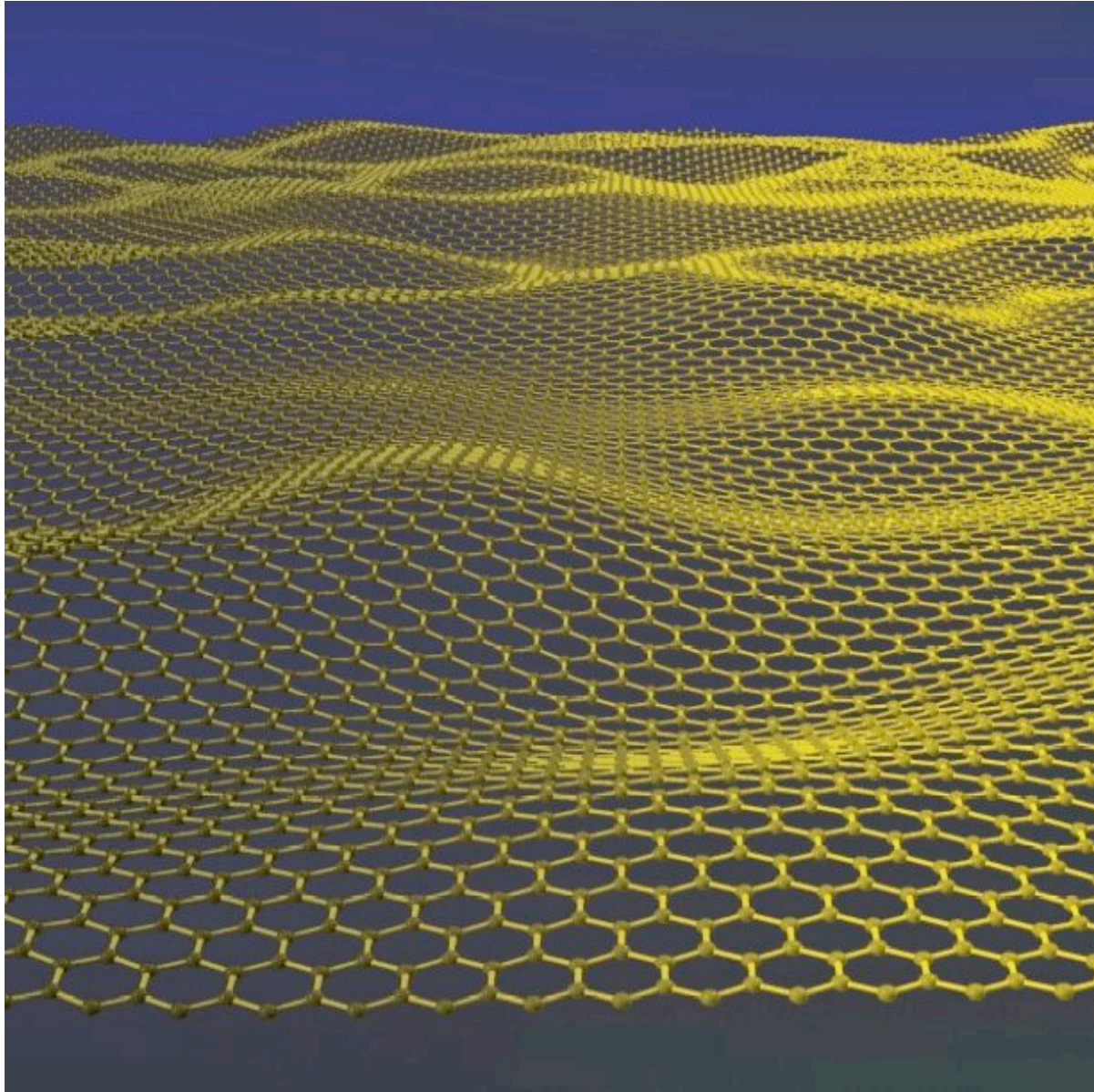
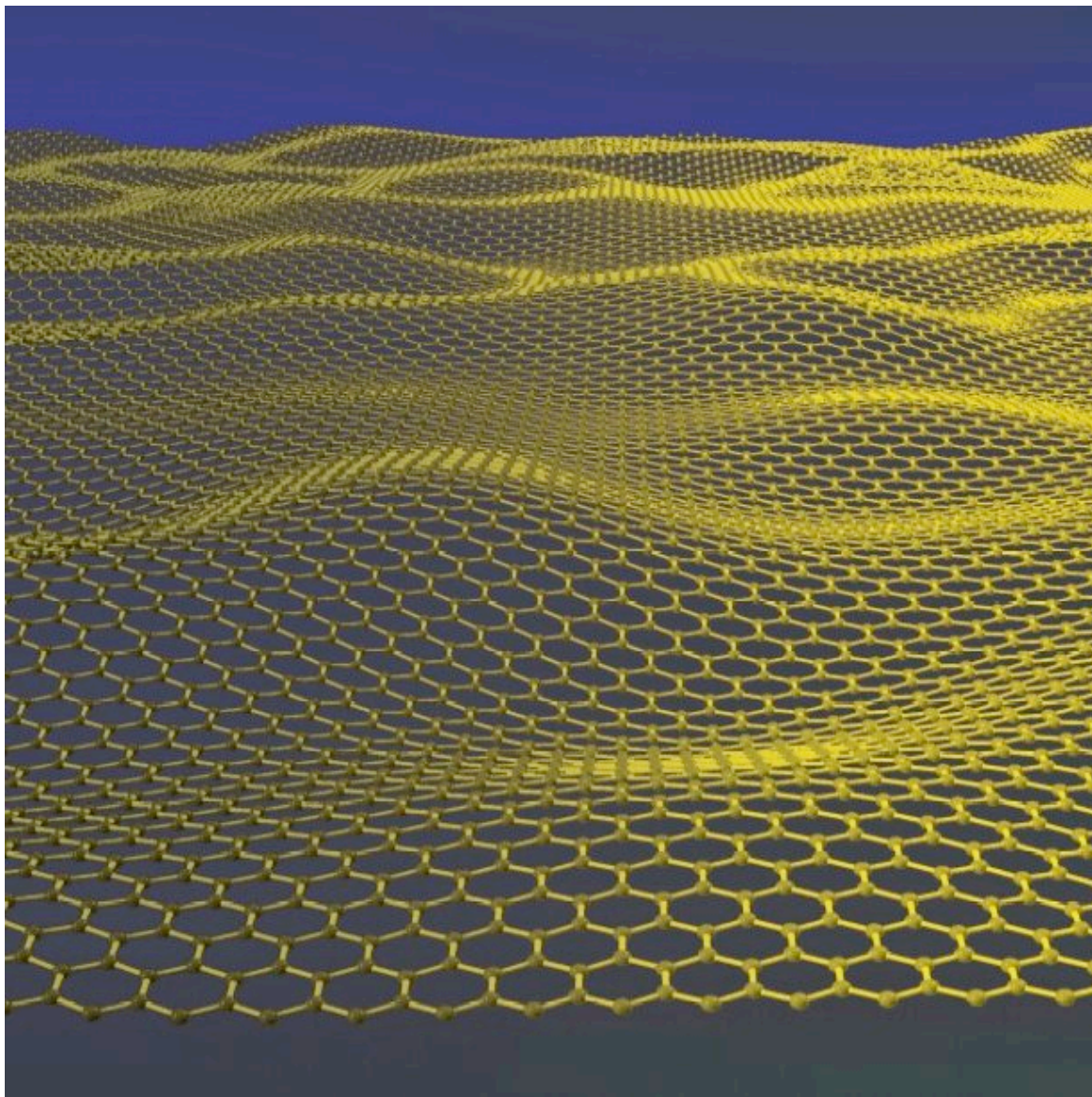


Carbon Flatland



Not your father's 2DEG



Socrates: Shall we set down astronomy among the subjects of study?

Glaucon: I think so, to know something about the seasons, the months and the years is of use for military purposes, as well as for agriculture and for navigation.

Socrates: It amuses me to see how afraid you are, lest the people should accuse you of recommending useless studies.

Socrates



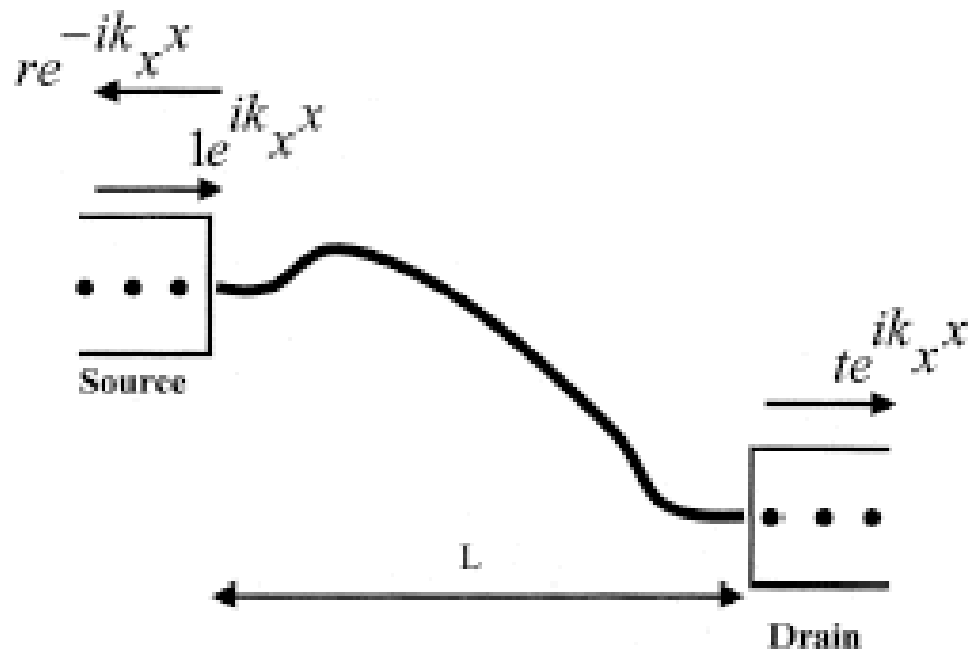
Glaucon



Semiconductor Roadmap



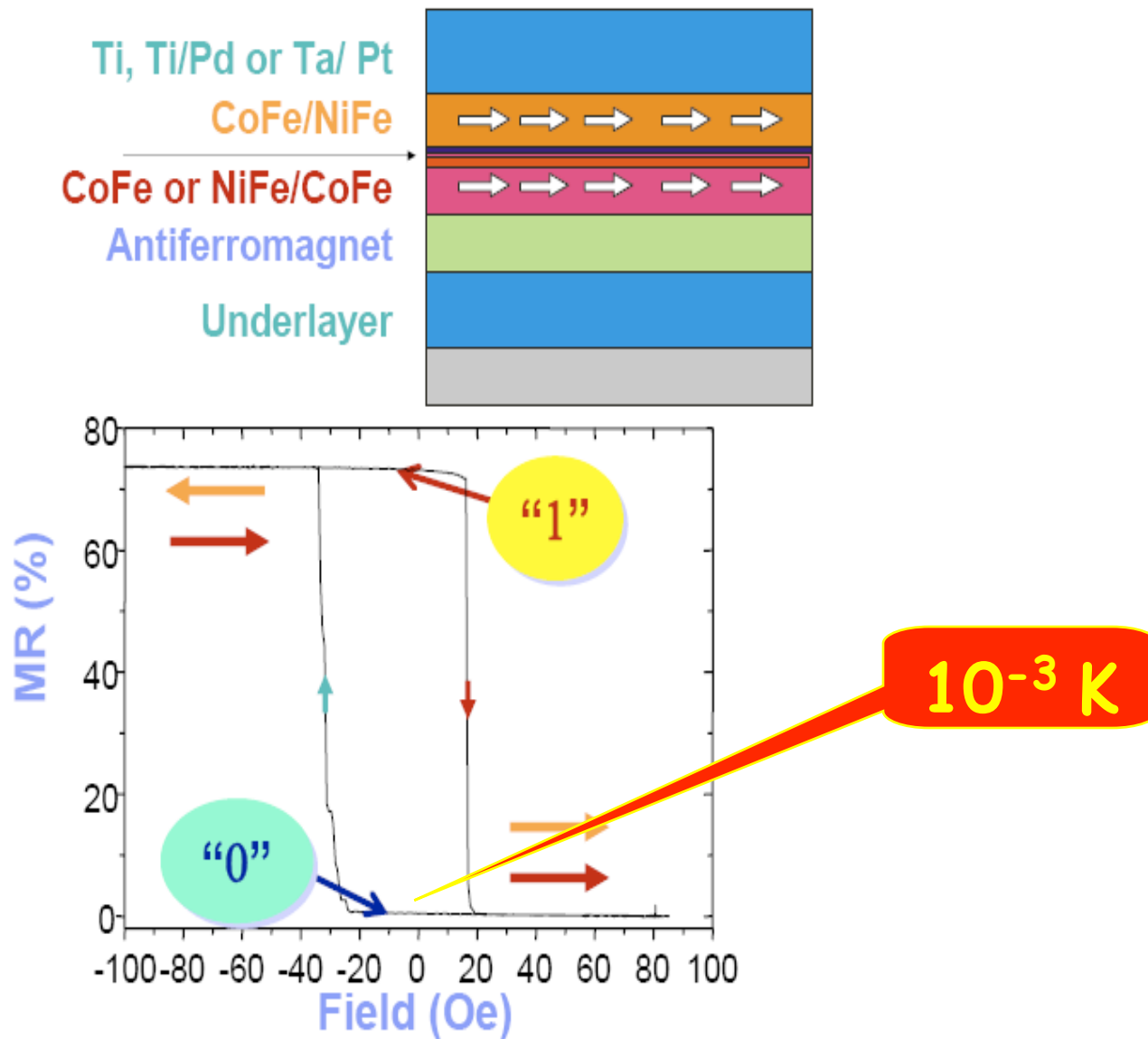
The Collective FET vision



$$I \sim I_{\text{sat}} (1 - \exp(-eV/k_B T))$$

$$eV/k_B T > \text{about } 10$$

The Collective FET vision



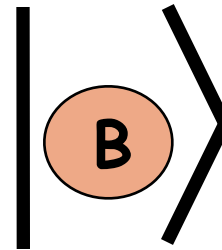
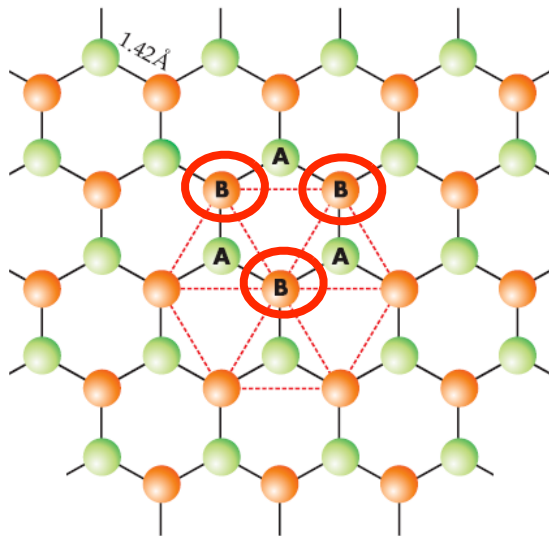
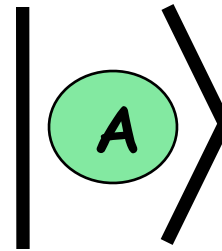
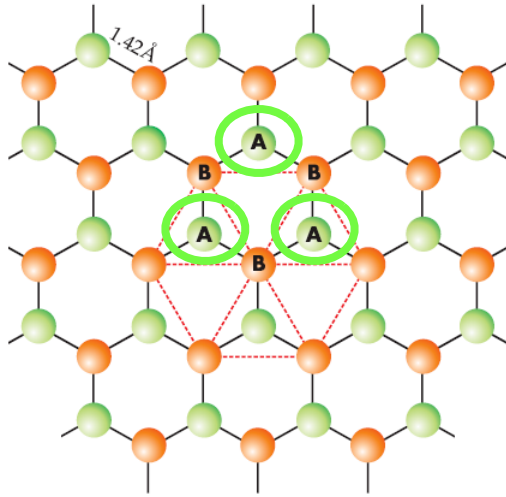
Pseudospins & Chirality

Bilayer Pseudospin Ferromagnetism

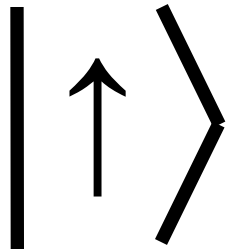
Bilayer Quantum Hall Effect

Chirality in Multilayers

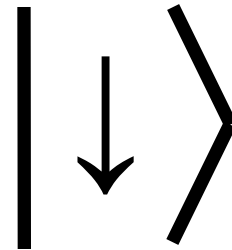
Pseudospins - Graphene



Electron Spins



spin-up



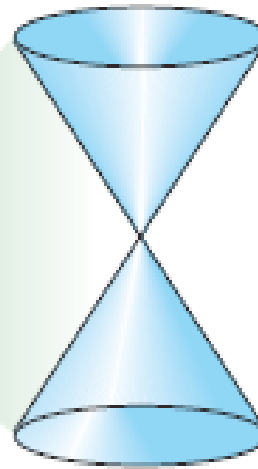
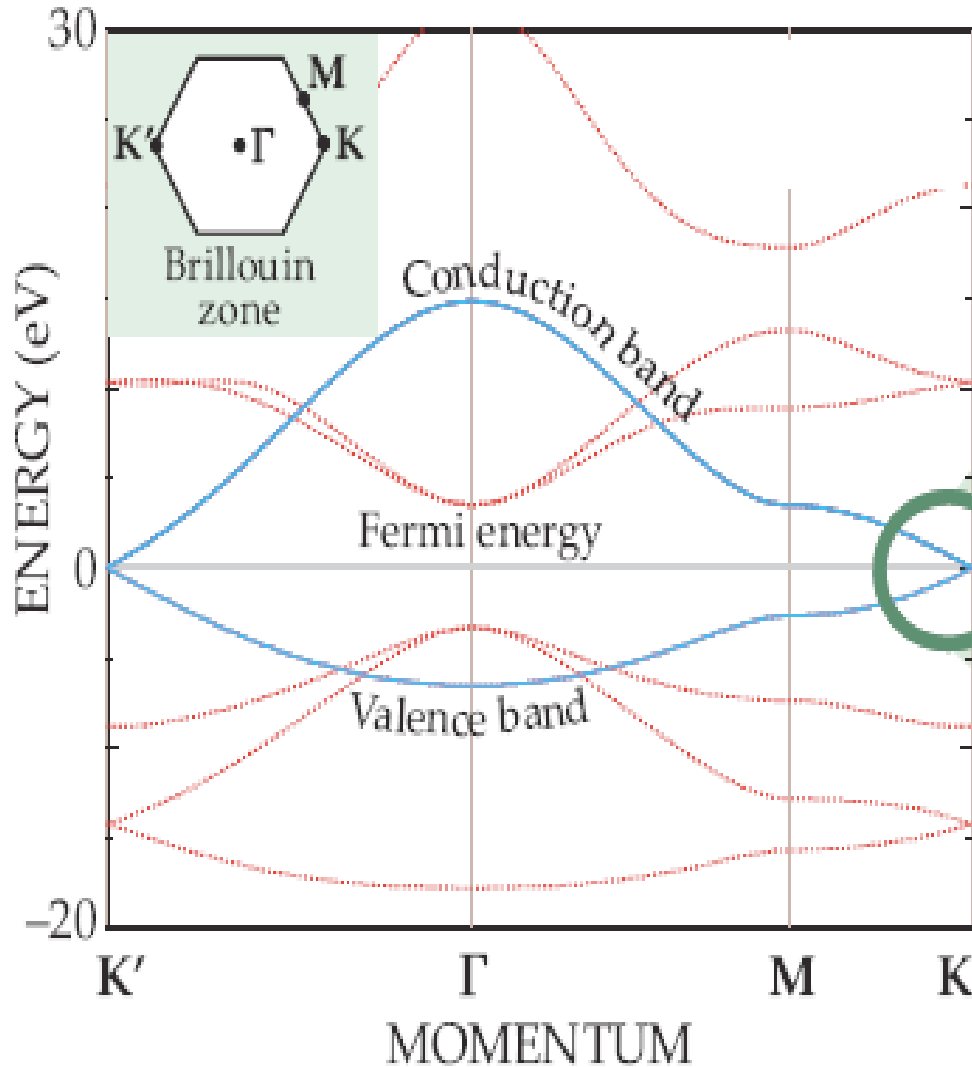
spin-down

Zeeman doublet

`two-valued quantum degree of freedom'

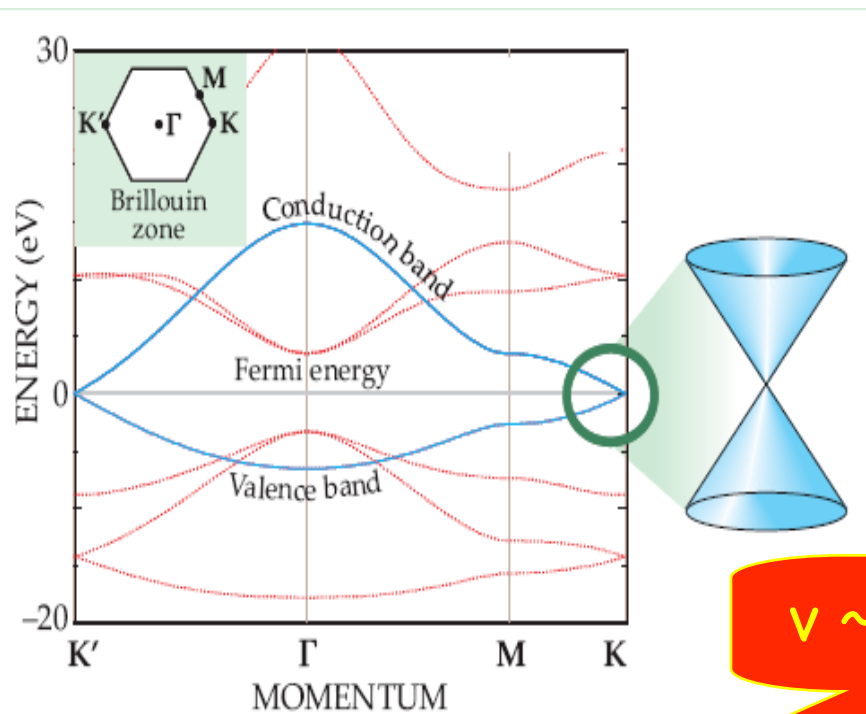
... Compton, Pauli, Goudsmit, Uhlenbech ...

flatland band structure



Phil Wallace
- Physical Review -
1947

2D Massless Dirac Equation



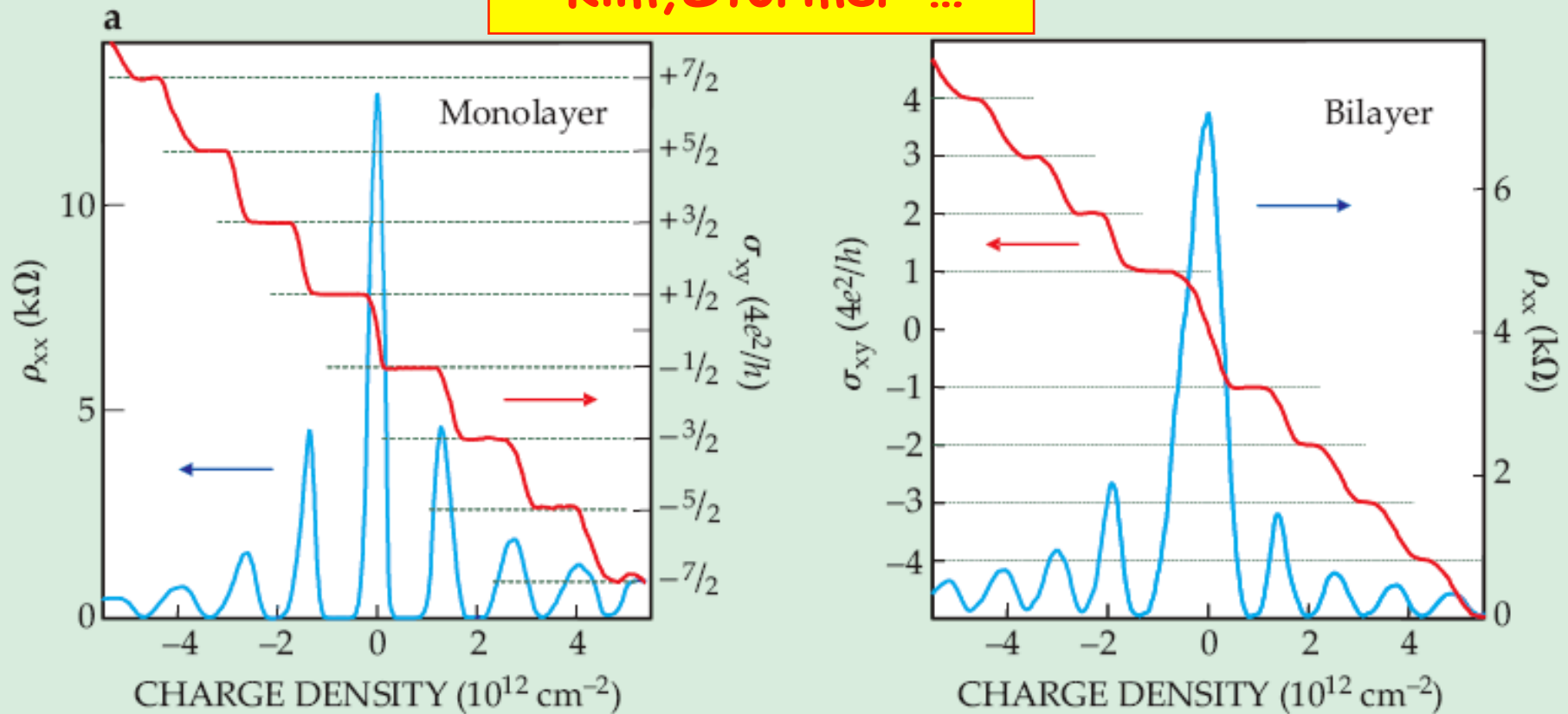
$$v \sim 100\text{nm}/(10^{-13}\text{ s})$$

$$H = -v\mathbf{k} \cdot \boldsymbol{\tau}$$

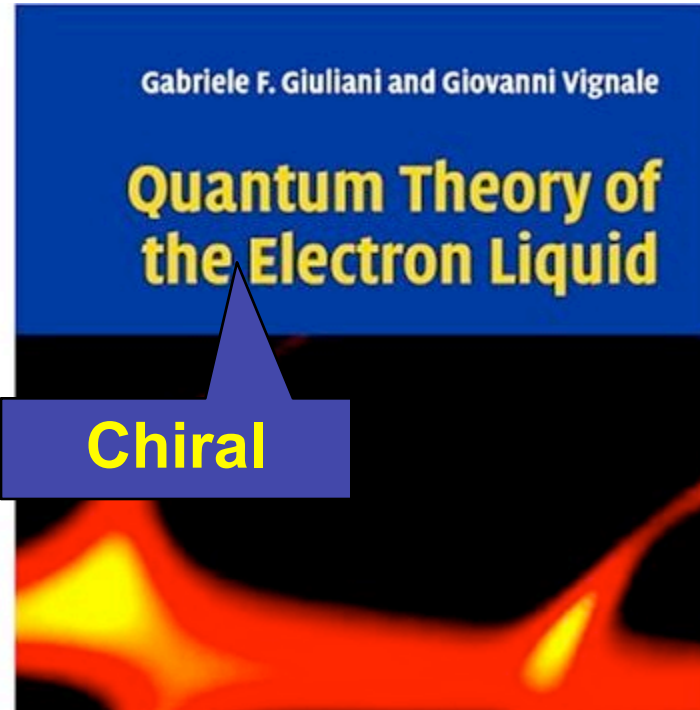
Andrey Geim and AHM - Physics Today - August 2007

Graphene Quantum Hall Effect

Geim, Novoselov ...
Kim, Stormer ...



Electron Gas Theory



$$k^2 \Rightarrow \sigma_x k_x + \sigma_y k_y$$

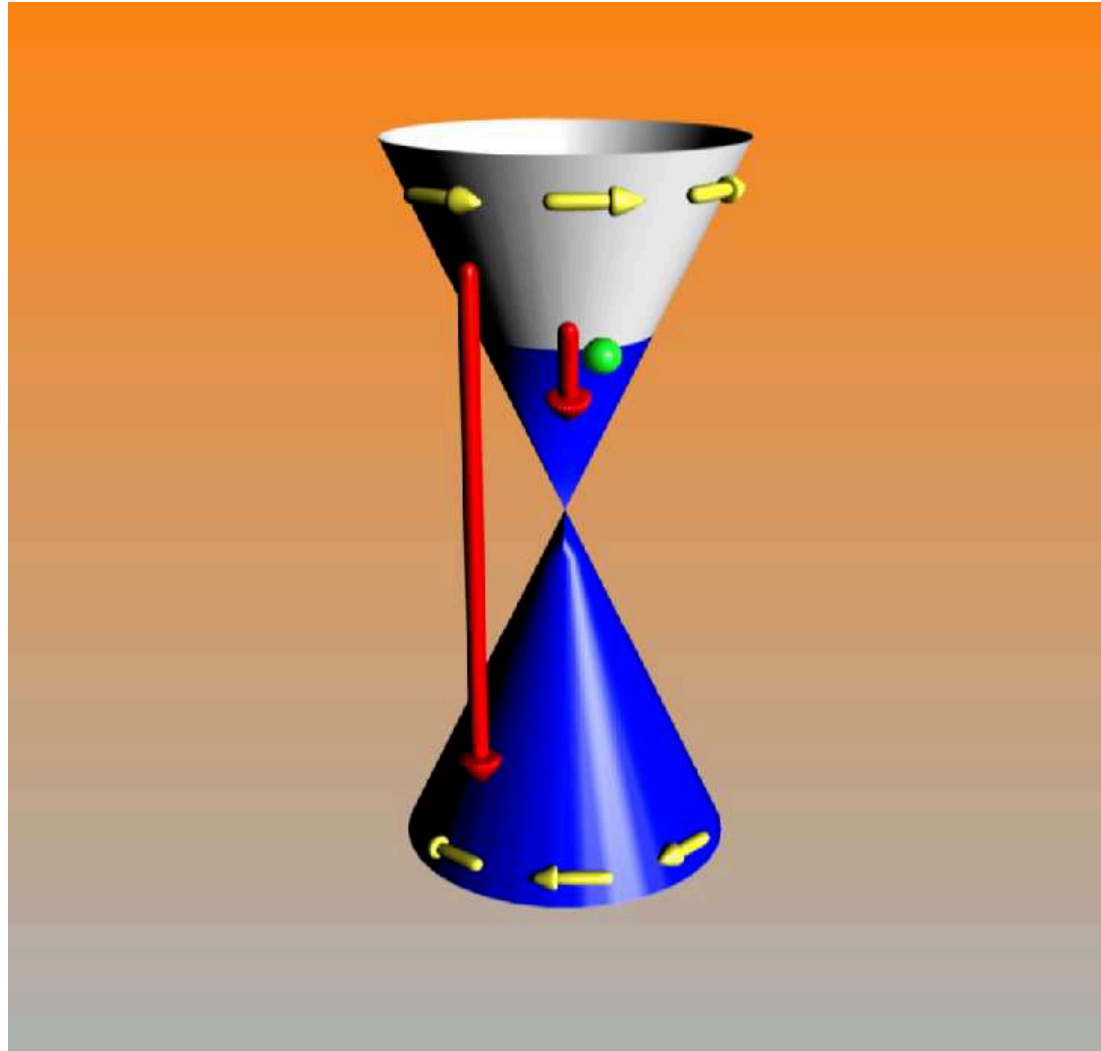
Electron-Electron Interactions



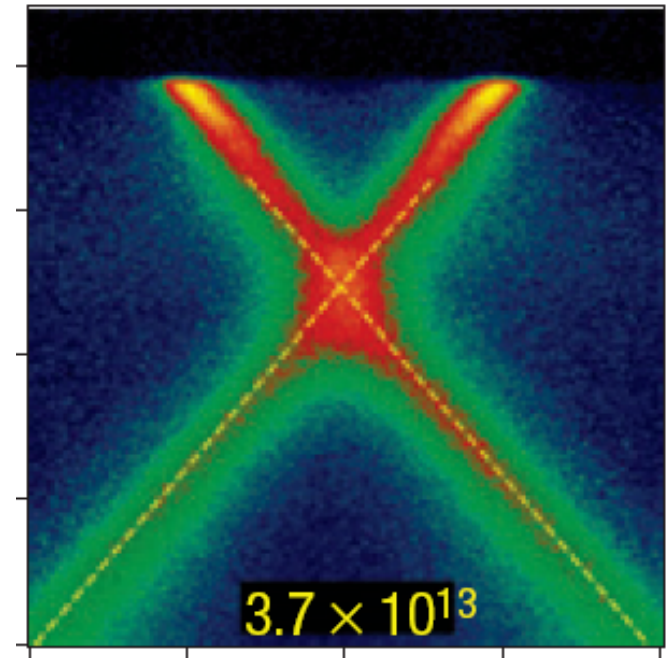
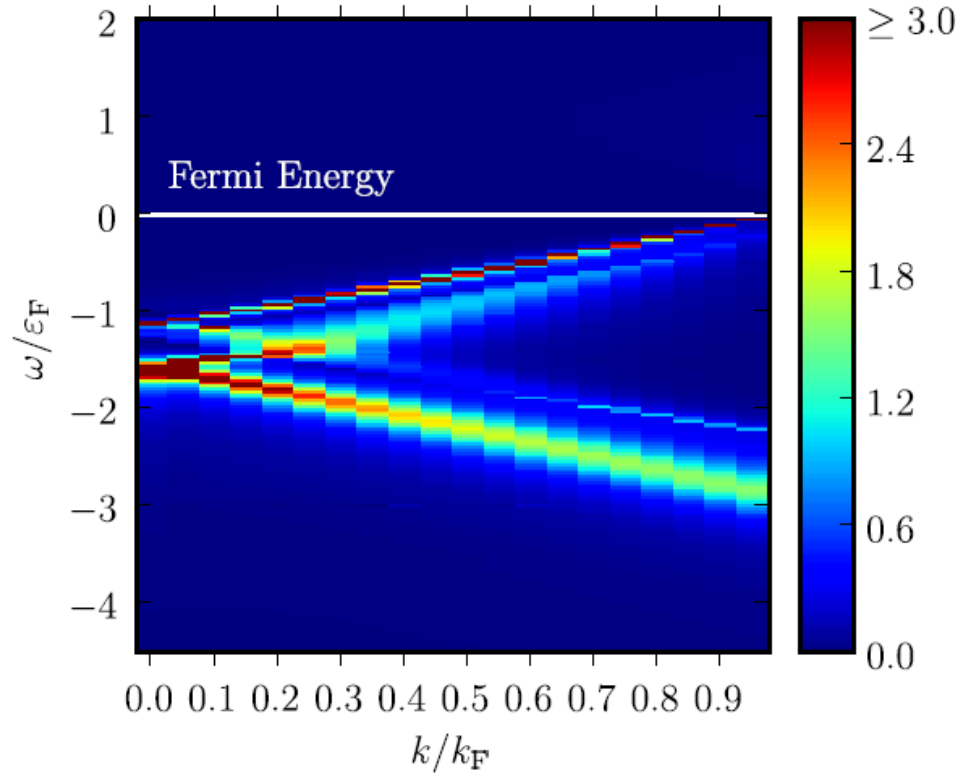
$$\alpha = e^2/\hbar c \approx 1/137$$

$$\alpha_{gr} = e^2/\epsilon\hbar v \approx 2$$

Chiral 2DES



ARPES



Theory
(Polini)

Experiment
(Rotenberg)

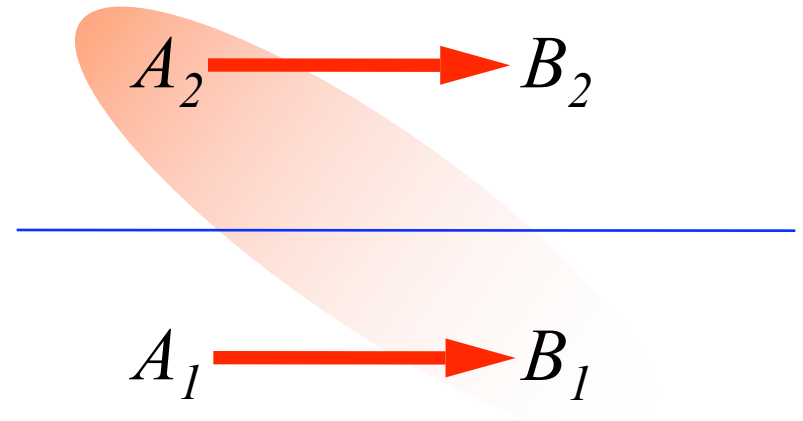
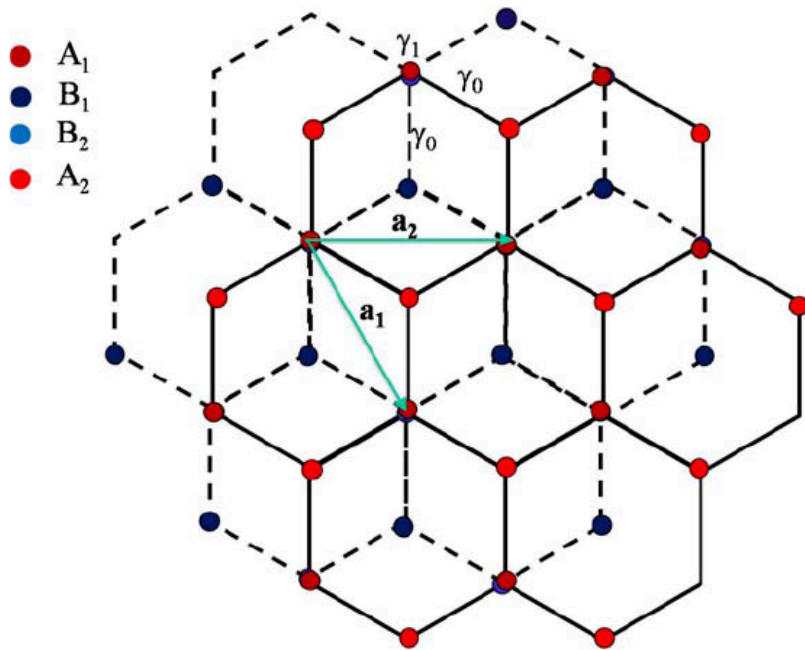
Pseudospins & Chirality

Bilayer Pseudospin Ferromagnetism

Bilayer Quantum Hall Effect

Chirality in Multilayers

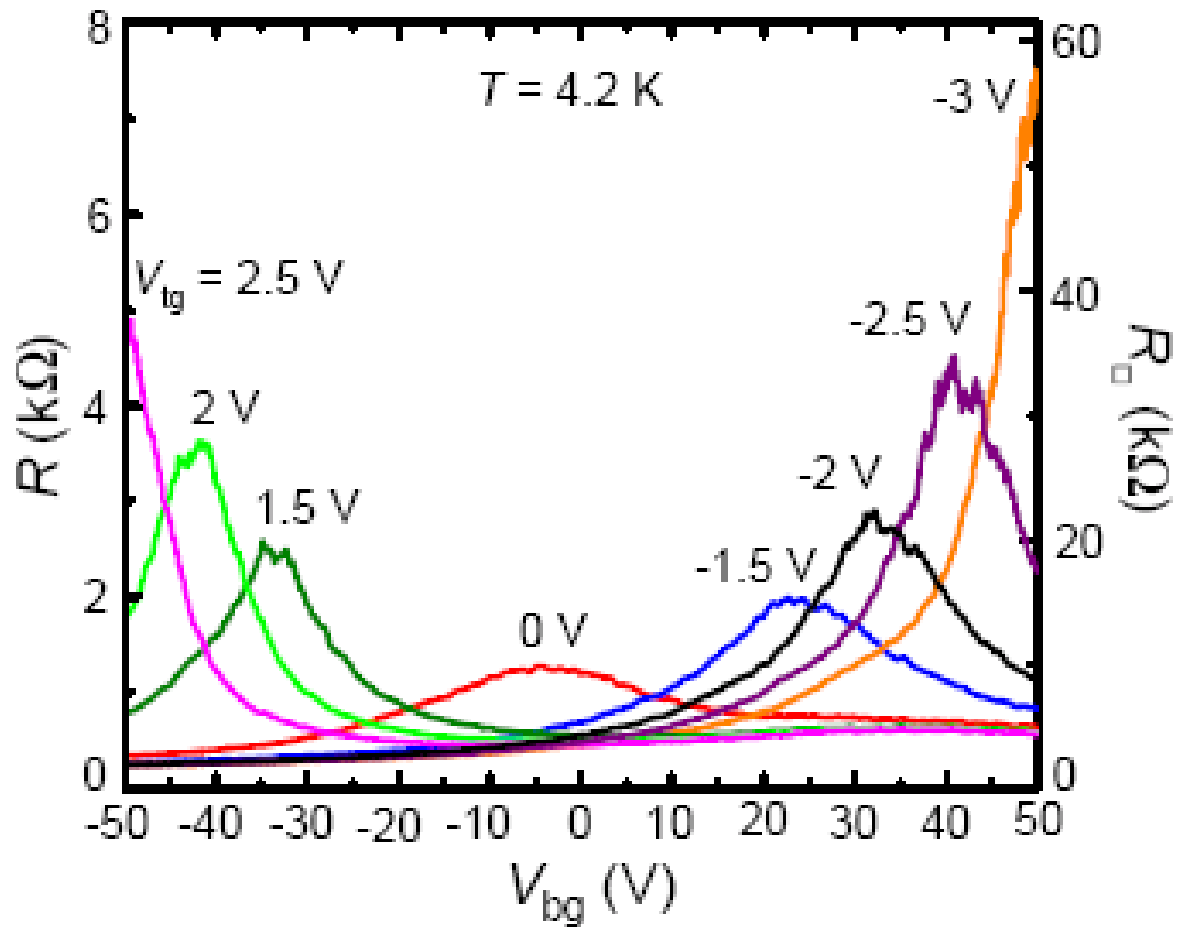
Bilayer AB stacking



$\pi = \pi_x + i\pi_y$
=perturbation

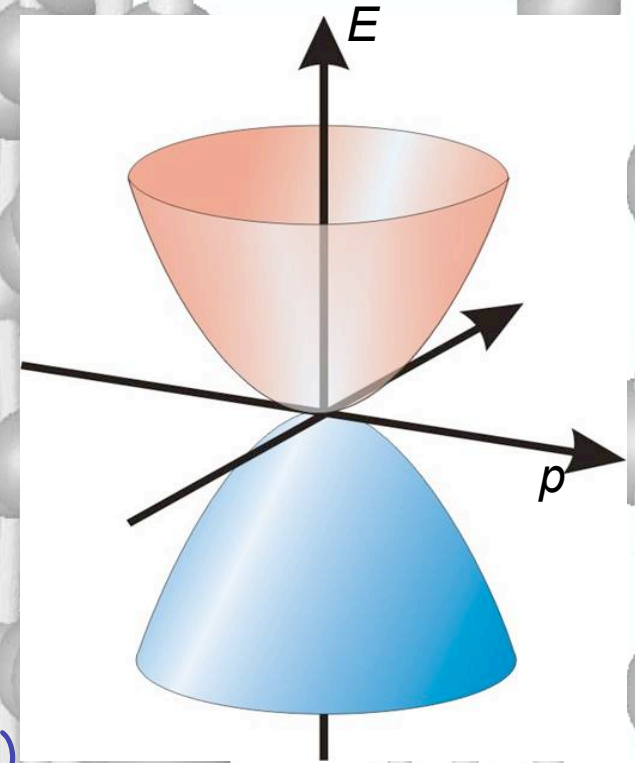
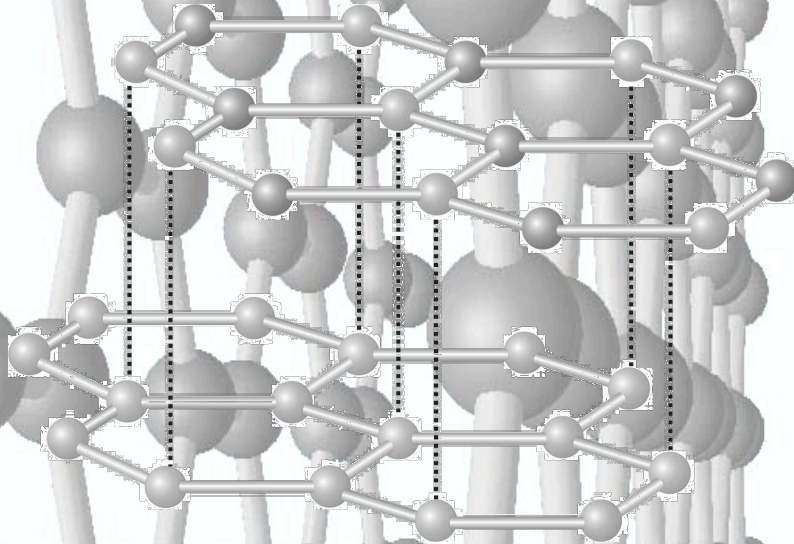
$$\mathcal{H}_{MD}(\mathbf{k}) = - \begin{pmatrix} 0 & v\pi^\dagger \\ v\pi & 0 \end{pmatrix} \begin{array}{l} |A\rangle \\ |B\rangle \end{array}$$

Bilayer Gaps



Vandersypen Delft

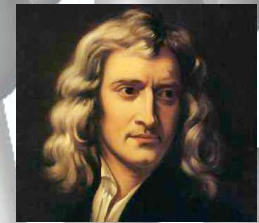
BILAYER GRAPHENE



McGann & Falko PRL 96 (2006)

Novoselov et al. Nature Phys 2, (2006)

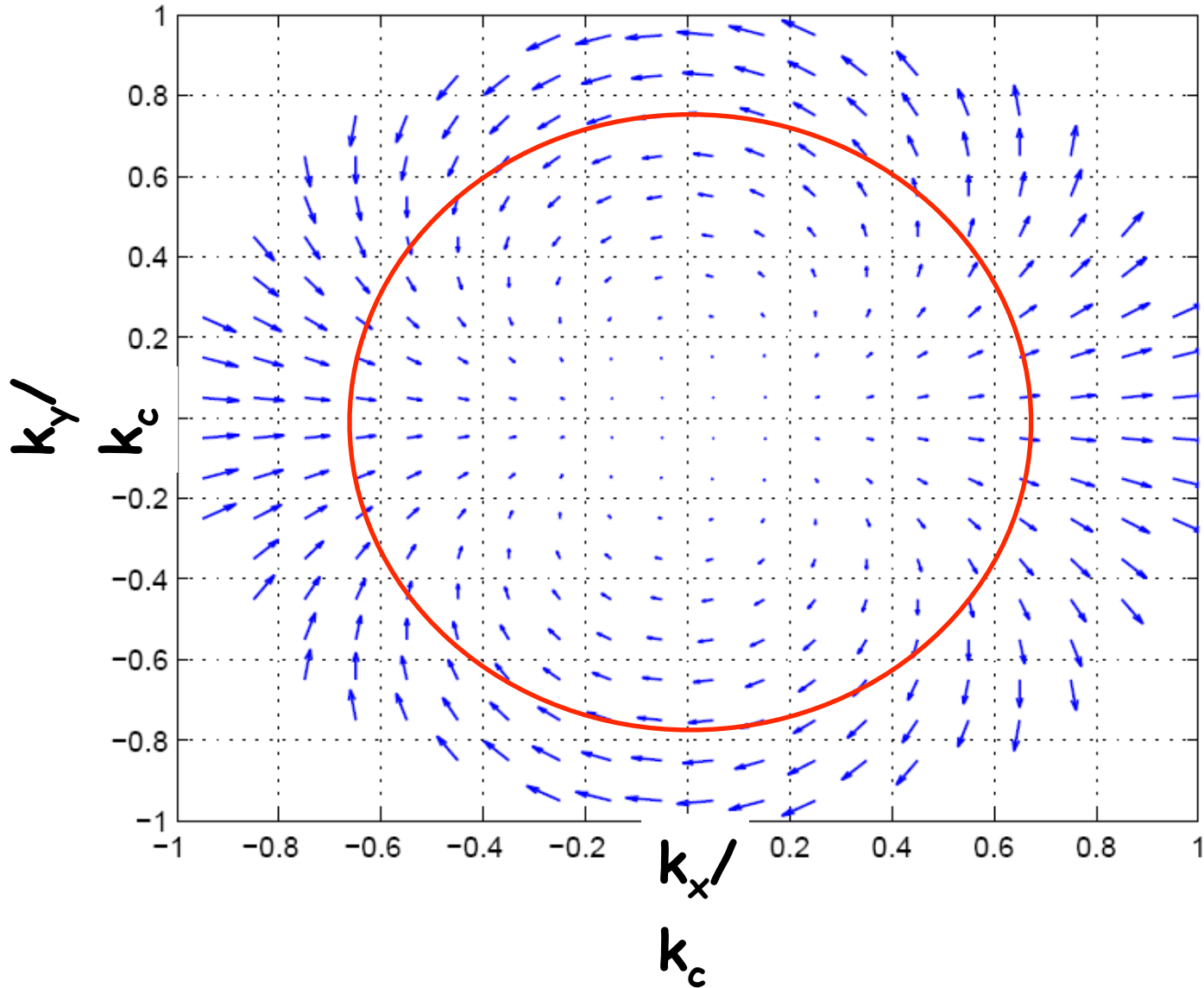
$$\vec{B}_{\text{band}}(\vec{k}) = \frac{\hbar^2 k^2}{2m^*} (\cos(2\phi_{\vec{k}}), \sin(2\phi_{\vec{k}}), 0)$$



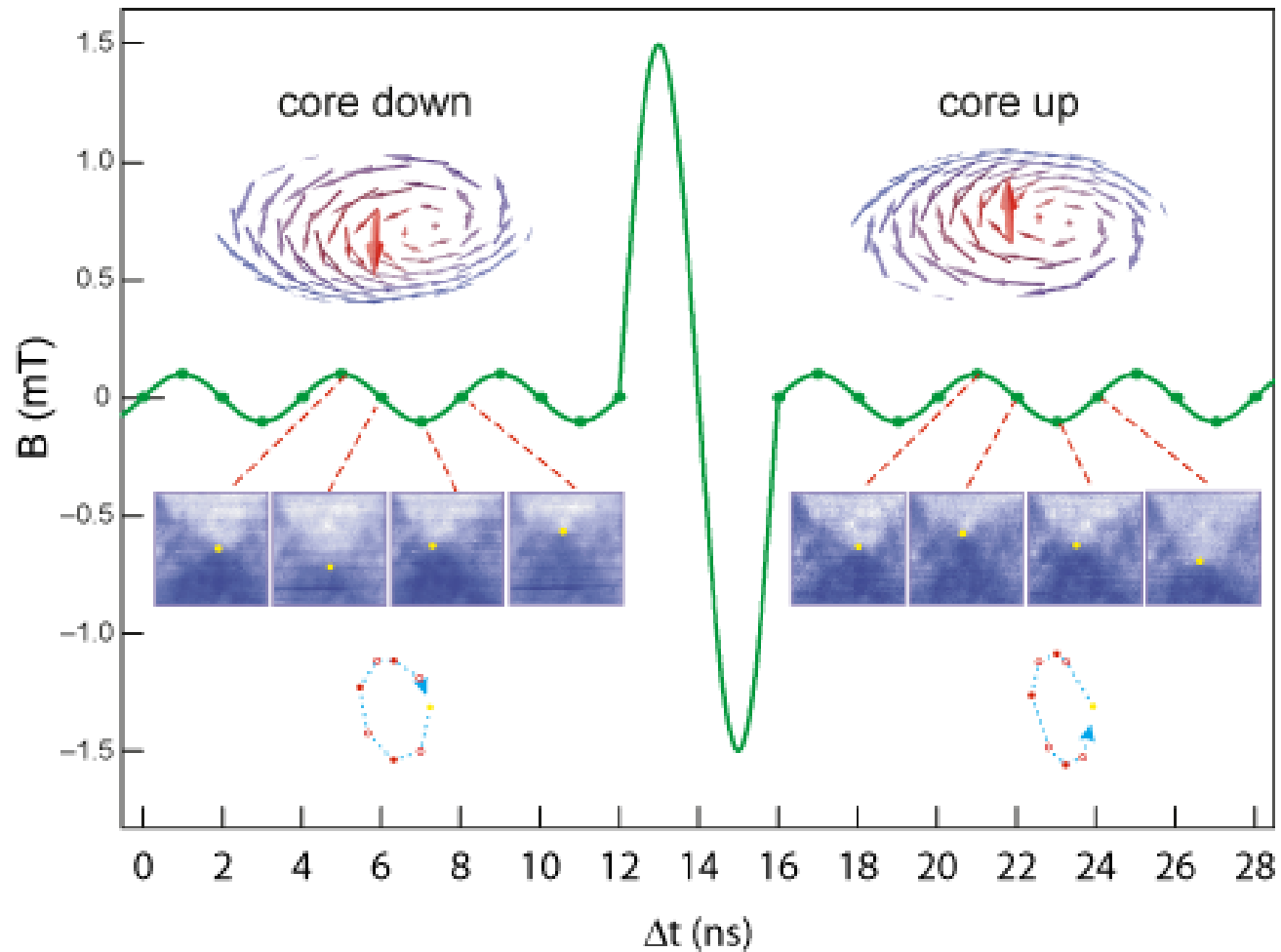
no hands!



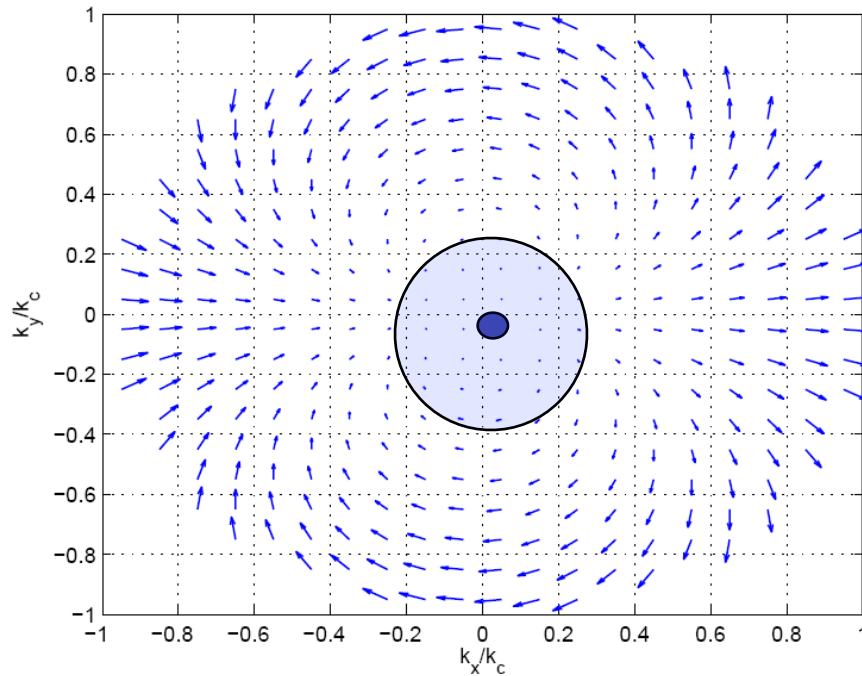
bilayer k-space vortex



Magnetic Vortex

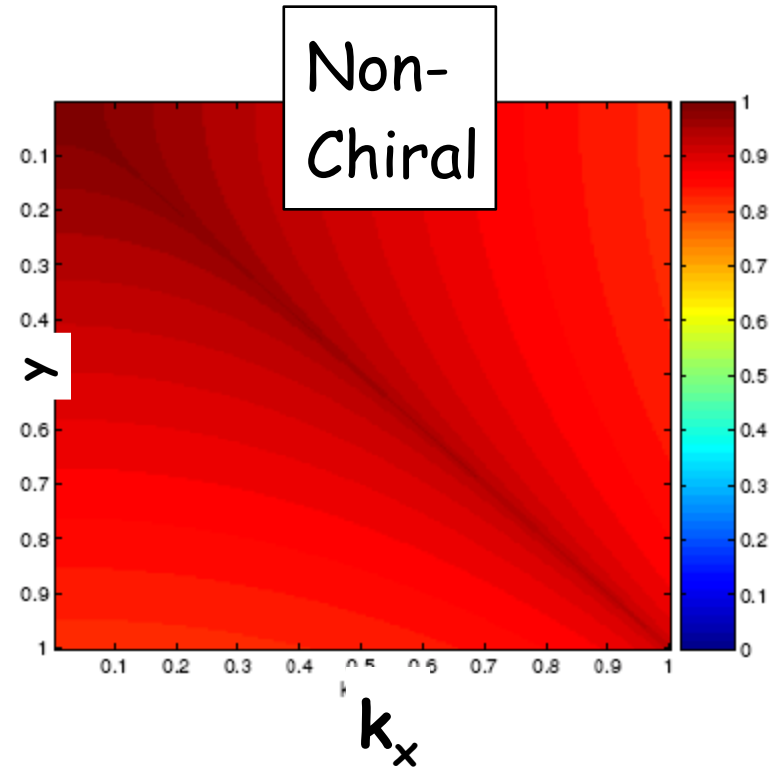
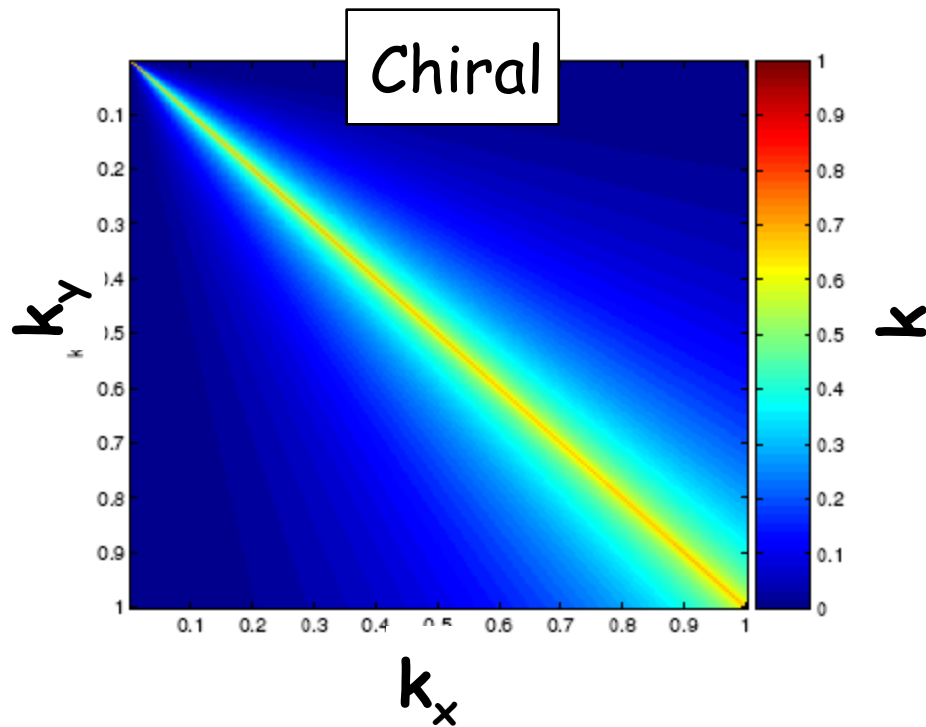


Pseudospin Exchange Fields



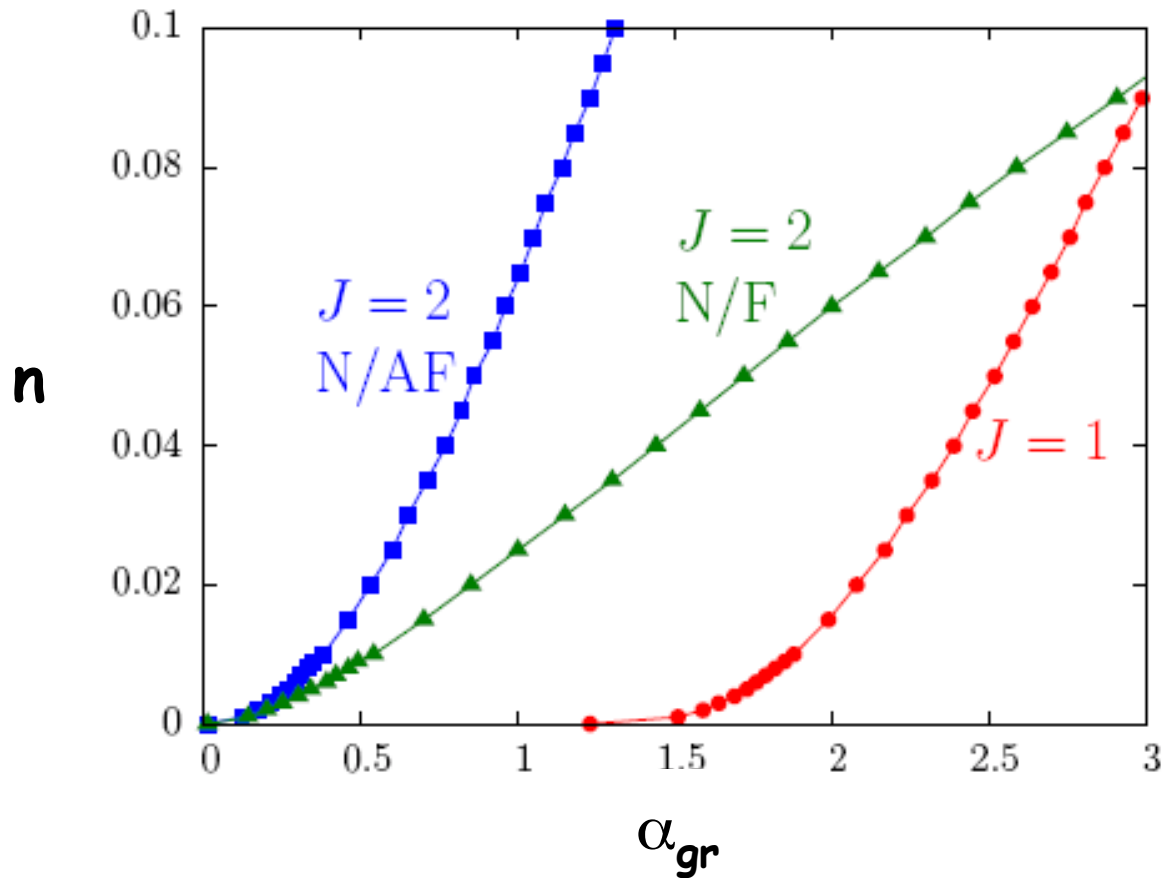
$$\vec{B}_x(\vec{k}) = \frac{1}{2A} \sum_{\vec{k}'} V(\vec{k} - \vec{k}') \hat{n}(\vec{k}')$$

Σ_{xy}/Σ_z - Bilayer



$$[\vec{B}_{\text{band}}(\vec{k}) + \vec{B}_x(\vec{k})] \times \hat{n}(\vec{k}) = 0$$

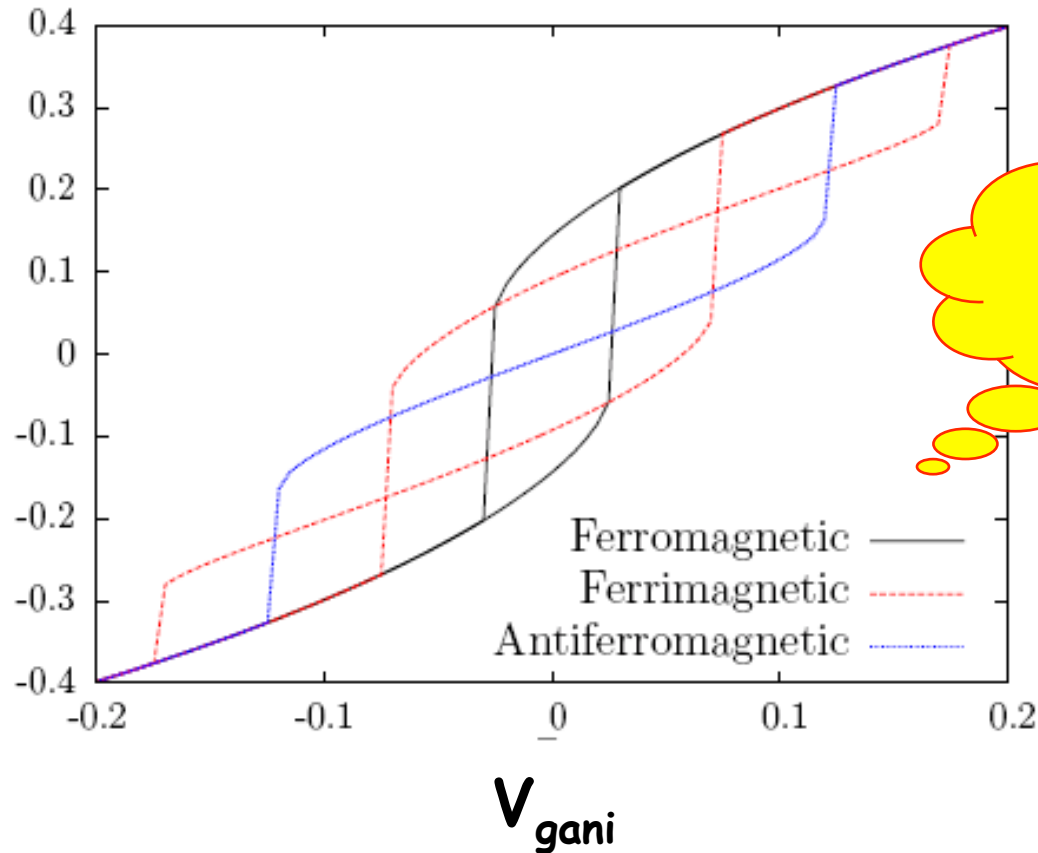
Phase Diagram



Min, Polini, Borghi, AHM - cond-mat/0707

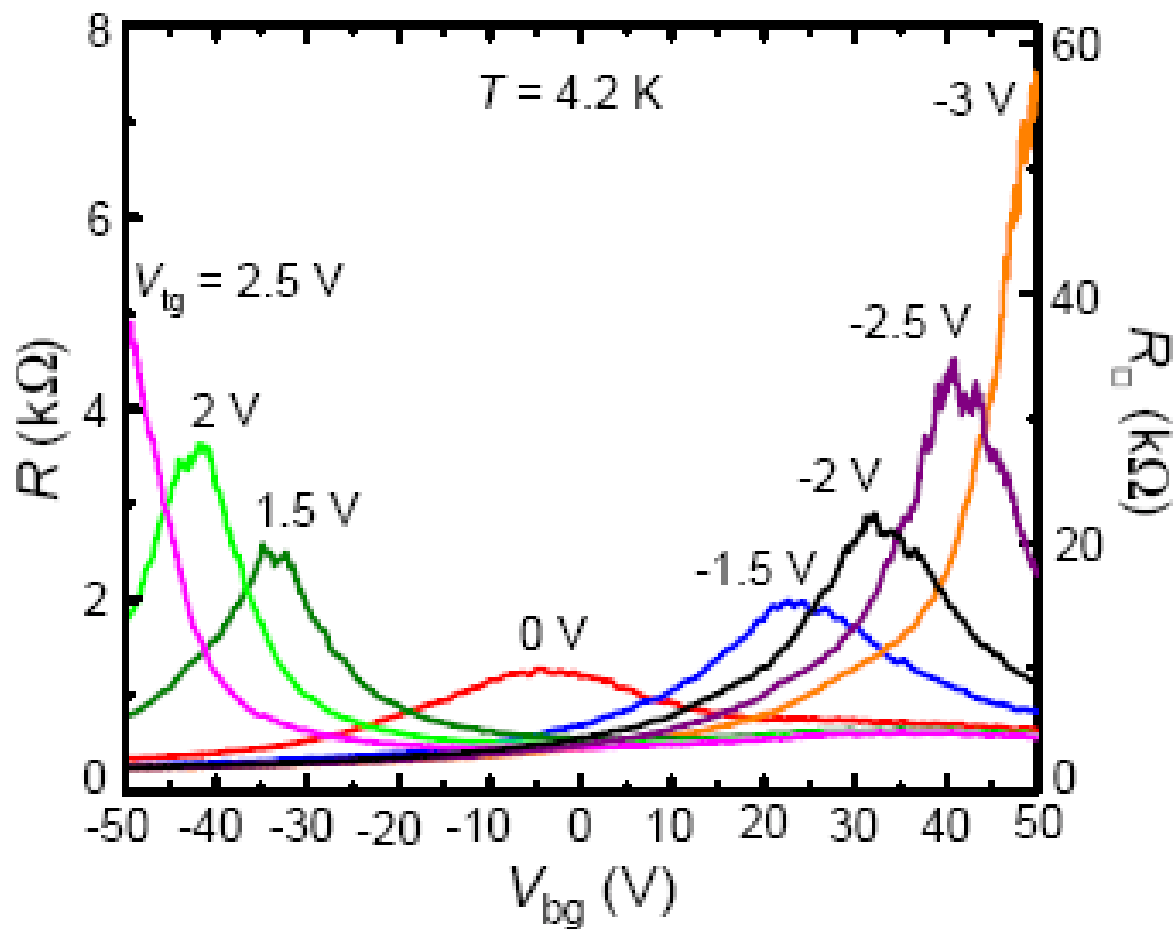
Gate Hysteresis

P
o
l
a
r
i
z
a
t
i
o
n



Collective
FET?

What about Lieven?



Vandersypen Delft


Pseudospins & Chirality

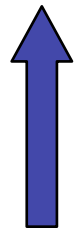
Bilayer Pseudospin Ferromagnetism

Bilayer Quantum Hall Effect

Chirality in Multilayers

N=4 Quantum Hall Ferromagnets

 = valley


 = spin

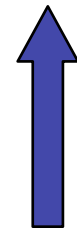
$$\mathcal{H}_{MD}(\mathbf{k}) = - \begin{pmatrix} 0 & v\pi^\dagger \\ v\pi & 0 \end{pmatrix} \begin{matrix} |A\rangle \\ |B\rangle \end{matrix}$$

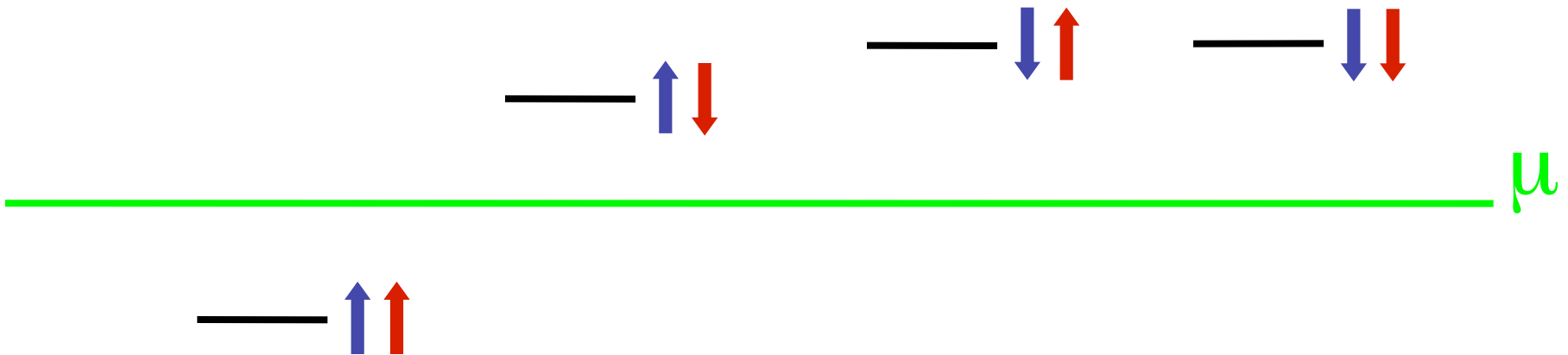


Levitov, Lee, Nomura, Alicea, Fisher, Balents, Sheng, Haldane
Columbia & Manchester

$$\nu = -1$$

 = valley

 = spin

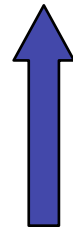


$$\nu =$$

0



= valley



= spin



μ

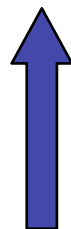


$\nu =$

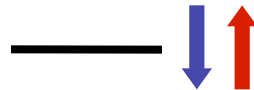
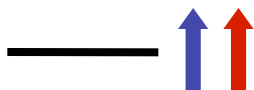
1





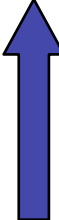
= valley



= spin



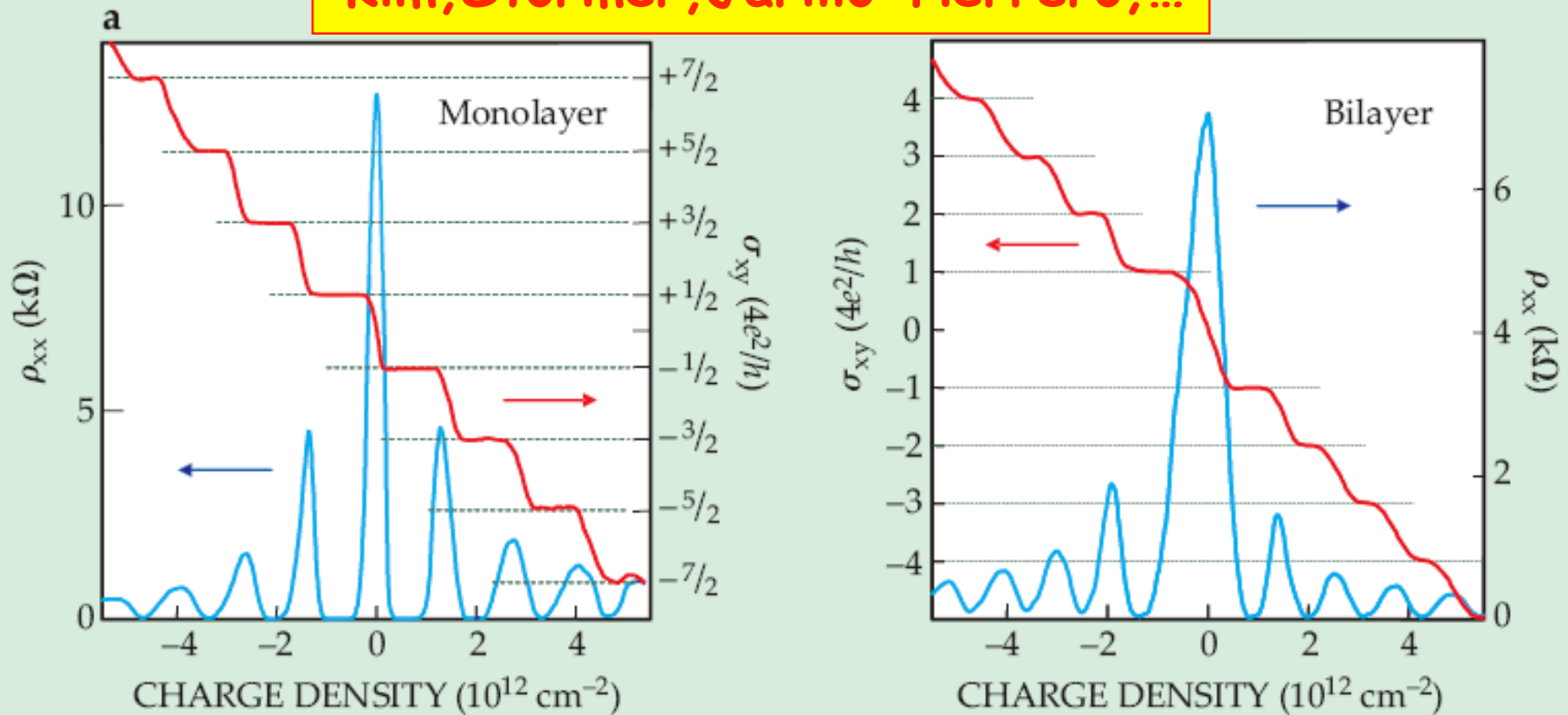
N=8 Quantum Hall Ferromagnets

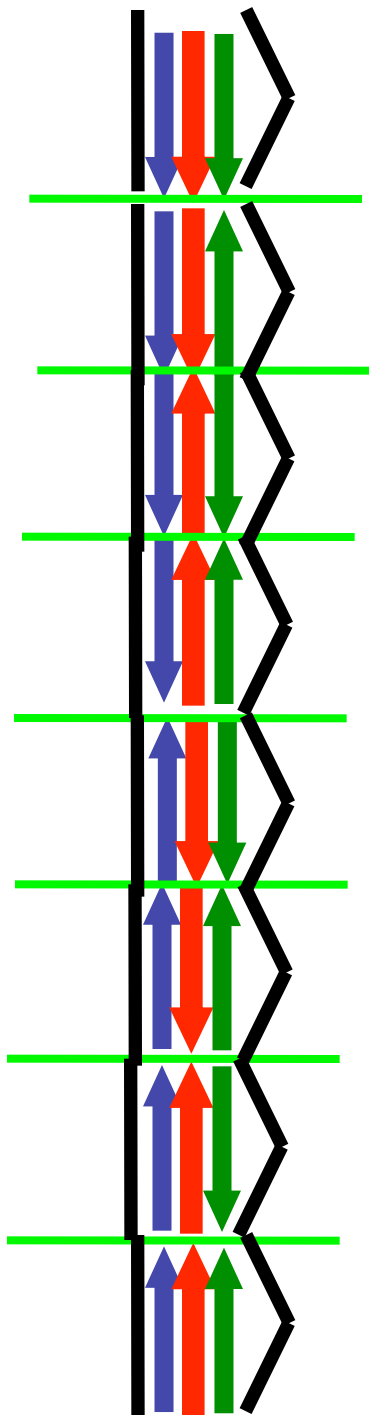
 = LL (0,1)  = layer  = spin

$$\mathcal{H} = -\hbar\omega_c \begin{pmatrix} 0 & a^2 \\ (a^\dagger)^2 & 0 \end{pmatrix} \begin{array}{l} | \text{A} \rangle \\ | \text{B} \rangle \end{array}$$

Graphene Quantum Hall Effect

Geim, Novoselov ...
Kim, Stormer, Jarillo-Herrero, ...





$$\nu = 3$$

$$\nu = 2$$

$$\nu = 1$$

$$\nu = 0$$

$$\nu = -1$$

$$\nu = -2$$

$$\nu = -3$$

Hunds Rules:

Spin

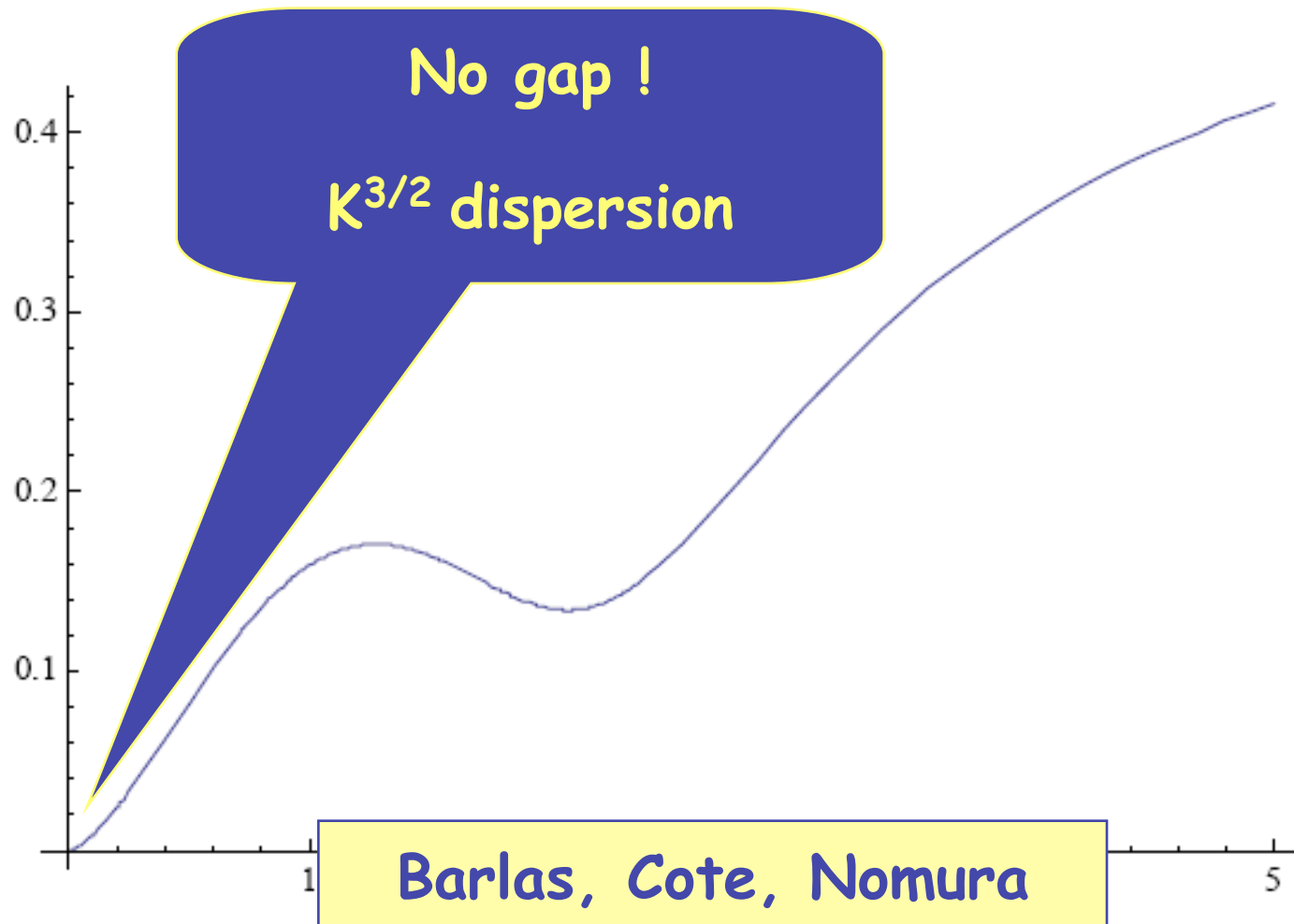
then

layer

then

LL

Bilayer $\nu = \text{odd}$ magnetorotons



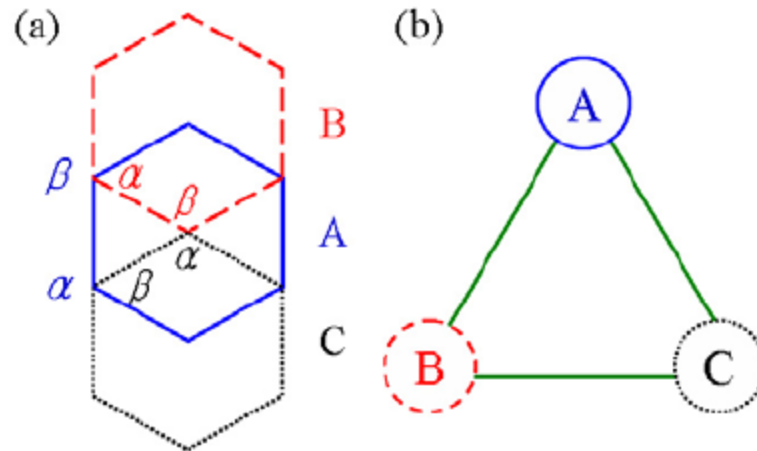
Pseudospins & Chirality

Bilayer Pseudospin Ferromagnetism

Bilayer Quantum Hall Effect

Chirality in Multilayers

Multilayer Stacking - A,B,C layers

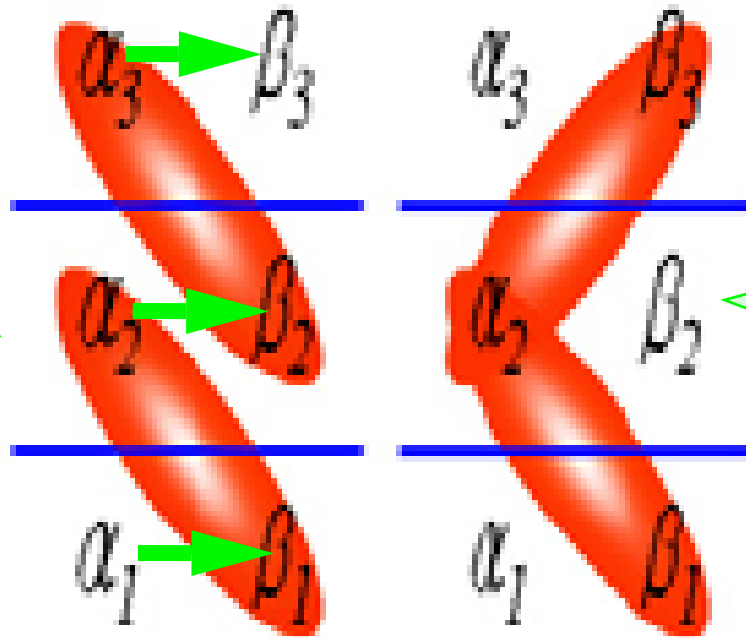


Koshino & Ando, PRB '07

Nakamura

3 layer stacks

$J=3$

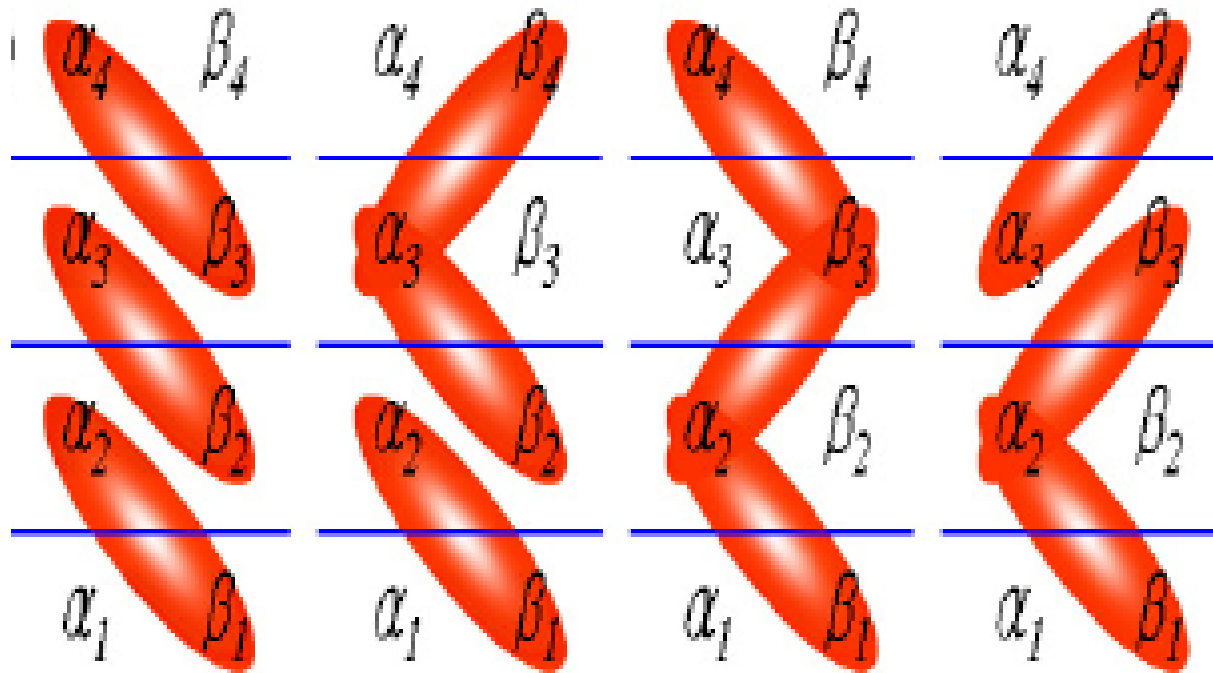


ABC

ABA

$J=1$
&
 $J=2$

4 layer stacks



ABCA

J=4

ABCB

J=1, J=3

ABAB

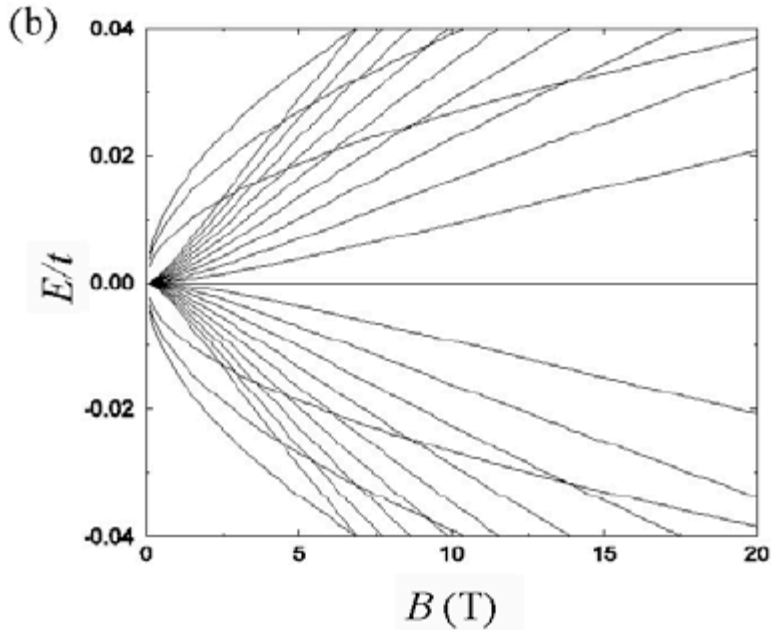
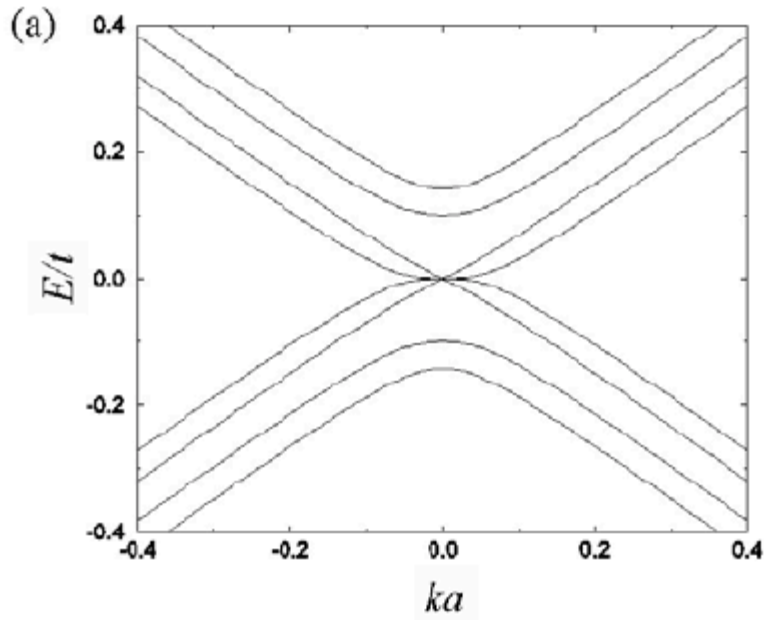
J=2, J=2

ABAC

J=1, J=3

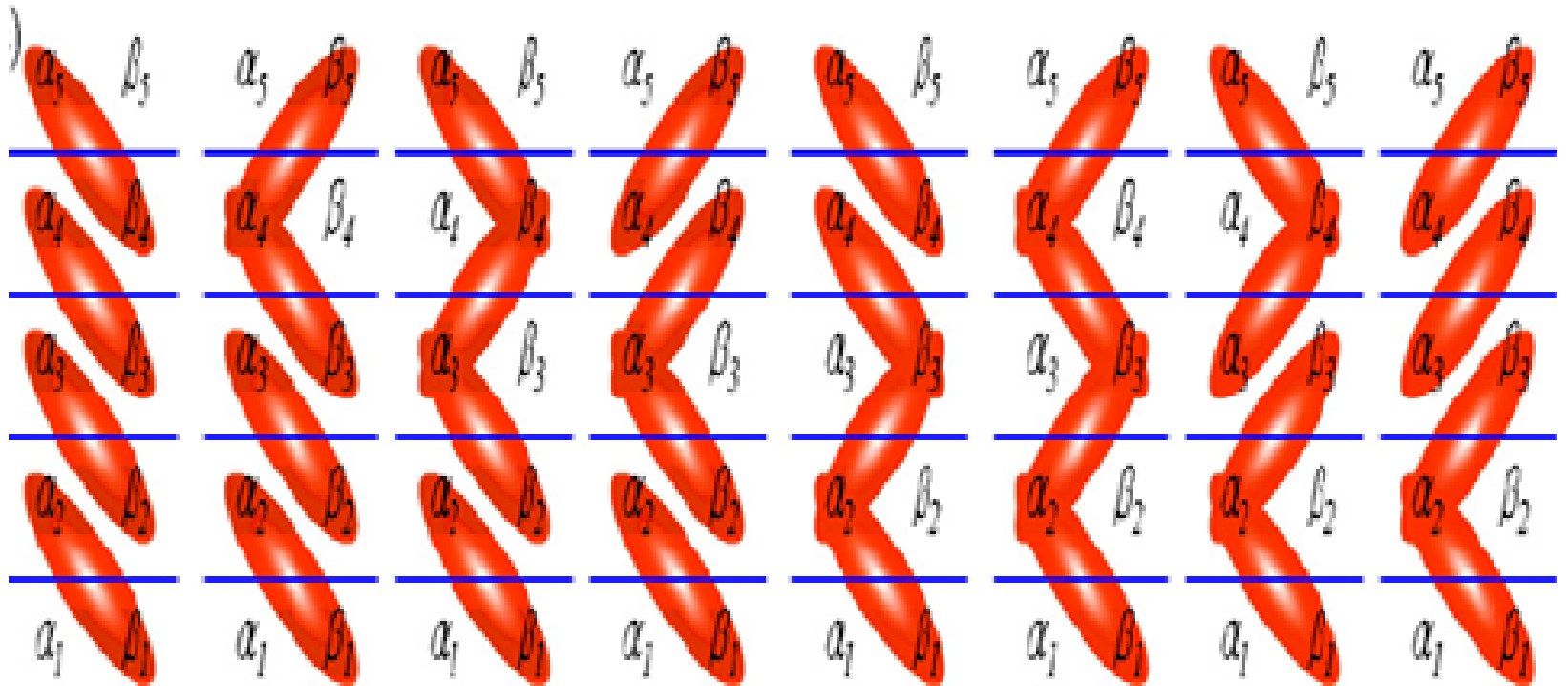
4-layer ABCB

$J=1,3$



$$E \sim k^J \sim B^{J/2}$$

5 layer stacks



J=5

J=1,
4

J=2,
3

J=1,
4

J=2,
3

J=2,2,1

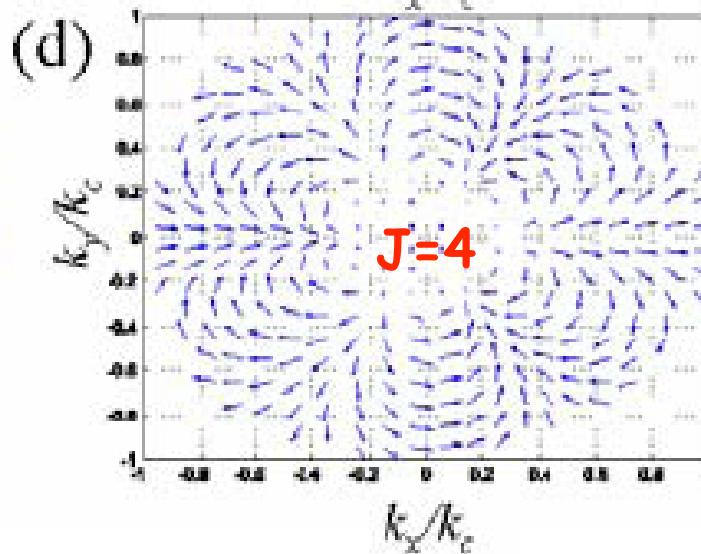
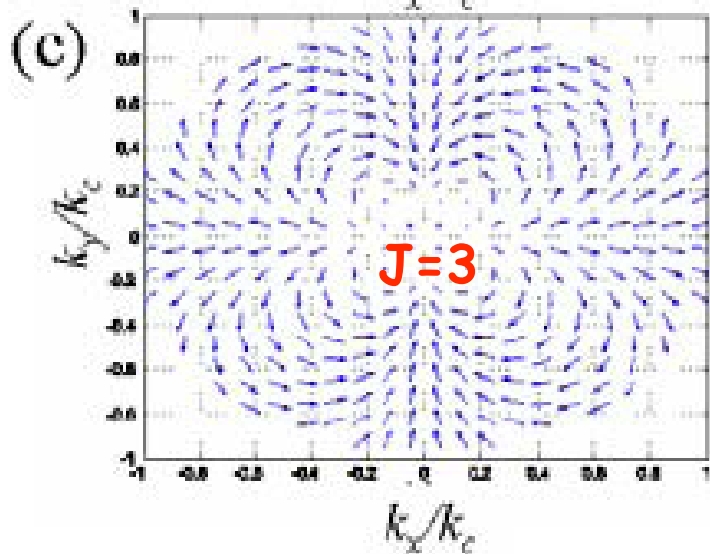
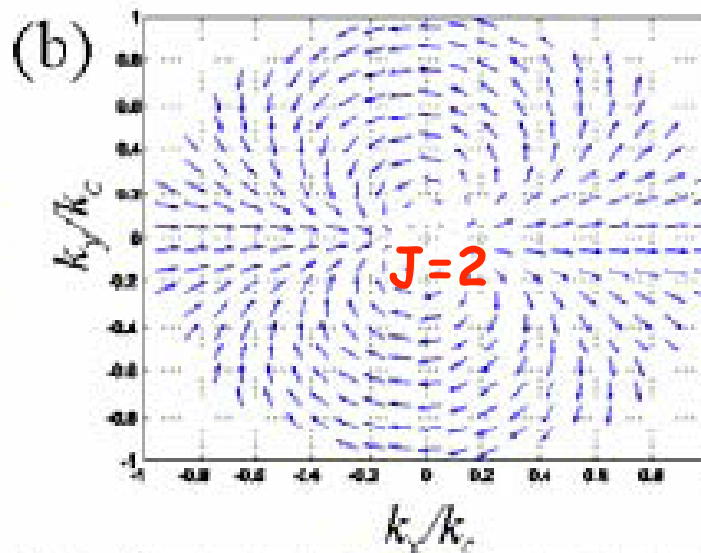
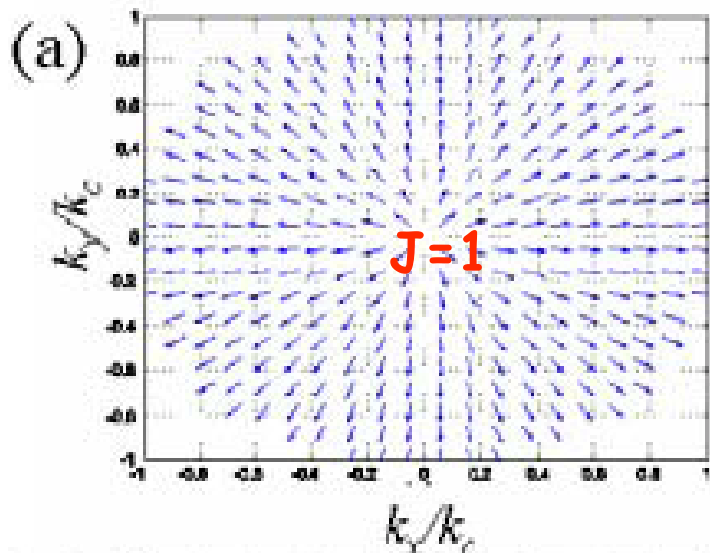
J=2,2,1

J=2,
3

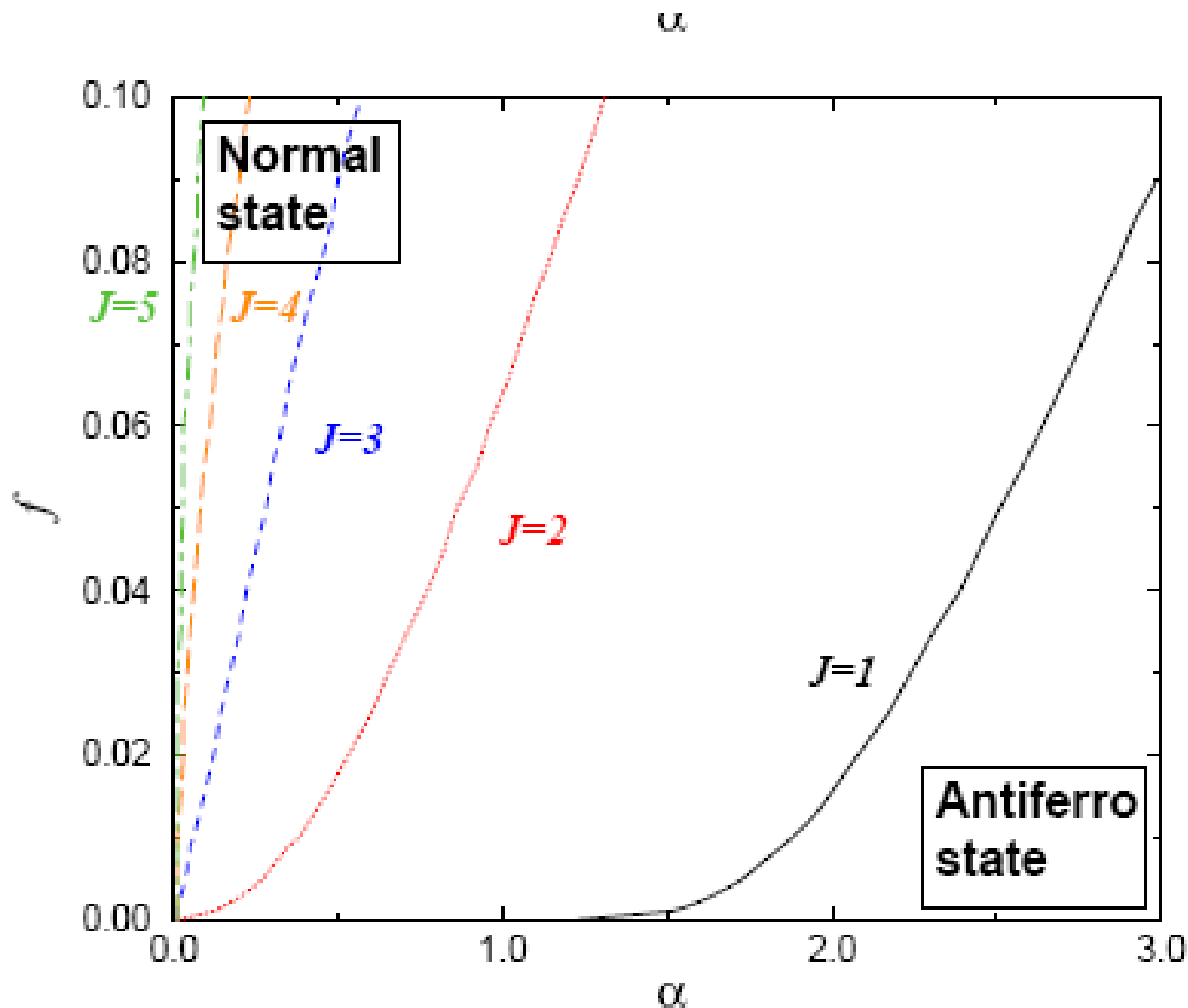
$$\sum_{i=1}^{N_D} J_i = N$$

Chiral Sum Rule

Higher J k -space vortices



Higher J Pseudospin Ferromagnets

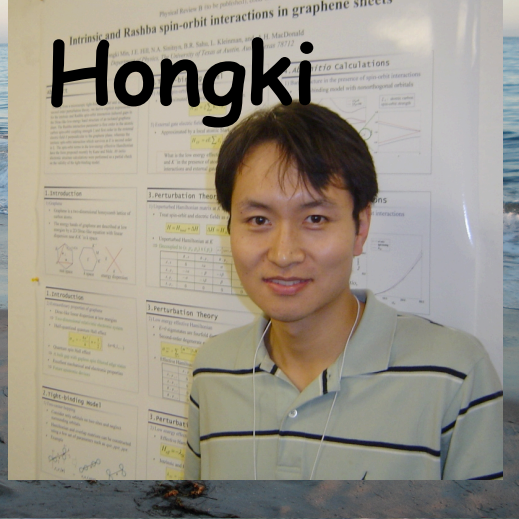




Jason



Sahu



Hongki



Wei-Cheng



Jej



Reza



Yafis



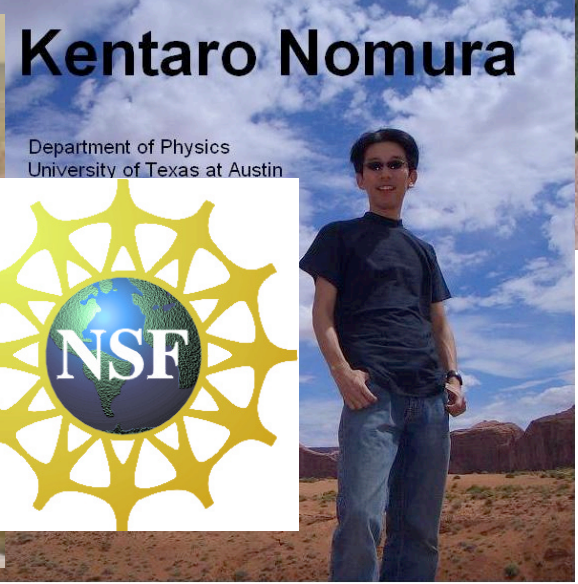
Marco



Nikolai



Tami



Kentaro Nomura



Massless Dirac Fermions in Flatland

