Ising model on random networks and canonical tensor model

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N. Sasakura, Y.S., PTEP 2014 (2014) 5, 053B03.

N. Sasakura, Y.S., 1402.0740 [hep-th] (accepted in SIGMA).

What happens when combining Quantum Mechanics and General Relativity?

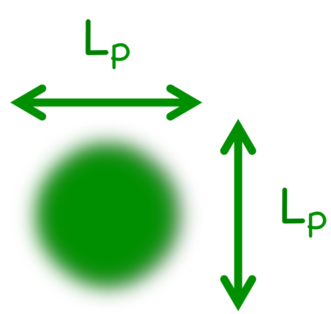
[Quantum mechanics ($\hbar$)] → $(\Delta X)\cdot(M\Delta V) \geq \hbar/2$

[Special Relativity (C)] → $\Delta V \leq C$

$(\Delta X) \geq \hbar/(2MC)$ Compton wavelength

[General Relativity (G_N)] → $M \leq \Delta X C^2/(2G_N)$

$(\Delta X) \geq [\hbar G_N/C^3]^{1/2} = L_P$ Planck length



We are ignorant of
a length shorter than L_P

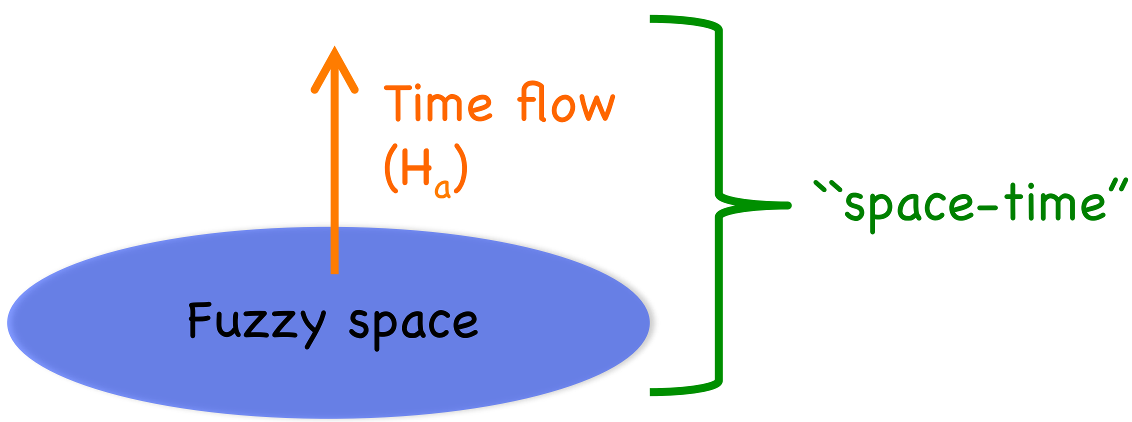
Canonical Tensor Model = Theory of dynamical fuzzy space [N. Sasakura, 2011]

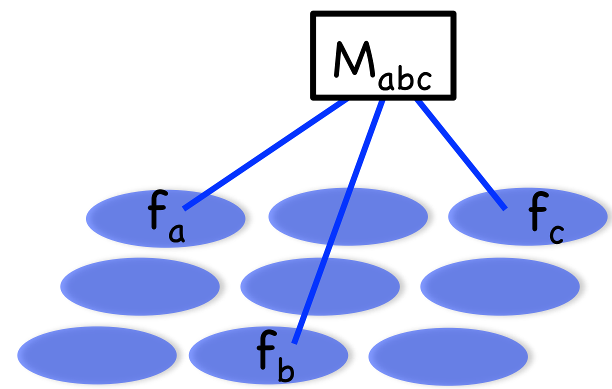
$$f_a \star f_b = M_{abc}\, f_c$$

$(a,b,c=1,\ldots,N)$

Fuzzy space is defined by functions $\{f_1, f_2, \ldots, f_N\}$ and the product, $f_a \star f_b$.

N = #[functions] = #["points" in space]





Canonical Tensor Model [N. Sasakura, 2011]

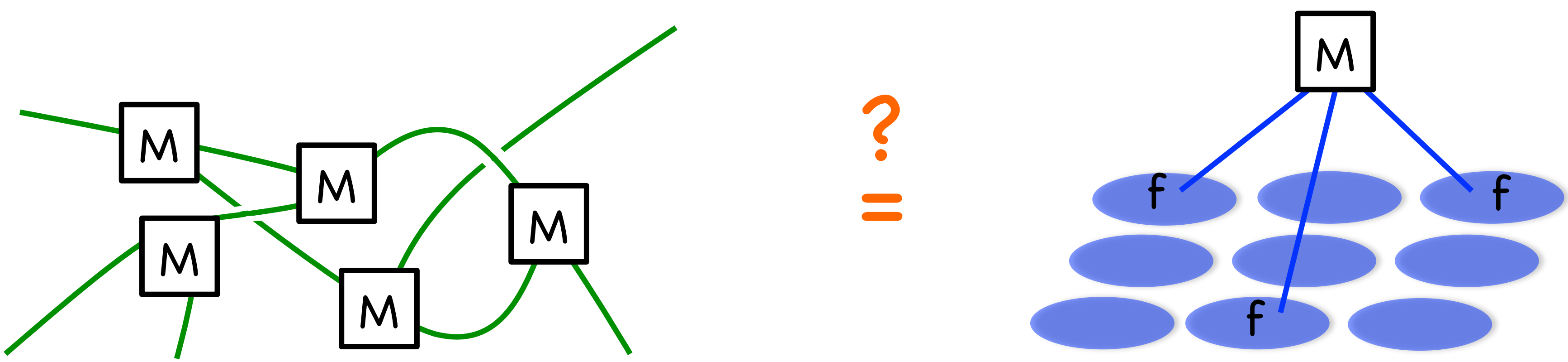
Our knowledge is quite limited...

N=1: General Relativity in minisuperspace [N. Sasakura and Y.S., 2014]
N=2: Locality is favoured [N. Sasakura, 2013]

Difficulty is tied to much DOF of M_{abc} $(a=1,\ldots,N)$.

Overcome

Statistical system on random networks [N. Sasakura and Y.S., 2014]



Statistical system on random network

Canonical tensor model

Canonical Tensor Model [N. Sasakura, 2011]

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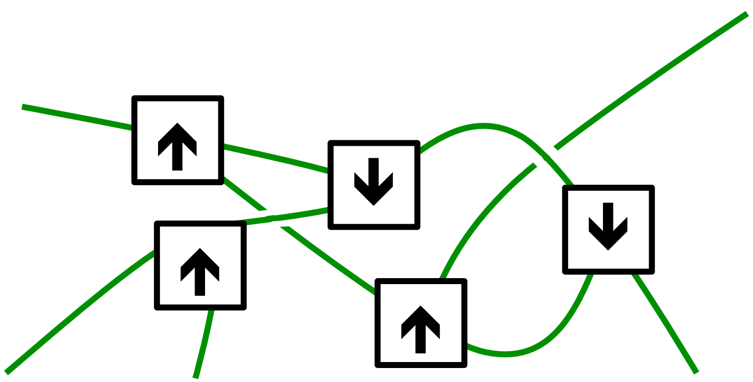

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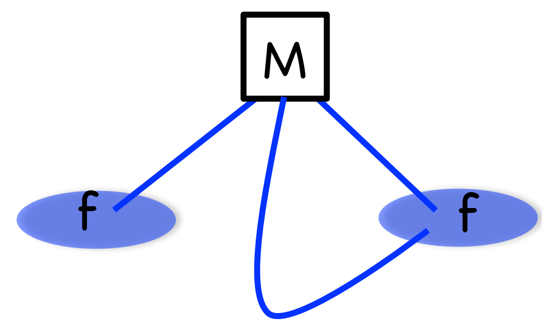
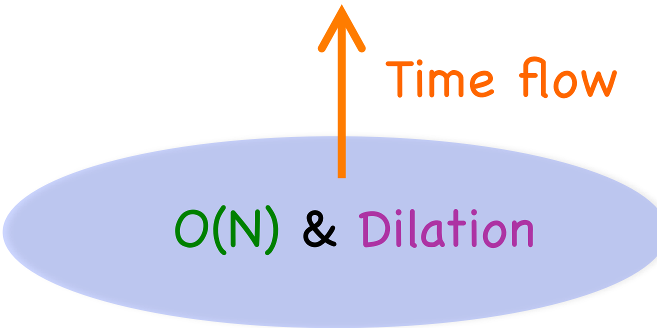


Ising model on random network

N=2 canonical tensor model

Phase transition line $\xleftrightarrow{\text{match}}$ Boundary of Hamiltonian flow

Canonical Tensor Model [N. Sasakura, 2011]



Fuzzy space

Hamiltonian:

$$H = N_a H_a + N_{[ab]} J_{[ab]} + N D$$

where

$$H_a = (1/2)\, P_{abc} P_{bde} M_{cde} \qquad \text{Generator for time flow}$$

$$J_{[ab]} = (P_{acd} M_{bcd} - P_{bcd} M_{acd})/4 \qquad \text{o(N) generator}$$

$$D = (1/6)\, M_{abc} P_{abc} \qquad \text{Dilation generator}$$

Poisson's bracket:

$$\{M_{abc}, P_{def}\} = \delta_{ad}\,\delta_{be}\,\delta_{cf} + \text{perm. of } (def)$$

Hamiltonian vector flow of N=2 model [N. Sasakura and Y.S., 2014]

Gauge fixing:

$$P_{111} = 1, \quad P_{112} = 0, \quad P_{122} = x, \quad P_{222} = y$$

[(M,P) are real & symmetric]

Hamiltonian vector flow:

$$H \propto A(x,y)\, \partial/\partial x + B(x,y)\, \partial/\partial y$$

[A(x,y) & B(x,y) are functions]

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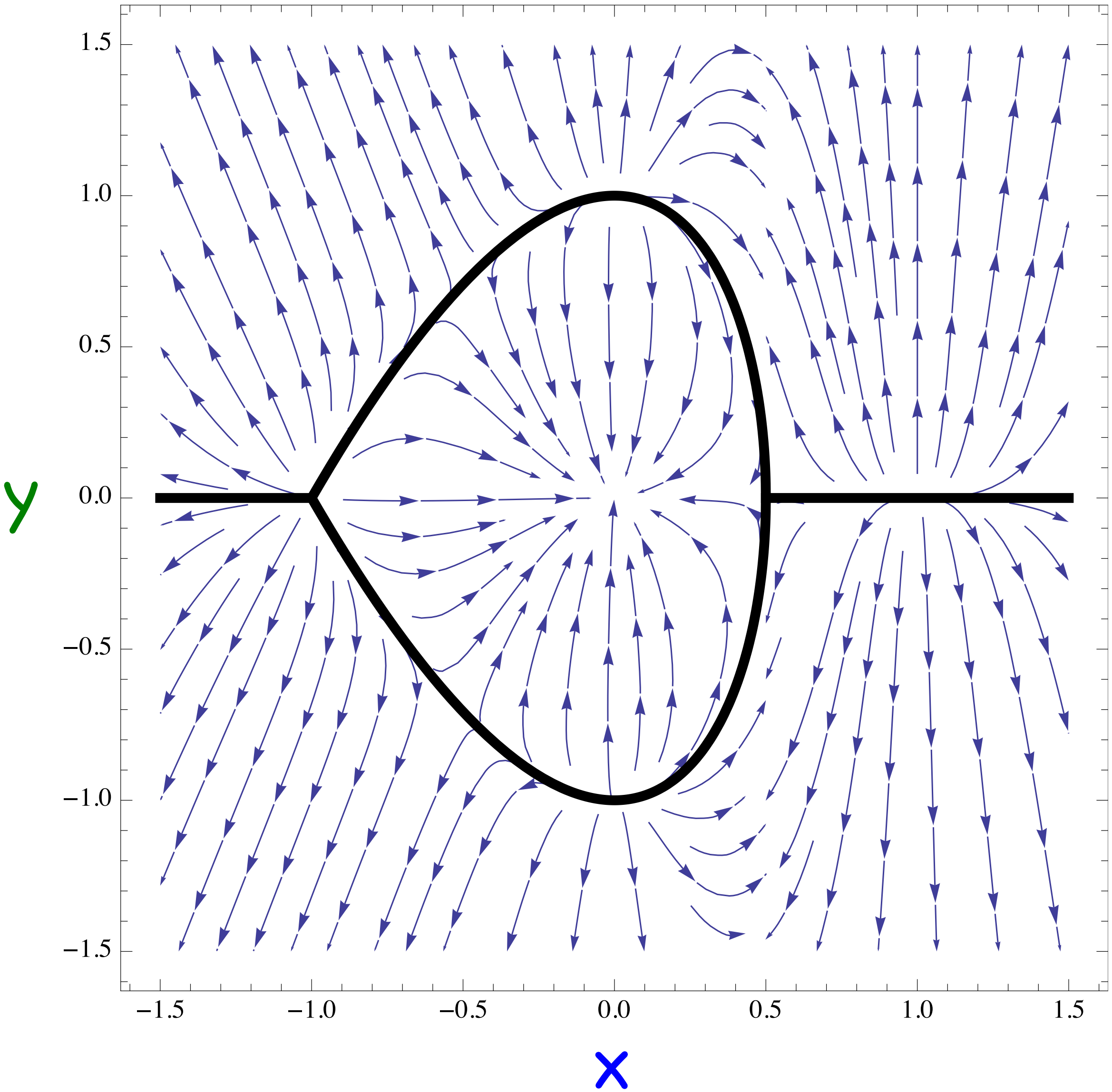


: Boundary of flow

Statistical systems on random network [N. Sasakura and Y.S., 2014]

Grand-type partition function:

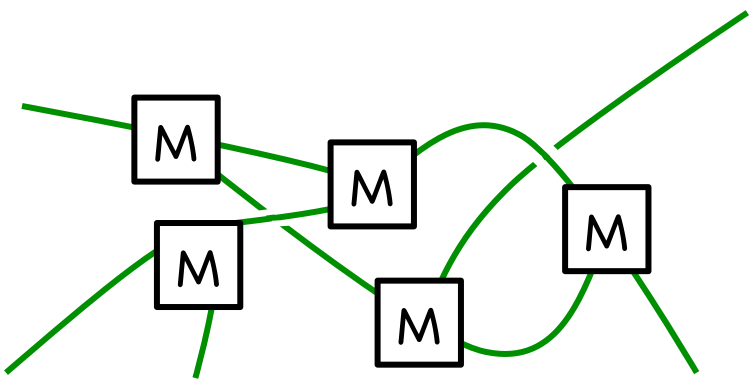
$$Z(M,t) = \int \prod_{d=1}^{N} d\Phi_d \exp[-\Phi_a \Phi_a + t M_{abc} \Phi_a \Phi_b \Phi_c]$$

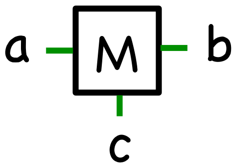
O(N) and dilation invariance:

$$Z(L(M),t) = Z(M,t) \qquad L(M)_{ace} = L_{a\acute{a}} L_{c\acute{c}} L_{e\acute{e}} M_{\acute{a}\acute{c}\acute{e}}$$

$$Z(e^{\sigma}M, e^{-\sigma}t) = Z(M,t)$$

Generate statistical system (M) on random network (Feynman graph):





:Rank-three tensor

Statistical systems on random network [N. Sasakura and Y.S., 2014]

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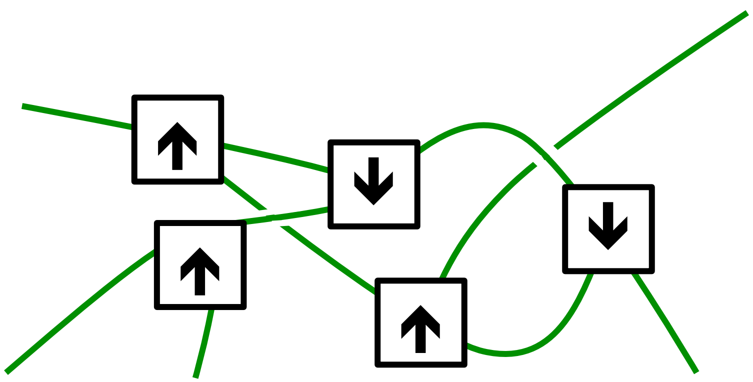


Ex.) N=2 (Ising model)

$$M_{abc} = \sum_i R_{ai}R_{bi}R_{ci}\, e^{H\sigma_i}$$

$$(R^T R)_{ij} = e^{J\sigma_i\sigma_j} \qquad (J \geq 0)$$

Statistical systems on random network [N. Sasakura and Y.S., 2014]

To check thermodynamic properties, introduce partition function:

$$Z(M,t) \cong \sum_{n=0}^{\infty} t^n Z_n(M)$$

$$= \sum_{n=0}^{\infty} t^n \left[(1/n!) \int \prod_{d=1}^{N} d\Phi_d \, (M_{abc}\Phi_a\Phi_b\Phi_c)^n e^{-\Phi_e\Phi_e}\right]$$

Define the free energy per vertex (n):

$$f_{2n} = -(1/2n) \log[Z_{2n}(M)]$$

[Z_n vanishes for n odd]

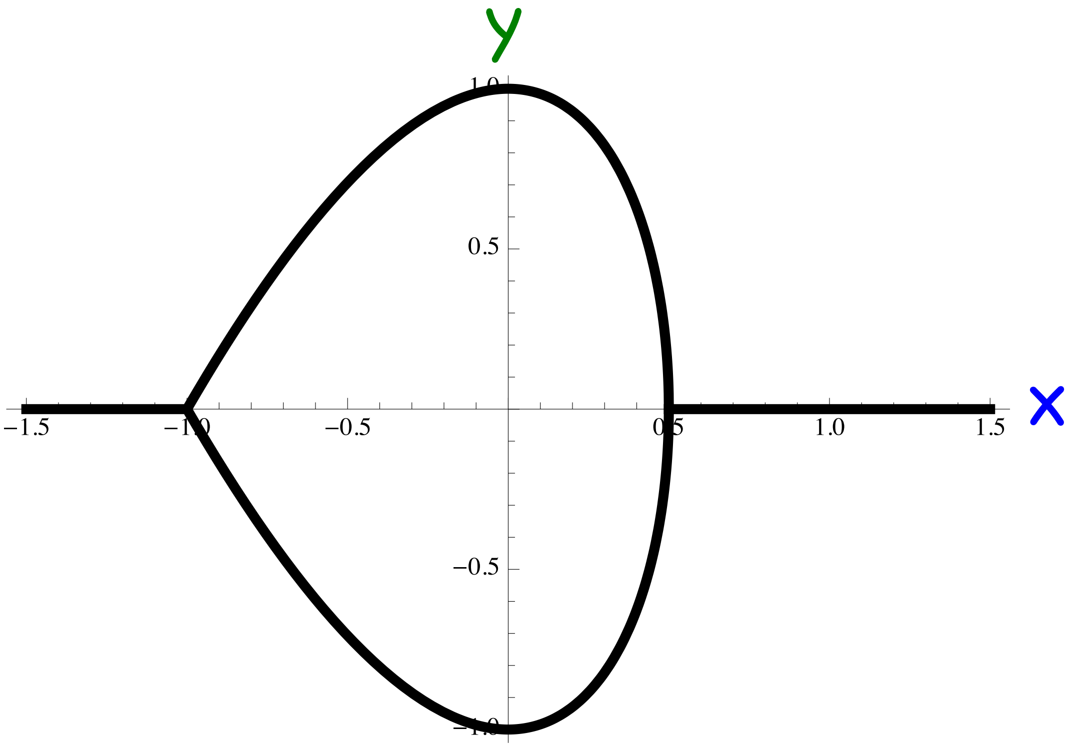
For N=2 (Ising model), check the behaviour of f_{2n} in the thermodynamic limit, $n \to \infty$.

$$M_{111} = 1, \quad M_{112} = 0, \quad M_{122} = x, \quad M_{222} = y$$

(Gauge fixing)

Statistical systems on random network [N. Sasakura and Y.S., 2014]

$$f_{2n}(x,y) = -(1/2n)\log[Z_{2n}(x,y)]$$



: Phase transition line

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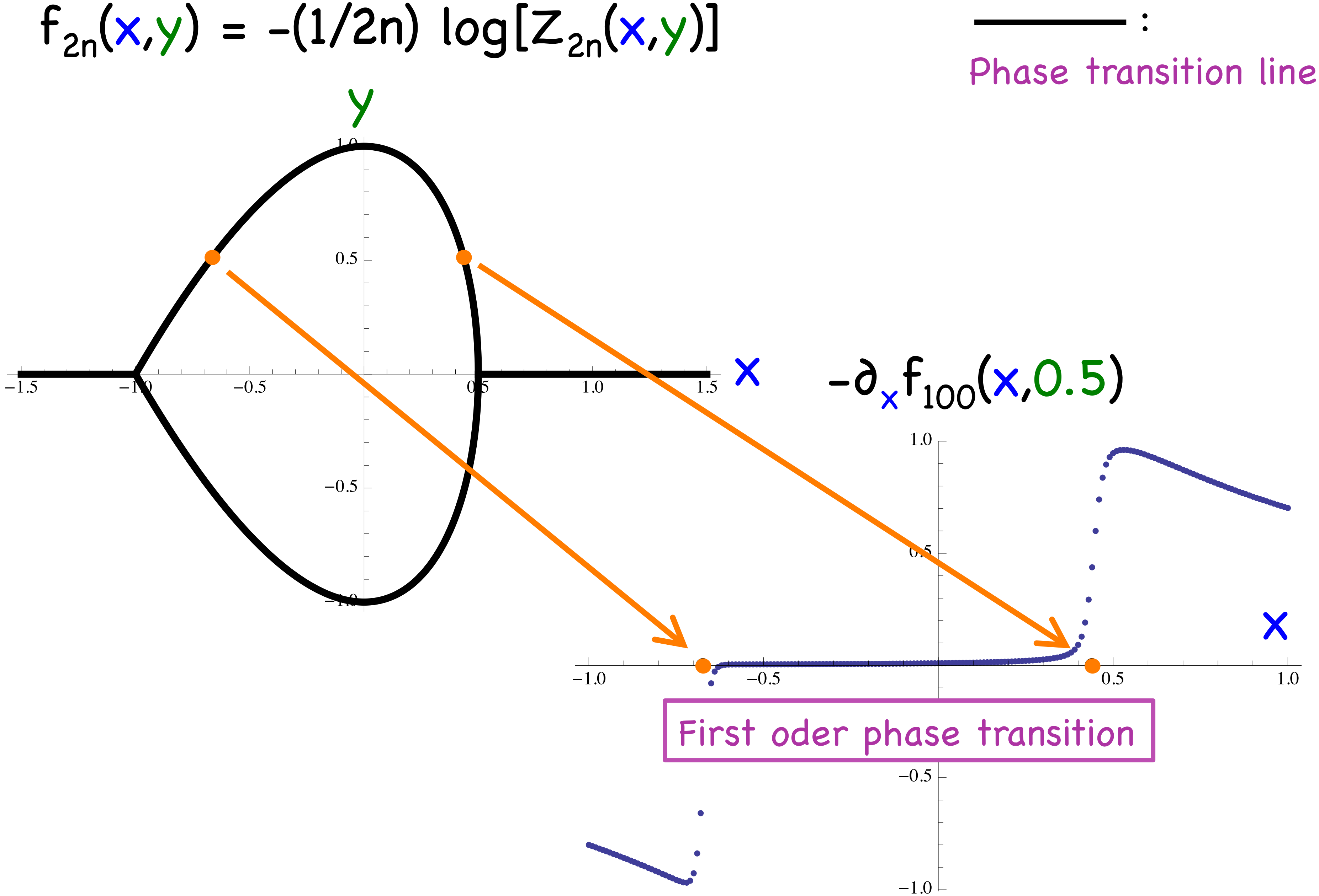

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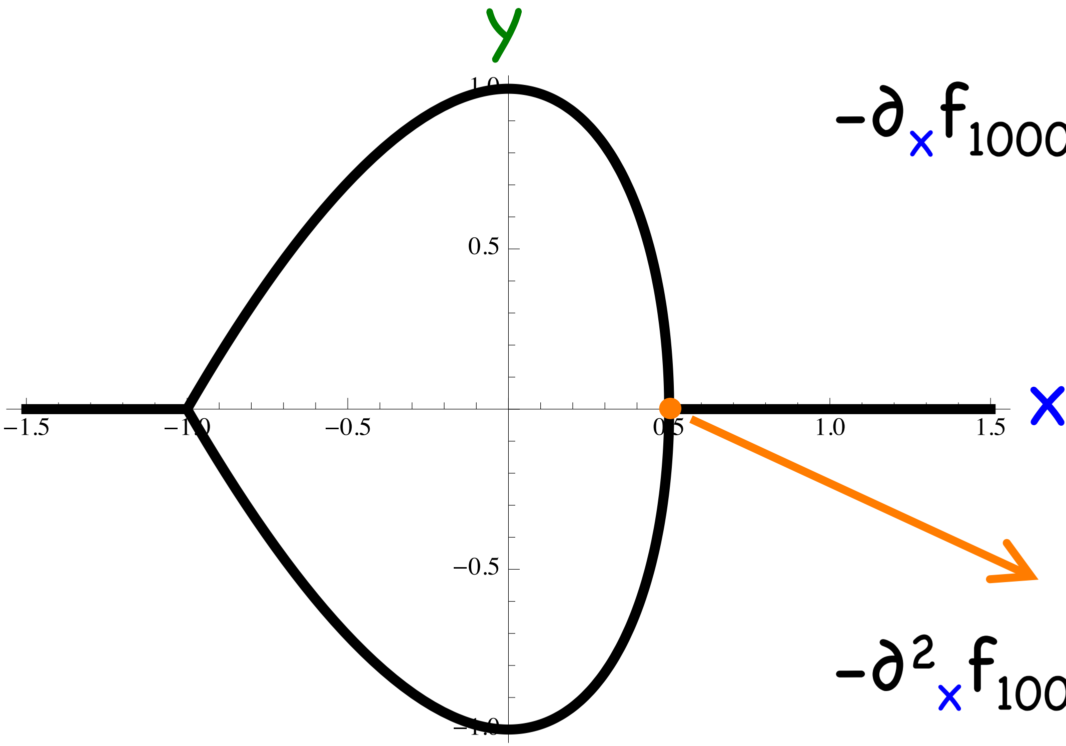
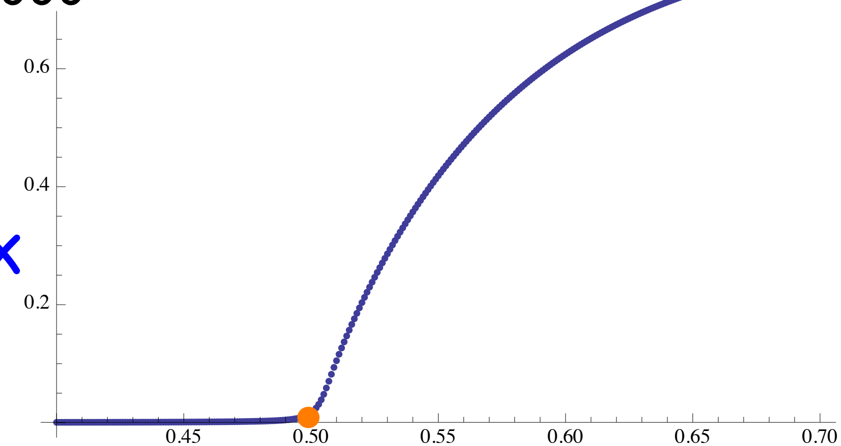
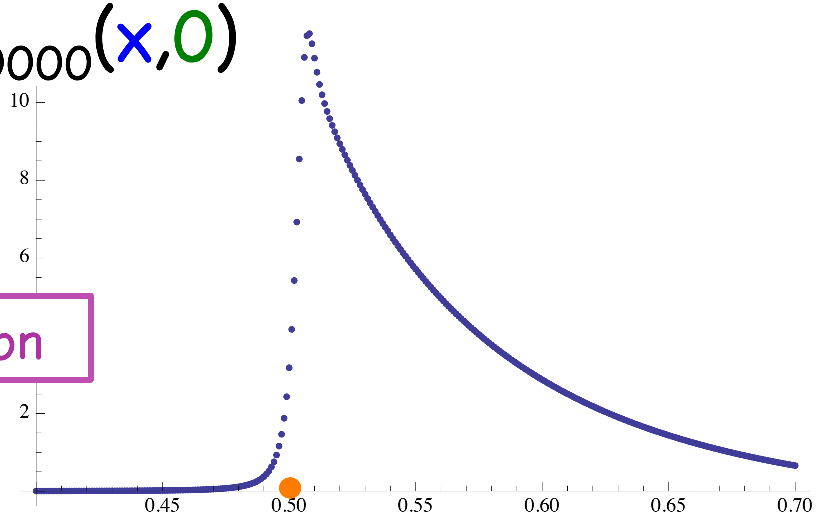


$-\partial_x f_{100000}(x,0)$



$-\partial^2{}_x f_{100000}(x,0)$



Second oder phase transition

Conclusion

[N=2 canonical tensor model]

Hamiltonian flow:

$H \propto A(x,y)\, \partial/\partial x + B(x,y)\, \partial/\partial y$

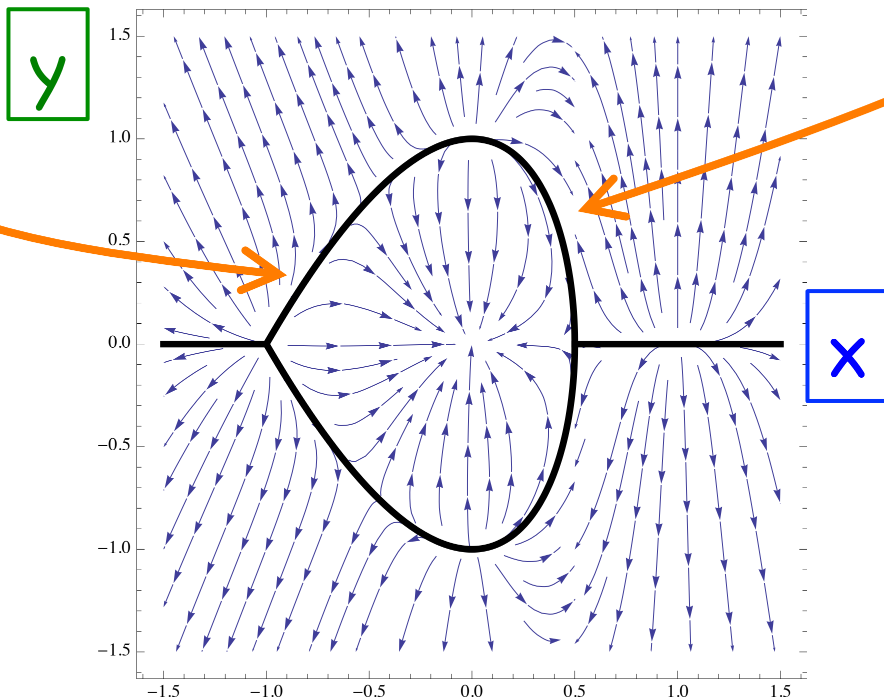
[Ising model on random networks]

Free energy:

$f(x,y) = \lim_{n \to \infty} -(1/2n) \log[Z_{2n}(x,y)]$

Boundary of flow

Phase transition line



Implication:
RG-like procedure may exist in Ising model on random networks

Large-N (#[points] are large) canonical tensor model
can be described by statistical systems on Random networks?

Works!

Exact solutions to the Wheeler-DeWitt equation
can be found in the language of random networks
[work in progress with G. Narain and N. Sasakura]

$$H_a \psi = J_{[ab]} \psi = D \psi = 0$$