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Spin Hall and quantum spin Hall effects

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Introduction – Spin Hall effect
 spin Hall effect in Pt
Quantum spin Hall phase
 2D
 3D
 phase transition from the ordinary insulator phase

Collaborators: N. Nagaosa, S.-C. Zhang, S. Iso, Y. Avishai, M. Onoda, G. Y. Guo, T. W. Chen

Intrinsic spin Hall effect

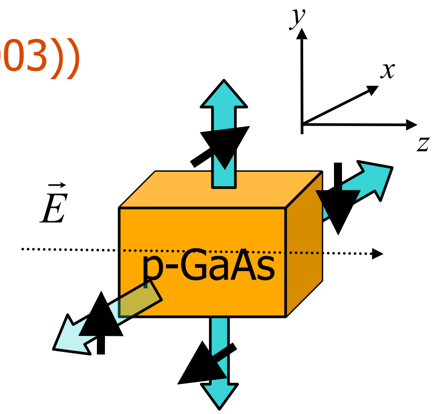
Cf. **Extrinsic spin Hall effect**
← impurity scattering

- p-type semiconductors (SM, Nagaosa, Zhang, Science (2003))

Luttinger model

$$H = \frac{\hbar^2}{2m}\left[\left(\gamma_1 + \frac{5}{2}\gamma_2\right)k^2 - 2\gamma_2\left(\vec{k}\times\vec{S}\right)^2\right]$$

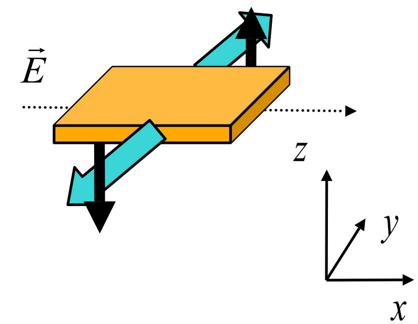
($\vec{S}$: spin-3/2 matrix)



- 2D n-type semiconductors in heterostructure (Sinova et al., PRL (2004))

Rashba model

$$H = \frac{k^2}{2m} + \lambda\left(\vec{\sigma}\times\vec{k}\right)_z$$



- ⋮

$$j_j^i = \sigma_s \varepsilon_{ijk} E_k$$

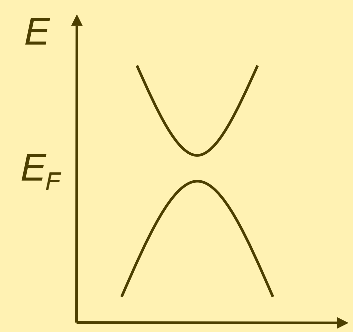
i: spin direction
j: current direction
k: electric field

σ_s : even under time reversal

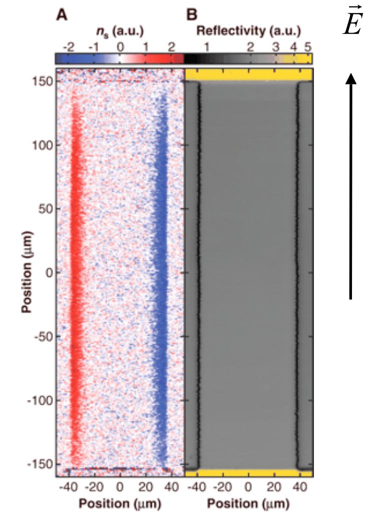
$$j_j^i \approx S^i v_j$$

- Nonzero in nonmagnetic materials.

Berry phase in k-space
← band crossing

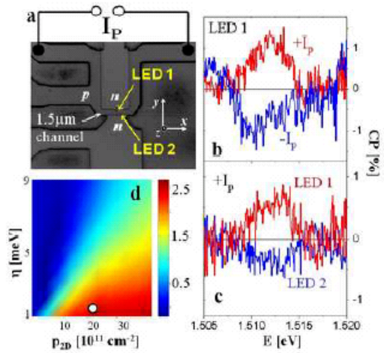


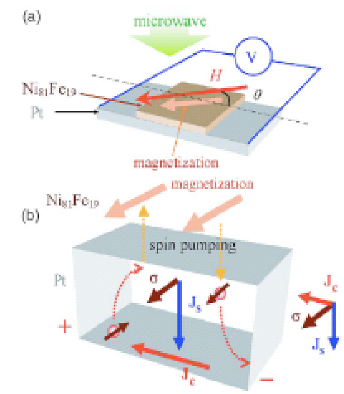
Experiments on spin Hall effect



- **3D n-type, Kerr rotation**
  - Y.K.Kato, R.C.Myers, A.C.Gossard, D.D. Awschalom, Science (2004)
  - Sih et al. , Nature Phys. (2005) n-GaAs, n- (In,Ga)As
  - Sih et al., PRL (2006)
  - Stern et al., PRL(2006) n-ZnSe **RT**

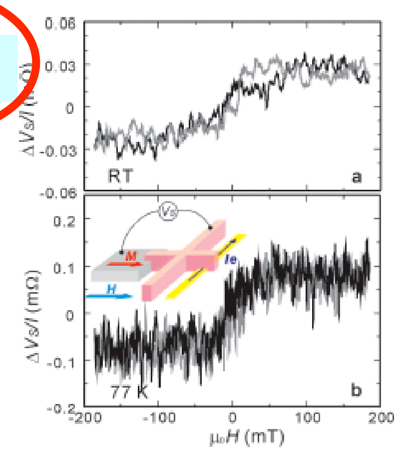
- **2D p-type, spin LED**
  - J. Wunderlich et al., PRL(2005)
    p-GaAs

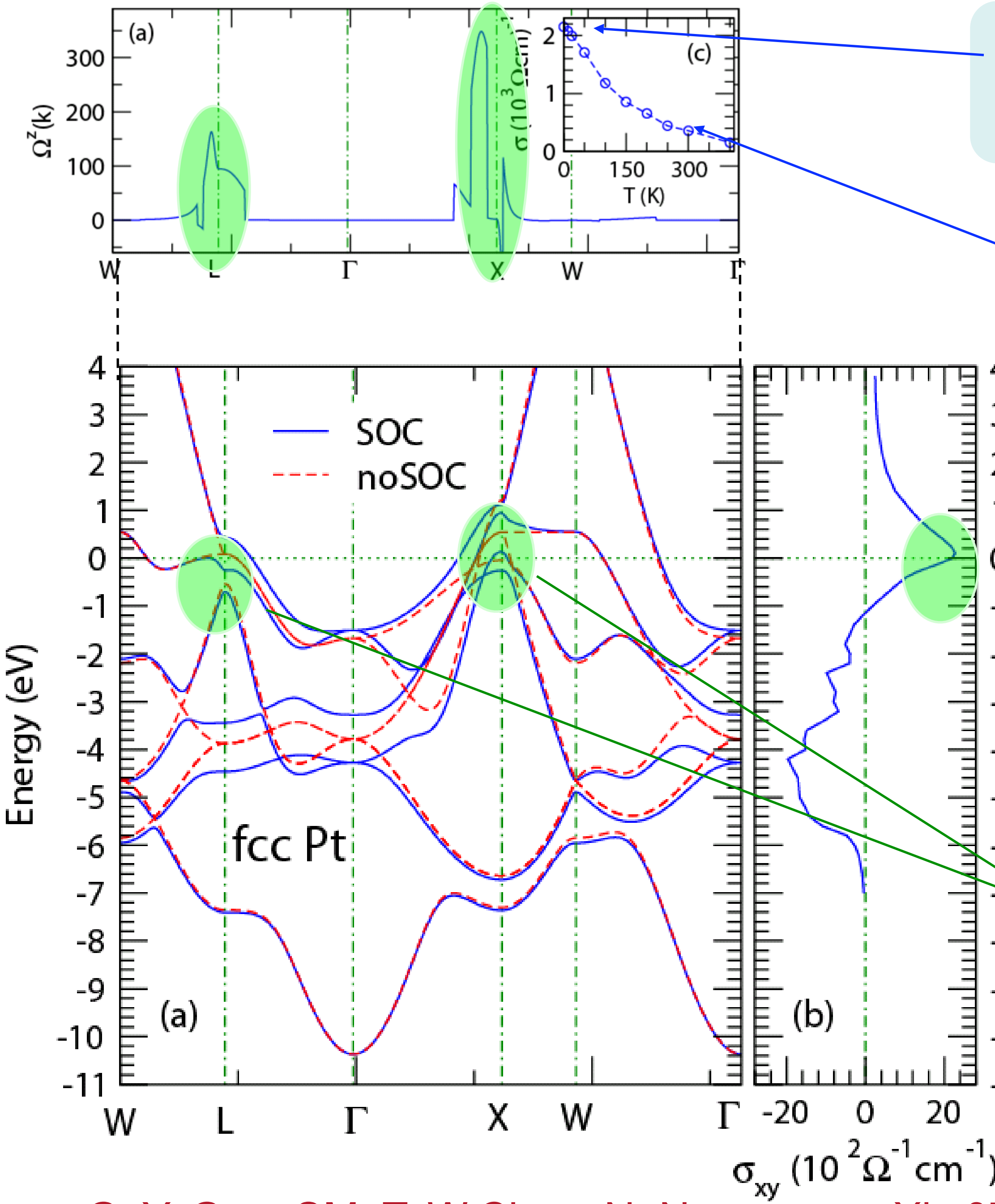




- **Metal (Pt, Al) -- Inverse SHE**
  - E. Saitoh, M. Ueda, H. Miyajima, G. Tatara, APL (2006) Pt **RT**
  - Valenzuela and Tinkham, Nature(2006) Al

- **Metal (Pt) -- SHE & Inverse SHE**
  - T. Kimura, Y. Otani, T. Sato,
    S. Takahashi, S. Maekawa, PRL (2007) Pt **RT**





$\sigma_S \approx 2000\ \Omega^{-1}cm^{-1}$ (T=0K)

$\sigma_S \approx 360\ \Omega^{-1}cm^{-1}$ at RT

↕ agree

Cf. Experiment (Kimura et al.)

$\sigma_S \approx 240\ \Omega^{-1}cm^{-1}$

Band crossing near Fermi energy (← d-orbitals)

G. Y. Guo, SM, T. W.Chen, N. Nagaosa, arXiv:0705.0409 (2007)

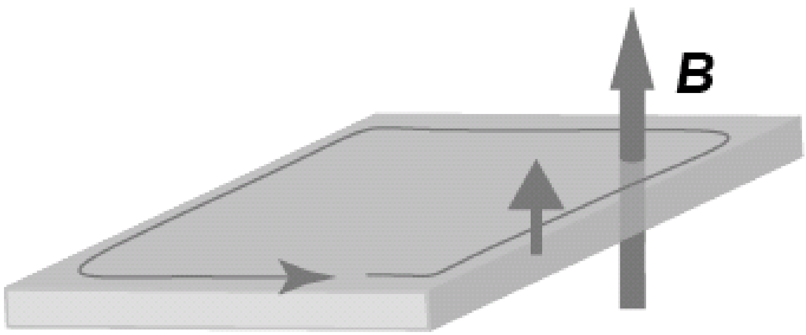
2D quantum spin Hall phase

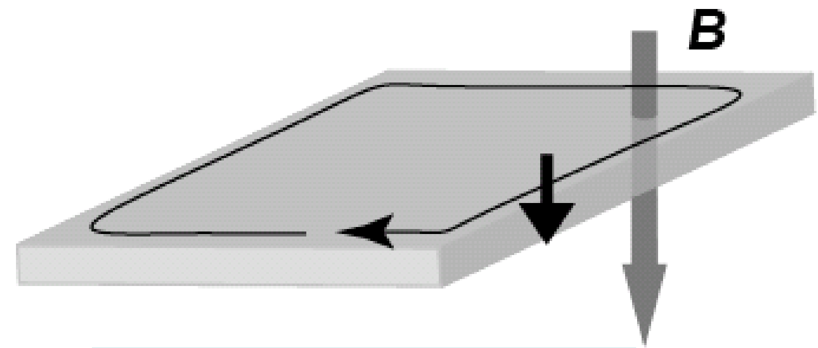
Quantum spin Hall phases

Bernevig and Zhang, PRL (2005)
Kane and Mele, PRL (2005),

- **bulk = gapped (insulator)**
- **gapless edge states -- carry spin current, topologically protected**

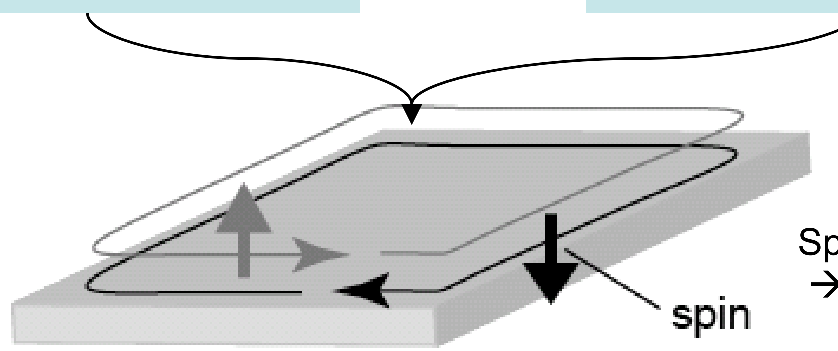
Quantum spin Hall state ≈ Quantum Hall state × 2





$$\sigma_{xy} = \frac{e^2}{h}$$ for up spin

$$\sigma_{xy} = -\frac{e^2}{h}$$ for down spin



Spin-orbit coupling
→ (spin-dependent) effective magnetic field

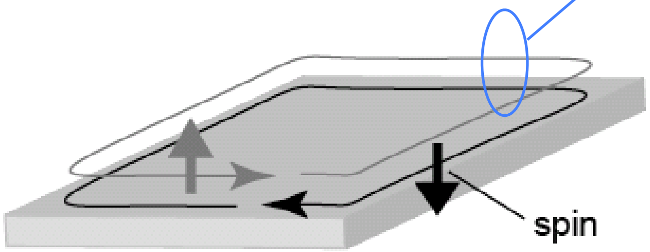
Z_2 topological number ν

Even → simple insulator
Odd → quantum spin Hall phase

Kane and Mele, PRL95, 146802 (2005).

(i) Quantum spin Hall phase (ν =odd)

no backscattering by nonmagnetic impurities.

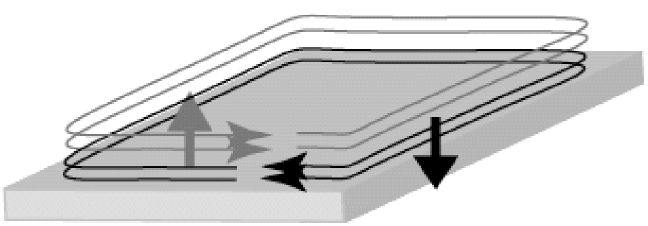


Odd number of Kramers pairs of edge states

Edge states
→ robust against { nonmagnetic impurities, interaction

Wu, Bernevig, Zhang, PRL (2006)
Xu, Moore, PRB (2006)

(ii) Simple insulator ν (



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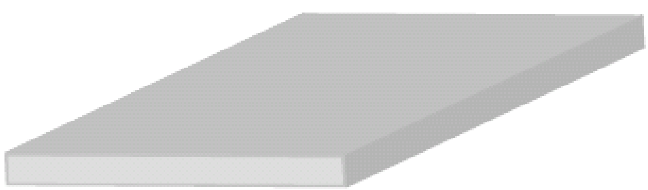


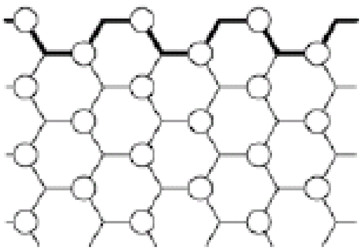
Even number of Kramers pairs of edge states

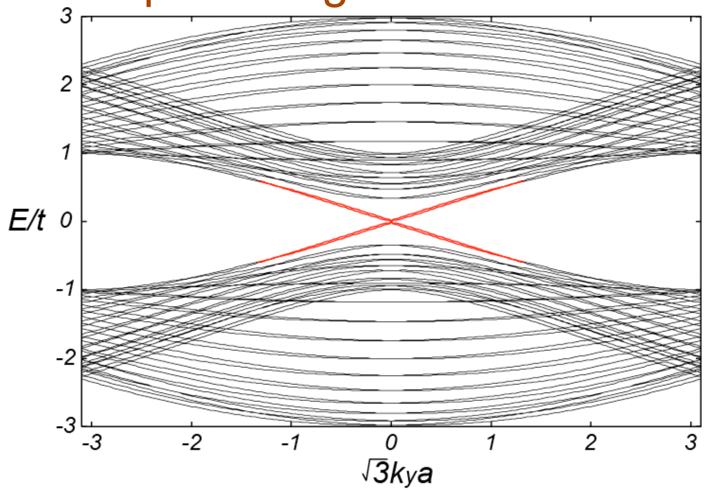
Kane-Mele model

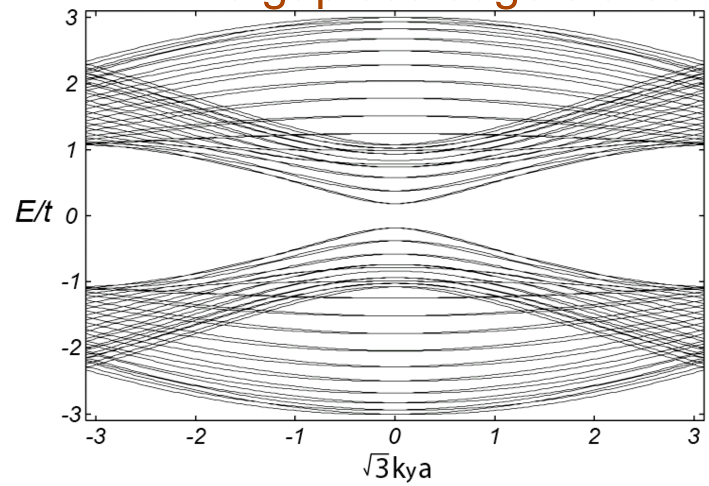
Quantum spin Hall phase $(\nu = \text{odd})$ → Gapless edge states

Simple insulator $(\nu = \text{even})$ → No gapless edge states

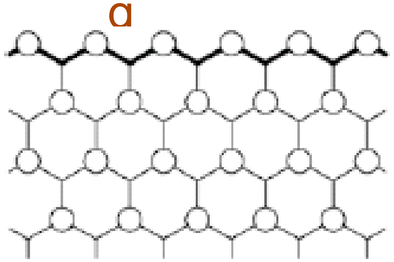
armchair

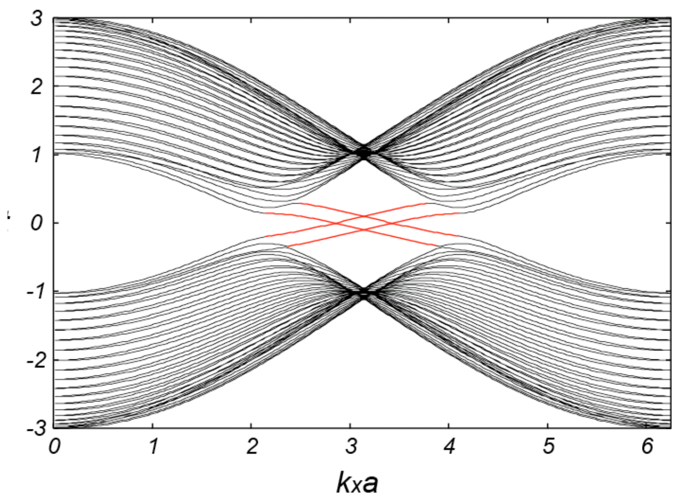


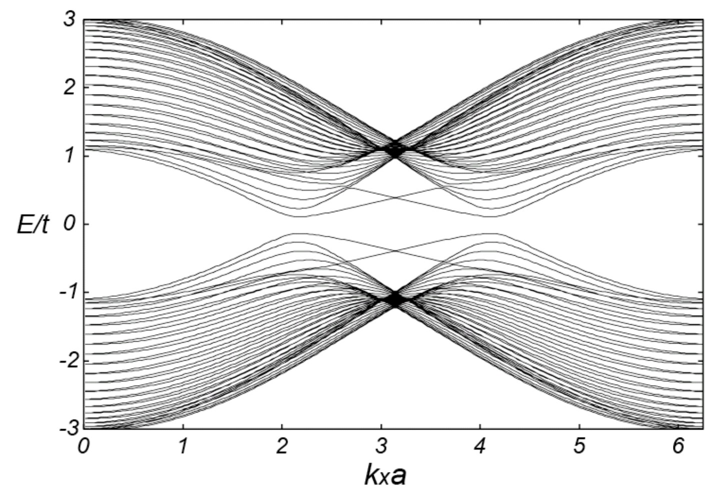




zigza
g







Gapless edge states exist irrespective of boundary conditions

topologically protected

Bulk topological order → manifest as edge states

In contrast : Graphene without spin-orbit → Edge states only on zigzag edges

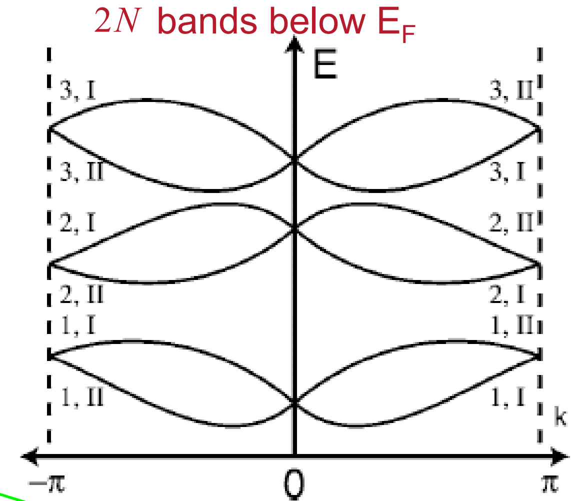
Z_2 topological number is a bulk property

Z_2 topological number in 2D systems without inversion-symmetry

Fu,Kane, PRB74,195312('06)

$$w_{mn}(\vec{k}) = \langle u_{-k,m} | \Theta | u_{k,n} \rangle$$ $2N \times 2N$ unitary

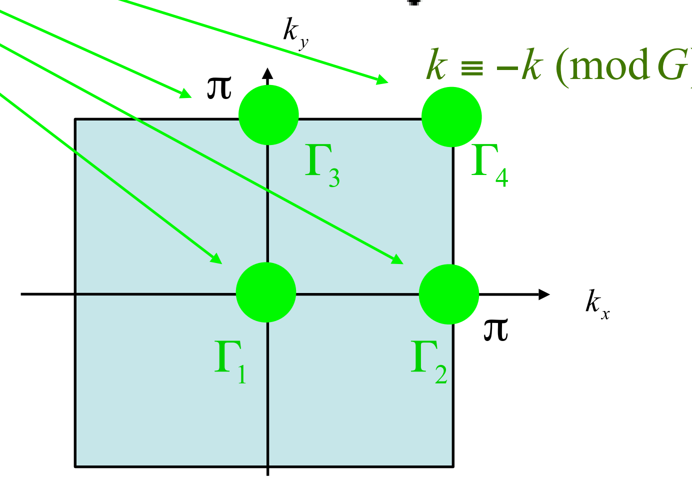
(& antisymmetric at $\Gamma_i = \frac{1}{2}\vec{G}$)



Z_2 topological number ν

$$(-1)^{\nu} = \prod_{i=1}^{4} \frac{\sqrt{\det[w(\Gamma_i)]}}{\mathrm{Pf}[w(\Gamma_i)]}$$

$= \pm 1$ -- Choice of branches of $\pm\sqrt{\det[w(\Gamma_i)]}$



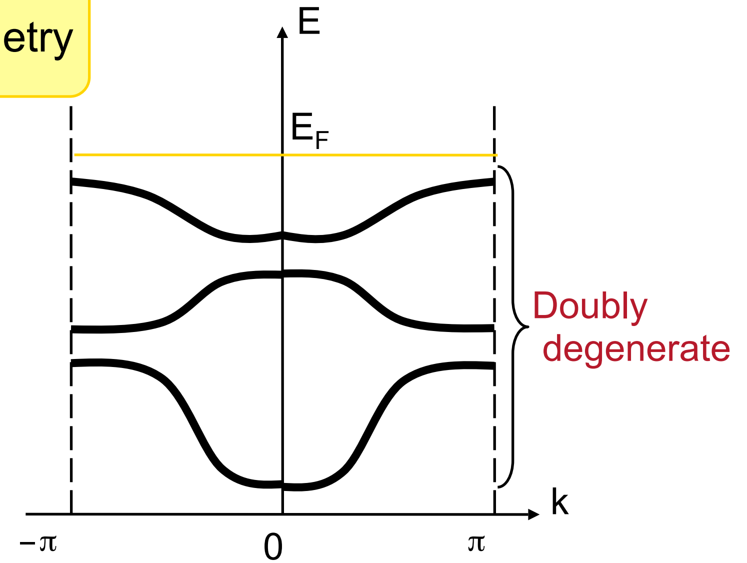
No inversion symmetry

→ Phases of the wavefunctions should be calculated for the whole BZ

Z_2 topological number in 2D systems with inversion-symmetry

Fu,Kane, PRB76, 045302 (2007)

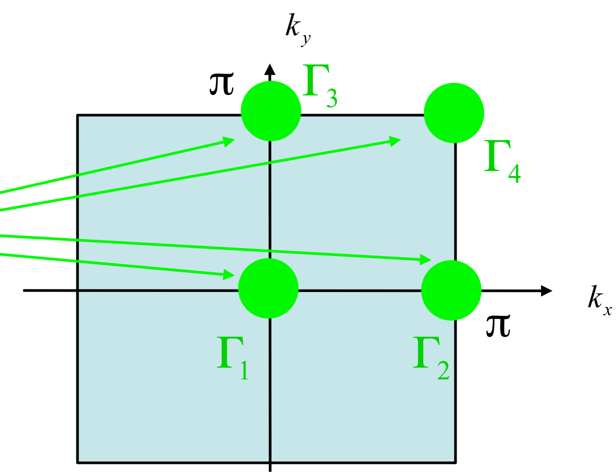
$2N$ bands below E_F (N Kramers pairs)



Z_2 topological number ν

$$(-1)^{\nu} = \prod_{i=1}^{4} \prod_{m=1}^{N} \xi_{2m}(\Gamma_i)$$

± 1 : parity eigenvalue of the Kramers pairs at $\Gamma_i = \frac{\vec{G}}{2}$



With inversion symmetry →

-- product of parity eigenvalues over filled Kramers pairs over Γ_i

easy to calculate !

Candidate systems for quantum spin Hall phase?

- Nonmagnetic insulator
- Z_2 topological number = odd

Spin Hall effect and Streda formula

Středa formula for Hall effect

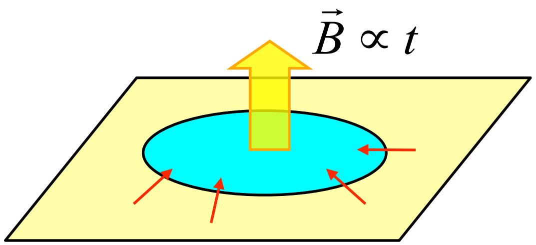
Středa (1982)

$$\sigma_{xy} = \sigma_{xy}^{I} + \sigma_{xy}^{II}$$

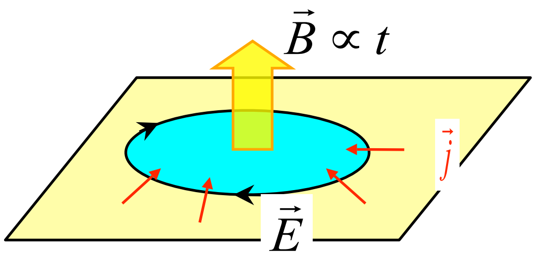
$$\sigma_{xy}^{I} = \frac{ie^2}{2} \mathrm{Tr}\left[v_i \frac{1}{E_F - H + i0} v_j \delta(E_F - H) - \mathrm{h.c.} \right]$$: Fermi-energy term, zero for insulator

$$\sigma_{xy}^{II} = e \left. \frac{dN}{dB} \right|_{E_F}$$

N : Number of states below E_F



$\left. \frac{dN}{dB} \right|_{E_F}$ electrons flow in.



$j = Ee \left. \frac{dN}{dB} \right|_{E_F}$: Hall current

Charge → spin

Středa formula for spin Hall effect

Expected result :

$$\sigma_s = \sigma_s^{I} + \sigma_s^{II}$$

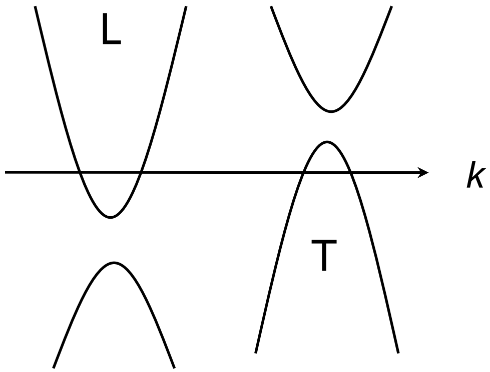
σ_s^{I} : zero for insulator

$\sigma_s^{II} = \left. \frac{dS_z}{dB} \right|_{E_F}$ **: Spin-orbit susceptibility** ← spin-orbit coupling

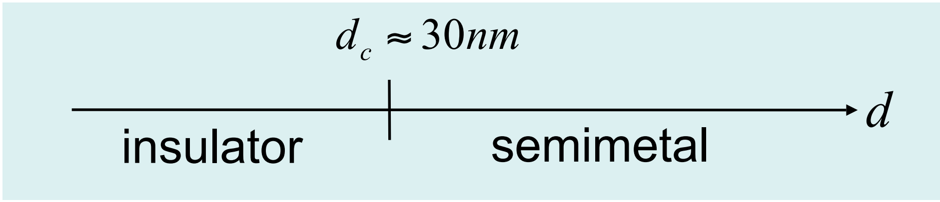
SM, PRL 97, 236805 (2006)

Candidate materials for QSH phase ?

- Bi – semimetal { hole pocket at T point; 3 electron pockets at L points



Thin film – vertical motion quantized → Gap opens



V. N. Lutskii (1965)
V. B. Sandomirskii(1967)
C. A. Hoffman et al. (1993)

Bi as a candidate material for 2D QSH phase

SM, PRL 97, 236805 (2006)

- **Enhanced diamagnetic susceptibility in Bi and $Bi_{1-x}Sb_x$**

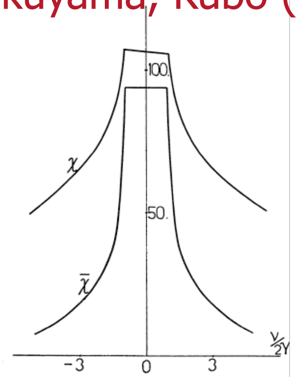
Interband matrix elements due to spin-orbit coupling

Theory

Fukuyama, Kubo (1970)

Experiment

Brandt, Semenov, Falkovsky (1977)



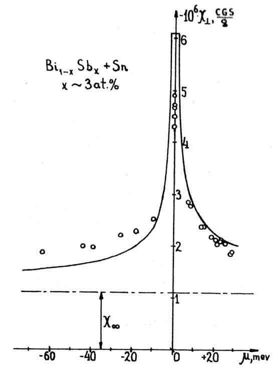


Fig. 6

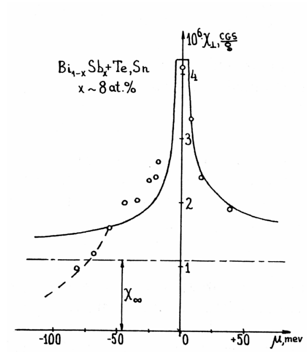
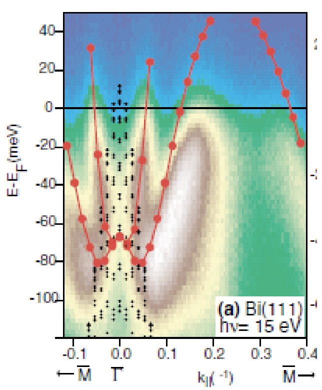


Fig. 7

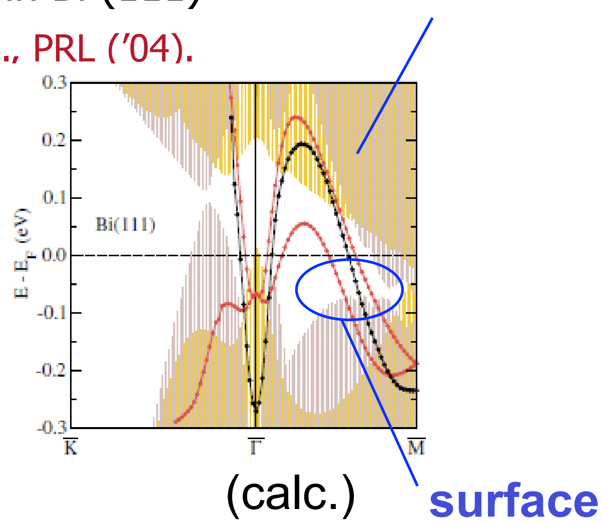
- **Surface states of Bi -- large spin splitting --**

Large spin splitting in Bi (111)

Koroteev et al., PRL ('04).



(exp.)



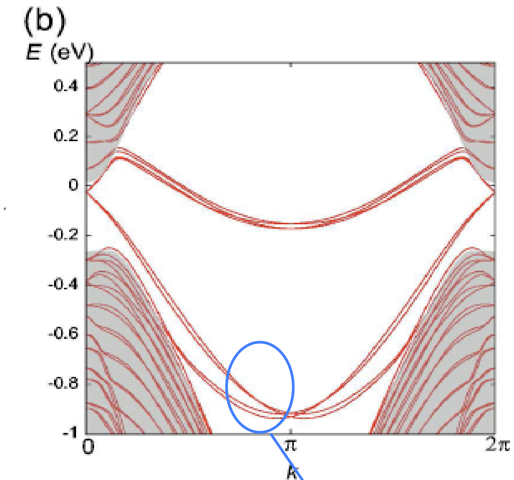
(calc.)

- large spin splitting ($\approx 0.1\mathrm{eV}$)
  = carry spin current
- surface states at E_F

2D bismuth as a candidate for QSH phase

SM
Phys. Rev. Lett.97, 236805(2006)

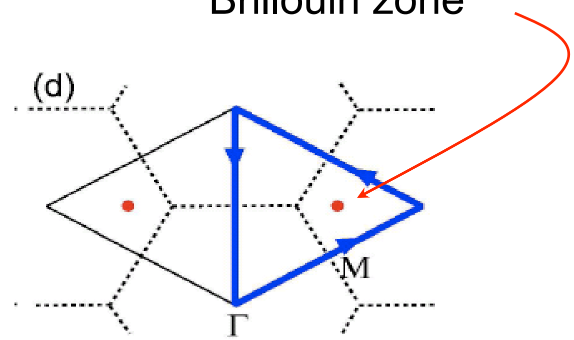
- **Band structure for 2D strip**



1 pair of edge states at each edge

- **Z_2 topological number**

zeros of the Pfaffian in half Brillouin zone



ν =odd

- **spin Chern number**

$$C_{sC} = -2$$

- **Parity eigenvalues and Z_2 topological number**

$$(-1)^{\nu} = \prod_{i=1}^{4}\prod_{j=1}^{N}\xi_j(\Gamma_i)$$

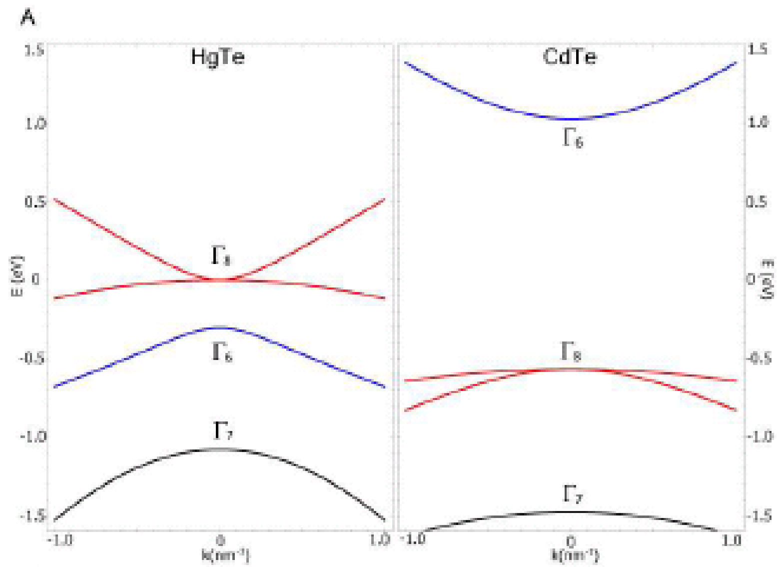
(🡐 Fu, Kane cond-mat/0611341)

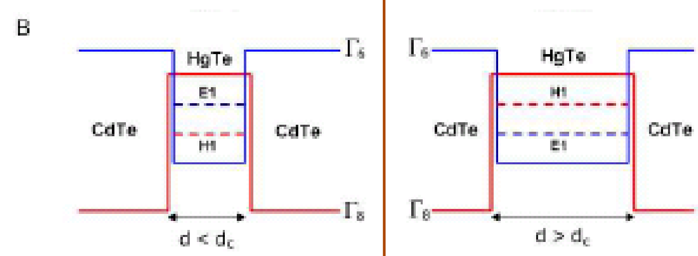
Conclusion

2D bismuth = good candidate for QSH phase !

Other candidate systems : CdTe/HgTe/CdTe quantum well

M B. A. Bernevig, T. L. Hughes, S.-C. Zhang
Science **314**, 1757(2007);





Simple insulator phase

$d < d_c$

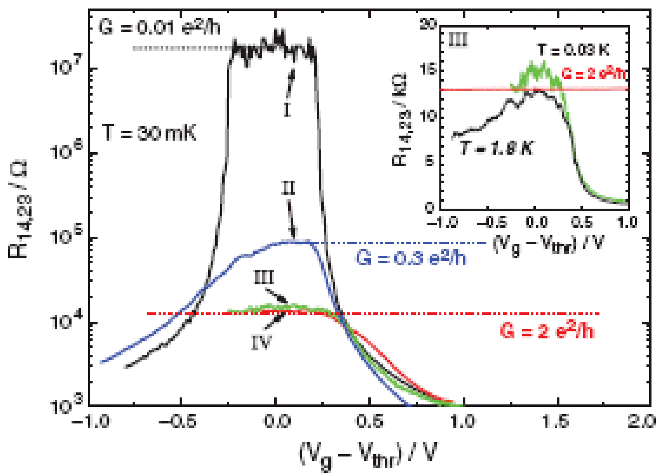
quantum spin Hall phase

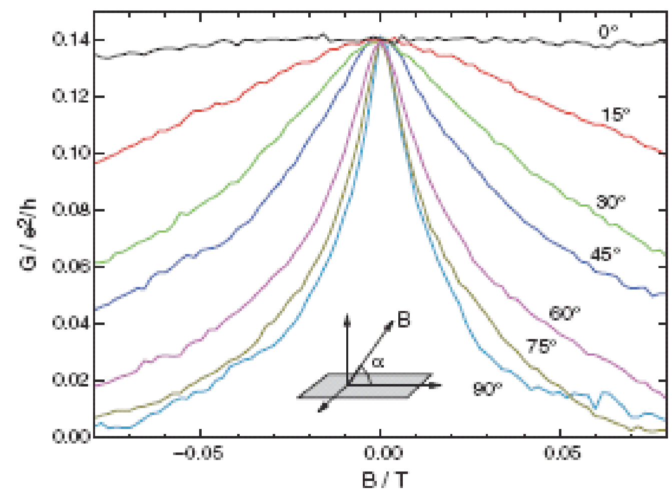
$d > d_c$

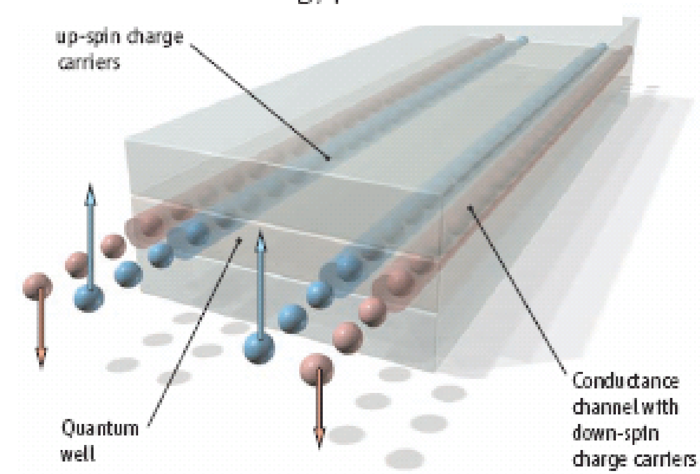
Phase transition at $d = d_c$

Observation of QSH state in CdTe/HgTe/CdTe quantum well

Markus König, *et al.*
Science **318**, 766 (2007);







Schematic of the spin-polarized edge channels in a quantum spin Hall insulator.

3D quantum spin Hall phase

Z_2 topological number in 3D

$$(-1)^{\nu_0} = \prod_{\vec{k}=\vec{G}/2} \delta_{\vec{k}}$$

No inversion symmetry $\delta_i = \dfrac{\sqrt{\det[w(\Gamma_i)]}}{\mathrm{Pf}[w(\Gamma_i)]}$

With inversion symmetry $\delta_i = \prod_{m=1}^{N} \xi_{2m}(\Gamma_i)$

- Only ν_0 is stable against disorder.

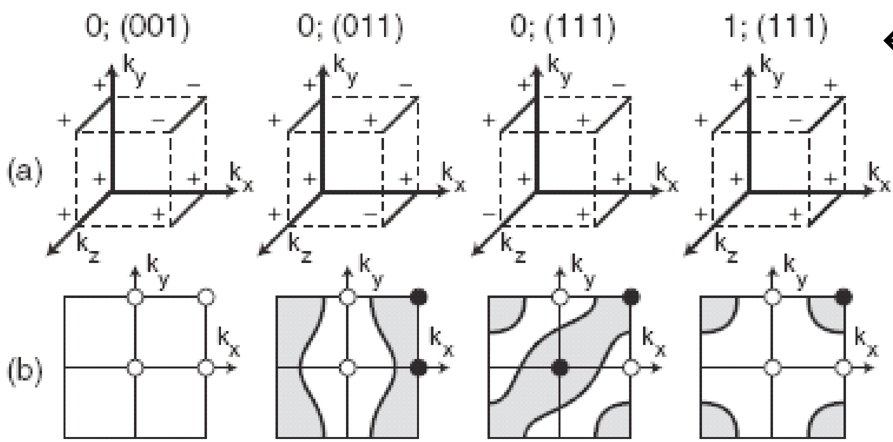
$\nu_0 = \text{even}$: weak topological insulator

$\nu_0 = \text{odd}$: strong topological insulator

Topological surface states

Materials for strong topological insulators (QSH phase) ?

Topology of the Fermi surfaces.



← Z_2 topological numbers

←Fermi surface of the surface states on (001) surface

Candidate materials for 3D QSH phase

Fu,Kane, PRB(2007)

$Bi_{1-x}Sb_x$ $(0.07<x<0.22)$

α-Sn, HgTe under uniaxial pressure .

$Pb_{1-x}Sn_xTe$ under uniaxial (111) strain

- Bi_2Te_3
- Kondo insulators
- skutterudites

(Thermoelectric materials ?)

Candidate materials for 3D QSH phase

Fu,Kane, PRB76, 045302 (2007)

$Bi_{1-x}Sb_x$

Bi & Sb → semimetal (direct gap >0 for every *k*)

Suppose the band overlap is lifted by perturbation

= Z_2 topological number defined

Calculation of ν_0 from parity

Bismuth

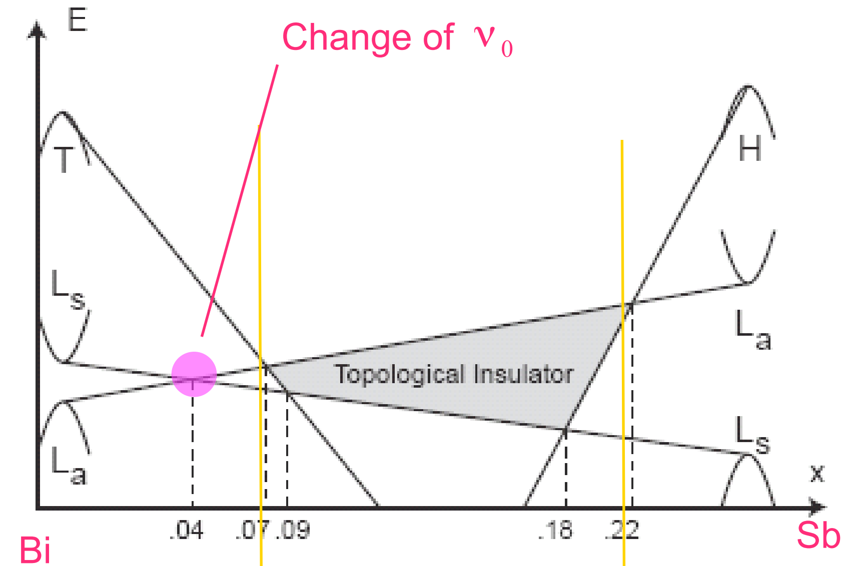
1Γ	Γ_6^+	Γ_6^-	Γ_6^+	Γ_6^+	Γ_{45}^+	$-$
$3L$	L_s	L_a	L_s	L_a	L_a	$-$
$3X$	X_a	X_s	X_s	X_a	X_a	$-$
$1T$	T_6^-	T_6^+	T_6^-	T_6^+	T_{45}^-	$-$
	Z_2 class					$+$

Even

Antimony

1Γ	Γ_6^+	Γ_6^-	Γ_6^+	Γ_6^+	Γ_{45}^+	$-$
$3L$	L_s	L_a	L_s	L_a	L_s	$+$
$3X$	X_a	X_s	X_s	X_a	X_a	$-$
$1T$	T_6^-	T_6^+	T_6^-	T_6^+	T_{45}^-	$-$
	Z_2 class					$-$

Odd
(but semimetal)



semimetal ← insulato
r → semimetal

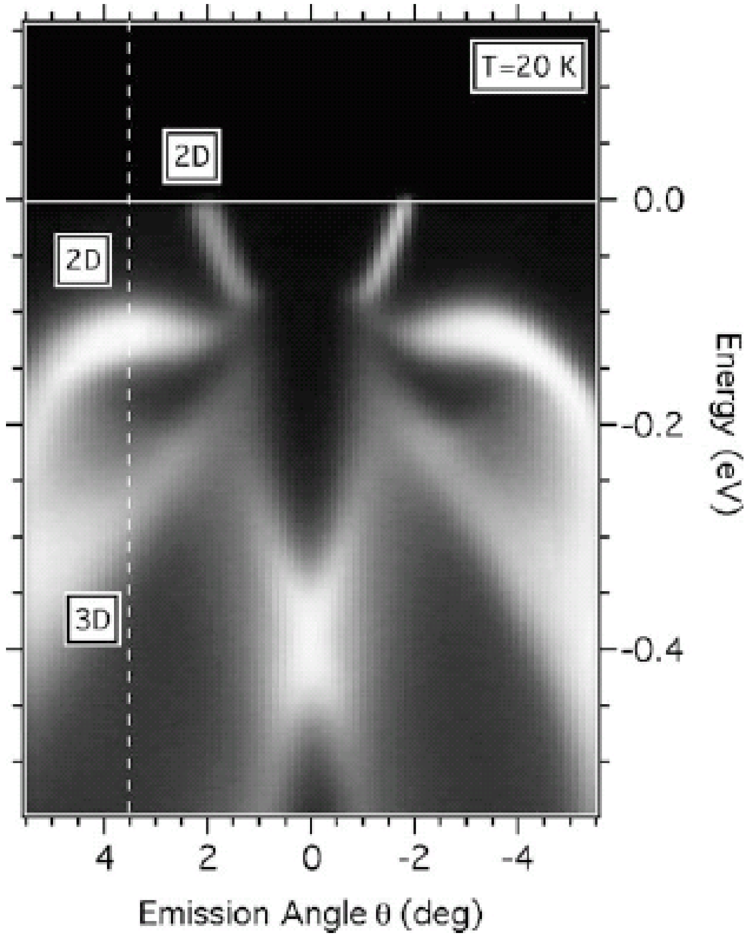
QSH phase
(0.07<x<0.22)

$Bi_{1-x}Sb_x$ (0.07<x<0.22) = QSH phase.

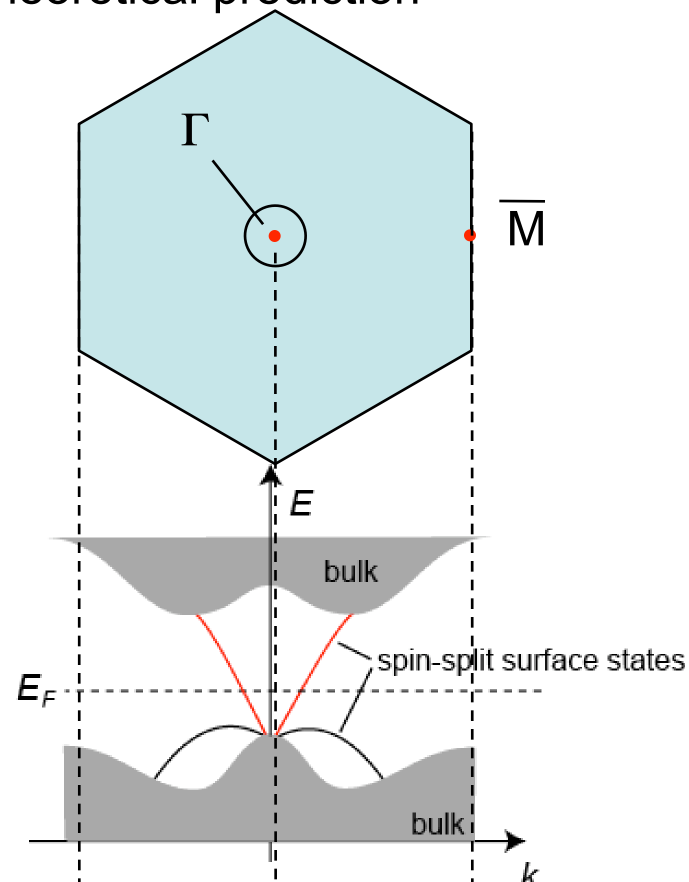
H. Höchst, S. A. Gorovikov

Journal of Electron Spectroscopy and Related Phenomena
144–147 (2005) 351–355

$Bi_{0.92}Sb_{0.08}$, (= theoretically in the QSH phase (Fu et al. (2006))



Theoretical prediction



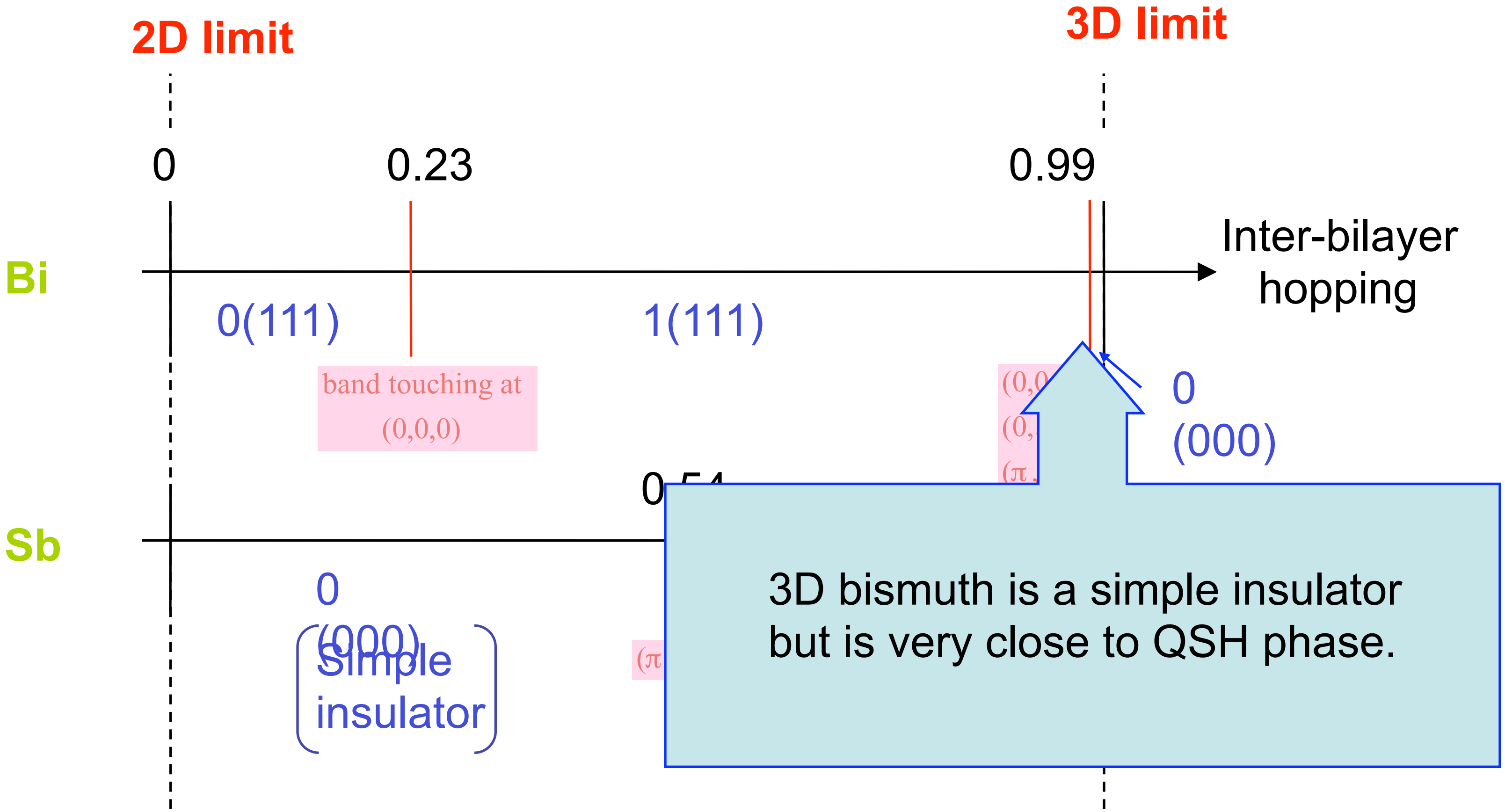
1 gapless fermion : Unique for a surface of 3D QSH system
cf. 2 gapless fermions in graphene (K and K' points)

Phase transitions

See Fukui, Hatsugai, (2006)

- 2D : Bi: QSH
  Sb: insulator
  (Murakami, PRL (2006))

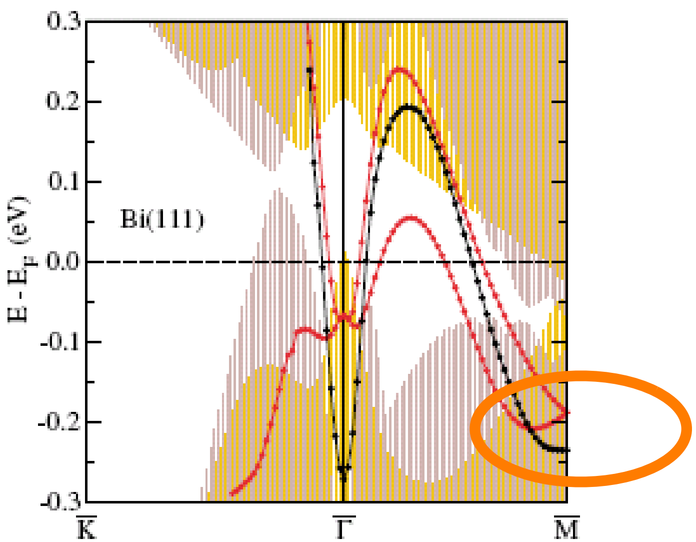
- 3D : Bi: insulator
  Sb: QSH
  (Fu,Kane,PRB(2007))



3D bismuth is a simple insulator but is very close to QSH phase.

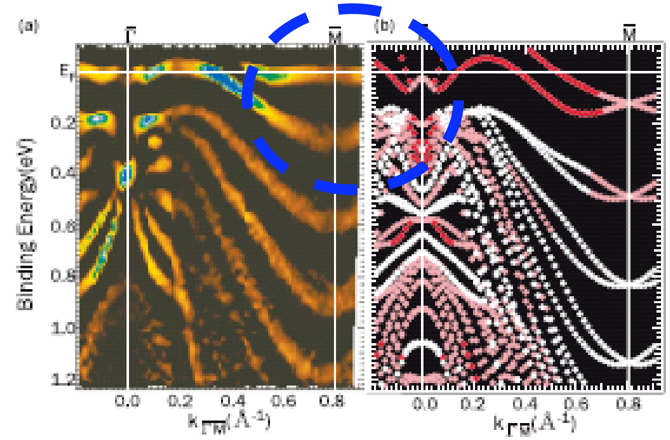
(111) surface

Koroteev et al., PRL (2004).



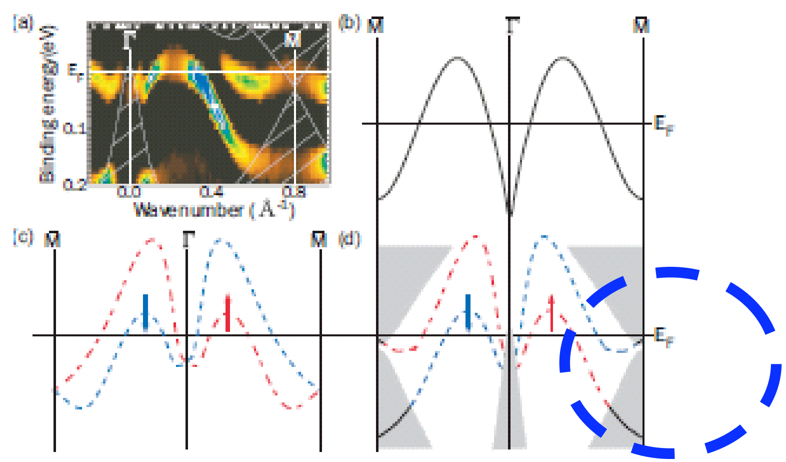
10-bilayer (111)-bismuth

Hirahara et al., PRL (2006)



Hirahara et al., preprint (2007)

Spin-resolved ARPES



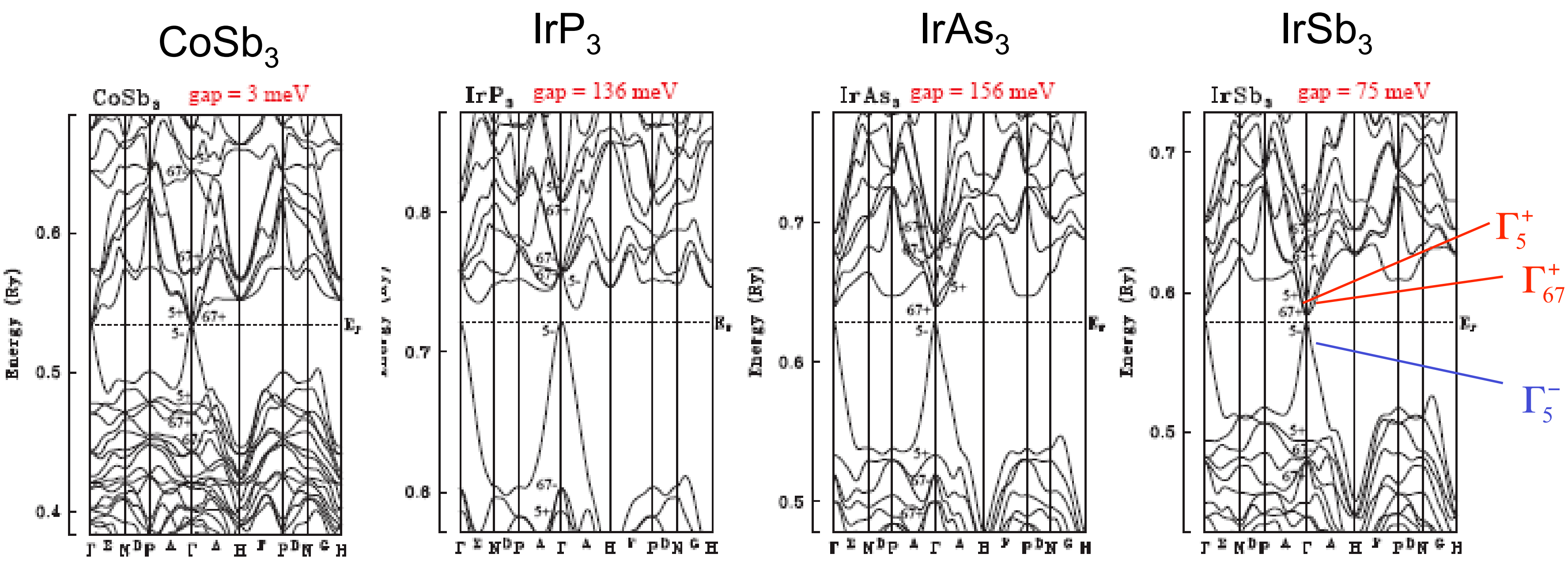
Rashba spin splitting

Rashba spin-split bands at Γ

→ Meet again at M

Do not meet at M

(Example) binary skutterudite (Thanks to K.Takegahara (Hirosaki))



$$\vec{k} = \vec{\Gamma}_i = \frac{1}{2}\left(n_1\vec{b}_1 + n_2\vec{b}_2 + n_3\vec{b}_3\right) \qquad (n_i = 0, 1) \longrightarrow \Gamma, H, N \times 6$$

Z_2 topological number ν

$$(-1)^{\nu} = \prod_{\Gamma, H, 6N} \prod_{m=1}^{N} \xi_{2m}(\Gamma_i) = +1$$

: Ordinary insulator

Parity (= ±1)

Phase transition
between the quantum spin Hall and insulator phases

- How does the gap close?

Z_2 topological number

global property of the BZ

↔

physics of gap closing

local in k-space

Fu, Kane, Mele
Moore, Balents
Zhang, Bernevig

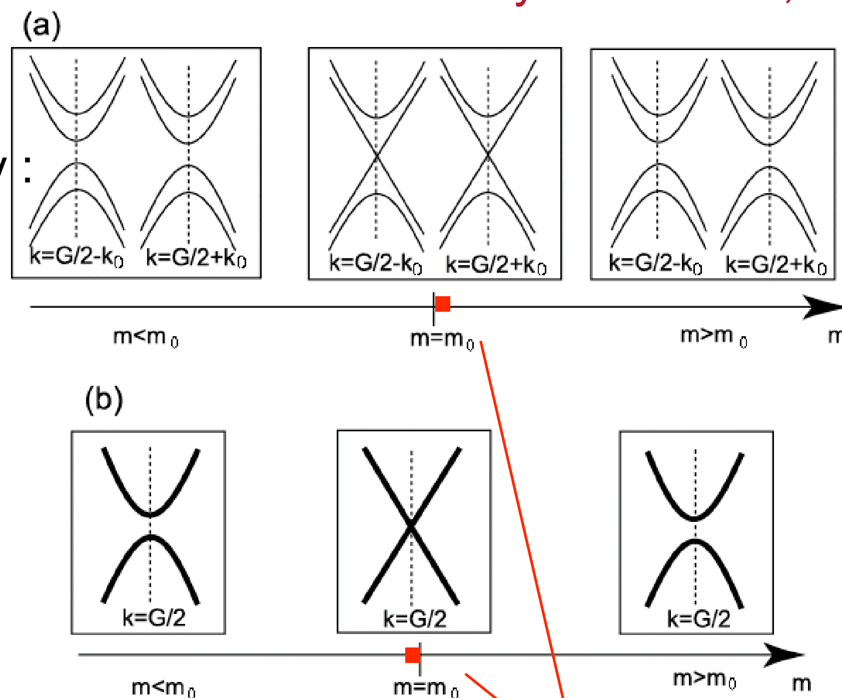
Phase transition
= Change of Z_2 topological number
← closing of bulk gap

Phase transition between QSH and insulator phases

(SM, Iso, Avishai, Onoda, Nagaosa, Phys. Rev. B76, 205304 (2007))

2D

- No inversion-symmetry :
- Inversion-symmetry :

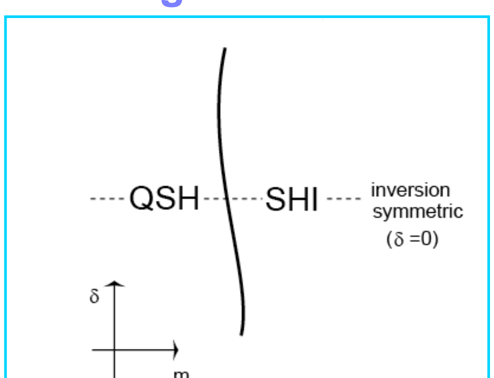


transition

(Honeycomb lattice model
Kane, Mele, PRL (2005))

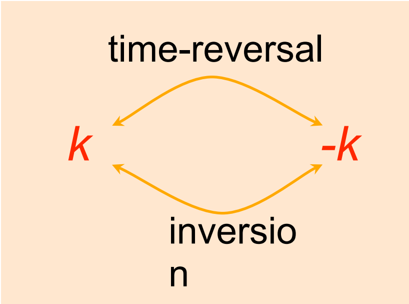
(CdTe/HgTe/CdTe QW
Bernevig et al., Science (2006))

Phase diagram



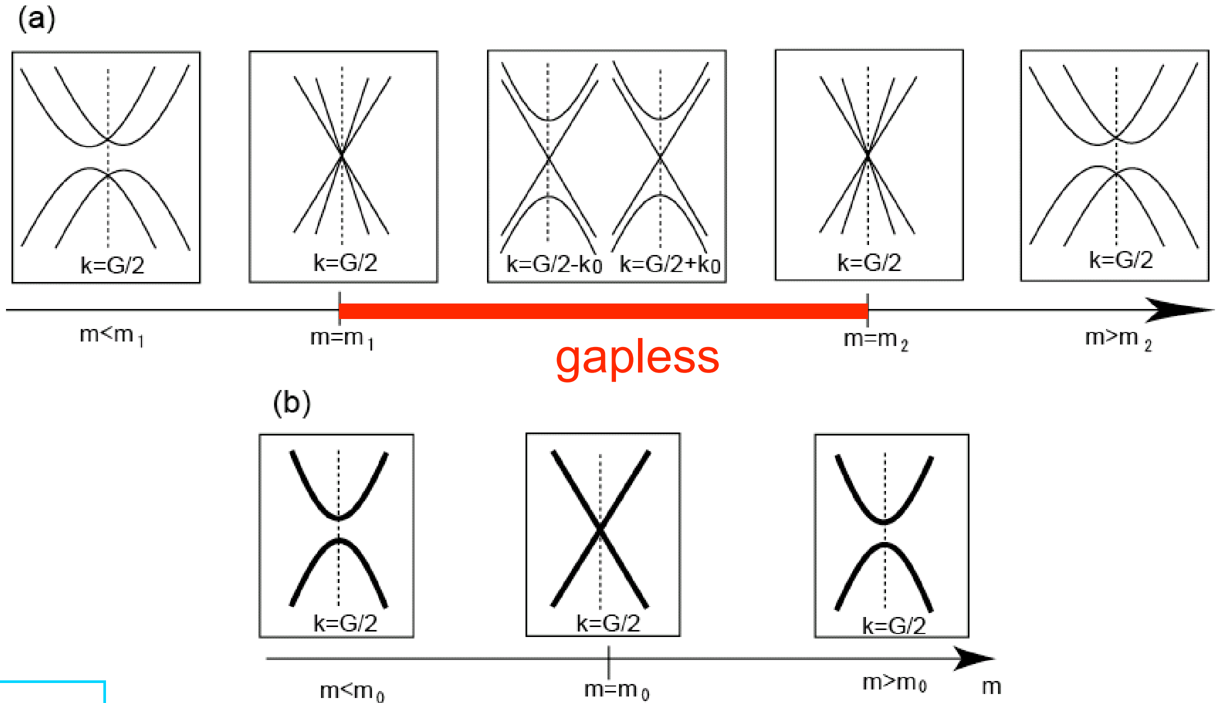
Phase transition between QSH and insulator phases

(SM, New J. Phys. (2007))

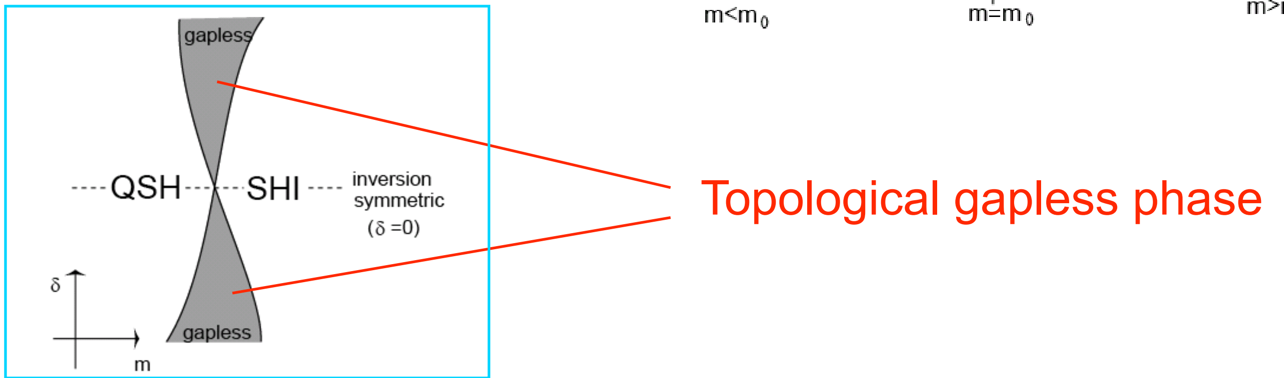


3D

- No inversion-symmetry :
- Inversion-symmetry :



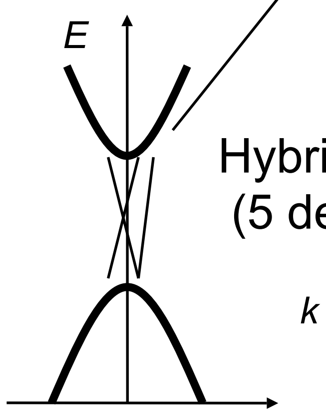
Phase diagram



Phase transition between QSH and insulator phases

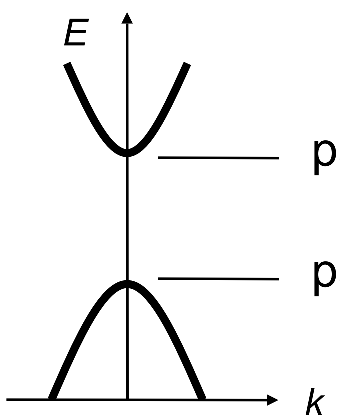
inversion-symmetric

Kramers degeneracy



Hybridization
(5 deg. of freedom)

→ One-parameter tuning
➔ no gap closing



parity =+1

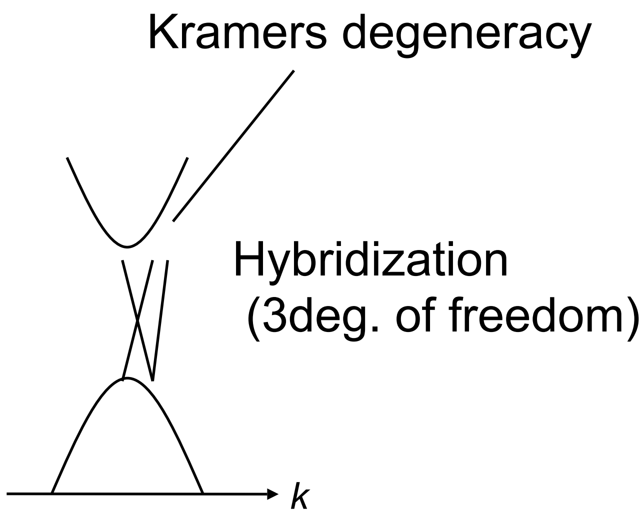
parity =-1

Some hybridization matrix elements vanish.

→ Can close the gap by one-parameter tuning

Phase transition between QSH and insulator phases

No inversion-symmetric



- 2D : 3 variables $\vec{k} = (k_x, k_y)\,,\, m$ Can close the gap by one-parameter tuning
- 3D : 4 variables $\vec{k} = (k_x, k_y, k_z)\,,\, m$ One extra parameter
→Gap closing = a curve in the hyperspace (k_x, k_y, k_z, m)

3D systems without inversion-symmetry

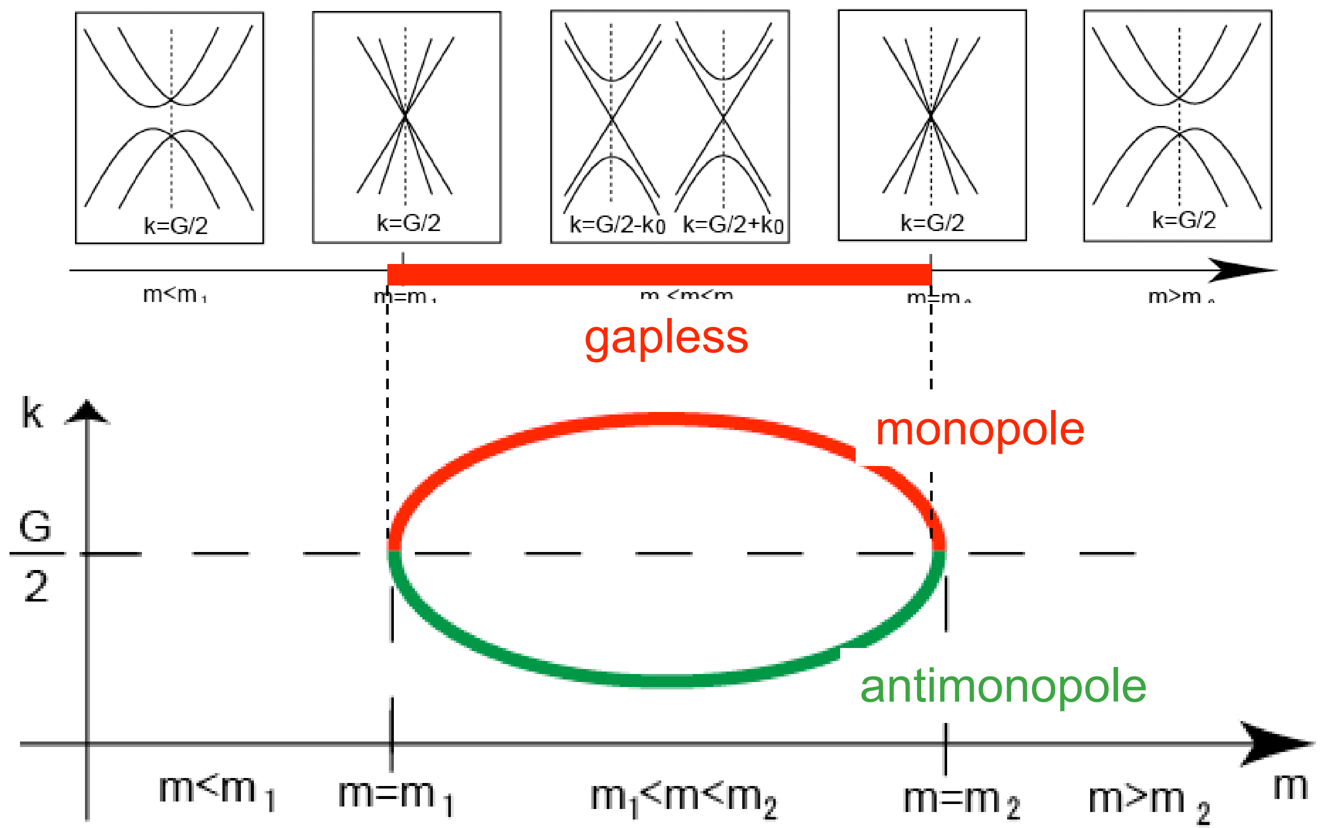
(SM, New J. Phys. (2007))

Phase transition at $m = m_0$: impossible

Instead, gapless phase appears between QSH and insulator phases.

$m_1 \leq m \leq m_2$

Pair creation of monopole and antimonopole in k-space



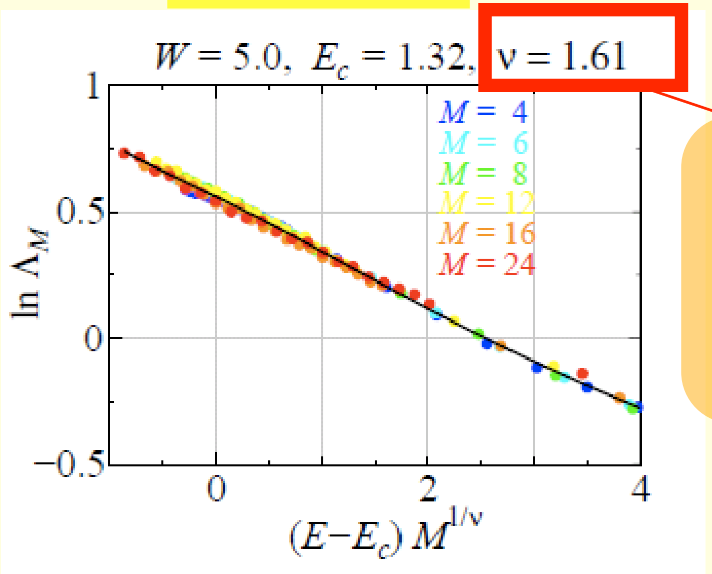
Universality classes of Anderson Localization

Time-reversal symmetric system
with the spin-orbit interaction
(symplectic universality class)

Scaling Analysis for QSH phase

Onoda, Avishai, Nagaosa, cond-mat/0605510

$$\xi \approx (E - E_c)^{-\nu}$$



Different from

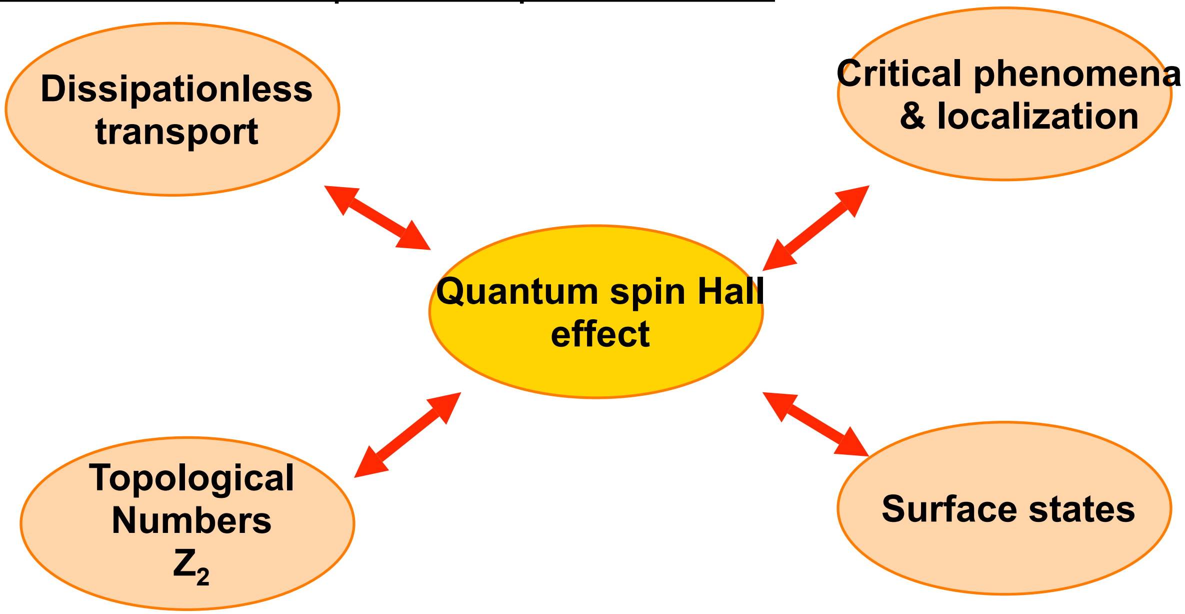
$\nu_{\text{symplectic}} = 2.73$

$\nu_{\text{unitary}} = 2.33$

New universality class?

Why is the quantum spin Hall effect interesting?

• Various fields are related with quantum spin Hall effect



- topological order in nonmagnetic insulators
- perfectly conducting channel

- No magnetic field required
  (cf. quantum Hall effect – requires strong magnetic field)

- Only 1 gapless fermion: Unique for a surface of 3D QSH system

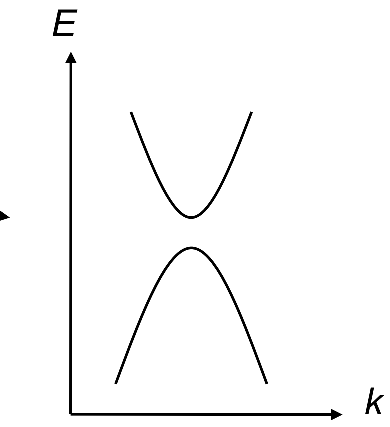
(cf. In 2D systems, number of fermions is even (fermion doubling)
2 gapless fermions in graphene)

Summary

Spin Hall effect in metals and semiconductors

- p-type >> n-type
- Enhanced at band crossing

  Pt : large spin Hall effect



Quantum spin Hall effect:

- 2D & 3D
- Edge states

  --- topologically protected
- Bismuth and surface states → spin current
- Phase transition between the quantum spin Hall phase and ordinary insulator phase.

  Topological gapless phase in 3D