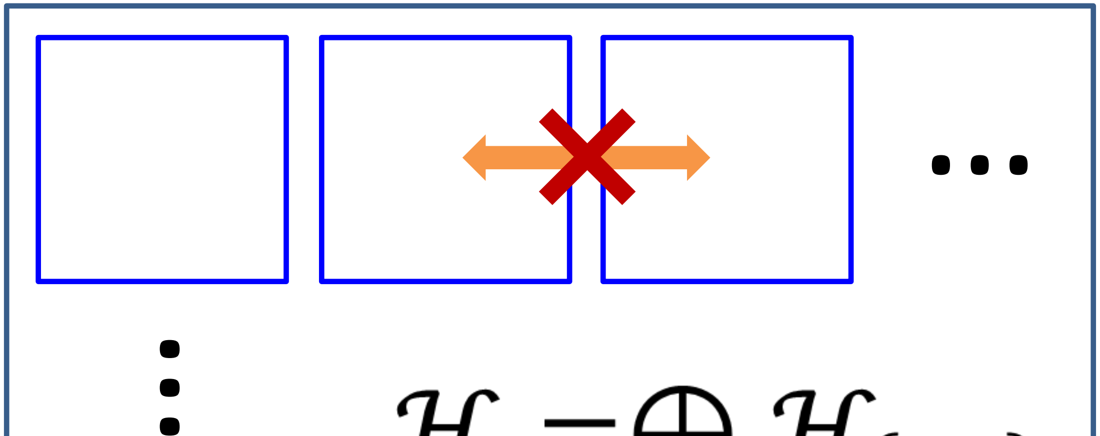
$$G_{\mathbf{n}} \left| phys \right\rangle = q_{\mathbf{n}} \left| phys \right\rangle$$

$$[G_n, H] = 0$$



$$\mathcal{H} = \bigoplus \mathcal{H}_{\{q_n\}}$$

QUANTUM SIMULATION
LOCAL GAUGE INVARIANCE

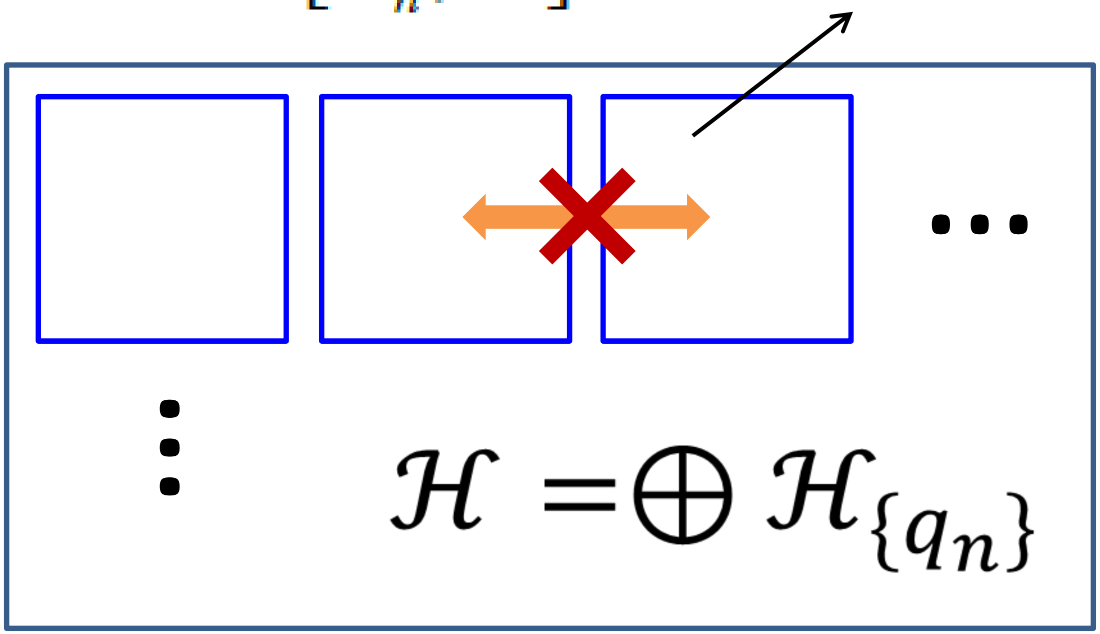
- Generators of gauge transformations:

$$(G_{\mathbf{n}})_a = \mathrm{div}_{\mathbf{n}} E_a - Q_{\mathbf{n}}$$

$$G_{\mathbf{n}} \left|phys\right\rangle = q_{\mathbf{n}} \left|phys\right\rangle$$

$$[G_n, H] = 0$$

local gauge invariance!!

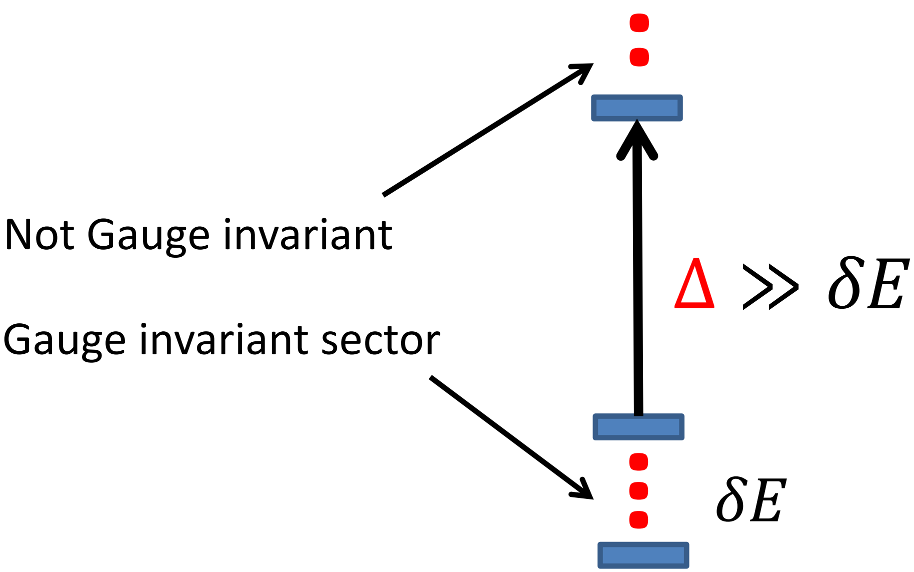


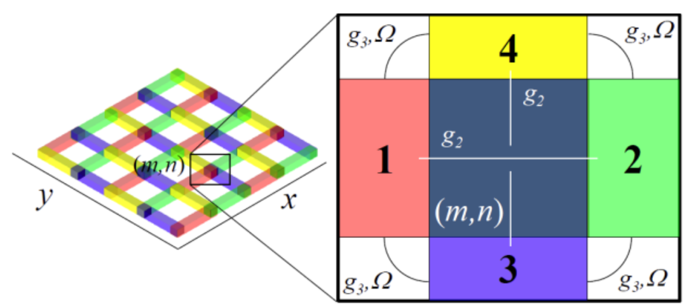
Local Gauge Invariance
at low enough energies

Gauss's law is added as a constraint.

Leaving the gauge invariant sector of Hilbert space costs too much Energy.

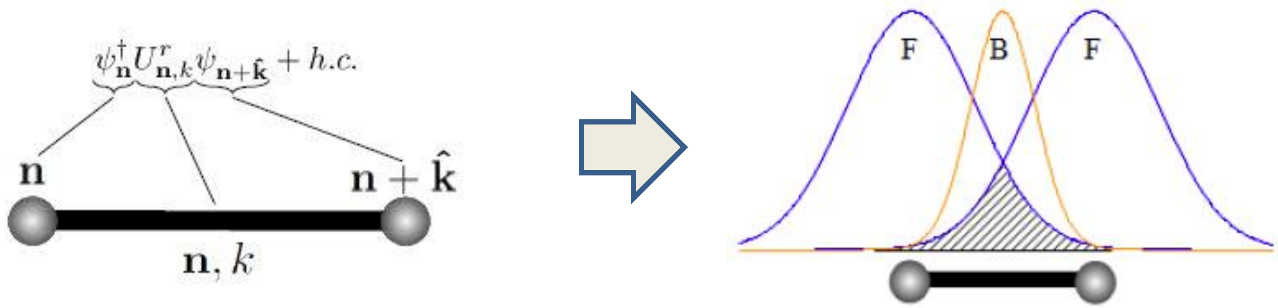
Low energy effective gauge invariant Hamiltonian.



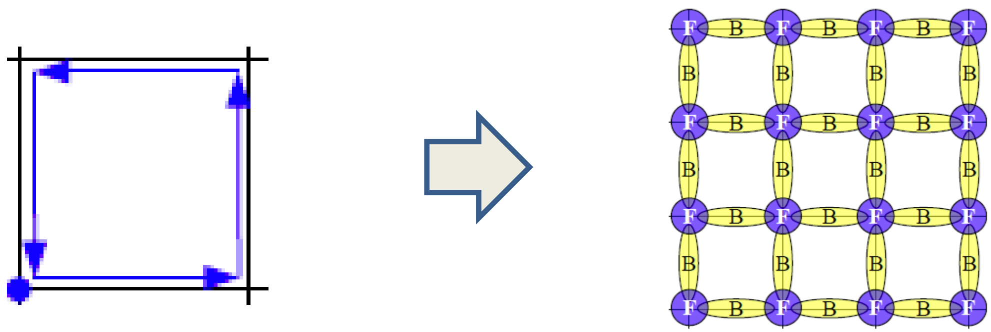


E. Zohar, BR, Phys. Rev. Lett. 107, 275301 (2011)

LGI is exact : emerging from some microscopic symmetries

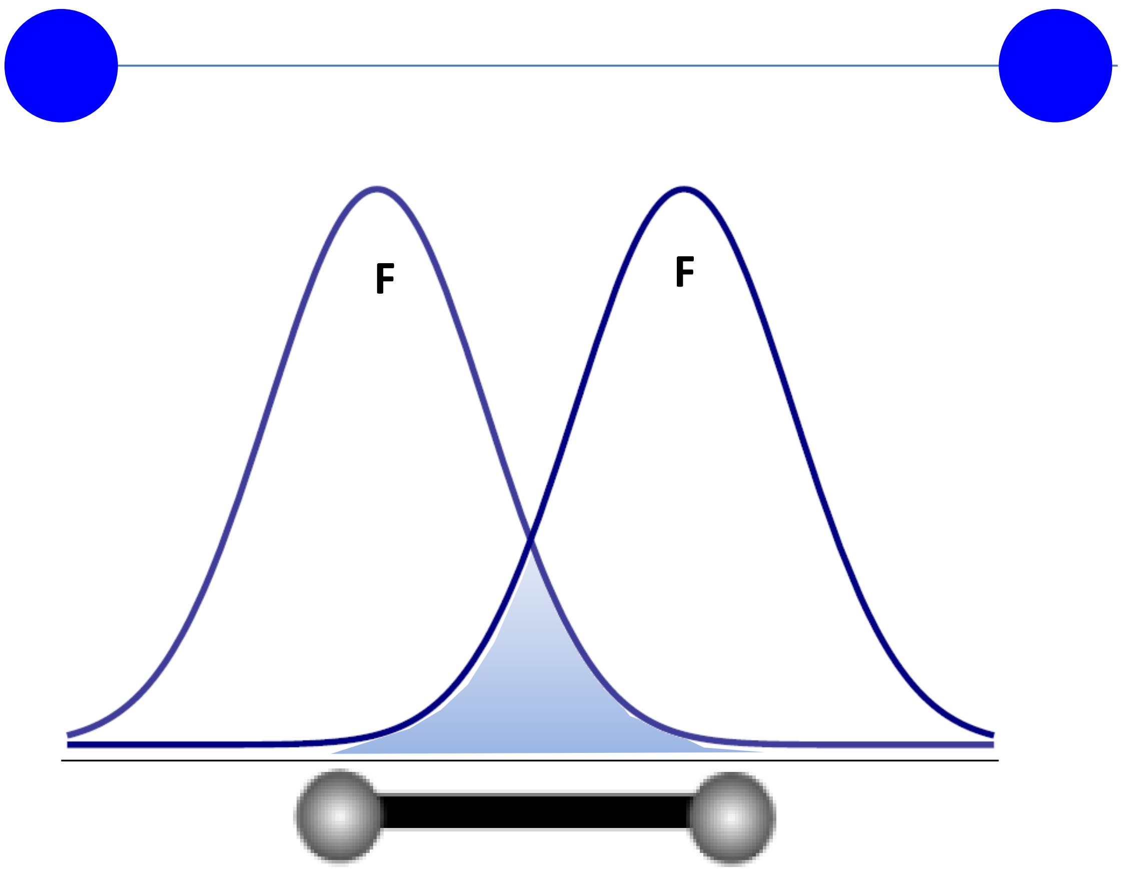


- Links $\leftrightarrow$ atomic scattering : gauge invariance is a <u>fundamental</u> symmetry



- Plaquettes $\leftrightarrow$ gauge invariant links $\leftrightarrow$ virtual loops of ancillary fermions.

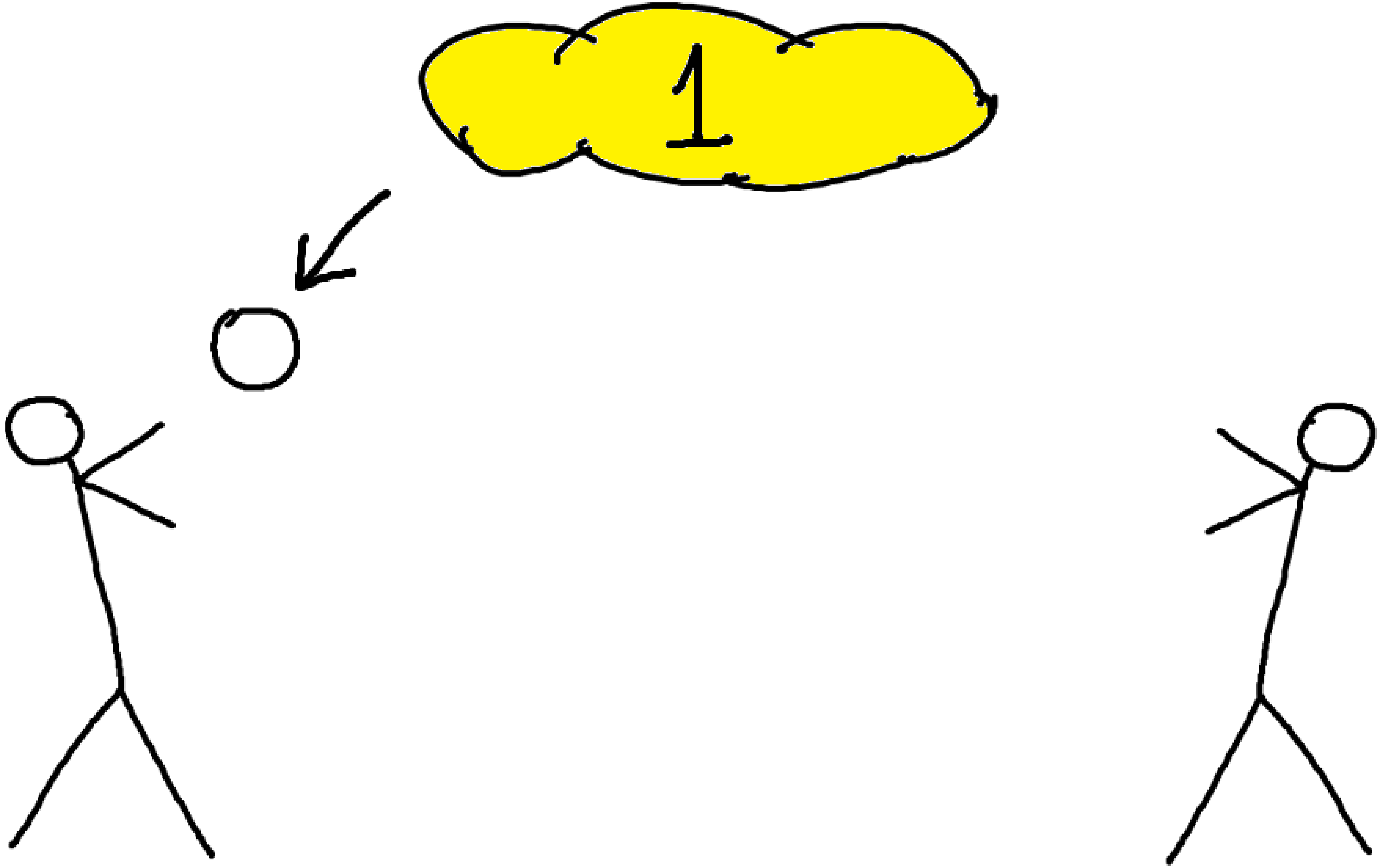
GLOBAL GAUGE INVRAIANT = FERMION HOPPING



GLOBAL GAUGE INVRAIANT = FERMION HOPPING

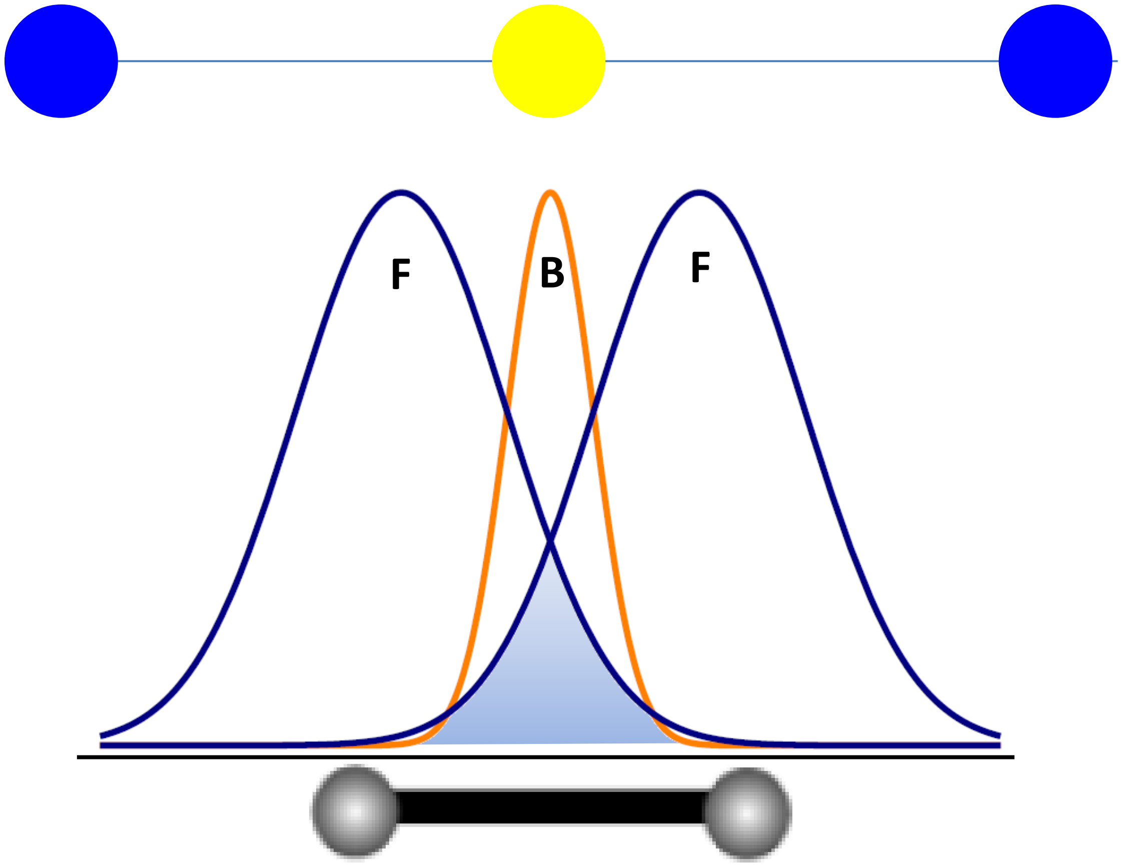
GLOBAL GAUGE INVRAIANT = FERMION HOPPING

LOCAL GAUGE INVARIANCE: ADD A MEDIATOR !

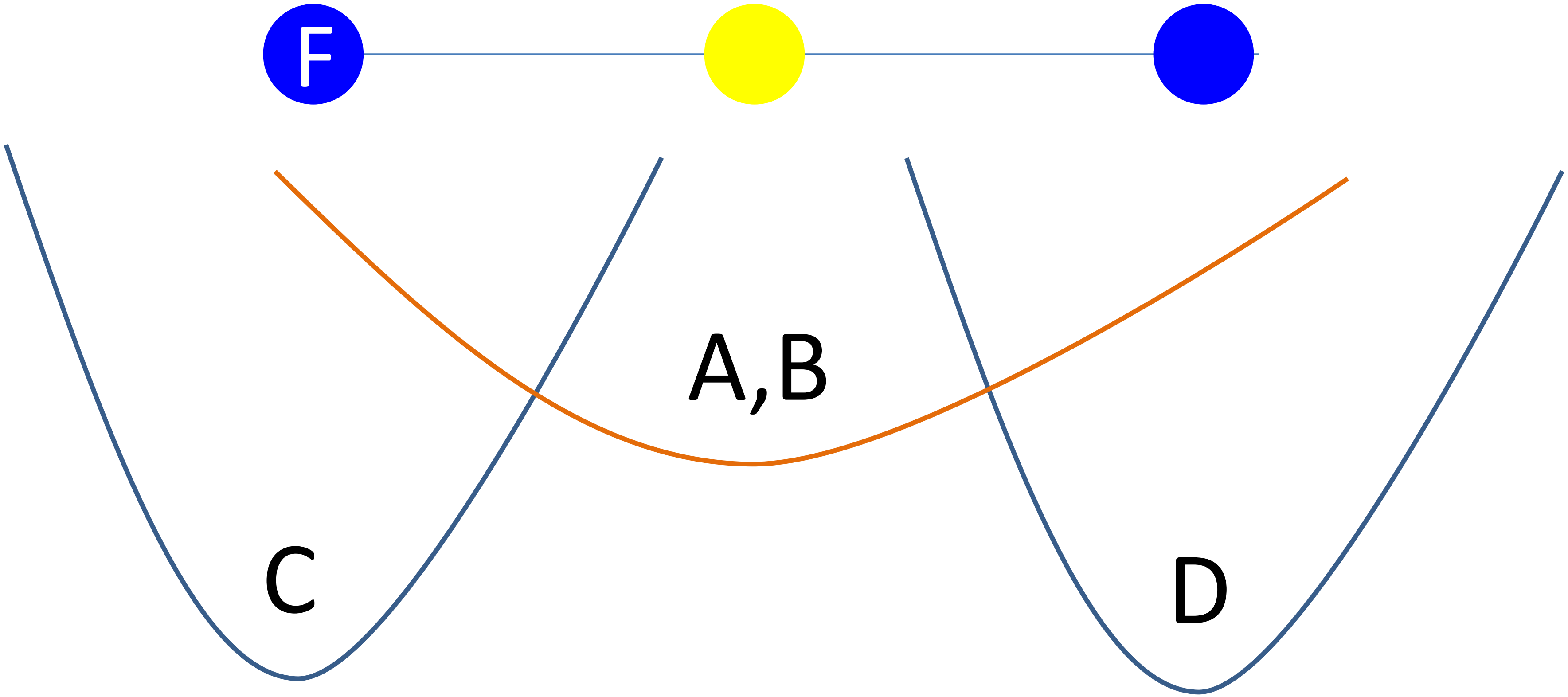


EXAMPLE – cQED

LINK INTERACTIONS

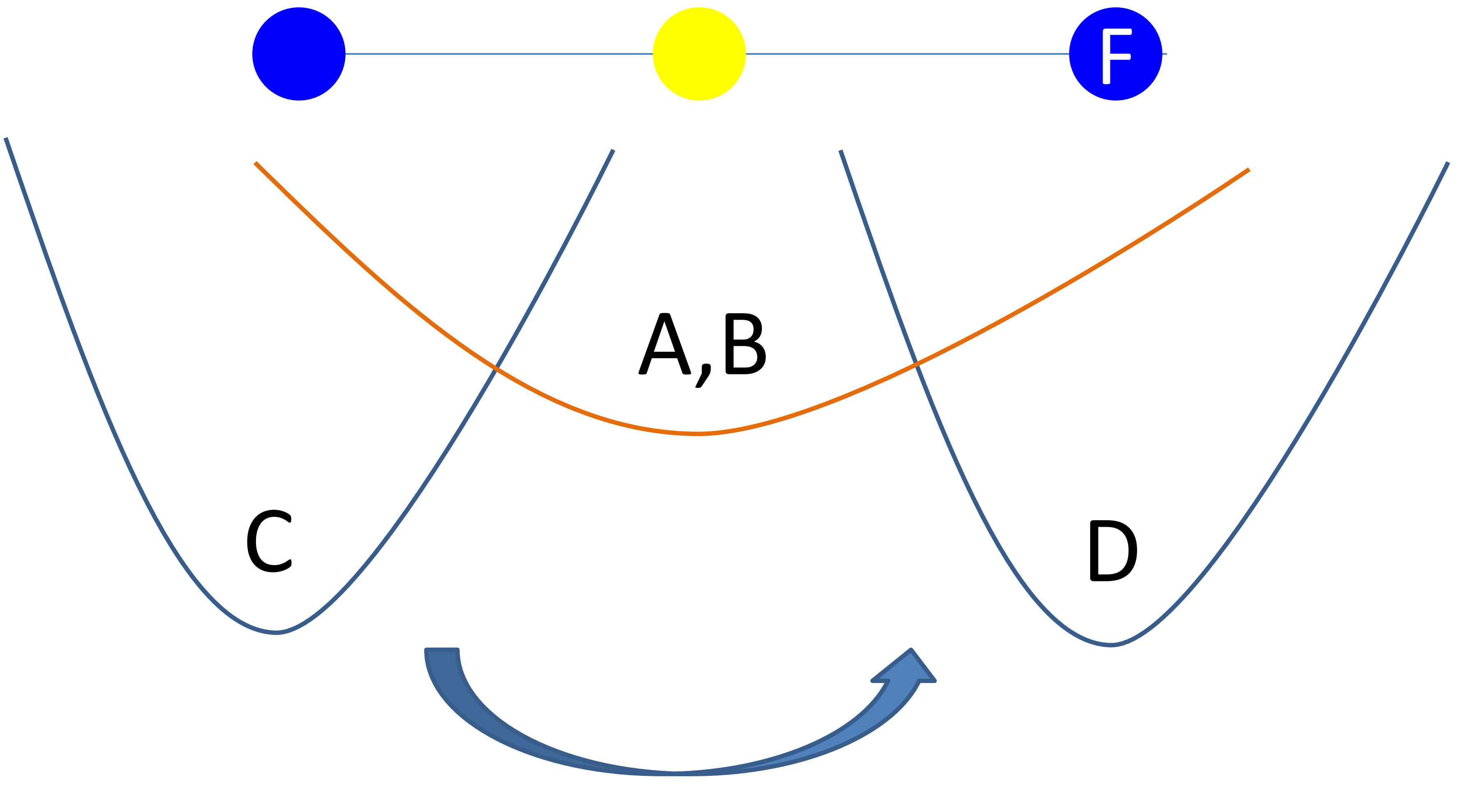


EXAMPLE – cQED LINK INTERACTIONS
LOCAL GAUGE INVARIANCE: ADD A MEDIATOR



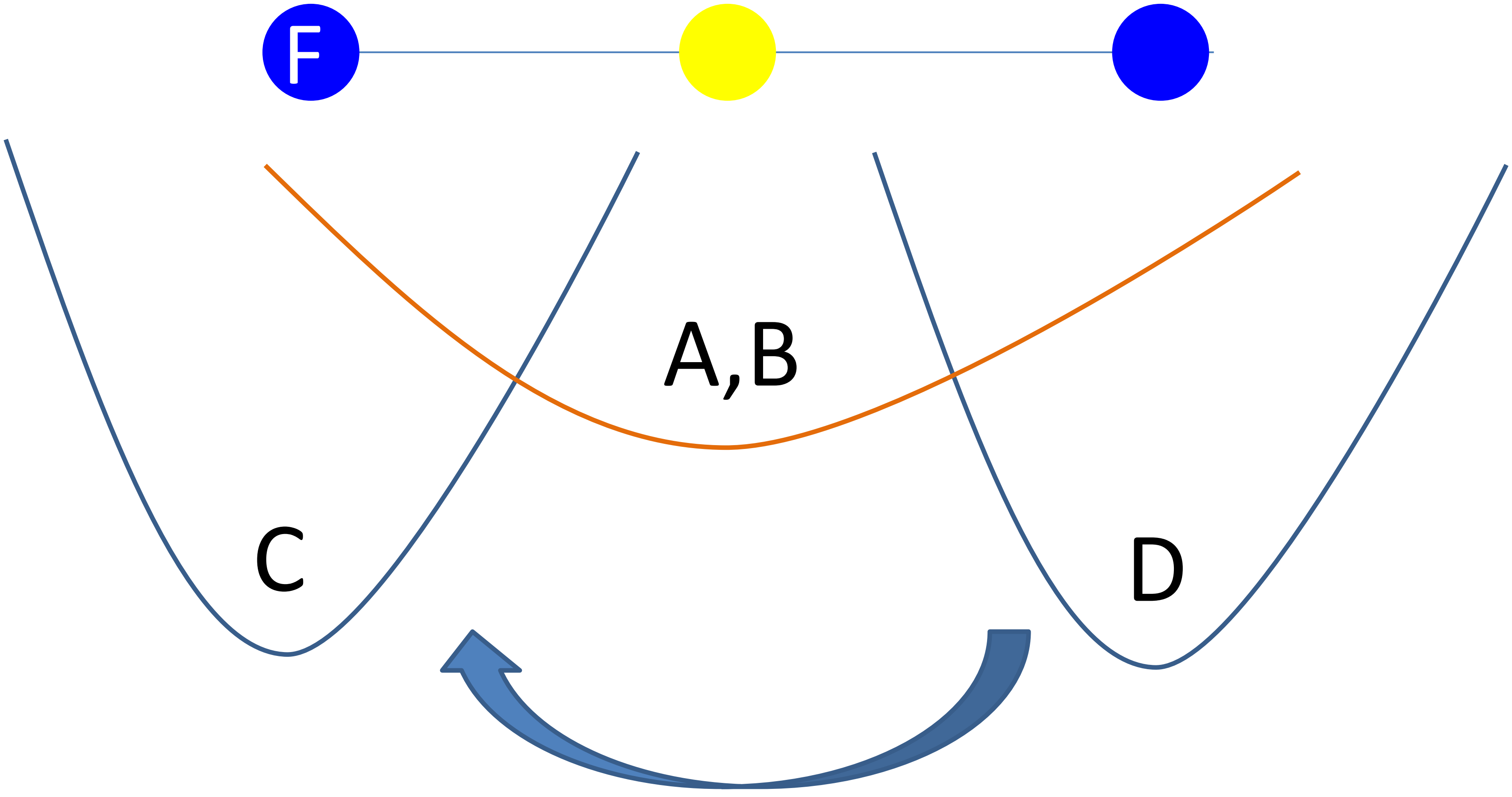
EXAMPLE – cQED
LINK INTERACTIONS

$$L \to L - 1$$

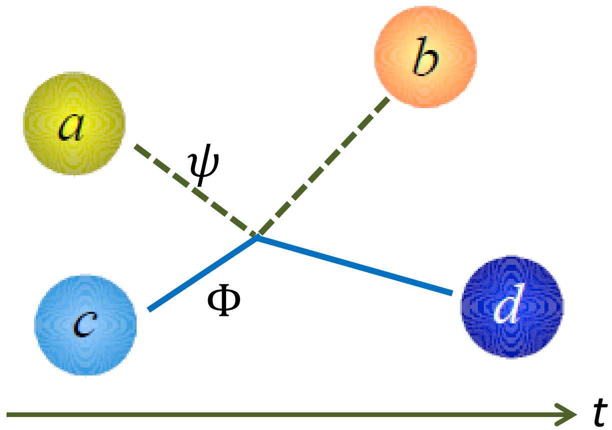


EXAMPLE – cQED
LINK INTERACTIONS

$$L \to L + 1$$



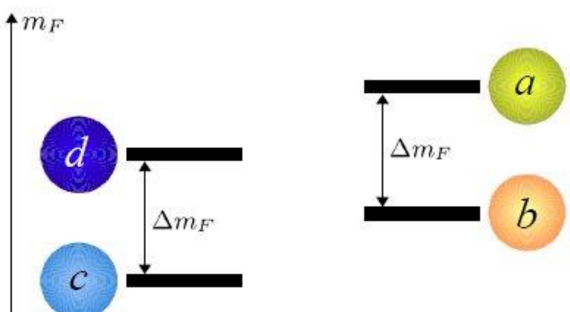
(HYPERFINE) ANGULAR MOMENTUM CONSERVATION ATOMIC SCATTERING



Hyperfine angular momentum conservation in atomic scattering.

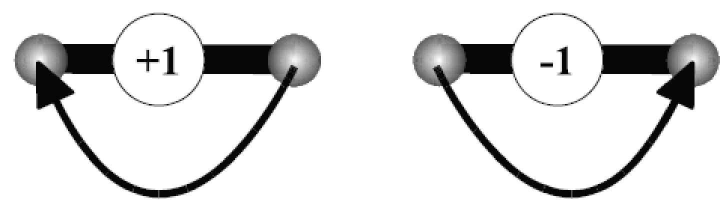
$$\int d^3\mathbf{x}\,\Psi^\dagger_\alpha(\mathbf{x})\,\Psi_\beta(\mathbf{x})\,\Phi^\dagger_\gamma(\mathbf{x})\,\Phi_\delta(\mathbf{x})$$

$$m_F(a) + m_F(c) = m_F(b) + m_F(d)$$

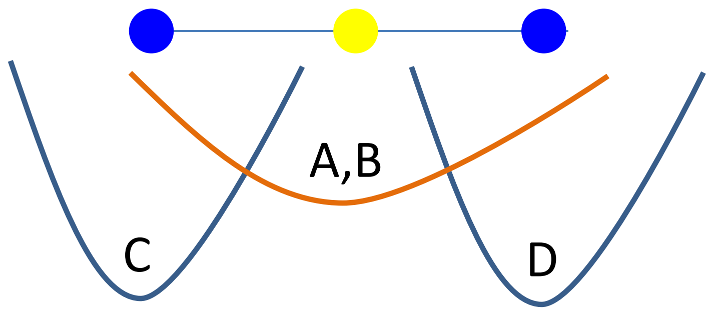


Fermionic atoms Bosonic atoms

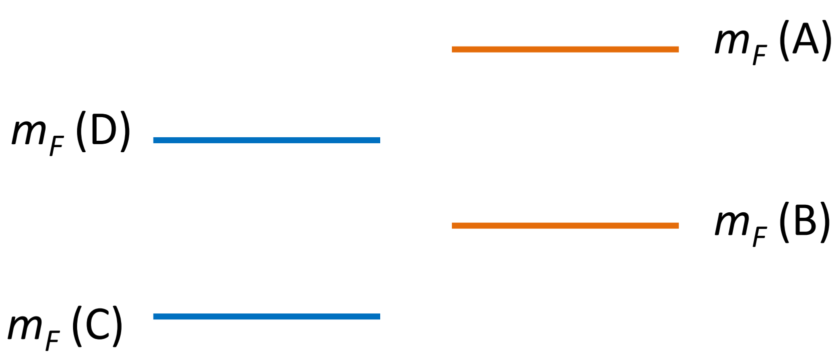
$$\underbrace{\psi^\dagger_{\mathbf{n}} e^{i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}+\hat{\mathbf{k}}}} + \underbrace{\psi^\dagger_{\mathbf{n}+\hat{\mathbf{k}}} e^{-i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}}}$$



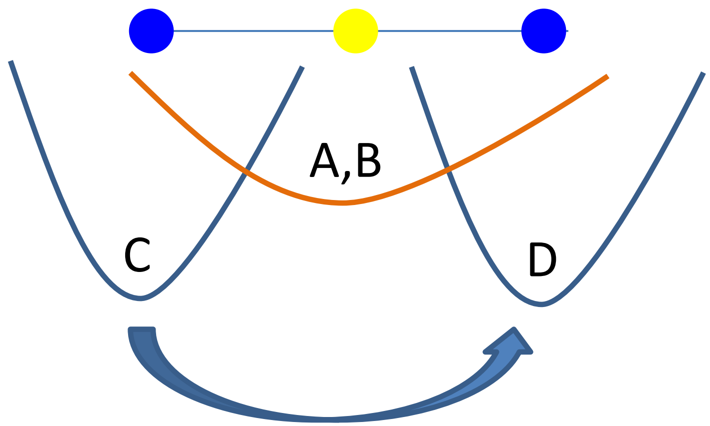
ANG. MOM. CONSERVATION $\leftrightarrow$ LOCAL GAUGE INVARIANCE



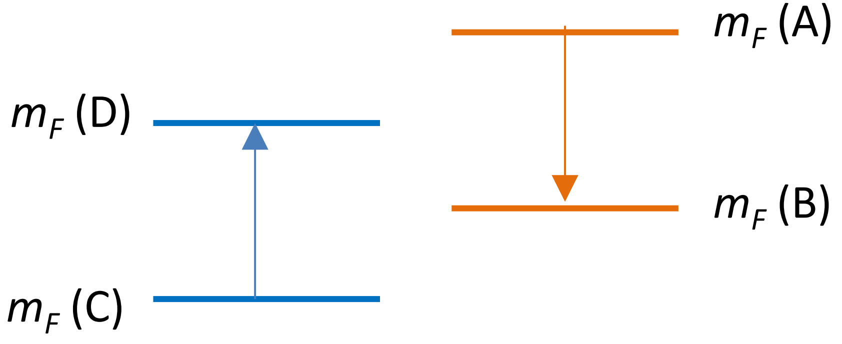
$$c^\dagger a^\dagger b d + d^\dagger b^\dagger a c$$



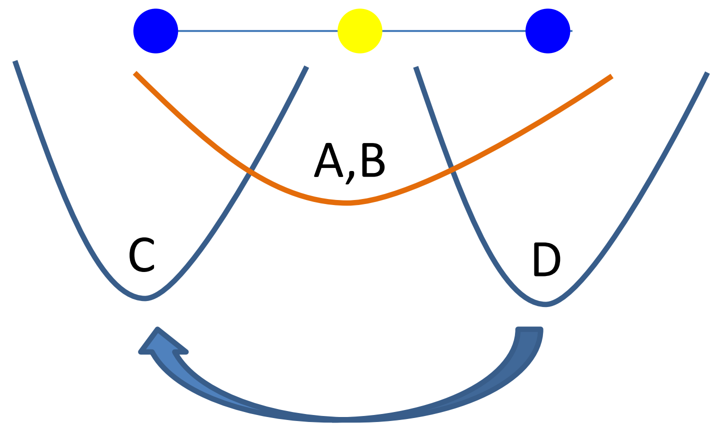
ANG. MOM. CONSERVATION $\leftrightarrow$ LOCAL GAUGE INVARIANCE



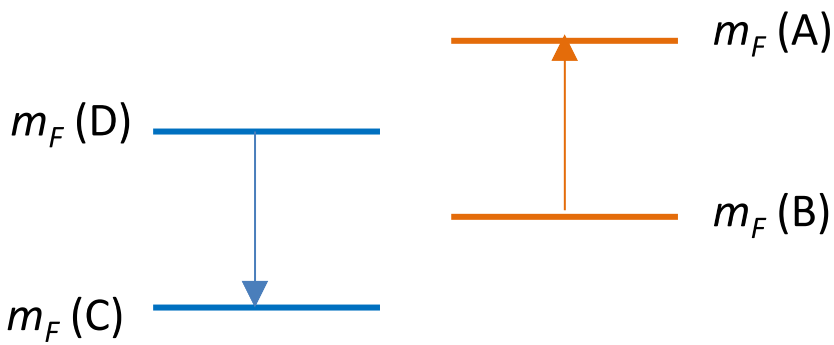
$$c^\dagger a^\dagger b d + d^\dagger b^\dagger a c$$



ANG. MOM. CONSERVATION $\leftrightarrow$ LOCAL GAUGE INVARIANCE



$$c^\dagger a^\dagger b d + d^\dagger b^\dagger a c$$



GAUGE BOSONS: SCHWINGER'S ALGEBRA

$$L_+ = a^\dagger b \qquad L_- = b^\dagger a$$

$$L_z = \frac{1}{2}\left(a^\dagger a - b^\dagger b\right) \qquad \ell = \frac{1}{2}\left(a^\dagger a + b^\dagger b\right)$$

and thus what we have is

$$c^\dagger a^\dagger b d + d^\dagger b^\dagger a c$$

$$\Downarrow$$

$$\psi_L^\dagger L_+ \psi_R + \psi_R^\dagger L_- \psi_L$$

GAUGE BOSONS: SCHWINGER'S ALGEBRA

$$L_+ = a^\dagger b \qquad L_- = b^\dagger a$$

$$L_z = \frac{1}{2}\left(a^\dagger a - b^\dagger b\right) \qquad \ell = \frac{1}{2}\left(a^\dagger a + b^\dagger b\right)$$

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GAUGE BOSONS: SCHWINGER'S ALGEBRA

$$\psi_L^\dagger L_+ \psi_R + \psi_R^\dagger L_- \psi_L$$

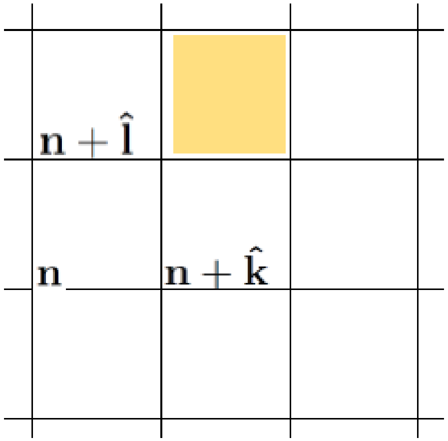
For large ℓ , $m \ll \ell$

$$L_+ = a^\dagger b \sim e^{i(\phi_a - \phi_b)} \equiv e^{i\phi} = U$$

$$\boxed{\psi_L^\dagger U \psi_R + \psi_R^\dagger U^\dagger \psi_L} \checkmark$$

Qualitatively similar results can be obtained with just two bosons on the link, as the U(1) gauge symmetry is ℓ-independent.

PLAQUETTES

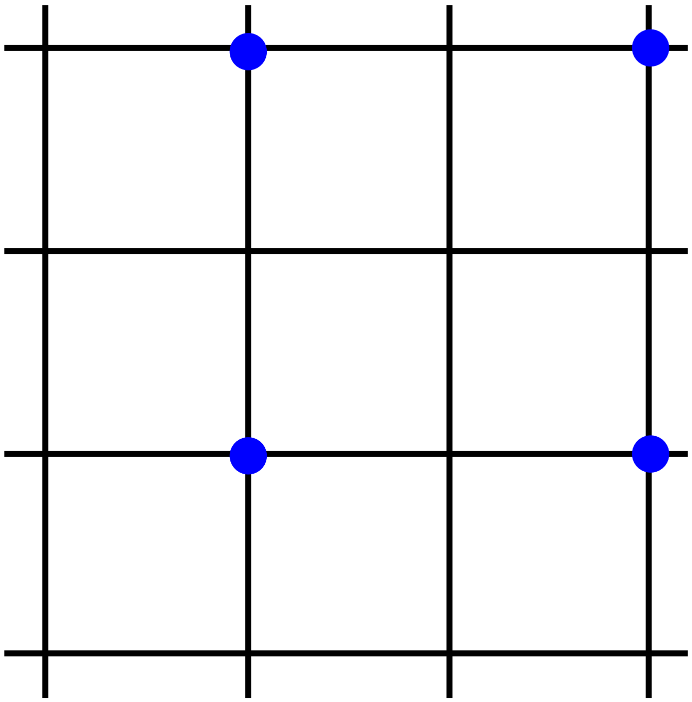


PLAQUETTES

1d elementary link interactions are **already gauge invariant**

Auxiliary fermions

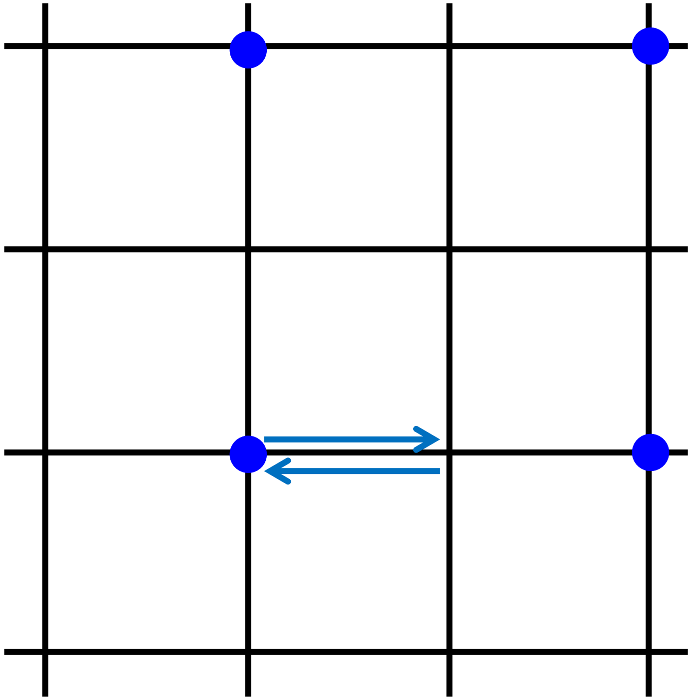
:= ●



PLAQUETTES

Auxiliary fermions

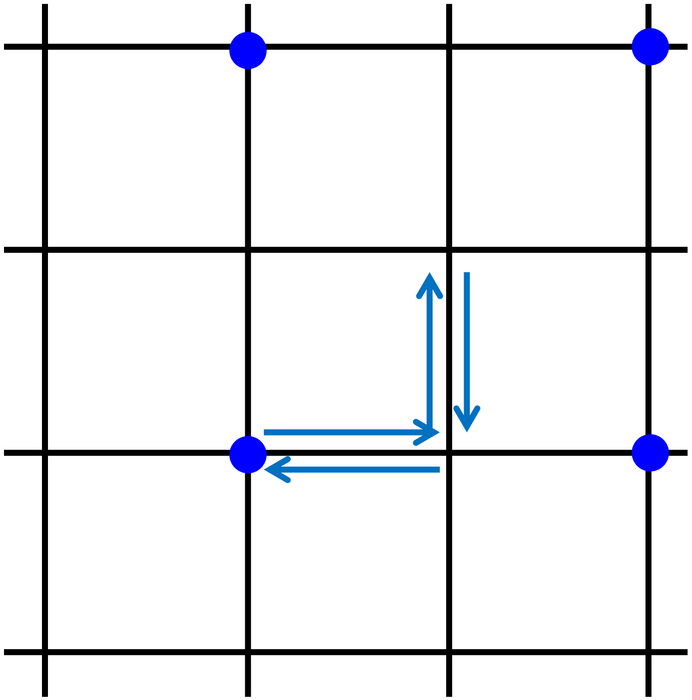
– virtual processes



PLAQUETTES

Auxiliary fermions

– virtual processes



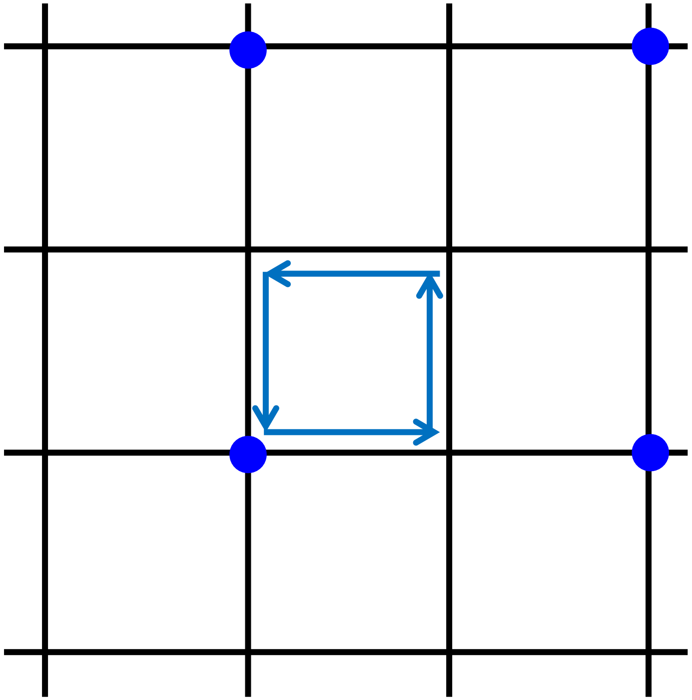
PLAQUETTES

Auxiliary fermions

– virtual processes

- plaquettes.

discrete, abelian
& non-abelian
groups



$$\sum_{\text{plaquettes}} \left(\mathrm{Tr} \left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$

cQED U(1) PLAQUETTES

$$H_B = -\frac{2\epsilon^4}{\lambda^3}\sum_{\mathbf{n}}\left(U_{\mathbf{n},1}U_{\mathbf{n}+\hat{\mathbf{1}},2}U^{\dagger}_{\mathbf{n}+\hat{\mathbf{2}},1}U^{\dagger}_{\mathbf{n},2} + h.c.\right) =$$

$$-\frac{4\epsilon^4}{\lambda^3}\sum_{\mathbf{n}}\cos\left(\phi_{\mathbf{n},1} + \phi_{\mathbf{n}+\hat{\mathbf{1}},2} - \phi_{\mathbf{n}+\hat{\mathbf{2}},1} - \phi_{\mathbf{n},2}\right)$$

λ is the "energy penalty" of the auxiliary fermion
ϵ is the "link tunneling energy".

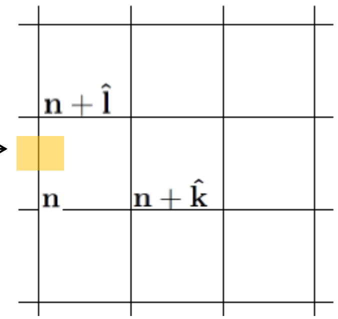
Only even orders contribute: effectively a second order process.

NON ABELIAN MODELS
YANG MILLS

LATTICE GAUGE THEORIES
HAMILTONIAN FORMULATION

Gauge group elements:

U^r is an element of the gauge group (in the representation *r*), on each link



Left and right generators:

$$[L_a, U^r] = T_a^r U^r \quad ; \quad [R_a, U^r] = U^r T_a^r$$

$$[L_a, L_b] = -i f_{abc} L_c \quad ; \quad [R_a, R_b] = i f_{abc} R_c \quad ; \quad [L_a, R_b] = 0$$

$$\sum_a L_a L_a = \sum_a R_a R_a \equiv \sum_a E_a E_a$$

$\left\{ \; |J, m, m'\rangle \; \right\}$

Gauge transformation:

Generators: $U^r_{\mathbf{n},k} \rightarrow V^r_{\mathbf{n}} U^r_{\mathbf{n},k} V^{\dagger r}_{\mathbf{n}+\hat{\mathbf{k}}}$

Dynamical!

$$\left(G_{\mathbf{n}}\right)_a = \mathrm{div}_{\mathbf{n}} E_a = \sum_k \left(\left(L_{\mathbf{n},k}\right)_a - \left(R_{\mathbf{n}-\hat{\mathbf{k}},k}\right)_a \right)$$

Left and right “electric” fields

SCHWINGER REPRESENTATION: SU(2)
PRE-POTENTIAL APPROACH

On each link – $a_{1,2}$ bosons on the left, $b_{1,2}$ bosons on the right

$$L_a = \frac{1}{2}\sum_{k,l} a_k^\dagger \, (\sigma_a)_{lk} \, a_l \; ; \; R_a = \frac{1}{2}\sum_{k,l} b_k^\dagger \, (\sigma_a)_{kl} \, b_l$$

$$[L_{n,a}, L_{n,b}] = -i\epsilon_{abc} L_{n,c}; \quad [R_{n,a}, R_{n,b}] = i\epsilon_{abc} R_{n,c}$$

In the fundamental representation -

$$U_L = \frac{1}{\sqrt{N_L + 1}} \begin{pmatrix} a_1^\dagger & -a_2 \\ a_2^\dagger & a_1 \end{pmatrix} \; ; \; U_R = \begin{pmatrix} b_1^\dagger & b_2^\dagger \\ -b_2 & b_1 \end{pmatrix} \frac{1}{\sqrt{N_R + 1}}$$

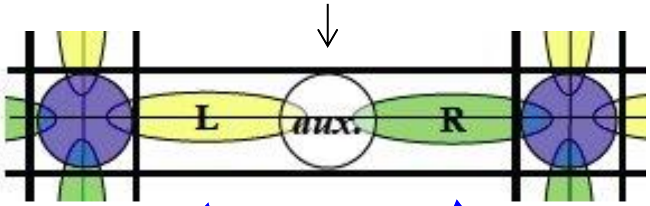
$$\boxed{U = U_L U_R}$$

M. Mathur, Journal of Physics A **38**, 10015 (2005)

SCHWINGER REPRESENTATION: SU(2)
REALIZATION

Ancillary “constraint” Fermion

“color” fermions



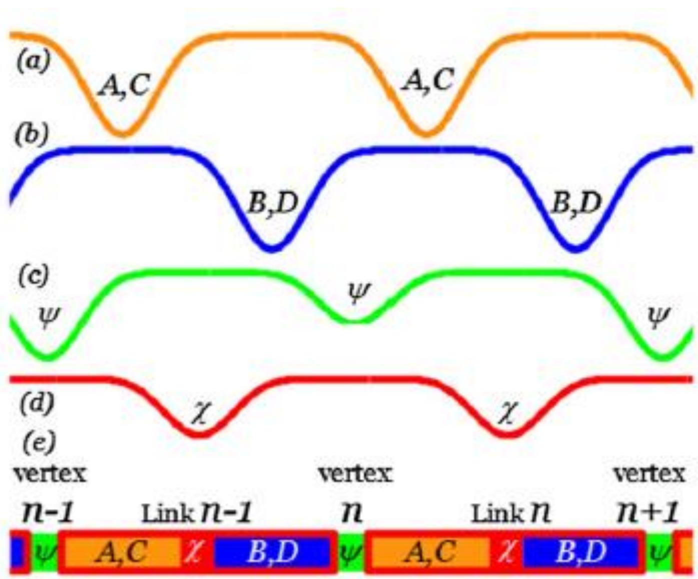
$$U_L = \frac{1}{\sqrt{N_L+1}} \begin{pmatrix} a_1^\dagger & -a_2 \\ a_2^\dagger & a_1 \end{pmatrix} \; ; \; U_R = \begin{pmatrix} b_1^\dagger & b_2^\dagger \\ -b_2 & b_1 \end{pmatrix} \frac{1}{\sqrt{N_R+1}}$$

$$\boxed{U = U_L U_R}$$

On each link – $a_{1,2}$ bosons on the left, $b_{1,2}$ bosons on the right

EXAMPLE: SU(2) IN 1+1

Superlattices →



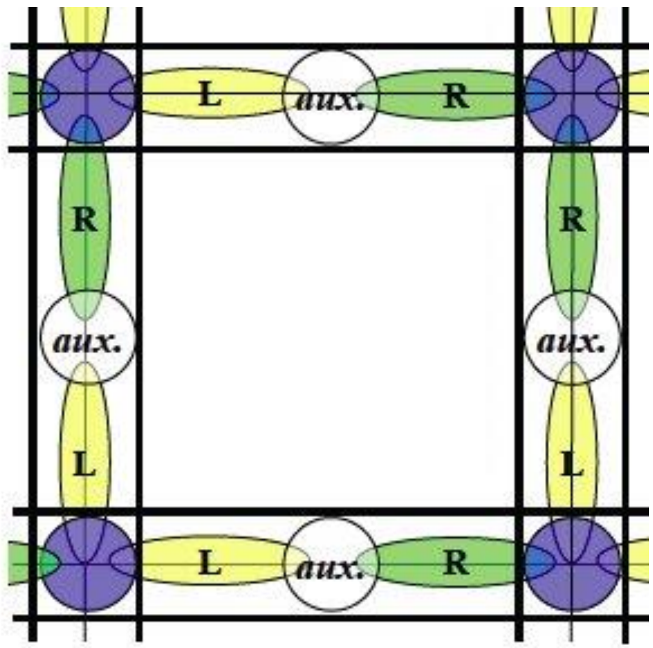
Fermions

L & R "gauge" bosons

$$\sum_{n,i,j}\left(\sqrt{N_{L,n}+1}\,(\psi_n^\dagger)_i\,(U_{L,n})_{ij}\,(\chi_n)_j+(\chi_n^\dagger)_i\,(U_{R,n})_{ij}\,(\psi_{n+1})_j\,\sqrt{N_{R,n}+1}+h.c.\right)$$

EXAMPLE: SU(2) IN 2+1

Non-abelian "charge"
Encoded in the relative
Rotation between R and L
("space and body frames"
of a rigid rotator)



FIRST STEPS

- Confinement in Abelian lattice models

- Toy models with "QCD-like properties" that capture the essential physics of confinement.

CONFINEMENT
Abelian TOY MODELS

- 1+1D: Schwinger's model.

- cQED: 2+1D: no phase transition
  Instantons give rise to confinement at $g < 1$ (Polyakov).
  (For T > 0: there is a phase transition also in 2+1D.)

- cQED: 3+1D: phase transition between a strong coupling confining phase, and a weak coupling coulomb phase.

- Z(N): for $\mathrm{N} \geq N_c$: Three phases: electric confinement, magnetic confinement, and non confinement.

LATTICE FERMIONS

- "Naïve" discretization of the Dirac field leads, in the continuum limit, to doubling of the fermionic species (double zeros in the fermionic Brillouin zone).
- There are several methods to solve this problem: Wilson fermions, Staggered (Kogut-Susskind) fermions, Domain- Wall fermions, …
- No-Go theorem (Nielsen and Ninomiya): any Hermitean, local and translationally invariant lattice theory leads to fermion doubling.
- Nice side effect: the chiral anomaly is cancelled.

STAGGERED (KOGUT-SUSSKIND) FERMIONS

- Doubling resolved by breaking translational invariance (in a very special manner).
- Each continuum spinor is constructed out of several lattice sites (depenging on the gauge group and the dimension).
- Continuum limit: Dirac field.

STAGGERED (KOGUT-SUSSKIND) FERMIONS IN 1+1d – MASS AND CHARGE

- The Hamiltonian: $\epsilon \sum_{\mathbf{n},k} \left(\psi^\dagger_{\mathbf{n}} e^{i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}+\hat{\mathbf{k}}} + \psi^\dagger_{\mathbf{n}+\hat{\mathbf{k}}} e^{-i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}} \right) + M \sum_n (-1)^n \psi^\dagger_n \psi_n$

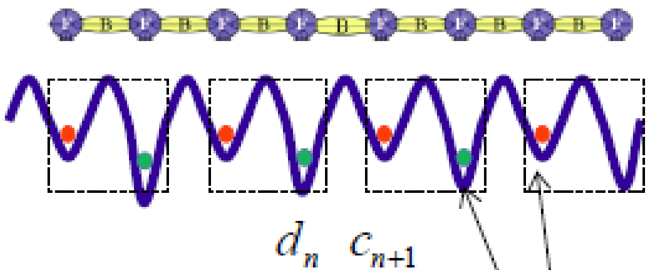
- Charge: $Q_n = \psi^\dagger_n \psi_n - \frac{1}{2}\left(1-(-1)^n\right)$

- Mass is measured relatively to $-M\left(1-(-1)^n\right)$

- Even n – particles: Q=N
  - 0 atoms: zero mass, zero charge
  - 1 atom: M, Q=1
- Odd n – anti-patrticles: Q=N-1
  - 1 atom: zero mass, zero charge (“Dirac sea”)
  - 0 atoms: mass M (relative to –M), charge Q=-1

QUANTUM SIMULATION

DYNAMICAL FERMIONS 1+1



$$H_M = M \sum_n (-1)^n \psi_n^\dagger \psi_n$$

internal states

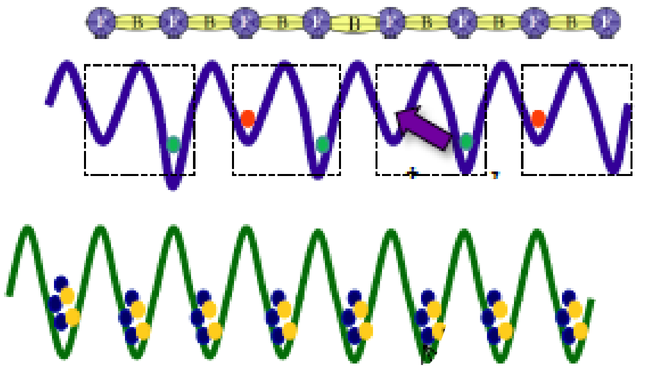
$$\{c_n, c_n^\dagger\} = \{d_n, d_n^\dagger\} = 1$$

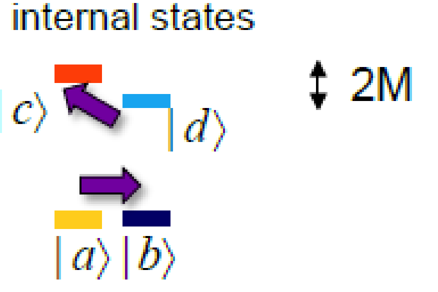
Staggered Fermions:

L. Susskind, Phys. Rev. D 16, 3031 (1977).

QUANTUM SIMULATION

SCHWINGER MODEL 1+1





$$\frac{\epsilon}{\sqrt{\ell(\ell+1)}}\sum_n \left(\psi_n^\dagger L_{+,n}\psi_{n+1} + h.c.\right)$$

STAGGERED (KOGUT-SUSSKIND) FERMIONS IN 1+1d – ELECTRIC FLUX TUBES, l = 1

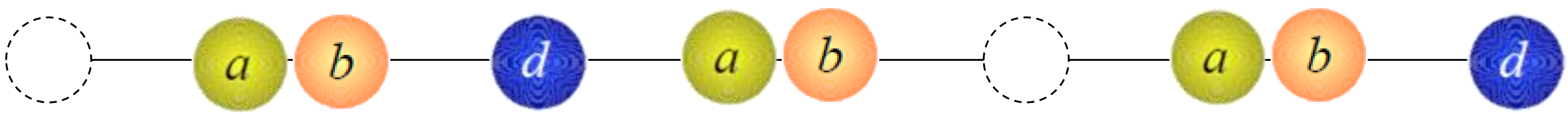
Dirac Sea

N_c	0
m	0
Q	0

N_d	1
m	0
Q	0

N_c	0
m	0
Q	0

N_d	1
m	0
Q	0



N_a	1
N_b	1
$L_z = ½(N_a\text{-}N_b)$	0
$l = ½(N_a+N_b)$	1

N_a	1
N_b	1
$L_z = ½(N_b\text{-}N_a)$	0
$l = ½(N_a+N_b)$	1

N_a	1
N_b	1
$L_z = ½(N_a\text{-}N_b)$	0
$l = ½(N_a+N_b)$	1

STAGGERED (KOGUT-SUSSKIND) FERMIONS IN 1+1d – ELECTRIC FLUX TUBES, l = 1

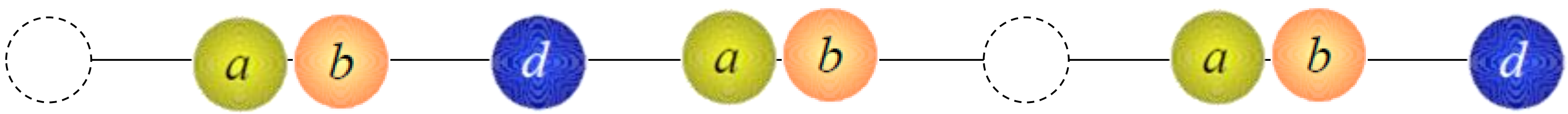
$c^\dagger a^\dagger b d$ **Act with** $c^\dagger a^\dagger b d$

N_c	0
m	0
Q	0

N_d	1
m	0
Q	0

N_c	0
m	0
Q	0

N_d	1
m	0
Q	0



N_a	1
N_b	1
$L_z = ½(N_a - N_b)$	0
$l = ½(N_a + N_b)$	1

N_a	1
N_b	1
$L_z = ½(N_b - N_a)$	0
$l = ½(N_a + N_b)$	1

N_a	1
N_b	1
$L_z = ½(N_a - N_b)$	0
$l = ½(N_a + N_b)$	1

STAGGERED (KOGUT-SUSSKIND) FERMIONS IN 1+1d – ELECTRIC FLUX TUBES, l = 1

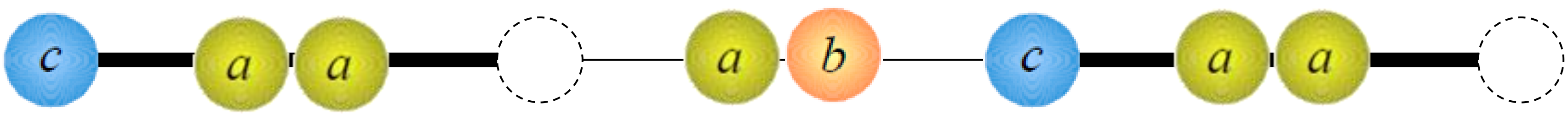
Two "mesons" (Flux tubes)

N_c	1
m	M
Q	+1

N_d	0
m	M
Q	-1

N_c	1
m	M
Q	+1

N_d	0
m	M
Q	-1



N_a	2
N_b	0
$L_z = ½(N_a\text{-}N_b)$	+1
$l = ½(N_a+N_b)$	1

N_a	1
N_b	1
$L_z = ½(N_b\text{-}N_a)$	0
$l = ½(N_a+N_b)$	1

N_a	2
N_b	0
$L_z = ½(N_a\text{-}N_b)$	+1
$l = ½(N_a+N_b)$	1

STAGGERED (KOGUT-SUSSKIND) FERMIONS IN 1+1d – ELECTRIC FLUX TUBES, l = 1

Act with

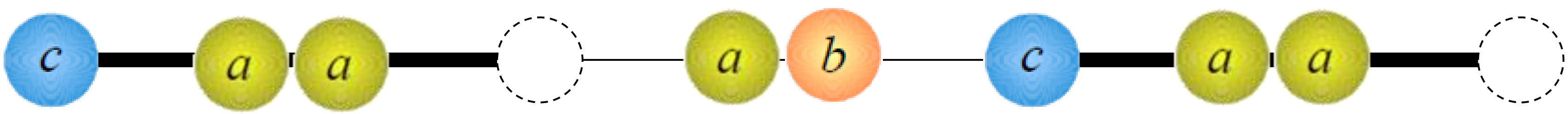
$$d^\dagger b^\dagger a c$$

N_c	1
m	M
Q	+1

N_d	0
m	M
Q	-1

N_c	1
m	M
Q	+1

N_d	0
m	M
Q	-1



N_a	2
N_b	0
$L_z = ½(N_a\text{-}N_b)$	+1
$l = ½(N_a+N_b)$	1

N_a	1
N_b	1
$L_z = ½(N_b\text{-}N_a)$	0
$l = ½(N_a+N_b)$	1

N_a	2
N_b	0
$L_z = ½(N_a\text{-}N_b)$	+1
$l = ½(N_a+N_b)$	1

STAGGERED (KOGUT-SUSSKIND) FERMIONS IN 1+1d – ELECTRIC FLUX TUBES, $l = 1$

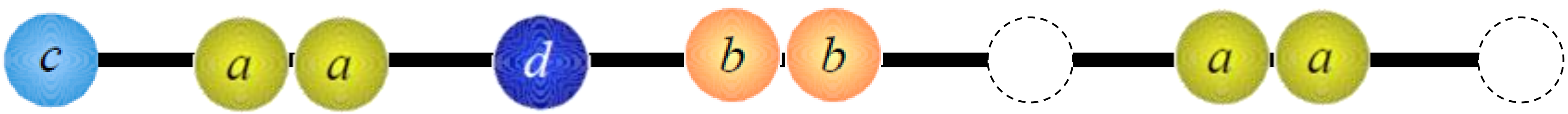
Longer meson

N_c	1
m	M
Q	+1

N_d	1
m	0
Q	0

N_c	0
m	0
Q	0

N_d	0
m	M
Q	-1

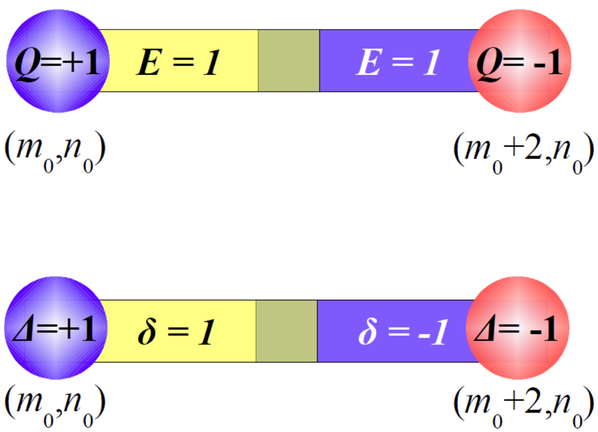


N_a	2
N_b	0
$L_z = ½(N_a\text{-}N_b)$	+1
$l = ½(N_a+N_b)$	1

N_a	0
N_b	2
$L_z = ½(N_b\text{-}N_a)$	1
$l = ½(N_a+N_b)$	1

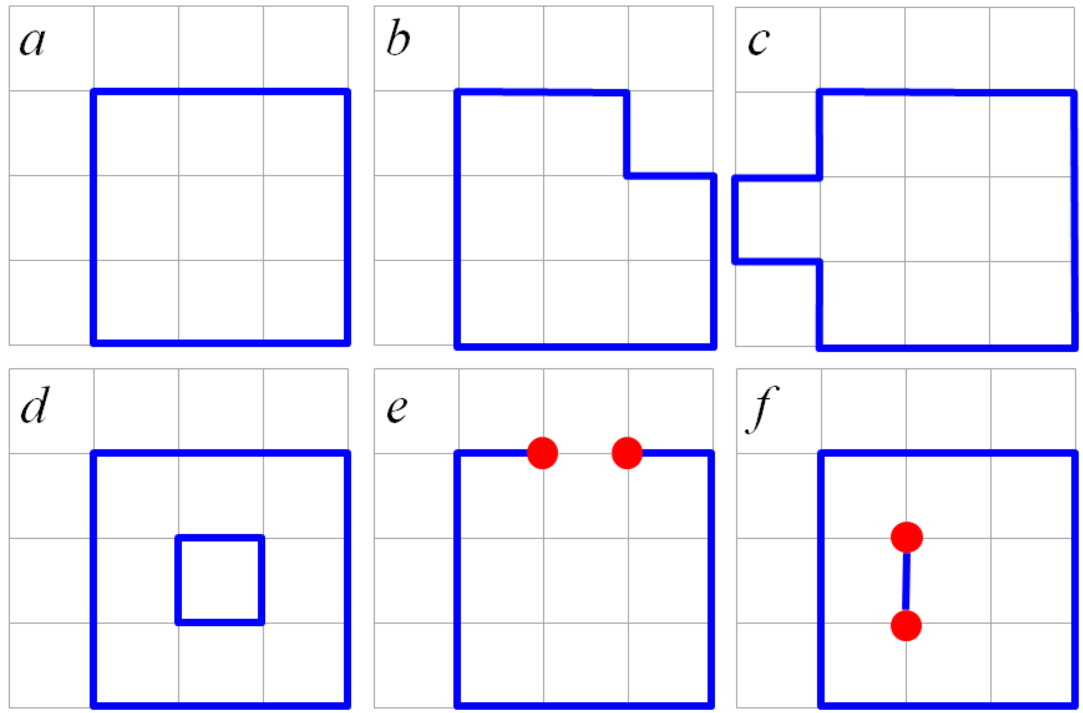
N_a	2
N_b	0
$L_z = ½(N_a\text{-}N_b)$	+1
$l = ½(N_a+N_b)$	1

Confinement, flux breaking & glueballs



Electric flux tubes

E. Zohar, BR,
Phys. Rev. Lett. 107, 275301 (2011).

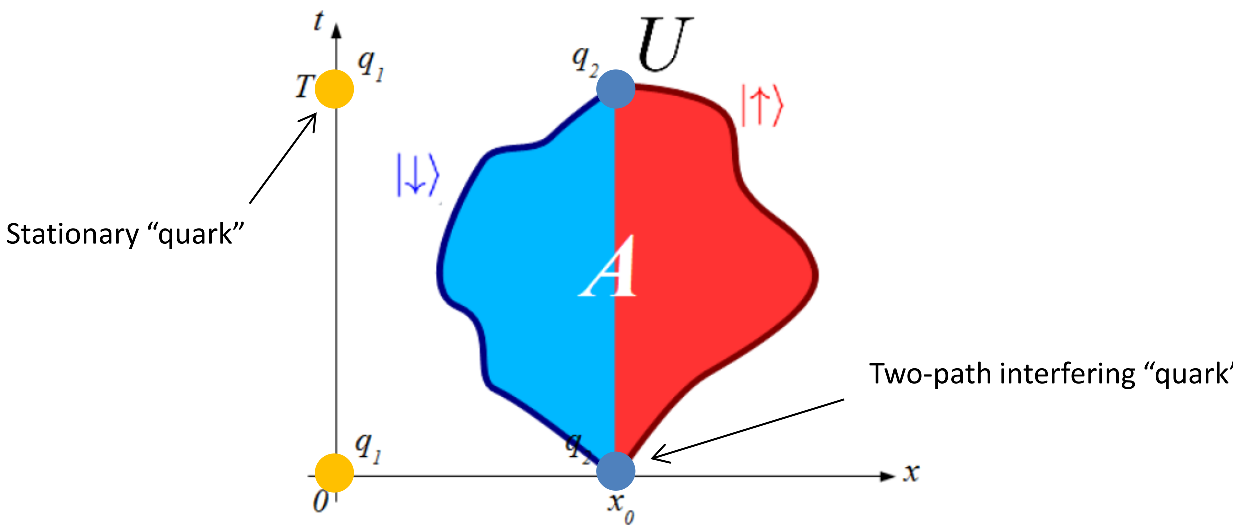


Flux loops deforming and breaking effects

E. Zohar, J. I. Cirac, BR,
Phys. Rev. Lett. 110, 055302 (2013)

WILSON LOOP MEASUREMENTS

$$W(C) = P\left(e^{i \oint_C A_\mu dx^\mu}\right) \xrightarrow{\text{Confining phase}} \text{“Erea law” dependence}$$



Detecting Wilson Loop’s area law by interference of “Mesons”.

This is equivalent to Ramsey Spectroscopy in quantum optics!

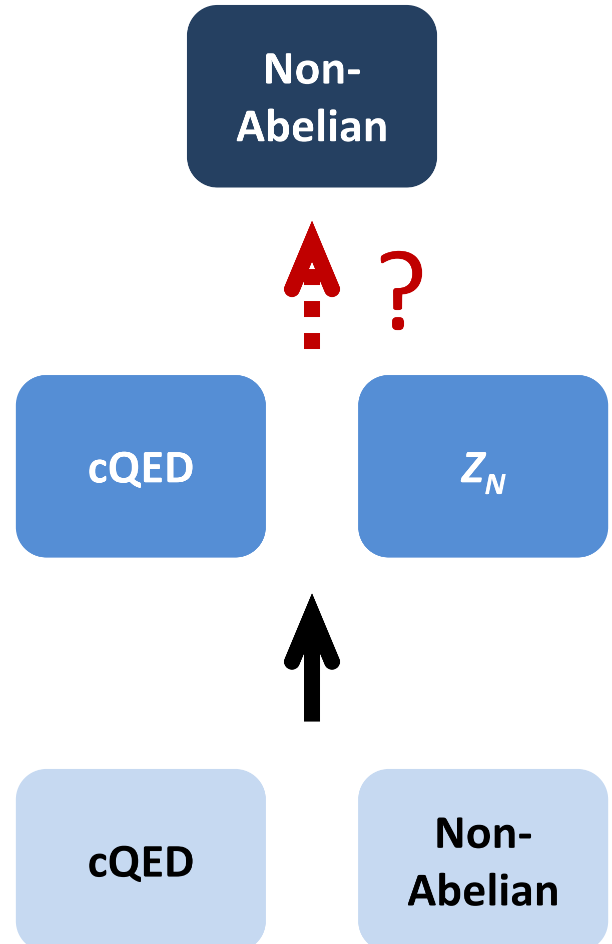
E. Zohar , BR, New J. Phys. 15 (2013) 043041

OUTLOOK

Non Abelian in Higher Dimensions

Plaquettes in 2+1 and 3+1
Abelian , cQED and Z(N)

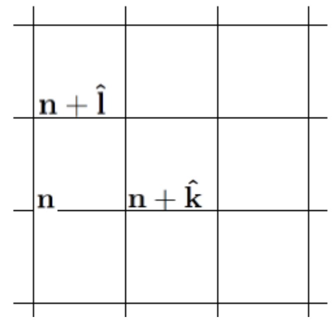
"Proof of principle" 1+1 toy models
Numerical comparison with DMRG

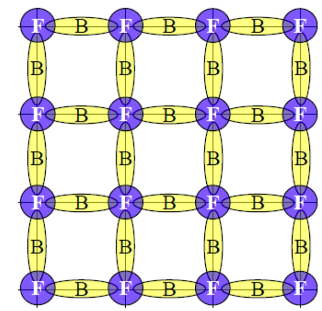


Decoherence, superlattices, scattering parameters control…

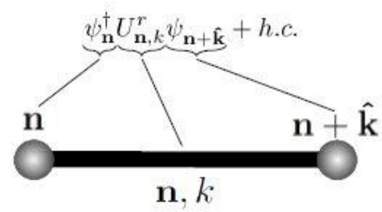
SUMMARY

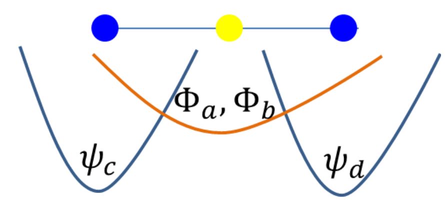
Lattice gauge theories can be mapped to an analog cold atom simulator.



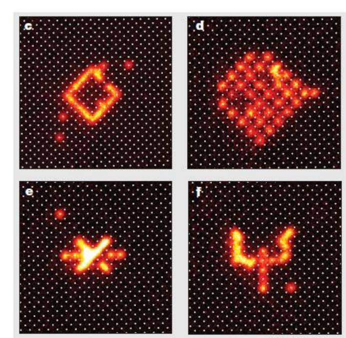


Atomic conservation laws can give rise to exact local gauge symmetry.

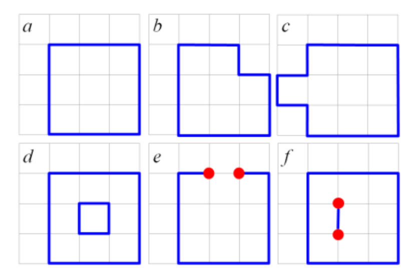




Near future experiments may be able to realize first steps in this direction, and offer a new types of LGT simulations.

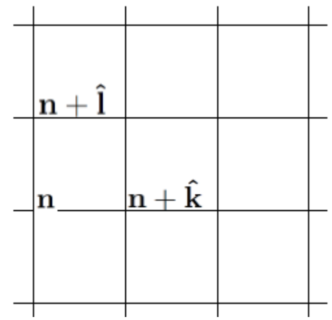


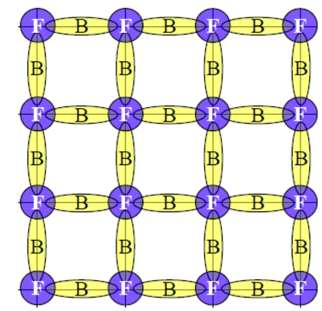
Weitenberg et. al., Nature, 2011



THANK YOU!

Lattice gauge theories can be mapped to an analog cold atom simulator.





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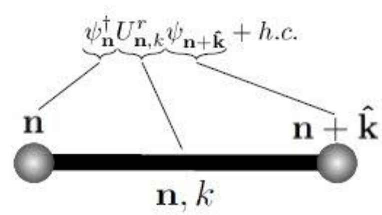

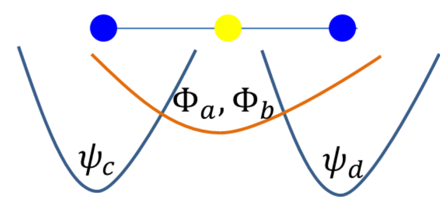

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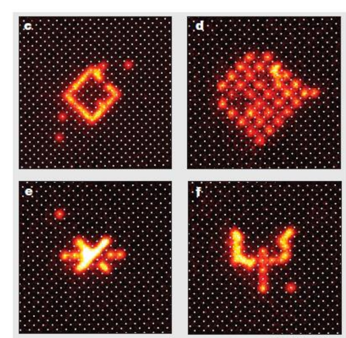
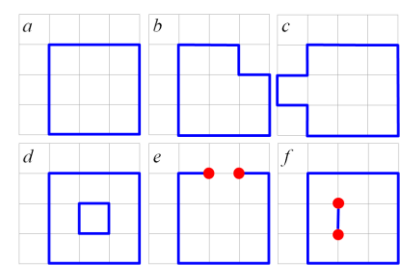


Weitenberg et. al., Nature, 2011



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E. Zohar, I. Cirac, BR, PRL **109**, 125302 (2012)

E. Zohar, BR, NJP **15**, 043041 (2013)

E. Zohar, I. Cirac, BR, PRL **110** 055302 (2013)

E. Zohar, I. Cirac, BR, PRL **110** 125304 (2013)

Detailed account → E. Zohar, I. Cirac, BR, PRA 88, 023617 (2013)), arxiv 1303.5040

Experimental progress

QUANTUM SIMULATIONS

COLD ATOMS – EXPERIMENTS

PRL 103, 080404 (2009) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2009

Experimental Demonstration of Single-Site Addressability in a Two-Dimensional Optical Lattice

Peter Würtz,[1] Tim Langen,[1] Tatjana Gericke,[1] Andreas Koglbauer,[1] and Herwig Ott[1,2,*]

[1]Institut für Physik, Johannes Gutenberg-Universität, 55099 Mainz, Germany

[2]Research Center OPTIMAS, Technische Universität Kaiserslautern, 67663 Kaiserslautern, Germany

(Received 18 March 2009; published 21 August 2009)

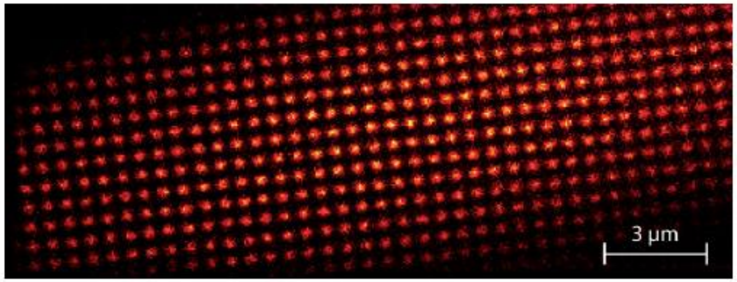

FIG. 1 (color online). Electron microscope image of a Bose-Einstein condensate in a 2D optical lattice with 600 nm lattice spacing (sum obtained from 260 individual experimental realizations). Each site has a tubelike shape with an extension of 6 μm perpendicular to the plane of projection. The central lattice sites contain about 80 atoms.

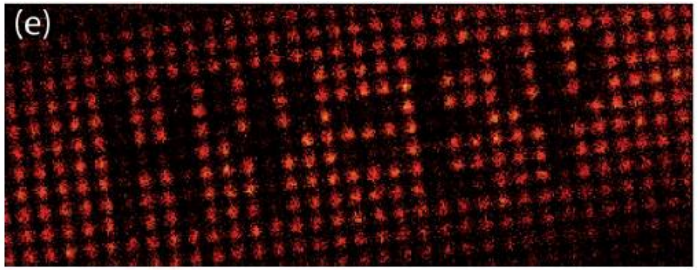

FIG. 2 (color online). Patterning a Bose-Einstein condensate in a 2D optical lattice with a spacing of 600 nm. Every emptied site was illuminated with the electron beam (7 nA beam current, 100 nm FWHM beam diameter) for (a),(b) 3, (c),(d) 2, and (e) 1.5 ms, respectively. The imaging time was 45 ms. Between 150 and 250 images from individual experimental realizations have been summed for each pattern.

QUANTUM SIMULATIONS

COLD ATOMS – EXPERIMENTS

doi:10.1038/nature09827

Single-spin addressing in an atomic Mott insulator

Christof Weitenberg[1], Manuel Endres[1], Jacob F. Sherson[1]†, Marc Cheneau[1], Peter Schauß[1], Takeshi Fukuhara[1], Immanuel Bloch[1,2] & Stefan Kuhr[1]

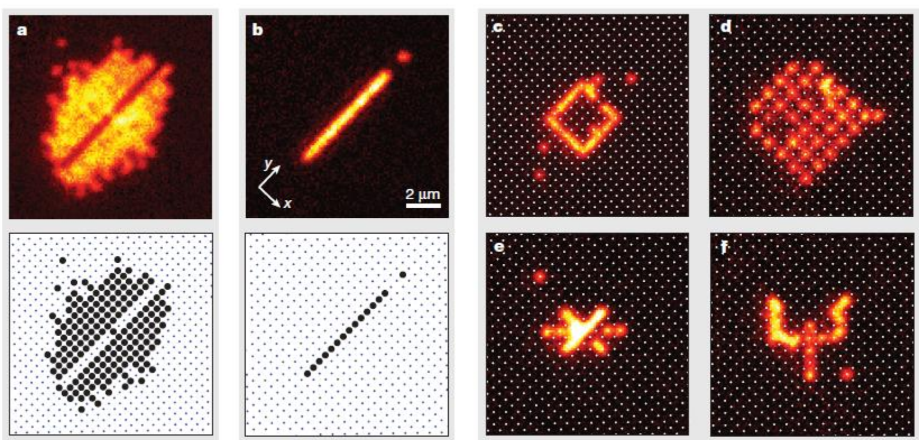

Figure 2 | Single-site addressing. a, Top, experimentally obtained fluorescence image of a Mott insulator with unity filling in which the spin of selected atoms was flipped from $|0\rangle$ to $|1\rangle$ using our single-site addressing scheme. Atoms in state $|1\rangle$ were removed by a resonant laser pulse before detection. Bottom, the reconstructed atom number distribution on the lattice. Each filled circle indicates a single atom; the points mark the lattice sites. **b**, Top, as for **a** except that a global microwave sweep exchanged the population in $|0\rangle$ and $|1\rangle$, such that only the addressed atoms were observed. Bottom, the reconstructed atom number distribution shows 14 atoms on neighbouring sites. **c–f**, As for **b**, but omitting the atom number distribution. The images contain 29 (**c**), 35 (**d**), 18 (**e**) and 23 (**f**) atoms. The single isolated atoms in **b**, **e** and **f** were placed intentionally to allow for the correct determination of the lattice phase for the feedback on the addressing beam position.