

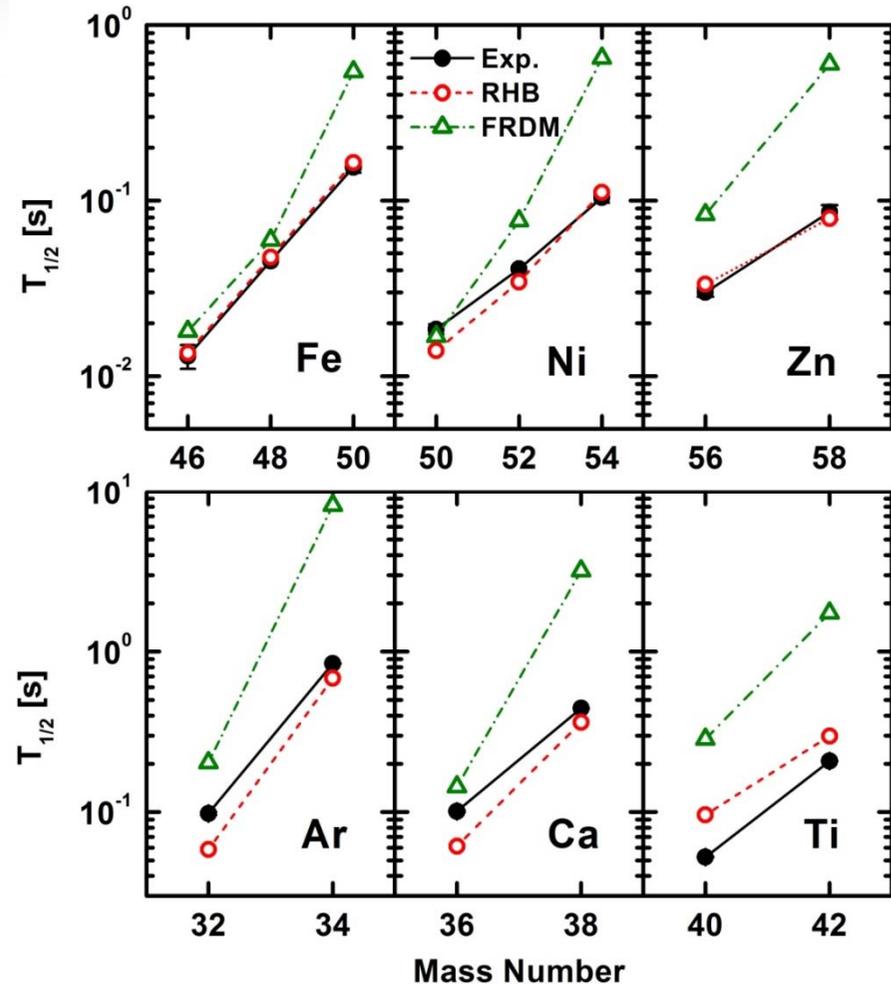


- For vibrations with **small amplitude**, it can be expanded in the vicinity of the ground state and a connection to the static energy density functional can be found.
- In the adiabatic approximation, i.e. by neglecting **the memory effects** and **the energy dependence** in Fourier space, one ends up with the **RPA** with a residual interaction derived as the second derivative of the static energy functional with the density.
- RPA provides a successful description of **the mean energies** of giant resonances in nuclei, but not able to reproduce the decay width of these excitations.
- For **the decay width** and **the fragmentation** of single-particle states, one has to go beyond mean field and to consider the energy dependence of the self-energy. This can be done within the particle-vibrational coupling (PVC) model.



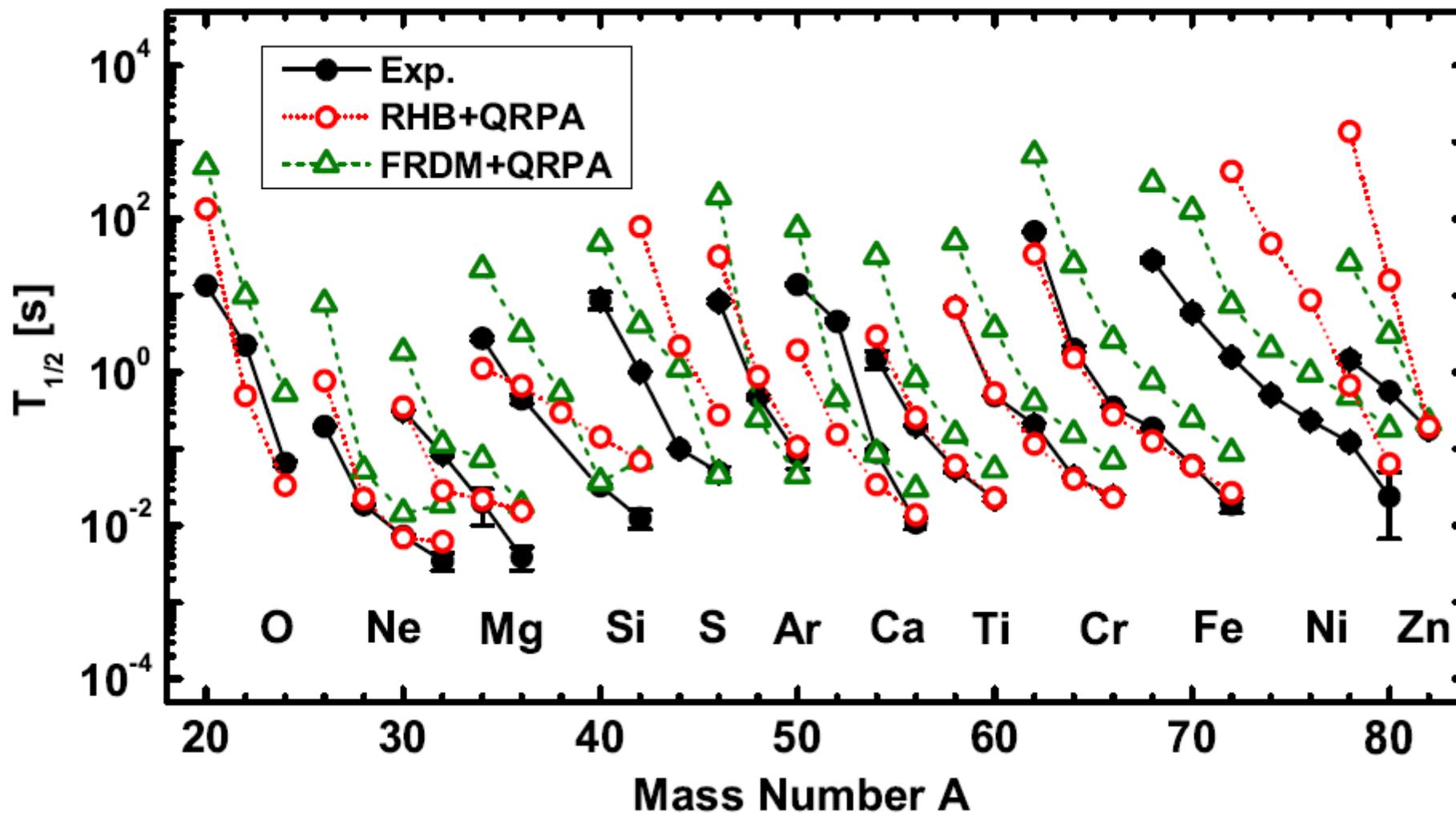
► RHB+QRPA: well reproduces the experimental half-lives for neutron-deficient Ar, Ca, Ti, Fe, Ni, and Zn isotopes by a universal  $T=0$  pairing strength.

► FRDM+QRPA: systematically overestimates the nuclear half-lives ← the pp residual interactions in the  $T=0$  channel are not considered.



Nuclear  $\beta^+$ /EC-decay half-lives calculated in RHB+QRPA model with the PC-PK1 parameter set.

**Niu et al., PRC 87, 051303(R) (2013)**



Z. Y. Wang, Z. M. Niu, Y. F. Niu, J. Y. Guo [arXiv:1503.01222](https://arxiv.org/abs/1503.01222)

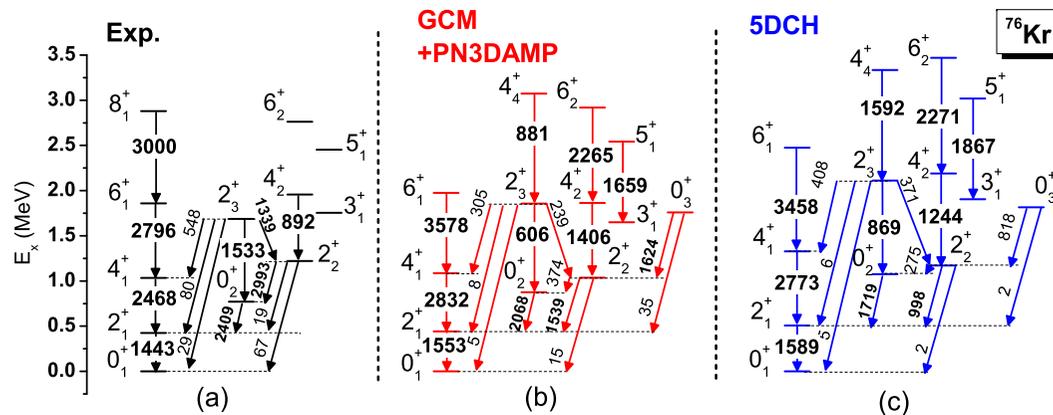
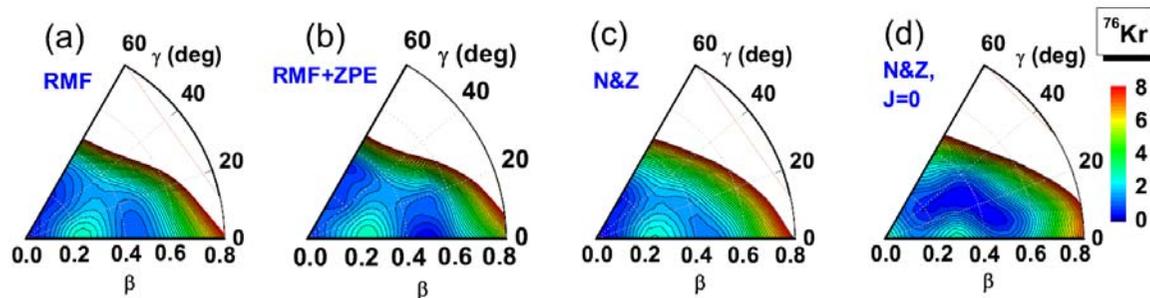
Nuclear  $\beta$ -decay half-lives in the relativistic point-coupling model



- Many-body energy takes the form of a functional of all transition density matrices between the various Slater determinants of different deformation and orientation. Mixing of these different configurations allows the **restoration of symmetries** and to take into account **fluctuations** around the mean-field equilibrium solution.
- In full space and with deformation degrees of freedom, such calculations require considerable **numerical efforts**, at the limit of present computer capabilities.
- A considerable simplification is by using the constraint calculations for the derivation of a **collective Hamiltonian** in these degrees of freedom.



7D GCM: two deformation parameters + projection 3DAM and 2PN



◆ The low-energy spectrum in  $^{76}\text{Kr}$  are well reproduced after including triaxiality in the full microscopic GCM+ PN3DAMP calculation based on the CDFT using PC-PK1.

◆ This study answers the important question of dynamic correlations and triaxiality in shape-coexistence nucleus  $^{76}\text{Kr}$  and provides the first benchmark for the EDF based collective Hamiltonian method.

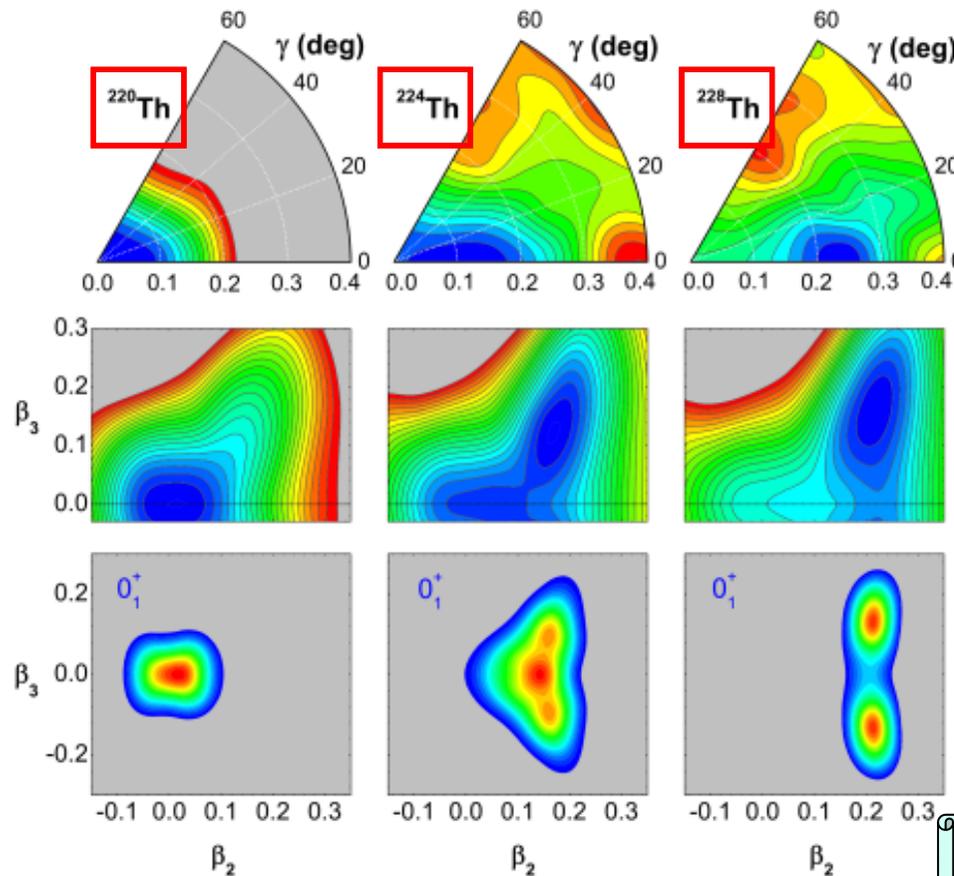
Yao, Hagino, Li, Meng, Ring, Phys. Rev. C 89, 054306 (2014)

Benchmark for the collective Hamiltonian in five dimensions



# Simultaneous shape phase transition

5DCH Calculations based on CDFT **PC-PK1** indicate a simultaneous quantum shape phase transition **from spherical to prolate shapes, and from reflection symmetric to octupole shapes.**



triaxial quadrupole energy surfaces

axially-symmetric quadrupole-octupole energy surfaces

probability density distributions for the ground states

Li, Song, Yao, Vretenar, and Meng, Phys. Lett. B  
726 (2013) 866



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# Application of CDFT in Nuclear astrophysics

# THE EVOLUTION OF ELEMENTS

Follow the Footprint of Nucleosynthesis

Courtesy of Zhu Lee

### Hydrogen Burning

pp-chain      CNO Cycle

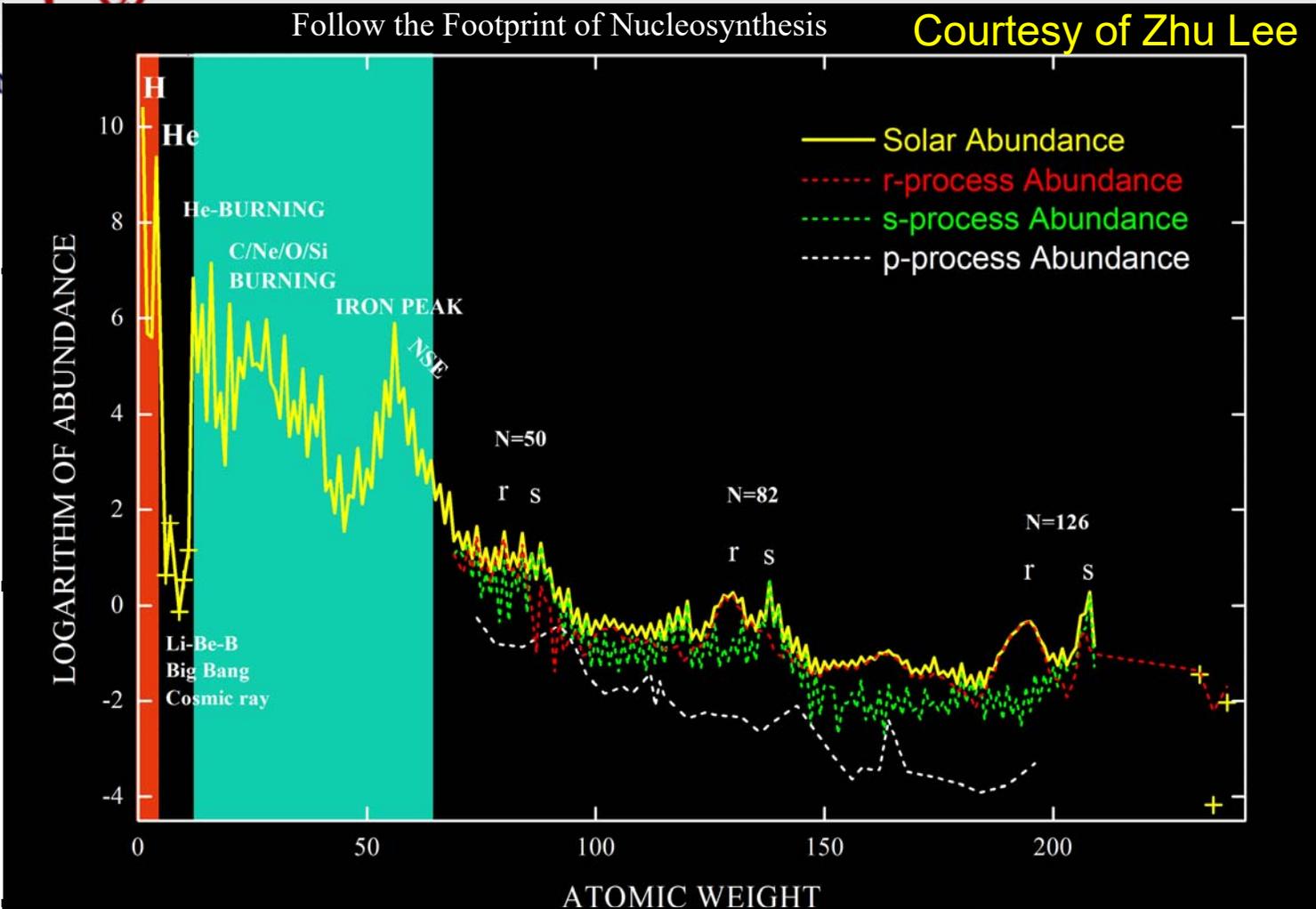
### Helium Burning

Triple-alpha      Hoyle State

### C/Ne/O/Si Burning

Fusion      Massive Star

### Neutron Capture



### 3 Mins After Big Bang

### Big Bang Nucleosynthesis

CMB

### Stellar Nucleosynthesis

Type I X-ray Burst      Explosive H-He Burning

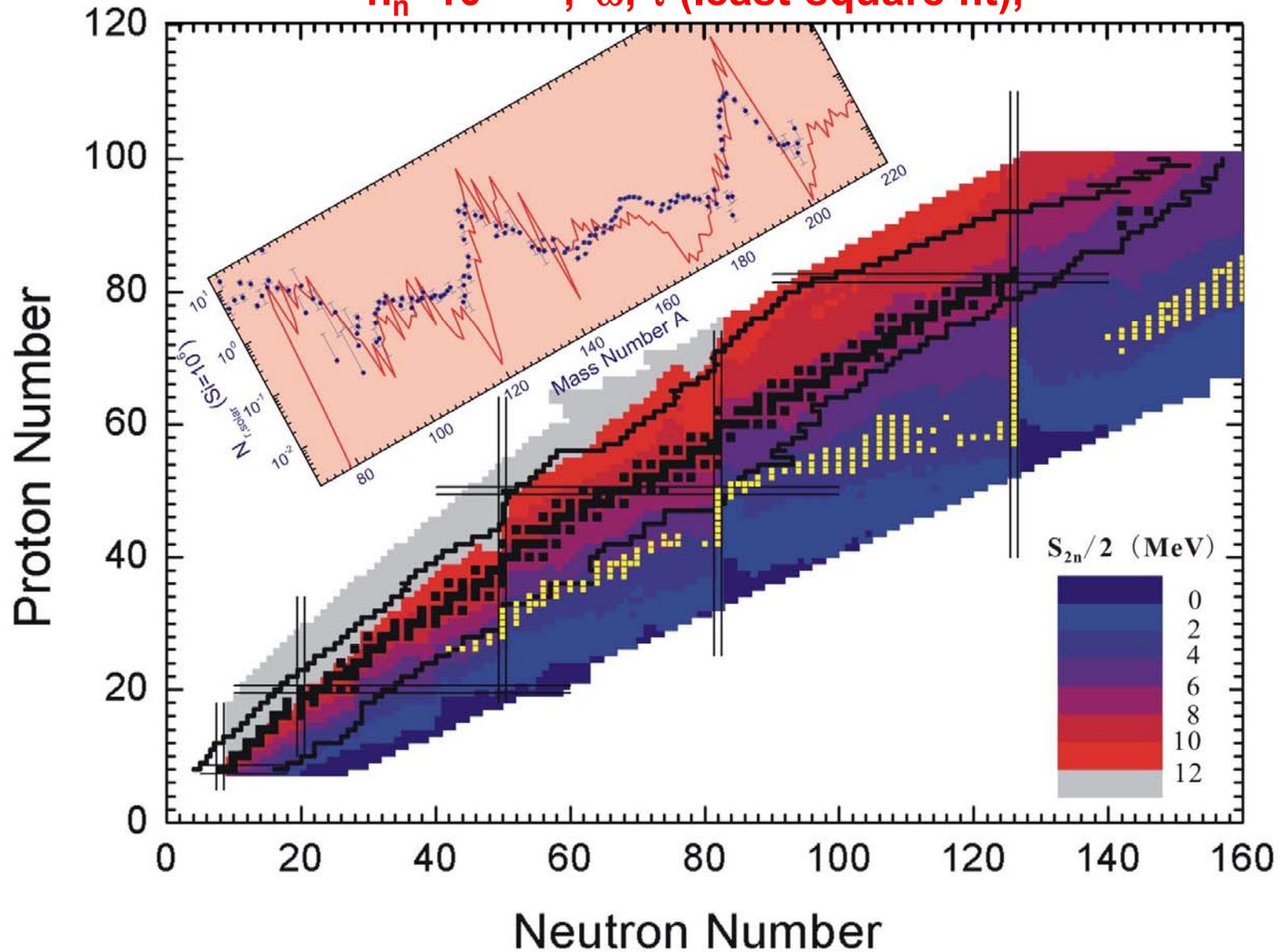
### From nucleus to star

Exotic Nuclei      Stellar Evolution



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nuclear inputs:  $S_n$ (RMF),  $T_{1/2}$ ( $\beta$ -decay),  $P_{1n}$ ,  $P_{2n}$ ,  
 $P_{3n}$  (FRDM), astrophysical parameters:  $T_9=1.5$ ,  
 $n_n=10^{20-28}$ ,  $\omega$ ,  $\tau$  (least-square fit),





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# Application of CDFT in Fundamental physics



## ➤ Cabibbo-Kobayashi-Maskawa matrix

- ✧ quark eigenstates of weak interaction  $\longleftrightarrow$  quark mass eigenstates
- ✧ Unitarity of CKM matrix  $\longrightarrow$  test of Standard Model

## ➤ To determine $|V_{ud}|$ in nuclear superallowed $\beta$ decays

- ✧ Experimental measurements *ft* survey:2009 by Hardy & Towner
- ✧ Theoretical corrections
  - radiative corrections Marciano:2006, Towner:2008
  - isospin symmetry-breaking corrections  $\delta_c$

## ➤ Definition of $\delta_c$ $|M_F|^2 = |\langle f | T_{\pm} | i \rangle|^2 = |M_0|^2(1 - \delta_c)$ ,

where  $|M_F|^2$  is the superallowed transition strength

and  $|M_0|^2$  is the strength with exact isospin symmetry

## ➤ Microscopic approaches for $\delta_c$

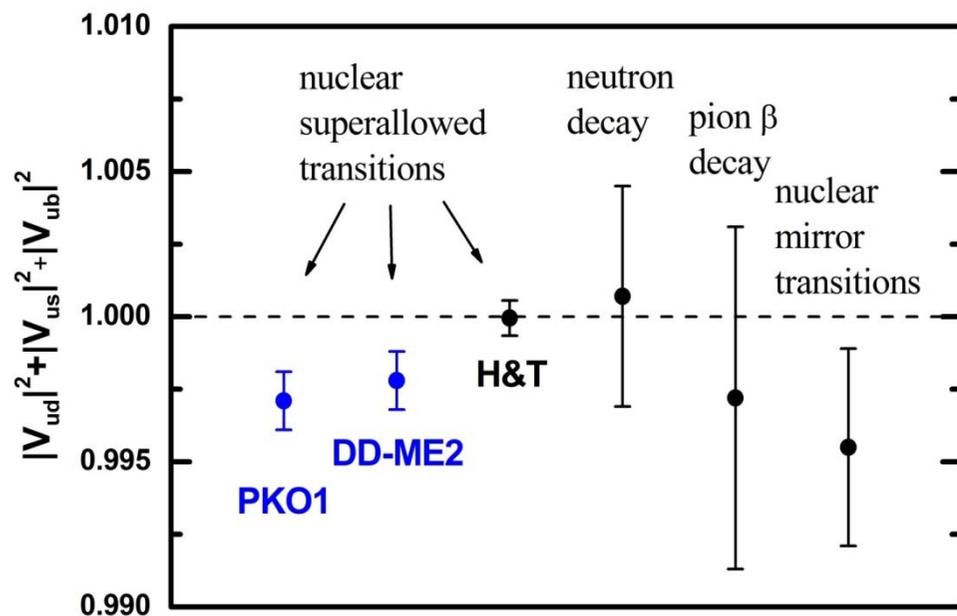
- ✧ shell model Towner:2008
- ✧ self-consistent charge-exchange RPA calculations e.g. Sagawa:1996

## ➤ Self-consistent relativistic RPA approaches: the isospin symmetry-breaking corrections $\delta_c$ .



## Up-down element and unitarity of the CKM matrix.

	PKO1	PKO2	PKO3	PKO1*	DD-ME1	DD-ME2	NL3	TM1
$ V_{ud} $	0.9727(3)	0.9728(3)	0.9727(3)	0.9730(3)	0.9731(3)	0.9731(3)	0.9730(3)	0.9731(3)
$ V_{ud} ^2 +  V_{us} ^2 +  V_{ub} ^2$	0.9971(10)	0.9971(10)	0.9971(10)	0.9977(10)	0.9978(10)	0.9978(10)	0.9975(10)	0.9978(10)



- ✓ Well agree with those obtained in neutron decay, pion  $\beta$  decay and nuclear mirror transitions.
- ✓ Somewhat deviate from the unitarity condition.

Liang, Van Giai, Meng, PRC (2009)

### Particle Data Group:

“We note, however, that the possibility of additional nuclear Coulombic corrections has been raised recently [Miller:2008, Auerbach:2009, Liang:2009].”

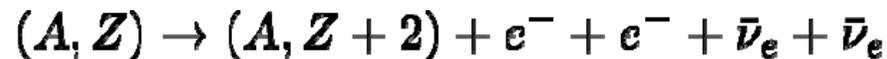


# Neutrinoless Double-beta decay

A second-order weak process : two protons are simultaneously transformed into two neutrons, or vice versa, inside an atomic nucleus.

❖ Two-neutrino double-beta ( $2\nu\beta\beta$ ) decay

*Goeppert-Mayer 1935, Phys. Rev. 48, 512*



❖ Neutrinoless double-beta ( $0\nu\beta\beta$ ) decay

*Majorana 1937, Nuovo Cim. 14, 171 Furry 1939, Phys. Rev. 56, 1184*



Majorana's theory of neutrinos  $\bar{\nu}_M = \nu_M$

The  $2\nu\beta\beta$  mode is allowed in SM while  $0\nu\beta\beta$  decay would go beyond SM. The  $0\nu\beta\beta$  decay occurs only if neutrinos are Majorana particles and lepton numbers can be violated.



The calculation of the NME requires two main ingredients :  
One is **the decay operator**, which reflects the mechanism governing the decay process. The other is **the wave functions of the initial and final states**.



# Nuclear matrix elements

❖ The  $0\nu\beta\beta$ -decay rate

$$\Gamma^{0\nu} = G^{0\nu}(Q_{\beta\beta}, Z) \times |M^{0\nu}|^2 \times |\langle m_\nu \rangle|^2 \quad \text{Unknown}$$

➤ Kinematic phase space factor  $G^{0\nu}(Q_{\beta\beta}, Z)$  can be accurately determined.

*Kotila & Iachello 2012, PRC 85, 034316*

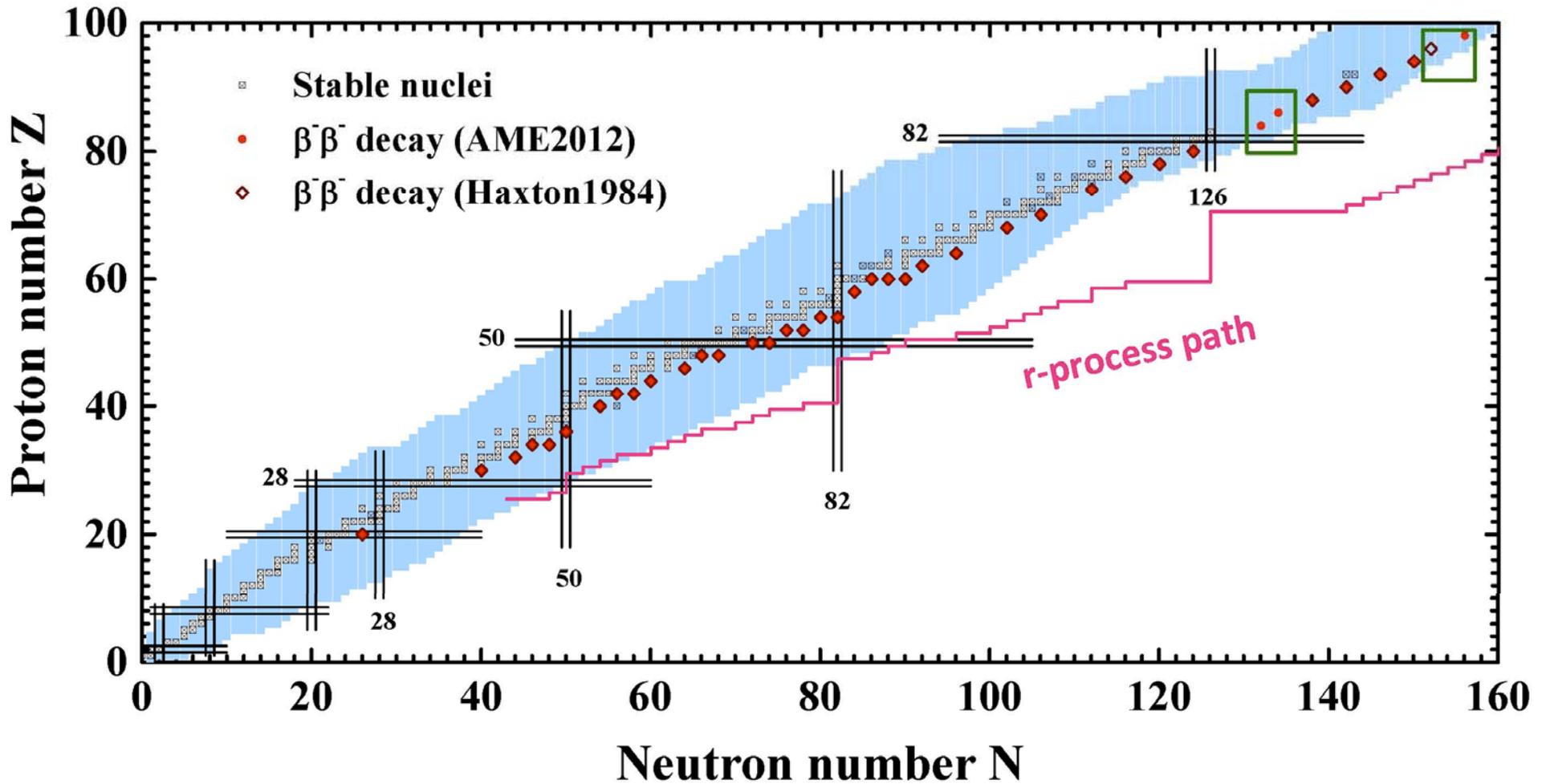
➤ Nuclear matrix element  $M^{0\nu}$  depend on nuclear structure models.

$$M^{0\nu} = \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle$$

**Accurate** nuclear matrix elements are crucial for extracting the **effective neutrino mass**.



# Nuclear neutrinoless double $\beta$ -decay





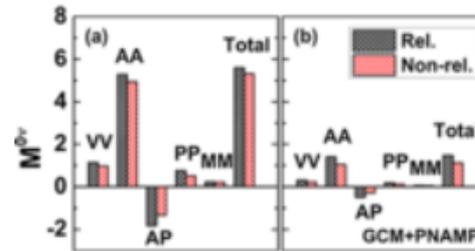
# Relativistic description of nuclear matrix elements in neutrinoless double- $\beta$ decay

Phys. Rev. C 90, 054309 – Published 10 November 2014  
Song, Yao, Ring, and Meng

Editors' Suggestion

## Relativistic description of nuclear matrix elements in neutrinoless double- $\beta$ decay

L. S. Song, J. M. Yao, P. Ring, and J. Meng  
Phys. Rev. C **90**, 054309 (2014) – Published 10 November 2014



Phys. Rev. C 91, 024316 (2015)  
Yao, Song, Hagino, Ring, and Meng

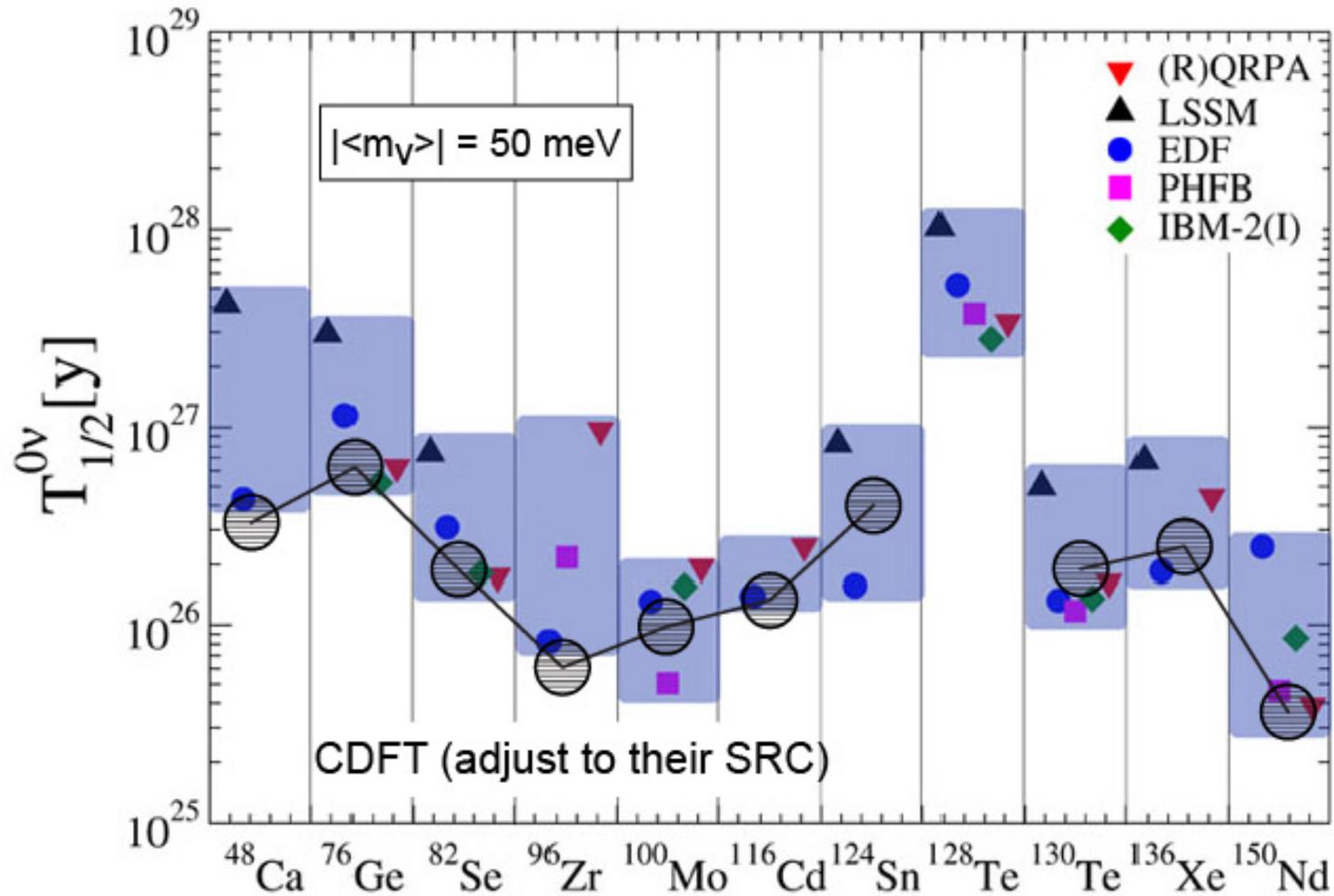
TABLE II. The calculated NME  $M^{0\nu}$  of the  $0\nu\beta\beta$  decay with the REDF (PC-PK1), in comparison with those by the NREDF (D1S), RQRPA, PHFB, ISM, and IBM2. Only the results considering the short-range correlation (SRC) effect by UCOM, except for the IBM2 where CCM is used and using the parameter  $R = 1.2A^{1/3}$  fm are adopted for comparison. The values in the parentheses are the results with additional pairing fluctuations.

The authors report a fully relativistic description based on a state-of-the-art nuclear structure and electric quadrupole transitions in both double- $\beta$  decay experiments.

Models	REDF(PC-PK1)	NREDF(D1S)	RQRPA (Tübingen)	PHFB	ISM	IBM2
$g_A(0)$	1.254	1.25	1.254	1.254	1.25	1.269
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	2.94	2.37 (2.23)			0.85	2.38
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	6.13	4.60 (5.55)	5.17		2.81	6.16
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	5.40	4.22 (4.67)	5.32		2.64	4.99
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	6.47	5.65 (6.50)	1.77	3.32		3.00
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	6.58	5.08 (6.59)	3.88	7.22		4.50
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	5.52	4.72 (5.35)	3.21			3.29
$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$	4.33	4.81 (5.79)			2.62	4.02
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	4.98	5.13 (6.40)	4.07	4.66	2.65	4.61
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	4.32	4.20 (4.77)	2.54		2.19	3.79
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	5.60	1.71 (2.19)		3.24		2.88



DOUBLE-BETA DECAY HALF-LIFE OF SEVERAL NUCLEI FROM DIFFERENT NUCLEAR MANY-BODY CALCULATIONS IN THE LIGHT NEUTRINO EXCHANGE SCENARIO



Vergados, Ejiri and Simkovic, Rep. Prog. Phys. 75 106301 (2012)

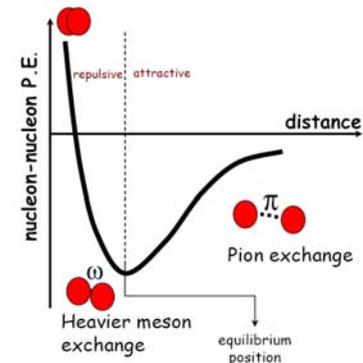
Song, Yao, Ring, Meng, Phys. Rev. C 95, 024305 (2017)



The effective interaction in current CDFT is not derived from the basic theory of the strong interaction --- QCD. As indicated by the successes achieved by the covariant density functional, the feasibility of a unified self-consistent description of nuclear ground state and excitation properties starting from an effective nucleon-nucleon interaction is anticipated.

In future, one needs to obtain properly the nucleon-nucleon interaction in nuclear medium starting from the quantum chromo-dynamics, eventually to build the standard model of nuclear structure that can implement the ab initio exploration for all nuclei in the nuclear chart.

Thank you for your attention!





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## Validity of CDFT



## ab initio----- “from the beginning”

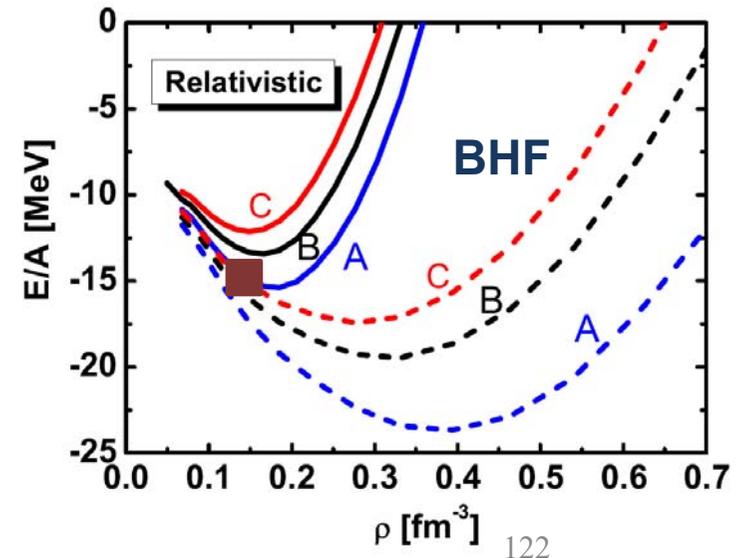
- without additional assumptions
- without additional parameters

## ab initio in nuclear physics

- with **realistic** nucleon-nucleon interaction
- with some **few-body** methods and **many-body** methods, such as Monte Carlo method, shell model and energy density functional theory

## ab initio in nuclear matter

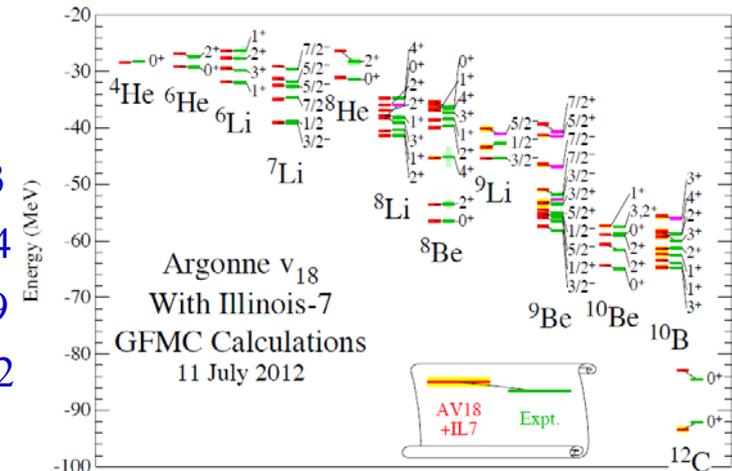
- Variational method Akmal PRC1998
- Green’s function method Dickhoff PPNP2004
- Chiral Perturbation theory Kaiser NPA2002
- Brueckner-Hartree-Fock (BHF) theory Baldo RPP2012
- **Relativistic BHF (RBHF) theory** Brockmann PRC1990
- .....





## ab initio calculation for light nuclei

- Gaussian Expansion Method Hiyama PPNP2003
- Green Function Monte Carlo Method Pieper PRC2004
- Lattice Chiral Effective Field Theory Lee PPNP2009
- No-Core Shell Model Barrett PPNP2012
- .....



Pieper, Wiringa, et al.

## ab initio calculation for heavier nuclei

- Coupled Channel method Hagen PRL2009
- BHF theory Hjorth-Jensen Phys.Rep.1995
  - With HJ potential Dawson Ann.Phys.1962
  - With Reid potential Machleidt NPA1975
  - With Bonn potentials Muether PRC1990

	Bonn C	Bonn B	Bonn A	Exp.
$\epsilon_{1s_{1/2}}$	-39.73	-44.37	-50.46	$-40 \pm 8$
$\epsilon_{1p_{3/2}}$	-16.98	-19.49	-22.89	-18.4
$\epsilon_{1p_{1/2}}$	-11.64	-13.24	-15.44	-12.1
$E$	-71.84	-85.60	-104.96	-127.68
$r_c$	2.465	2.380	2.291	2.737

**$^{16}\text{O}$  in BHF method in Bonn potential**



## Relativistic Brueckner Hartree-Fock: Dirac BHF (RBHF)

- Nuclear matter [Anastasio PRep 1978](#) [Brockmann PLB 1984](#) [ter Haar PRep. 1987](#)
- Defining an effective medium dependent meson-exchange interaction based upon the nuclear matter G matrix [Brockmann PRC1990](#) [Brockmann PRL 1992](#) [Fritz PRL 1993](#)

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## *ab initio* calculation attempt with CDFT: extracted interaction from the *ab initio* calculation in nuclear matter

- Density-dependent relativistic mean field theory [Brockmann PRL1992](#)
- Density-dependent relativistic Hartree-Fock theory [Fritz PRL1993](#)



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# First Full Relativistic Brueckner Hartree-Fock calculation for finite nucleus

CHIN. PHYS. LETT. Vol. 33, No. 10 (2016) 102103

Express Letter

## Relativistic Brueckner–Hartree–Fock Theory for Finite Nuclei \*

Shi-Hang Shen(申时行)<sup>1,2</sup>, Jin-Niu Hu(胡金牛)<sup>3</sup>, Hao-Zhao Liang(梁豪兆)<sup>2,4</sup>, Jie Meng(孟杰)<sup>1,5,6\*\*</sup>, Peter Ring<sup>1,7</sup>, Shuang-Quan Zhang(张双全)<sup>1</sup>

<sup>1</sup>State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871

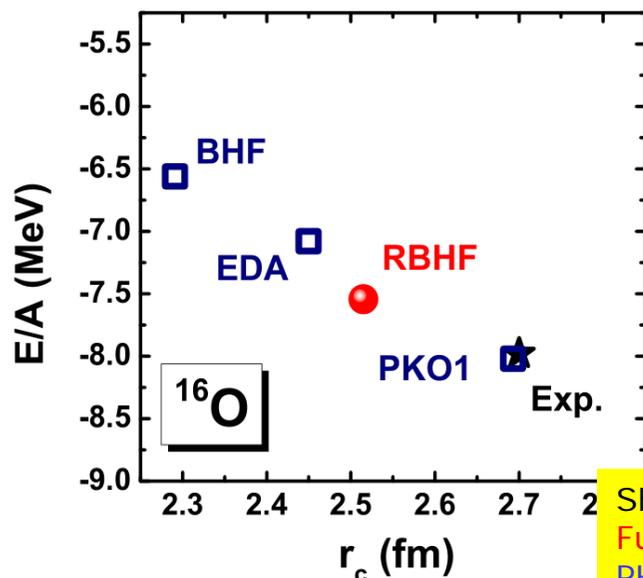
<sup>2</sup>RIKEN Nishina Center, Wako 351-0198, Japan

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<sup>4</sup>Department of Physics, Technische Universität München, D-85748 Garching, Germany

Shi-Hang Shen, Jin-Niu Hu, Hao-Zhao Liang, Jie Meng, Peter Ring, Shuang-Quan Zhang, Relativistic Brueckner–Hartree–Fock Theory for Finite Nuclei . Chin. Phys. Lett. 33 (2016) 102103

Technischen Universität München, D-85748 Garching, Germany



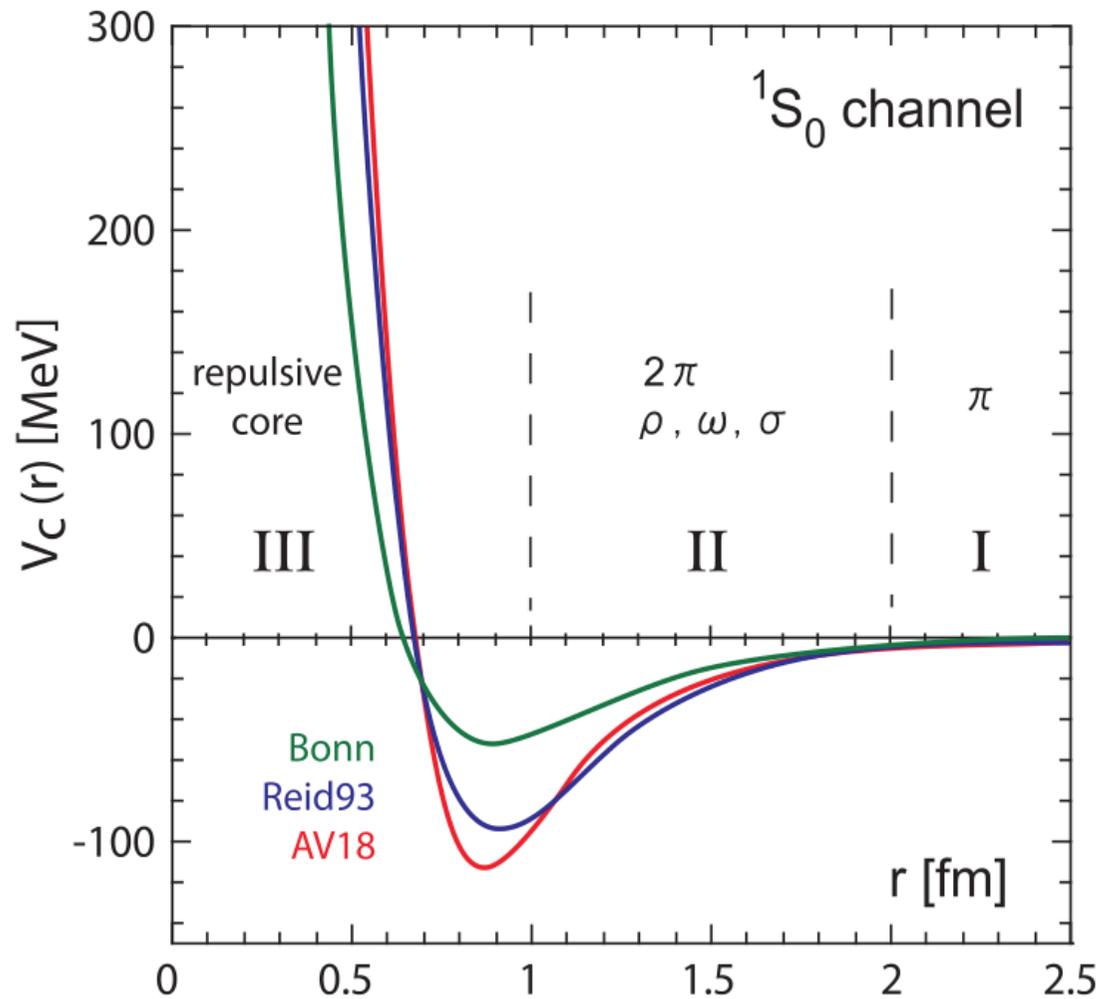
“This is the best paper on the topic and the only one with fully self-consistent solutions. To my opinion this paper is important to theoretical nuclear physics.”

Long paper: [arXiv:1705.01691](https://arxiv.org/abs/1705.01691)

Shi-Hang Shen, Hao-Zhao Liang, Jie Meng, Peter Ring, Shuang-Quan Zhang, Fully self-consistent relativistic Brueckner-Hartree-Fock theory for finite nuclei. PHYSICAL REVIEW C 96, 014316 (2017)

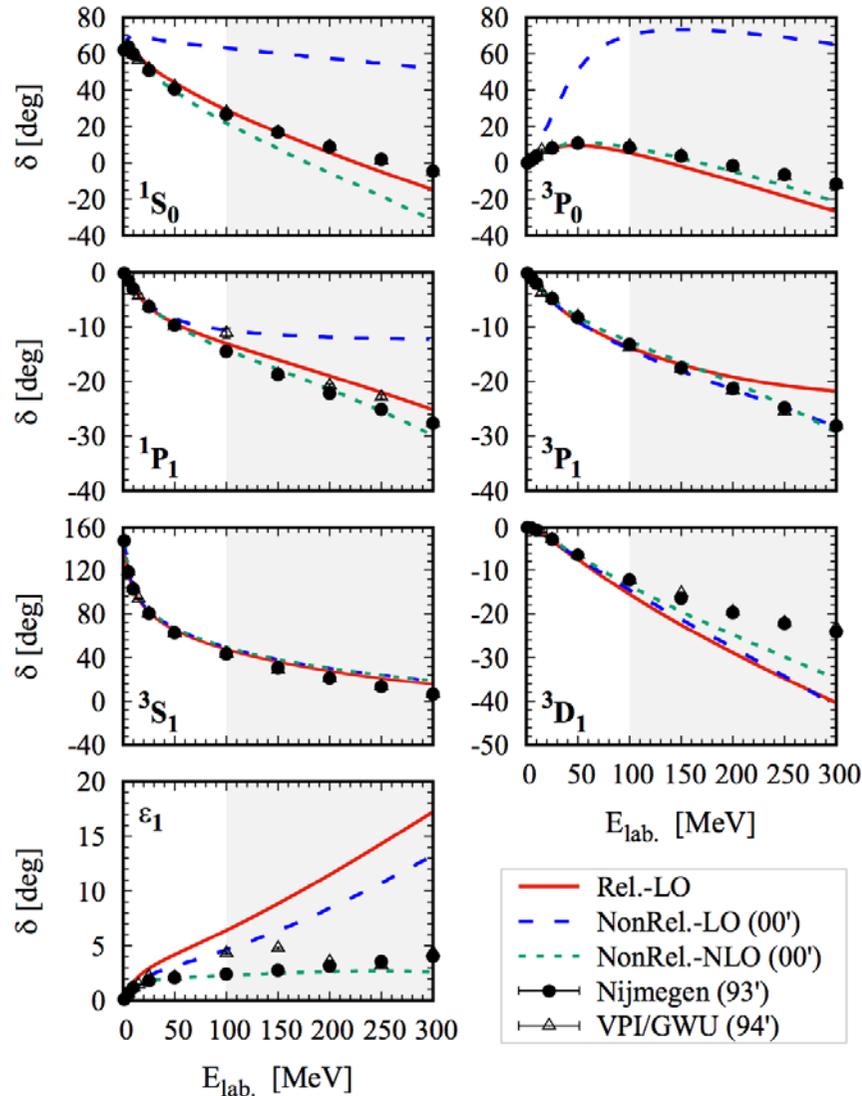


Two nucleon interaction: Nuclear Force from Lattice QCD





## Toward microscopic nucleon interaction



Leading order covariant  
chiral nucleon-nucleon  
interaction

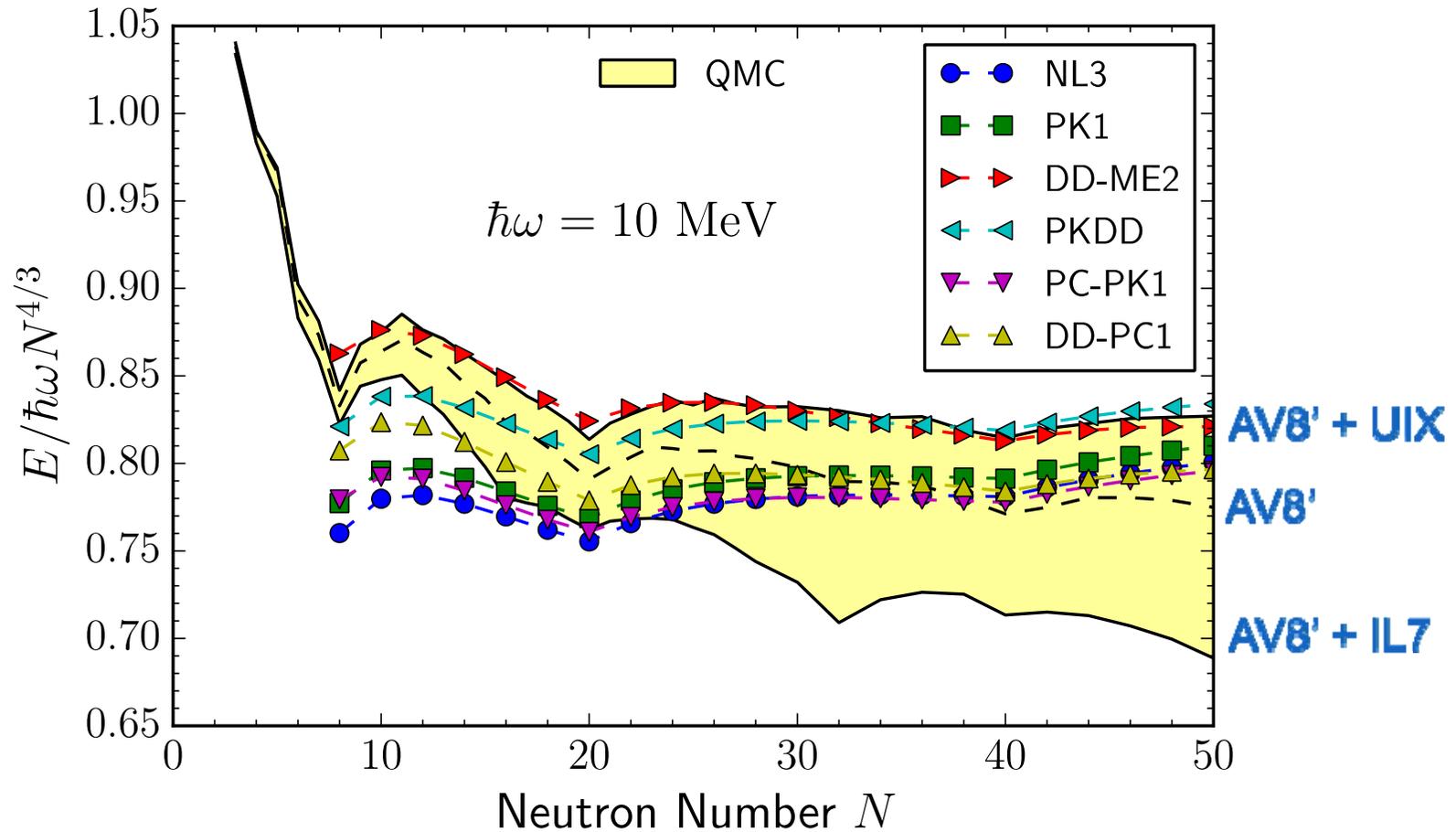
**The relativistic framework presents a more efficient formulation of the chiral nuclear force.**

Ren, Li, Geng, Long, Ring, and Meng,  
**Chinese Physics C** 42 (2018) 014103

arXiv:1611.08475



# Energy variation in Neutron drop

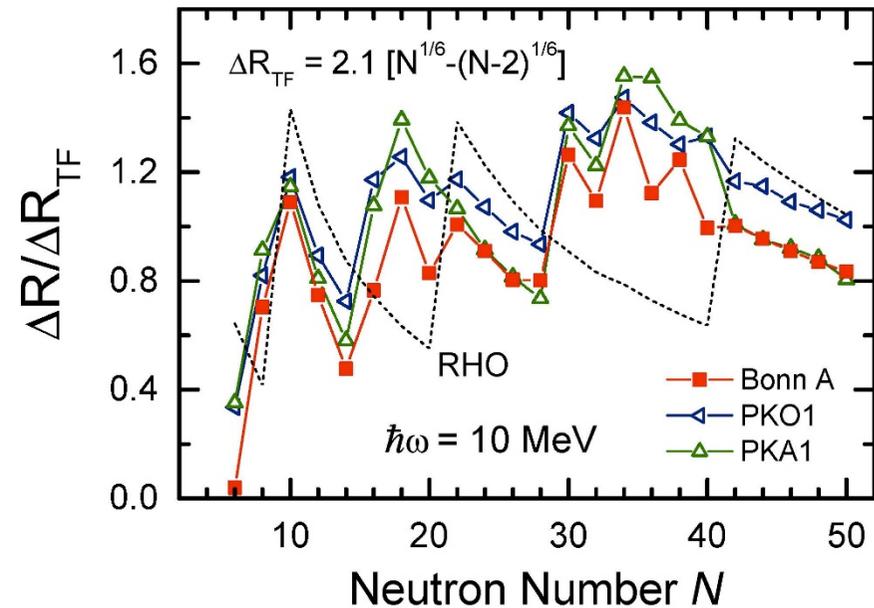
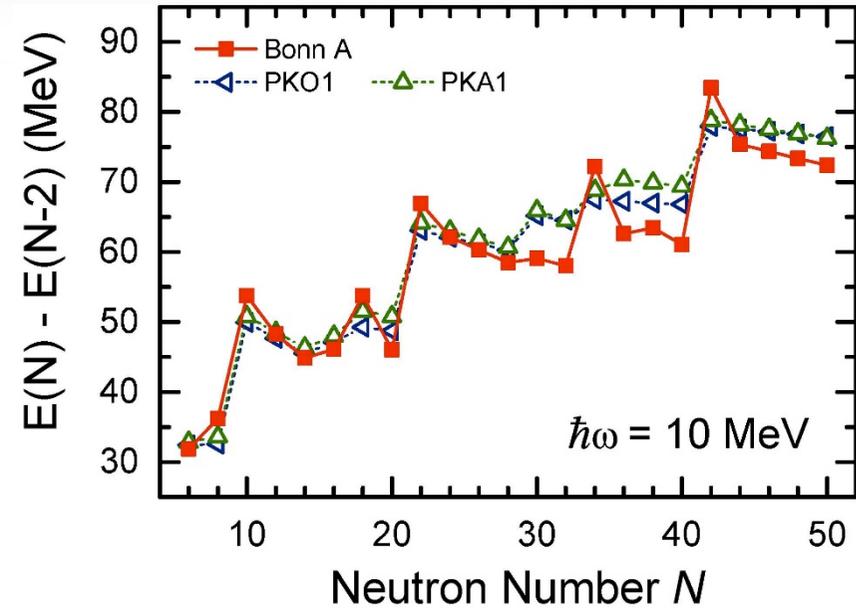
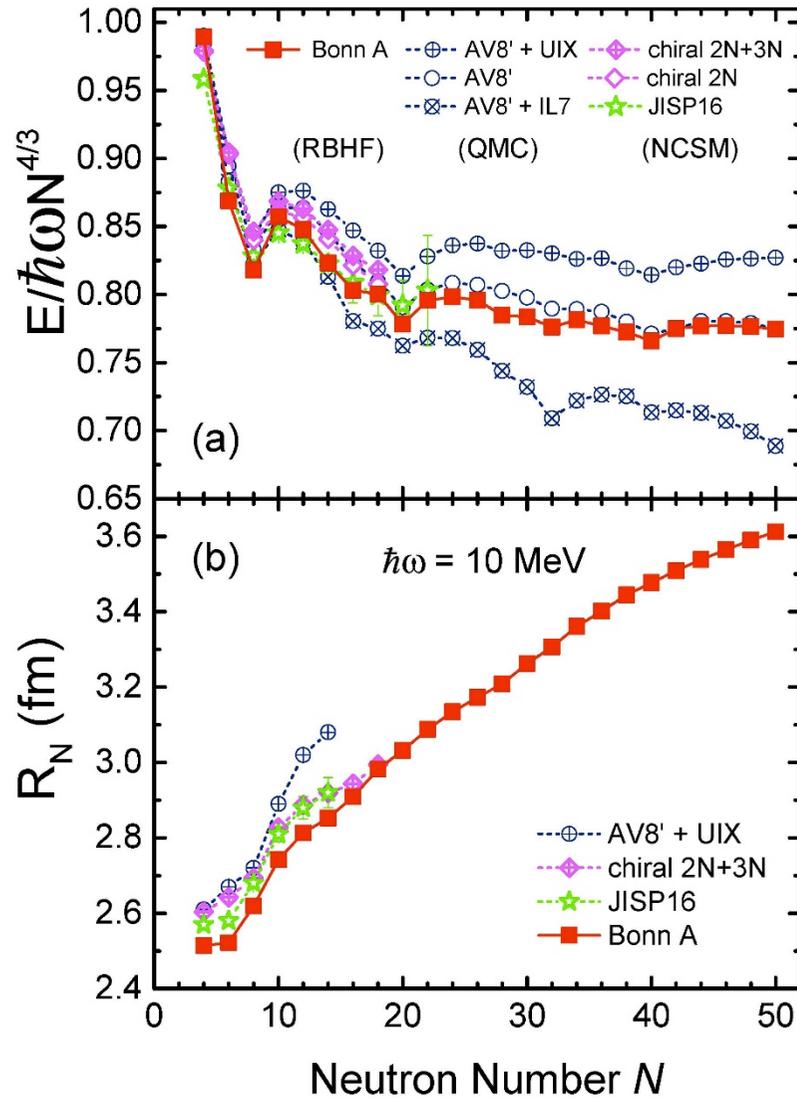


Zhao and Gandolfi PRC 94 (2016) 041302(R)

The three-body forces and isovector density functionals are still very unclear !



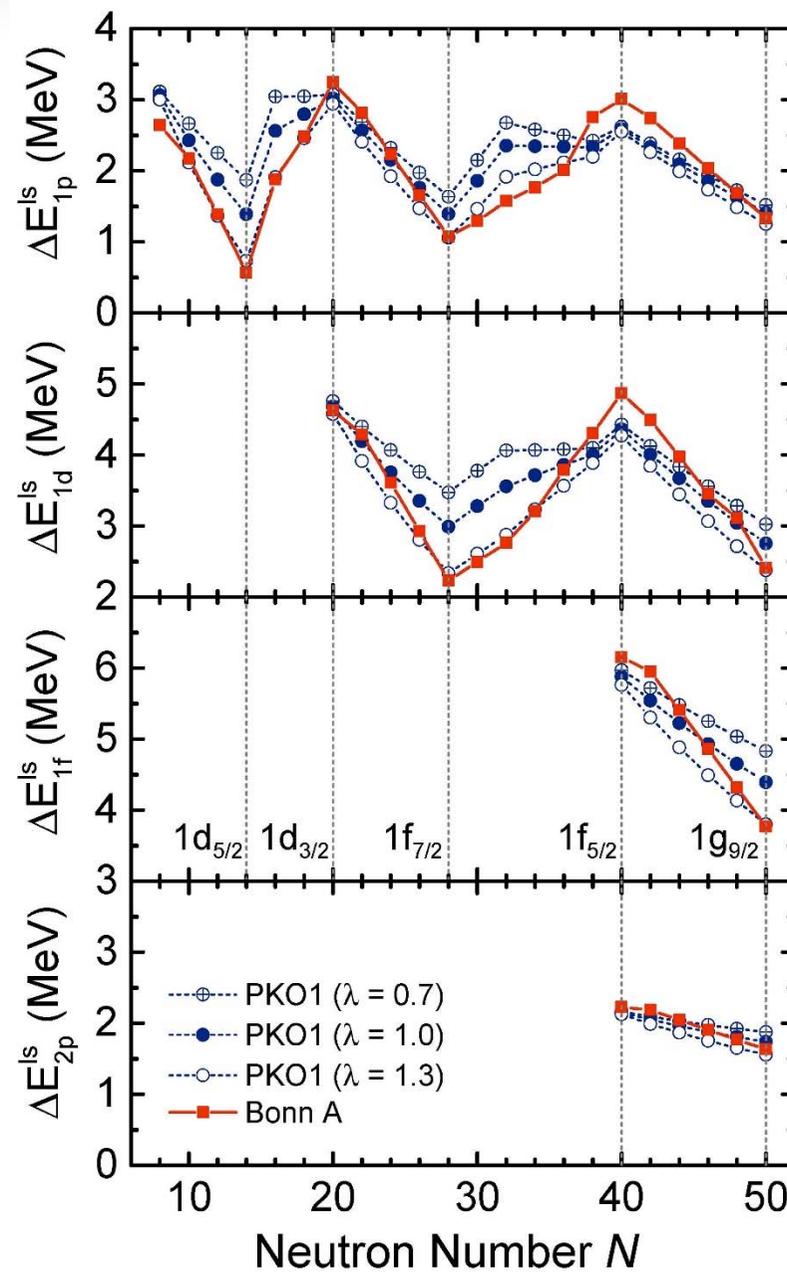
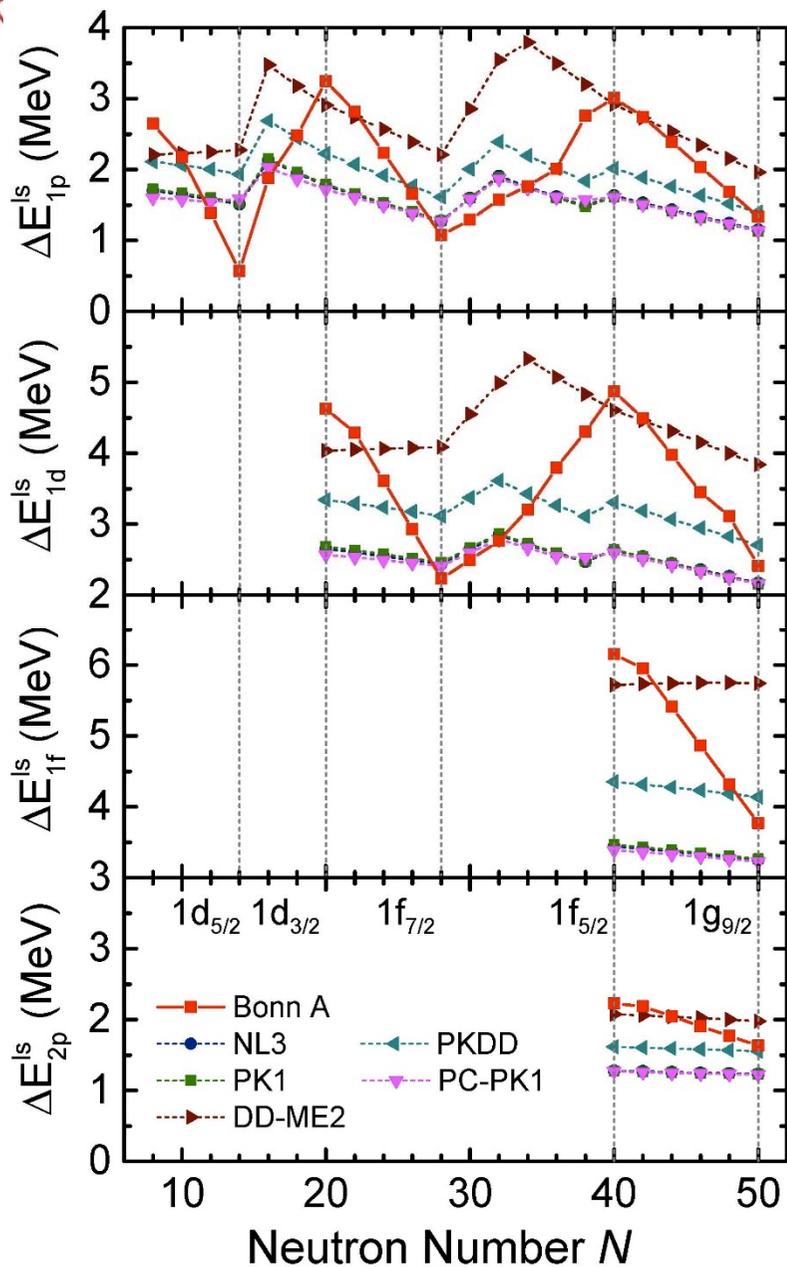
# Energy and drop size in Neutron drop





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# Tensor force and so splitting in Neutron drop





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Thank you for your attention!