# Nuclei: Quantum Finite Many-Body System with Two Fermions

## The Overview of the Nuclear Structure and Excitations

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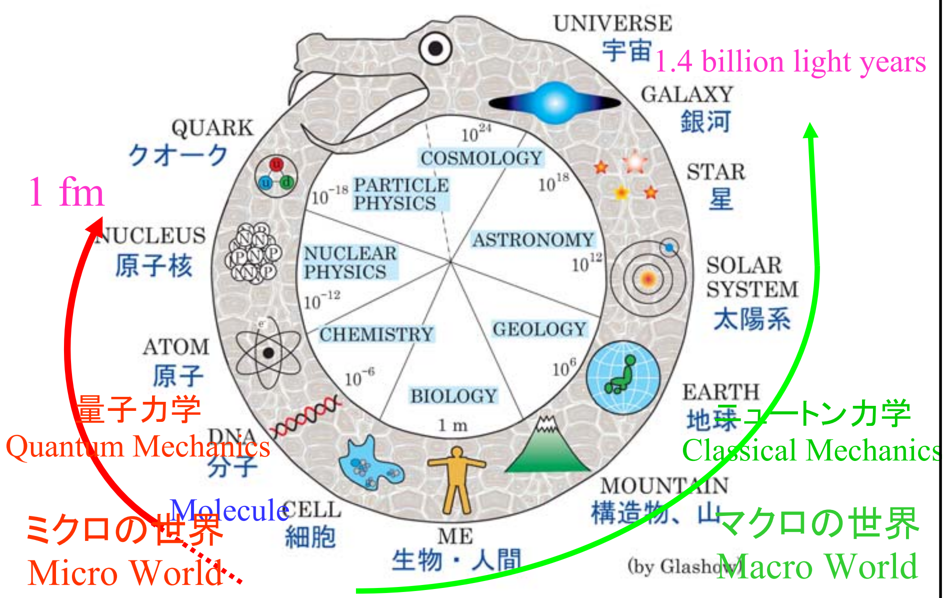
## 自然の階層性（ウロボロスの蛇）

Layer Structure of Nature (Snake of Uroboros)

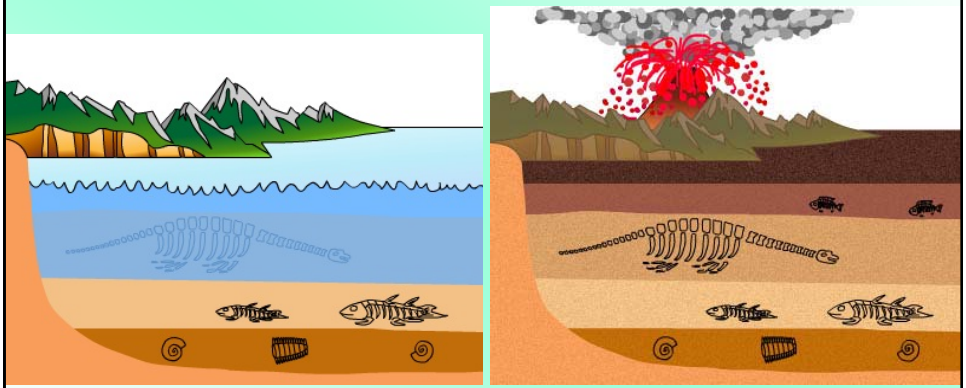



Ski is fun !
野沢温泉スキー場
SNOW RESORT
標高 1,650m
毛無山山頂


The Nature is alive!
（自然は生きている！）
La Clusaz

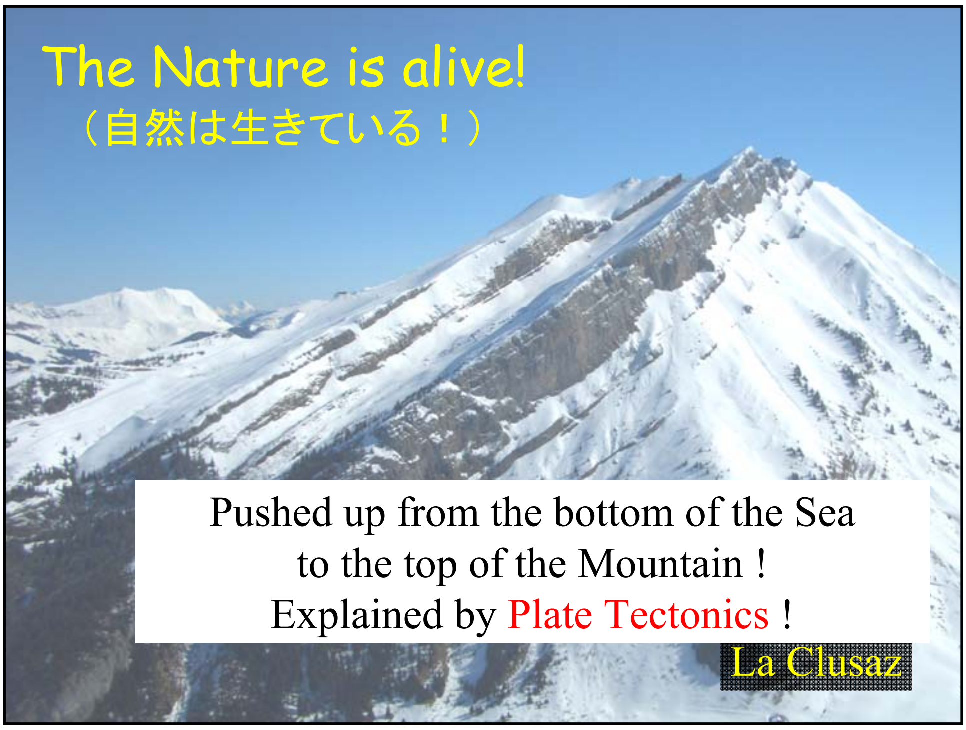
## How is the layer (stratum) structure made?

- elementary school 6th grade -



1)侵食作用で、土砂が海や湖の底にたまります。
2)時には火山の爆発でも地層ができます。

学研のホームページより



## The Nature is alive!

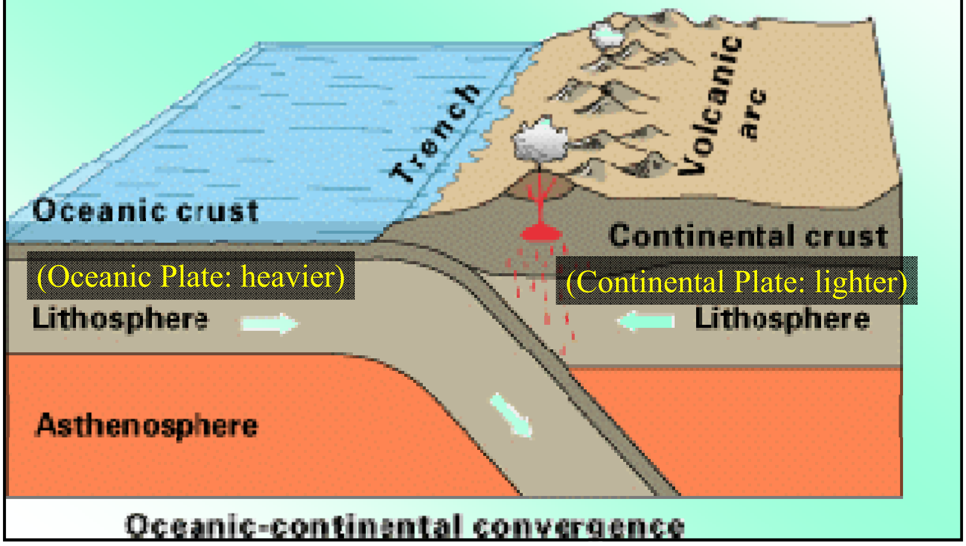
（自然は生きている！）

Pushed up from the bottom of the Sea
to the top of the Mountain !
Explained by Plate Tectonics !

La Clusaz

# Plate Tectonics

The model is build on the idea of "Continental Drift."

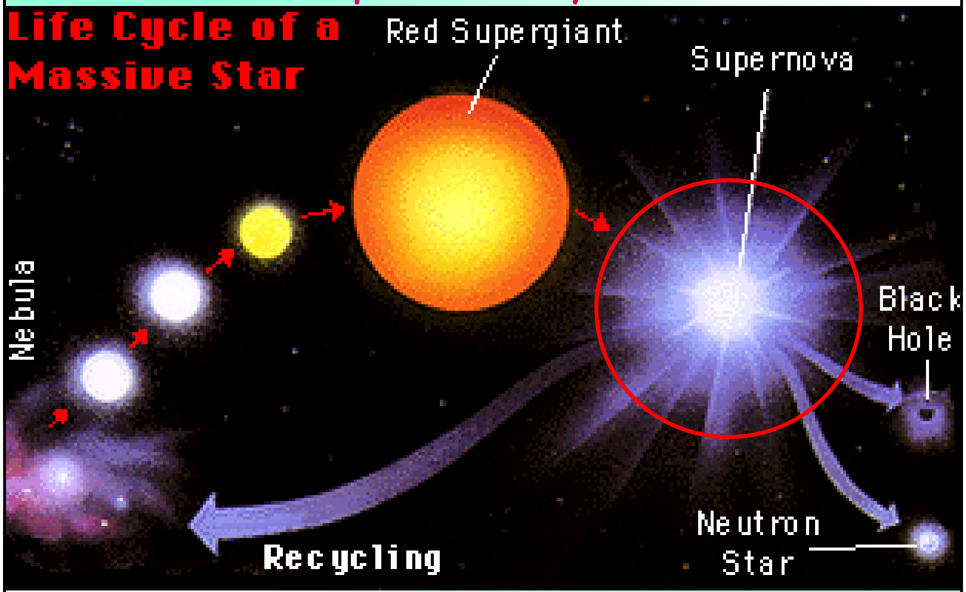



# Eruption of Kilauea, Hawaii (1970's)

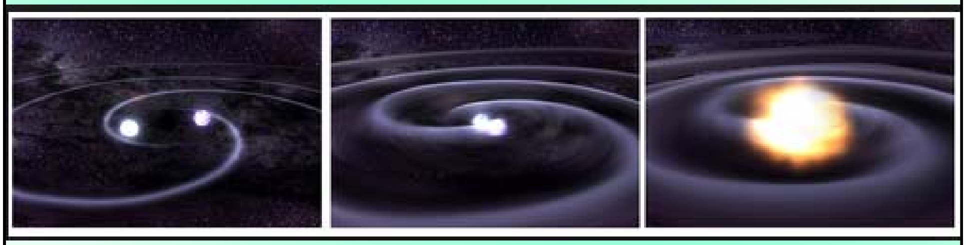
地熱:ウラン等、放射性元素の核分裂の熱

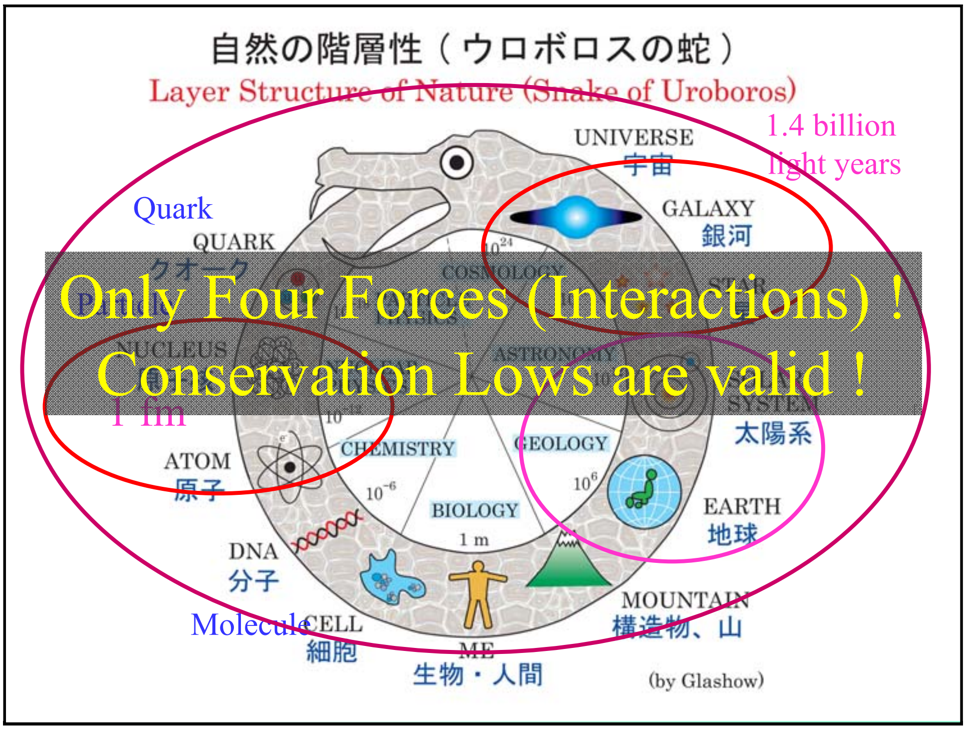
Terrestrial heat: originates from Radio Activity like $^{235}$U

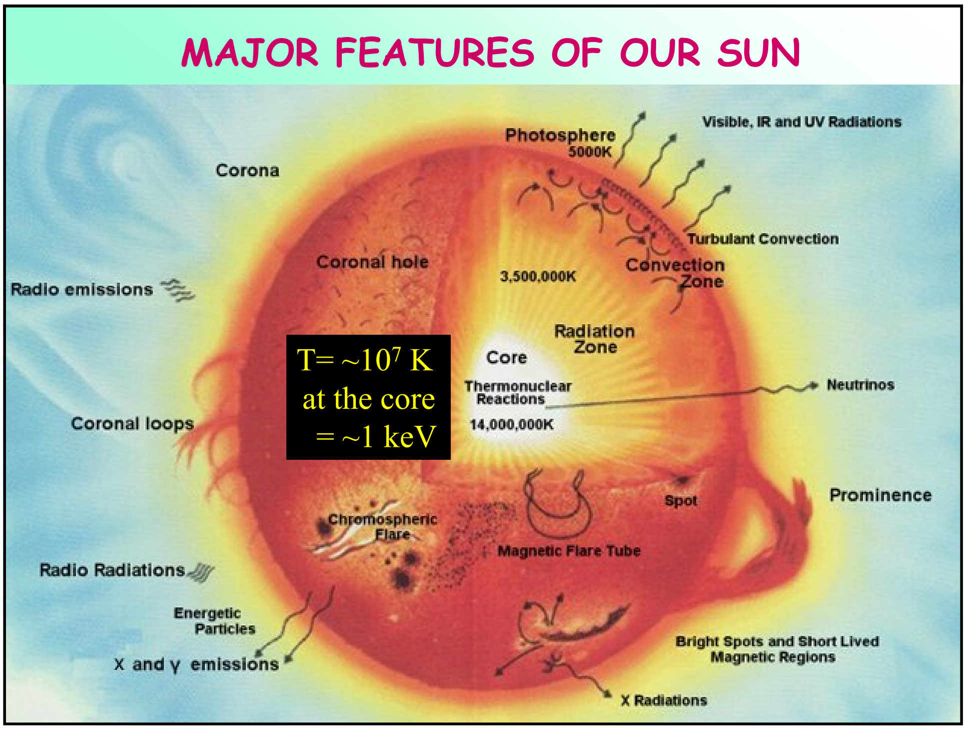
## Supernova Cycle

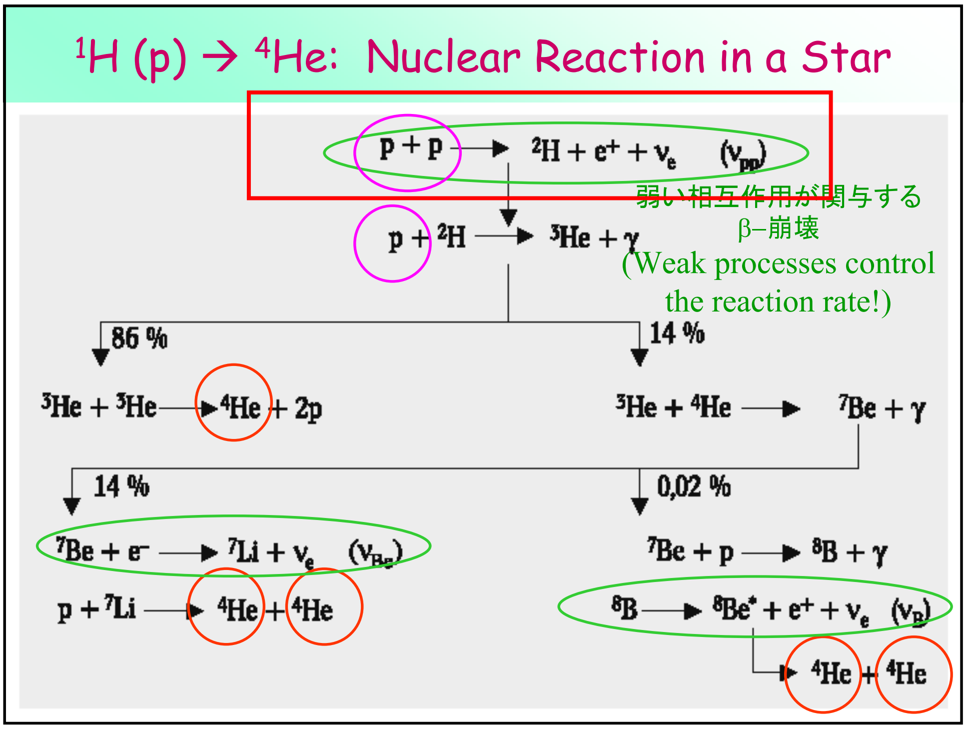


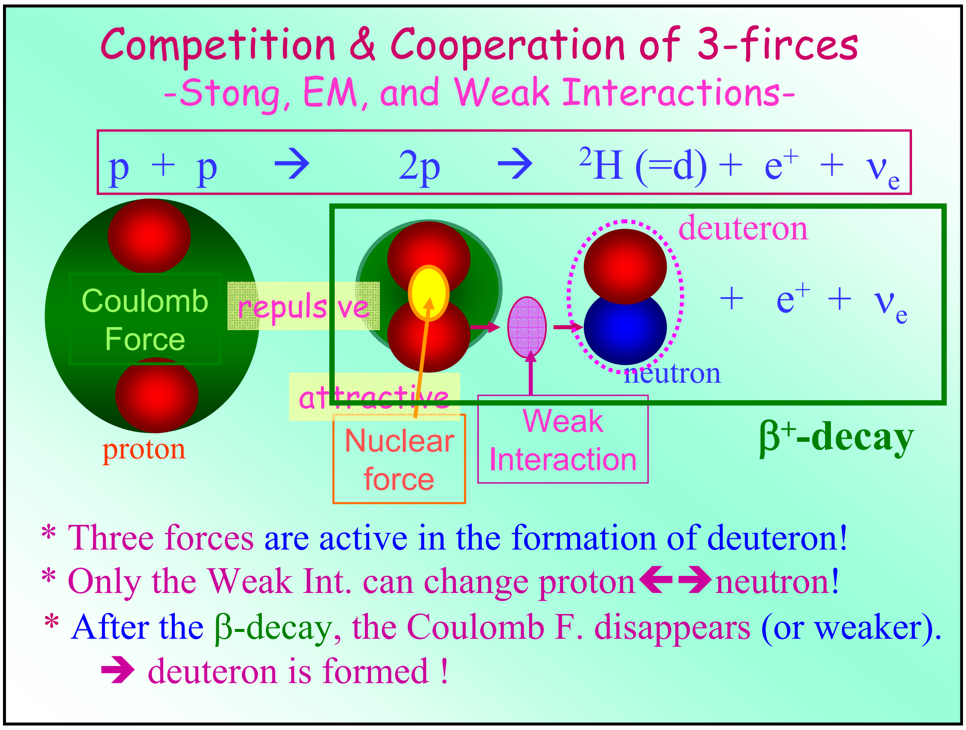

## Neutron-star Merger




自然の階層性（ウロボロスの蛇）
Layer Structure of Nature (Snake of Uroboros)
Only Four Forces (Interactions) !
Conservation Lows are valid !
UNIVERSE
宇宙
1.4 billion
light years
GALAXY
銀河
Quark
QUARK
クオーク
1 fm
ATOM
原子
DNA
分子
Molecule
CELL
細胞
ME
生物・人間
MOUNTAIN
構造物、山
EARTH
地球
太陽系
CHEMISTRY
BIOLOGY
GEOLOGY
1 m
(by Glashow)


MAJOR FEATURES OF OUR SUN
Photosphere
5000K
Visible, IR and UV Radiations
Corona
Turbulant Convection
Coronal hole
3,500,000K
Convection
Zone
Radio emissions
Radiation
Zone
Core
T= ~$10^7$ K
at the core
= ~1 keV
Thermonuclear
Reactions
Neutrinos
14,000,000K
Coronal loops
Prominence
Spot
Chromospheric
Flare
Magnetic Flare Tube
Radio Radiations
Energetic
Particles
X and γ emissions
Bright Spots and Short Lived
Magnetic Regions
X Radiations


1H (p) → 4He: Nuclear Reaction in a Star
p + p → 2H + e+ + νe (νpp)
弱い相互作用が関与する
β−崩壊
(Weak processes control
the reaction rate!)
p + 2H → 3He + γ
86 %
14 %
3He + 3He → 4He + 2p
3He + 4He → 7Be + γ
14 %
0,02 %
7Be + e− → 7Li + νe (νBe)
7Be + p → 8B + γ
p + 7Li → 4He + 4He
8B → 8Be* + e+ + νe (νB)
4He + 4He

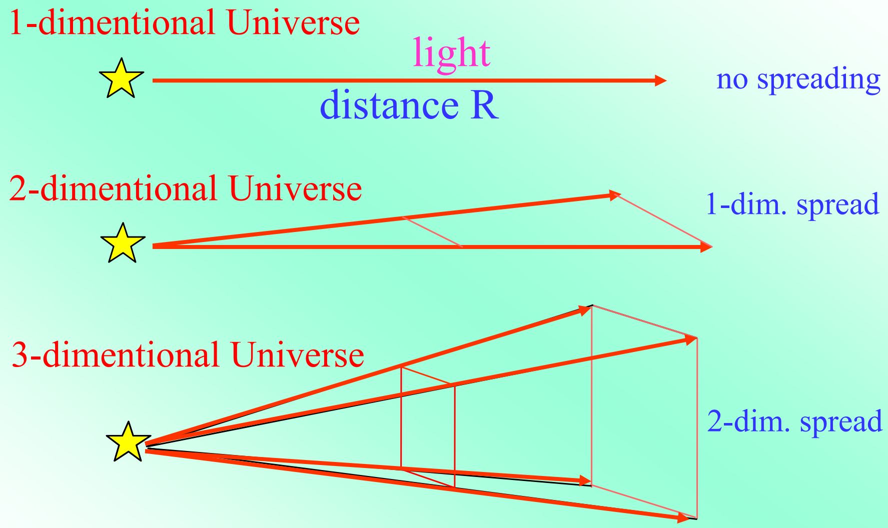

Competition & Cooperation of 3-firces
-Stong, EM, and Weak Interactions-
p + p → 2p → 2H (=d) + e+ + νe
Coulomb Force
proton
repulsive
attractive
Nuclear force
Weak Interaction
deuteron
neutron
+ e+ + νe
β+-decay
* Three forces are active in the formation of deuteron!
* Only the Weak Int. can change proton ↔ neutron!
* After the β-decay, the Coulomb F. disappears (or weaker).
➔ deuteron is formed !

# 4-fundamental interactions (forces)

| Interactions | Example | Transmitter |
|---|---|---|
| Strong int.* | Nuclear Force | Meson* (Gluon) |
| EM int.# | Coulomb Force | Photon# |
| Weak int.* | Beta-decay | W-, Z-boson* |
| Gravitational# | Apple falls! | Graviton# |

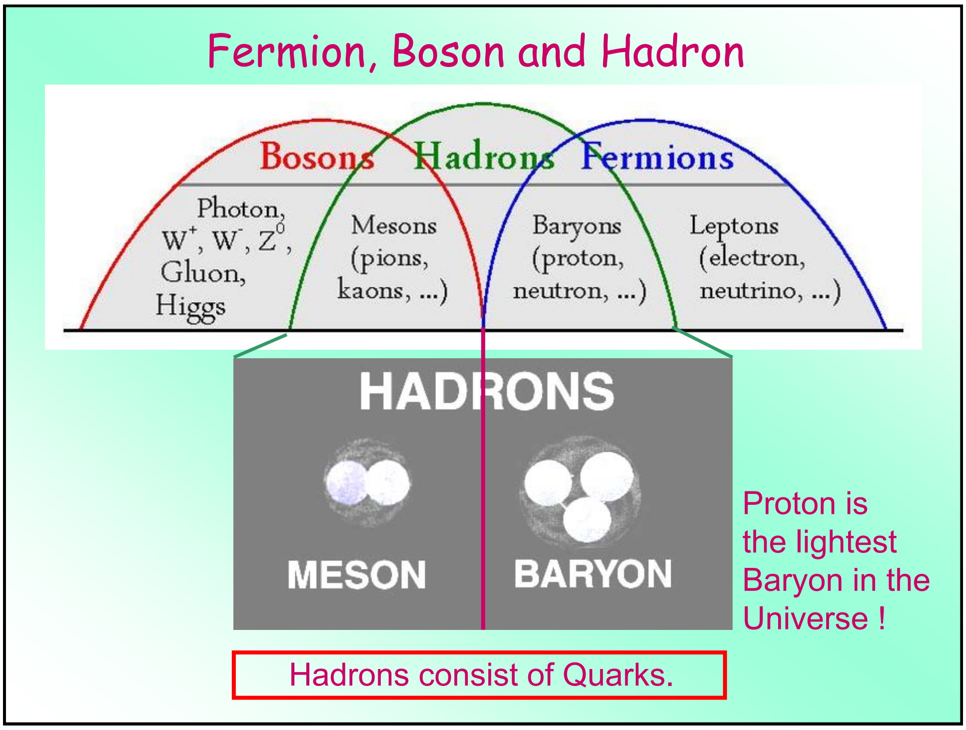
*short-range #long-range *with mass #mass-less

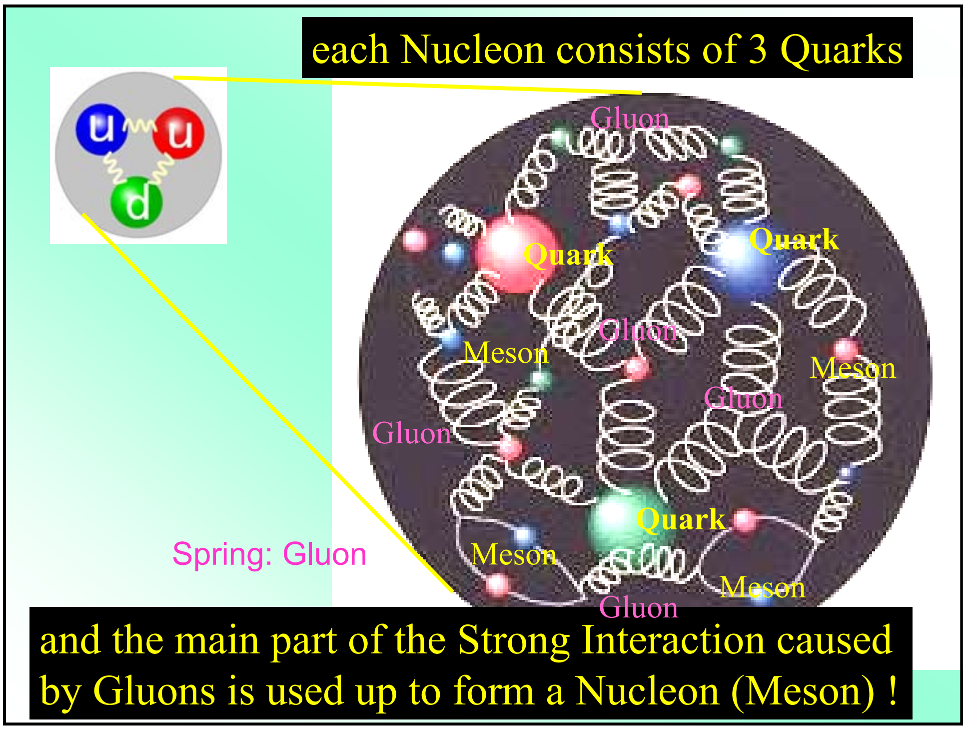
# Long and short-range forces



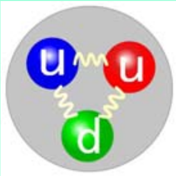

brightness of a star $\propto 1/R^2$

i.e., inversely prop. to the expansion of the space

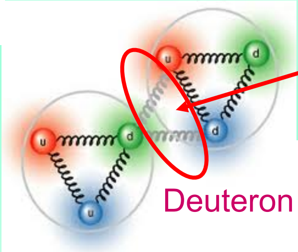

Fermion, Boson and Hadron
Bosons
Hadrons
Fermions
Photon, W+, W-, Z0, Gluon, Higgs
Mesons (pions, kaons, ...)
Baryons (proton, neutron, ...)
Leptons (electron, neutrino, ...)
HADRONS
MESON
BARYON
Proton is the lightest Baryon in the Universe !
Hadrons consist of Quarks.


each Nucleon consists of 3 Quarks
u
u
d
Gluon
Quark
Quark
Gluon
Meson
Meson
Gluon
Gluon
Quark
Spring: Gluon
Meson
Meson
Gluon
and the main part of the Strong Interaction caused by Gluons is used up to form a Nucleon (Meson) !

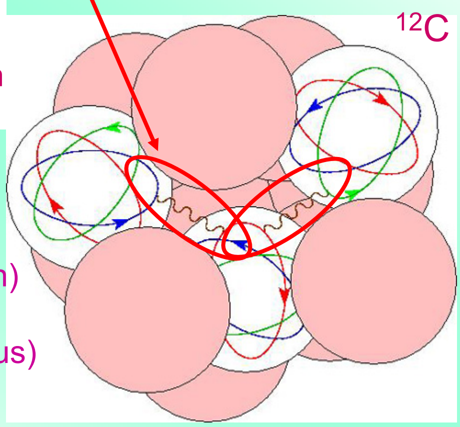
# Image of Nuclei




Proton



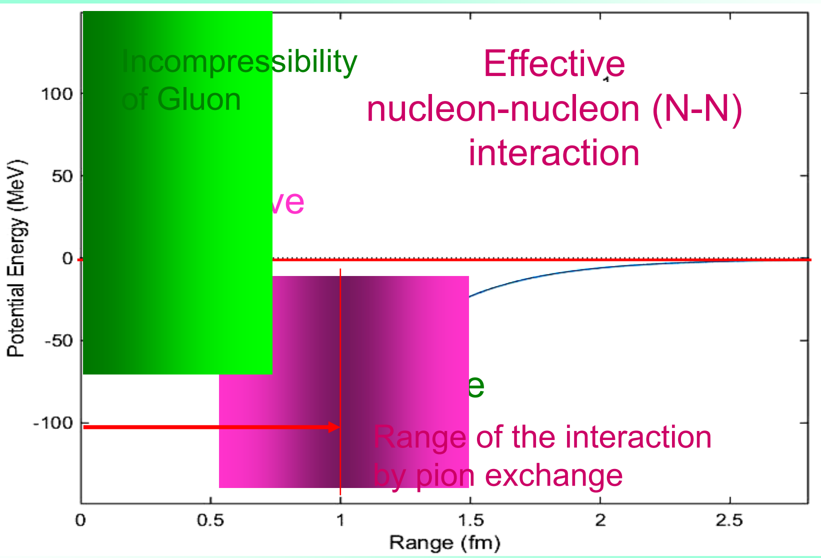

Effective nucleon-nucleon (N-N) force i.e., Nuclear Force !



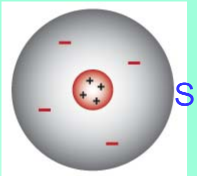

Strong Interaction is the source for binding

1) Quarks (and forms a nucleon) as well as
2) Nucleons (and forms a nucleus)

# Nucleon-Nucleon Interaction



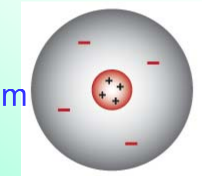
## Similarity of NN-force with the Van der Waals force (intermolecular force)



Simple atom

$F_C \propto 1/R^2$

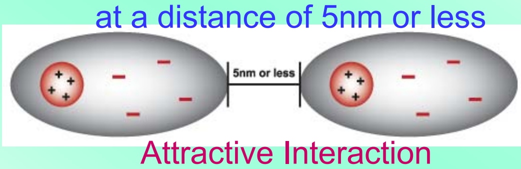
Simple atom



$F_C \propto 1/R^2$

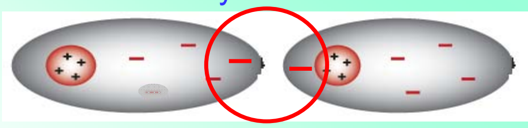
No Interaction

at a distance of 5nm or less




Attractive Interaction

at a very close distance



Repulsive Interaction

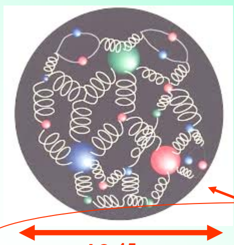
## Importance of field (potential E): nucleon mass vs. quark mass

proton =uud neutron =udd


**nucleon mass : ~940 MeV**
**pion mass : ~140 MeV**

**u-quark mass : 3 MeV**
**d-quark mass : 6 MeV**

The quark mass is only ~1% of the nucleon mass !
Where is the other part of the mass (energy)? ($E = mc^2$)
Ans: Potential Field ➔ **The field of the strong interaction (gluon field) carries the main part of the energy !**



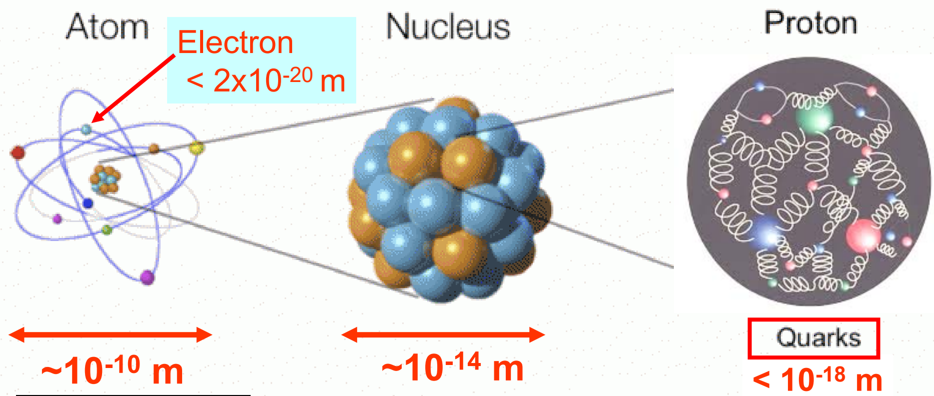
Nucleon mass
= kinetic energy of quarks
+ potential energy (gluon cloud)
+ (small) quark mass

**~$10^{-15}$ m**

**(quark size < $10^{-18}$ m)**

Nucleons are almost empty !

## How empty are we ?




Atom is almost empty !

Protons and Neutrons are almost empty !

"Matter" is almost empty !

## Roles of 3 forces

in Nuclear excitation & decay

| | |
|---|---|
| Strong: | nuclear reactions |
| | [(p, p'), ($\alpha$, $\alpha$'),.., (p, n), ($^3$He,t) etc] |
| | particle decays |
| EM: | (e, e'), Coulomb ex., $\gamma$-decay |
| Weak: | $\nu$-induced reactions, $\beta$-decay |
| | [($\nu_x$, $\nu_x$'), ($\nu_e$, $e^-$),…] |

*if Strong can play a role, other two are hidden!
*if EM , Weak
*if Strong and EM cannot play roles,
then Weak will appear on the stage.

# ***How Do We See Nuclei?

## Uniqueness of Nuclei

Nucleus :

Quantum Finite Many-body System with 2 Fermions

a Quantum System where

3 interactions out of 4 are active

Strong, Weak, EM

(Gravitational force is too weak!)

Conservation Laws are valid

energy

momentum

angular momentum ( *L*, *S*, parity)

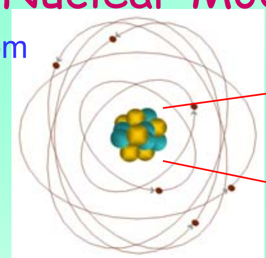
charge

hadron number, lepton number,

## How do we see nuclei?
## How do we see Table Mountain?

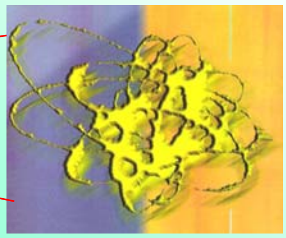
## Nuclear Models & Nuclear Structure

Atom



Nucleus

size difference of $10^4$



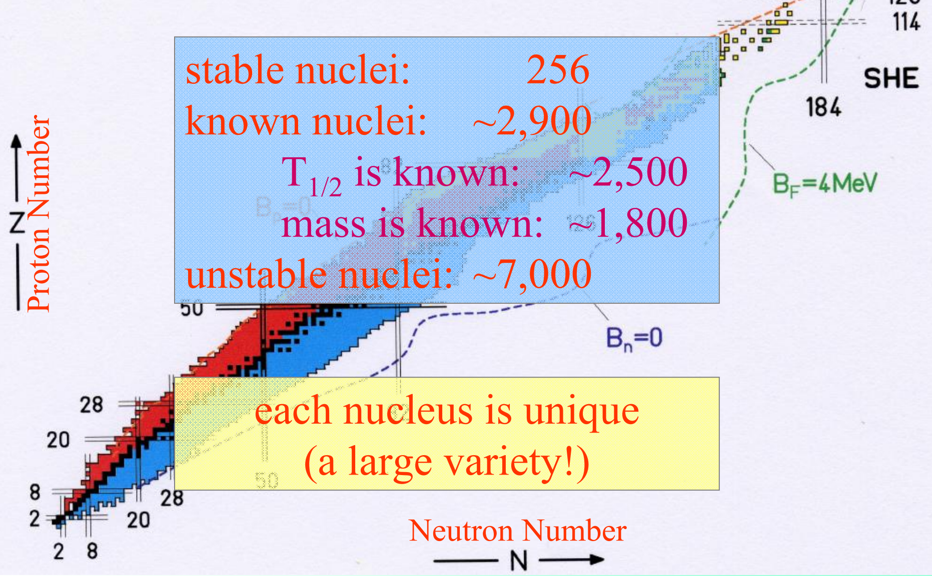
*Movement of electrons in a Coulomb potential well defined by the nucleus in the center.

Perturbation method works well for the calculation.
Accurately described !

*Movement of nucleons in a self-consistent potential formed by the moving nucleons themselves.
Nuclear force is not well defined.
Recoil effect should be taken into account.

Taking the "essential parts"
Nuclear Models are constructed !

## Nuclear Chart




## How Nuclei are defined ?

*Quantum Finite Many-body System

=> quantum numbers are important

L, S, J, K, T

=> selection rules of Q-numbers are importnat

*Active forces in nuclei:

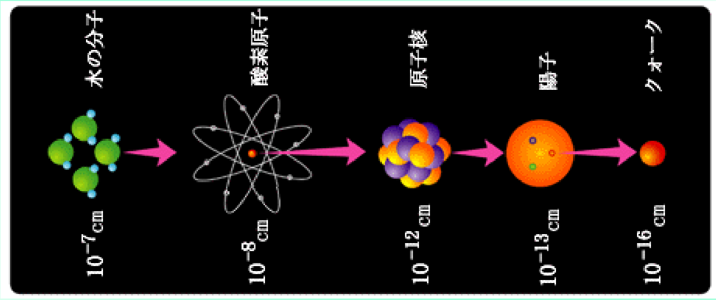
3 out of 4 fundamental forces

| | | | |
|---|---|---|---|
| strength: | **strong** >> | **electro-magnetic** >> | **weak** |
| time : | fast | middle | slow |
| | $(\sim 10^{-20}\text{s})$ | $(\sim 10^{-15}\text{s})$ | $(\sim 10^{-1}\text{s})$ |

*they struggle to make their territory larger !

➔phenomena from 3 forces can be combined for the study of nuclei !

## Uniqueness of Nuclei




Nucleus: Quantum Finite System

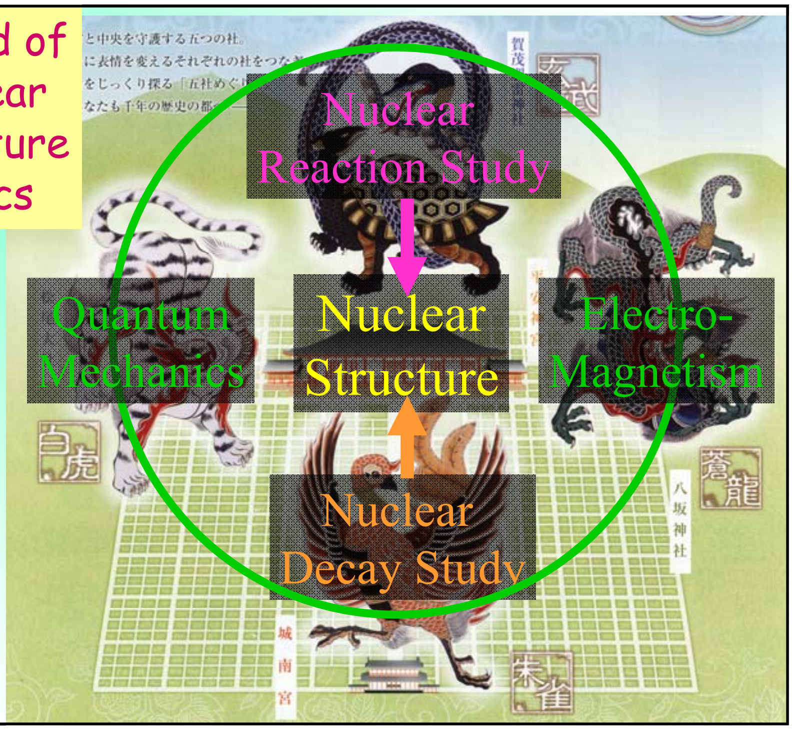
freedom: protons Z & neutrons N

further freedom: mesons

further more freedom: quarks

Shell model

## **World of Nuclear Structure Physics



## Structure information form Transitions

Nuclear Transitions give us Structure information

*Transition strength: proportional to $|\langle f|\, \boldsymbol{Op}\, |i\rangle|^2$

$\mathbf{H}_i |i\rangle = E_i |i\rangle, \quad \mathbf{H}_f |f\rangle = E_f |f\rangle$

*Studied by: Nuclear Reactions, Decays

Reaction: Excitation + Spectroscopy

Decay: Spectroscopy

*Mode of Excitation ←→ ***Op***

## For the study of Nuclear Structure

We have two different tools!

1) Decay Studies

γ-decay: in beam γ-study, source study

β-decay: β-ray study, β-delayed **γ, *p*** or ***n***

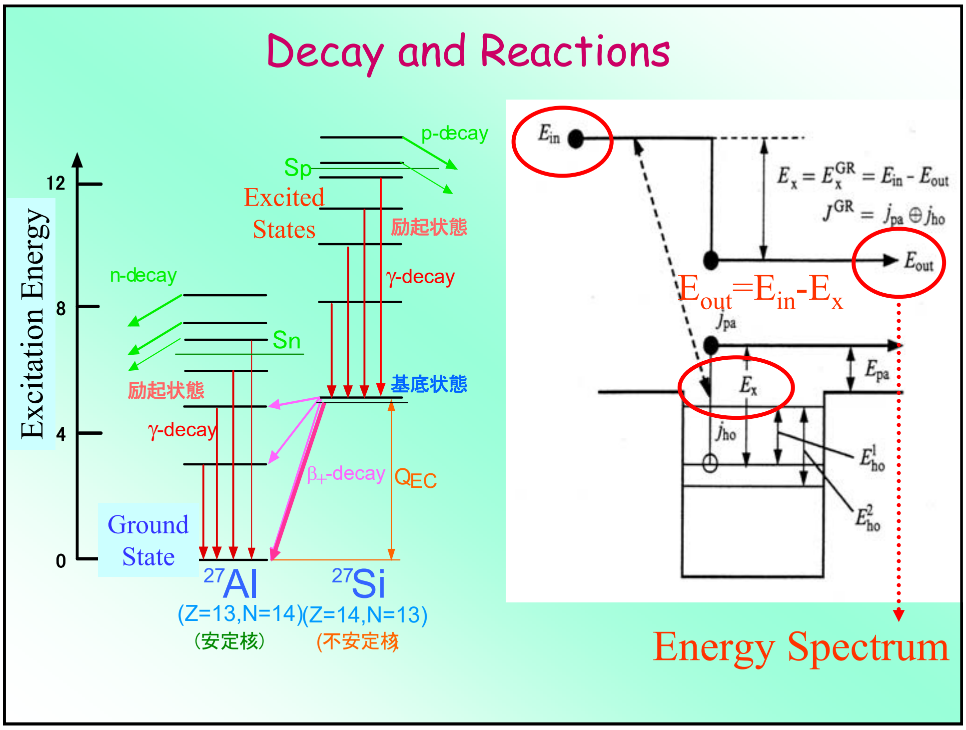
2) Reaction Studies

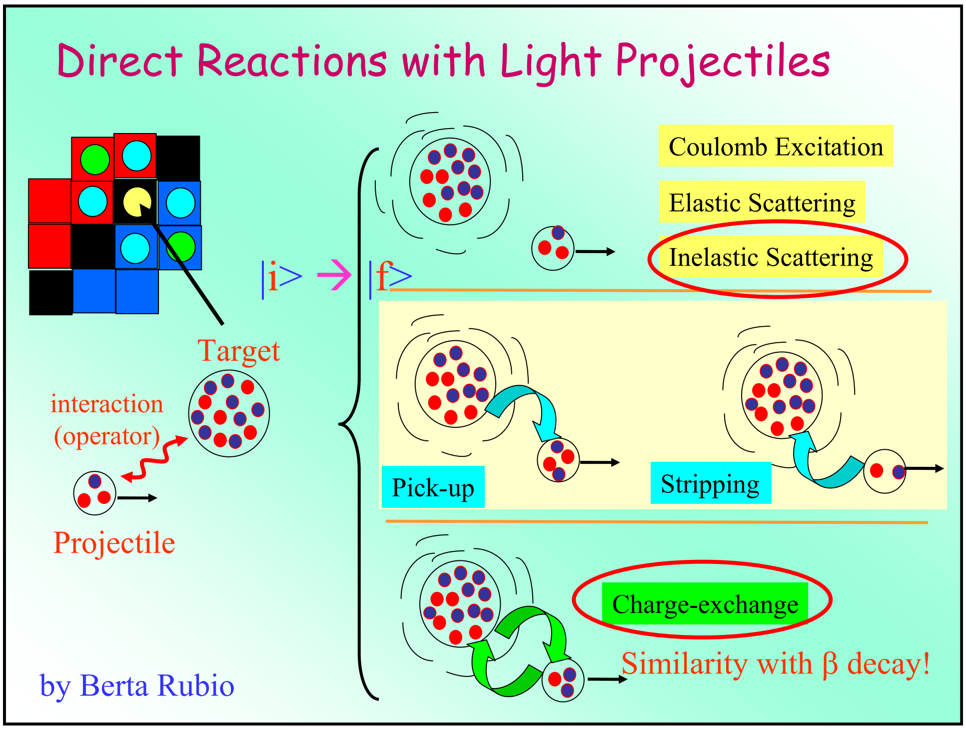
Inelastic Scattering: simply giving Energy

Charge Exchange Reaction:

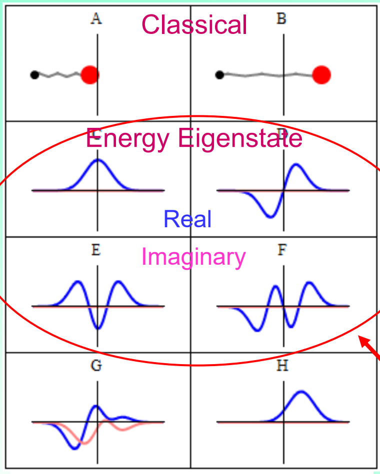
charge-exchange & giving Energy

Pick-up Reaction, Transfer Reaction, …

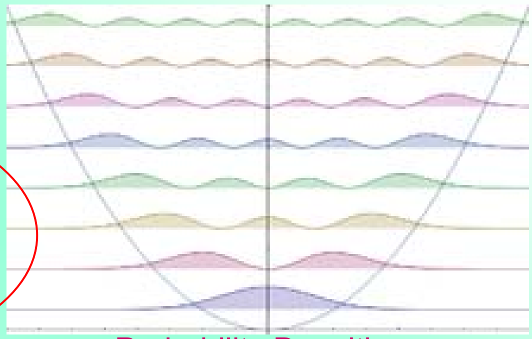

Decay and Reactions
Excitation Energy
12
8
4
0
p-decay
Sp
Excited States
励起状態
γ-decay
n-decay
Sn
励起状態
γ-decay
基底状態
β+-decay
QEC
Ground State
27Al
(Z=13,N=14)
(安定核)
27Si
(Z=14,N=13)
(不安定核
E_in
E_x = E_x^GR = E_in - E_out
J^GR = j_pa ⊕ j_ho
E_out
E_out=E_in-E_x
j_pa
E_pa
E_x
j_ho
E^1_ho
E^2_ho
Energy Spectrum


Direct Reactions with Light Projectiles
|i> → |f>
Target
interaction (operator)
Projectile
Coulomb Excitation
Elastic Scattering
Inelastic Scattering
Pick-up
Stripping
Charge-exchange
Similarity with β decay!
by Berta Rubio

## Harmonic Oscillator




$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$



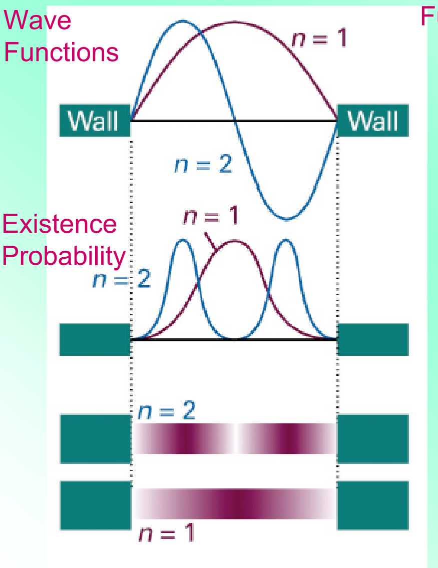
Probability Densities

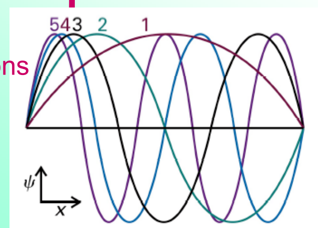
Quantum **n**-phonon Oscillation

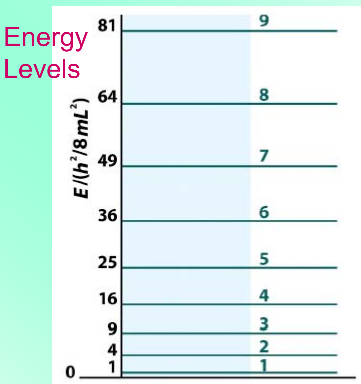
$$E_n = \hbar\omega\left(n + \frac{1}{2}\right).$$

from Wikipedia

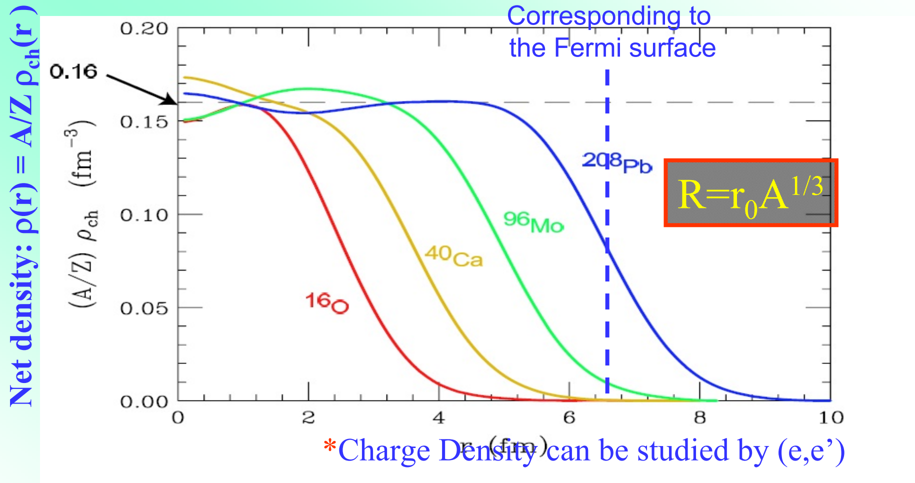
## Particles in a infinite potential



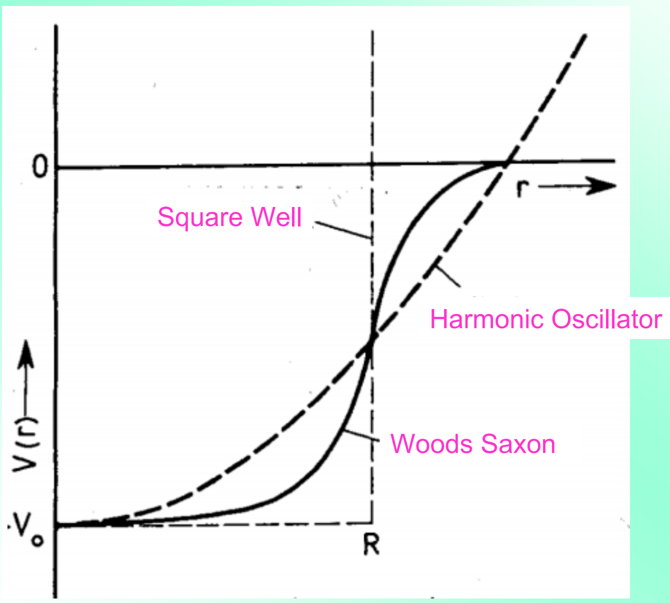
## Saturation of Nucleon Density in Nuclei

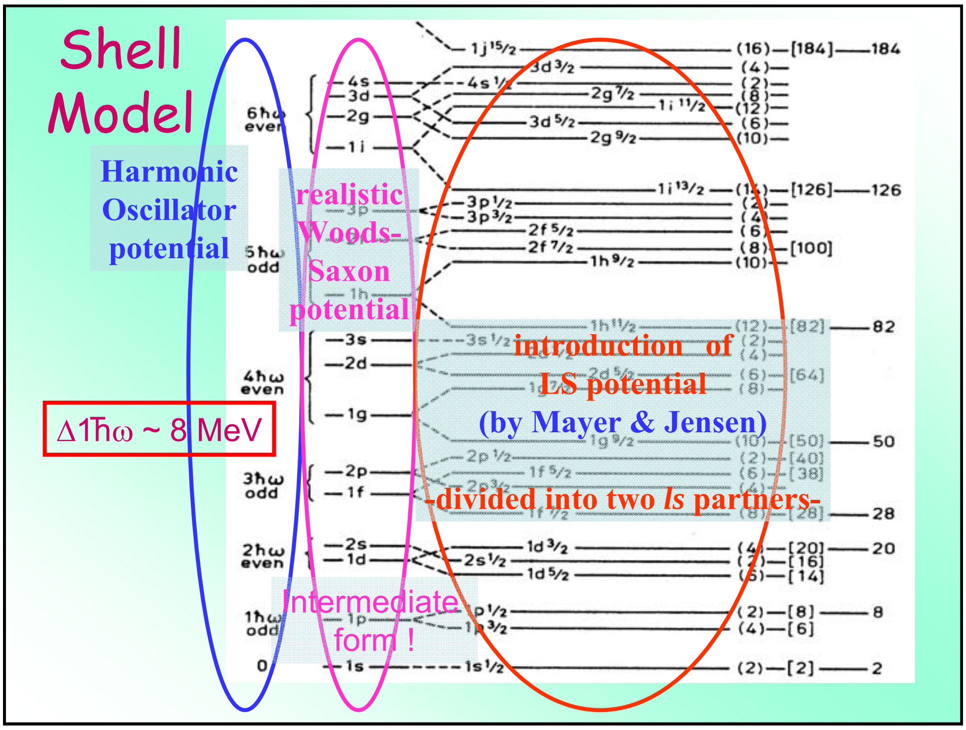


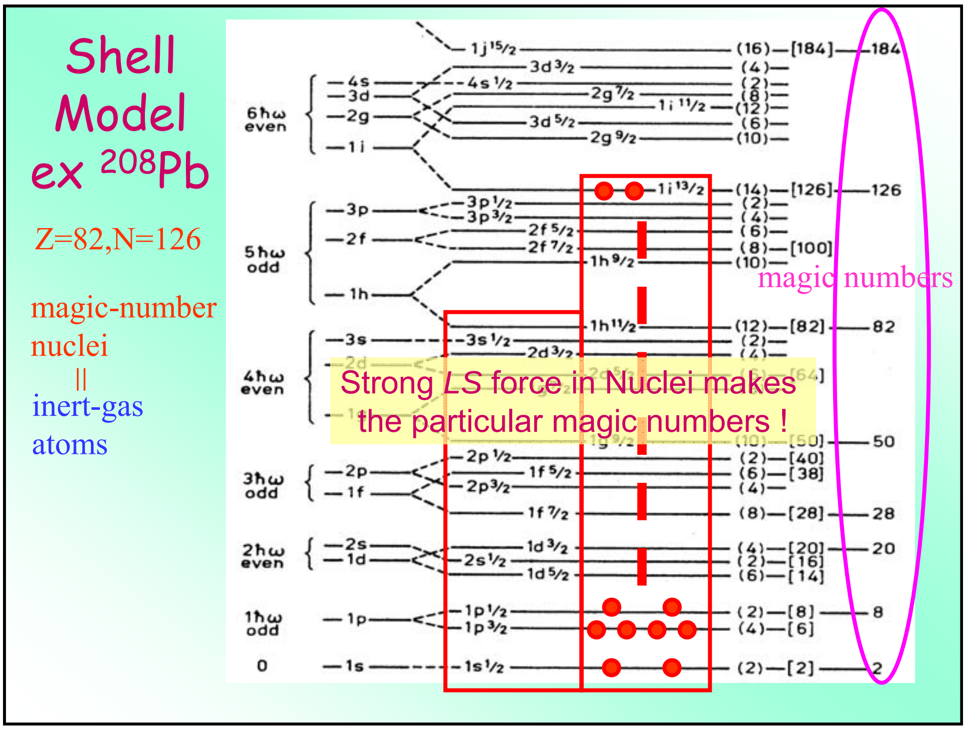

*Charge Density can be studied by (e,e')

- → due to the "short range" nature of nuclear interaction
- → due to the intermediate mass of pion (~135 MeV)

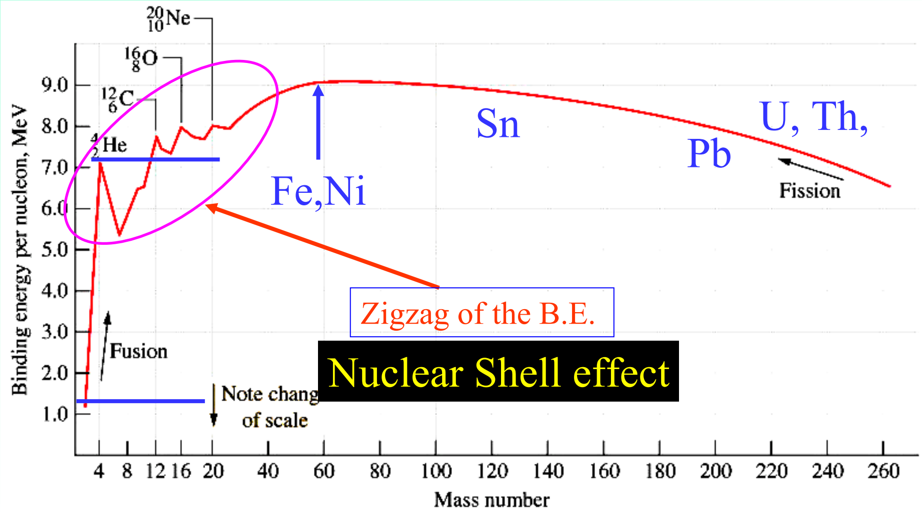
## Nuclear Potentials



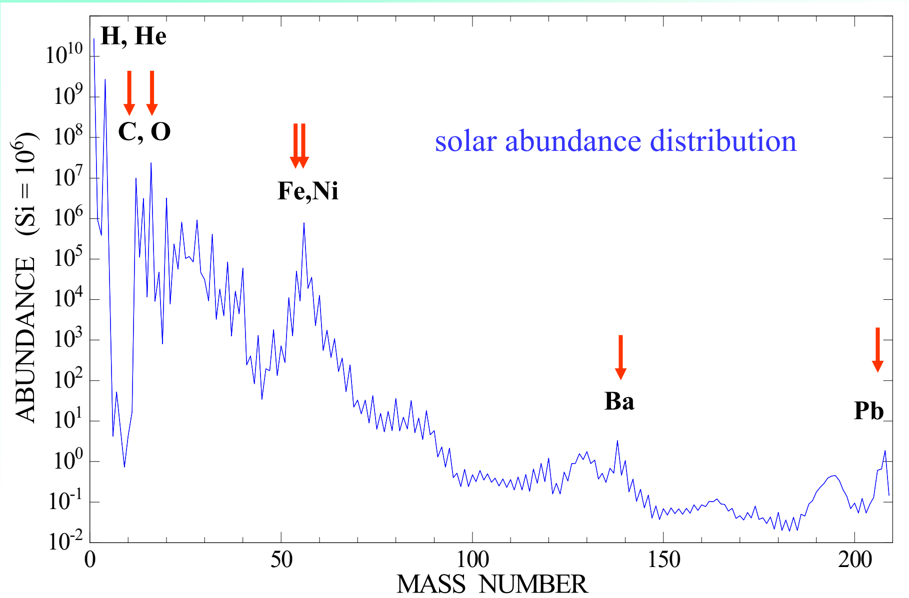

Shell
Model
Harmonic
Oscillator
potential
Δ1ħω ~ 8 MeV
realistic
Woods-
Saxon
potential
introduction of
LS potential
(by Mayer & Jensen)
-divided into two ls partners-
Intermediate
form !


Shell
Model
ex 208Pb
Z=82,N=126
magic-number
nuclei
||
inert-gas
atoms
Strong LS force in Nuclei makes
the particular magic numbers !
magic numbers

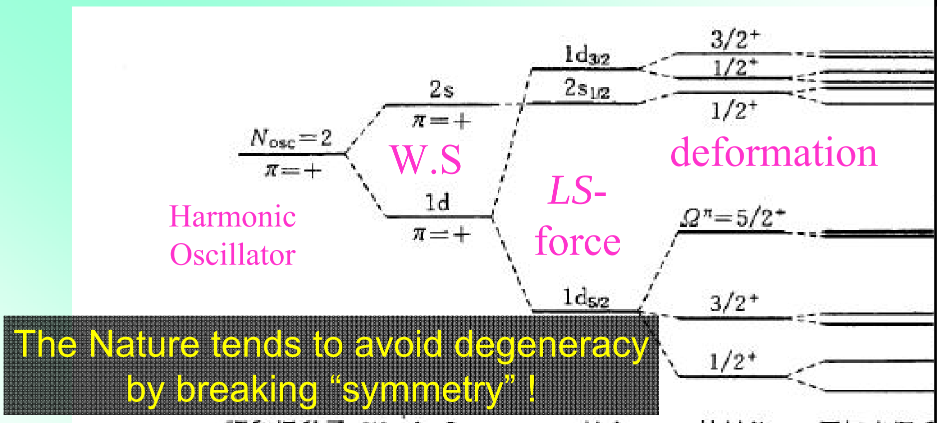
# Nuclear Binding Energy
## -Shell Effect-




# Solar Abundance



## Degeneracy of single particle states

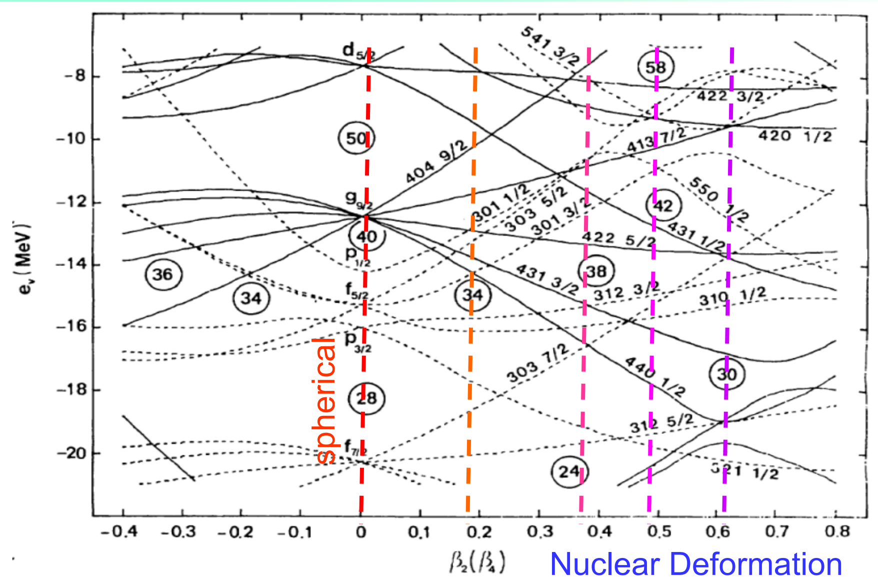



The Nature tends to avoid degeneracy by breaking "symmetry" !

| | 調和振動子ポテンシャル | Woods-Saxon ポテンシャル | j-j結合殻モデル | 軸対称変形 | 回転座標系 |
|---|---|---|---|---|---|
| Q-numbers : | $(N_{osc}, \pi)$ | $(n, l, \pi)$ | $(n, l, j, \pi)$ | $(\Omega, \pi)$ | $(\alpha, \pi)$ |
| Degeneracy | $(N_{osc}+1)(N_{osc}+2)$ | $2(2l+1)$ | $2j+1$ | 2 | 1 |

## Nilsson Orbit in Deformed Nuclei

Magic Numbers depend on Nuclear Deformation !



## ***Rotational Bands

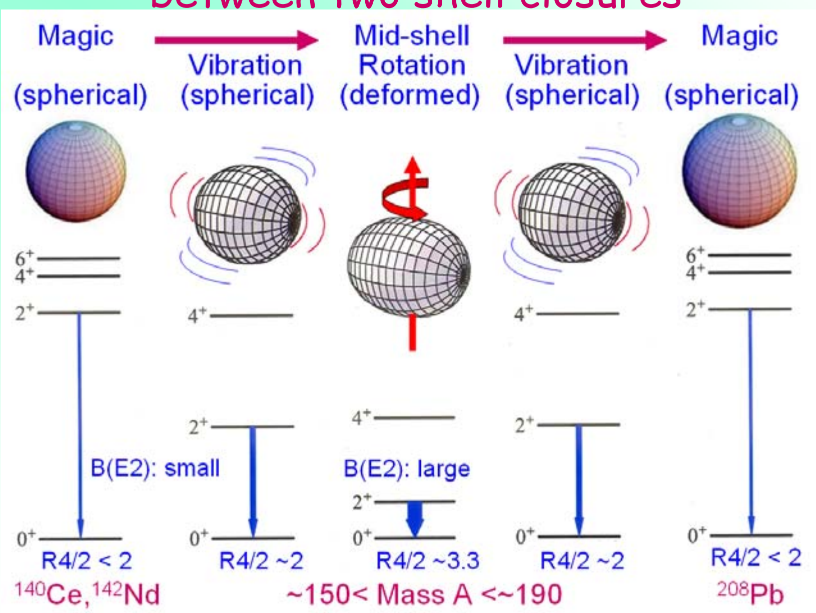
- in deformed nuclei -

Rotational Energy

| in Classical Mechanics | in Quantum Mechanics |
| --- | --- |
| $E_R = J^2 / 2\mathcal{J}$ | $E_R = J(J+1) / 2\mathcal{J}$ |

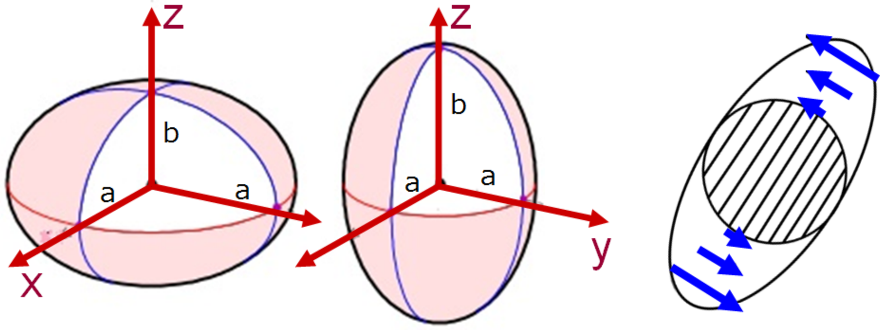
$\mathcal{J}$ : Moment of Inertia

## Evolution of Nuclear Structure -between two shell closures-



## Rotation of a Quantum System

Oblate & Prolate Nuclei and Quantum Rotation




$\beta_2 < 0$
Q-moment < 0

$\beta_2 > 0$
Q-moment > 0

Center part: Irrotational
Outer part : Rotational

Experimentally observed
“Moment of Inertia”
is smaller than is expected !

## ***Residual Interactions

## Note on SM & "Residual Interactions" (I)

In the Shell-Model nucleons are treated as independent.

Single-particle phenomena are usually well described.

*ex.* $J^\pi$ values of ground states of odd nuclei

| Nucleus | $J^\pi$ |
|---|---|
| ${}^{3}_{2}He_{1}$ | $1/2^+$ |
| ${}^{7}_{3}Li_{4}$ | $3/2^-$ |
| ${}^{17}_{8}O_{9}$ | $5/2^+$ |
| ${}^{41}_{20}Ca_{21}$ | $7/2^-$ |

The $J^\pi$ values : determined by Valence Nucleons !

Shells deep inside are treated as inert.

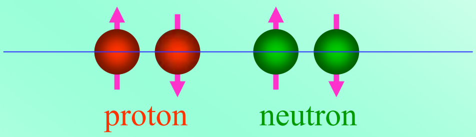
Doubly magic nuclei form "inert core".

*ex.* ${}^{4}He$ (N=Z=2), ${}^{16}O$ (N=Z=8), ${}^{40}Ca$ (N=Z=20), ${}^{56}Ni$ (N=Z=28)

## Note on SM & "Residual Interactions" (II)

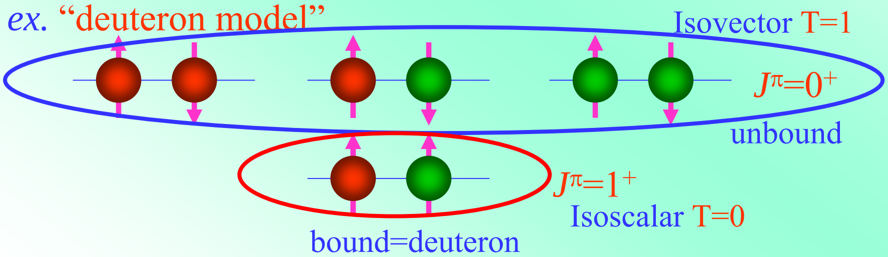
However, $J^\pi$ values of g.s. in even-even nuclei are $J^\pi=0^+$.

➡ We notice the importance of the pairing interaction

*ex.* $1s_{1/2}$  $= {}^{4}He = \alpha$ $J^\pi=0^+$

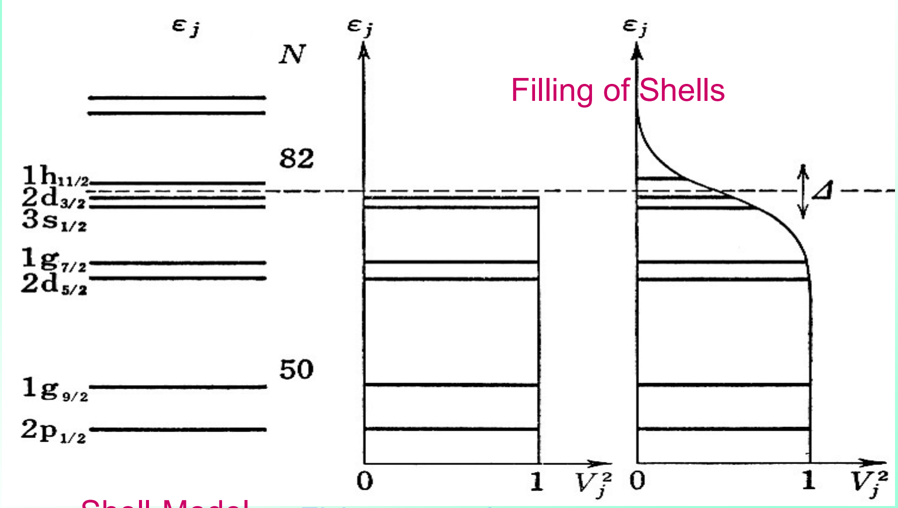

In general, interactions that are not included in a model are called "residual interactions"

*ex.* "deuteron model"



## Pure Shell-Model & +Residual Int.

(e.g. Pairing Int.)




Shell-Model States

Filling assuming "Ideal Shell-Model"

Shell-Model +Residual Interaction

## Note on SM & "Residual Interactions" (III)

We first assume a model.
*ex.* Harmonic Oscillator Model,
Shell Model.

Remaining part of nucleon-nucleon interactions that are not included in the model are treated as "residual (or effective) interactions."

Residual interactions between valence nucleons play important roles to form nuclear structure.

Mainly 2-body int., but also 3-body int.

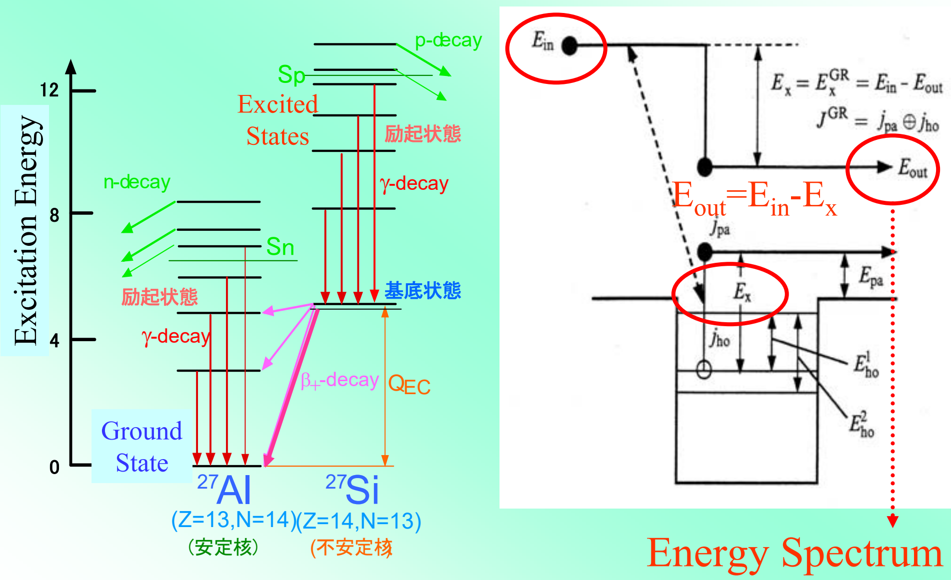
***Observing Residual Interactions

-seeing is believing-

*** "Nuclear Excitations"

"Nuclear De-excitations"

## Decay and Reactions




## ***Observation of Nuclear Excitations

### Nuclear Reaction and Decay Studies

**Nuclear Reaction**

In-coming particle with $E_{in}$

Out-going particle with $E_{out}$

*out-going particles are analyzed

by energy

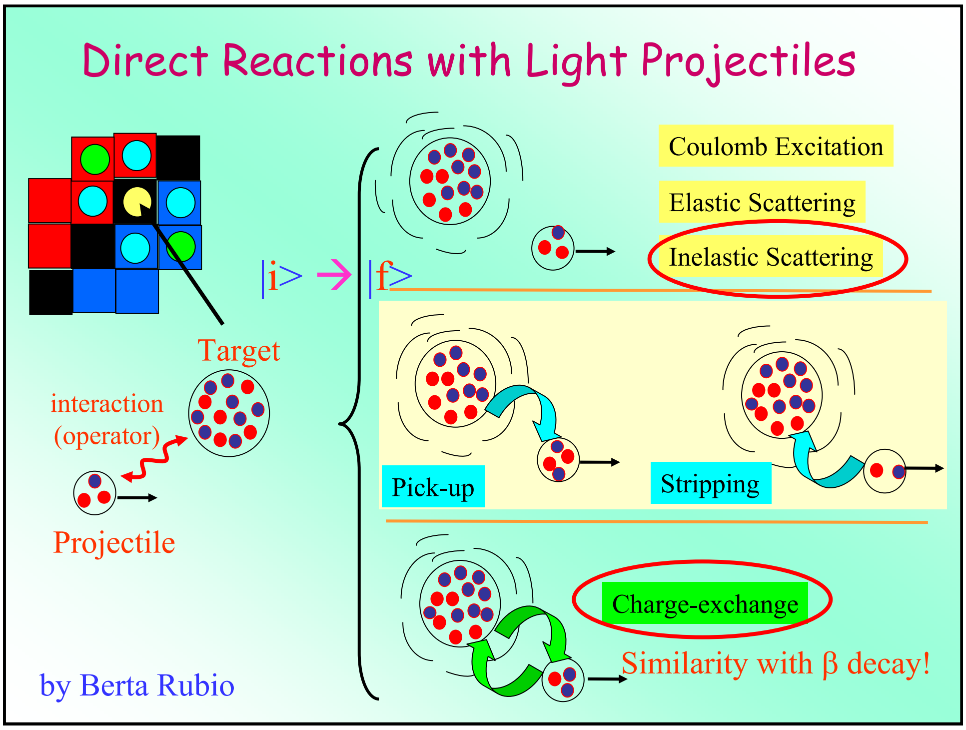
by angular distribution…

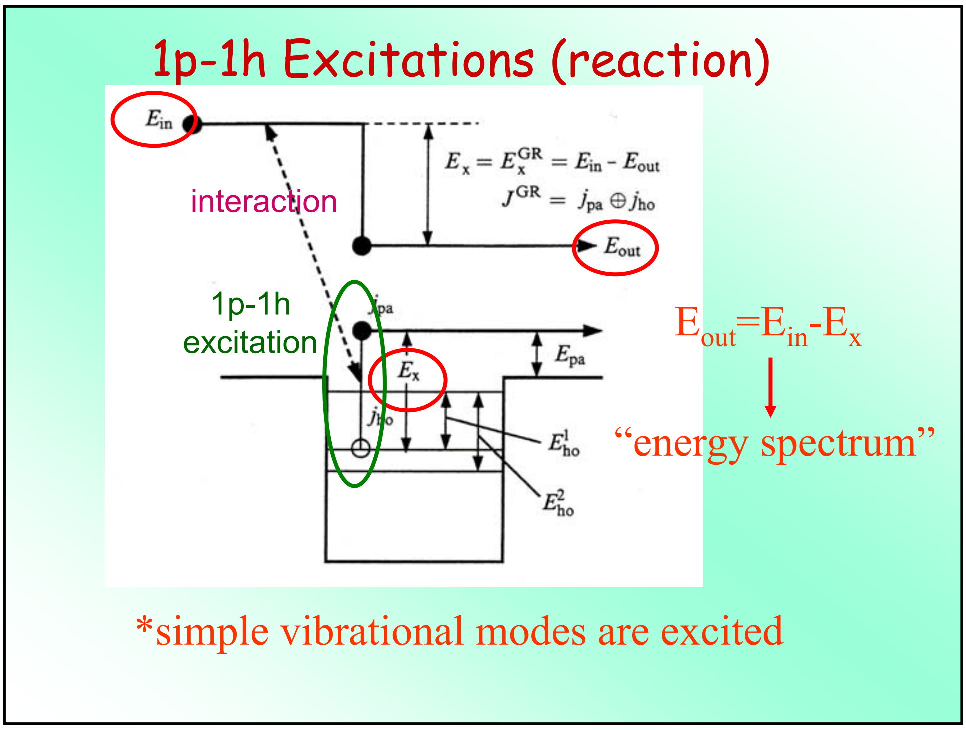
**Decay Studies**

β decay, γ decay, particle decay

*unstable nuclei are first produced

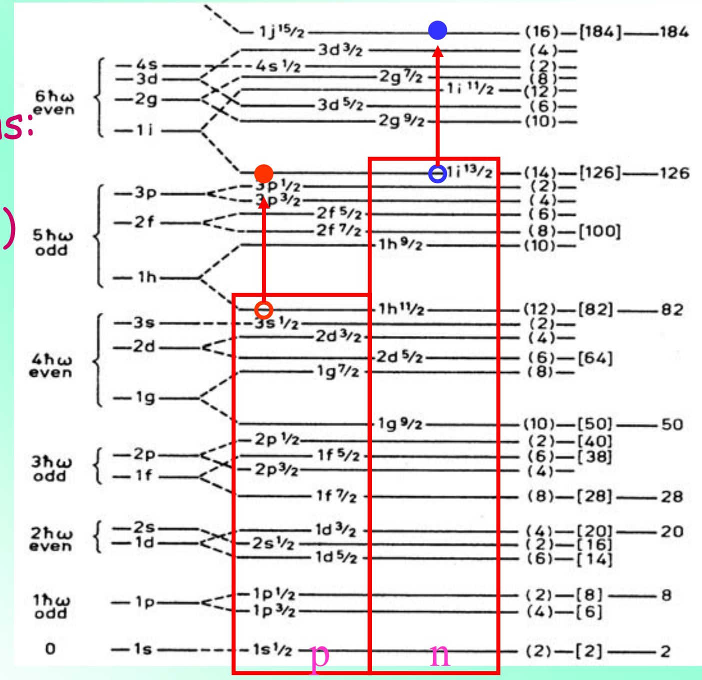
*then decays are measured


Direct Reactions with Light Projectiles
Coulomb Excitation
Elastic Scattering
Inelastic Scattering
|i> → |f>
Target
interaction
(operator)
Projectile
Pick-up
Stripping
Charge-exchange
Similarity with β decay!
by Berta Rubio

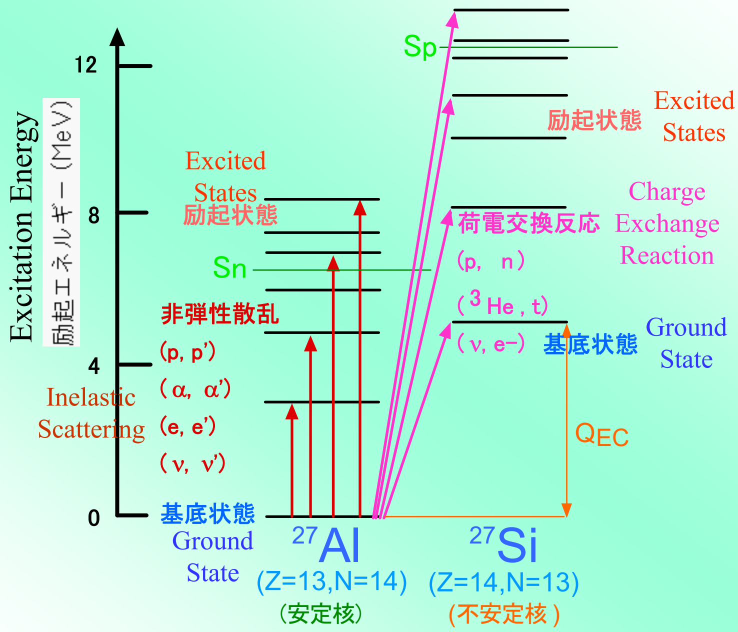

1p-1h Excitations (reaction)
E_in
interaction
E_x = E_x^GR = E_in - E_out
J^GR = j_pa ⊕ j_ho
E_out
1p-1h
excitation
j_pa
E_pa
E_x
j_ho
E_ho^1
E_ho^2
E_out=E_in-E_x
"energy spectrum"
*simple vibrational modes are excited

Basic Excitations: 1p-1h (inelastic)

Z=82,N=126
$^{208}$Pb

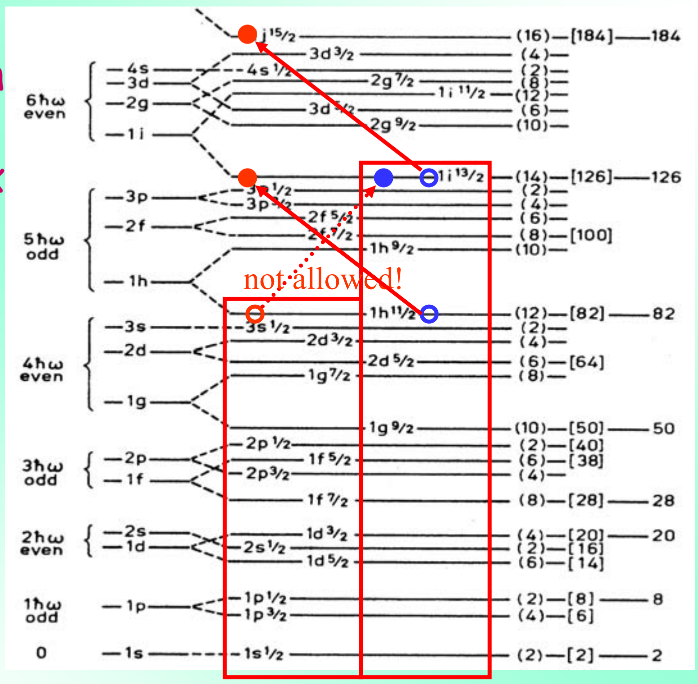



# Nuclear Excitations by Reactions



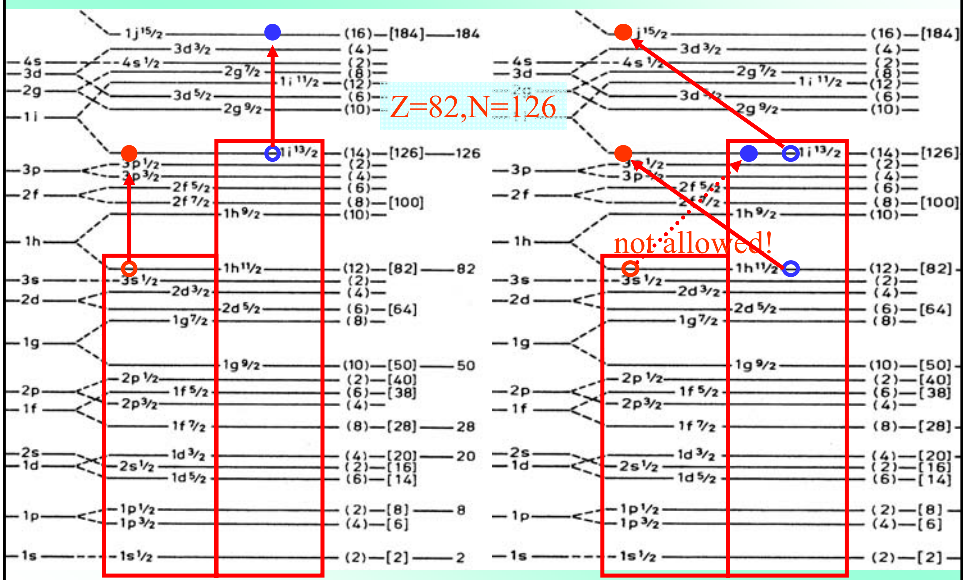
## 1p-1h Excitation in $^{208}$Bi (charge ex

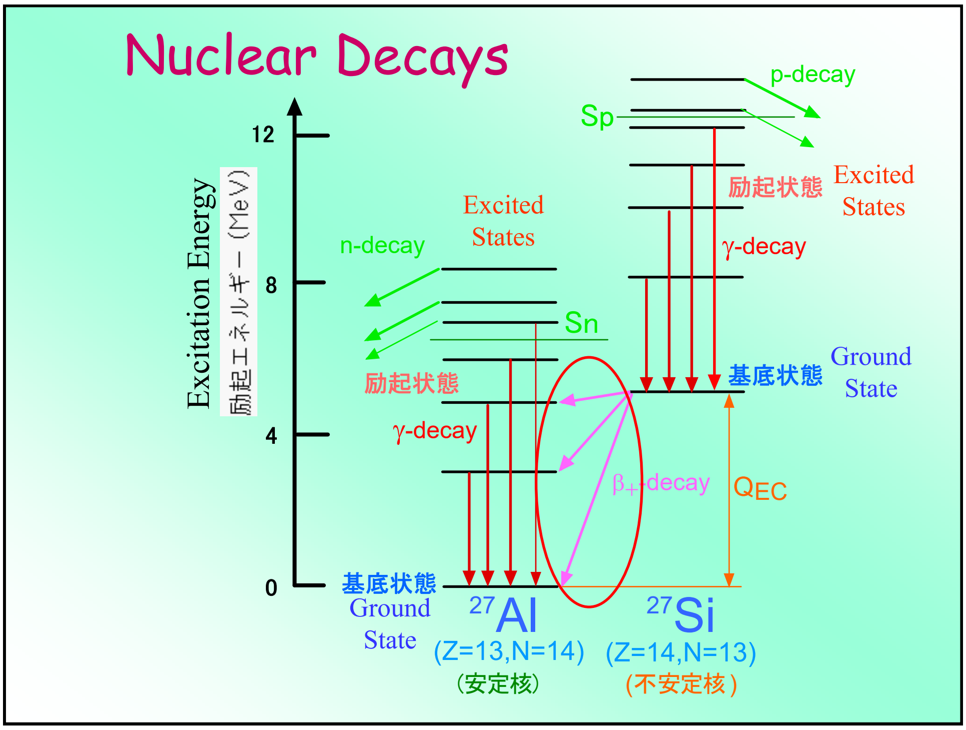
Z=83, N=125

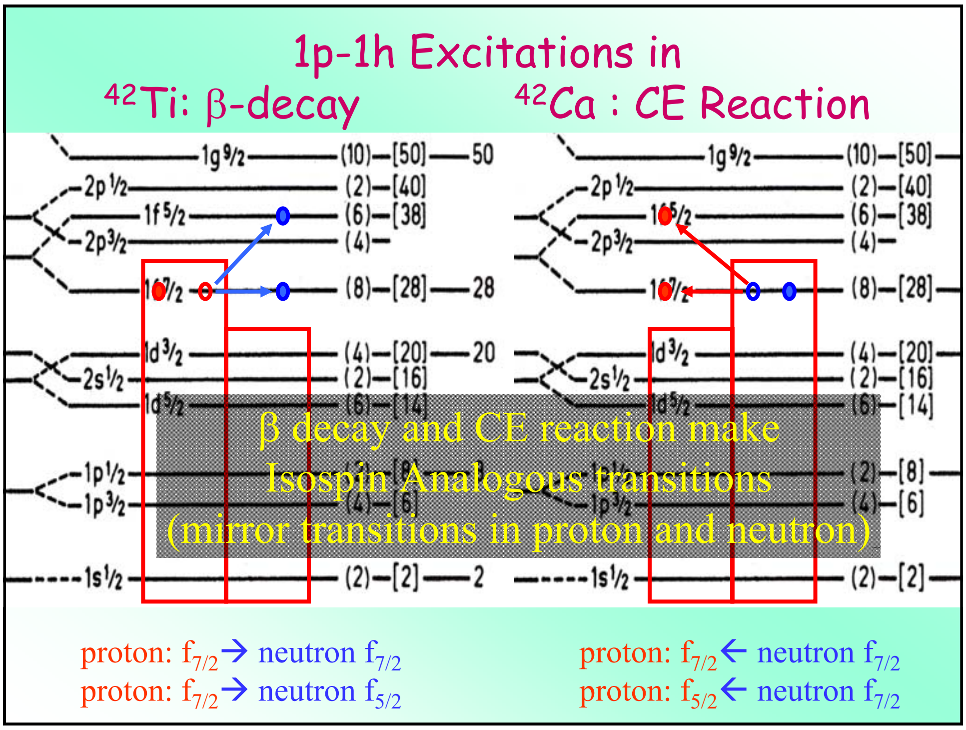



## 1p-1h Excitations in $^{208}$Pb

(inelastic) (charge-exchange)




Nuclear Decays
Excitation Energy
励起エネルギー（MeV）
12
8
4
0
p-decay
Sp
Excited States
励起状態
γ-decay
n-decay
Sn
Ground State
基底状態
β+ decay
QEC
27Al
(Z=13,N=14)
(安定核)
27Si
(Z=14,N=13)
(不安定核)


1p-1h Excitations in
42Ti: β-decay
42Ca : CE Reaction
1g9/2
2p1/2
1f5/2
2p3/2
1f7/2
1d3/2
2s1/2
1d5/2
1p1/2
1p3/2
1s1/2
(10)—[50]—50
(2)—[40]
(6)—[38]
(4)—
(8)—[28]—28
(4)—[20]—20
(2)—[16]
(6)—[14]
(2)—[8]
(4)—[6]
(2)—[2]—2
β decay and CE reaction make
Isospin Analogous transitions
(mirror transitions in proton and neutron)
proton: f7/2→ neutron f7/2
proton: f7/2→ neutron f5/2
proton: f7/2← neutron f7/2
proton: f5/2← neutron f7/2

## Structure information form Transitions

Nuclear Transitions give us Structure information

*Transition strength: proportional to $|\langle f|\, \boldsymbol{Op}\, |i\rangle|^2$

$\mathbf{H}_i |i\rangle = E_i |i\rangle$, $\mathbf{H}_f |f\rangle = E_f |f\rangle$

*Studied by: Nuclear Reactions, Decays
Reaction: Excitation + Spectroscopy
Decay: Spectroscopy

*Mode of Excitation ←→ ***Op***

## What do we observe?

Observed Strength
= reaction mechanism
⊗ operator
⊗ structure
* integration of 3-quantities!

| | mechanism | operator (interaction) |
|---|---|---|
| γ decay: | $\boxed{\text{simple}}$ | EM |
| β decay: | $\boxed{\text{simple}}$ | weak |
| reaction: | complicated | strong |

*to study Structures, other 2 should be simple!

## Case 1 : $\gamma$-decay & $\beta$-decay

*both have very simple mechanism.
(people even don't think of "mechanism !")

*Operators are relatively simple!
Weak : Gamow-Teller, Fermi
EM : E1, E2,… M1, M2,…

matrix element & $t_{1/2}$

$$(1 / t_{1/2}) = \text{Coup.Const.} \times \text{PhaseSpaceFac.} \times |\langle \mathbf{f} \,|\, \boldsymbol{Op} \,|\, \mathbf{i} \rangle|^2$$

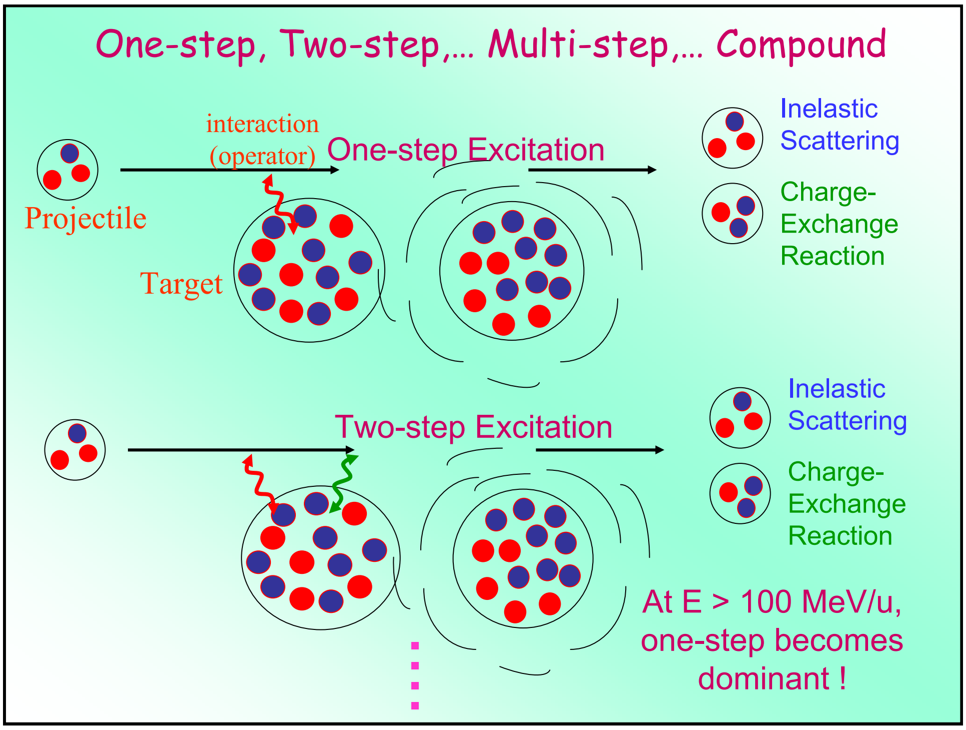
*if *Op* is specified, w.f.(=structures) are studied !
(*Op* specification is not always easy!)
* highly *Ex* region cannot be reached !

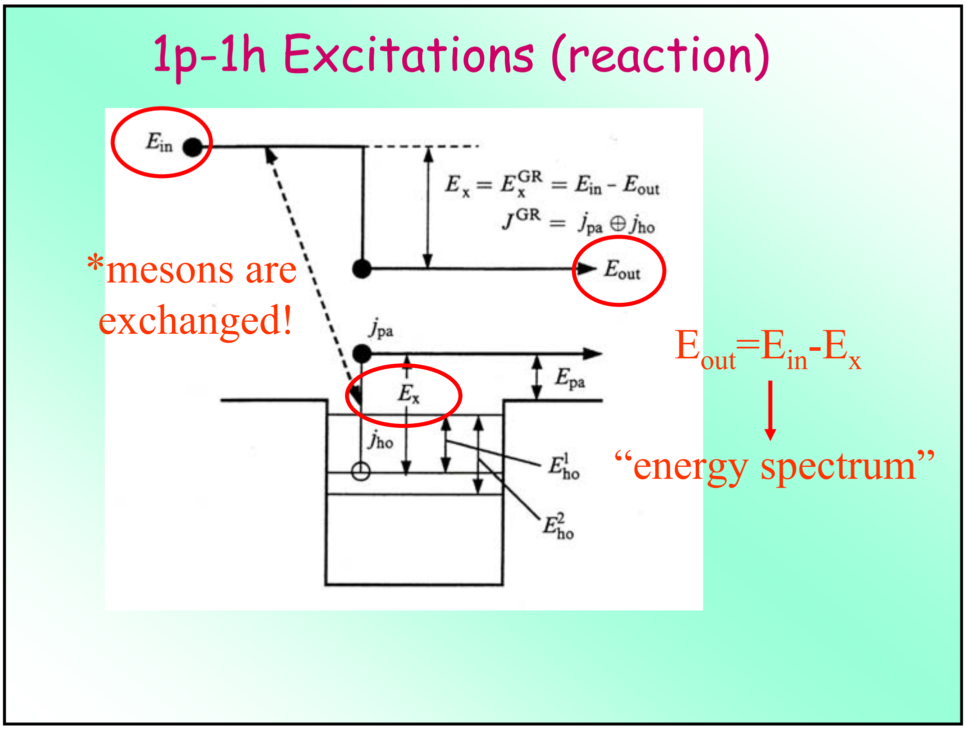
## Case 2 : Study by Nuclear Reactions

*we have to think of mechanisms seriously.
1) one-step, two-step,…
2) direct, exchange

*we have to think of operators (modes) seriously.
*separation of excitation modes is the main subject.
various reactions (e.g. charge exch., inelastic,..)
using different particles,
at different incident energies.
angular distribution analysis ( "*L*" analysis)

*complicated, but highly *Ex* region can be reached!
(reaction study is a dirty business, but effective!)


One-step, Two-step,… Multi-step,… Compound
interaction
(operator)
One-step Excitation
Projectile
Target
Inelastic Scattering
Charge-Exchange Reaction
Two-step Excitation
Inelastic Scattering
Charge-Exchange Reaction
At E > 100 MeV/u, one-step becomes dominant !


1p-1h Excitations (reaction)
$E_{in}$
$E_x = E_x^{GR} = E_{in} - E_{out}$
$J^{GR} = j_{pa} \oplus j_{ho}$
$E_{out}$
*mesons are exchanged!
$j_{pa}$
$E_{pa}$
$E_x$
$j_{ho}$
$E_{ho}^1$
$E_{ho}^2$
$E_{out}=E_{in}-E_x$
“energy spectrum”

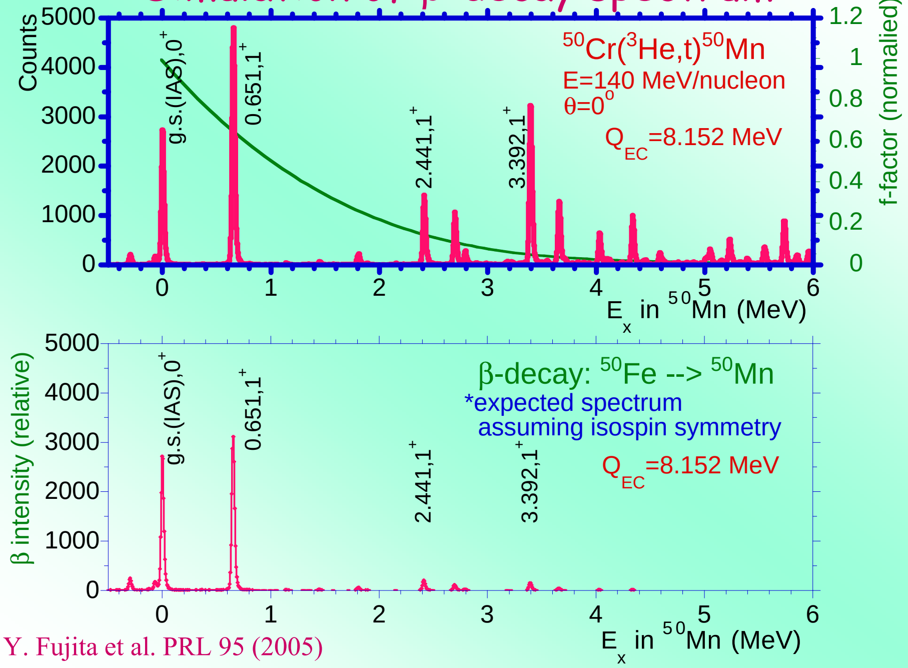
## β-decay transition rate and B(GT)

*β-decay GT tra. rate = $\dfrac{1}{t_{1/2}} = f\dfrac{\lambda^2}{K}\boxed{B(\mathrm{GT})}$

Where                                                    constant !

* $1/t_{1/2}$ : speed of decay rate
* $B$(GT) : reduced GT transition strength
  $\propto$ (matrix element)$^2$ $= |\langle f\|\ \sigma\tau\ \|i\rangle|^2$
* $f$ : phase-space factor
  at Q=0 ➔ $f = 0$

## Simulation of β-decay spectrum




Y. Fujita et al. PRL 95 (2005)

# Reduced transition strength *B* (Op )

換算遷移確率

A value proportional to (matrix element)$^2$

$|\langle \mathbf{f} \mid \boldsymbol{Op} \mid \mathbf{i} \rangle|^2$

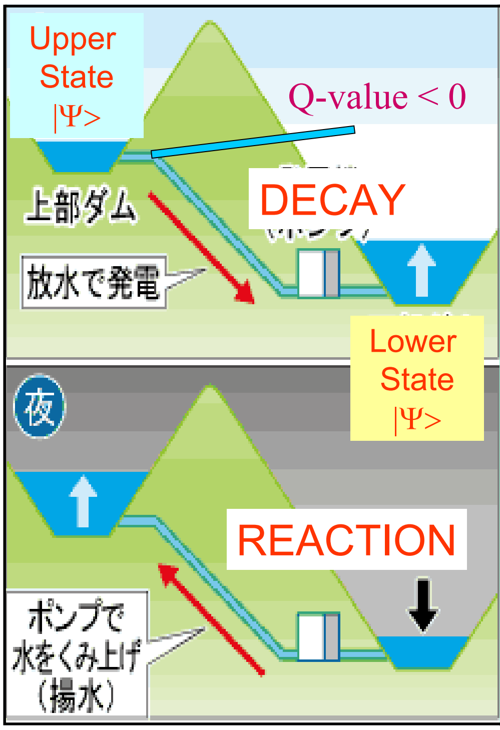
is called "reduced transition strength"

*ex.* $B$(GT), $B$(F), $B$($M1$), $B$($E2$),….

*representing only the structure part
for a specific operator!

*reaction mechanism part is removed !

*coupling constant part is removed !




## A Pump-up Dam

$B$(**)s are proportional to (matrix element)$^2$

$|\langle \mathbf{f} \mid \boldsymbol{Op} \mid \mathbf{i} \rangle|^2$

*ex.* $B$(GT), $B$(F), $B$($M1$), $B$($E2$),….

Corresponding to the SIZE of WATER PIPE !

If the pipe is horizontal (Q=0) or upward (Q<0), there is no water flow.

But a pipe does exist !

## Reduced transition strength *B* (*Op* )

A value proportional to (matrix element)$^2$

$|\langle \mathbf{f} \,|\, \boldsymbol{Op} \,|\, \mathbf{i} \rangle|^2$

is called “reduced transition strength”

*ex.* $B$(GT), $B$(F), $B$($M1$), $B$($E2$),….

*representing only the structure part
for a specific operator!

*reaction mechanism parts
(kinetic-energy, Q-value, mass A parts)
are all removed!

## ***Operators and Nuclear Excitations

- Nuclear surface vibrations -

## ***Operators = Hammers ??

## Various Operators / Various Hammers!

wooden hammers

metal hammers

hammers
=operators

The sound from the bell is different depending on hammers!

The mode of nuclear excitation is determined by an operator!

## Different Reaction Mechanism
## / How and Where you hit the bell!

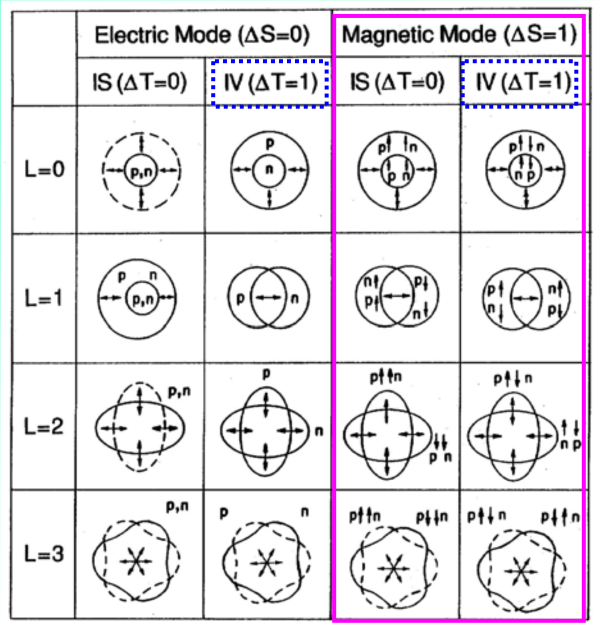
how and where you hit
=reaction mechanism

The sound from the bell is different how and where you hit!

The strength of nuclear excitation is dependent on them!

## ***Operators causing Excitations (De-excitations)

## Vibration Modes in Nuclei (Schematic)




## Vibration Modes in Nuclei (Operators)

Microscopic classification of giant resonances

| | $\Delta S = 0$ $\Delta T = 0$ | $\Delta S = 0$ $\Delta T = 1$ | $\Delta S = 1$ $\Delta T = 0$ | $\Delta S = 1$ $\Delta T = 1$ |
|---|---|---|---|---|
| $\Delta L = 0$ | | $\sum \tau_i$ IAS | | $\sum \vec{\sigma}_i \tau_i$ GTR |
| 2[nd] order | $\sum r_i^2$ ISGMR | $\sum r_i^2 \tau_i$ IVGMR | $\sum r_i^2 \vec{\sigma}_i$ ISSMR | $\sum r_i^2 \vec{\sigma}_i \tau_i$ IVSMR |
| $\Delta L = 1$ | | $\sum r_i Y_m^1 \tau_i$ IVGDR | $\sum r_i Y_m^1 \vec{\sigma}_i$ ISSDR | $\sum r_i Y_m^1 \vec{\sigma}_i \tau_i$ IVSDR |
| 2[nd] order | $\sum r_i^3 Y_m^1$ ISGDR | | | |
| $\Delta L = 2$ | $\sum r_i^2 Y_m^2$ ISGQR | $\sum r_i^2 Y_m^2 \tau_i$ IVGQR | $\sum r_i^2 Y_m^2 \vec{\sigma}_i$ ISSQR | $\sum r_i^2 Y_m^2 \vec{\sigma}_i \tau_i$ IVSQR |
| $\Delta L = 3$ | $\sum r_i^3 Y_m^3$ ISGOR | $\sum r_i^3 Y_m^3 \tau_i$ IVGOR | $\sum r_i^3 Y_m^3 \vec{\sigma}_i$ ISSOR | $\sum r_i^3 Y_m^3 \vec{\sigma}_i \tau_i$ IVSOR |

$\Delta S$=1:
spin excitation

$\Delta T$=1:
IV excitation
(isospin related!)

Operators
$\sigma$: spin
$\tau$: isospin
$r$: radial
$Y^l_m$: Spherical Harmonic

## ***Giant Resonances***

Vibrational Modes
-collective excitations-

- absorbs a large fraction of
  the total sum rule strength -

## Gamow-Teller Resonances for A>90 Nuclei

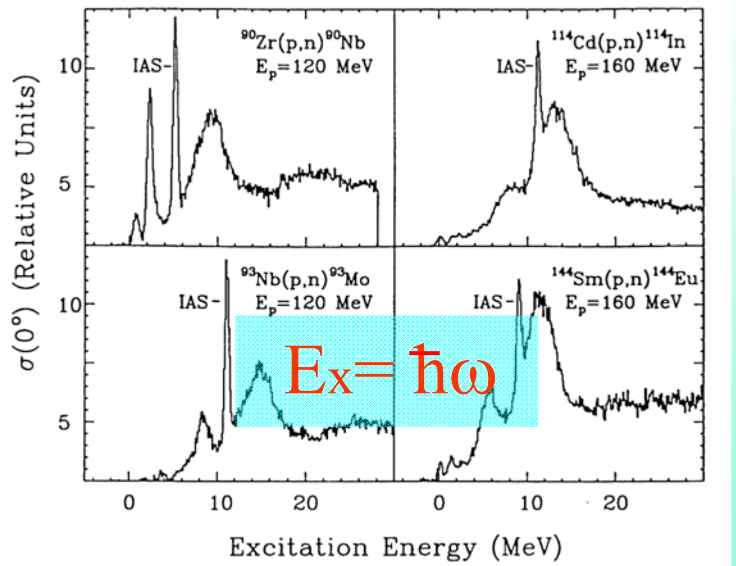
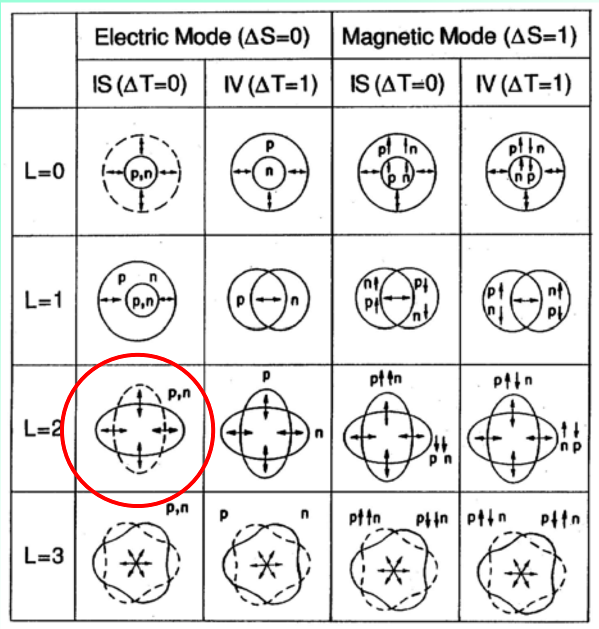



*Figure 10* Zero-degree $(p,n)$ spectra for medium $A$-mass nuclei at the indicated incident energies.

**IS Electric Giant Resonances

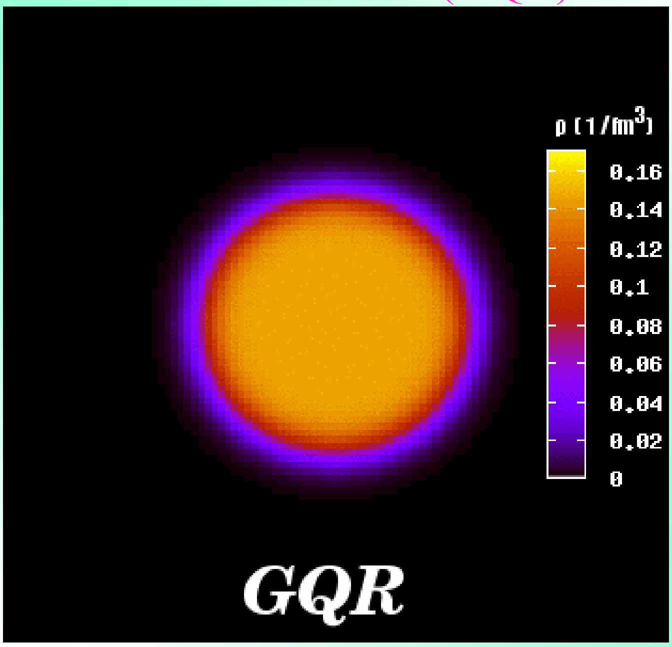
## Vibration Modes in Nuclei (Schematic)




IS-Giant Quadrupole Resonance (GQR)

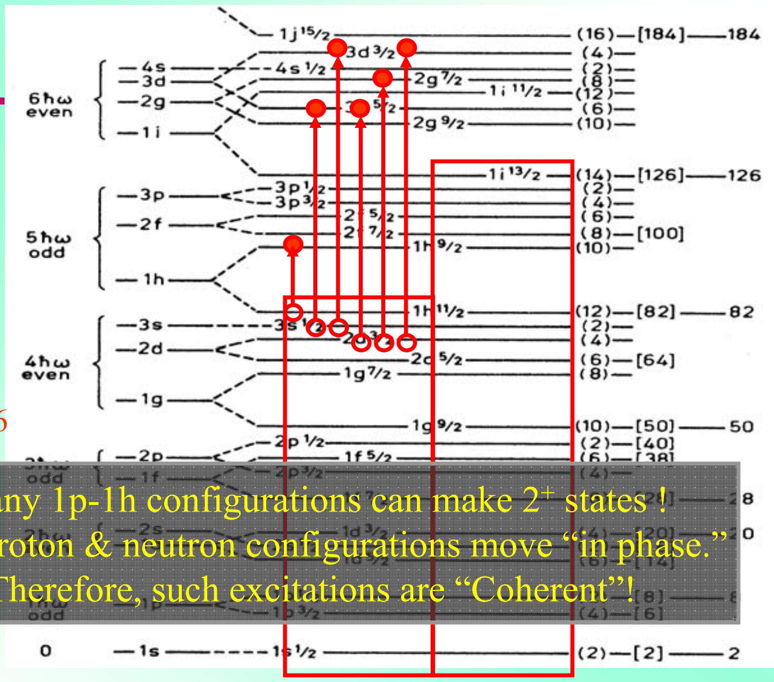
## Giant Resonance (GQR)

by M. Itoh




## 1p-1h Configu-rations making GQR in $^{208}$Pb

Z=82,N=126




Many 1p-1h configurations can make 2$^+$ states !

Both proton & neutron configurations move “in phase.”

Therefore, such excitations are “Coherent”!

# Compression Modes of Nuclei

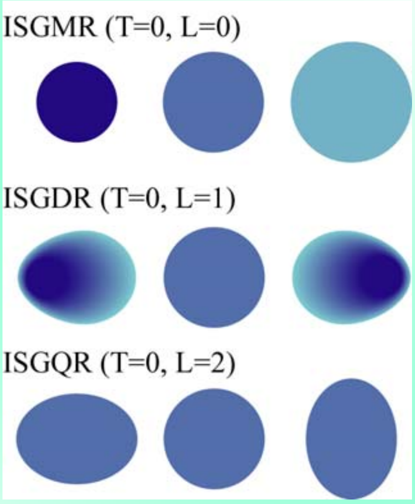
**Macroscopic Picture/Hydrodynamic models/Giant Resonances**

**Coherent vibrations of nucleonic fluids in a nucleus.**

**Compression modes : ISGMR, ISGDR**

$$E_{ISGMR} = \hbar\sqrt{\frac{K_A}{m\langle r^2\rangle}}$$

$$E_{ISGDR} = \hbar\sqrt{\frac{7}{3}\frac{K_A + \frac{27}{25}\varepsilon_F}{m\langle r^2\rangle}}$$

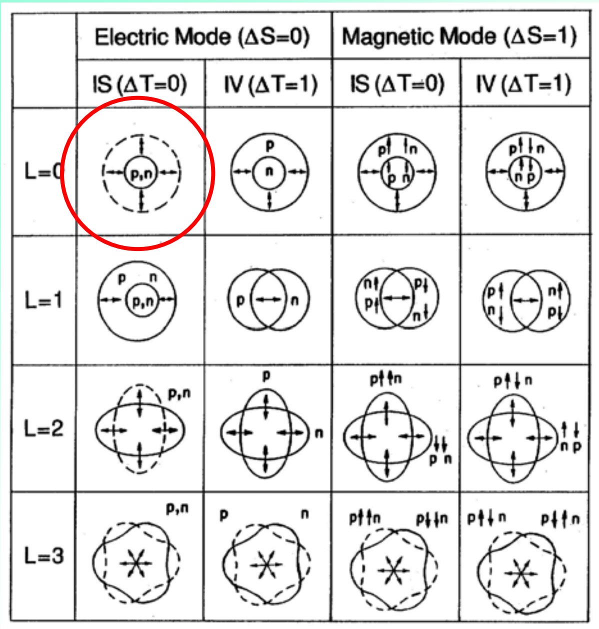



**The nucleus incompressibility:**

$$K_A = \left[r^2 (d^2(E/A)/dr^2)\right]_{r=R_0}$$

by M. Harakeh

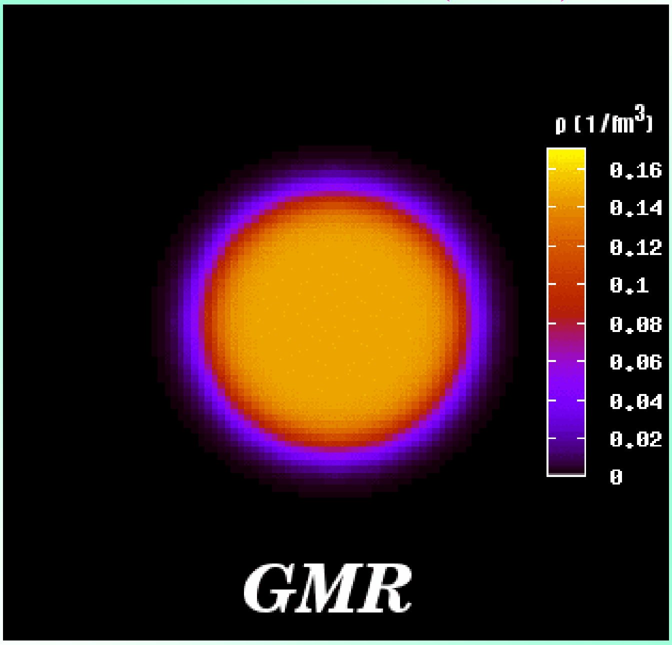
# Vibrational Modes in Nuclei (Schematic)



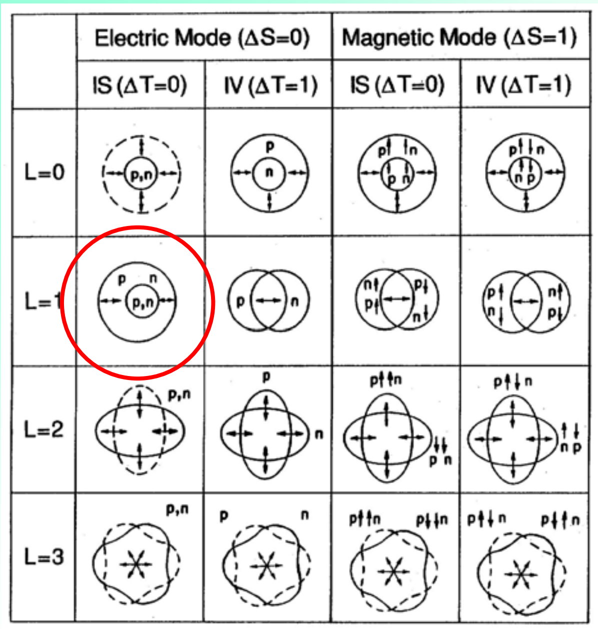

IS-Giant Monopole Resonance (GMR)

## Giant Resonance (GMR)

by M. Itoh

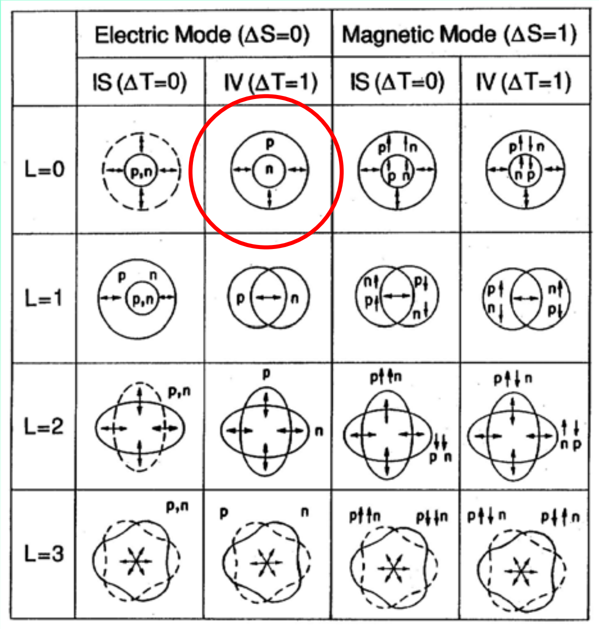



## Vibration Modes in Nuclei (Schematic)




IS-Giant Dipole Resonance (GDR)

## Vibration Modes in Nuclei (Schematic)




IV-Giant Monopole Resonance (IVGMR)

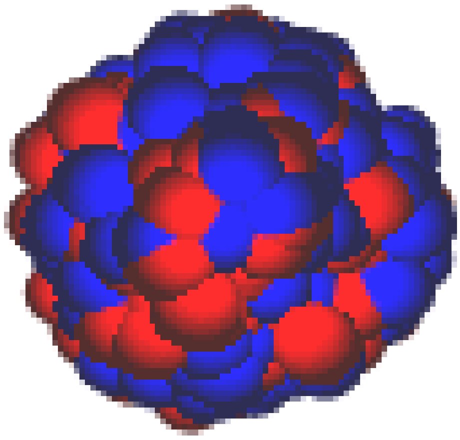
## Vibration Modes in Nuclei (Operators)

Microscopic classification of giant resonances

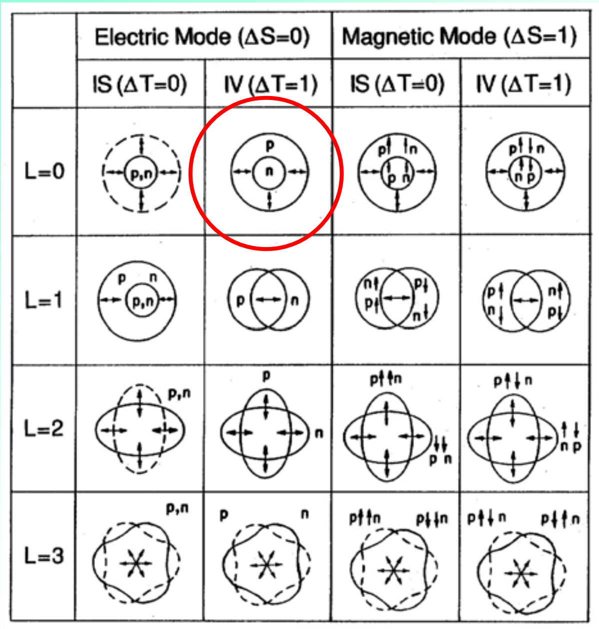
| | $\Delta S = 0$, $\Delta T = 0$ | $\Delta S = 0$, $\Delta T = 1$ | $\Delta S = 1$, $\Delta T = 0$ | $\Delta S = 1$, $\Delta T = 1$ |
|---|---|---|---|---|
| L = 0 | | $\sum \tau_i$ IAS | | $\sum \vec{\sigma}_i \tau_i$ GTR |
| 2nd order | $\sum r_i^2$ ISGMR | $\sum r_i^2 \tau_i$ IVGMR | $\sum r_i^2 \vec{\sigma}_i$ ISSMR | $\sum r_i^2 \vec{\sigma}_i \tau_i$ IVSMR |
| L = 1 | | $\sum r_i Y_m^1 \tau_i$ IVGDR | $\sum r_i Y_m^1 \vec{\sigma}_i$ ISSDR | $\sum r_i Y_m^1 \vec{\sigma}_i \tau_i$ IVSDR |
| 2nd order | $\sum r_i^3 Y_m^1$ ISGDR | | | |
| L = 2 | $\sum r_i^2 Y_m^2$ ISGQR | $\sum r_i^2 Y_m^2 \tau_i$ IVGQR | $\sum r_i^2 Y_m^2 \vec{\sigma}_i$ ISSQR | $\sum r_i^2 Y_m^2 \vec{\sigma}_i \tau_i$ IVSQR |
| L = 3 | $\sum r_i^3 Y_m^3$ ISGOR | $\sum r_i^3 Y_m^3 \tau_i$ IVGOR | $\sum r_i^3 Y_m^3 \vec{\sigma}_i$ ISSOR | $\sum r_i^3 Y_m^3 \vec{\sigma}_i \tau_i$ IVSOR |

## IV Giant Monopole Resonance (IVGMR)

by P. Adrich



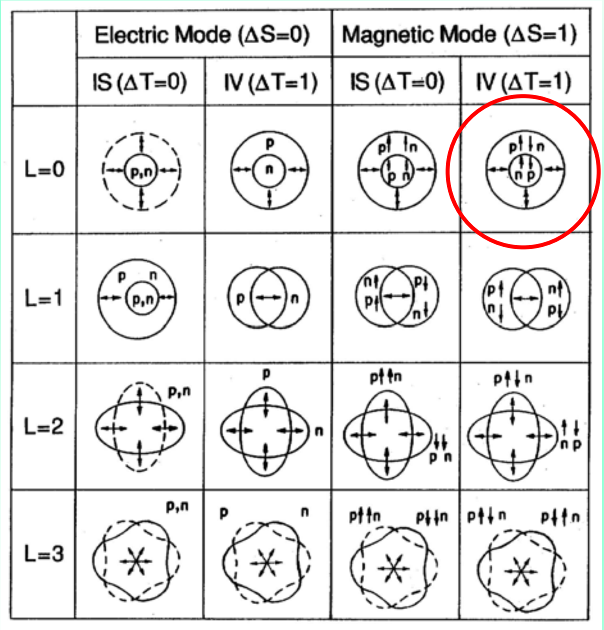
## Vibration Modes in Nuclei (Schematic)




Fermi mode

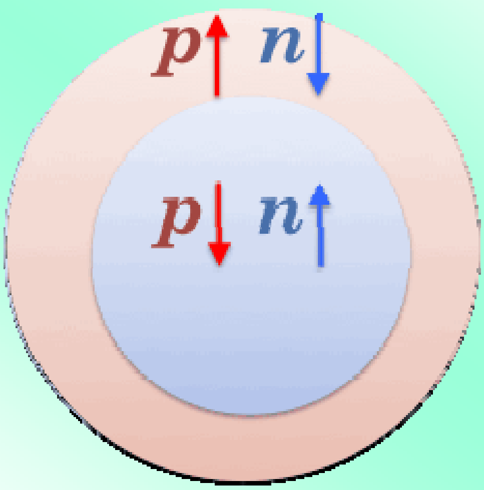
(τ)

## Vibration Modes in Nuclei (Schematic)



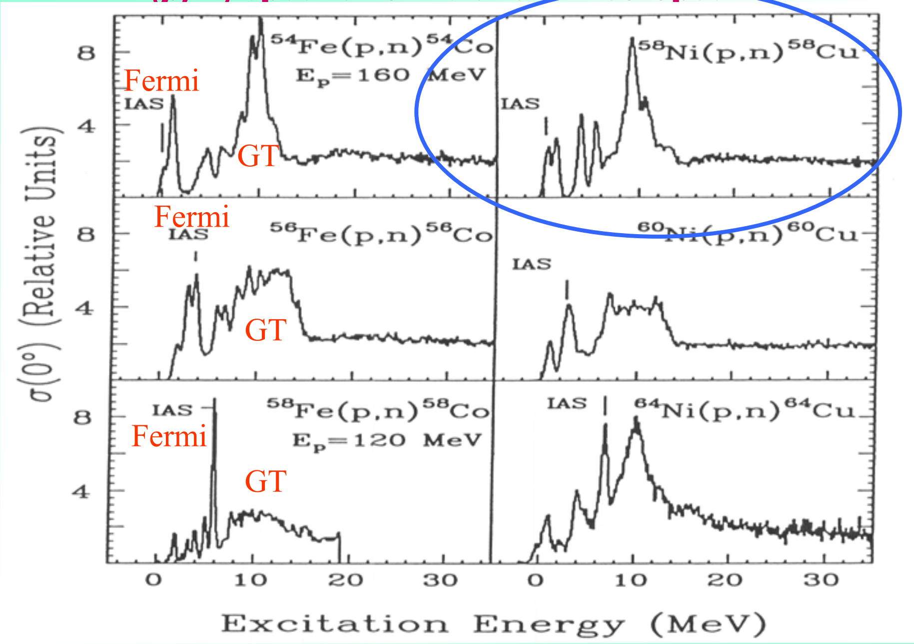

Gamow-Teller mode ($\sigma\tau$)

## IV Spin Monopole Mode



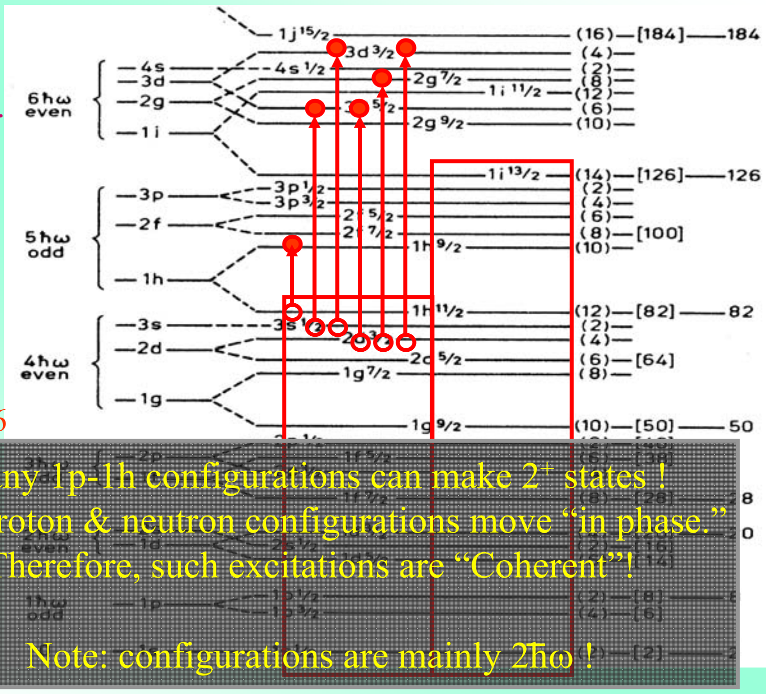

WEB, Dpt. Phys. Tokyo U

## (p, n) spectra for Fe and Ni Isotopes



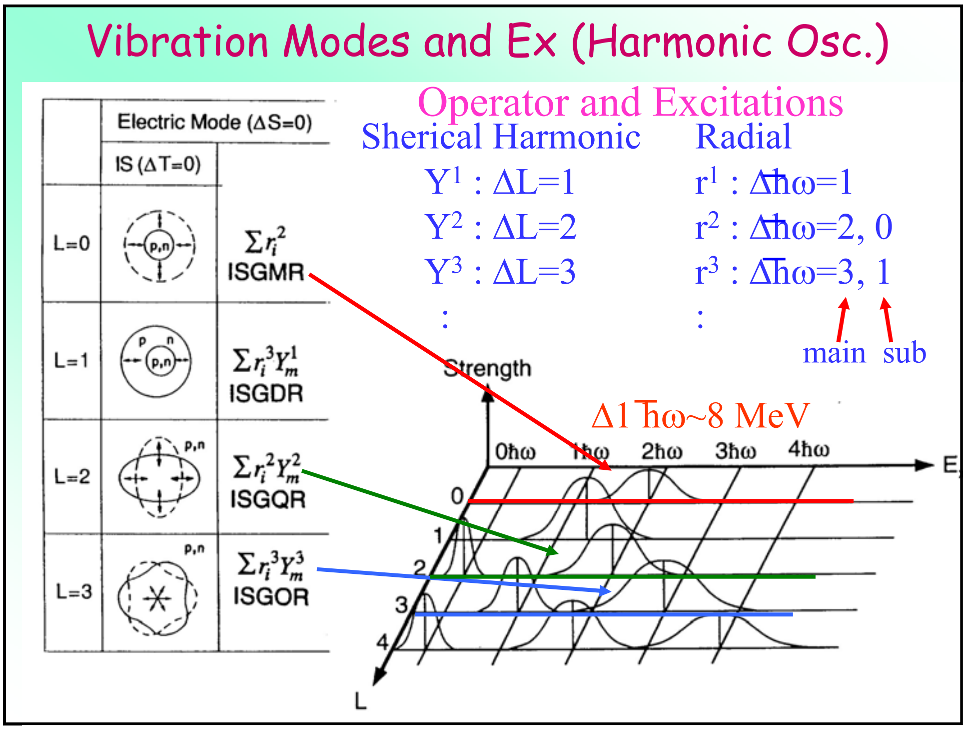

## 1p-1h Configurations making GQR in $^{208}$Pb

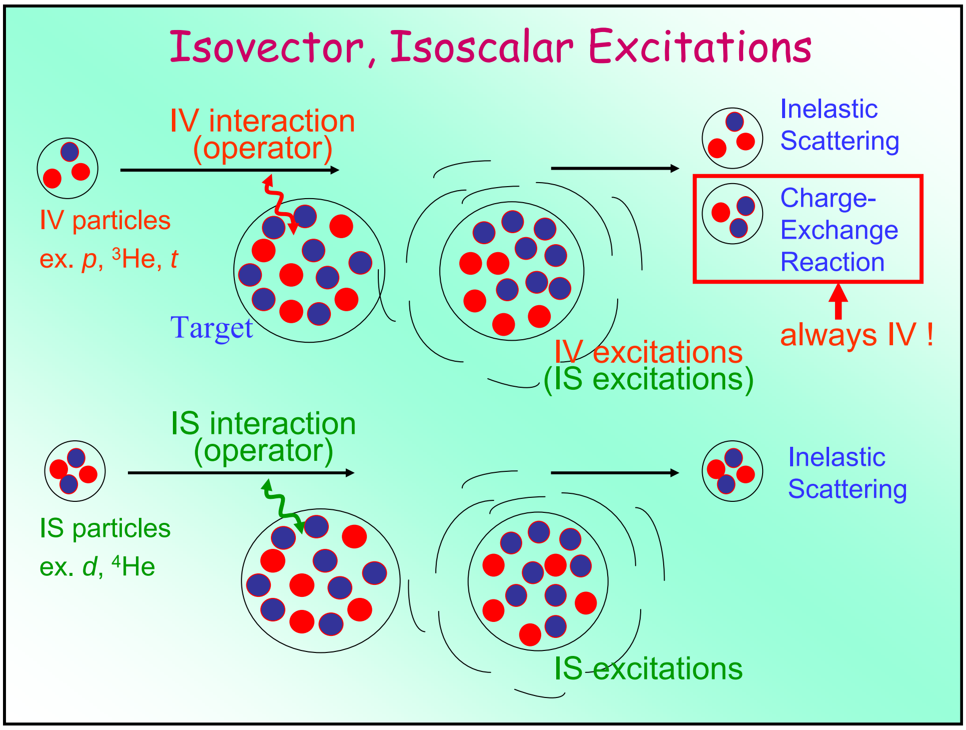
Z=82,N=126

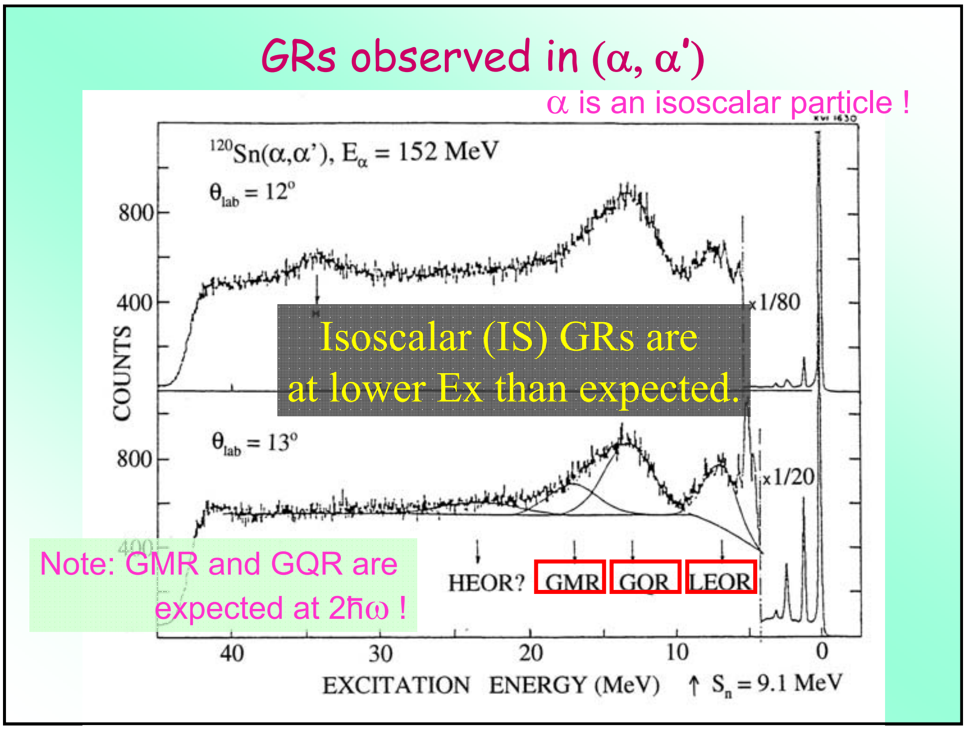


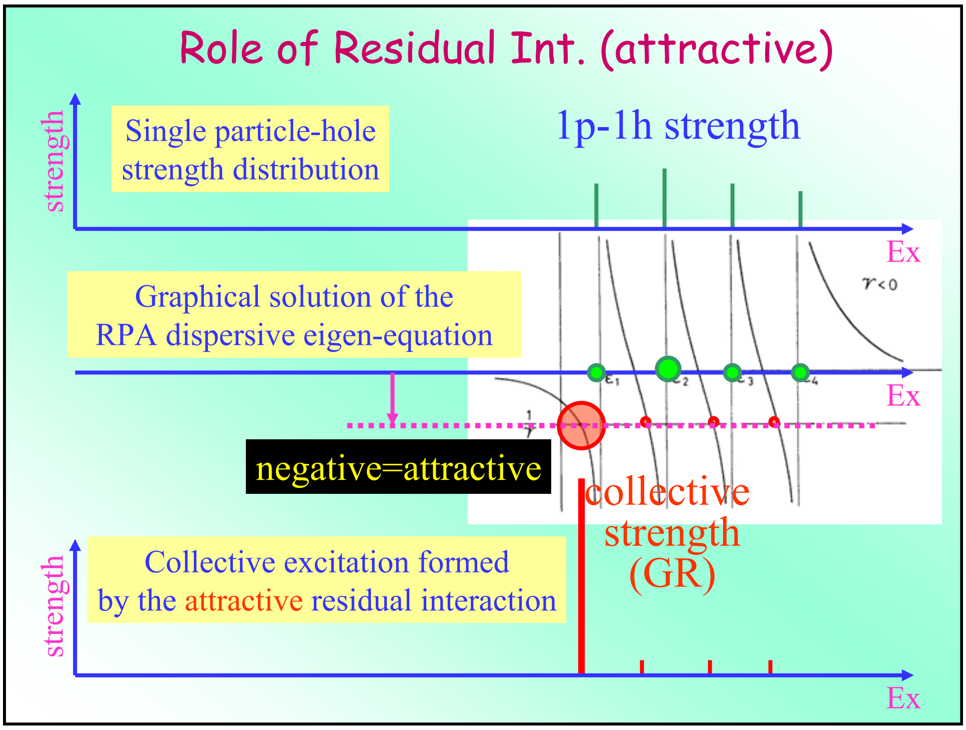

Many 1p-1h configurations can make 2$^+$ states !
Both proton & neutron configurations move “in phase.”
Therefore, such excitations are “Coherent”!

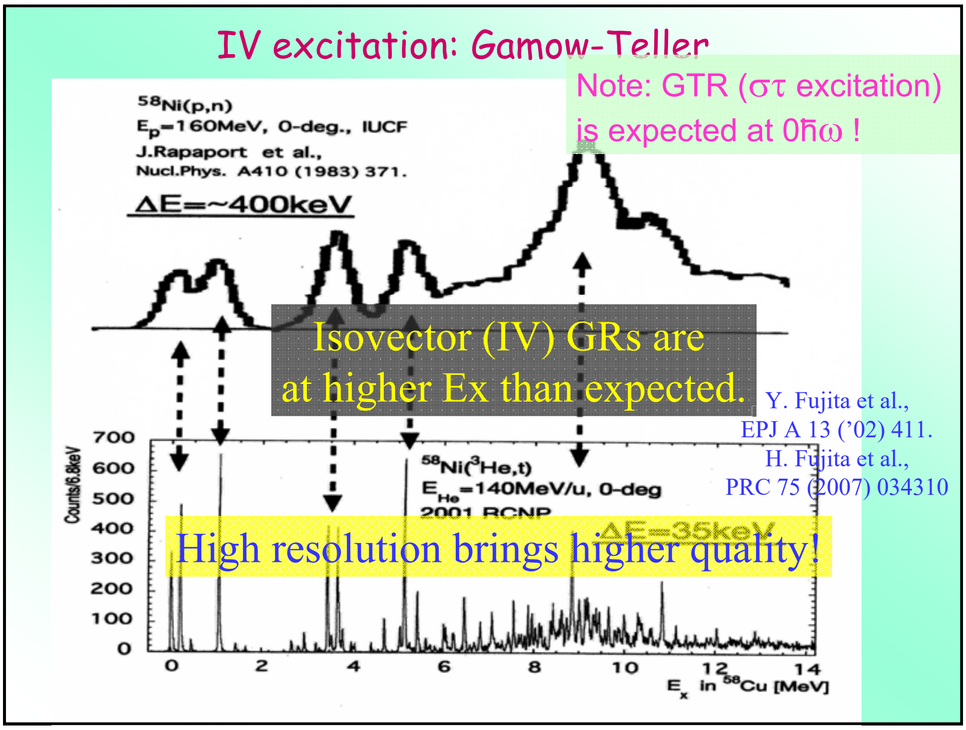
Note: configurations are mainly 2ħω !

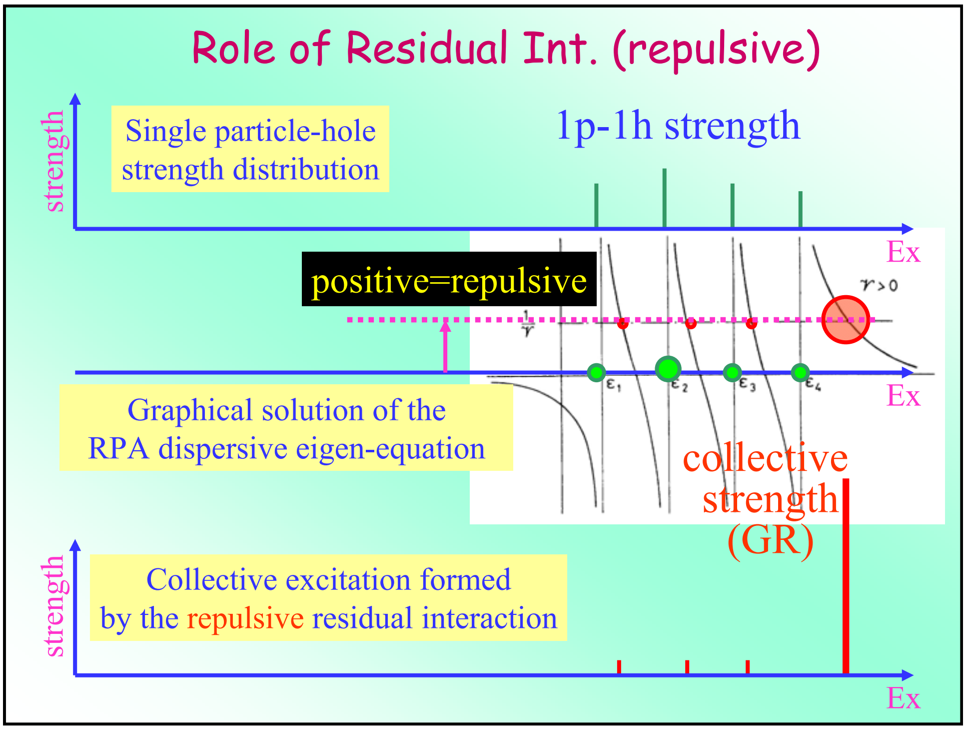

Vibration Modes and Ex (Harmonic Osc.)
Electric Mode (ΔS=0)
IS (ΔT=0)
L=0
L=1
L=2
L=3
$\Sigma r_i^2$
ISGMR
$\Sigma r_i^3 Y_m^1$
ISGDR
$\Sigma r_i^2 Y_m^2$
ISGQR
$\Sigma r_i^3 Y_m^3$
ISGOR
Operator and Excitations
Sherical Harmonic
Radial
$Y^1$ : ΔL=1
$Y^2$ : ΔL=2
$Y^3$ : ΔL=3
$r^1$ : Δħω=1
$r^2$ : Δħω=2, 0
$r^3$ : Δħω=3, 1
main
sub
Strength
Δ1 ħω~8 MeV
0ħω
1ħω
2ħω
3ħω
4ħω
E
L


Isovector, Isoscalar Excitations
IV interaction
(operator)
IV particles
ex. p, 3He, t
Target
IV excitations
(IS excitations)
Inelastic
Scattering
Charge-
Exchange
Reaction
always IV !
IS interaction
(operator)
IS particles
ex. d, 4He
IS excitations
Inelastic
Scattering


GRs observed in (α, α')
α is an isoscalar particle !
$^{120}$Sn(α,α'), $E_α$ = 152 MeV
$θ_{lab}$ = 12°
800
400
COUNTS
x1/80
Isoscalar (IS) GRs are
at lower Ex than expected.
$θ_{lab}$ = 13°
800
x1/20
Note: GMR and GQR are
expected at 2ħω !
HEOR?
GMR
GQR
LEOR
40
30
20
10
0
EXCITATION ENERGY (MeV)
↑ $S_n$ = 9.1 MeV


Role of Residual Int. (attractive)
strength
Single particle-hole
strength distribution
1p-1h strength
Ex
Graphical solution of the
RPA dispersive eigen-equation
Ex
negative=attractive
collective
strength
(GR)
strength
Collective excitation formed
by the attractive residual interaction
Ex


IV excitation: Gamow-Teller
Note: GTR (στ excitation) is expected at 0ħω !
58Ni(p,n)
Ep=160MeV, 0-deg., IUCF
J.Rapaport et al.,
Nucl.Phys. A410 (1983) 371.
ΔE=~400keV
Isovector (IV) GRs are at higher Ex than expected.
Y. Fujita et al.,
EPJ A 13 ('02) 411.
H. Fujita et al.,
PRC 75 (2007) 034310
58Ni(3He,t)
EHe=140MeV/u, 0-deg
2001 RCNP
ΔE=35keV
High resolution brings higher quality!
Counts/6.8keV
Ex in 58Cu [MeV]


Role of Residual Int. (repulsive)
strength
Single particle-hole strength distribution
1p-1h strength
Ex
positive=repulsive
Ex
Graphical solution of the RPA dispersive eigen-equation
collective strength (GR)
strength
Collective excitation formed by the repulsive residual interaction
Ex

## ***Decay and Widths of States

## Relationship: Decay and Width

Heisenberg's Uncertainty Principle

$$\Delta x \cdot \Delta p \approx \hbar$$

$$\Delta t \cdot \Delta E \approx \hbar$$

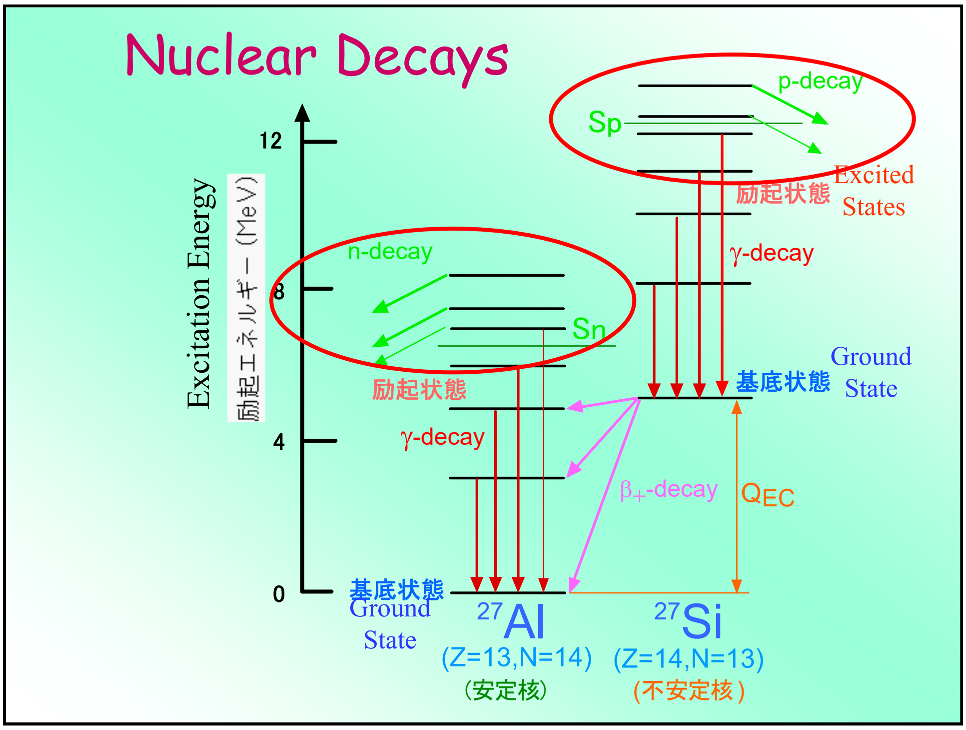
Width $\Gamma = \Delta E$

*if: Decay is Fast,

then: Width of a State is Wider !

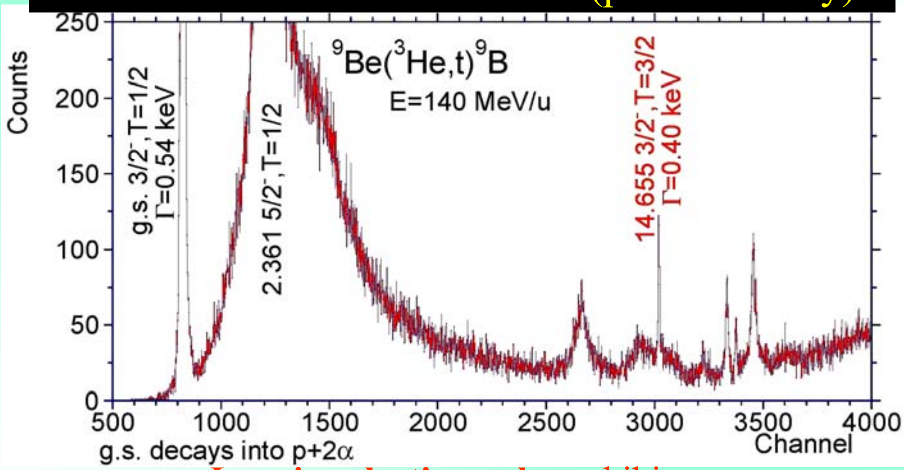
*if $\Delta t = 10^{-20}$ sec → $\Delta E$ ~100 keV (particle decay

$\Delta t = 10^{-15}$ sec → $\Delta E$ ~ 1 eV (fast $\gamma$ decay)

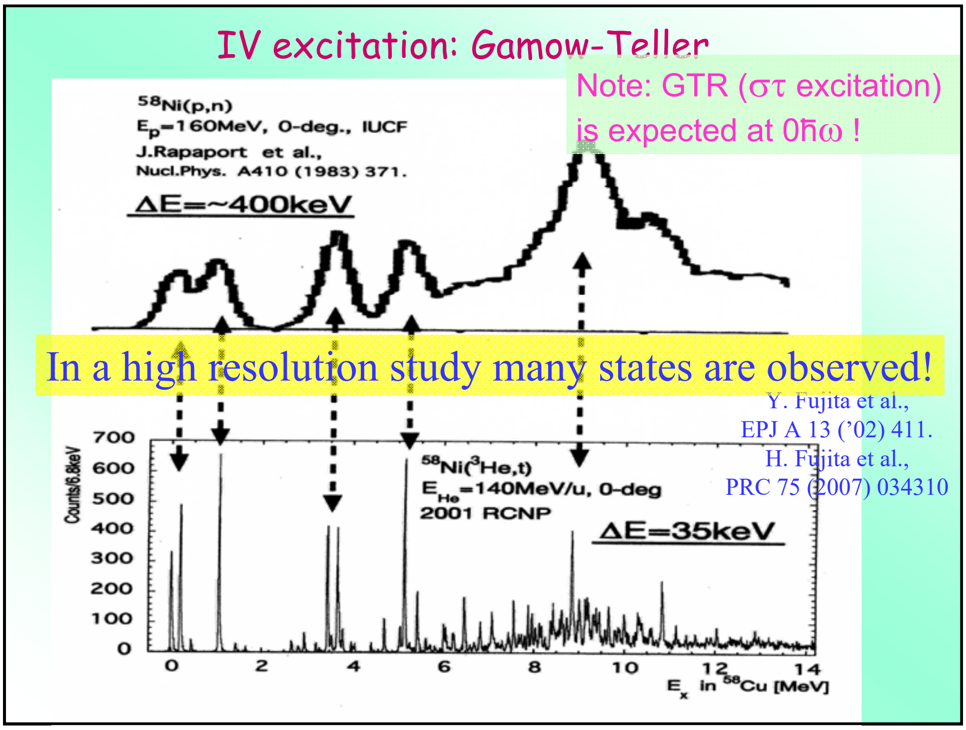


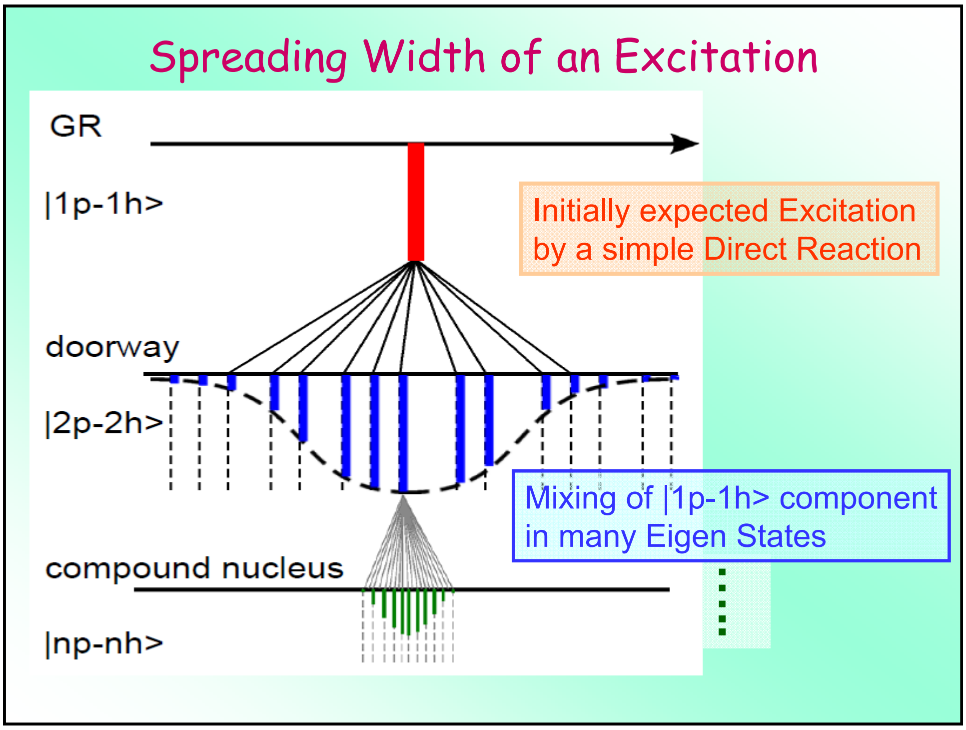

Width $\Gamma = \Delta E$

*if $\Delta t = 10^{-18}$ sec → $\Delta E$ ~1 keV (particle decay)

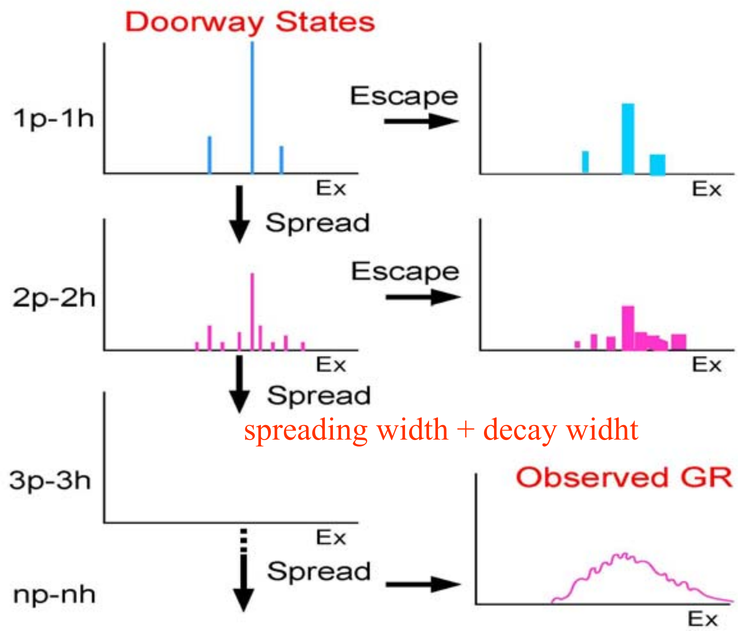



**Isospin selection rule** prohibits
**proton decay** of **T=3/2** state!

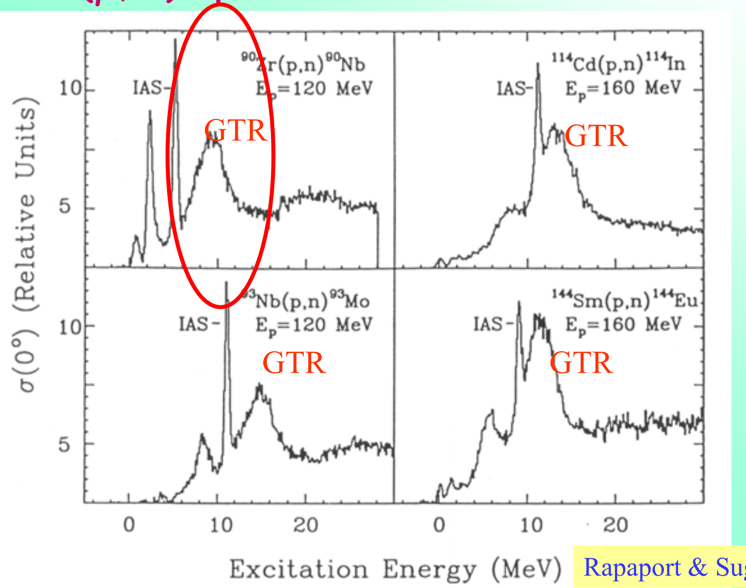

IV excitation: Gamow-Teller
Note: GTR (στ excitation) is expected at 0ħω !
58Ni(p,n)
Ep=160MeV, 0-deg., IUCF
J.Rapaport et al.,
Nucl.Phys. A410 (1983) 371.
ΔE=~400keV
In a high resolution study many states are observed!
Y. Fujita et al.,
EPJ A 13 ('02) 411.
H. Fujita et al.,
PRC 75 (2007) 034310
58Ni(3He,t)
EHe=140MeV/u, 0-deg
2001 RCNP
ΔE=35keV
Counts/6.8keV
700
600
500
400
300
200
100
0
0
2
4
6
8
10
12
14
Ex in 58Cu [MeV]


Spreading Width of an Excitation
GR
|1p-1h>
Initially expected Excitation by a simple Direct Reaction
doorway
|2p-2h>
Mixing of |1p-1h> component in many Eigen States
compound nucleus
|np-nh>

## Giant Resonances: Decay & Width



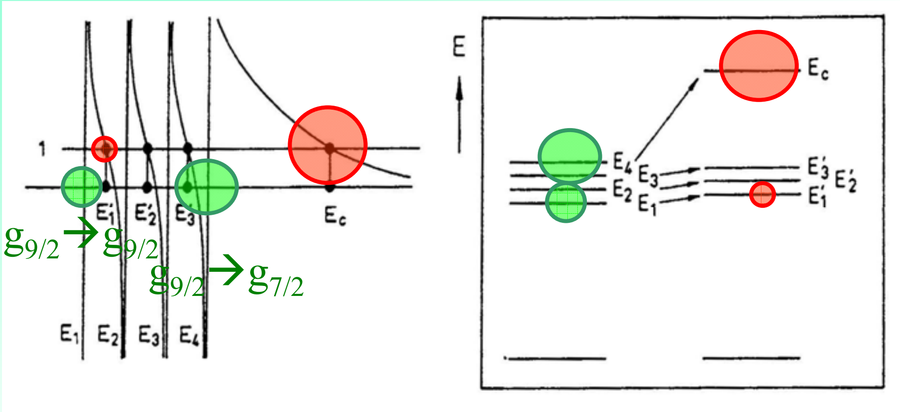

## (p, n) spectra for A>90 Nuclei




Rapaport & Sugerbaker

*Figure 10* Zero-degree (*p*,*n*) spectra for medium *A*-mass nuclei at the indicated incident energies.

## Formation of GT-GR in $^{90}$Nb




*in $^{90}Zr \rightarrow ^{90}Nb$ transitions
*$\sigma\tau$ int. : repulsive nature
*both configurations : p-h nature (repulsive)

## Summary: how do we see Nuclei?

*Quantum Finite Many-body System of Fermions
=> quantum numbers are important
L, S, J, T
=> selection rules of Q-numbers are important

*Active forces in nuclei:
3 out of 4 fundamental forces
strength: **strong** >> **electro-magnetic** >> **weak**
*they struggle to make their territory larger !
*studies using nuclear reactions and decays are fruitful.

## Summary: Operator, Width

Both Decay and Reaction studies are important.

What we observe =
reaction mechanism
$\otimes$ operator $\otimes$ structure

Operators: IS, IV, Electric, Magnetic

Life time ←→ decay width ←→ interaction strength

Fragmentation Process of Collective Excitations