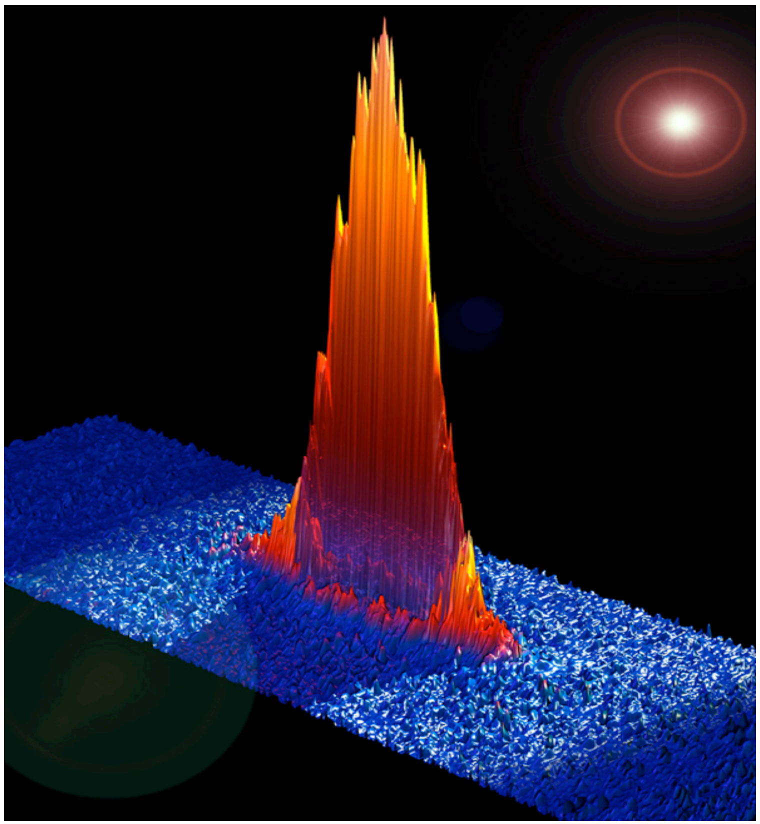
The University of Hong Kong

Exploring Novel Fermionic Superfluidity with Imbalanced Populations in Optical Lattice



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Yukawa Workshop, Kyoto, Nov. 19, 2007

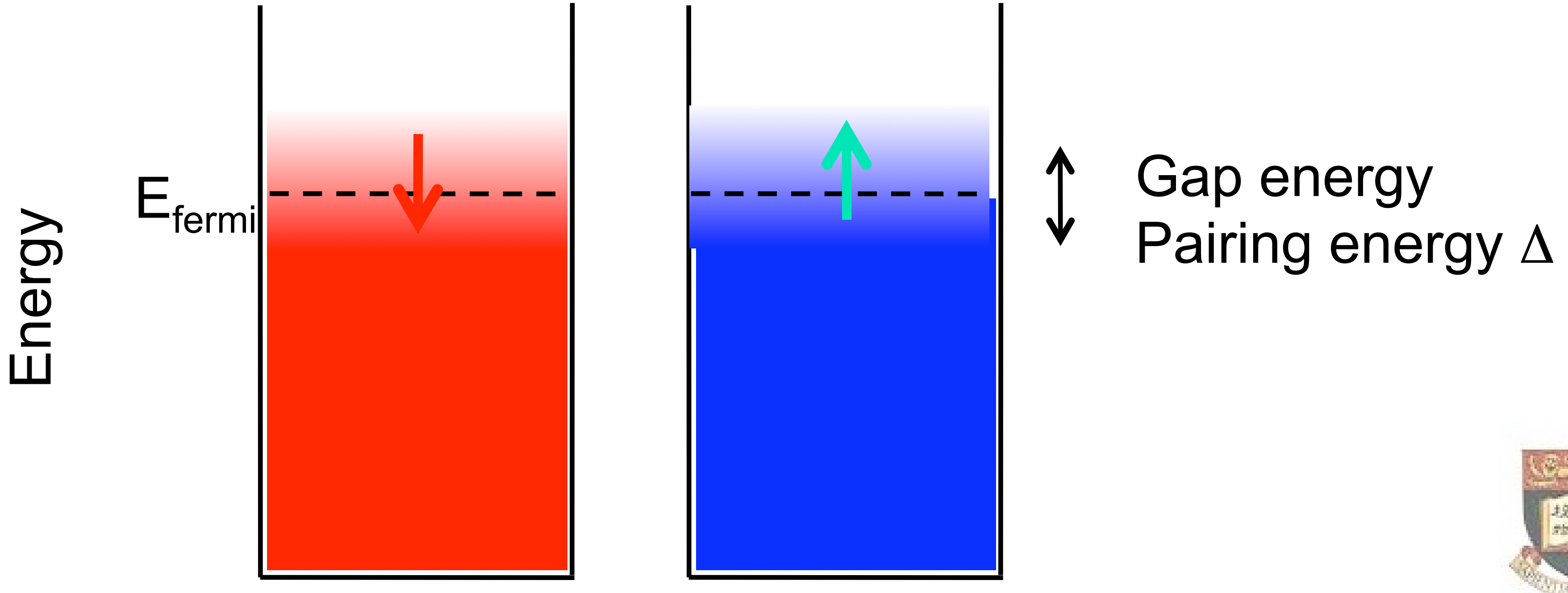
Outline

- Introduction: Imhomogenous **Fulde-Ferrell-Larkin-Ovchinnikov** state
- Ultra-cold atoms and optical lattice
- Theoretical formalism
- Main Results
- Summary

BCS Pairing of Fermions

Pairing costs kinetic energy, but there is gain in potential energy

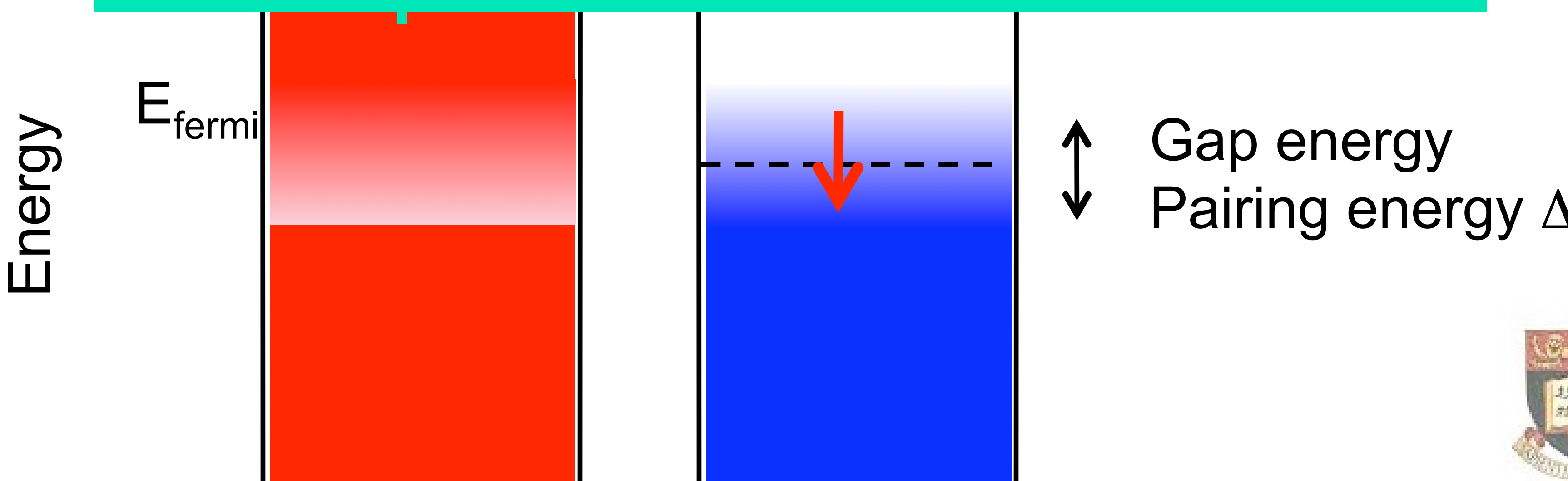
An equal number of spin-up and spin-down particles form Cooper pairs



BCS Pairing of Fermions

Unequal Fermi energies:
Formation of pairs with zero momentum costs extra kinetic energy $E_{fermi1} - E_{fermi2}$

Breakdown of the BCS state when $\Delta \approx E_{fermi1} - E_{fermi2}$



Why this study interesting?

- New superfluid states predicted:

FFLO (Fulde-Ferrell-Larkin-Ovchinnikov) – state (1964)

Cooper pairs have non-zero momentum; Pairing gap becomes a spatially varying function; Translational invariance broken

- Superfluidity in quarks (color superconductivity) involves unequal Fermi surfaces (due to different quark masses)

Ongoing search for FFLO state in superconductors:
- highly anisotropic organic superconductors
- layered heavy fermion superconductors ($CeInO_5$)

FFLO State

FF state: $\Delta = \Delta_0 e^{i\vec{Q}\bullet\vec{r}}$

- LO state: $\Delta = \Delta_0 \cos(\vec{Q}\bullet\vec{r})$
  - Excess spins stay near nodal lines of the order parameter, spin density is periodic in space
  - Existence of low-lying excitations near the nodal lines
  - In electron systems, imbalance requires a magnetic field, usually zero or in the form of vortex. Not easy to realize.

Phase diagram of atomic Fermi gases in free space,

Sheehy and Radzihovsky, 2006

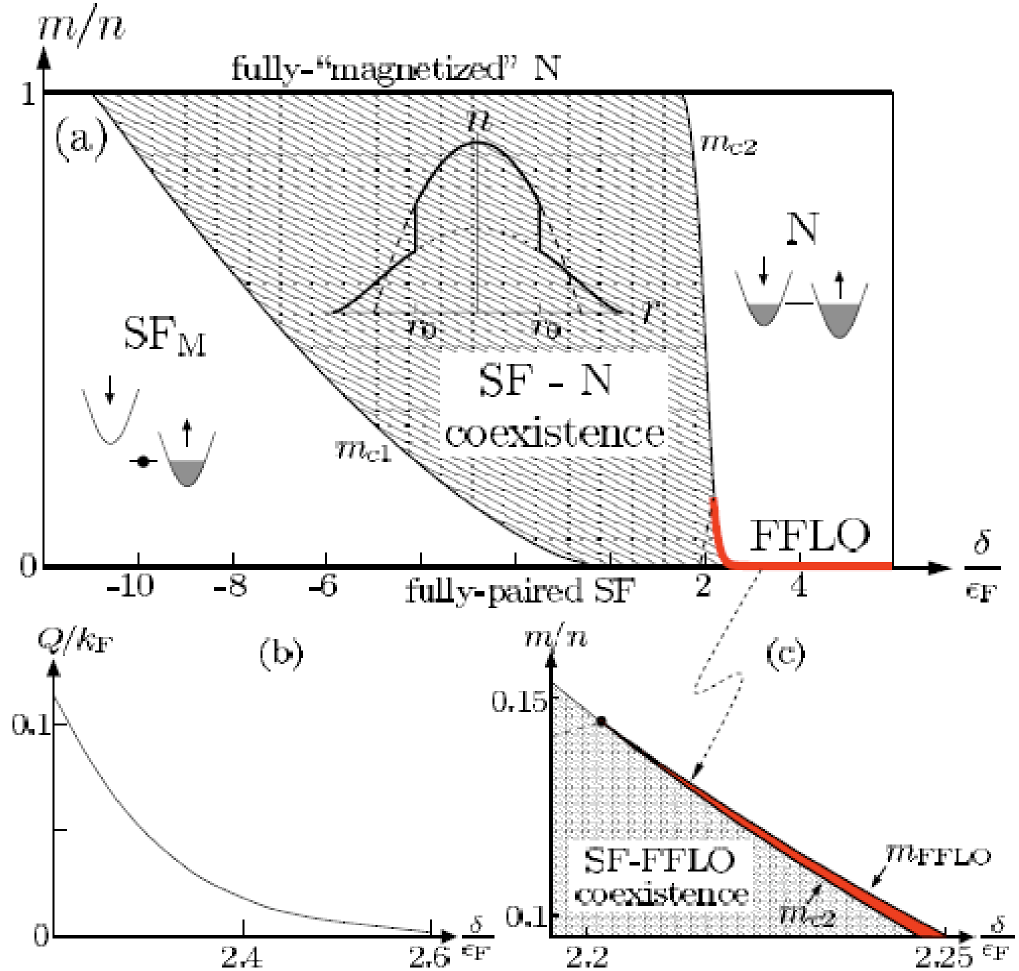
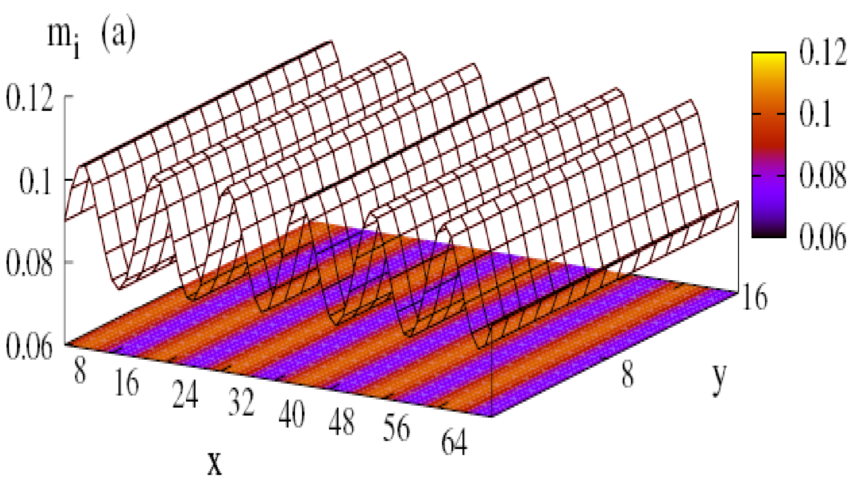


FIG. 1 (color online). Detuning, δ-population difference, $m/n = (n_\uparrow - n_\downarrow)/$
$(n_\uparrow + n_\downarrow)$ phase diagram (for coupling $\gamma = 0.1$) in (a) displaying "normal" (N), magnetized superfluid (SF_M), FFLO (thick red line) and SF-N coexistence states, (b) showing the FFLO wave vector $Q(\delta)$ along the FFLO-N phase boundary, and (c) zoom-in on the FFLO state, stable only for $\delta > \delta_* \simeq 2.2\epsilon_F$. To the right of the dashed lines in (a) and (c), the SF-N coexistence undergoes a transition to SF-FFLO coexistence.

Superconductivity (s-wave) & magnetization

- For an *s*-wave supercon-ductor, spatial profile of the order parameter shows 1D stripes:

On nodal lines Δ_{ii} =0.



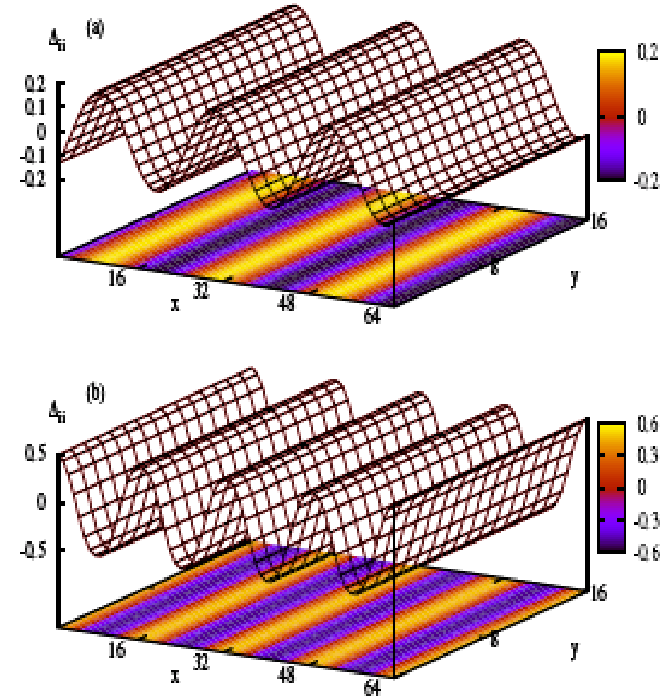
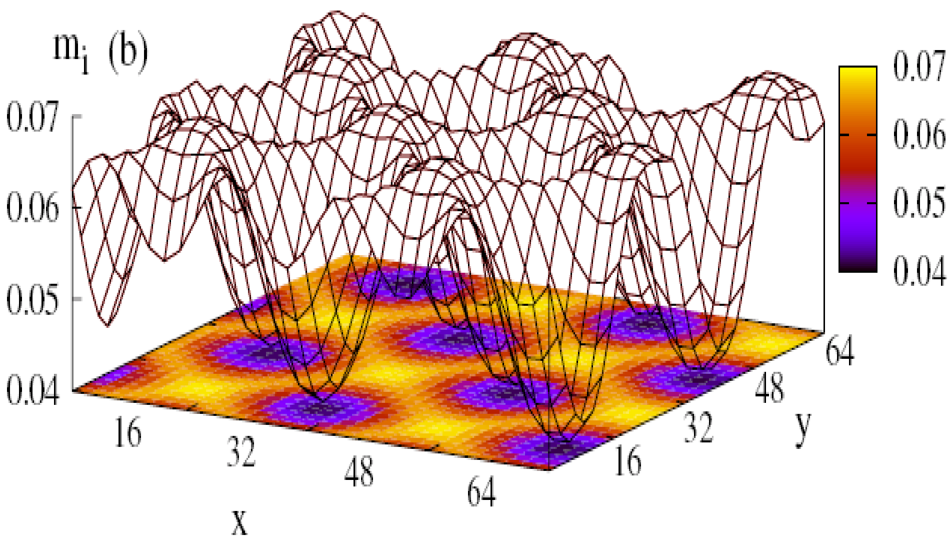


FIG. 1: Plots of the OP Δ_{ii} as a function of position for an s-SC. For (a), $V = 2.0$, $h = 0.25$; for (b), $V = 2.5$, $h = 0.4$. Here the chemical potential $\mu = -0.4$.

Q. Wang et al., Phys. Rev. Lett. 96, 117006 (2006).

SC versus Magnetization (d-wave)

- For a *d*-wave superconductor, spatial profile of the order parameter shows a 2D structure.



lower energy than stripe state

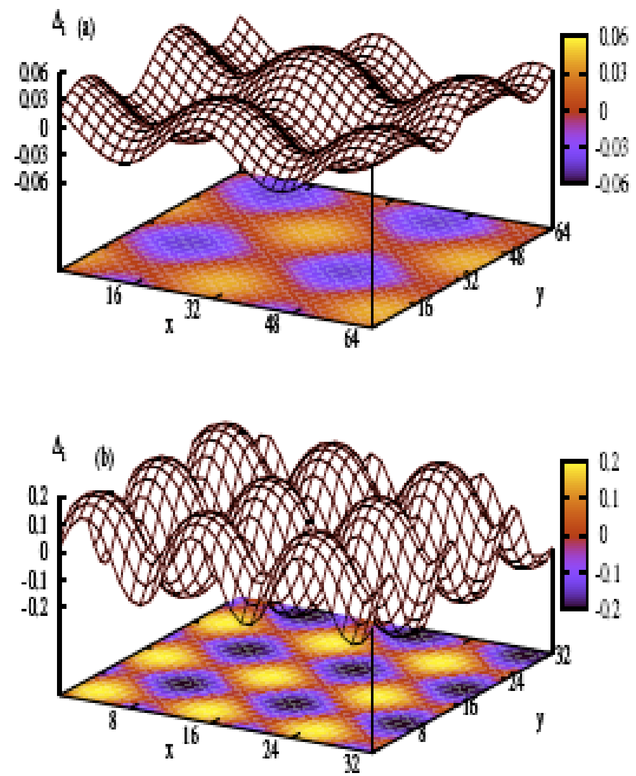


FIG. 2: Plots of the OP Δ_i as a functions of position for a d-SC. For (a), $V = 1.0$, $h = 0.15$ and $\mu = -0.4$; for (b), $V = 2.0$, $h = 0.4$ and $\mu = -0.4$.

Ultra-cold atoms and optical lattice

- Trapped fermionic atoms interacting via Feshbach resonance
- Possible unequal populations of two pairing hyperfine states

  -- already realized in two experiments
- Optical lattice generated by standing light wave in a Guassian potential (harmonic) confinement
- Provided interacting systems to study FFLO state
- Unequal fermionic atoms in optical lattice are accessible

Atomic Fermi gases with imbalanced

M.Zwierlein et al., Science 311, 492 (2006).
M.Zwierlein et al., Nature 442, 54 (2006).
Y. Shin, et al., Phys. Rev. Lett. 97, 030401 (2006).
G. Partridge et al., Science 311, 503 (2006).
G. Partridge et al., Phys. Rev. Lett. 97, 190407 (2006).

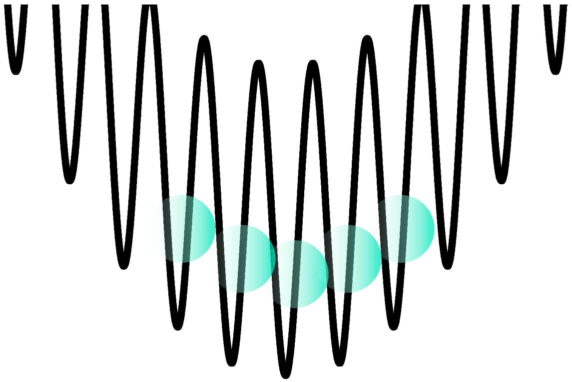
Theories: Sheehy and Radzihovsky, Ho, Machida, Muller and de Silva …..

Fermions in an optical lattice

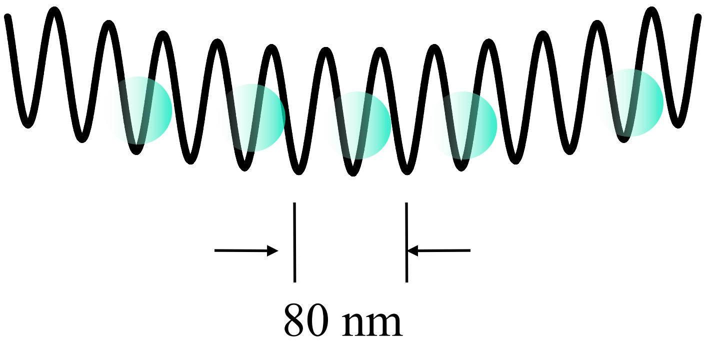
Confining Potential:

Magnetic Trap + Standing wave minimum

tight confinement



weak confinement

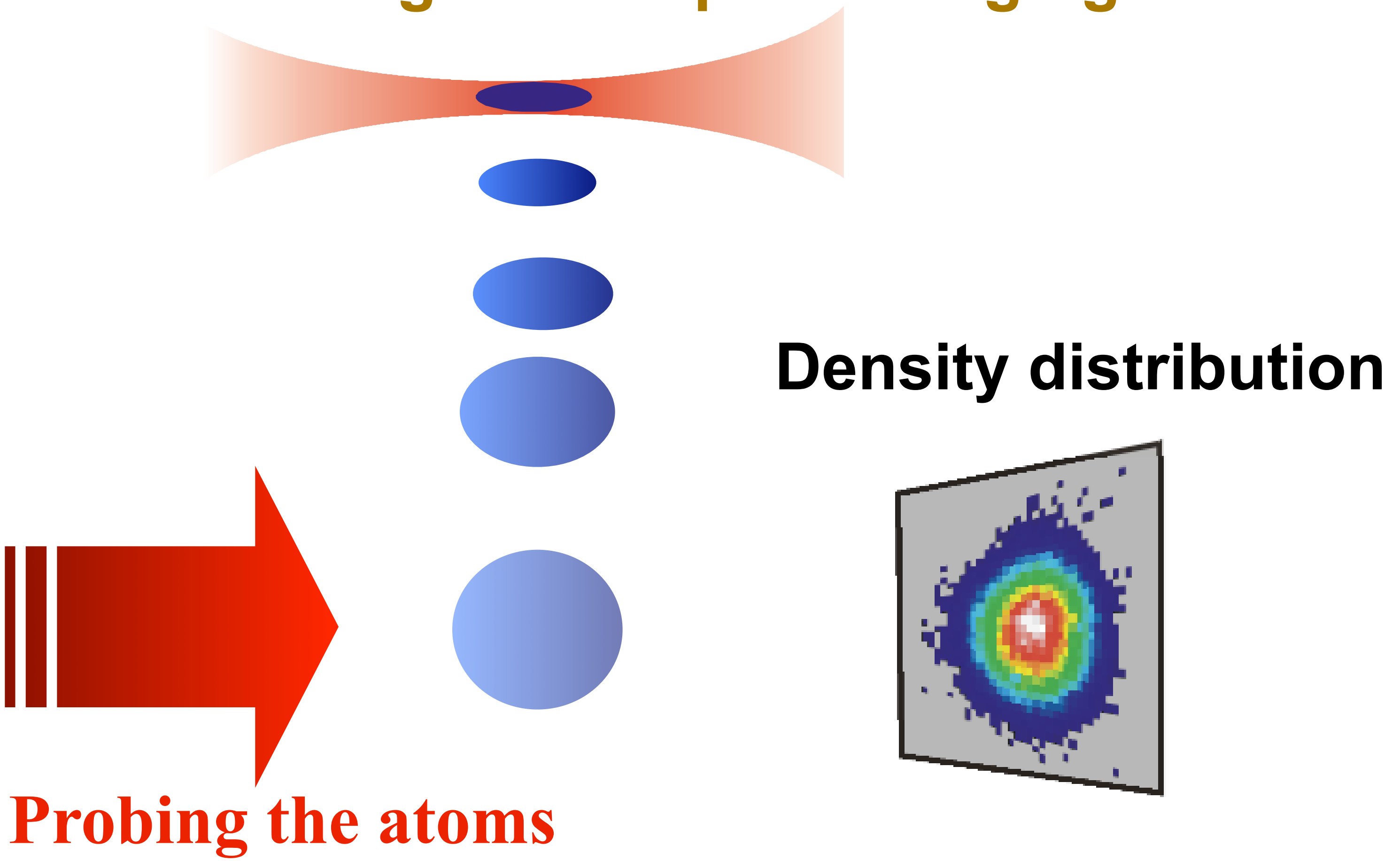


One can effectively change the band width or filling factor by tuning the amplitude or frequency of laser beam!!

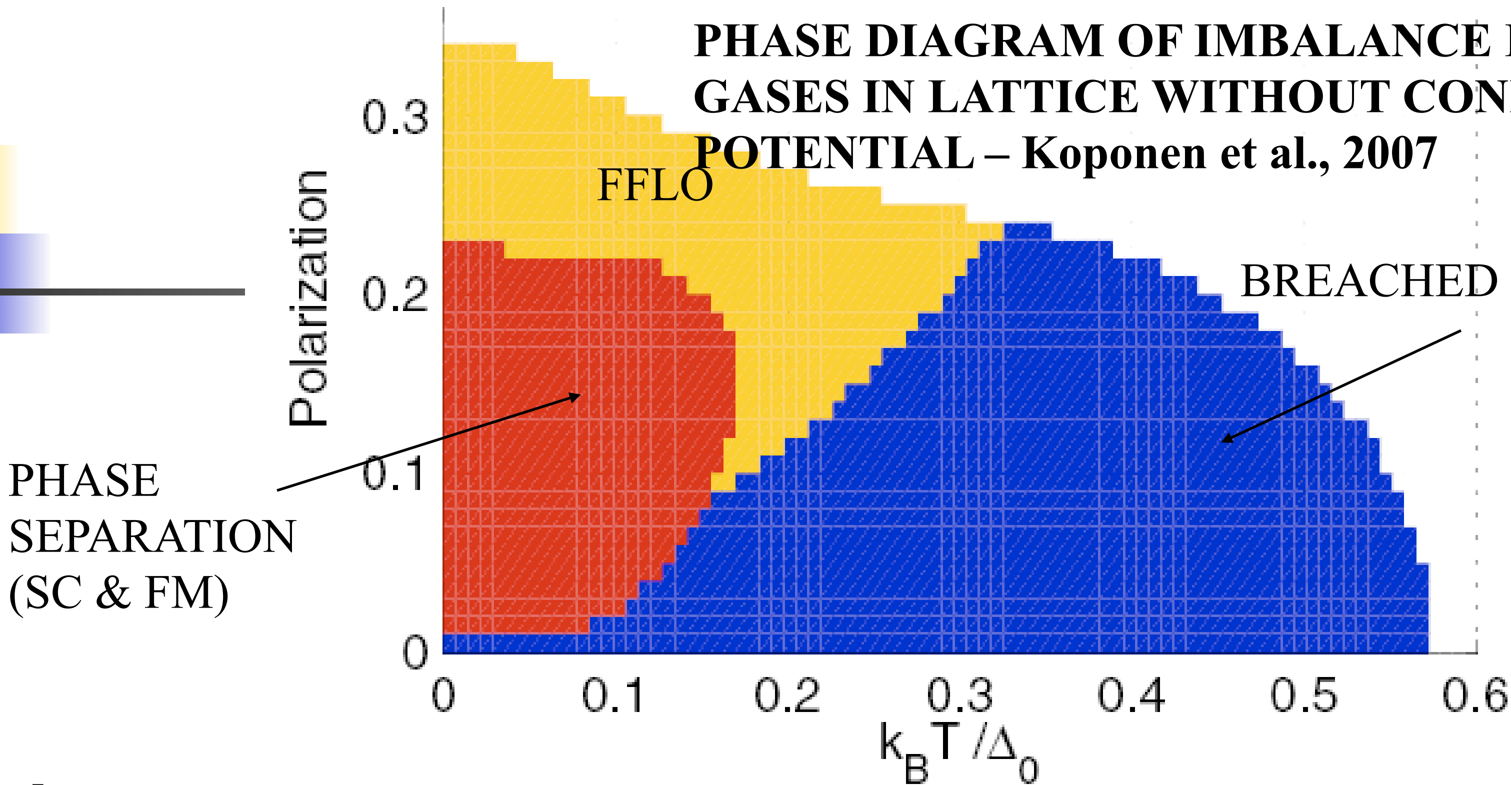
M. Rigol and A. Muramatsu PRA 69, 053612 (2004)

Imaging the ultracold gases

Time-of-flight absorption imaging



PHASE DIAGRAM OF IMBALANCE FERMI GASES IN LATTICE WITHOUT CONFINING POTENTIAL – Koponen et al., 2007

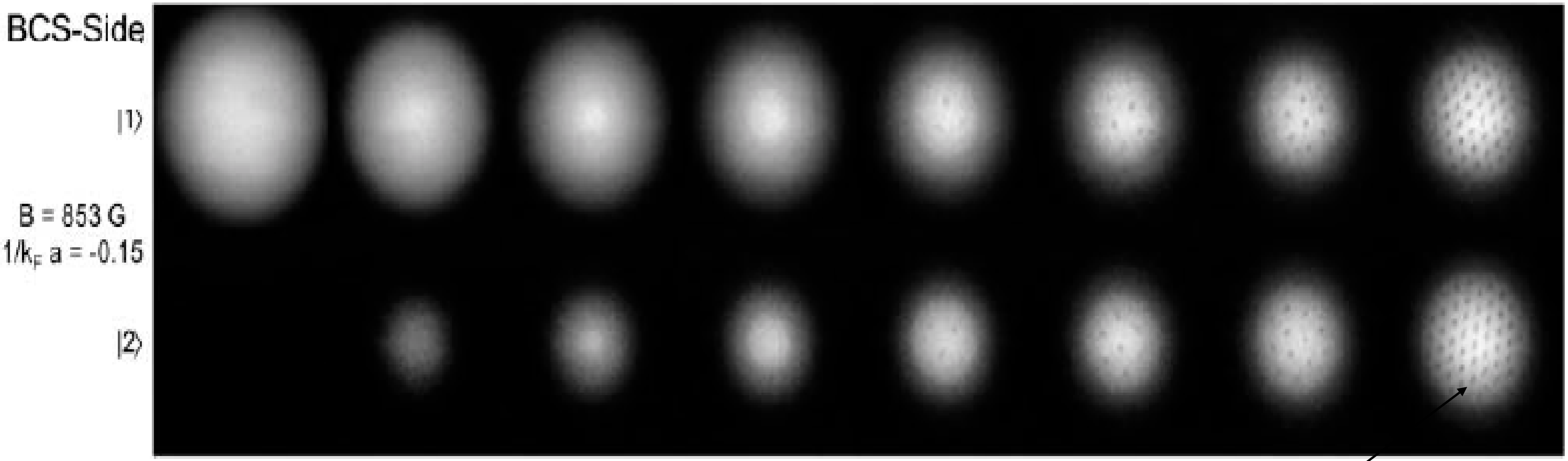


Breached pair state: BCS + spin polarized q.p.

FIG. 1 (color online). The phase diagram of the imbalanced Fermi gas in a lattice. Colors (or shading): BP = blue or dark gray (at $P = 0$ this is actually BCS), FFLO = yellow or light gray, PS = red or gray, normal = white. The average filling is 0.2 atoms/lattice site in each component, $J = 0.07E_R$, and $U = -0.26E_R$, where $E_R = \hbar^2 k^2/2m$ is the recoil energy, $k = 2\pi/\lambda$ and $\lambda = 1030$ nm. Here Δ_0 means the gap at $T = 0$ and $P = 0$. The T_C at $P = 0$ is 41 nK.

MIT Experiment

- Uses rotation to create vortices and measures density profiles after expansion and magnetic field sweep:



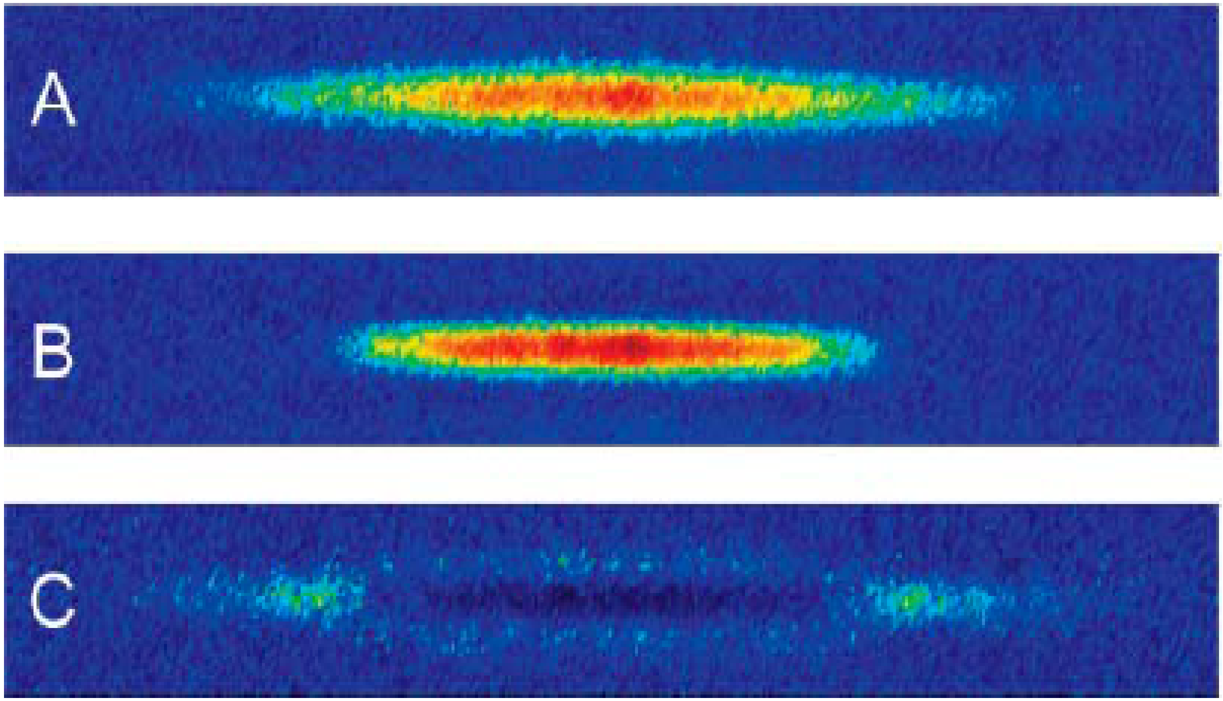
Vortex in rotating system

M. Zwierlein et al., Science 311, 492 (2006)

Rice Experiment (I)

SC in imbalanced Fermi gas, phase separation. Not clear if any FFLO

- Uses in-situ imaging:



6Li

G.B. Partridge, W. Li, R.I. Kamar, Y. Liao, and R.G. Hulet, Science (2006).

Theoretical Formalism

Model and Hamiltonian

2D lattice model for interplay among pairing, ferromagnetism, as well as confining harmonic potential

$$\hat{H} = -t \sum_{\langle ij \rangle, \sigma} \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} - \sum_{i\sigma} \mu_\sigma \hat{n}_{i\sigma} + \sum_{i\sigma} \epsilon_i \hat{n}_{i\sigma} \quad (1)$$

$$+ \sum_{\langle ij \rangle} \left[\Delta_{ij} c^{\dagger}_{i\uparrow} c^{\dagger}_{j\downarrow} + \mathrm{H.c} \right].$$

where $\mu_\sigma = \mu - \sigma h$ and $\epsilon_i = \frac{1}{2} m \omega^2 |\mathbf{r_i} - \mathbf{r_0}|^2$

h : effective field, or imbalance of 2 fermions, r_0 : trap center

an approach beyond “local density approximation”, spatial dependence follows contours of trapping potential in uniform systems

Solving Bogoliubov-de-Gennes equations

$(u^n_{j\sigma}, v^n_{j\bar{\sigma}})$: the Bogoliubov quasiparticle amplitudes on the *j*-th site.

The superconducting order parameter satisfies self-consistency condition:

$$\sum_j \begin{pmatrix} H_{ij,\sigma} & \Delta^*_{ii} \\ \Delta_{ii} & -H^*_{ij,\bar{\sigma}} \end{pmatrix} \begin{pmatrix} u^n_{j\sigma} \\ v^n_{j\bar{\sigma}} \end{pmatrix} = E_n \begin{pmatrix} u^n_{i\sigma} \\ v^n_{i\bar{\sigma}} \end{pmatrix}$$

$$H_{ij,\sigma} = -t_{ij} - (\mu + \sigma h)\delta_{ij}$$

$$\Delta_{ii} = \frac{V}{4} \sum_n (u^n_{i\uparrow} v^{n*}_{i\downarrow} + u^n_{i\uparrow} v^{n*}_{i\downarrow}) \tanh(E_n/2k_B T)$$

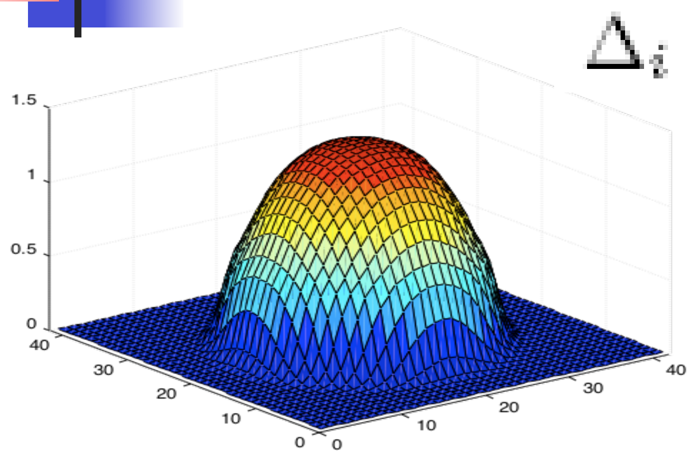
Consider 42X 42 square lattice use t=1 as energy unit, V=4, T=0.001.

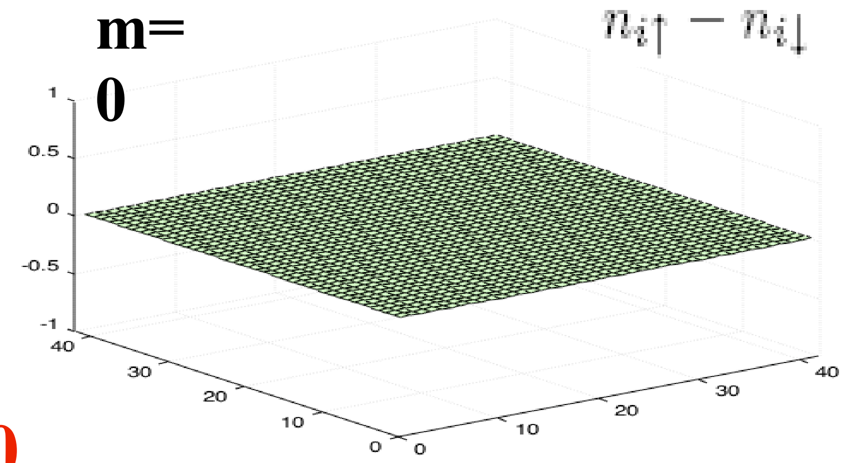
Variables: magnetic field h, total electron density.

Main Results

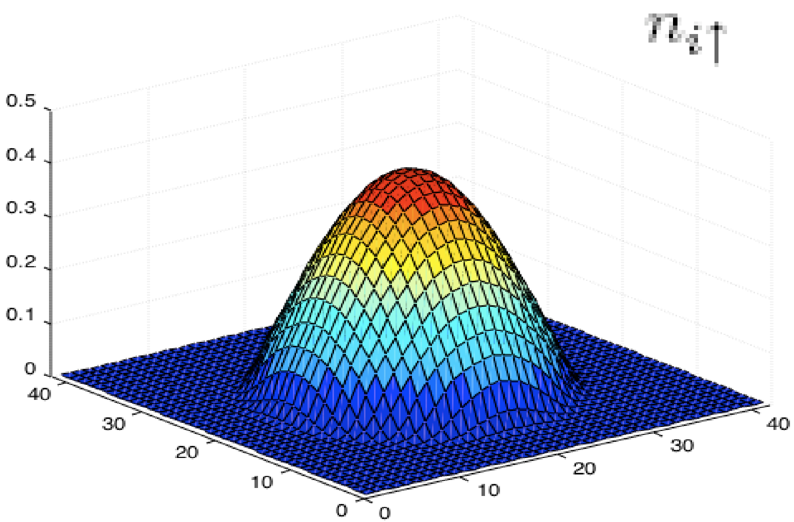
Low Density (n=200, filling ~0.12): h=0 case

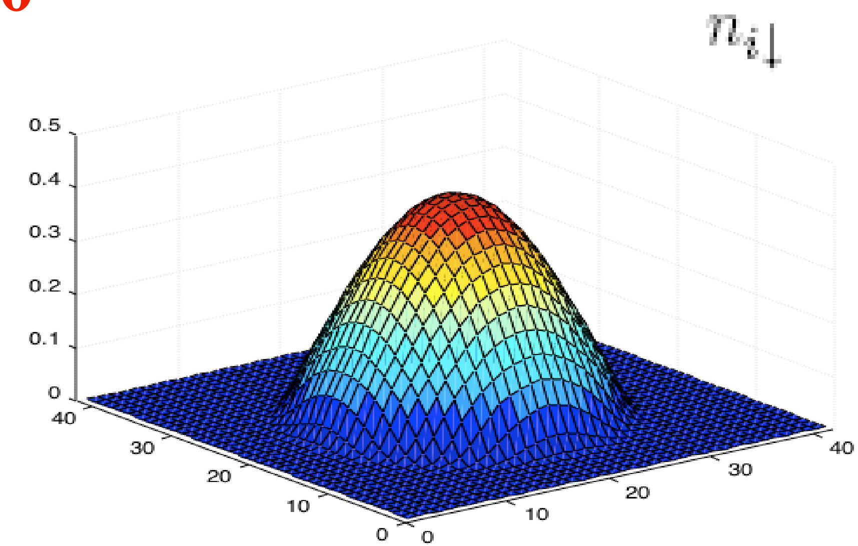
no magnetism, trapping potential results in electron accumulation around the center





h=0.0



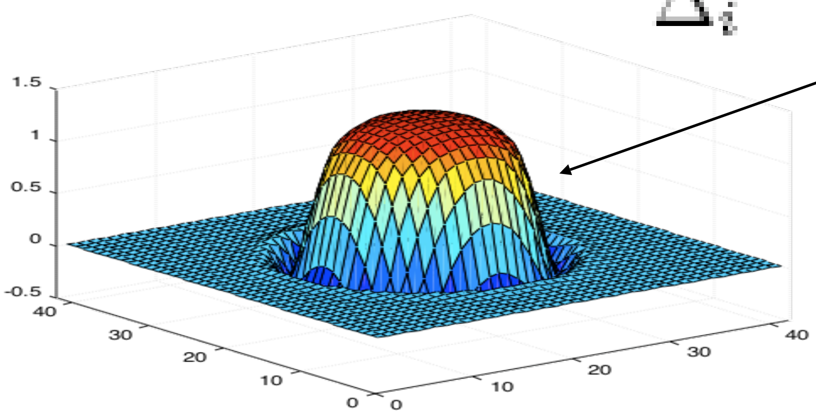


Low density (filling ~0.12): medium field

SC & FM compete. 3 regions: SF with equal population, partially polarized shell, fully polarized

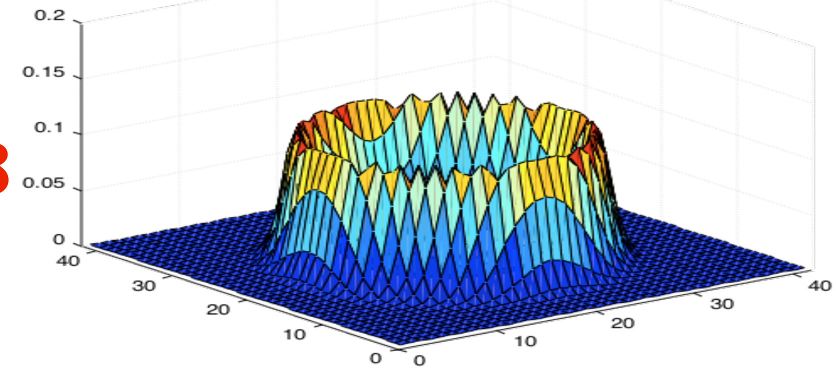
Δ_i

Radial

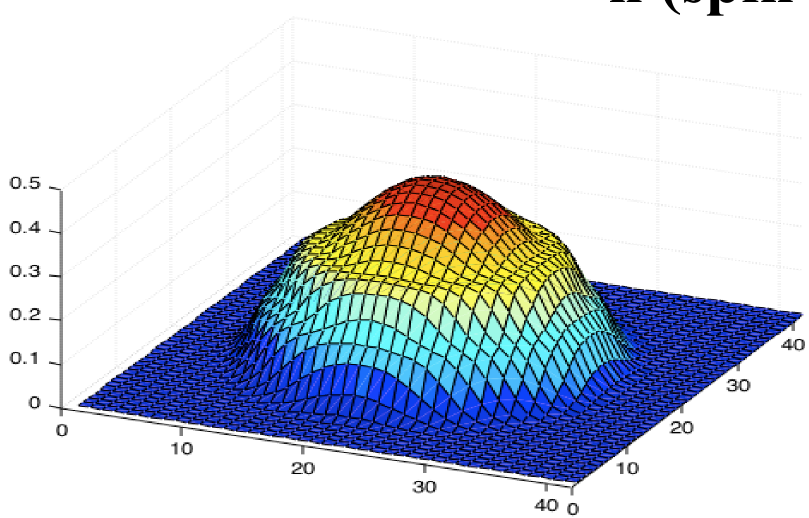


h=0.8

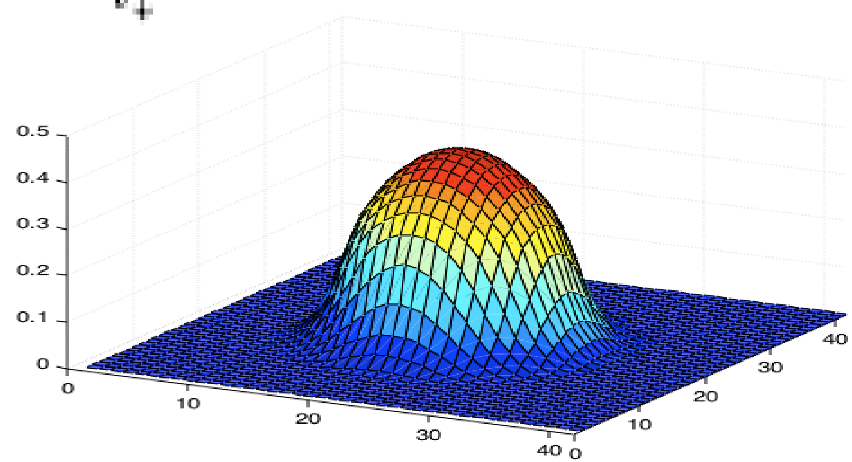
$n_{i\uparrow} - n_{i\downarrow}$



n (spin up)

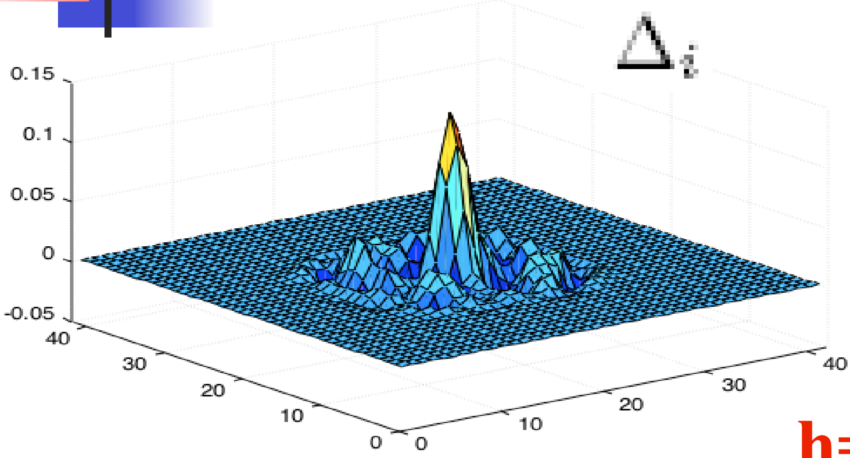


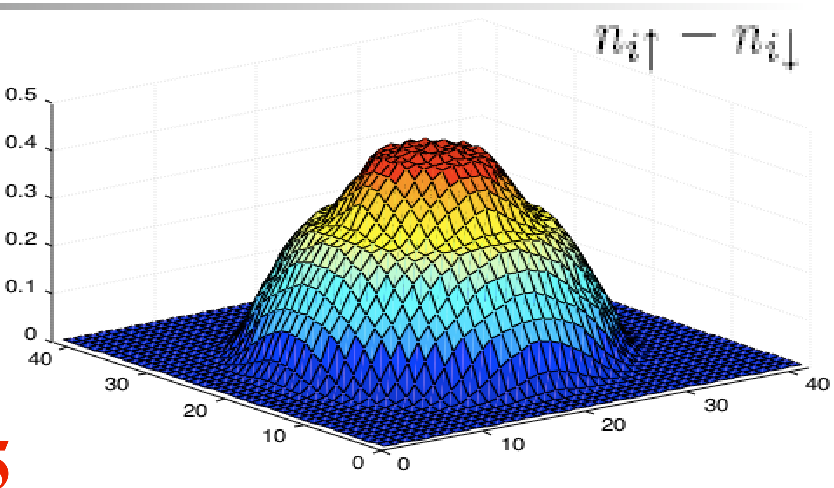
$n_{i\downarrow}$: squeezed to center



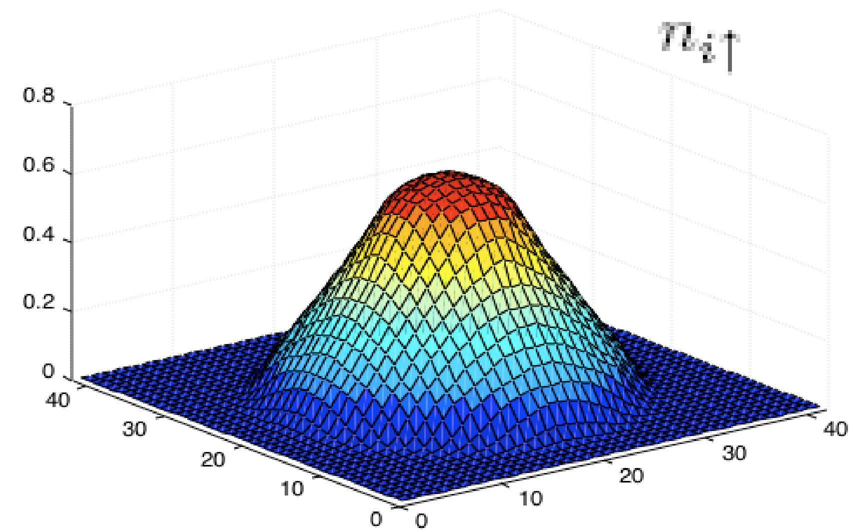
Low Density (filling~0.12): high field

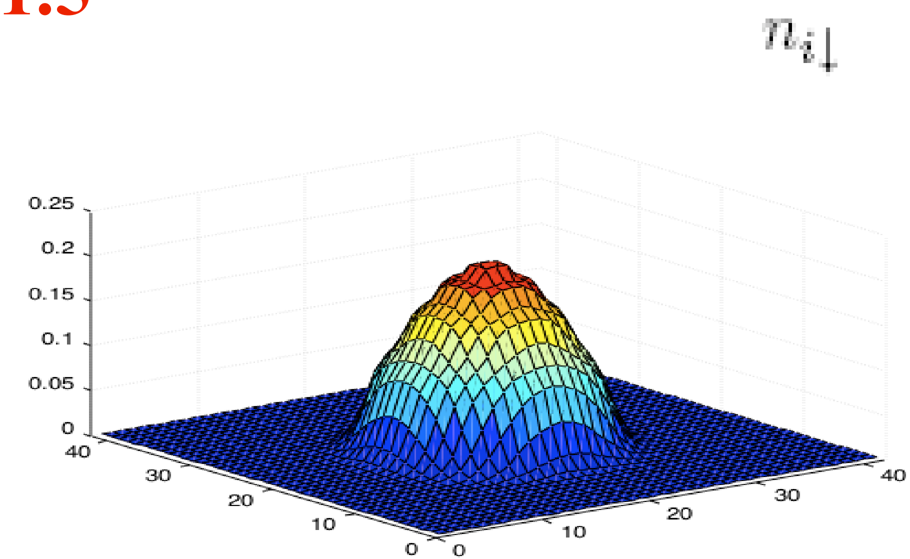
(critical field hc=1.6, no SC, Clogston limit)





h=1.5





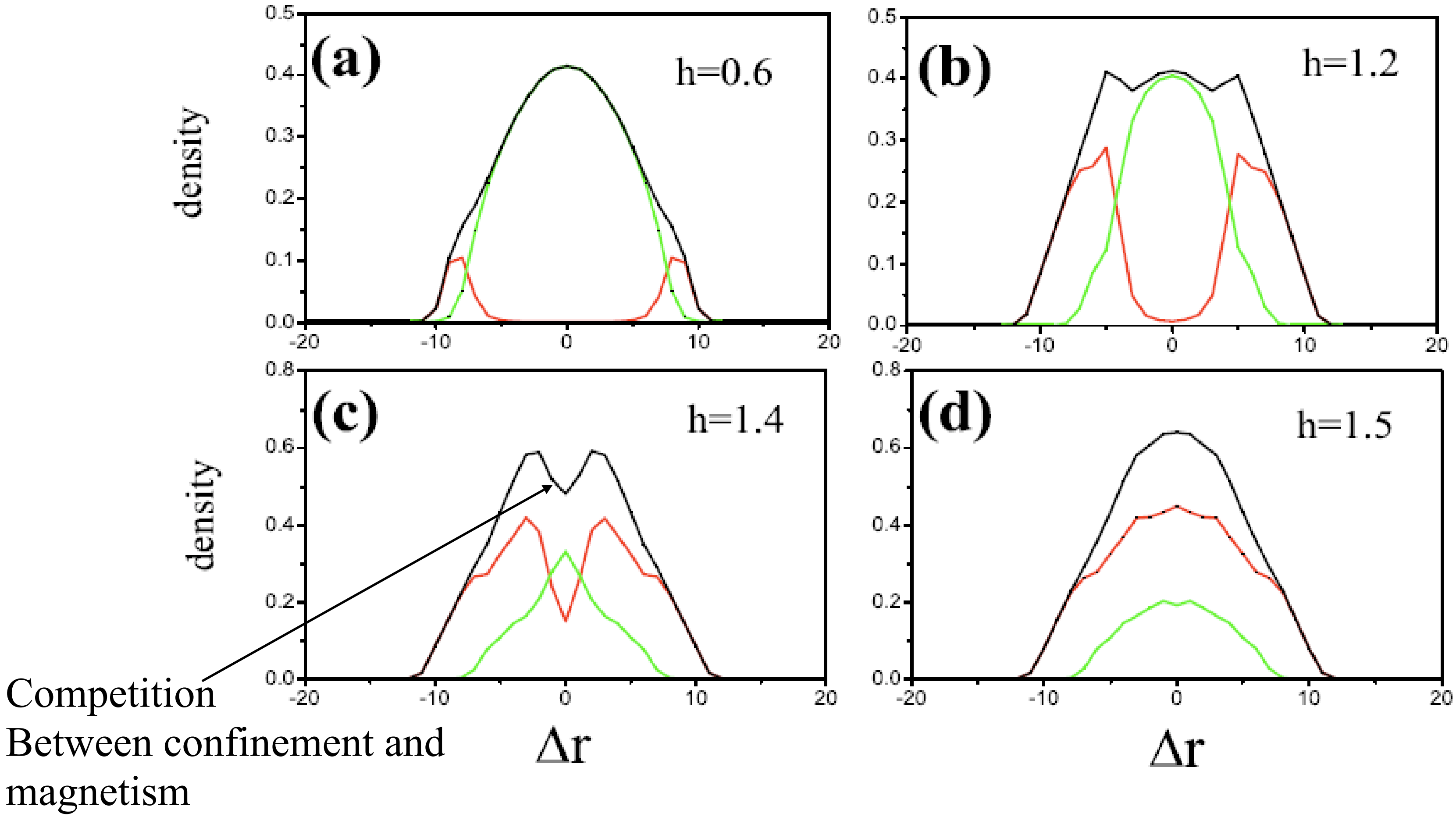
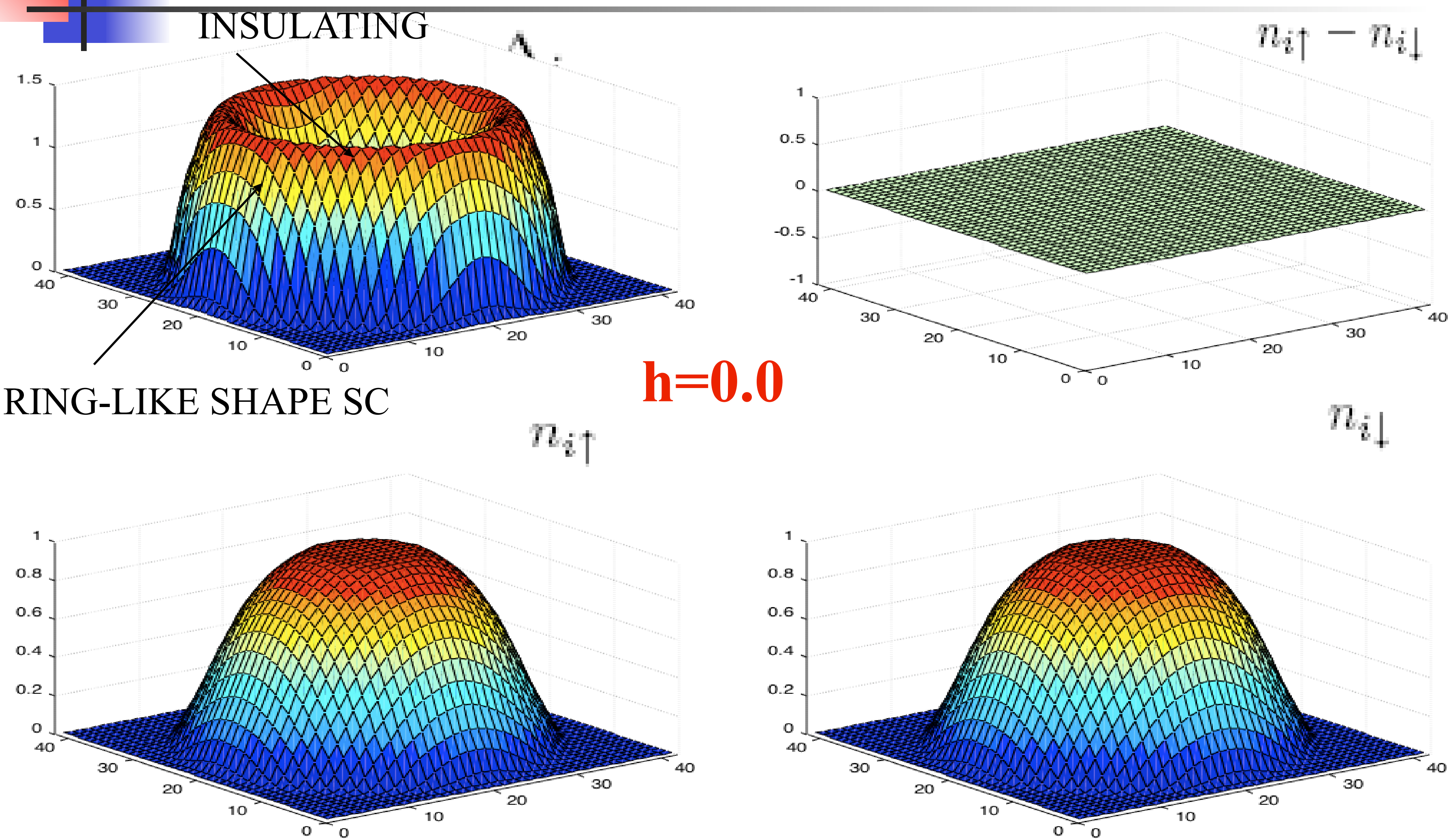


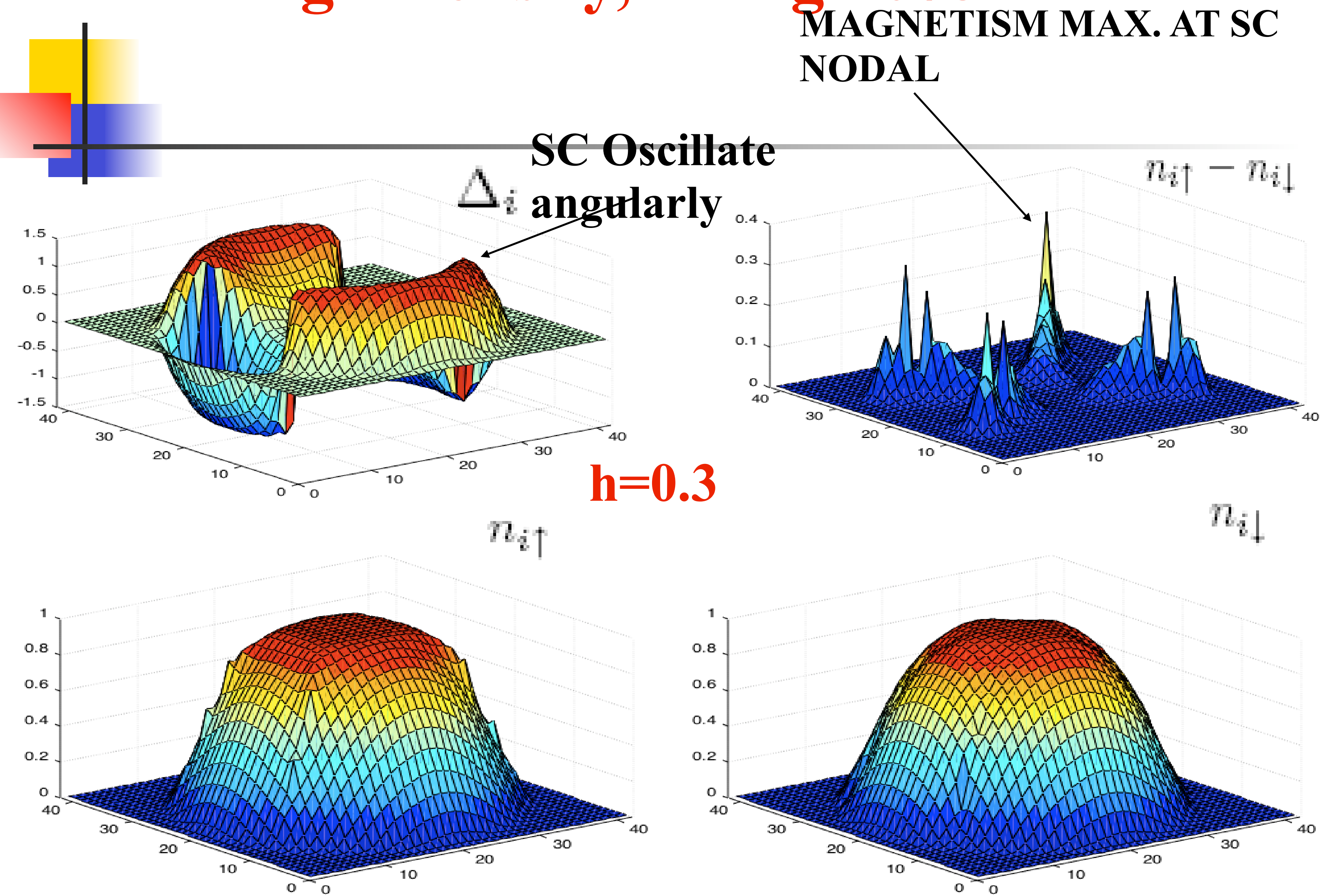
FIG. 2: (Color online) Density profiles along the diagonal of the lattice. Δr is the distance from the trap center. Red line denotes the magnetization while the black and green lines represent the spin-up and spin-down densities, respectively.

High density (n=1200, filling~0.75)

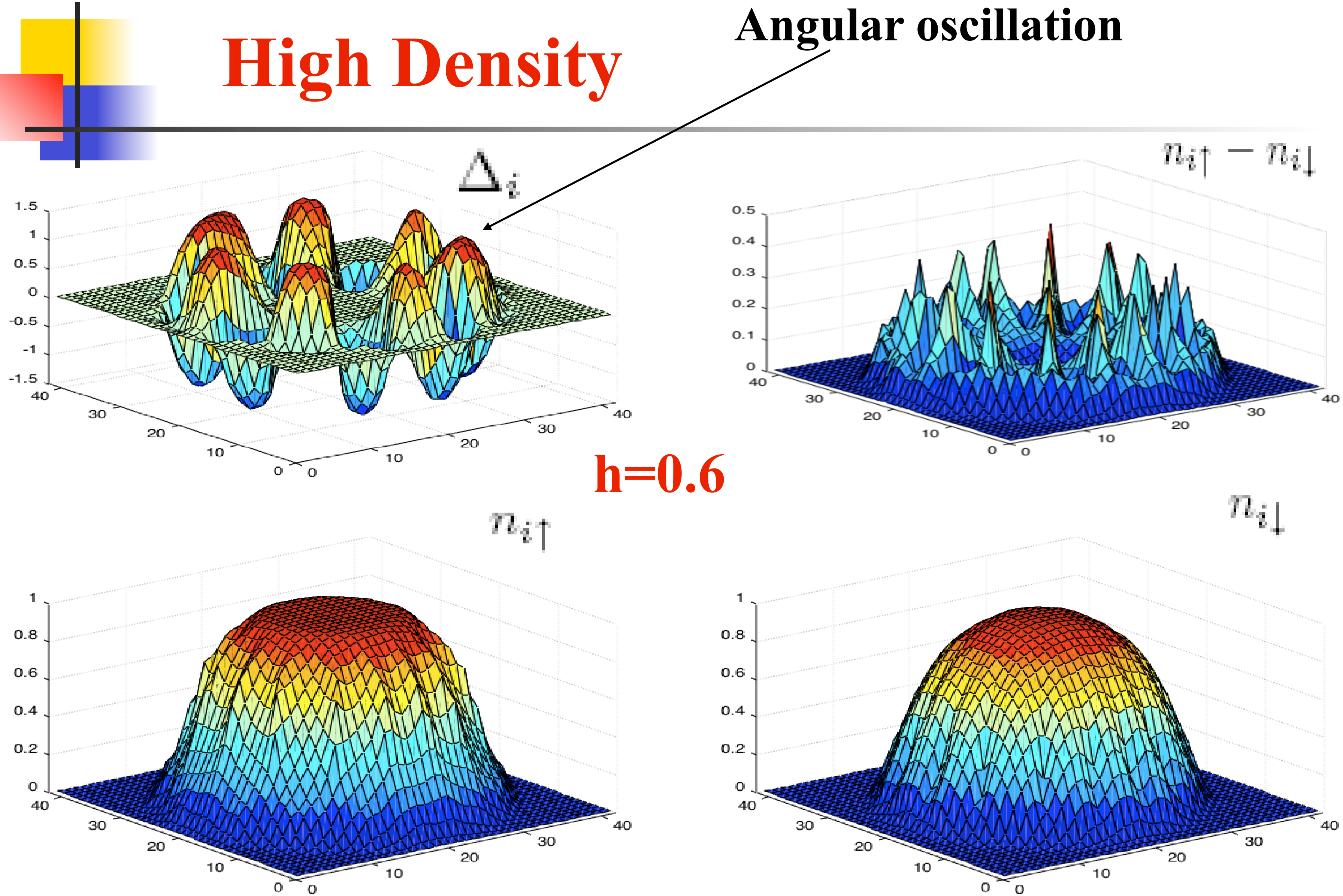
band-like insulator around center for both spin states fully occupied



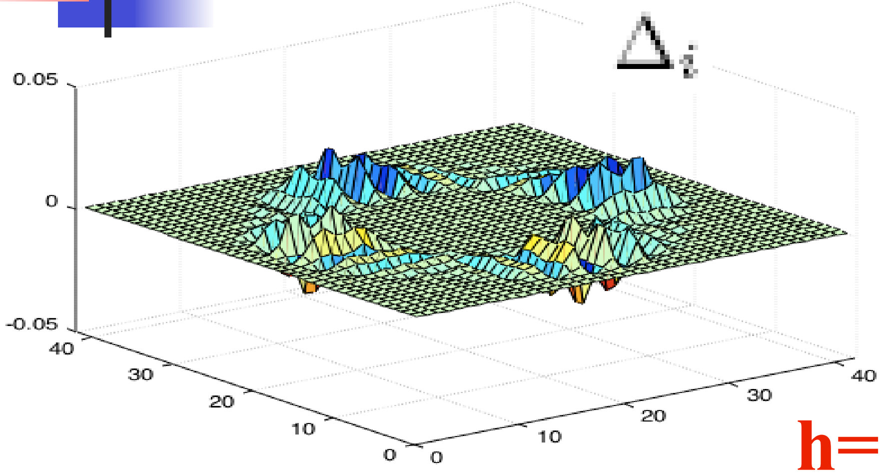
High Density, filling ~075

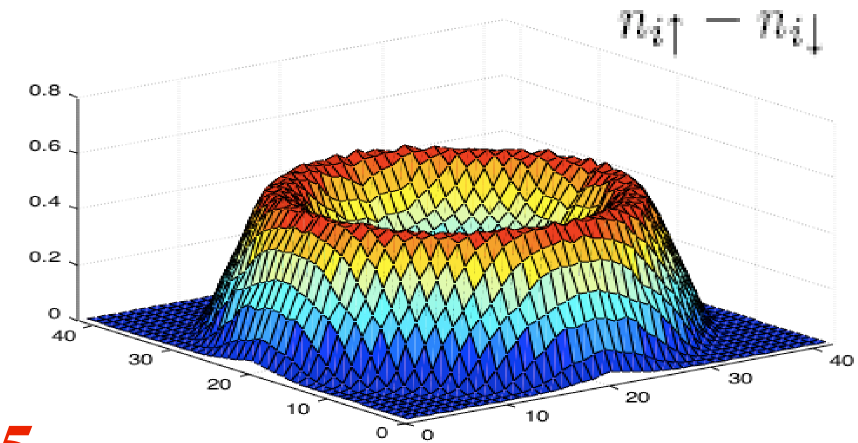


High Density

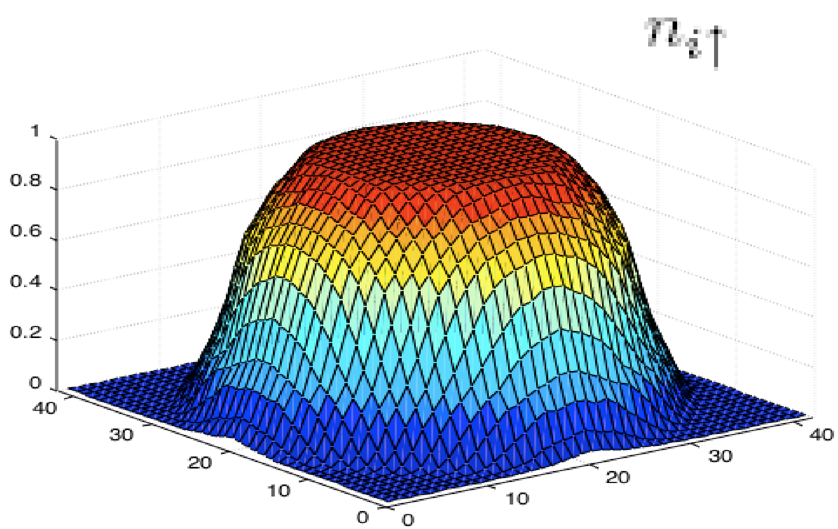


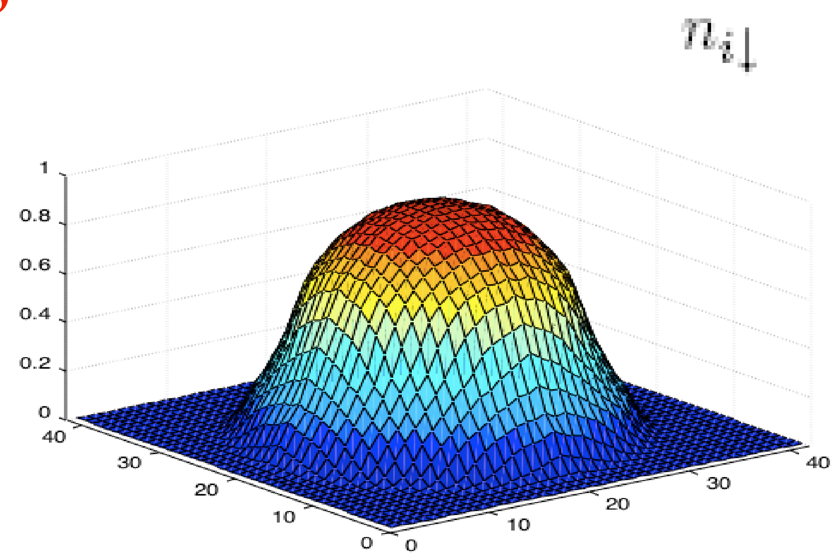
High Density. SC almost vanishes, Insulating at core and FM on a shell





h=1.5

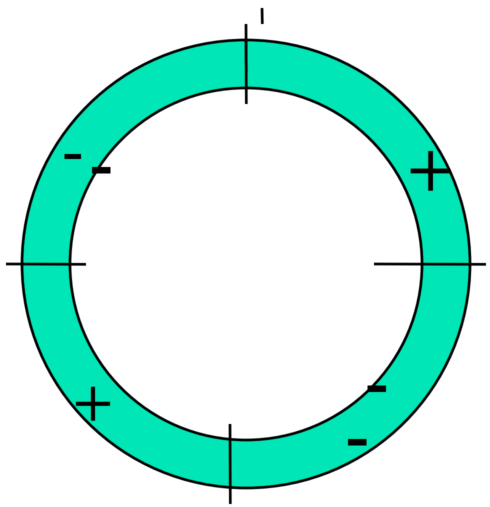




Qualitative explanation for angular oscillation

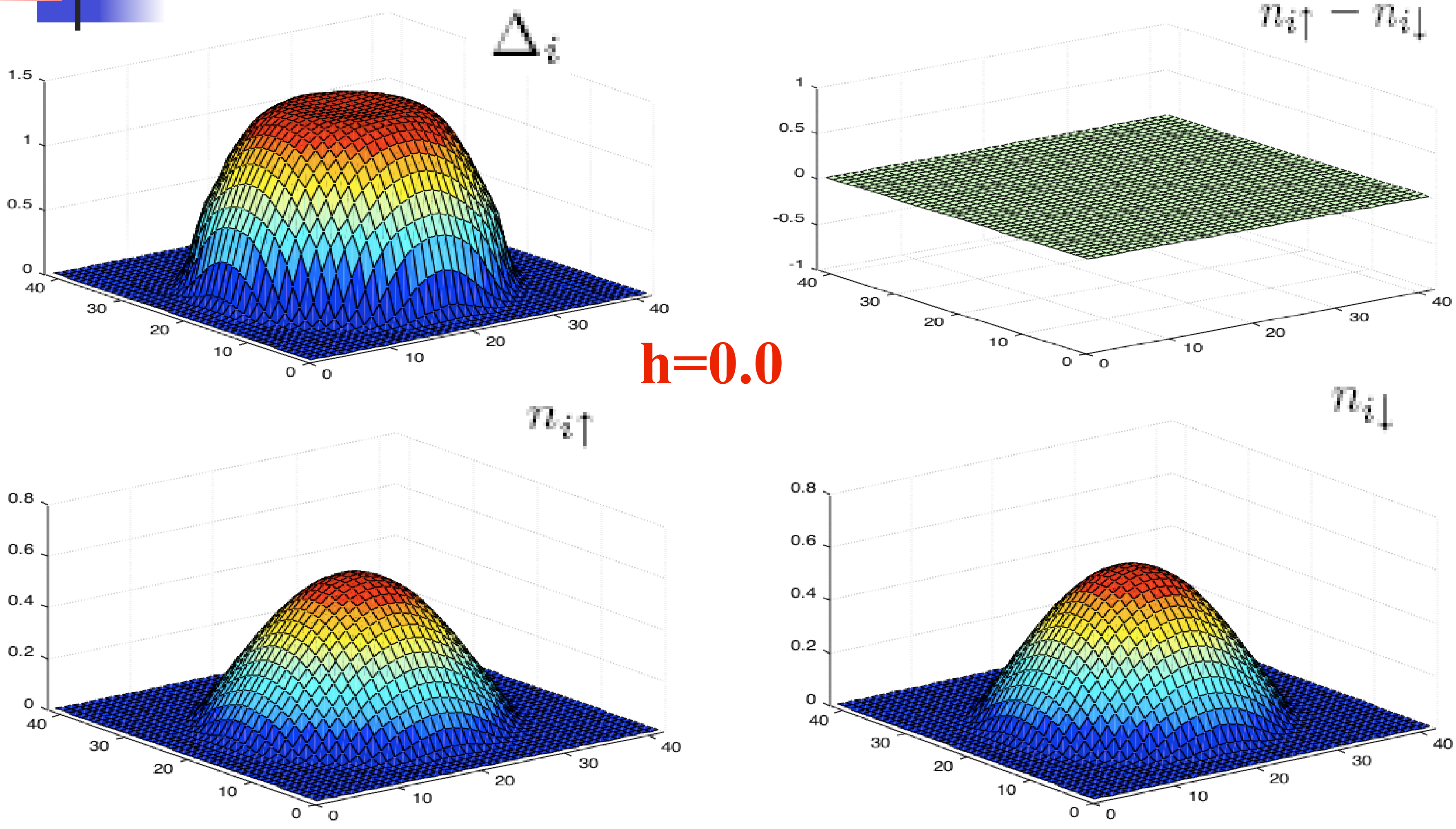
- Center area, fully occupied, insulating

2D becomes an effective “ring-like”

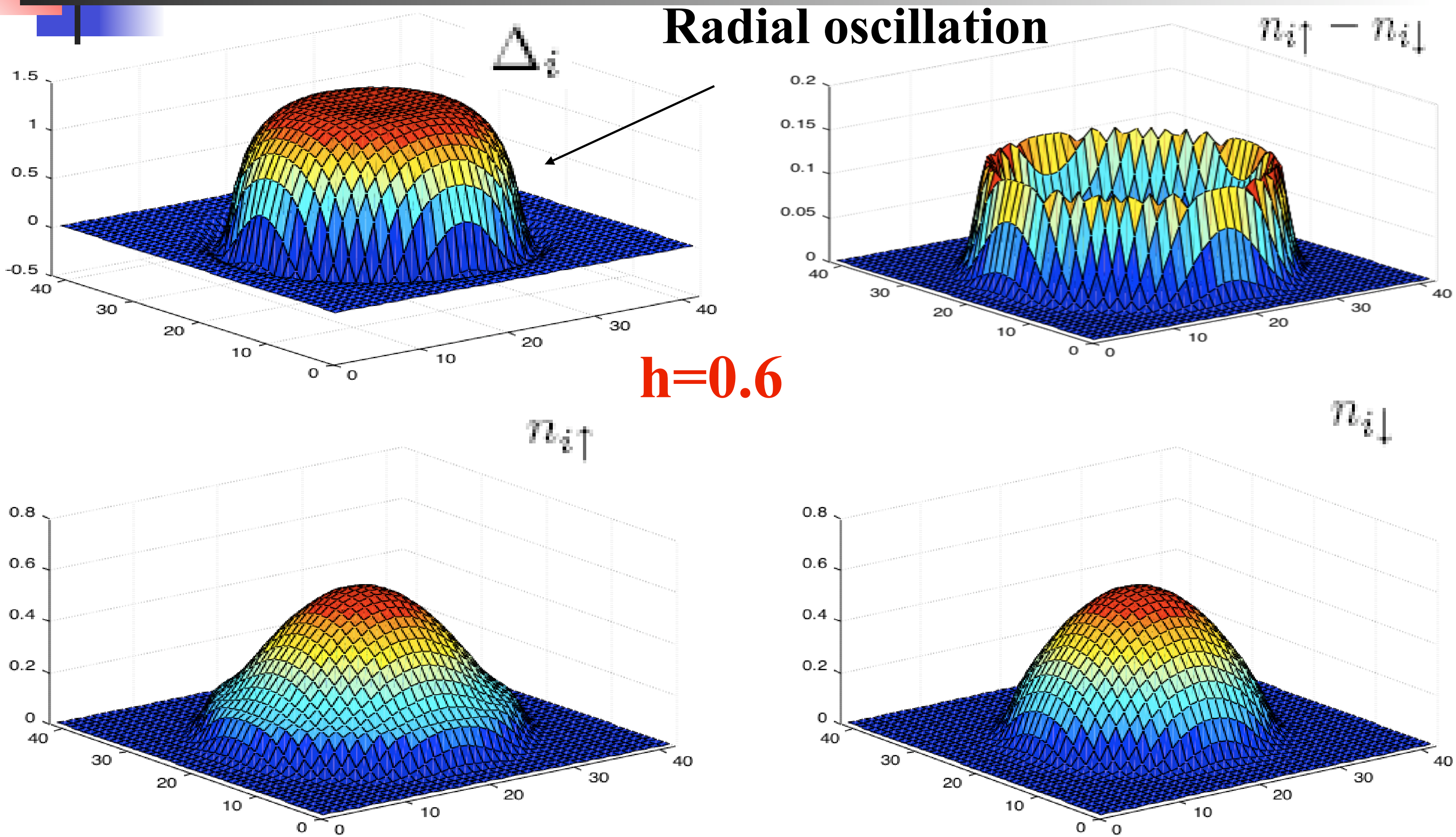


•Magnetic field tends to generate FM region or nodal in SC
• reduce Free energy for zeroes accross ring for short distance
• Sign change at nodal line to reduce “kinetic energy”

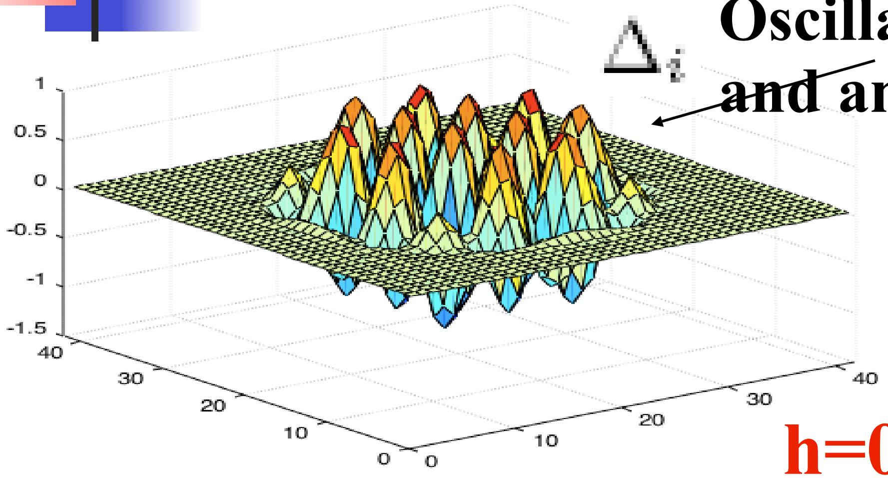
Medium Density (n=400) (1)



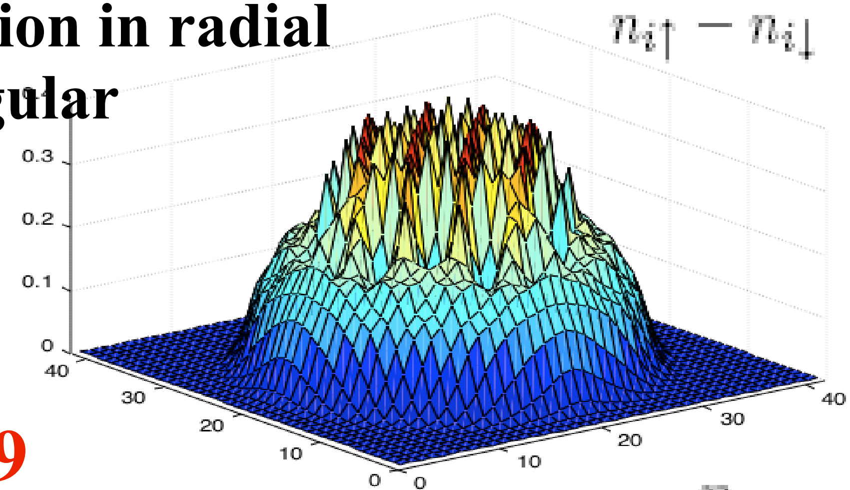
Medium Density (n=400)



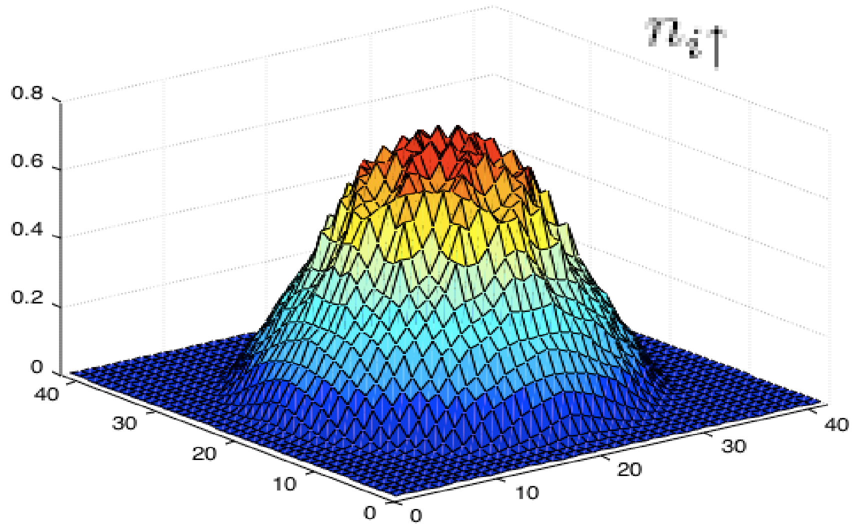
Medium Density (n=400)

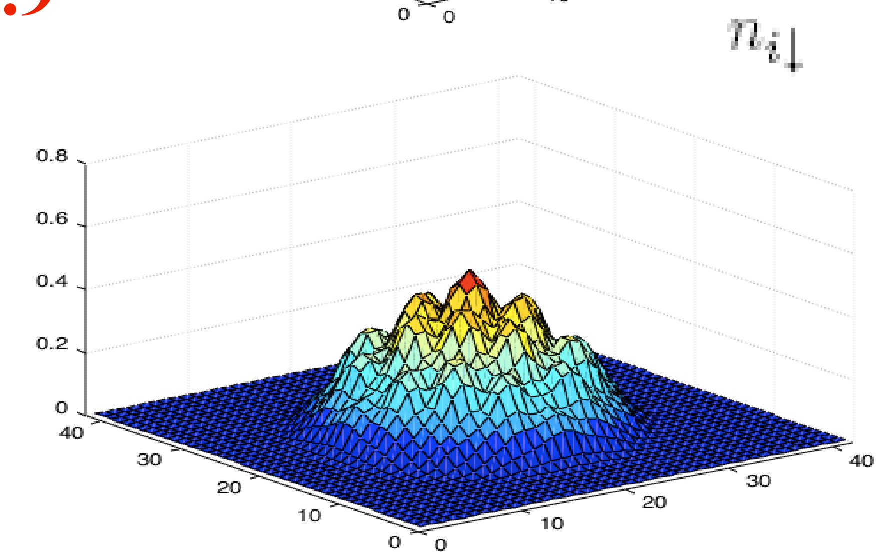


Oscillation in radial and angular

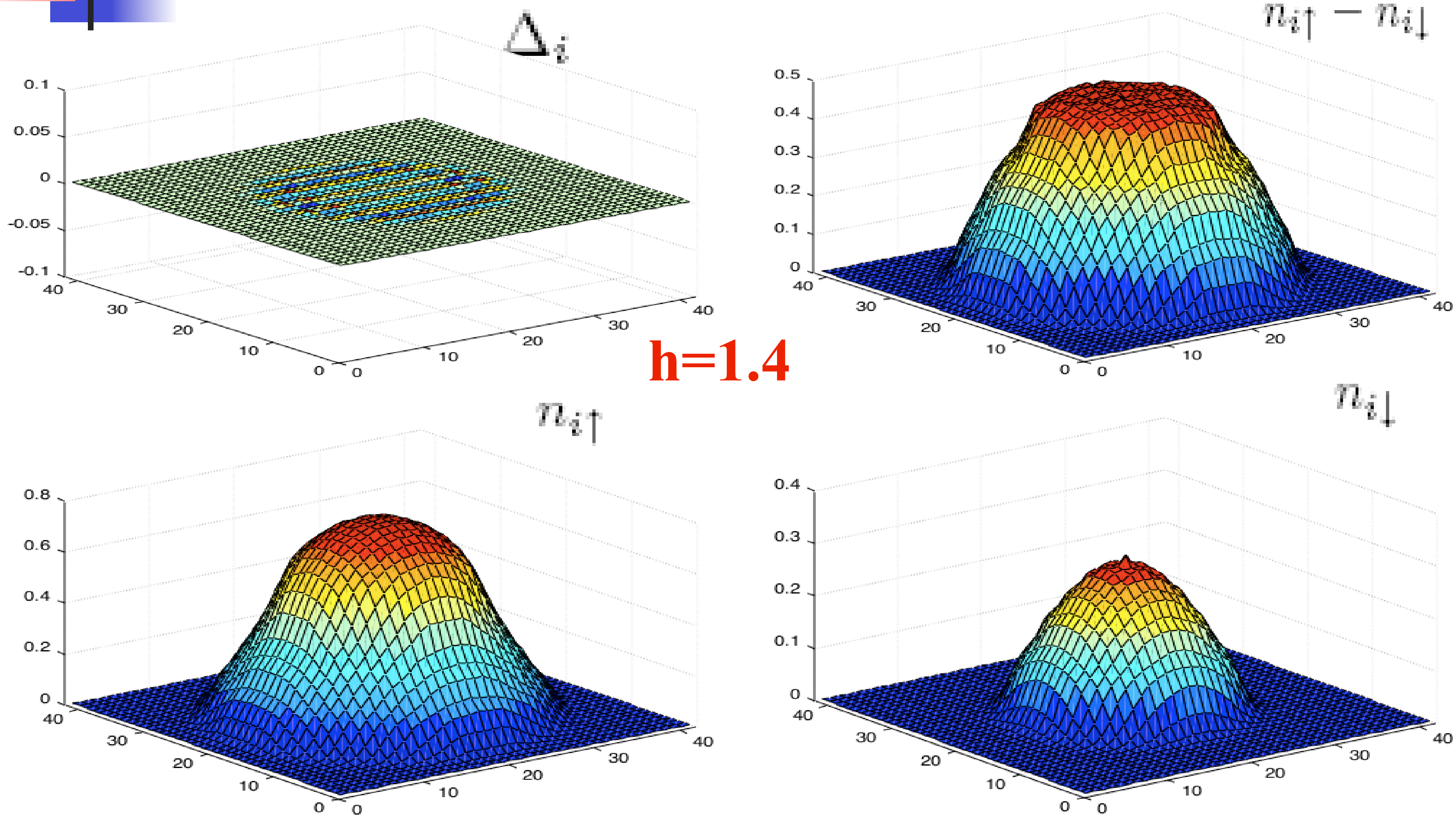


h=0.9





Medium Density (n=400)



Proposed experiments for angular FFLO state

1) **Trapped fermionic atoms in 2D optical lattice in high filling regime.**
   Very recently, the MIT group produced preliminary experimental evidence for superfluidity of ultracold ^{6}Li atoms in optical lattice. Future experiment for imbalanced fermions would be conducted.

2) **In a thin superconducting ring of quasi-2D heavy fermion materials.**
   By applying strong magnetic field, angular-dependent FFLO state may show up due to the special topology of superconducting ring. Detection of the real space modulation of superconducting gap as well as magnetization can be achieved by using SQUID or STM techniques.

V. Conclusions and Outlook

- Our results provide a qualitative understanding of main features in MIT and Rice experiments. The obtained density profiles (low density regime) show striking resemblance to the experimentally obtained ones.
- We present some theoretical predictions for imbalanced Fermi gases in optical lattices, in particular, a new type of FFLO state.
- More systematic studies will be undertaken to get a detailed phase diagram. We are working on this!

Thanks for your attention!!!