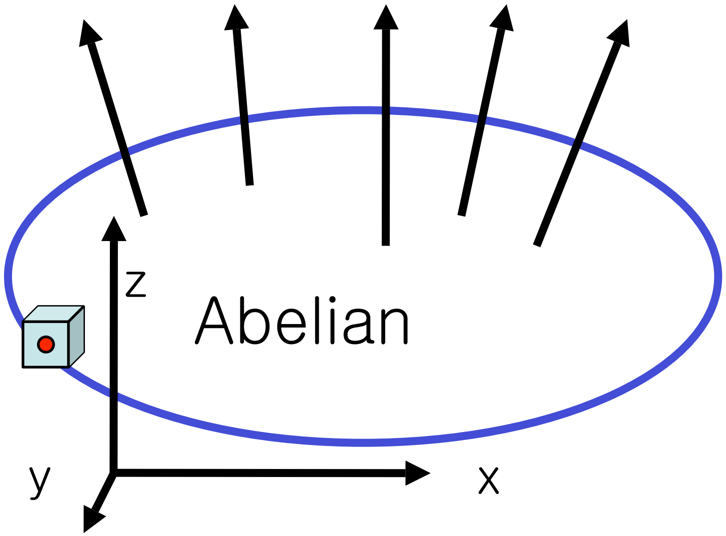
Matrix Berry phases of n-type semiconductor dots

- Coherent electric control of electrons by adiabatic means in semiconductor nano systems.
- Applies to II-VI and III-V n-type semiconductor dots and rings (Rashba and Dresselhaus spin-orbit couplings).
- Many-body effects: exchange has no effect on matrix Berry phase and correlation effects can be included.
- Applies to quantum pumping: a manifestation of matrix Berry phase (dots without spin-orbit).

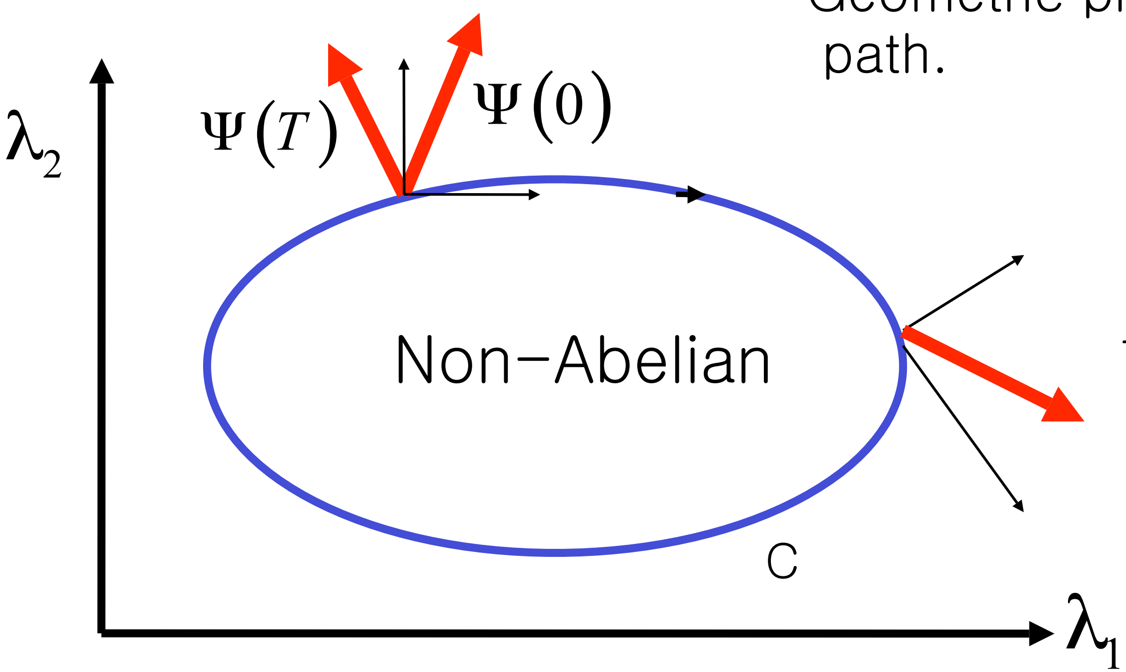
1. Introduction to matrix Berry phase



$$\Psi' = e^{i\int \vec{A}d\vec{l}}\,\Psi = e^{i\int \vec{B}d\vec{S}}\,\Psi$$

- $H(\lambda_1, \lambda_2)$: adiabatic path in parameter space.

- Geometric phase is dependent on path.



Non-Abelian vector potential

$$\Psi' = Pe^{i\int A_\mu d\lambda_\mu}\,\Psi$$

Wilczek and Zee ('84)

Derivation of matrix phase in degenerate energy shell

$$H\left(\lambda_1(t),\lambda_2(t)\right)$$

- Symmetry: degeneracy intact.
- Adiabatic: does not leave deg. energy shell.

$$\Psi(t) = c_1(t)\Psi_1(t) + c_2\Psi_2(t)$$

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = H\Psi(t),$$

$$H(t)\Psi_i = E(t)\Psi_i, \quad i = 1,2$$

$$\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix}, \begin{pmatrix}\Psi'_1\\ \Psi'_2\end{pmatrix}, \begin{pmatrix}\Psi''_1\\ \Psi''_2\end{pmatrix}, \cdots$$

$c_1(t)$ ® $c_1(t)e^{-iEt/\hbar}$

$$i\hbar\dot{c}_1 = -i\hbar\left\langle\Psi_1\middle|\frac{\partial\Psi_1}{\partial\lambda_p}\right\rangle\dot{\lambda}_p c_1 - i\hbar\left\langle\Psi_1\middle|\frac{\partial\Psi_2}{\partial\lambda_p}\right\rangle\dot{\lambda}_p c_2$$

$$\begin{pmatrix} \dot{c}_1 \\ \dot{c}_2 \end{pmatrix} = i\sum_p A_p \dot{\lambda}_p \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \quad A_P = i\begin{pmatrix} \left\langle \Psi_1 \middle| \frac{\partial \Psi_1}{\partial \lambda_P} \right\rangle & \left\langle \Psi_1 \middle| \frac{\partial \Psi_2}{\partial \lambda_P} \right\rangle \\ \left\langle \Psi_2 \middle| \frac{\partial \Psi_1}{\partial \lambda_P} \right\rangle & \left\langle \Psi_2 \middle| \frac{\partial \Psi_2}{\partial \lambda_P} \right\rangle \end{pmatrix}$$

U(2) gauge symmetry

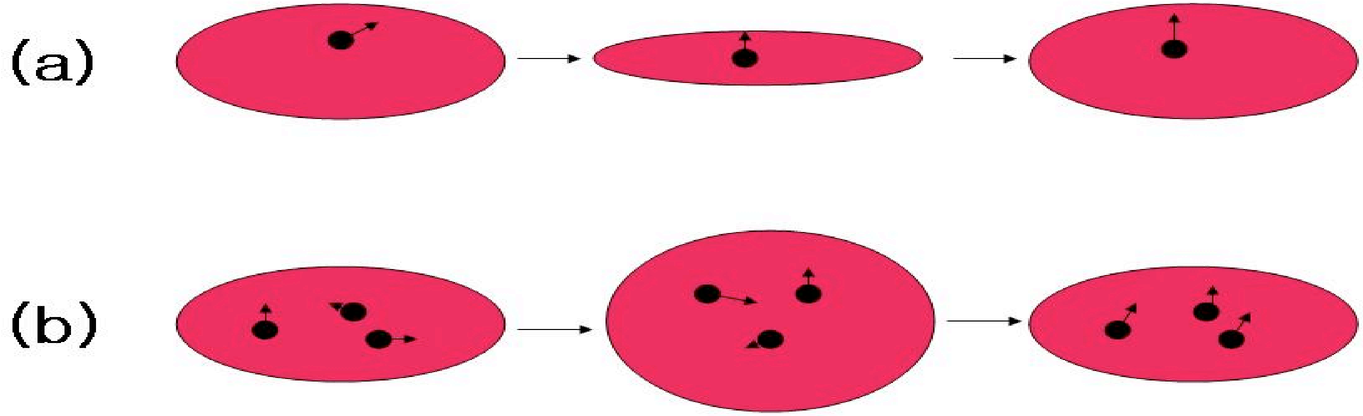
$$\left|\Psi_i^{'}\right\rangle = \sum_j U_{ij}^{*} \left|\Psi_j\right\rangle \ , \quad A_k^{'} = U A_k U^{+} + iU \frac{\partial U^{+}}{\partial \lambda_k}$$

$$\Psi(T) = \Theta\Psi(0), \ \Theta \text{ matrix Berry phase}$$

Matrix Berry phase not ‘gauge invariant’: basis dependent.

2. Matrix Berry phase of II-VI and III-V n-type semiconductor dots.

Confined electrons in a 3D electric potential

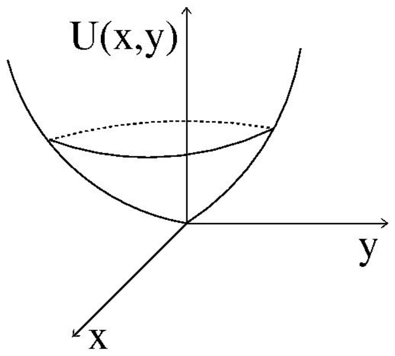


Yang and Hwang ('06)

•Time reversal symmetry

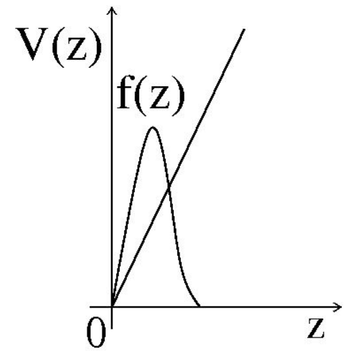
- Effective Hamiltonian $H = H_0 + [V(z) + U(x,y)] + H_R + H_D$

λ_1



(b)

λ_2



(a)

Change shape electrically: $\lambda_1 \quad \lambda_2$

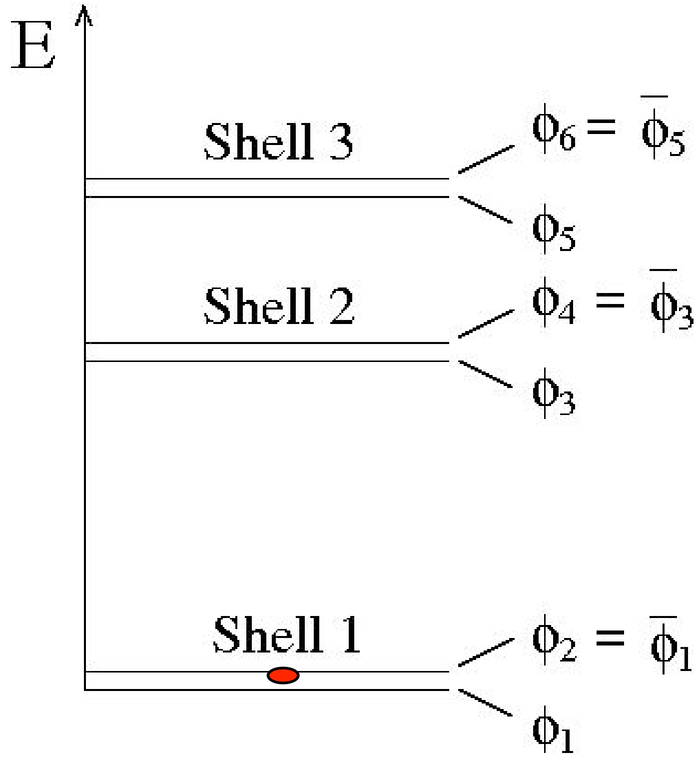
Rashba: $$H_R = c_R(\sigma_x k_y - \sigma_y k_x) \ .$$

Dresselhaus $$H_D = c_D\{[\sigma_x k_x(k_y^2 - k_z^2)] + [\sigma_y k_y(k_z^2 - k_x^2)]\} \ .$$

•No inversion symmetry but time-reversal symmetry

$$\mathrm{T}\phi_\mathrm{i} = \bar{\phi}_i \qquad \phi_i = \begin{pmatrix} F_\uparrow \\ F_\downarrow \end{pmatrix} \quad \bar{\phi}_i = \begin{pmatrix} -F_\downarrow^* \\ F_\uparrow^* \end{pmatrix}$$

•Degeneracy


E
Shell 3
Shell 2
Shell 1
$\phi_6 = \bar{\phi}_5$
ϕ_5
$\phi_4 = \bar{\phi}_3$
ϕ_3
$\phi_2 = \bar{\phi}_1$
ϕ_1

• Matrix Berry phase and Inversion symmetry of lateral potential

•Inversion symmetry of lateral potential

$$U(x, y) = U(-x, -y)$$

$$\varphi = \begin{pmatrix} F_e \\ F_o \end{pmatrix} ® \begin{pmatrix} F_e \\ -F_o \end{pmatrix} \text{ , type A} \qquad \bar{\varphi} = T\varphi = \begin{pmatrix} -F_o^* \\ F_e^* \end{pmatrix} ® \begin{pmatrix} F_o^* \\ F_e^* \end{pmatrix} \text{ , type B}$$

Off-diagonal non-Abelian vector potential

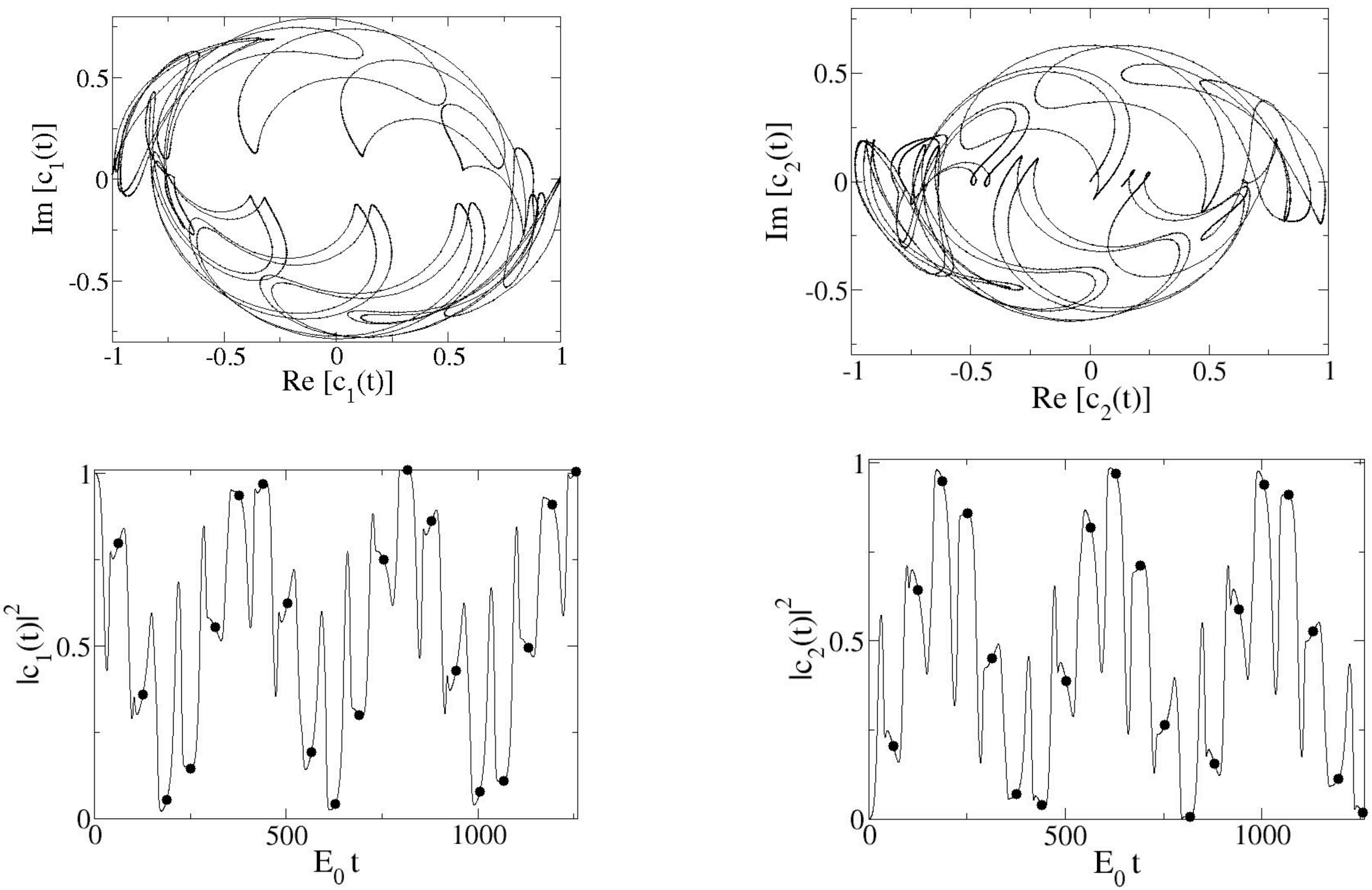
$$\left(a_k\right)_{12} = \left\langle \varphi \right| \frac{\partial}{\partial \lambda_i} \left| \bar{\varphi} \right\rangle = 0$$

• Inversion symmetry broken

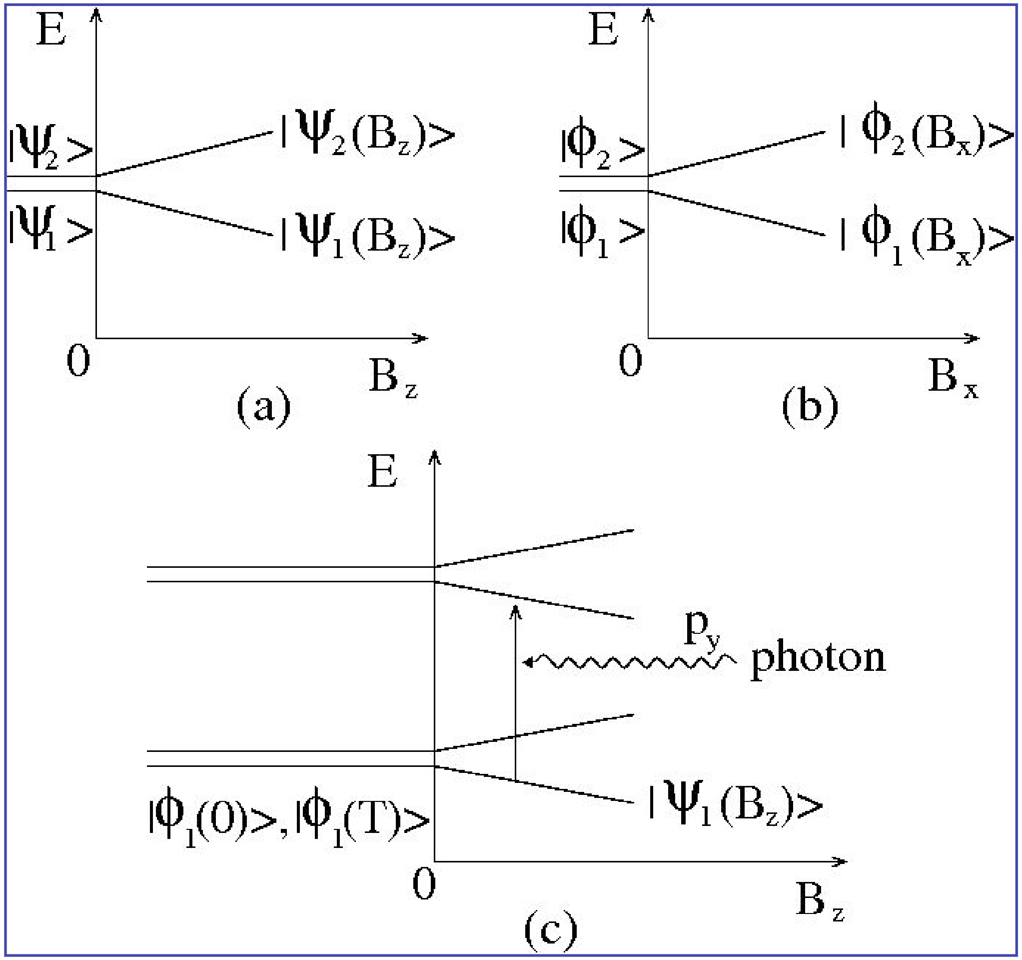
$$\left(a_k\right)_{12} \neq 0$$

• Time dependent behavior between integer multiple periods

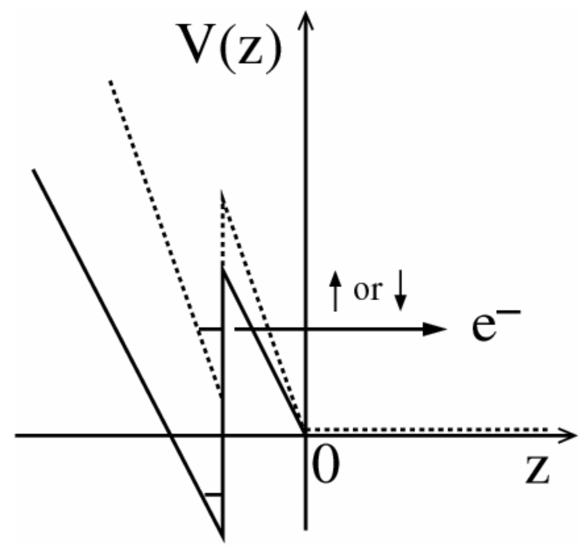
Rashba and Dresselhaus



• Detection of matrix Berry phase: optical dipole transitions & transport



$$\frac{I(T)}{I(0)} = \frac{|c_1(T)|^2}{|c_1(0)|^2}$$



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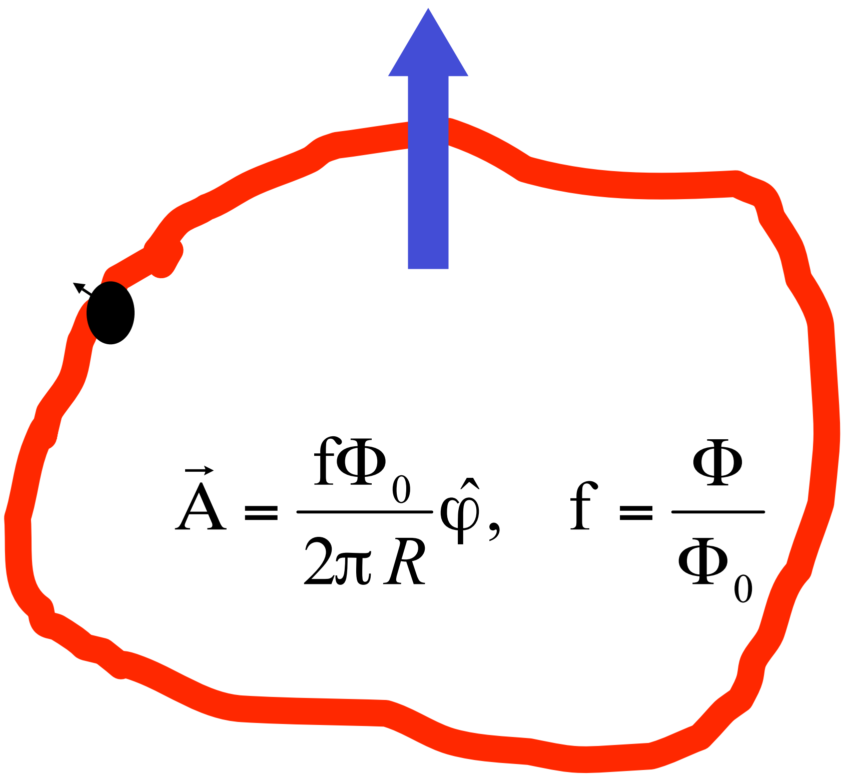
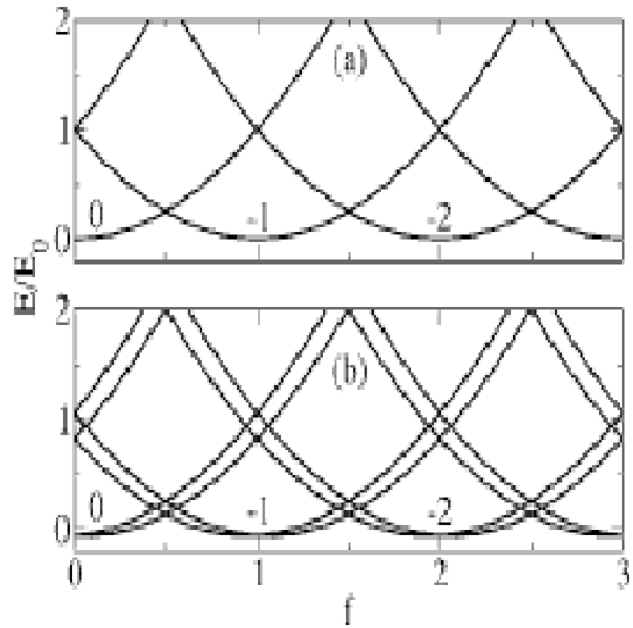
3.Matrix Berry phase of quantum ring threaded with integer or half-integer flux



$$\vec{A} = \frac{f\Phi_0}{2\pi R}\hat{\varphi}, \quad f = \frac{\Phi}{\Phi_0}$$



Circular ring

Doubly degenerate states for any shape and disorder

Yang ('06)

- Time reversal combined with large gauge transformation

$$H = \frac{1}{2m^*}\vec{\Pi}^2 + U(r) + c_R(\sigma_x \Pi_y - \sigma_y \Pi_x),$$

$$\vec{\Pi} = \vec{p} + \frac{e}{c}\vec{A}.$$ Canonical momentum

- Time reversal operation $\vec{p} \rightarrow -\vec{p}$.
- Large gauge transformation

$$\delta\vec{\mathrm{A}} = \frac{-2\,\mathrm{f}\,\Phi_0}{2\pi R}\hat{\varphi},$$

$$\vec{\mathrm{A}} + \delta\vec{\mathrm{A}} = -\vec{\mathrm{A}} \text{ and } \Psi' = e^{i2\,\mathrm{f}\,\varphi}\Psi.$$

Single valued wavefunctions-> f integer or half integer

4. Matrix Berry phase and many-body effects

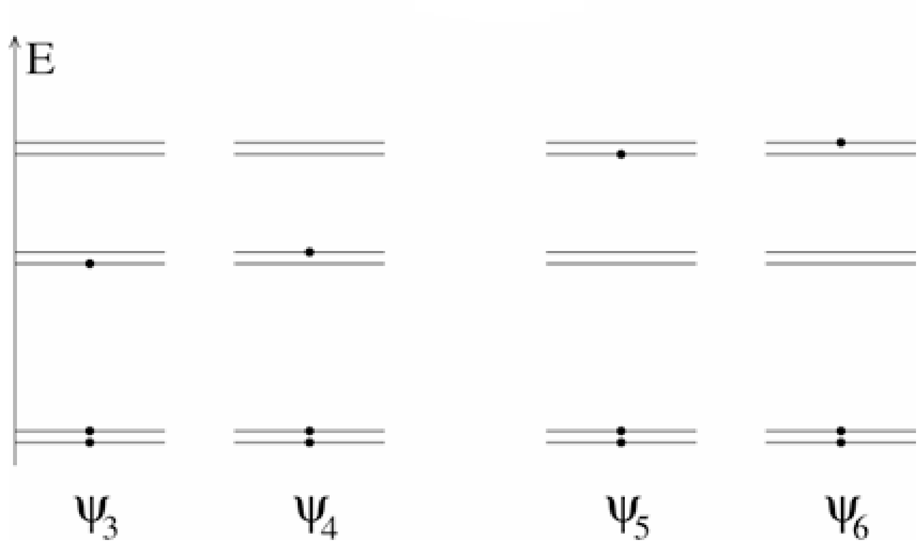
Yang ('07)

Odd number of electrons: groundstate degenerate

$$|\Phi\rangle = \sum_i c_i |\Psi_i\rangle$$

$$|\bar{\Phi}\rangle = T\Phi = \sum_i c_i^* |\bar{\Psi}_i\rangle = \sum_i d_i |\Psi_i\rangle$$

Slater determinant basis states



- No exchange effect

$$\left(B_k\right)_{ij} = i\left\langle \Psi_i \middle| \frac{\partial \Psi_j}{\partial \lambda_k} \right\rangle = \begin{cases} 0, & \Psi_i \neq a_p^{+} a_q \Psi_j \\ (a_k)_{pq}, & i \neq j \\ \sum_{r \in occ} (a_k)_{rr}, & i = j \end{cases}$$

Intra shell

$$(a_P)_{3,4} = i\left\langle \phi_3 \middle| \frac{\partial \phi_4}{\partial \lambda_P} \right\rangle$$

Inter-shell

$$(a_P)_{4,5} = i\left\langle \phi_4 \middle| \frac{\partial \phi_5}{\partial \lambda_P} \right\rangle$$

- Many-body correlation effects

$$(A_k)_{1,2} = i < \Phi \,|\, \frac{\partial}{\partial \lambda_k} \,|\, \bar{\Phi} >=$$

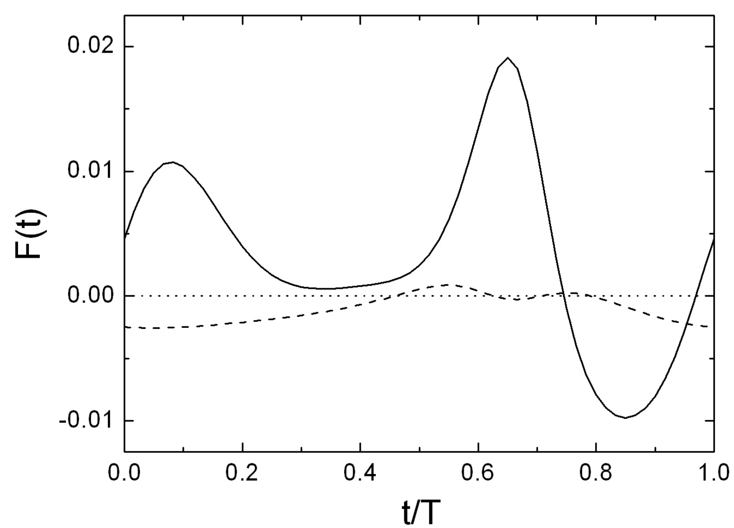
$$i \sum_i c_i^* \frac{\partial d_i}{\partial \lambda_k} + \sum_{i \neq j} c_i^* d_j (B_k)_{ij}$$

When lateral potential has inversion symmetry: matrix Berry phase exactly zero.

When lateral potential breaks inversion symmetry: matrix Berry phase present.

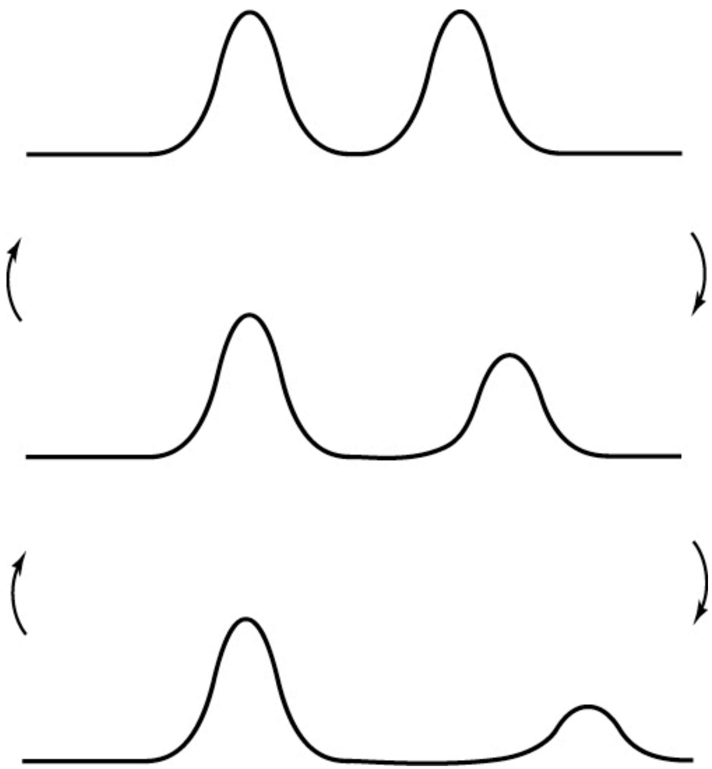
$$\begin{pmatrix} c_1(T) \\ c_2(T) \end{pmatrix} = \begin{pmatrix} \cos\chi & i\sin\chi \\ i\sin\chi & \cos\chi \end{pmatrix} \begin{pmatrix} c_1(0) \\ c_2(0) \end{pmatrix}$$

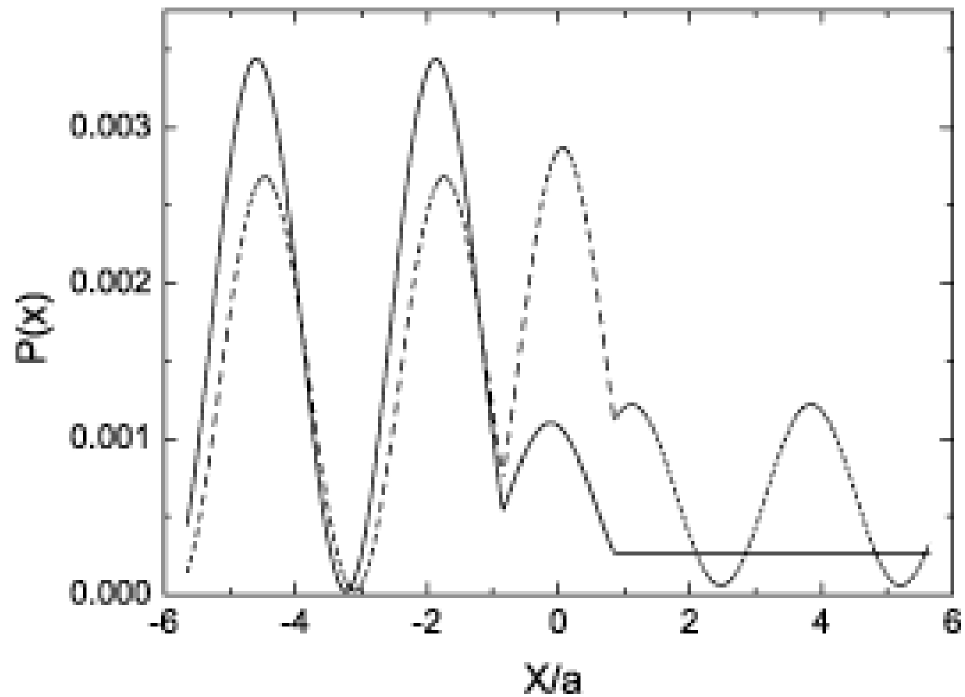
$$\chi = \int_0^T F(t)\,dt$$ 'gauge invariant'



5. Matrix Berry phase and pumping in quantum dots

No spin-orbit terms

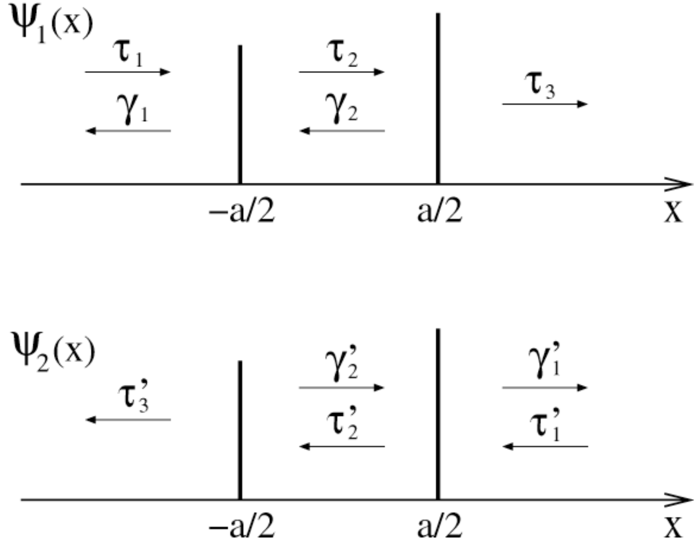




$$Q = \frac{e}{\pi}\int_A d\lambda_1 d\lambda_2 \,\mathrm{Im}\left(\frac{\partial s_{11}^*}{\partial \lambda_1}\frac{\partial s_{11}}{\partial \lambda_2} + \frac{\partial s_{12}^*}{\partial \lambda_1}\frac{\partial s_{12}}{\partial \lambda_2}\right)$$

Brouwer ('98)

Inversion symmetry of vertical potential broken

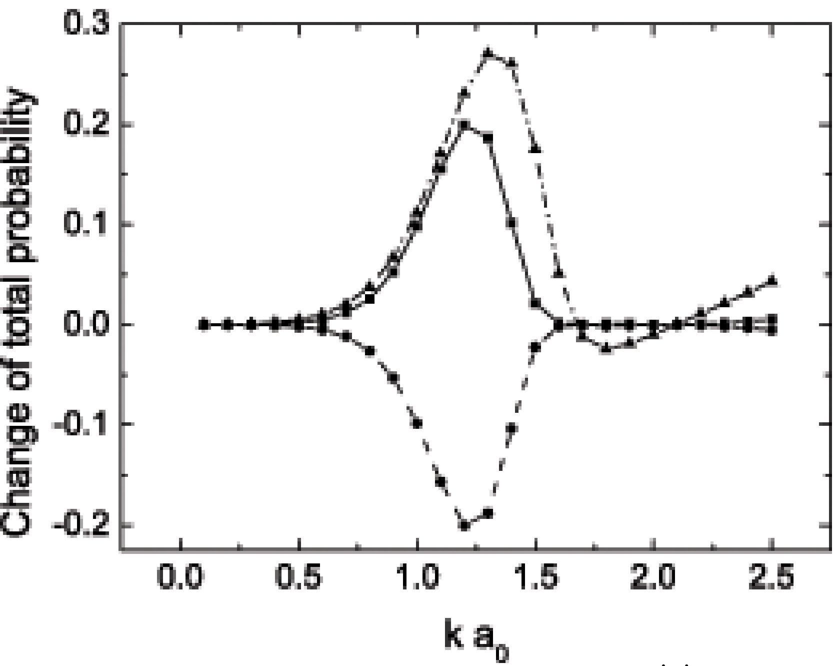


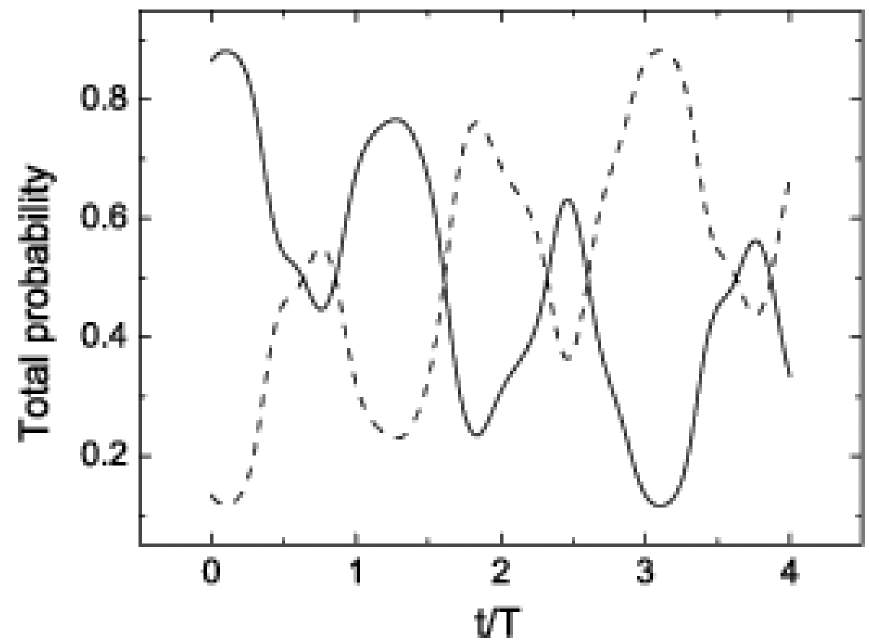
Since $T(t+\delta t,\, t) \simeq 1 - \dfrac{i\delta t}{\hbar} H(t)$

$$\left\langle \Psi_2(t+\delta t) \middle| T(t+\delta t,\, t) \middle| \Psi_1(t) \right\rangle \simeq$$

$$i\frac{\delta t}{\hbar}\left\langle \Psi_2(t) \middle| \delta V(t) \middle| \Psi_1(t) \right\rangle \neq 0$$

$$\left(i\left\langle \Psi_2 \middle| \frac{\partial}{\partial \lambda_k} \middle| \Psi_1 \right\rangle \neq 0 \right)$$





Hwang, Kim, Park, and Yang ('07)

6.Conclusions

Common mathematical properties

	Non-Abelian systems		
Systems	dot with spin-orbit	Ring with spin-orbit	quantum pump no spin-orbit
Degeneracy	Time reversal	Time reversal & large gauge	Left and right incoming scattering states
Gauge group: basis transformation	U(2)	U(2)	U(2)
Breaking of inversion symmetry	lateral potential (xy-plane)	lateral potential (xy-plane)	vertical potential (z-direction)

- Need non-trivial degeneracy.
- Matrix phases are non-integrable.
- Adiabatic electric control of spin is possible in II-VI & III-V semiconductor quantum dots and rings.
- Pumping is manifestation of matrix Berry phase.
- Experiment would be most interesting.

•Contents

1. Introduction to matrix Berry phases.
2. Matrix Berry phase of II–VI and III–V n–type semiconductor dots.
3. Matrix Berry phase of II–VI and III–V n–type quantum ring threaded with integer or half–integer flux.
4. Matrix Berry phase and many–body effects: exchange and correlation effect.
5. Matrix Berry phase and quantum pumping.
6. Conclusions.

- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$

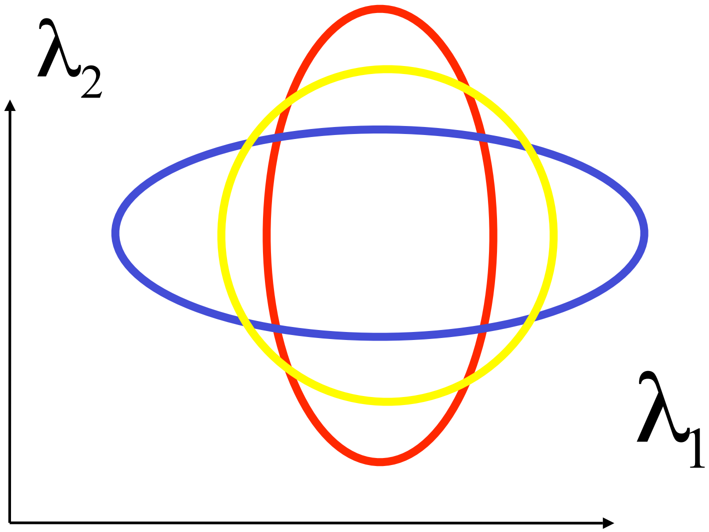
$$[A_\mu, A_\nu] = 0 \rightarrow \Phi_C(1) = e^{i\int_S F_{12}\, dS},$$

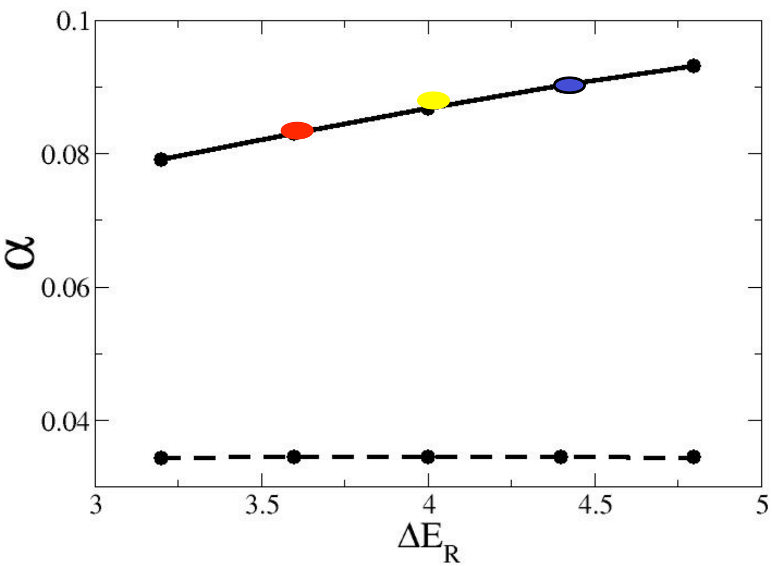
$$[A_\mu, A_\nu] \neq 0 \rightarrow \Phi_C(1) = e^{iF_{12}S}, \quad \text{S small}$$

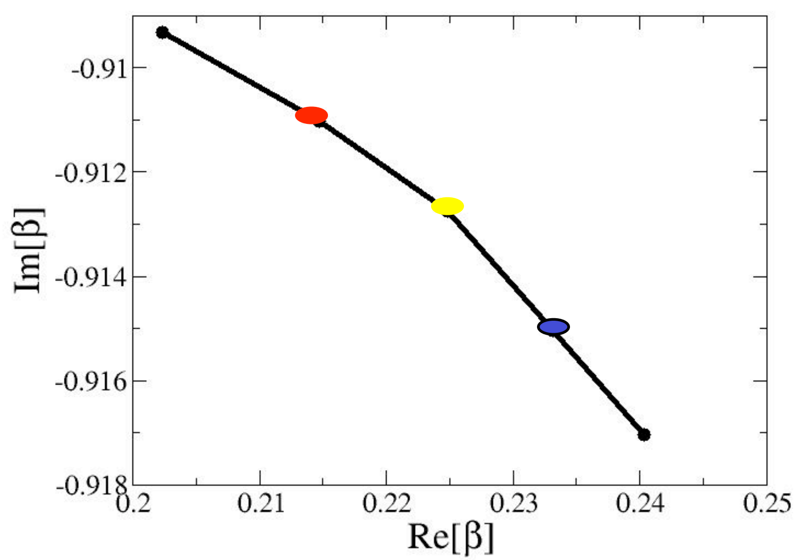
Matrix Berry phase and shape of path

$$\bullet\,\Phi_C(1) = P e^{i\int A_\mu d\lambda_\mu} = \exp\left(\frac{i}{2}(2\alpha)\vec{m}\bullet\vec{\sigma}\right),$$

$$\vec{m} = \left(\mathrm{Re}(\beta), -\mathrm{Im}(\beta), \sqrt{1-|\beta|^2}\right).$$







Elementary derivation

$$\Psi(t) = c_1(t)\Psi_1(t) + c_2\Psi_2(t)$$

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = H\Psi(t)$$

$$i\hbar\left\langle \Psi_1 \,|\, \frac{\partial\Psi(t)}{\partial t}\right\rangle = \left\langle \Psi_1 \,|\, H \,\middle|\, \Psi(t)\right\rangle$$

$$i\hbar\left(\dot{c}_1 + c_1\left\langle \Psi_1 \middle| \dot{\Psi}_1\right\rangle + c_2\left\langle \Psi_1 \middle| \dot{\Psi}_2\right\rangle\right) = Ec_1$$

$$c_1(t)\ \text{®}\ c_1(t)e^{-iEt/\hbar}$$

$$i\hbar\dot{c}_1 = -i\hbar\left\langle \Psi_1 \middle| \dot{\Psi}_1\right\rangle c_1 - i\hbar\left\langle \Psi_1 \middle| \dot{\Psi}_1\right\rangle c_2$$

$$i\hbar\dot{c}_1 = -i\hbar\left\langle \Psi_1 \middle| \frac{\partial\Psi_1}{\partial\lambda_p}\right\rangle \dot{\lambda}_p c_1 - i\hbar\left\langle \Psi_1 \middle| \frac{\partial\Psi_2}{\partial\lambda_p}\right\rangle \dot{\lambda}_p c_2$$

$$\begin{pmatrix}\dot{c}_1\\ \dot{c}_2\end{pmatrix} = i\sum_p A_p\dot{\lambda}_p\begin{pmatrix}c_1\\ c_2\end{pmatrix},\quad \begin{pmatrix}c_1(T)\\ c_2(T)\end{pmatrix} = Pe^{i\int A_\mu d\lambda_\mu}\begin{pmatrix}c_1(0)\\ c_2(0)\end{pmatrix}$$

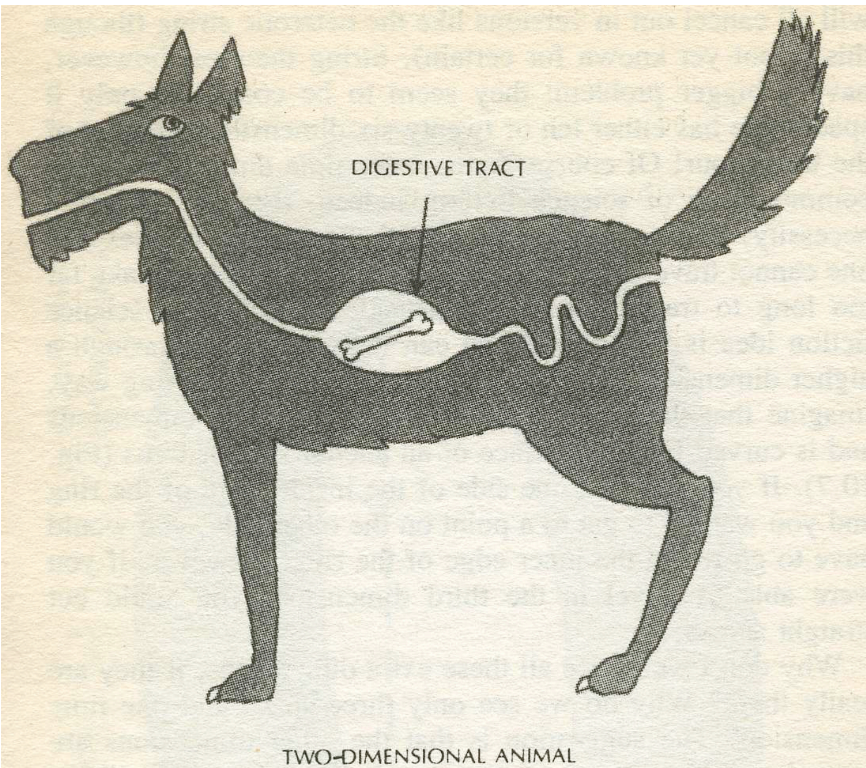
$$A_P = i\begin{pmatrix}\left\langle \Psi_1 \middle| \frac{\partial\Psi_1}{\partial\lambda_P}\right\rangle & \left\langle \Psi_1 \middle| \frac{\partial\Psi_2}{\partial\lambda_P}\right\rangle \\ \left\langle \Psi_2 \middle| \frac{\partial\Psi_1}{\partial\lambda_P}\right\rangle & \left\langle \Psi_2 \middle| \frac{\partial\Psi_2}{\partial\lambda_P}\right\rangle\end{pmatrix}$$

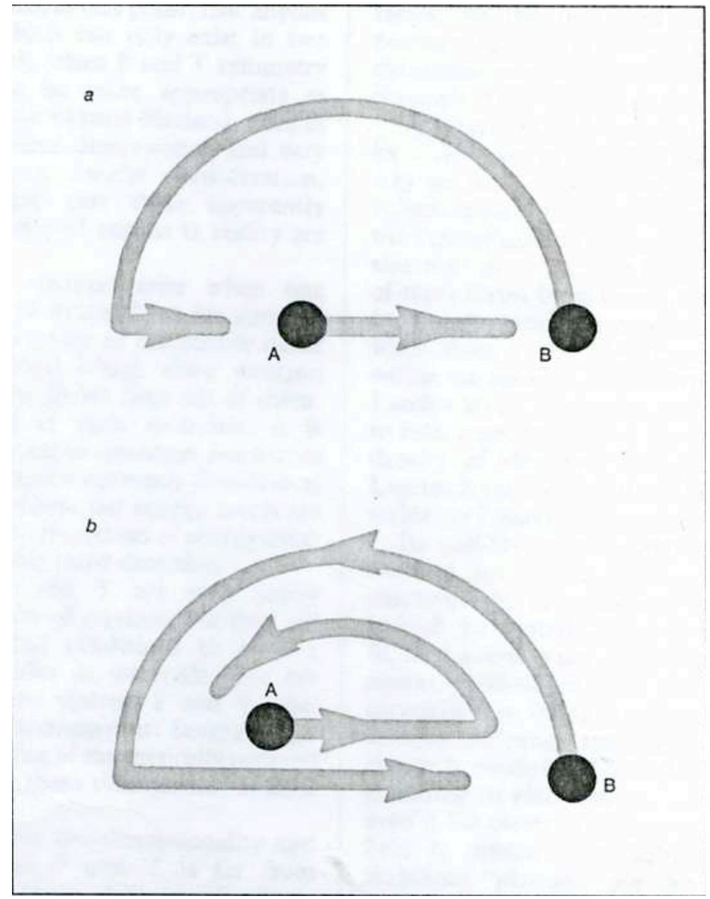
U(2) gauge symmetry

$$\left|\Psi'_i\right\rangle = \sum_j U^*_{ij}\left|\Psi_j\right\rangle,\quad A'_k = UA_kU^+ + iU\frac{\partial U^+}{\partial\lambda_k}$$

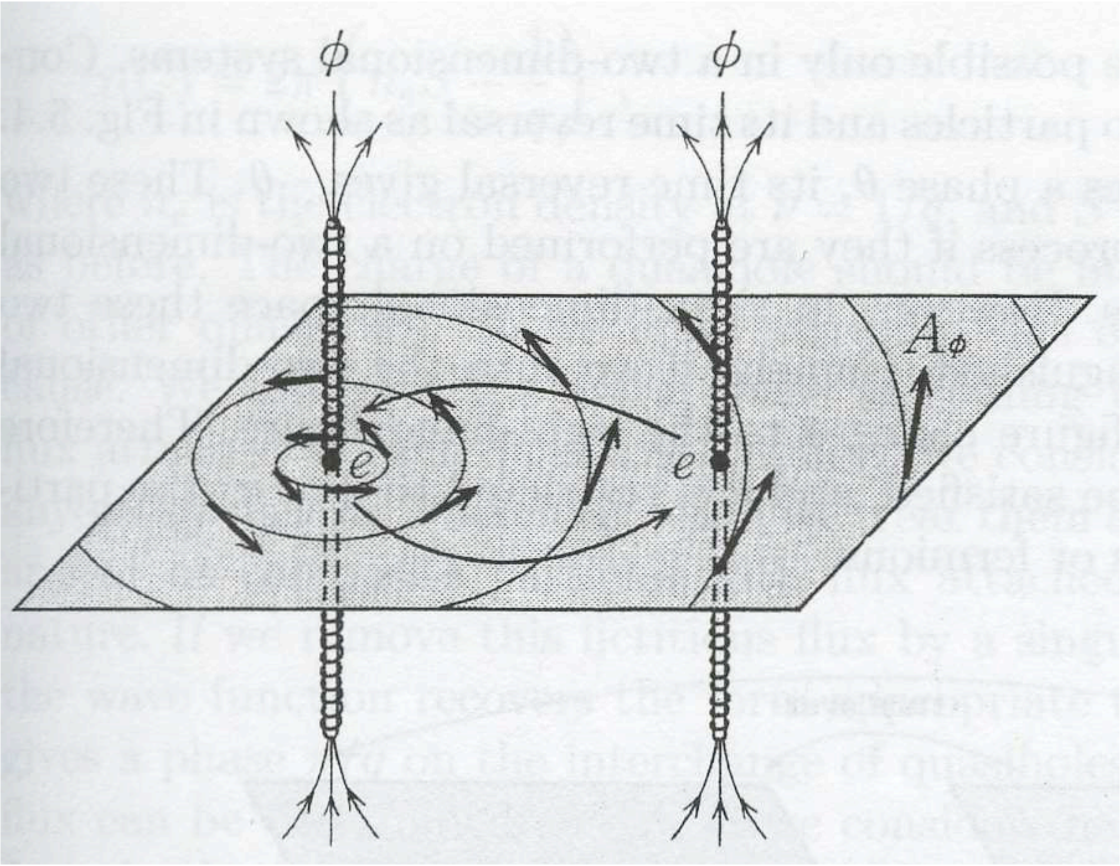
11.Anyons with fractional and non-Abelian statistics

- Double exchange is not trivial direct process

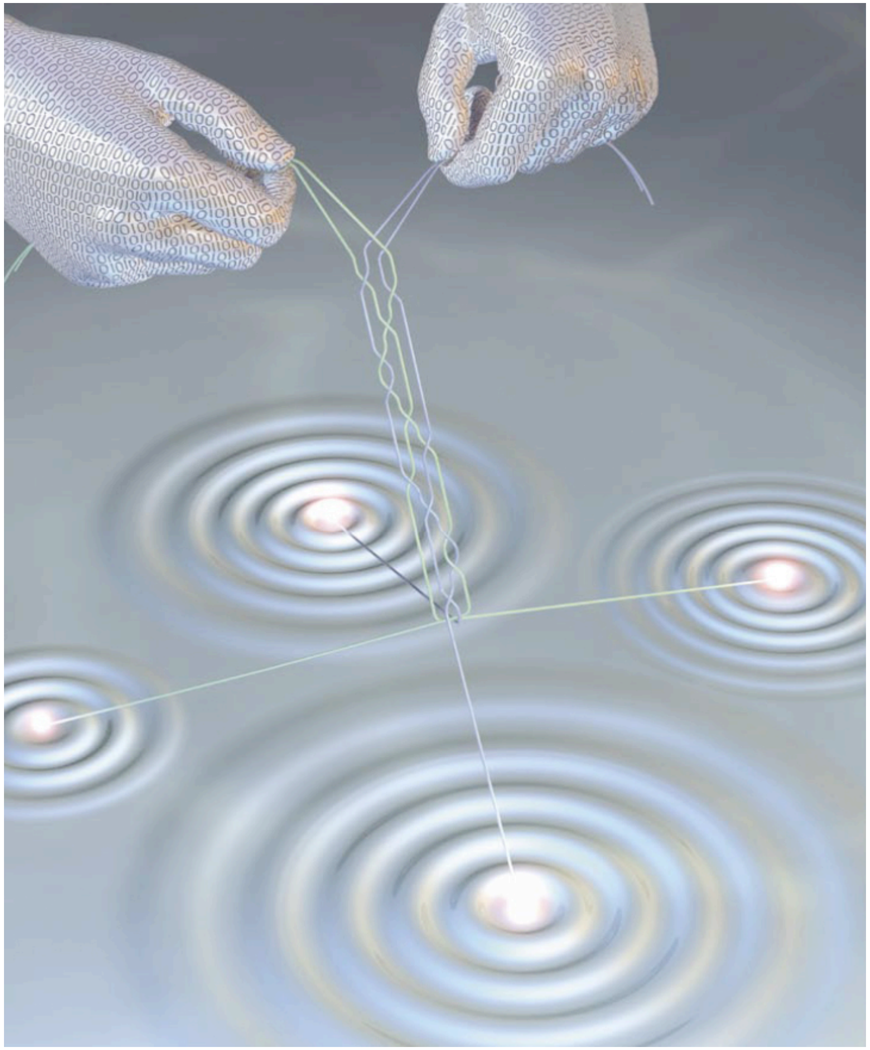


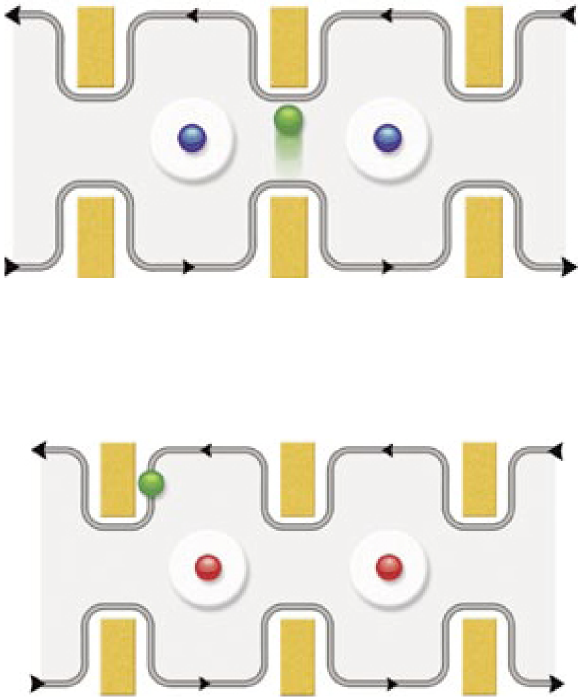


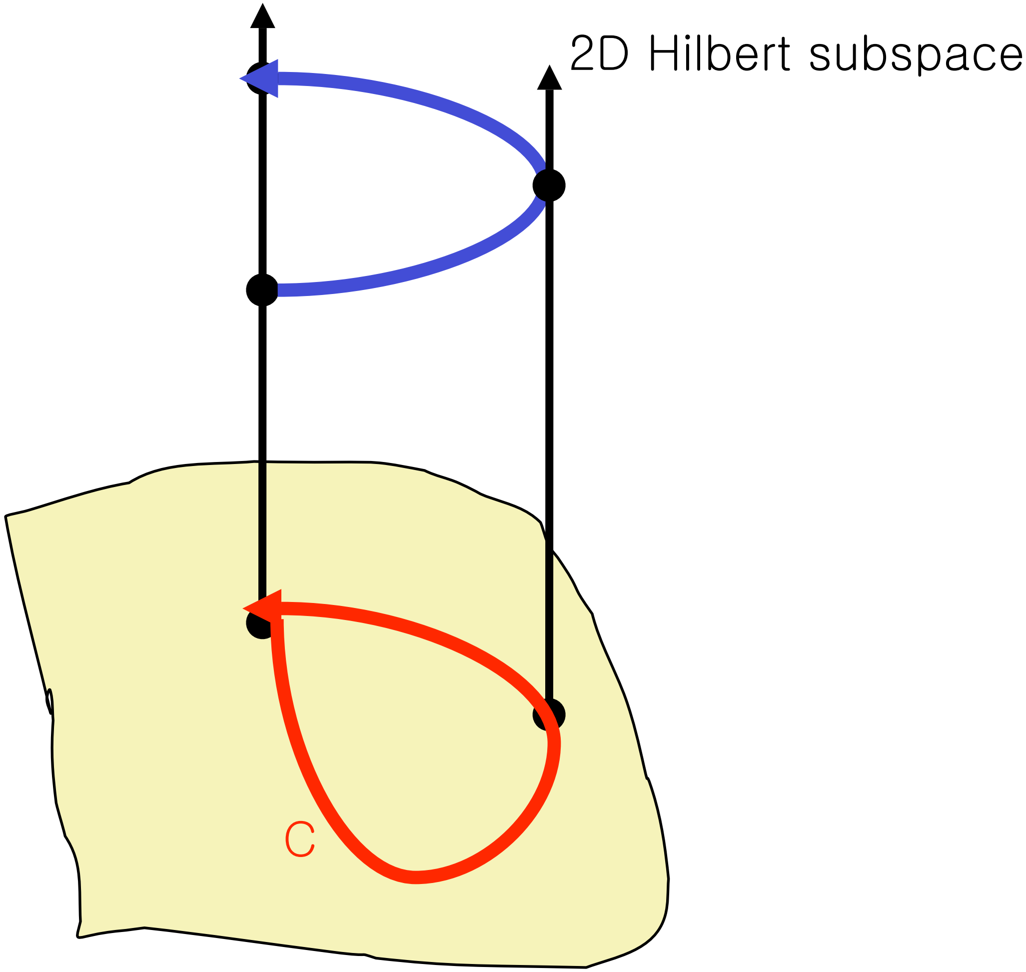
Anyon is a fractional charge with a flux tube attached



Non-Abelian statistics of anyons of the Moore-Read state: degenerate states

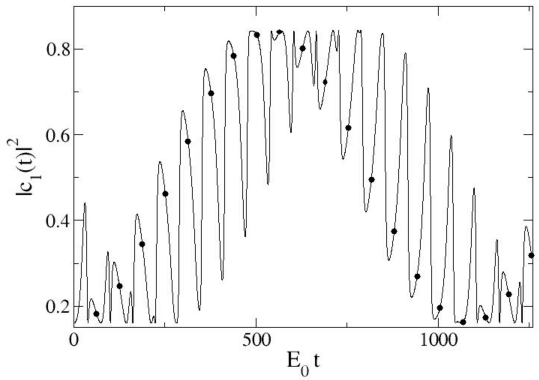


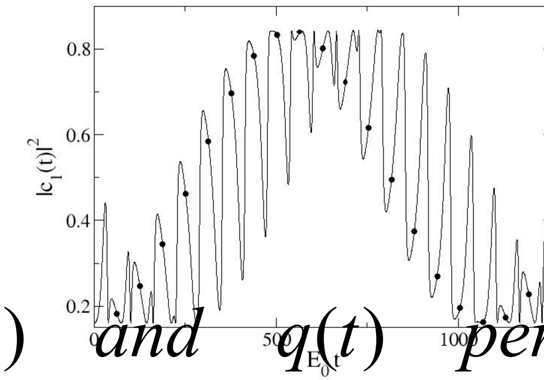




Wilczek and Zee ‘84

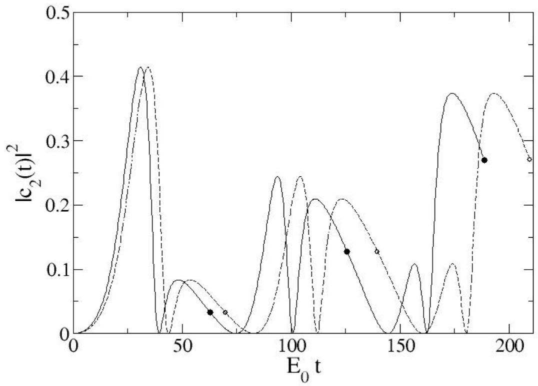
Basis transformation in degenerate Hilbert subspace:





$$p(t) \quad and \quad q(t) \quad periodic$$

$$c_1(nT) = P(0)e^{i\omega nT} + Q(0)e^{-i\omega nT}$$



$$\Phi_C(n) = \begin{pmatrix} A\sin(\alpha n) + e^{-i\alpha n} & -B^*\sin(\alpha n) \\ B\sin(\alpha n) & A^*\sin(\alpha n) + e^{i\alpha n} \end{pmatrix}$$

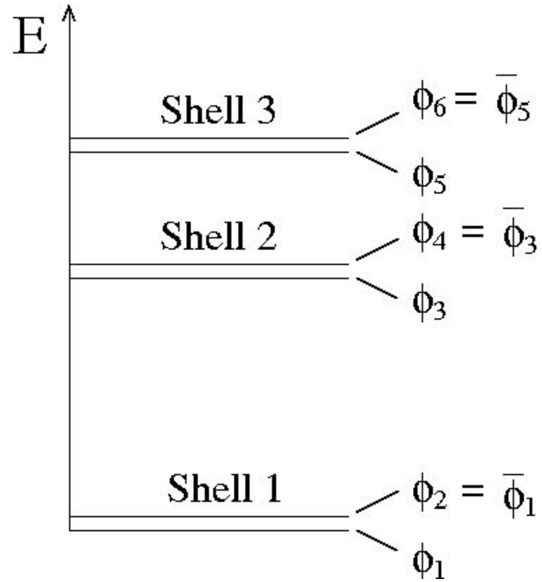
$$\ddot{c}_1 + p(t)\dot{c}_1 + q(t)c_1 = 0$$

- Large gauge transformation & time reversal:

$$\begin{pmatrix} F_{\uparrow}(\varphi) \\ F_{\downarrow}(\varphi) \end{pmatrix} \ \text{®} \ e^{i2k\varphi} \begin{pmatrix} F_{\uparrow}(\varphi) \\ F_{\downarrow}(\varphi) \end{pmatrix} \ \text{®} \ e^{-i2k\varphi} \begin{pmatrix} -F_{\downarrow}^{*}(\varphi) \\ F_{\uparrow}^{*}(\varphi) \end{pmatrix}$$

$$\Theta_i = c_1 \Psi_1 + c_2 \bar{\Psi}_1, \quad \left\langle \Psi_1 \,|\, \mathrm{H} \,|\, \bar{\Psi}_1 \right\rangle = 0$$

Ψ_1 and $\bar{\Psi}_1$ are eigenstates!



- Can be written in terms of intra-shell and inter-shell single electron non-Abelian vector potentials.

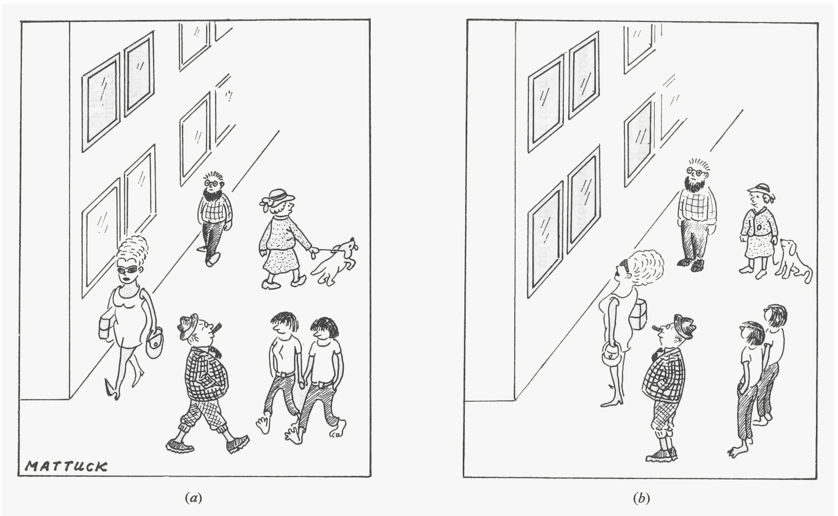
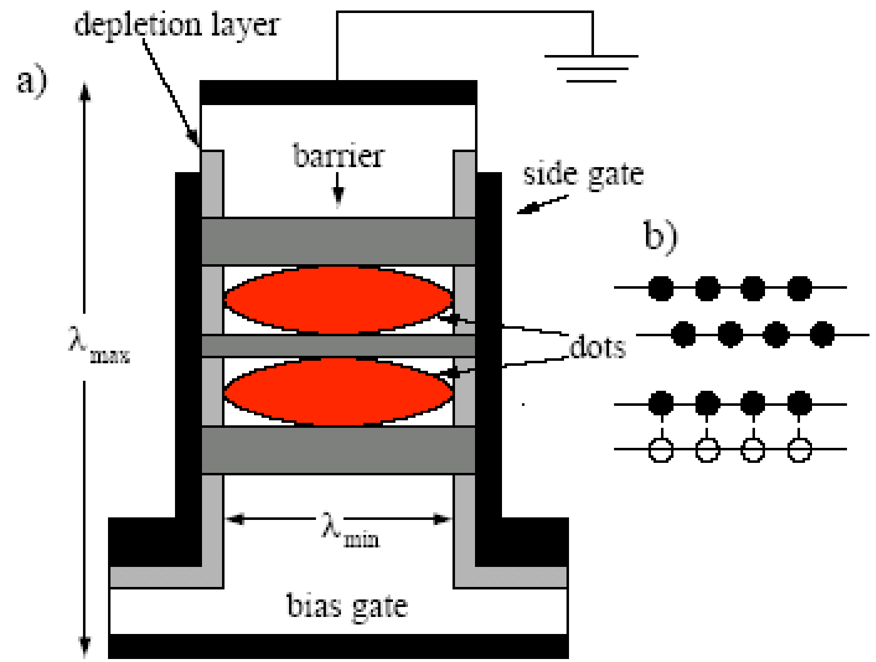


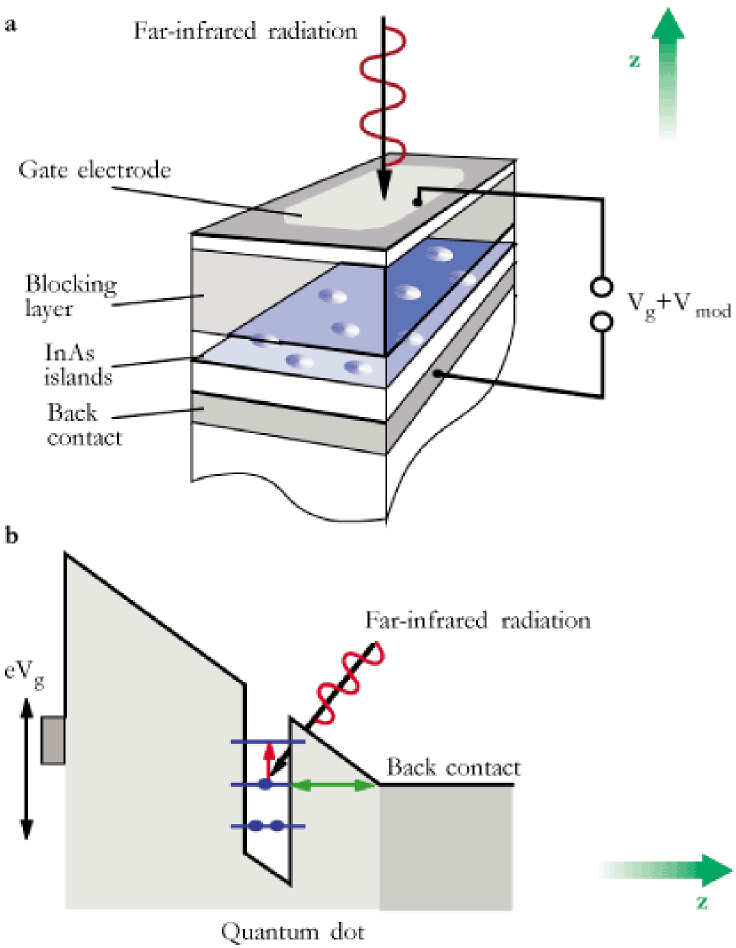
Fig. 17.2. '*Staring Crowd' Analogue of Phase Transition. (a) 'Normal' Phase. (b) 'Condensed' Phase*

Quantum control by decoherence-free subspaces

- Qubits protected by condensate with interlayer phase coherence.
- Matrix Berry phase.



- Why semiconductor quantum dots.
- Coherent control of electron spin electrically.
- Matrix Berry phase and holonomy.
- Quantum ring and large gauge transformation.
- General structure of matrix Berry phase in doubly degenerate subspace.
- Strongly correlated quantum dot.
- Topological quantum computing using non-Abelian statistics.



$$(A_P)_{i,j} = i\left\langle \Theta_i \middle| \frac{\partial \Theta_j}{\partial \lambda_P} \right\rangle = i\left\langle \Psi_i \middle| \frac{\partial \Psi_j}{\partial \lambda_P} \right\rangle$$

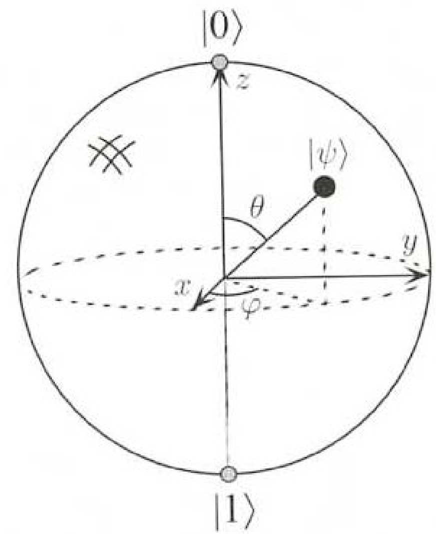
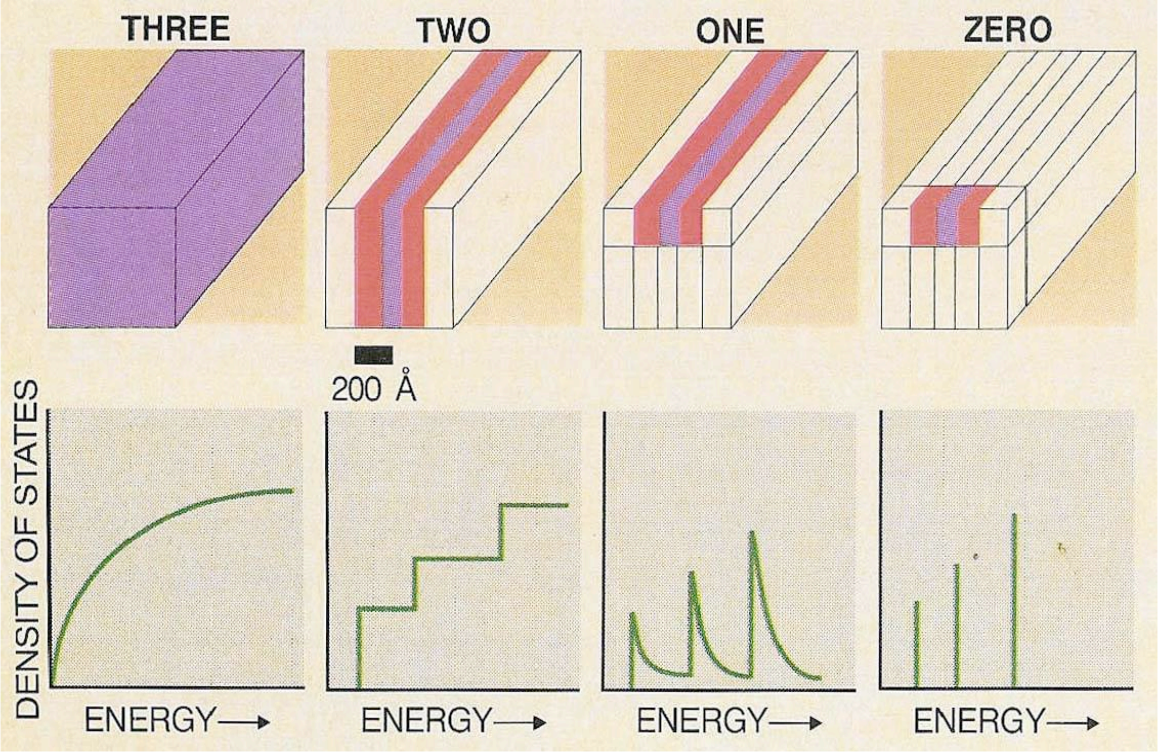


Figure 1.3. Bloch sphere representation of a qubit.

1.Non-Abelian Berry phases, semiconductor quantum dots



- We propose a new possibility: adiabatic electric control via matrix Berry phase.

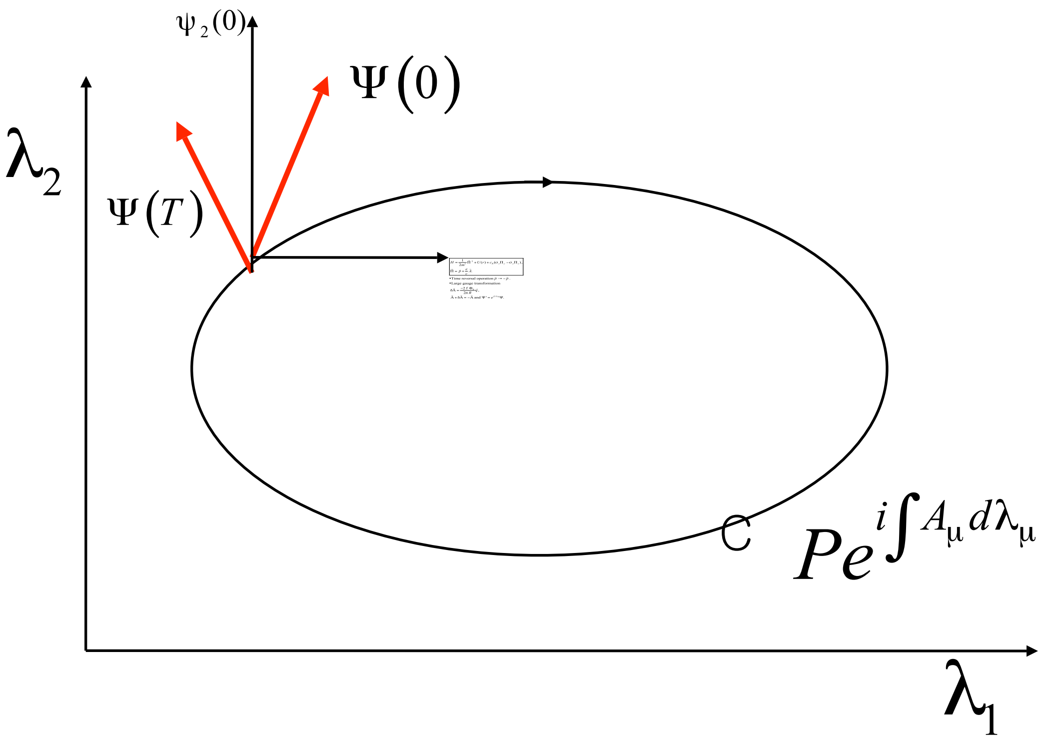
- Applies to all II-VI and III-V semiconductor quantum dots.

2.How to implement coherent control of electron spins in n-type semiconductor quantum dots?

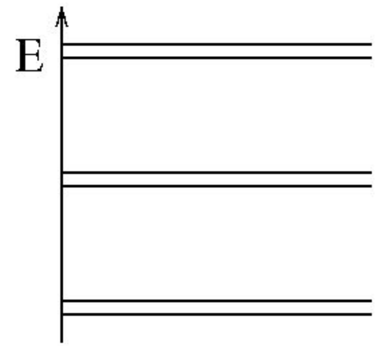
- Single electron transistor.
- Photons, time dependent magnetic field, etc have been used.
- We propose a new possibility: adiabatic electric control via matrix Berry phase.
- Applies to all II-VI and III-V semiconductor quantum dots.
- Spintronics & quantum information technology.

Strongly correlated
and
non-perturbative

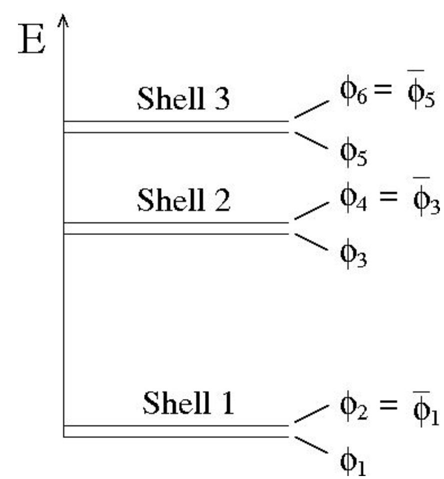
- S.-R. Eric Yang and N.Y. Hwang, Phys. Rev. B, 73, 125330 (2006).
- S.-R. Eric Yang, Phys. Rev. B, 74, 075315 (2006).
- S.-R. Eric Yang, cond-mat/0701318.
- S.-R. Eric Yang and N.Y. Hwang, preprint.
- More papers to follow.



$$Pe^{i\int A_\mu d\lambda_\mu} \approx e^{iA_\mu(t_N)d\lambda_\mu(t_N)} \ldots\ldots e^{iA_\mu(t_1)d\lambda_\mu(t_1)}$$



(c)



$$\Phi_C(n) = \begin{pmatrix} \cos(\alpha n) + i\sqrt{1-4|\beta|^2}\,\sin(\alpha n) & 2i\beta\,\sin(\alpha n) \\ 2i\beta^*\sin(\alpha n) & \cos(\alpha n) - i\sqrt{1-4|\beta|^2}\,\sin(\alpha n) \end{pmatrix}$$

1.Single Electron Transistors

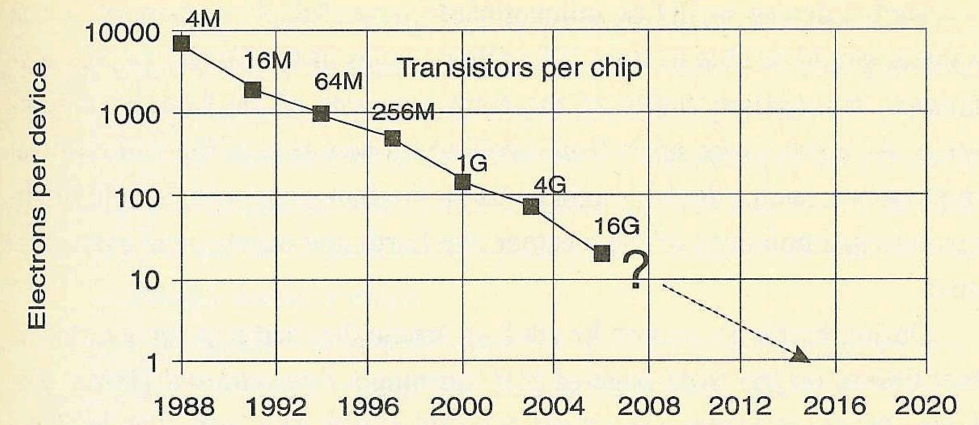


Figure 1.3. Number of electrons per transistor

Switching logic states in today's silicon chips involves the movement of hundreds of electrons, but the number is rapidly diminishing with time. The labels 4M to 16G refer to memory chips ranging from 4 million to 16 billion bits.

New electric control of spins in dots

- Spintronics
- Holonomic unitary transformation

- S.-R. Eric Yang and N.Y. Hwang, Phys. Rev. B, 73, 125330 (2006).
- S.-R. Eric Yang, Phys. Rev. B, 74, 075315 (2006).
- S.-R. Eric Yang, cond-mat/0701318.
- S.-R. Eric Yang and N.Y. Hwang, preprint.

- Adiabatic electric control of spin is possible in II-VI & III-V semiconductor quantum dots.
- May lead to new applications in spintronics and quantum computing.