Yukawa International Seminar 2007 (YKIS2007)
Interaction and Nanostructural Effects in Low-Dimensional Systems
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Spin Effects in Coherent Transport

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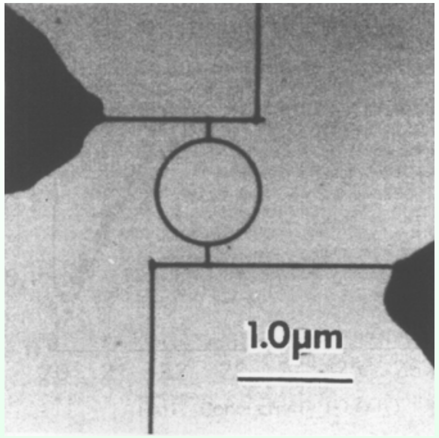
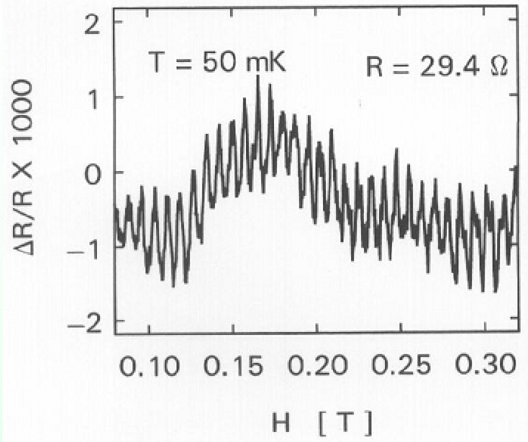
T. Nakanishi, T. Ando, M. Eto

Outline

1. Spin-orbit Berry phase in Aharonov-Bohm (AB) type oscillation
2. Effect of spin scattering on orbital coherence
3. The Fano effect in AB and T-type interferometers with quantum dots
4. The Fano-Kondo effect in T-type and AB interferometers
5. Introduction
6. The Kondo effect and the RKKY interaction in two quantum dot system

Aharonov-Bohm (AB) ring

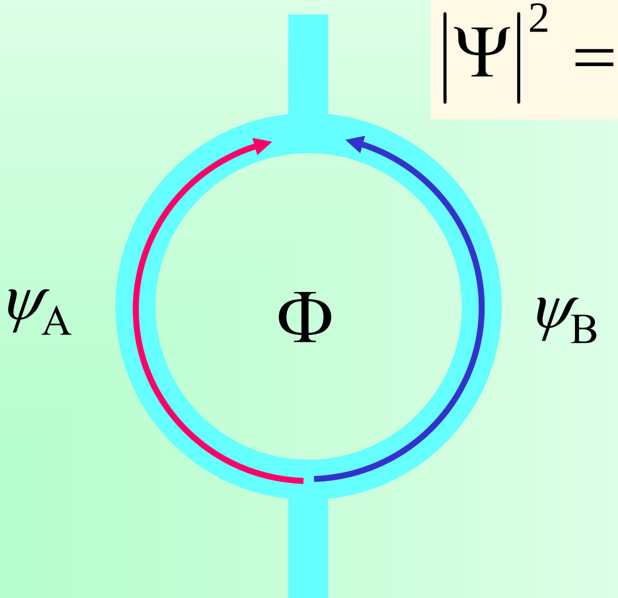




Richard A. Webb

R. A. Webb et al. PRL 54, 1610 (1985).

$$|\Psi|^2 = |\psi_A|^2 + |\psi_B|^2 + 2|\psi_A||\psi_B|\cos\theta$$

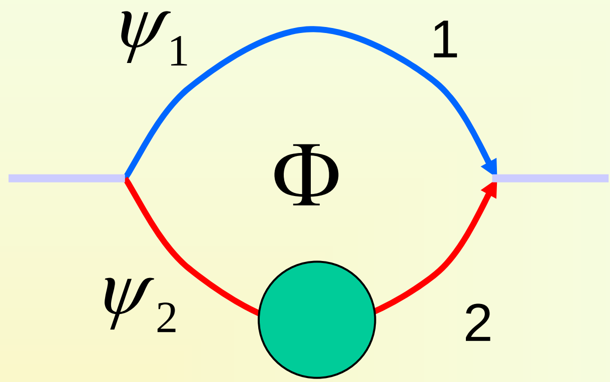


Electron spin and coherent transport

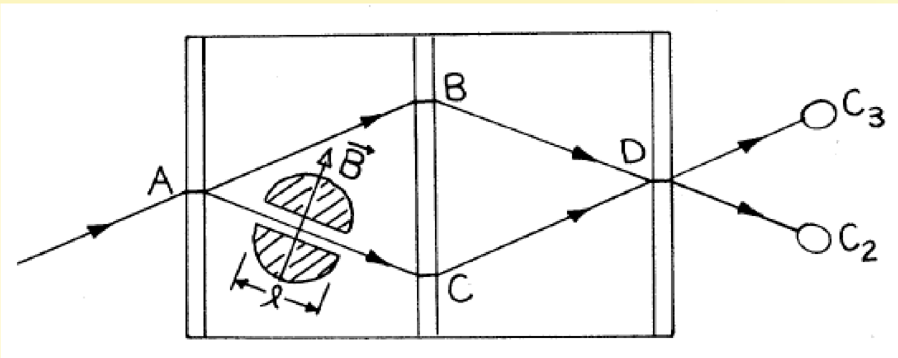
wavefunction $\psi = \underline{\chi(s)}\,\underline{\phi(r)}$

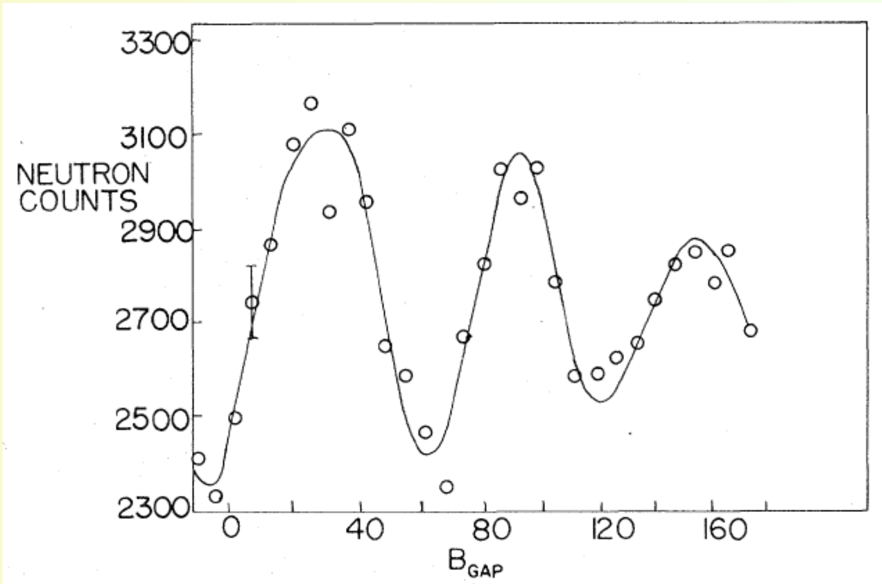
spin part orbital

$$\langle \psi_1 | \psi_2 \rangle = \underline{\int \chi_1^* \chi_2 ds} \int \phi_1^* \phi_2 dr$$

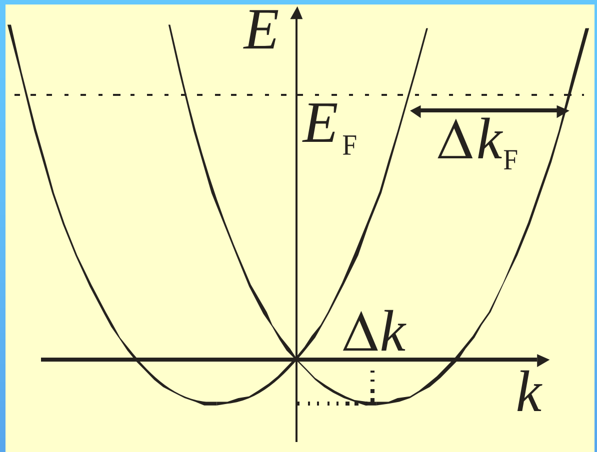


S. A. Werner et.al., Phys. Rev. Lett. 35, 1053 (1975)

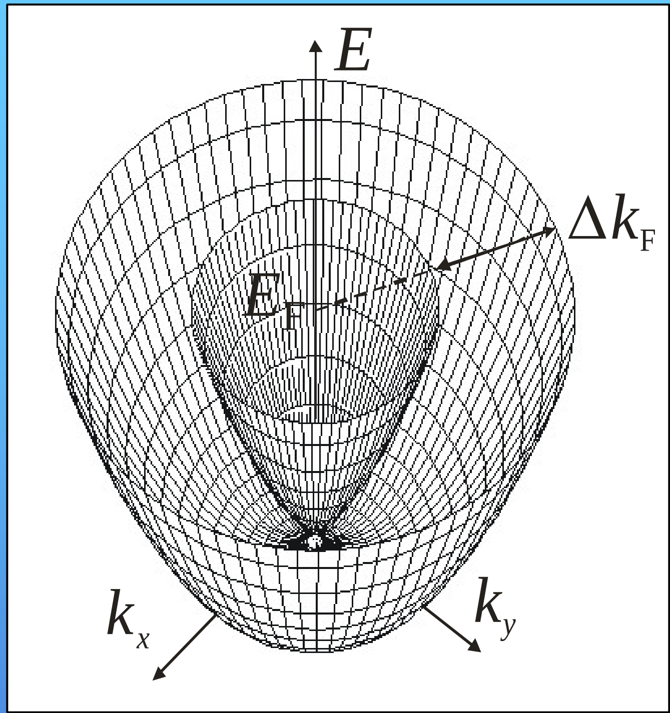




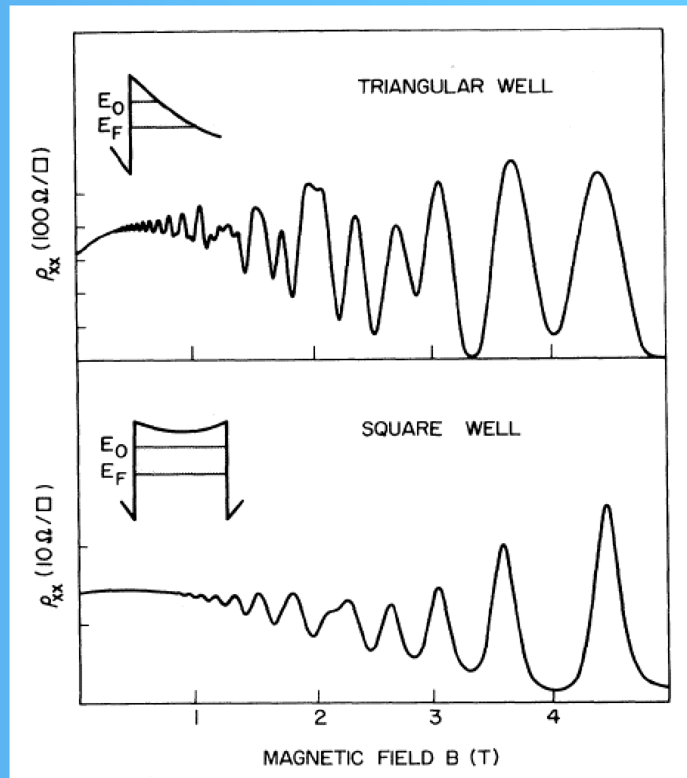
Spin-orbit interaction in two-dimensional systems

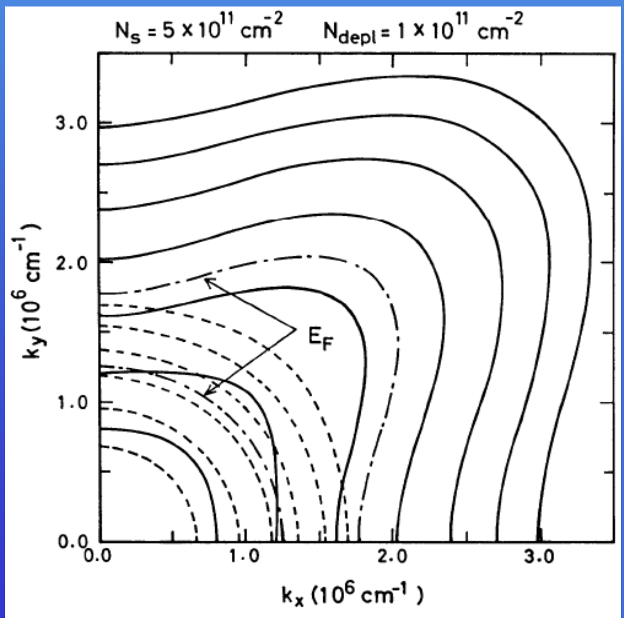


Rashba type
spin-orbit interaction



Double Fermi contour





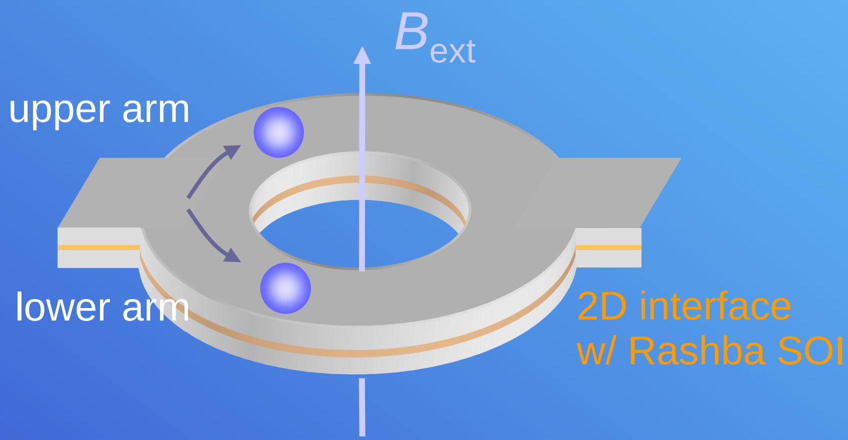
In reality:
Dresselhause contribution

Lattice anisotropic effect

T. Ando,
JPSJ54, 1528 ('85)

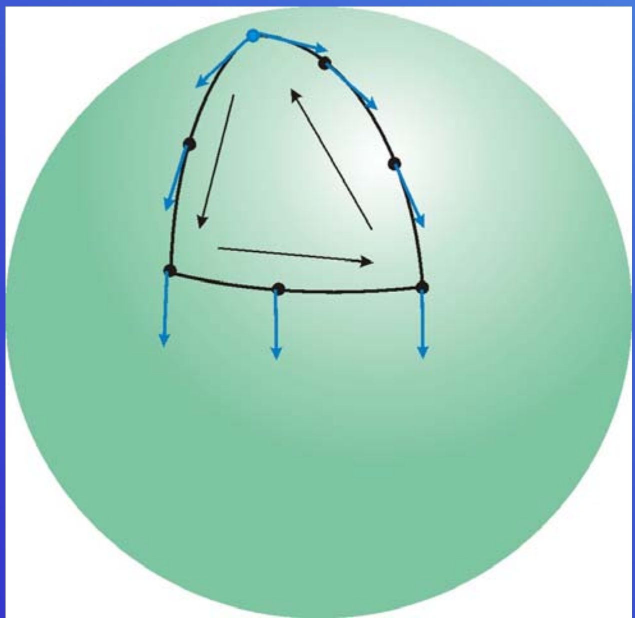
J. Eisenstein et al. PRL 53, 2759 ('86)

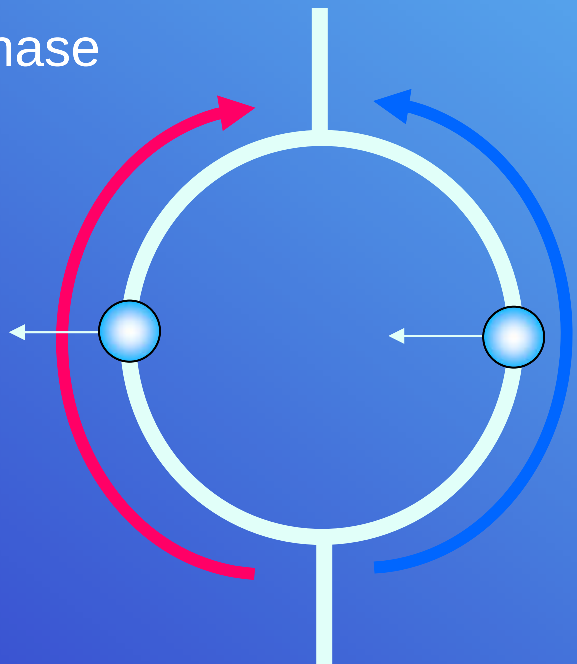
Berry phase in a single mesoscopic ring

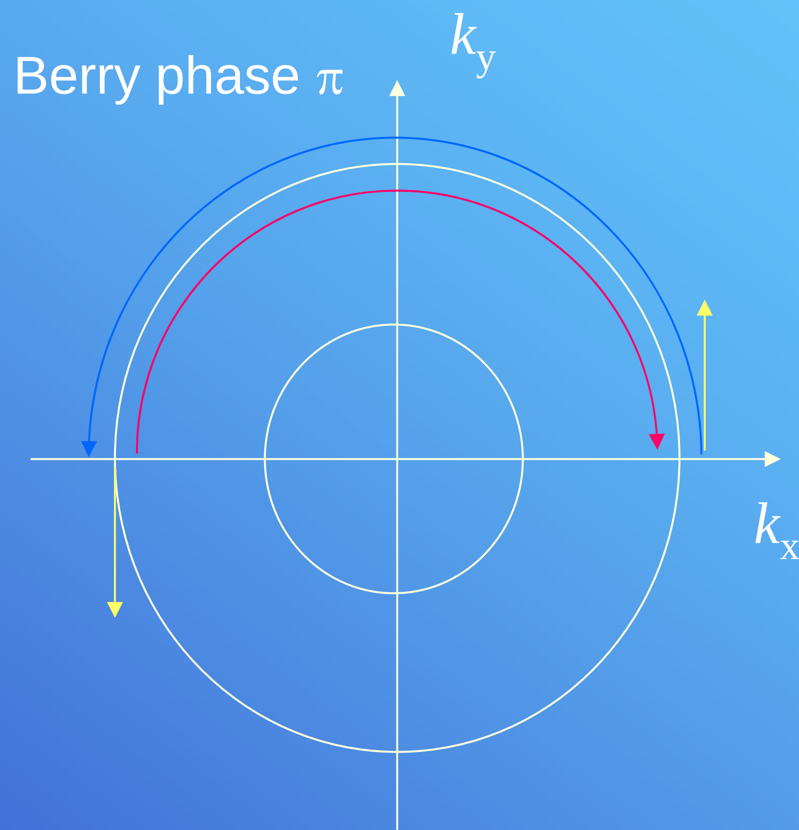


D. Loss et al., PRL 65, 1655 (1990)
A. Aronov & Y. Lyanda-Geller, PRL70, 343 (1993)
T. Quian & Z. Su, PRL 72, 2311 (1994)
Y. Meir et al., PRL 63, 798 (1989)

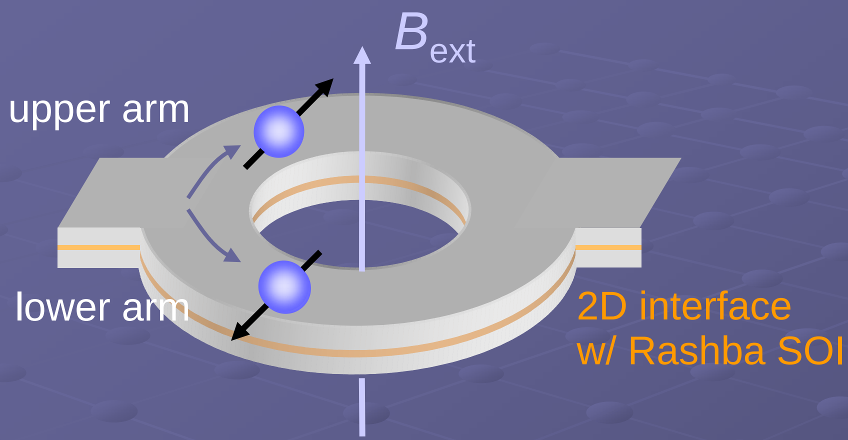
Constraint+
Adiabatic approx -> Berry phase

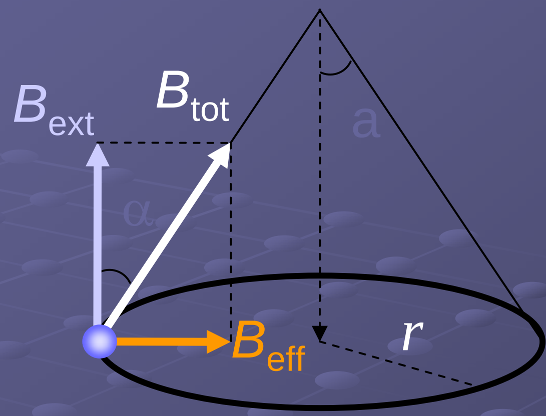


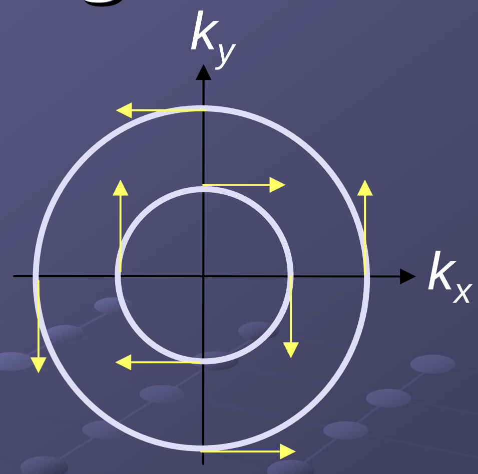




Berry phase in a single mesoscopic ring







AB phase $\theta_{AB} = 2\pi \frac{\Phi_{ext}}{\Phi_0}$ **ALWAYS**

Effective when the carrier has

Band-splitting $\Delta\theta_k = \pi\, r \Delta k_F \sin\alpha$ → **Difference in k_F**

Berry phase $\Delta\theta_B = \pi(1-\cos\alpha)$ → **Difference in spins**

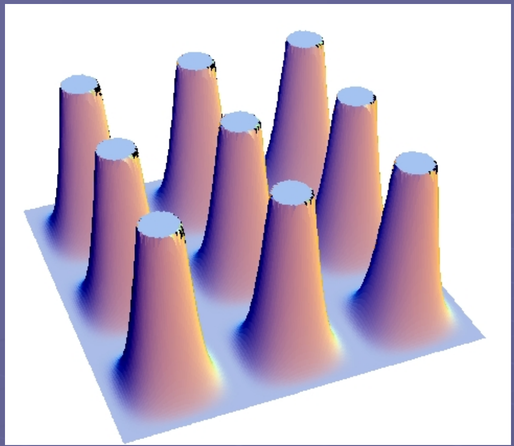
in the upper and lower arms

EXCLUSIVE

Overall phase $\theta_{AB} \pm \Delta\theta_k, \quad \theta_{AB} \pm \Delta\theta_B$

Antidot lattice (ADL)

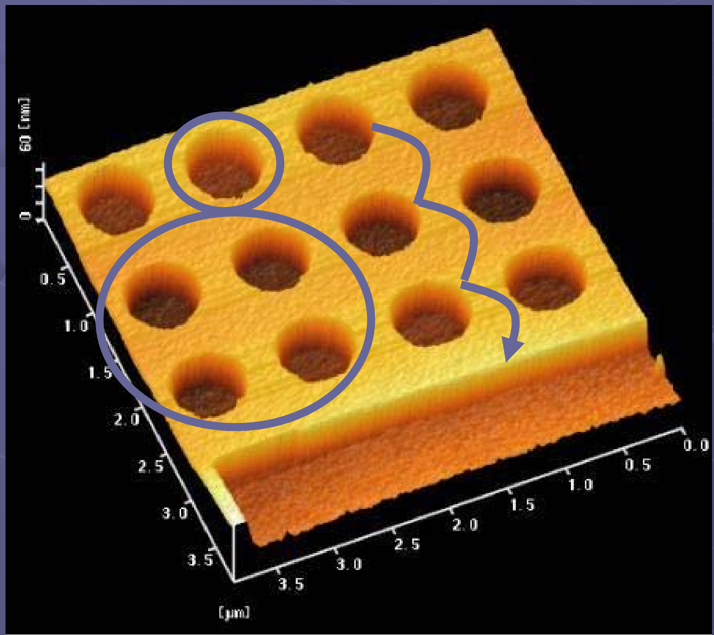
Potential landscape



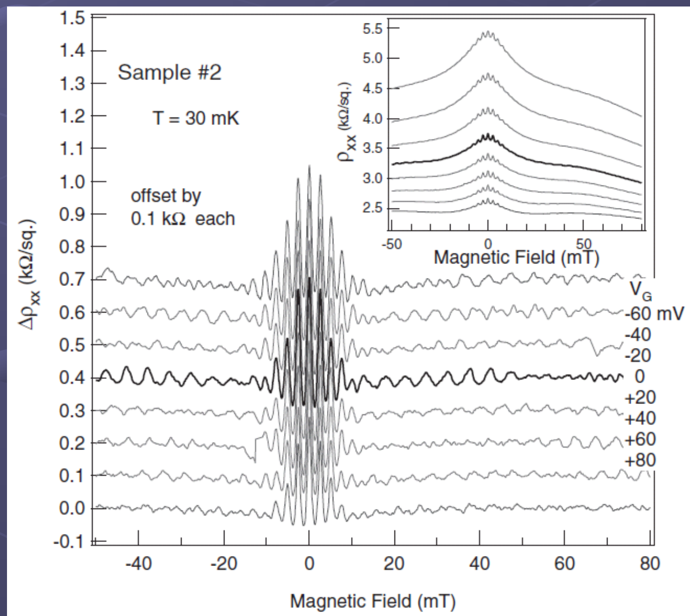
AAS oscillation: *h*/2*e* period

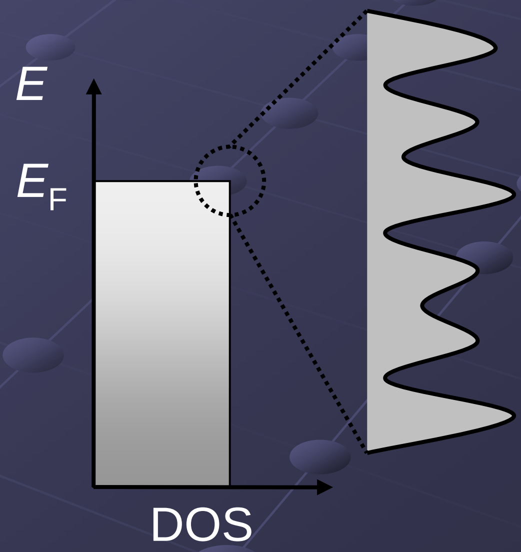
AB-type oscillation: *h*/*e* period

- survives even when the ordinary AB phases are averaged out due to random phasing
- presumably manifests the oscillatory structure in the DOS, but it is not obvious the Berry phase still appears in it
- random sample-specific effects are suppressed



Y. Iye et al. JPSJ 73, 3370 (2004)

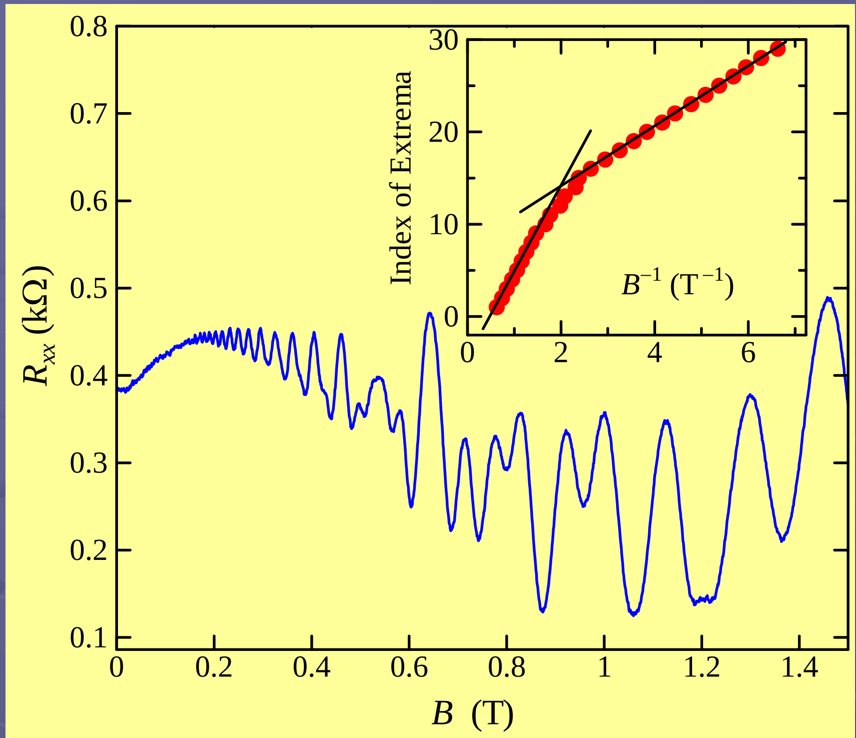




Commensurability peak:

- appears when the carrier cyclotron orbit is commensurate with an ADL
- is also as a result of 'pinball' transport

Sample



Two-dimensional Hole gas

(001) $Ga_{0.65}Al_{0.35}As/GaAs$

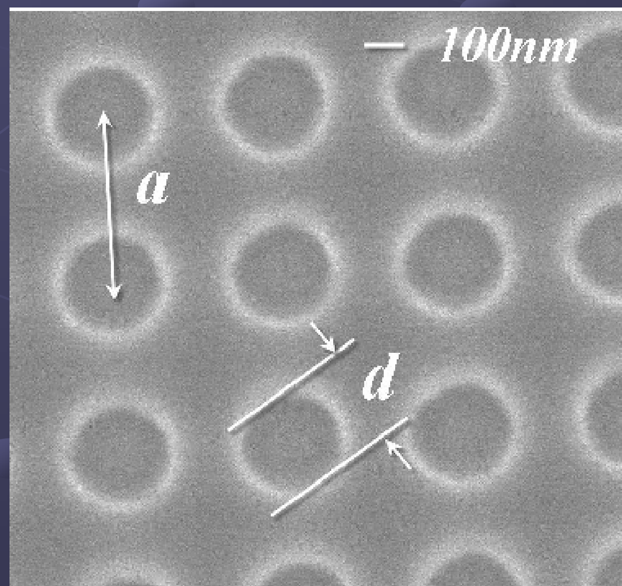
Hole concentration from SdH $p_l=0.79\times10^{11}cm^{-2}$

$p_h=1.5\times10^{11}cm^{-2}$

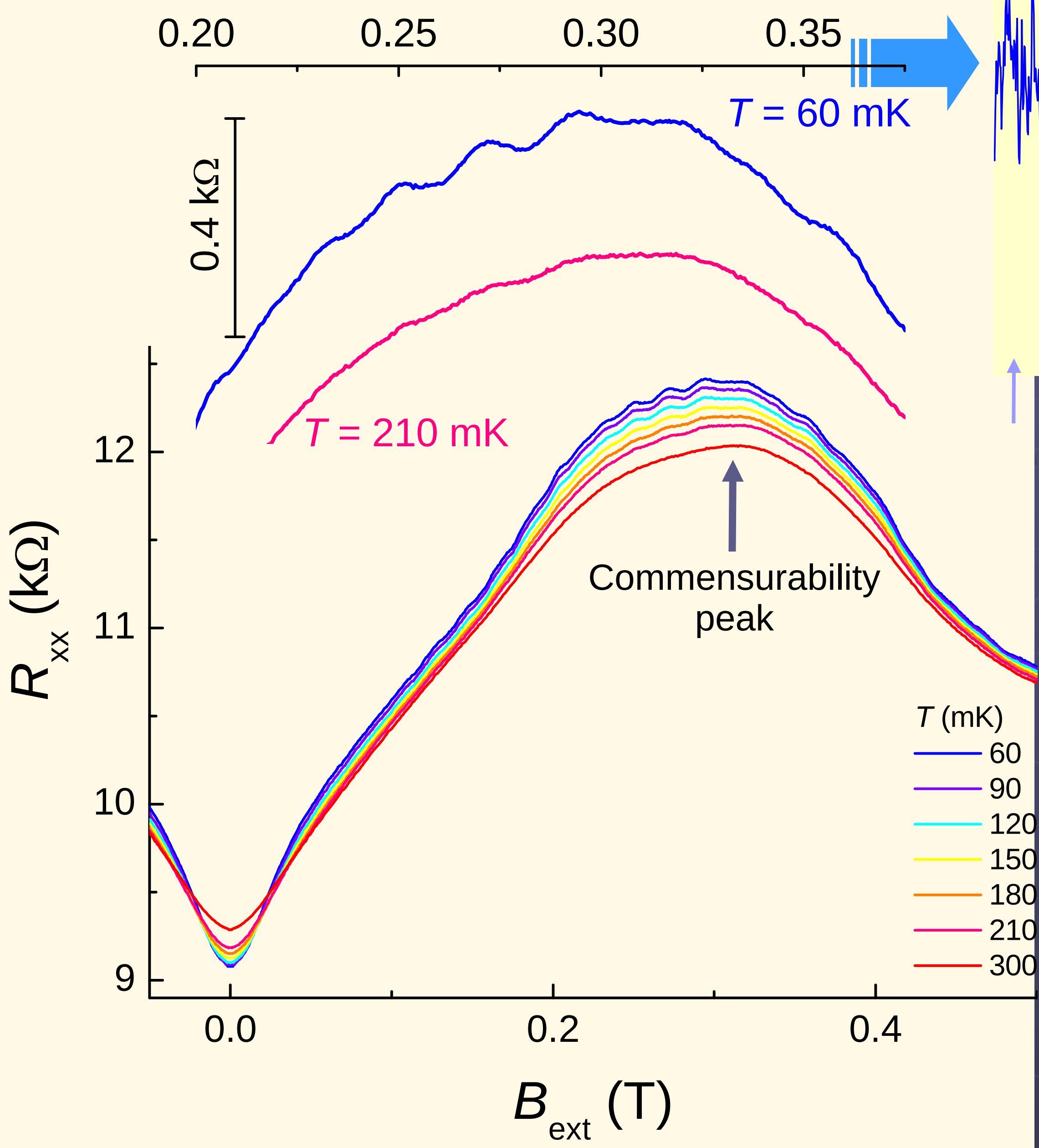
Hall concentration $p=2.3\times10^{11}cm^{-2}$

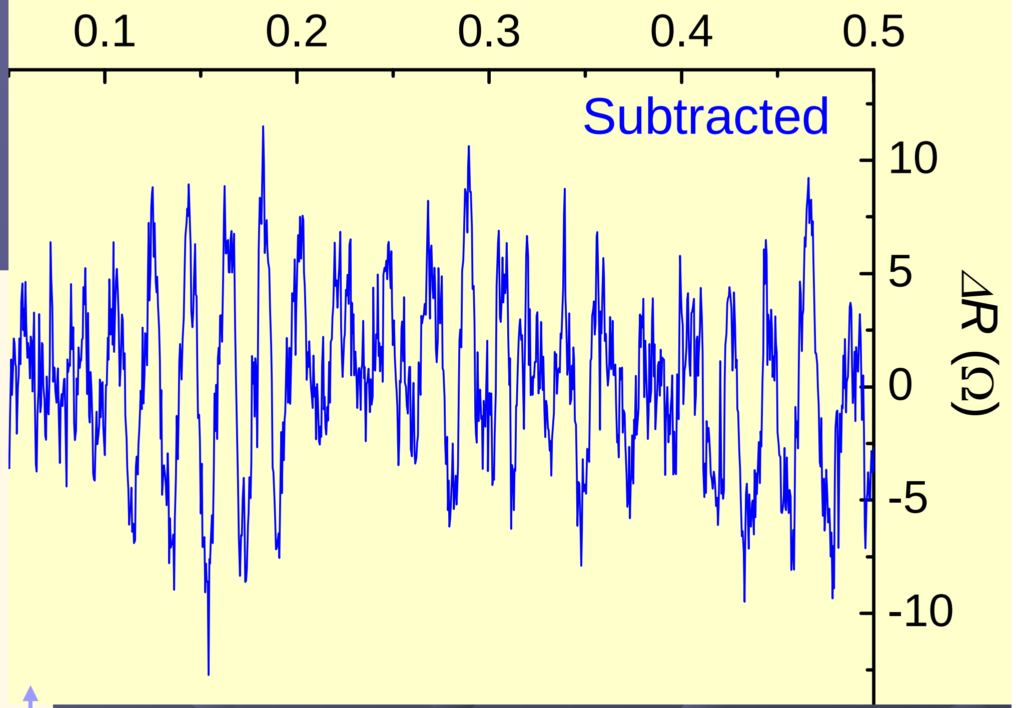
Mobility $\mu=6.8\times10^4 cm^2(Vs)^{-1}$

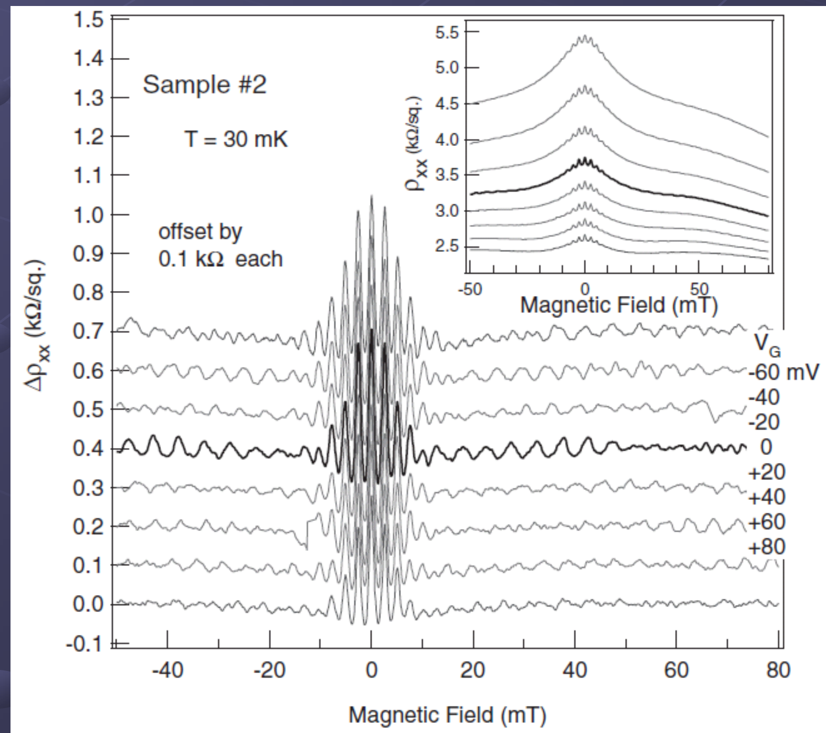
Sample	Diameter d (nm)	Period a (nm)	Lattice structure
SL	250	1000	Square
SS	250	500	Square
TL	250	1000	Triangular
TS	250	750	Triangular



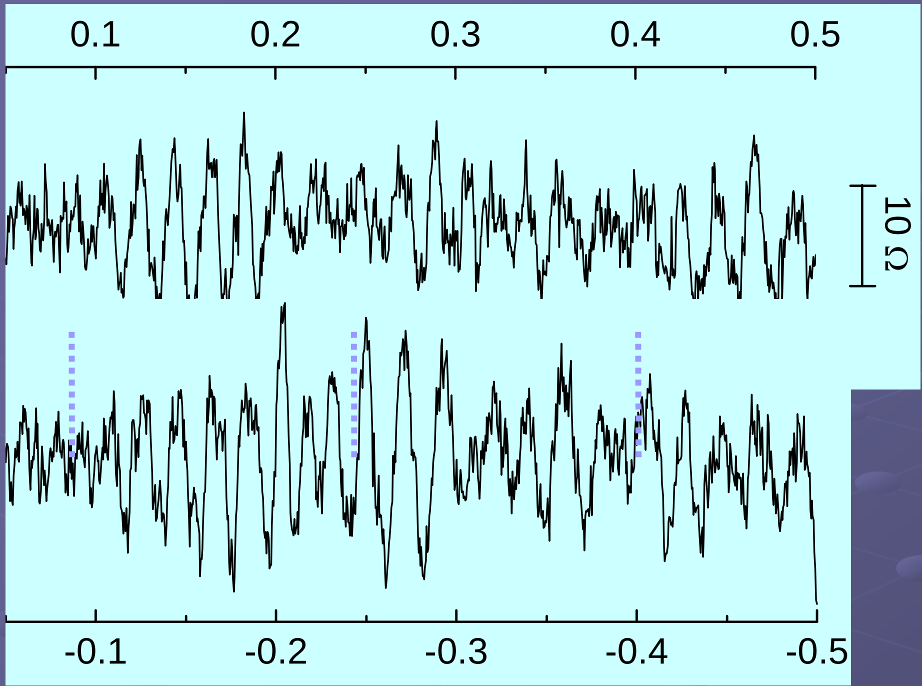
Magnetoresistance

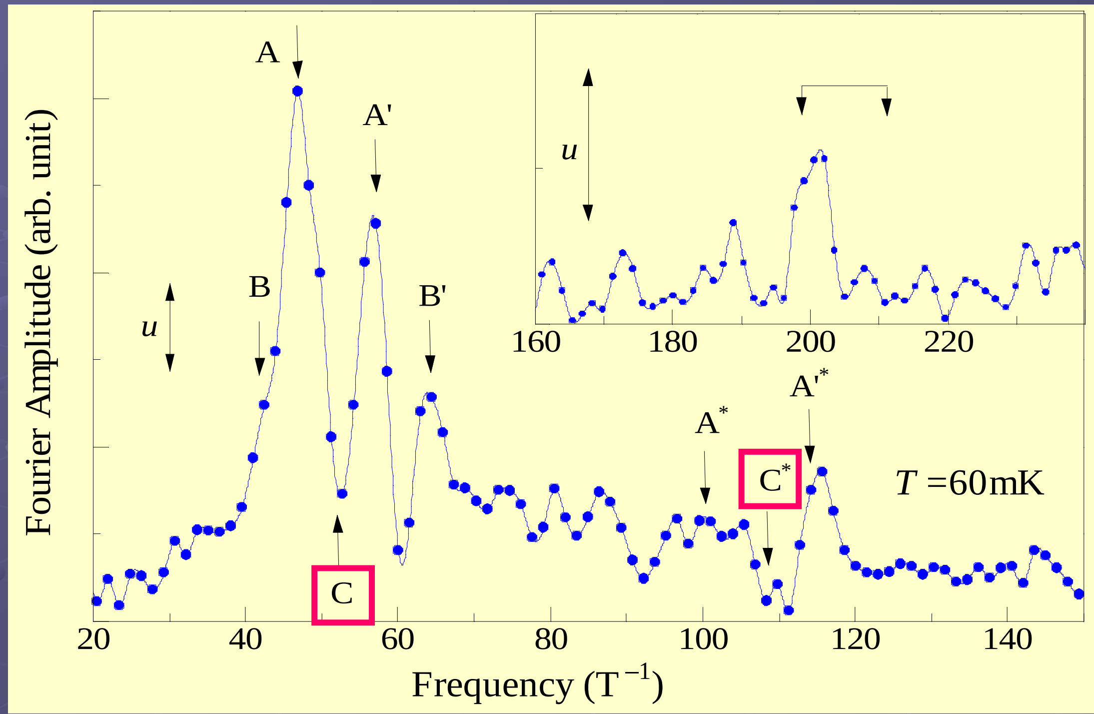




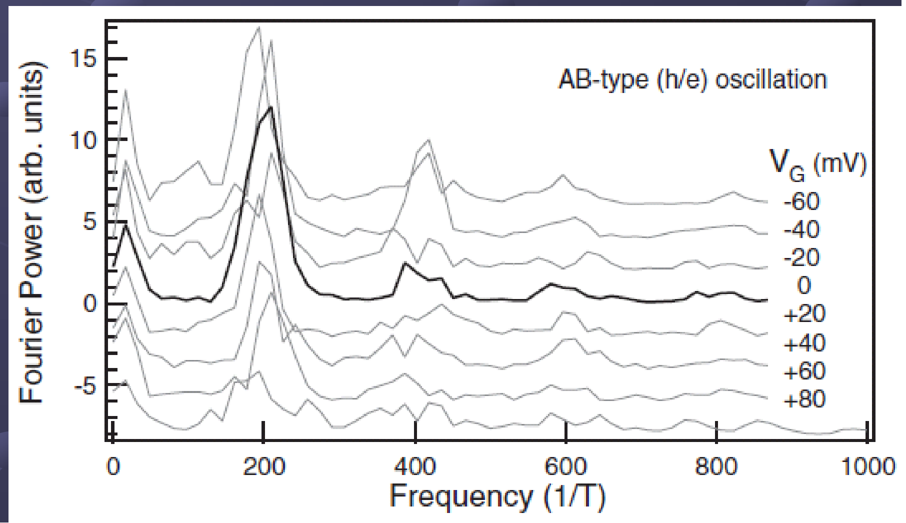


Fourier spectrum of the AB-type oscillation





n-type



- Dip C corresponds to *h*/*e* through a single antidot cell
- The main peak splits into peaks A, A’ and B’, and a shoulder B
- Dip C* corresponds to *h*/2*e* with split-peaks A* and A*’

Quantum Entanglement

$$|\psi\rangle = |A\rangle + |B\rangle \qquad |\varphi\rangle = |1\rangle + |2\rangle$$

$\|A\rangle$	$\|B\rangle$	
$\|A\rangle\|1\rangle$		$\|1\rangle$
	$\|B\rangle\|2\rangle$	$\|2\rangle$

Direct product $|\Psi\rangle = |\psi\rangle \otimes |\varphi\rangle = |A\rangle|1\rangle + |A\rangle|2\rangle + |B\rangle|1\rangle + |B\rangle|2\rangle$

Maximally entangled state $|\Phi\rangle = |A\rangle|1\rangle + |B\rangle|2\rangle$

Quantification of Entanglement?

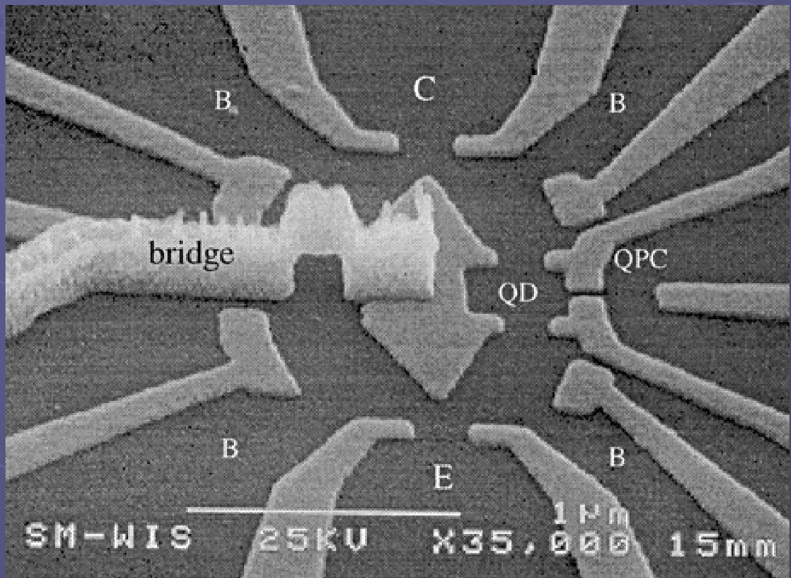
What is “measurement”?

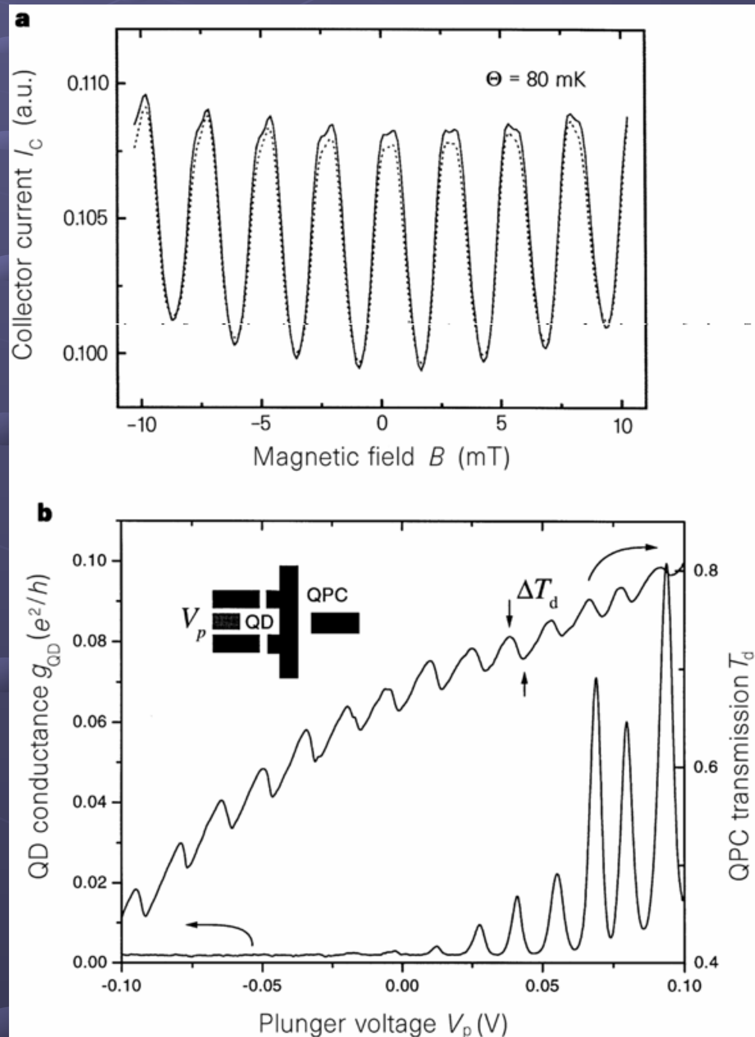
$$\left|\psi\right\rangle = \left|\psi_A\right\rangle + \left|\psi_B\right\rangle$$

$$\left|\Psi\right\rangle = \left|\psi_A\right\rangle\left|A\right\rangle + \left|\psi_B\right\rangle\left|B\right\rangle$$

State entangled with macroscopically distinguishable states |A> and |B>

“Collapse” of wavefunction into ψ_A (or ψ_B).



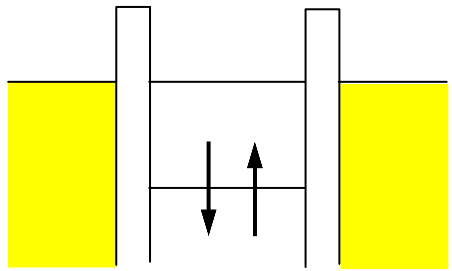


Buks et al. Nature 391, 871 (`98)

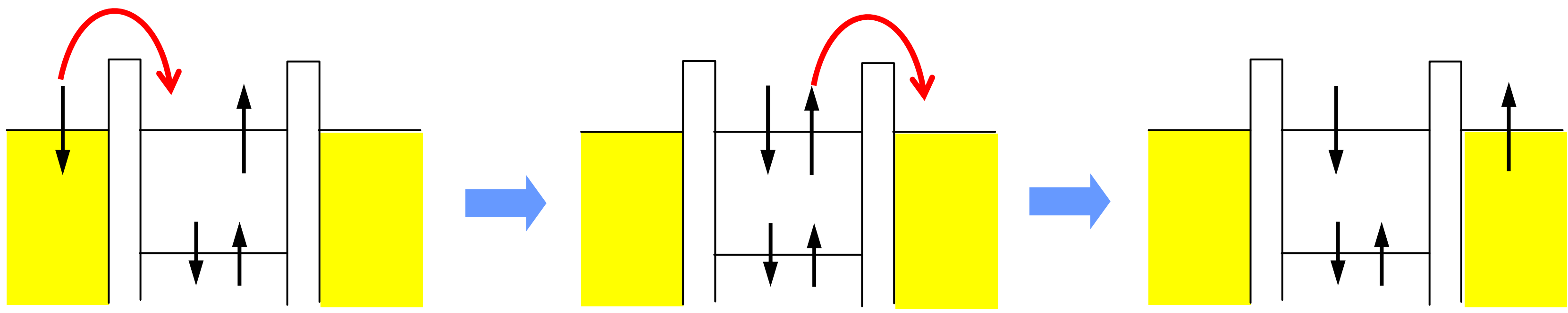
Spin state and quantum decoherence

Akera PRB **59**, 9802(`99), König & Gefen PRB**65**, 045316 (`02)

discrete levels by quantum confirmation



- When the number of electrons is odd:



Spin state and quantum decoherence

Akera PRB **59**, 9802(`99), König & Gefen PRB**65**, 045316 (`02)

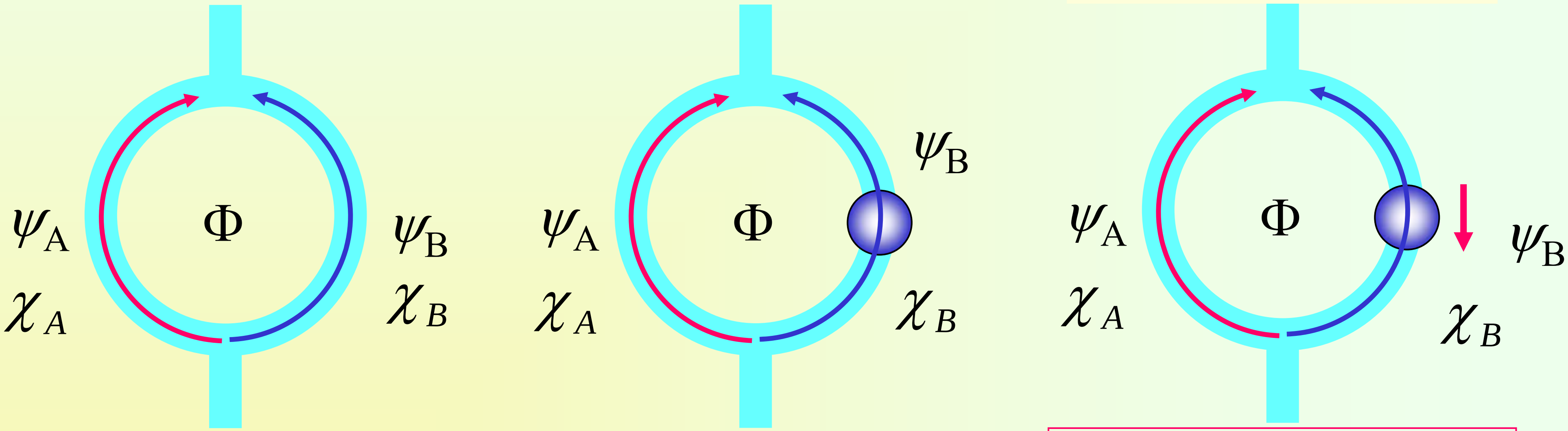
χ_A:spin-up χ_B:spin-down

interference term : 0

$$2|\psi_A||\psi_B|\cos\theta\int d\xi\chi_A(\xi)\chi_B^*(\xi)$$

$$2|\psi_A||\psi_B|\cos\theta$$

Partial coherence

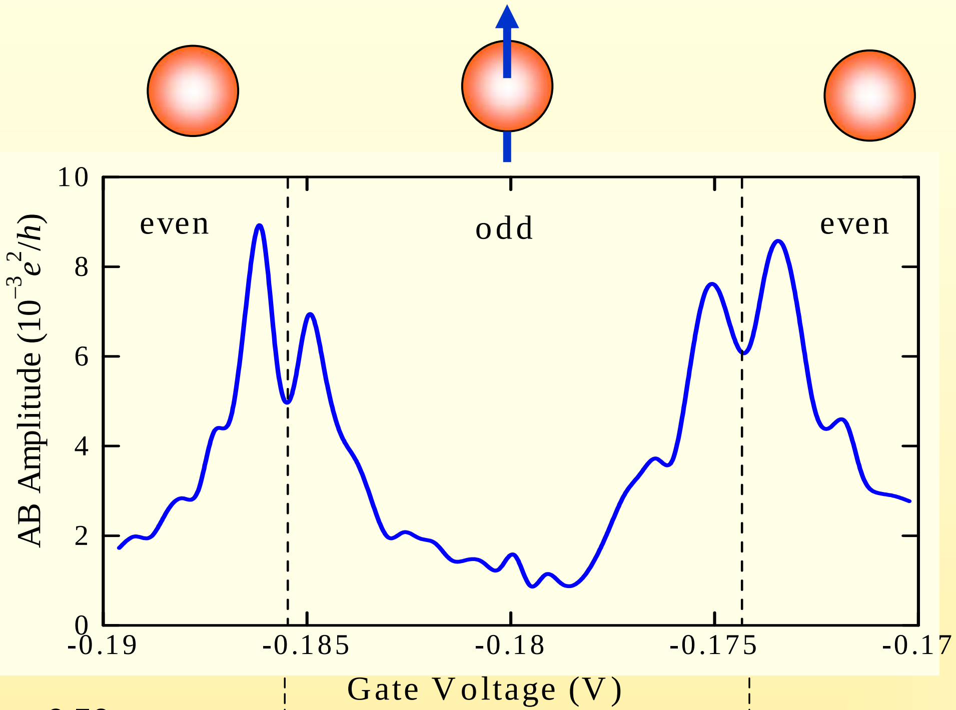


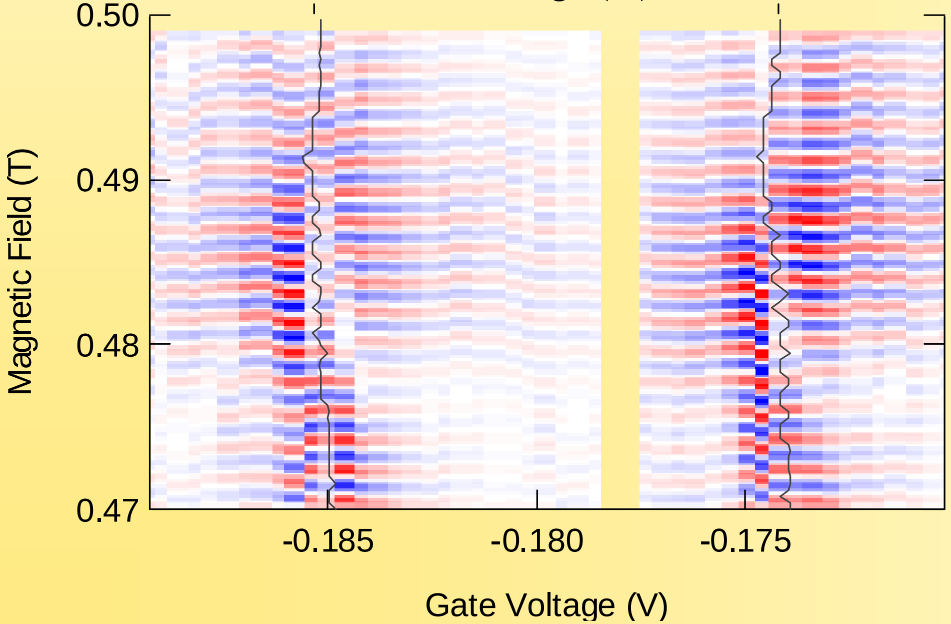
N:even
no spin-flip scattering

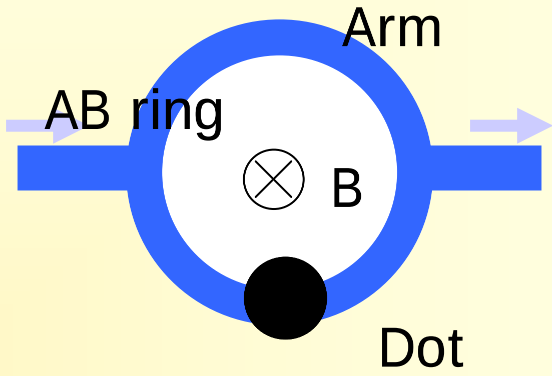
N:odd
spin-flip scattering exists

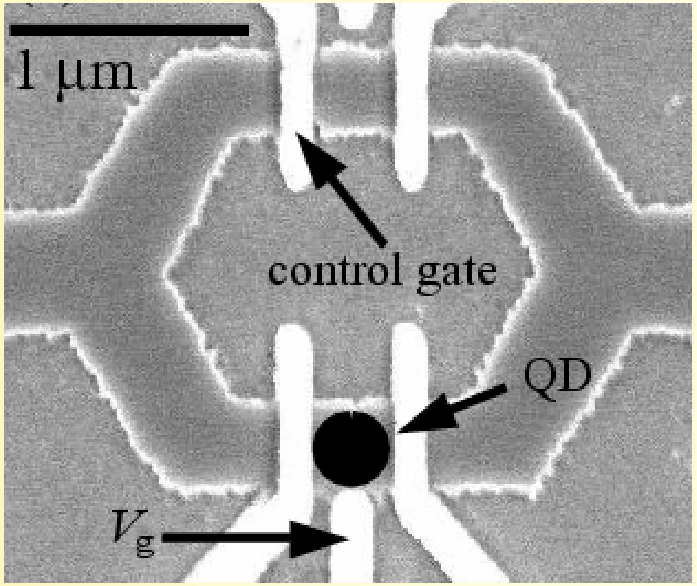
Spin-flip process reduces quantum coherence

AB amplitude for a spin-pair





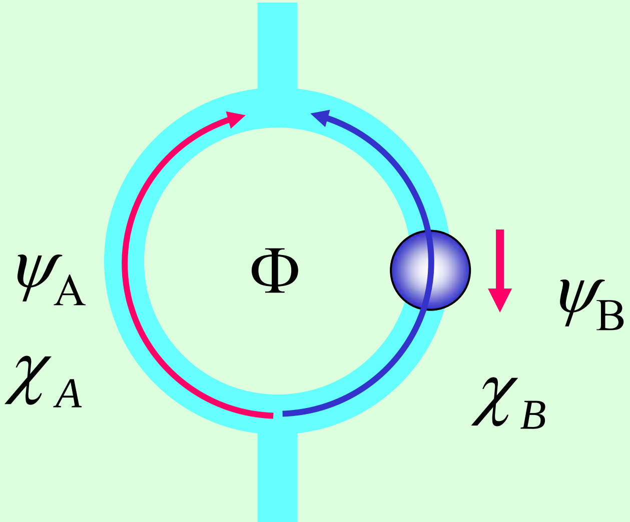




H. Aikawa et al. PRL **92**, 176802 (`04)

Quantum entanglement and decoherence

$$|\langle A|B\rangle|^2 = \frac{|\langle \psi_A|\psi_B\rangle|^2}{2}$$



$$\psi_A : |\psi_A \uparrow\rangle|d\downarrow\rangle$$

$$\psi_B : \frac{1}{\sqrt{2}}\left(|\psi_B \uparrow\rangle|d\downarrow\rangle - |\psi_B \downarrow\rangle|d\uparrow\rangle\right)$$

Qauntum dot: creates entanglement between spin freedom and orbital freedom (A or B)

Spatially localized interaction causes entanglement with the orbital freedom

Decoherence occured when the dot freedom is traced out

↓

Suggestion: Degree of entanglement can be mearued by decoherence when the freedoms in the other system are integrated out.

Question: Is this really decoherence?

Schmidt decomposition

Two systems $\mathcal{H}_\mathrm{A}, \mathcal{H}_\mathrm{B}$ states of them can be written as

$$|A\rangle = \sum_i^{d_\mathrm{A}} c_i |\eta_i\rangle, \quad |B\rangle = \sum_j^{d_\mathrm{B}} c_j |\xi_j\rangle$$

ex) Direct product (no entanglement)

$$|A\rangle \otimes |B\rangle = \sum_{i,j} c_i c_j |\eta_i\rangle |\xi_j\rangle$$

In general

$$|\psi_\mathrm{AB}\rangle = \sum_{i,j}^{d_\mathrm{A}, d_\mathrm{B}} c_{ij} |\eta_i, \xi_j\rangle$$

Diferent basis *u*, *v* (Schmidt decomposition)

$$|\psi_\mathrm{AB}\rangle = \sum_{k=1}^{d} d_k |u_k, v_k\rangle, \quad \sum_{k=1}^{d} d_k^2 = 1 \quad (d = \min(d_\mathrm{A}, d_\mathrm{B}))$$

Density matrix after tracing out of each other's degree of freedom

$$\rho_\mathrm{A} = \sum d_k^2 |u_k\rangle\langle u_k|, \quad \rho_\mathrm{B} = \sum d_k^2 |v_k\rangle\langle v_k|$$

Quantification of Entanglement

$$\rho_A = \sum d_k^2 |u_k\rangle\langle u_k|, \quad \rho_B = \sum d_k^2 |v_k\rangle\langle v_k|$$

"Entanglement entropy" or "von Neumann entropy"

$$E(|\psi_{AB}\rangle) = S(\rho_A) = S(\rho_B) = -\sum_{k=1}^{d} d_k^2 \log_2(d_k^2)$$

$$S(\rho) = -\mathrm{Tr}\rho \log \rho$$

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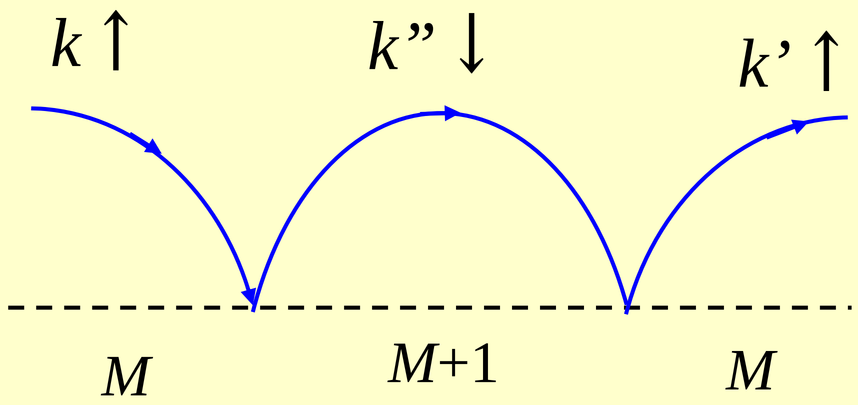
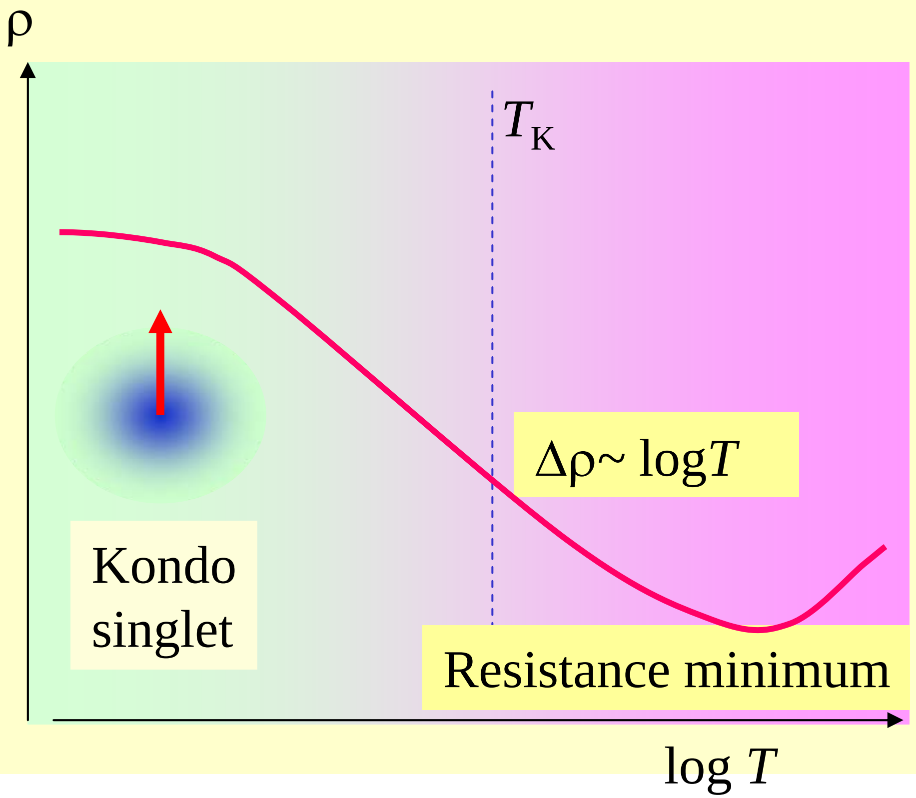
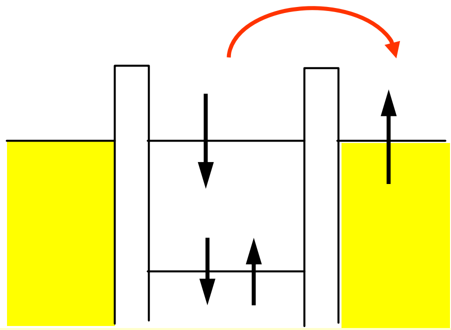
ex) $|\Psi_s\rangle = \frac{1}{\sqrt{2}}\left(|\phi_\downarrow\rangle|\chi_\uparrow\rangle - |\phi_\uparrow\rangle|\chi_\downarrow\rangle\right)$ $\quad |\phi_\downarrow\rangle = \frac{1}{\sqrt{N}}\sum_{k>k_F}\Gamma_{\mathbf{k}} c^\dagger_{\mathbf{k}\downarrow}|F\rangle$

$$\rho_{im} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad S(\rho_{im}) = 1 \qquad \text{Maximally entangled}$$

The Kondo Effect







Jun Kondo

Really decoherence?

$$\frac{1}{\sqrt{2}}\left(\left|s\uparrow\right\rangle\left|d\downarrow\right\rangle-\left|s\downarrow\right\rangle\left|d\uparrow\right\rangle\right)$$

Spin-flip scattering

Shield of local moment

Kondo singlet

Recovery of coherence?

PHYSICAL REVIEW VOLUME 168, NUMBER 2 10 APRIL 1968

Closed-Form Solution for the Collective Bound State due to the *s-d* Exchange Interaction

Akio Yoshimori
Institute for Solid State Physics, University of Tokyo, Tokyo, Japan

(Received 6 September 1967)

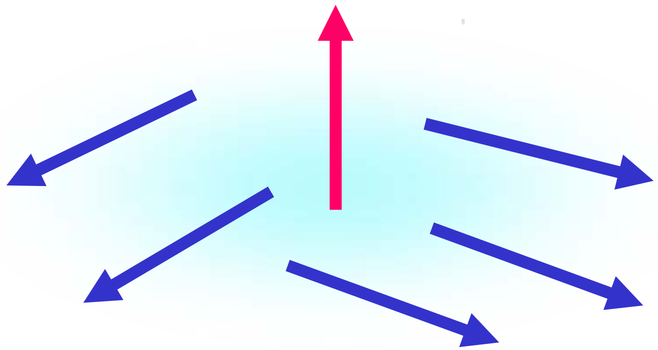
$$\psi=\Big\{\sum_k\left[\Gamma_k{}^{\alpha}a_{k\downarrow}{}^{\dagger}\alpha+\Gamma_k{}^{\beta}a_{k\uparrow}{}^{\dagger}\beta\right] \longrightarrow \Big(|s\uparrow\rangle|d\downarrow\rangle-|s\downarrow\rangle|d\uparrow\rangle\Big)$$

$$+\sum_{k_1k_2k_3}\left[\Gamma_{k_1k_2k_3}{}^{\alpha\downarrow}a_{k_1\downarrow}{}^{\dagger}a_{k_2\downarrow}{}^{\dagger}a_{k_3\downarrow}\alpha+\Gamma_{k_1k_2k_3}{}^{\beta\uparrow}a_{k_1\uparrow}{}^{\dagger}a_{k_2\uparrow}{}^{\dagger}a_{k_3\uparrow}\beta\right.$$

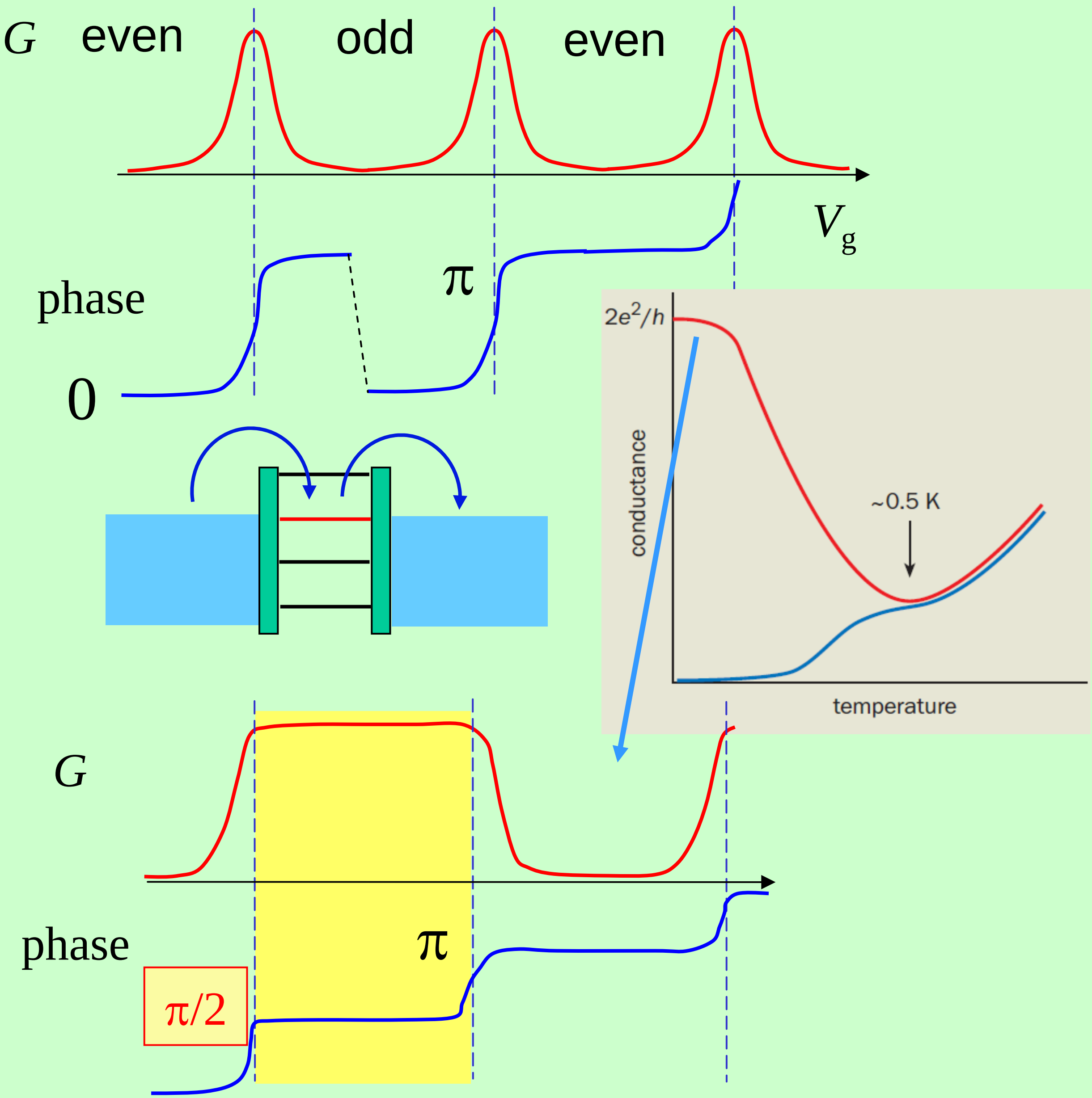
$$\left.+\Gamma_{k_1k_2k_3}{}^{\alpha\uparrow}a_{k_1\downarrow}{}^{\dagger}a_{k_2\uparrow}{}^{\dagger}a_{k_3\uparrow}\alpha+\Gamma_{k_1k_2k_3}{}^{\beta\downarrow}a_{k_1\uparrow}{}^{\dagger}a_{k_2\downarrow}{}^{\dagger}a_{k_3\downarrow}\beta\right]$$

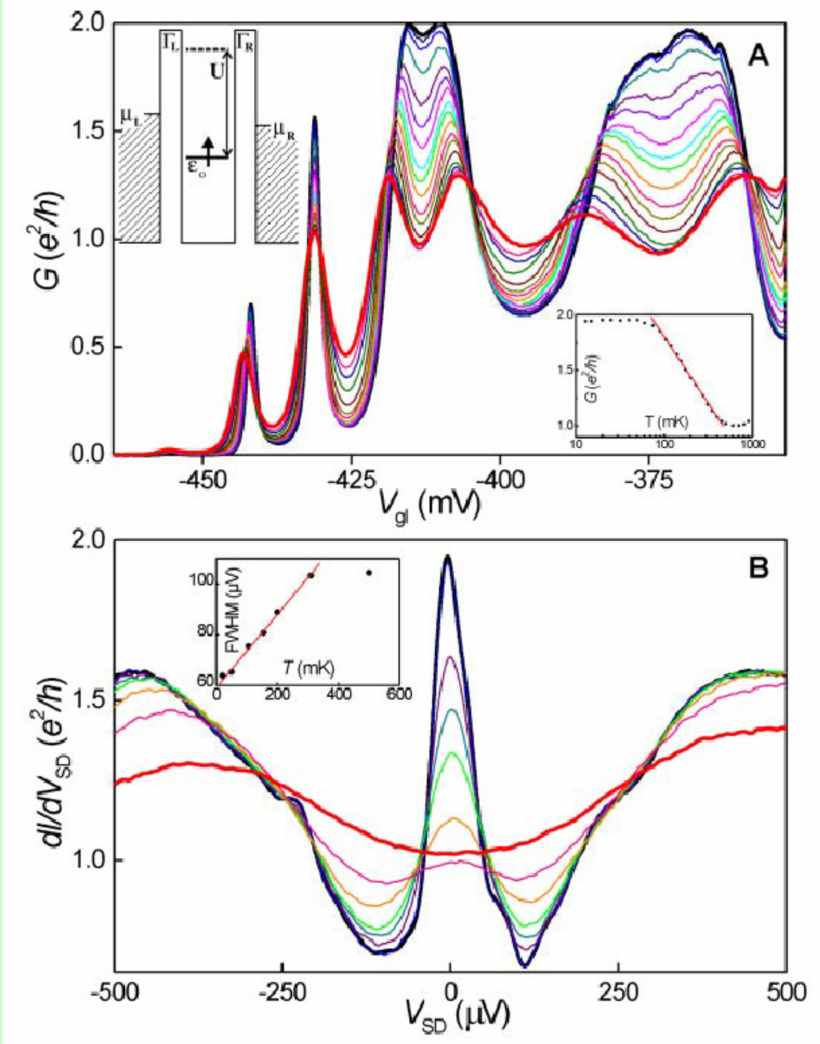
$$+\cdots\Big\}\psi_v, \quad (1)$$

Fermi State



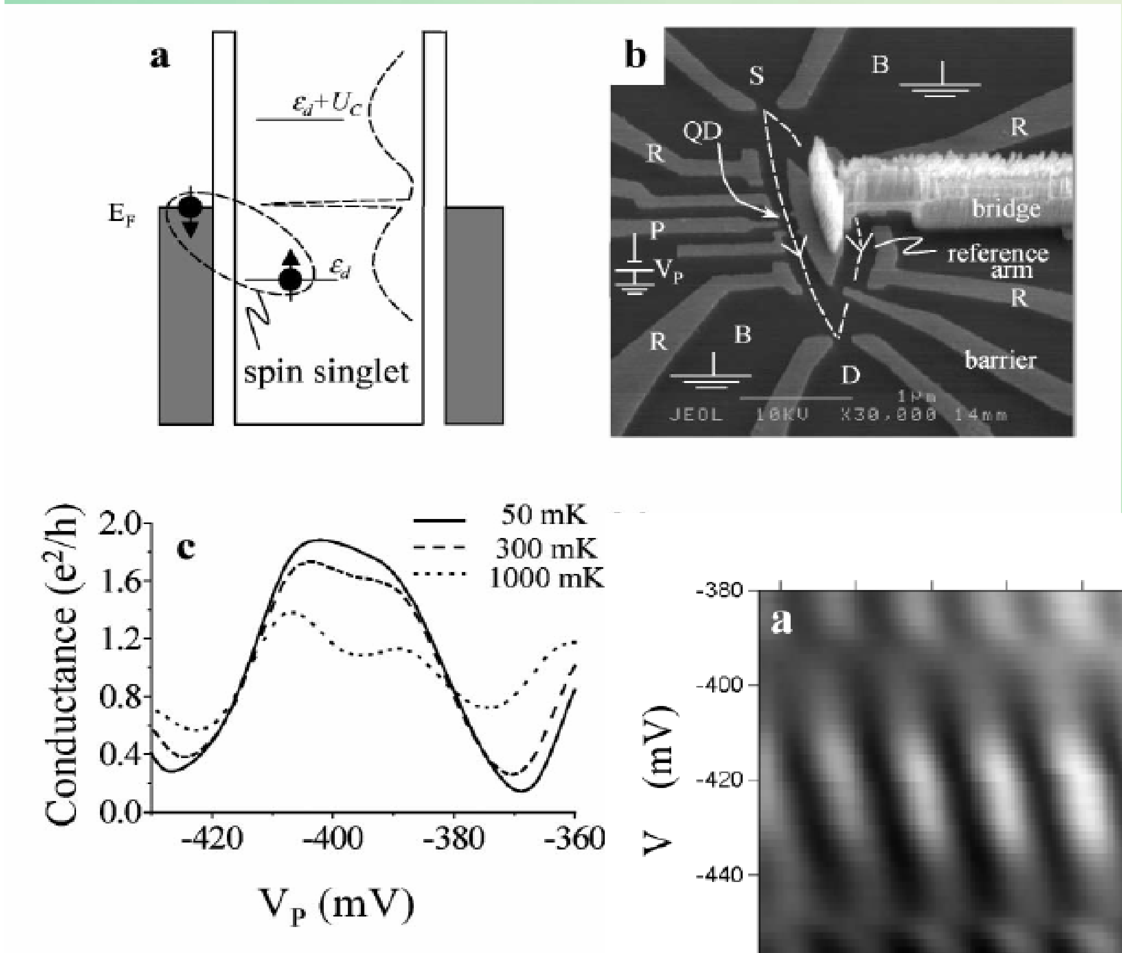
The Kondo Effect in a Quantum Dot System





W. G. van der Wiel et al.
Science **289**, 2105 (2000).

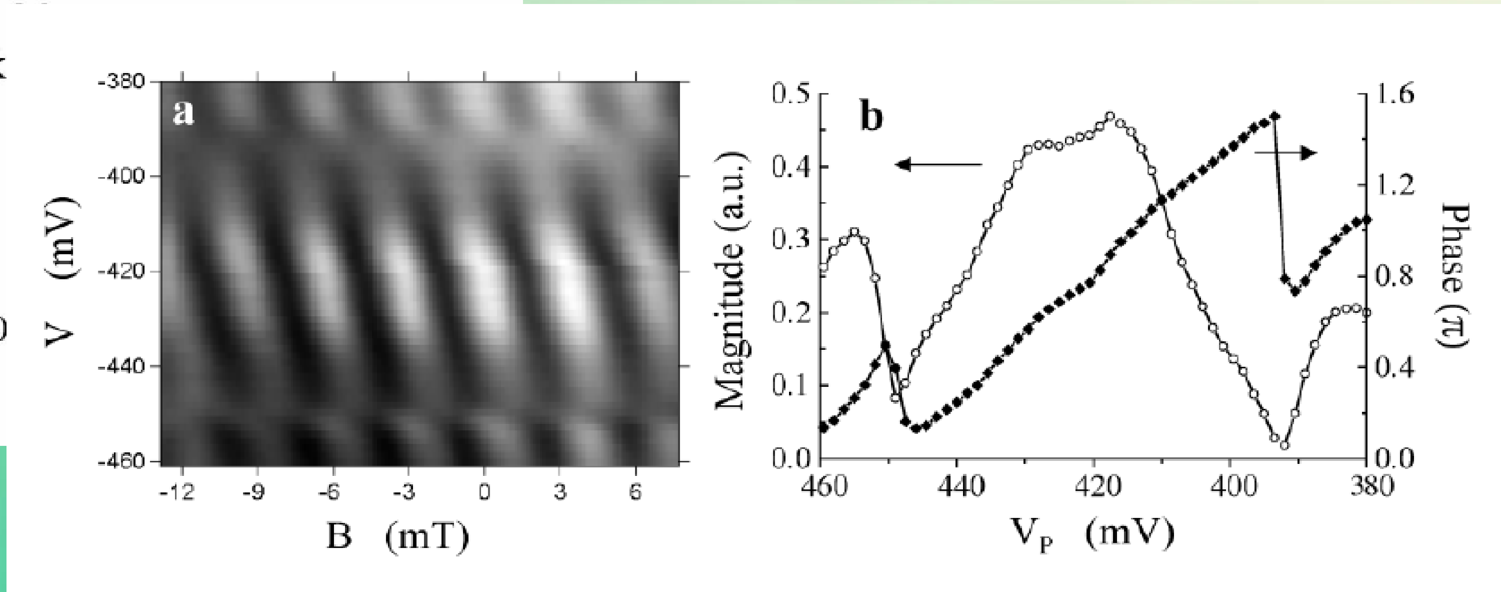
"Phase Sensitive" Measurement



No phase shift locking to $\pi/2$?

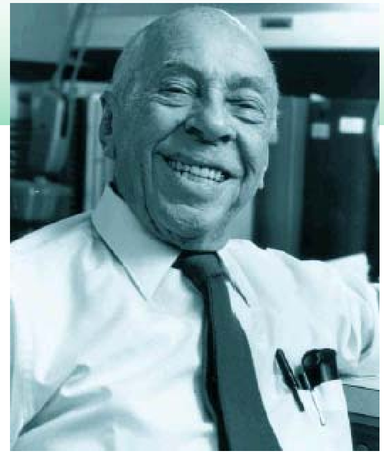
Breakdown of Anderson impurity model?

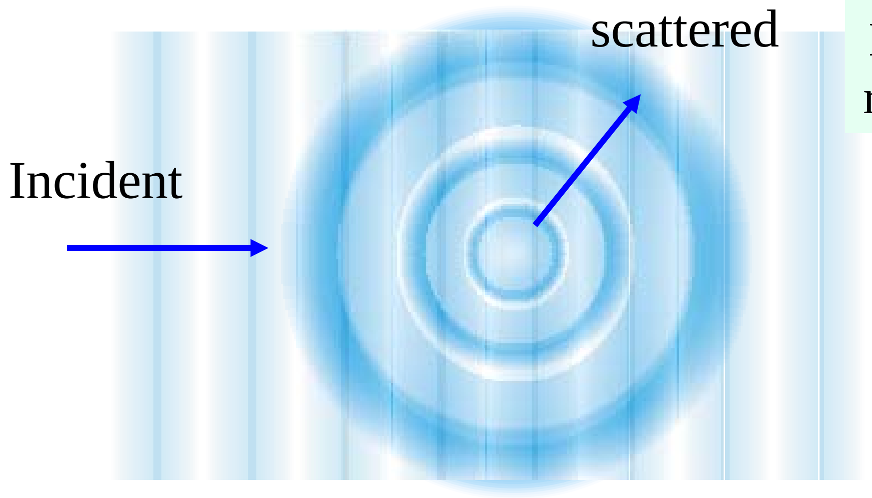
Y. Ji et al. PRL 88, 076601 (2002)



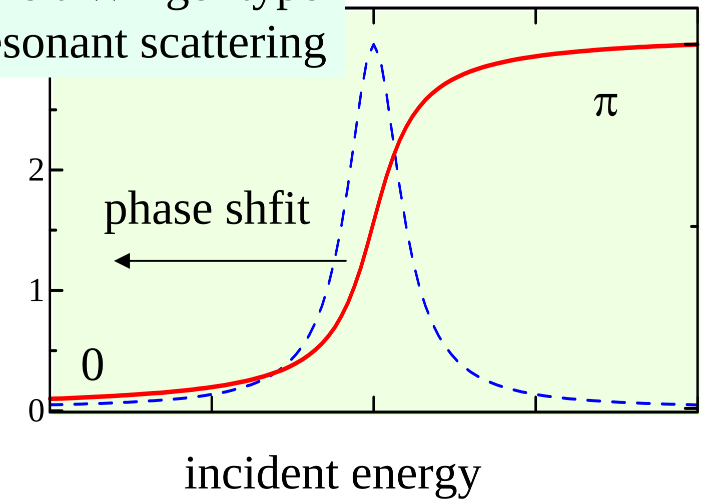
The Fano effect

Ugo Fano





Breit-Winger type resonant scattering

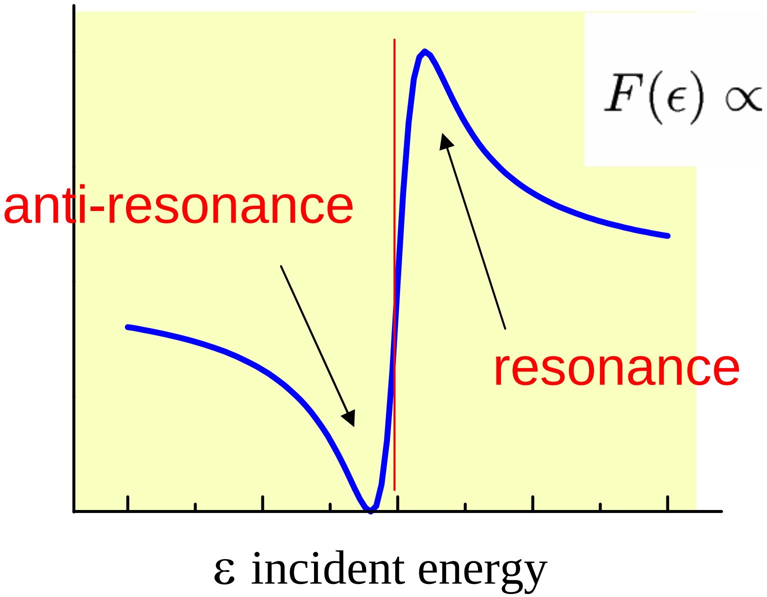


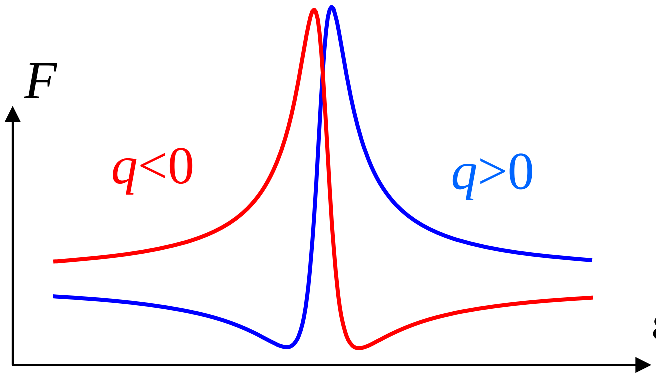
interference

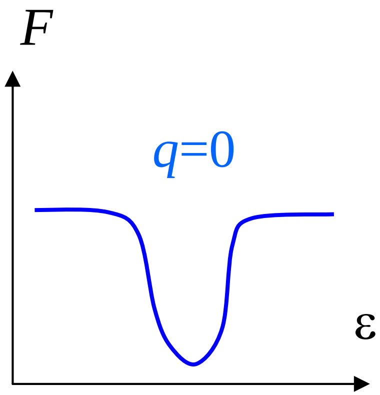
F (ε) : transition probability

Fano parameter

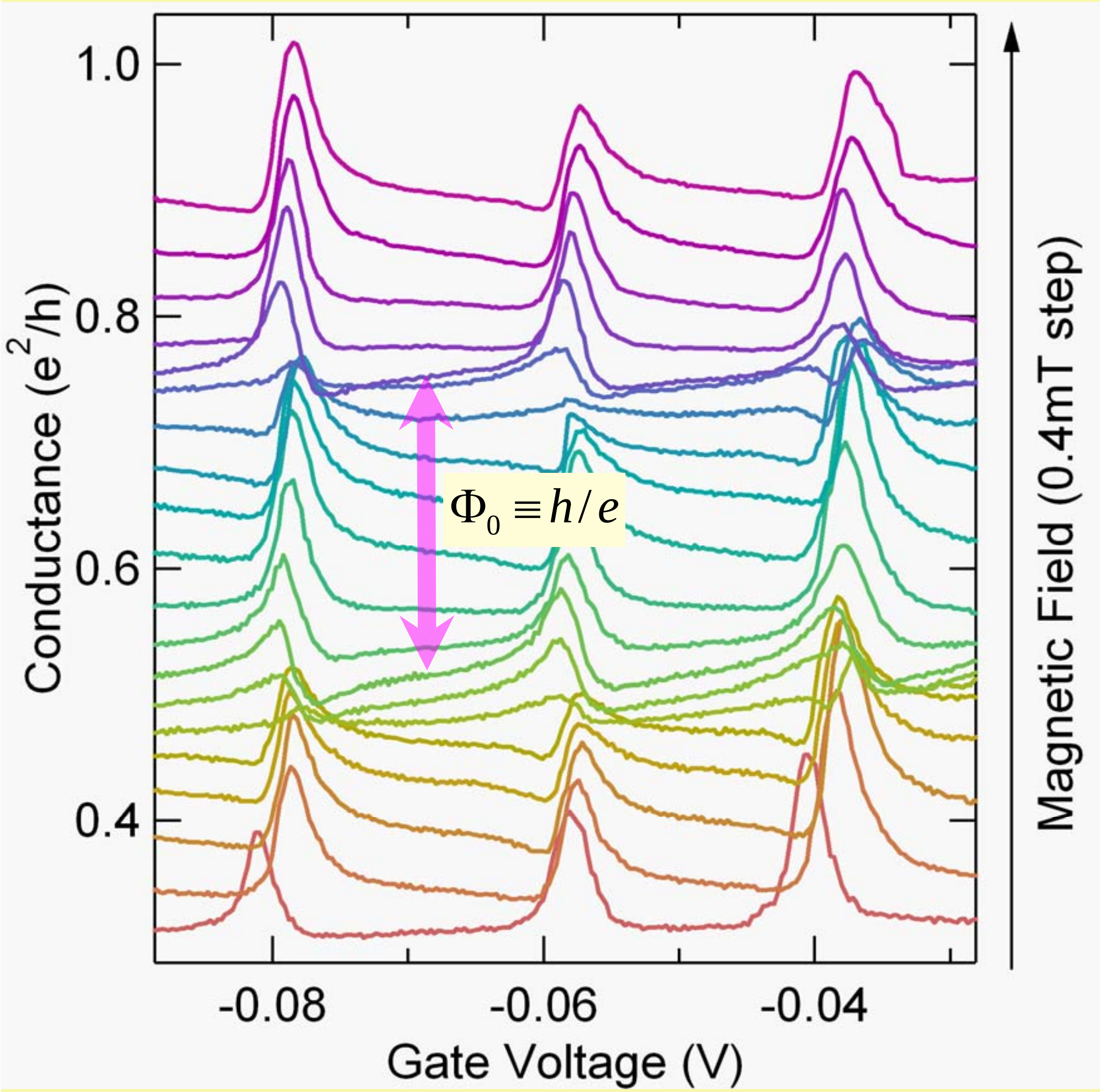
$$F(\epsilon) \propto \frac{(\tilde{\epsilon} + q)^2}{(\tilde{\epsilon}^2 + 1)}, \quad \tilde{\epsilon} = \frac{\epsilon - \epsilon_c}{\Gamma/2}$$

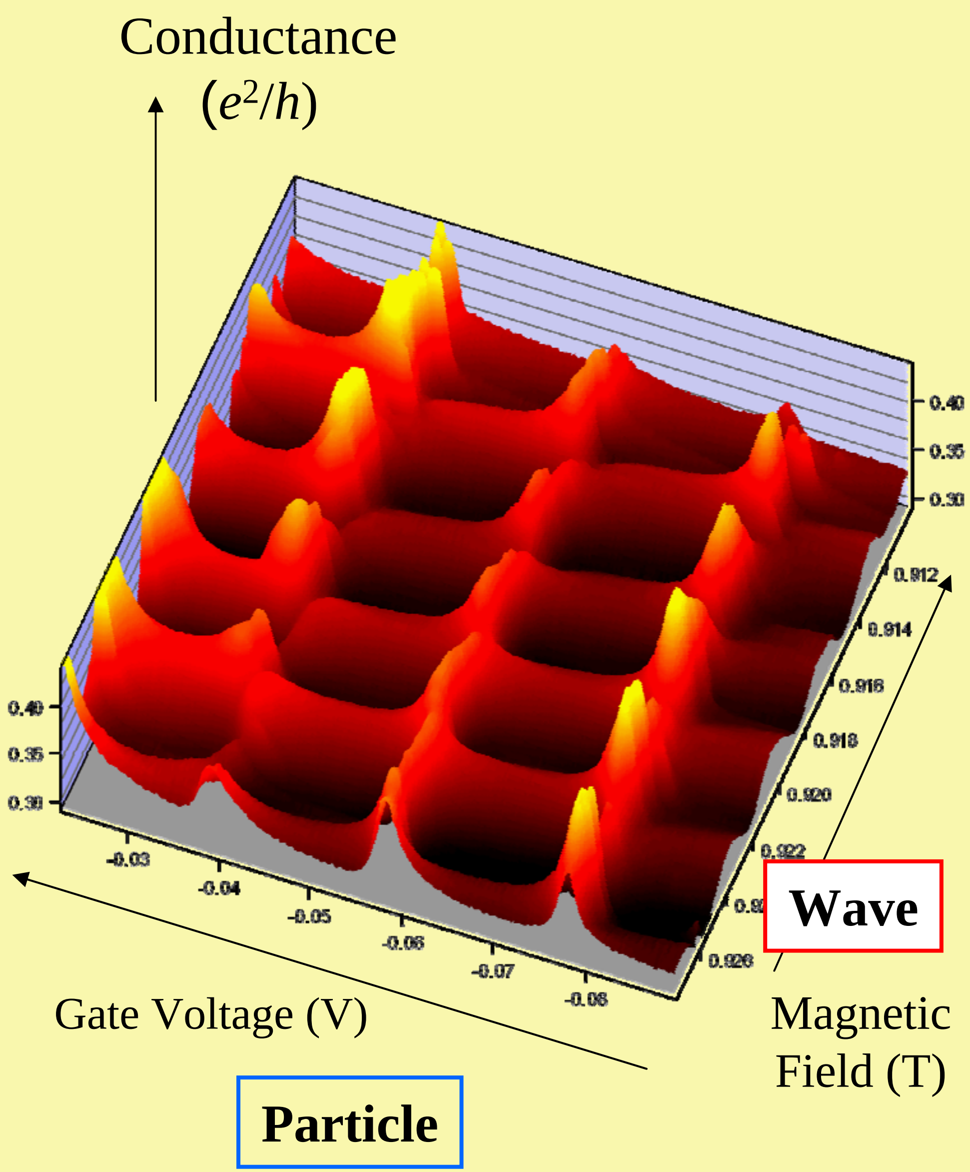






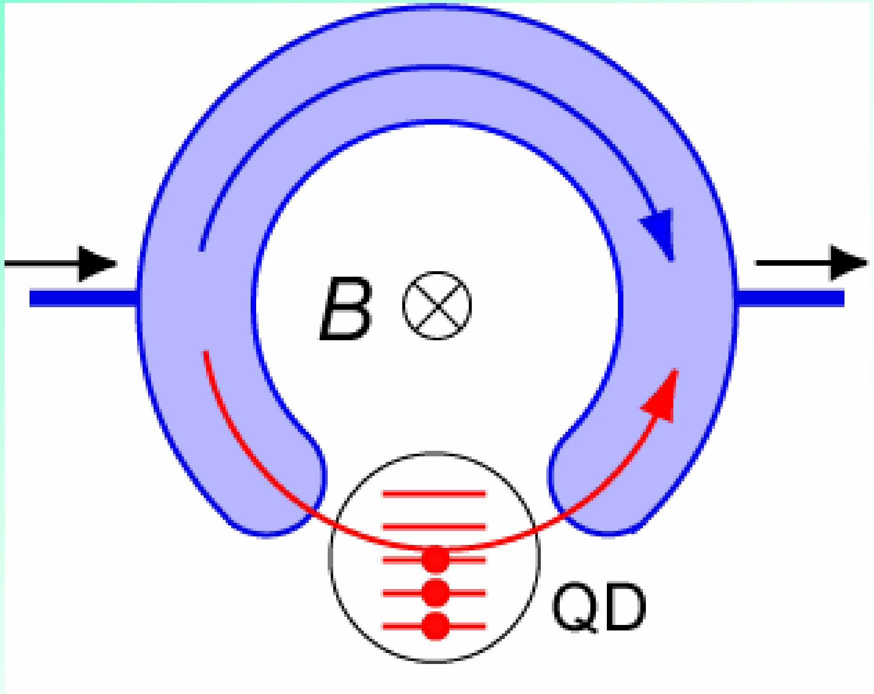
Effect of magnetic flux

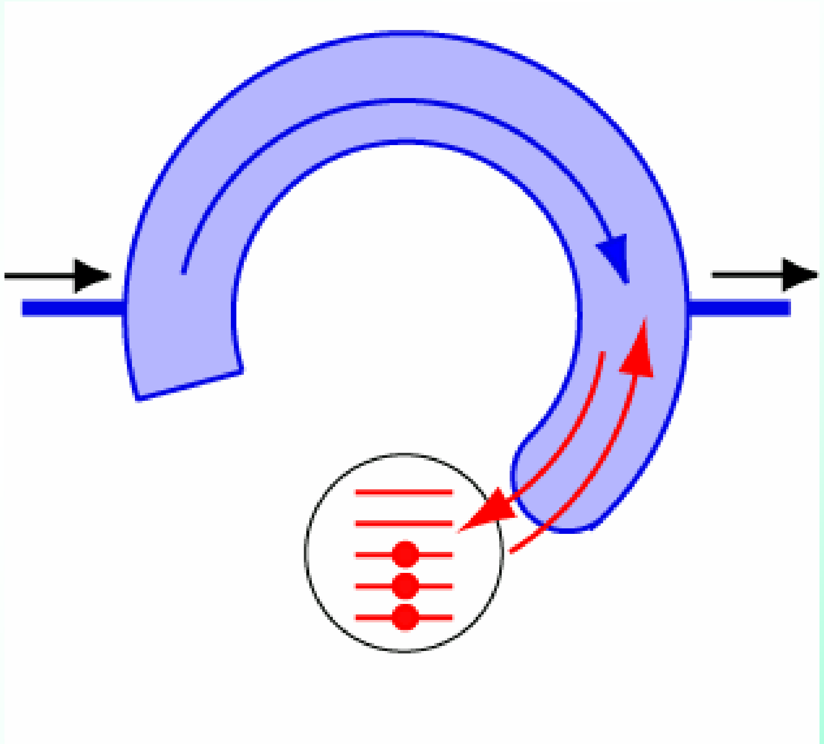




K. Kobayashi et al. PRL 88, 256806 (`02)

Fano effect in side-coupled dot geometry





QD-AB-ring system

Fano effect

in the transmission mode.

(Mach-Zender-like)

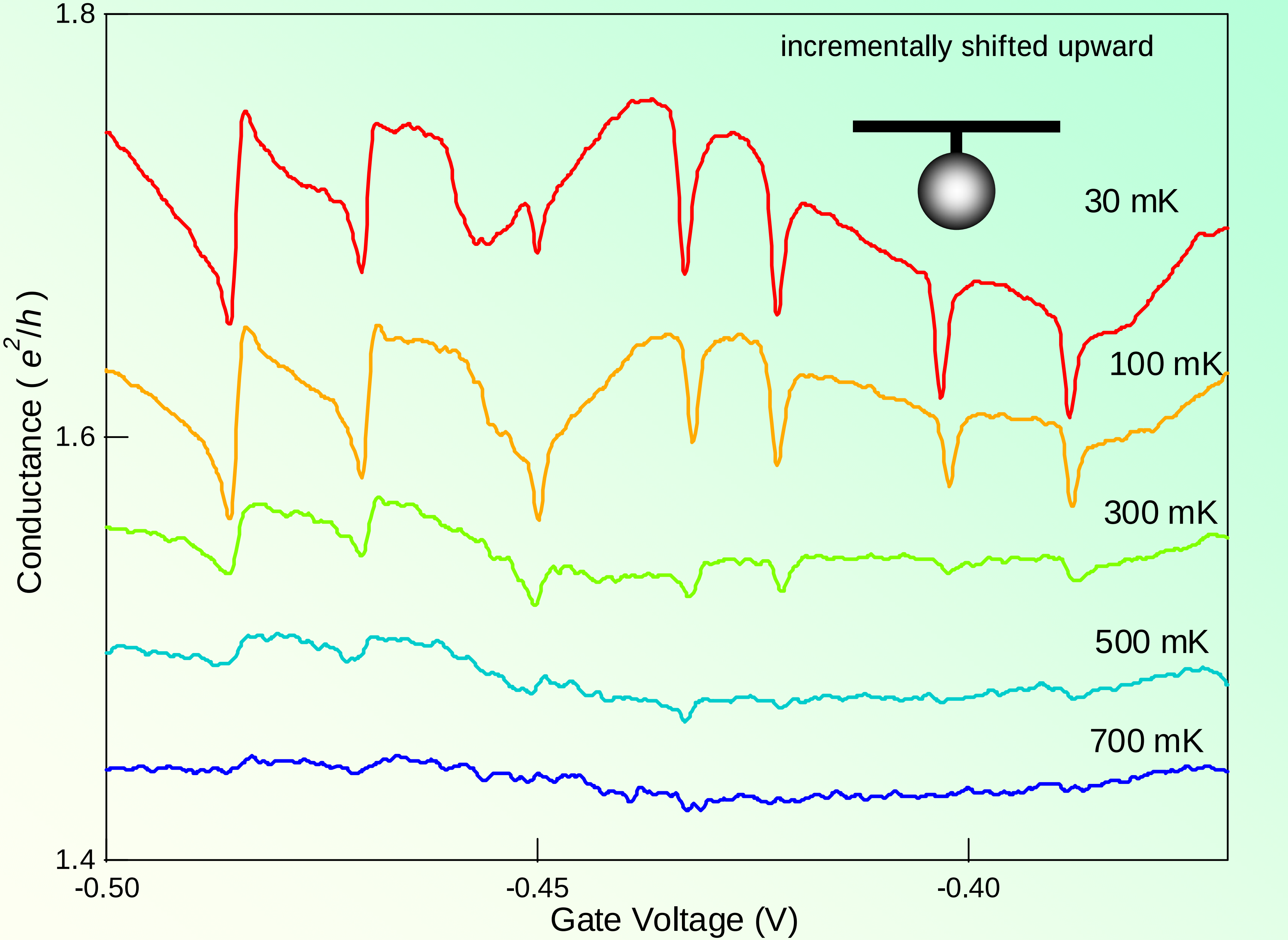
T-coupled quantum dot

Fano effect

in the reflection mode.

(stub-type or Michelson-type)

Emergence of non-local Coulomb "dips" with Fano distortion

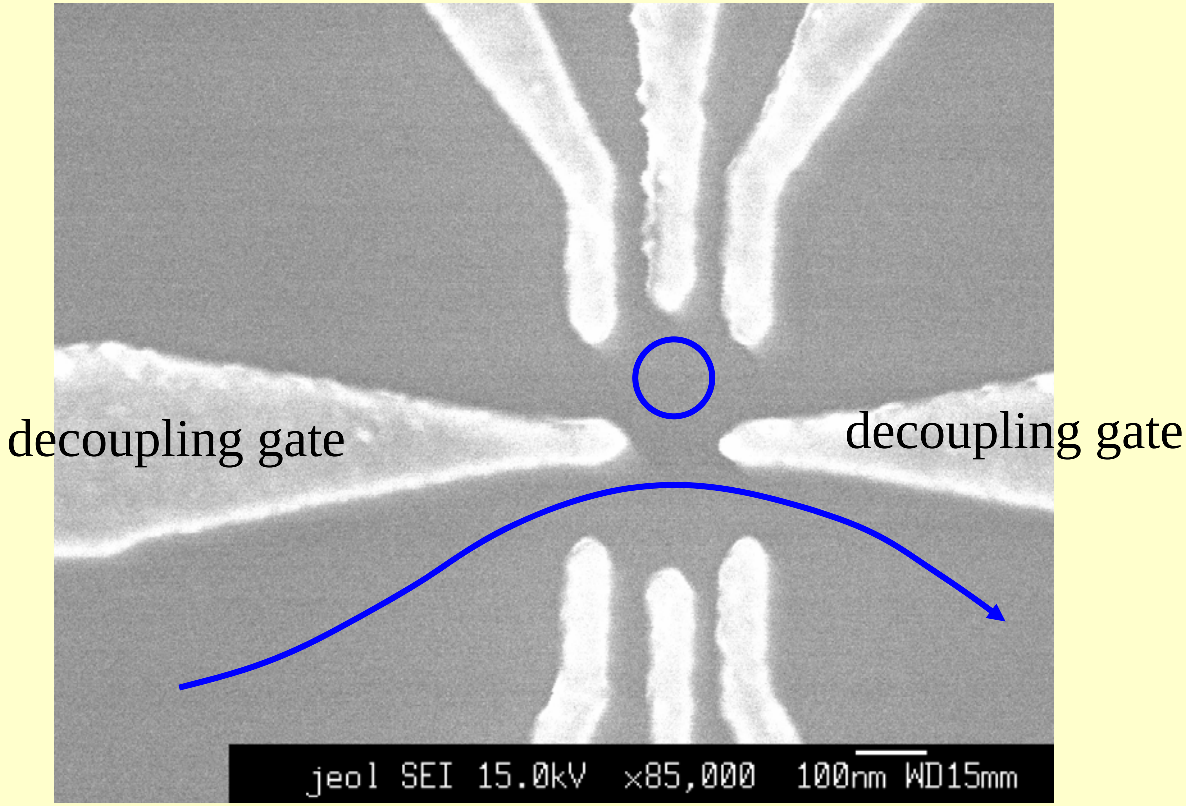


T-coupled Quantum Dot-Wire Hybrid

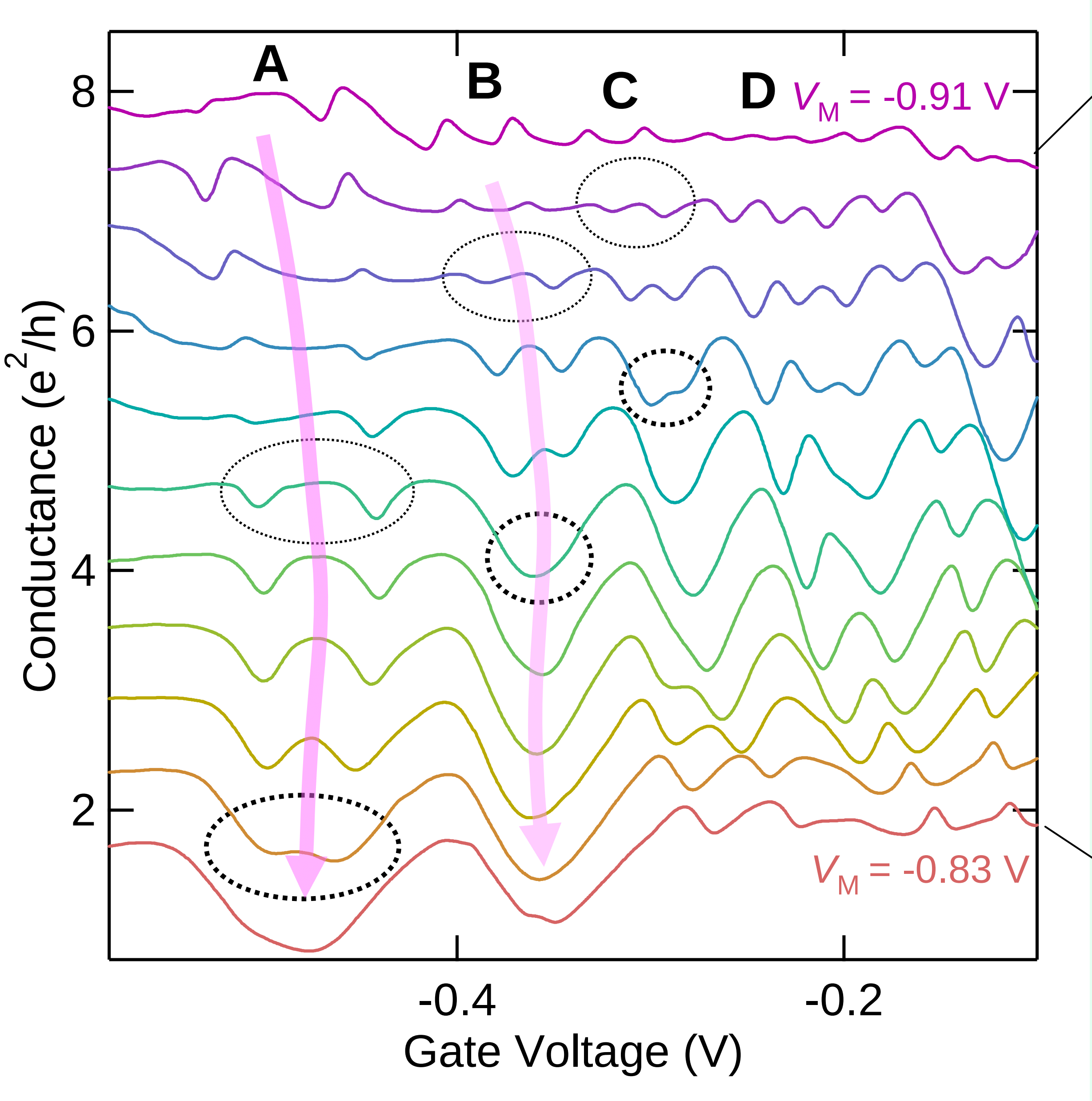
- U = 0.3 - 0.7meV
- Δ = 0.3 - 0.5meV
- Dot diameter ~ 50nm

Spatially compact
-> high coherence

Single connection point
-> small dot size is available



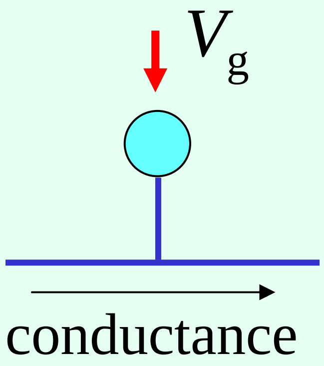
Coupling strength dependence of anti-resonance



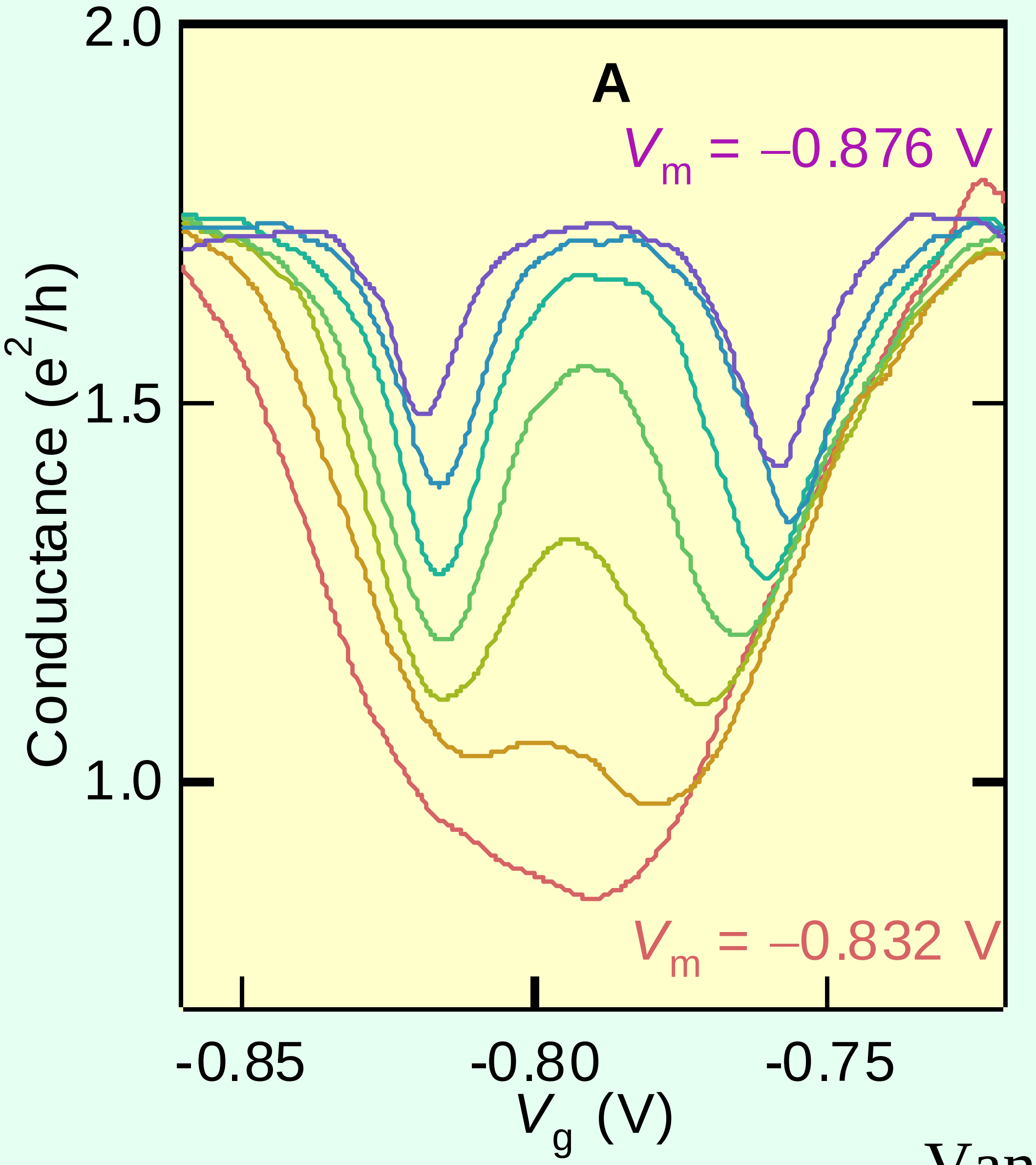
coupling : weak

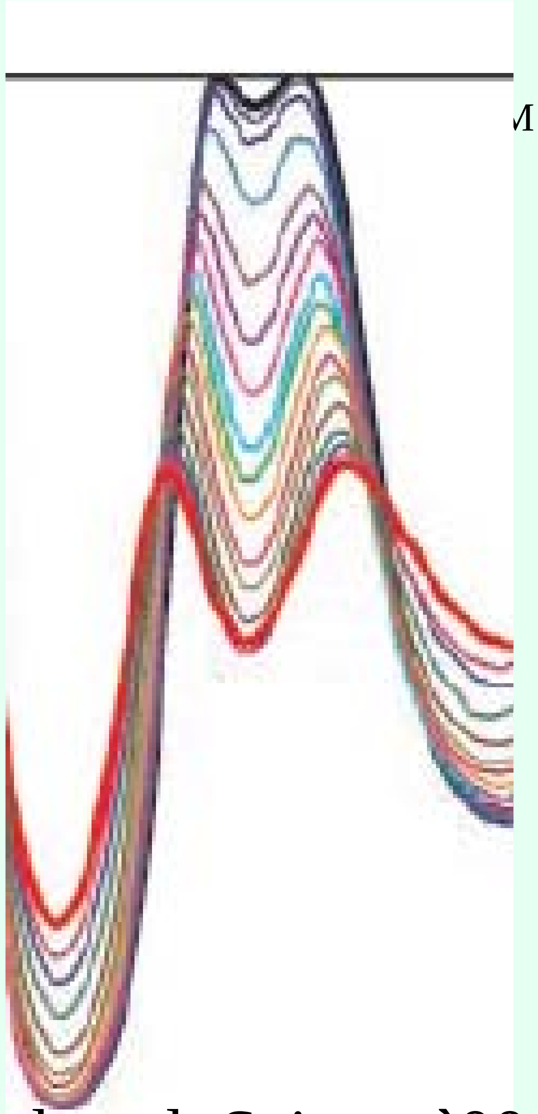
coupling : strong

Decoupling gate V_M
: 8mV pitch



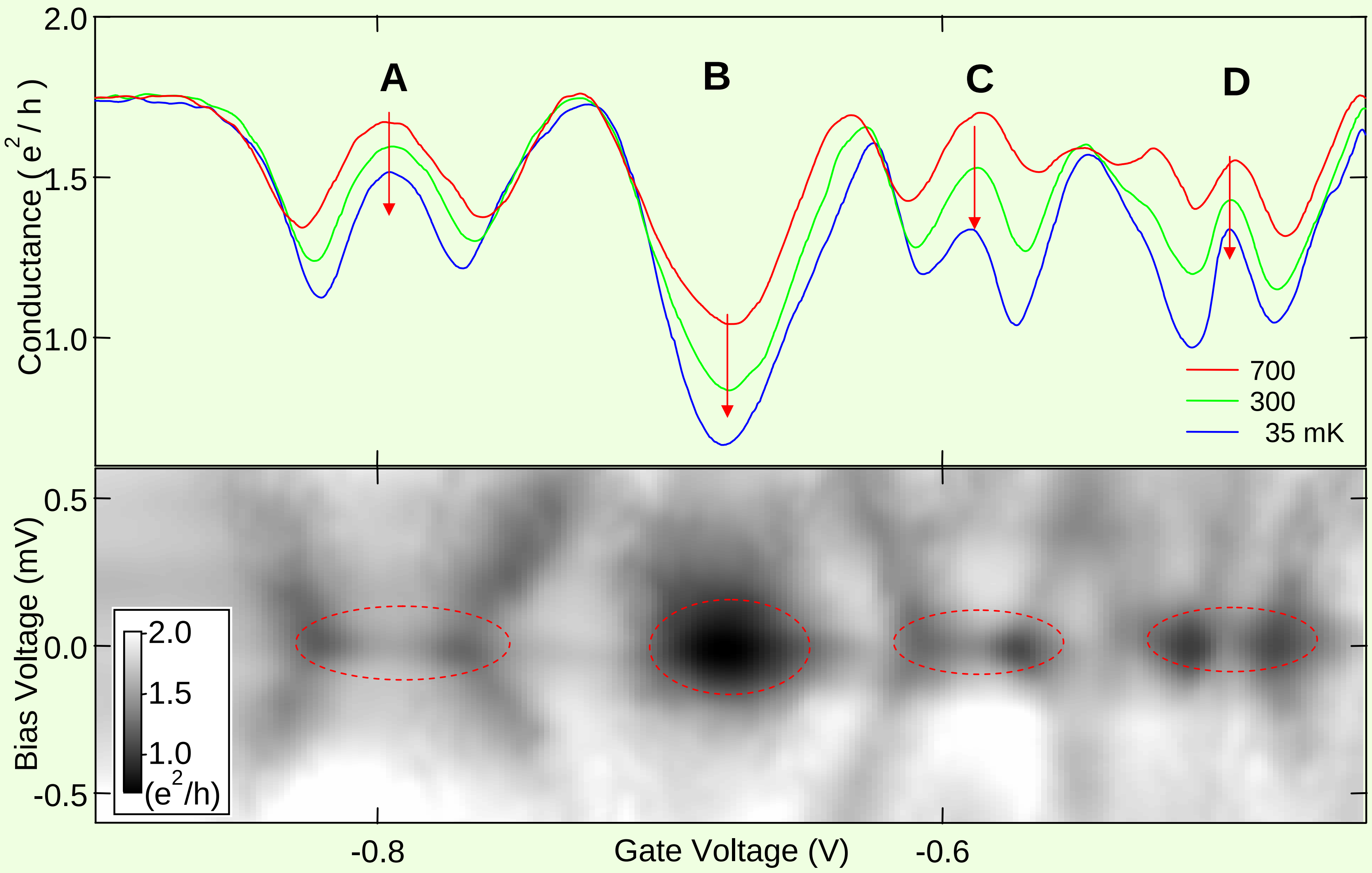
Coupling strength dependence of anti-resonance





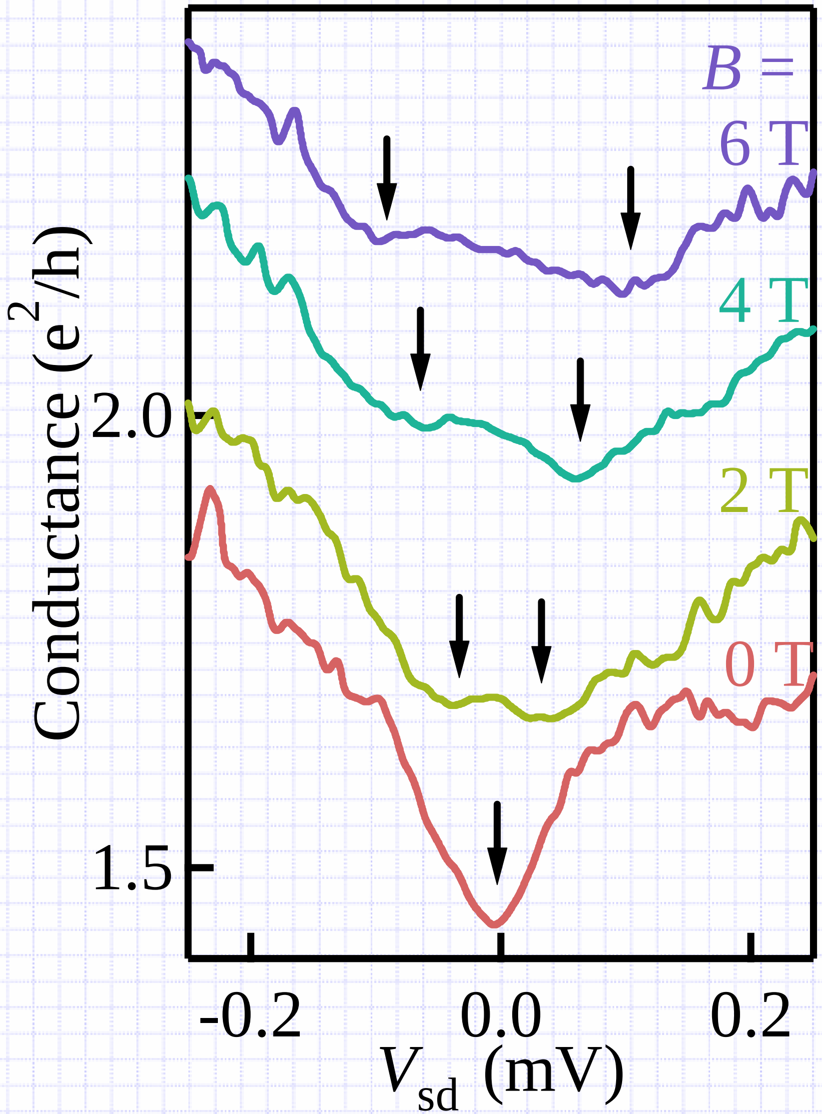
Van der Wiel et al. Science`00

Observation of Fano-Kondo anti-resonance

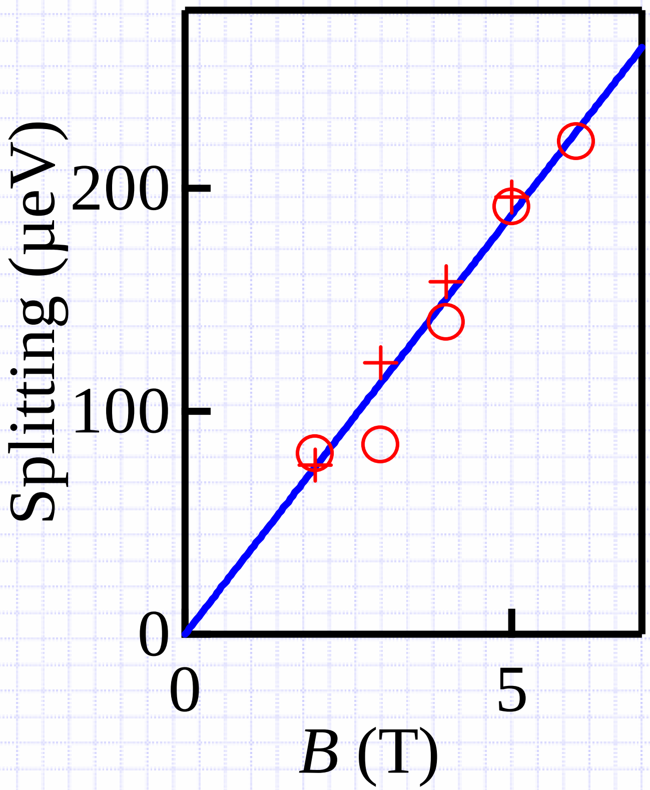


M. Sato et al. PRL.

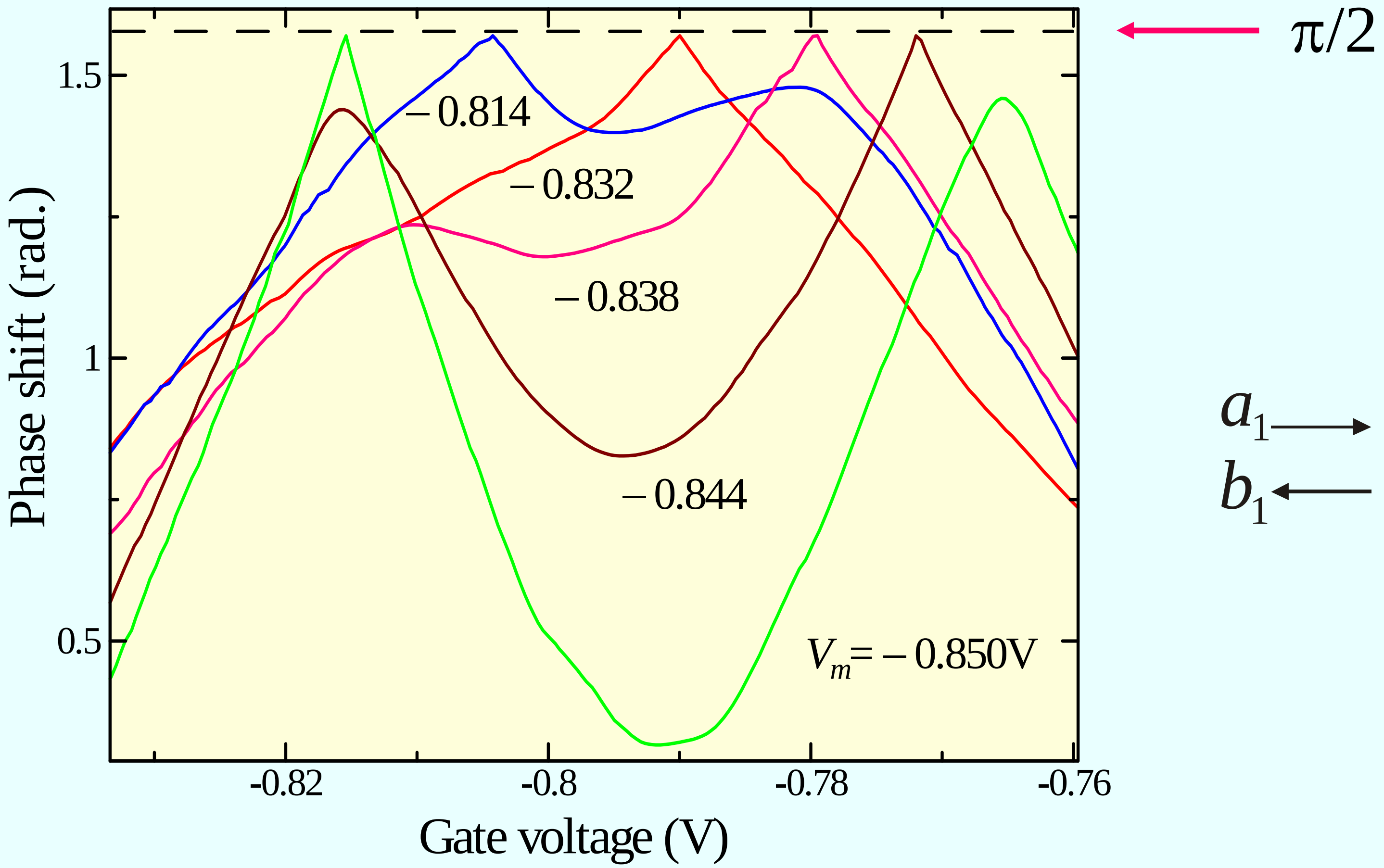
Zeeman splitting

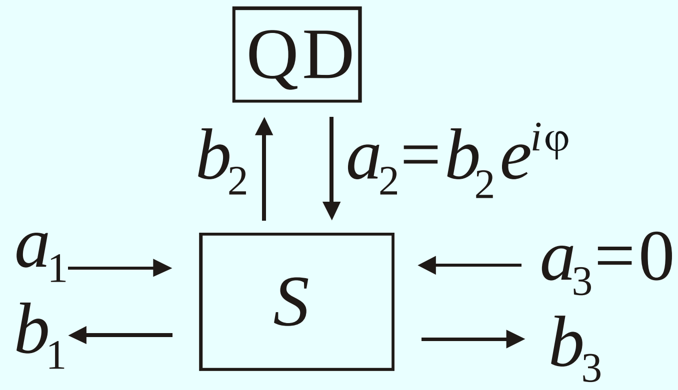


Zeeman splitting of zero bias dip proportional to *B* (|g|=0.33)



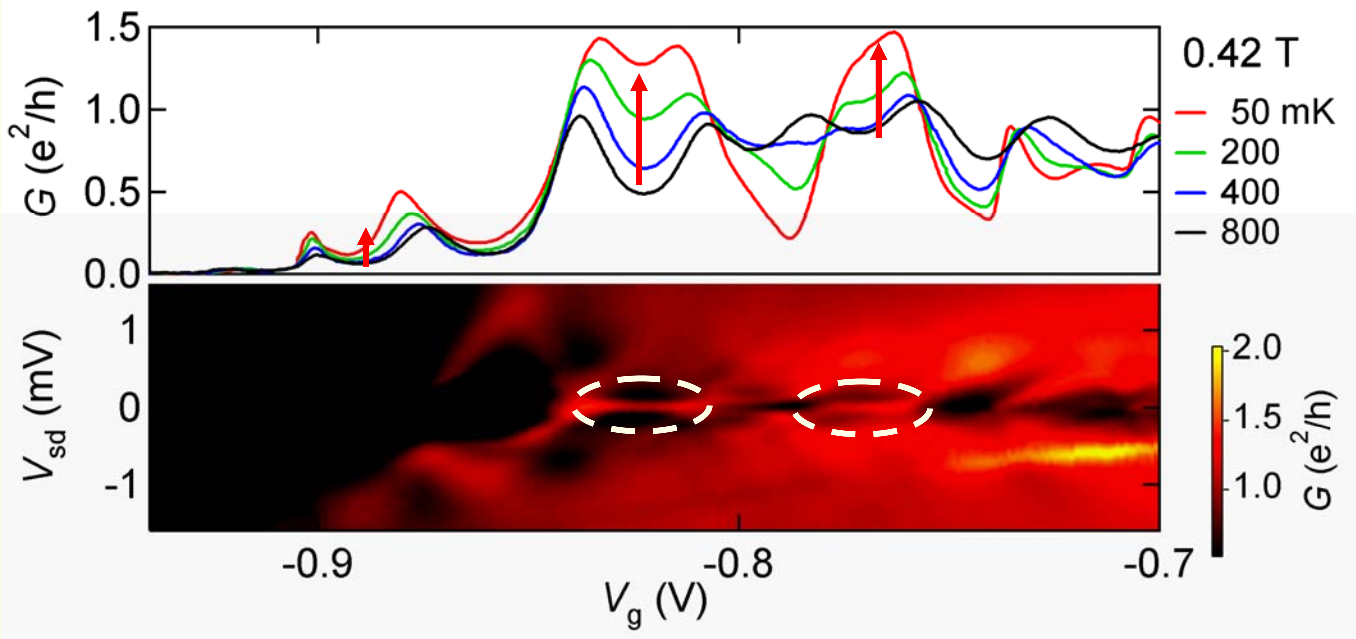
Phase shift locking to π/2



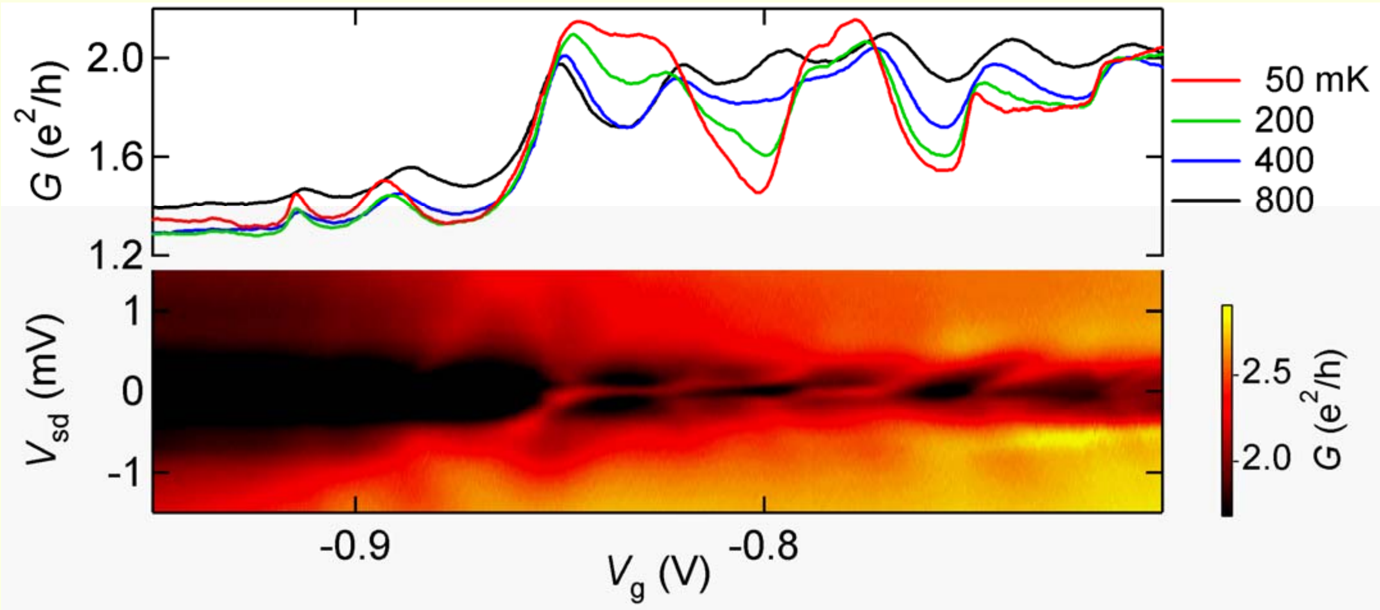


The Kondo Effect

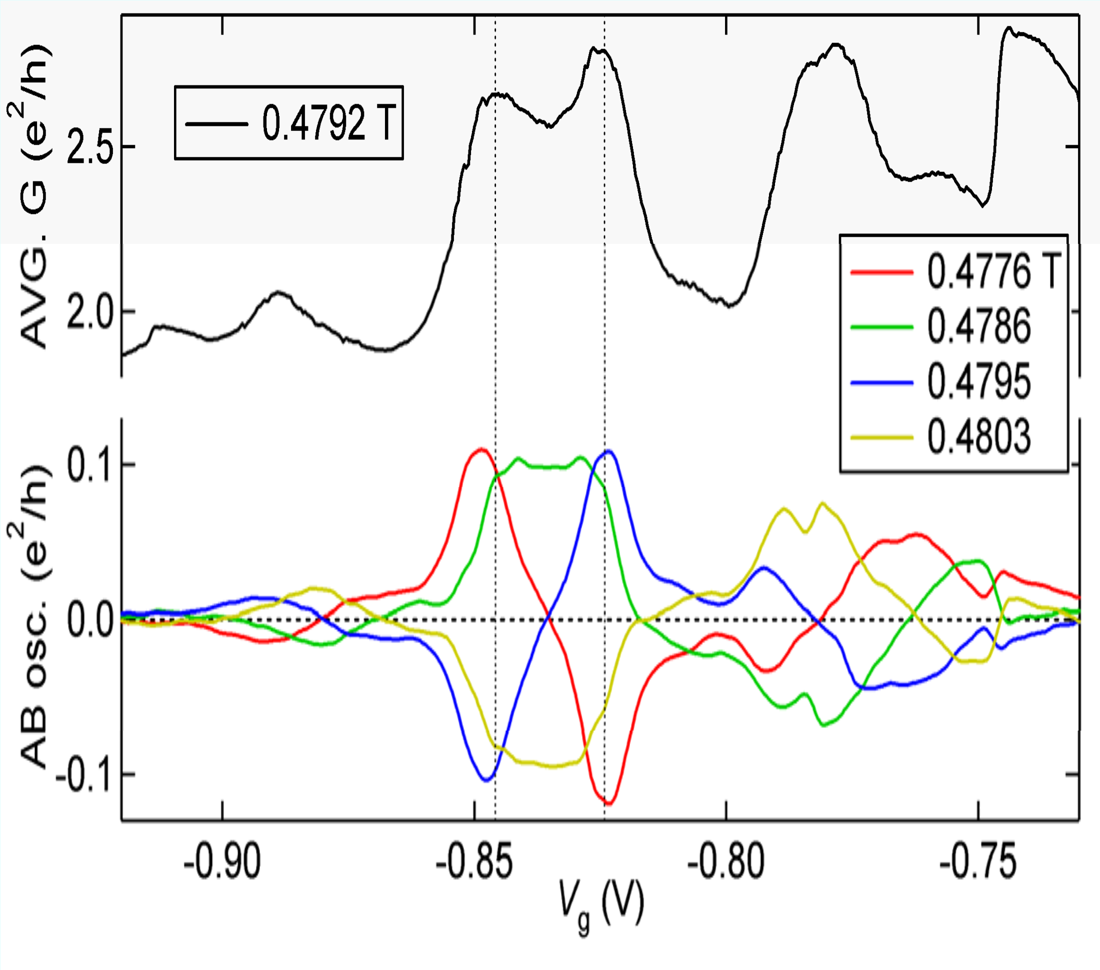
Without reference

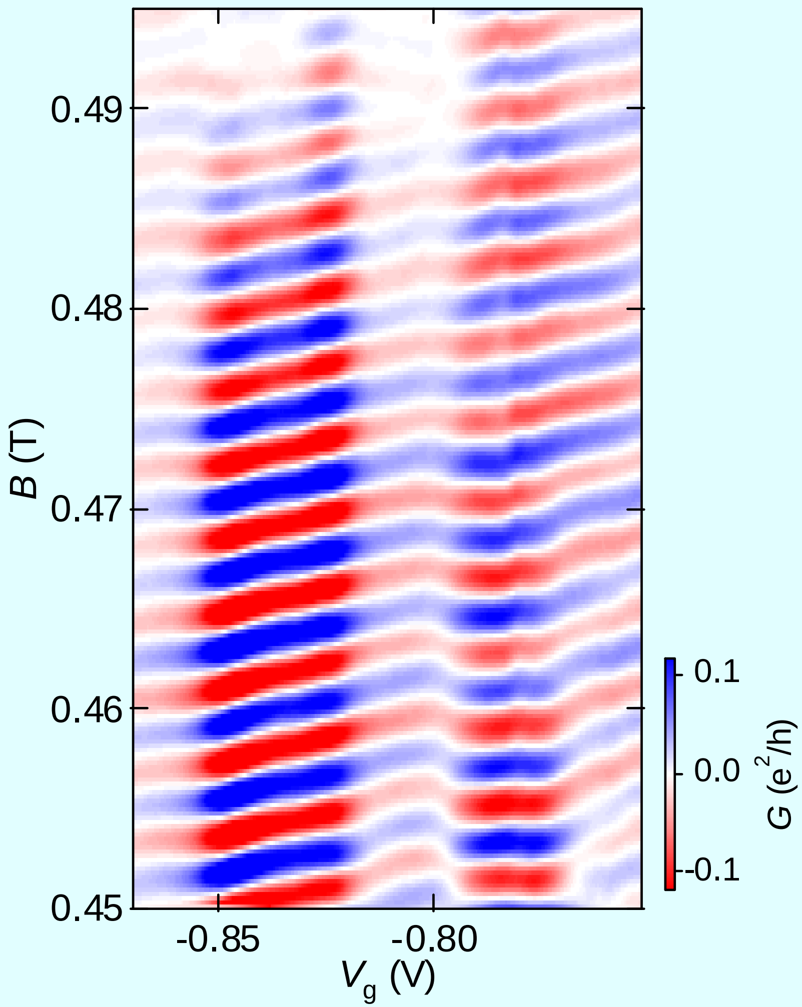


With reference

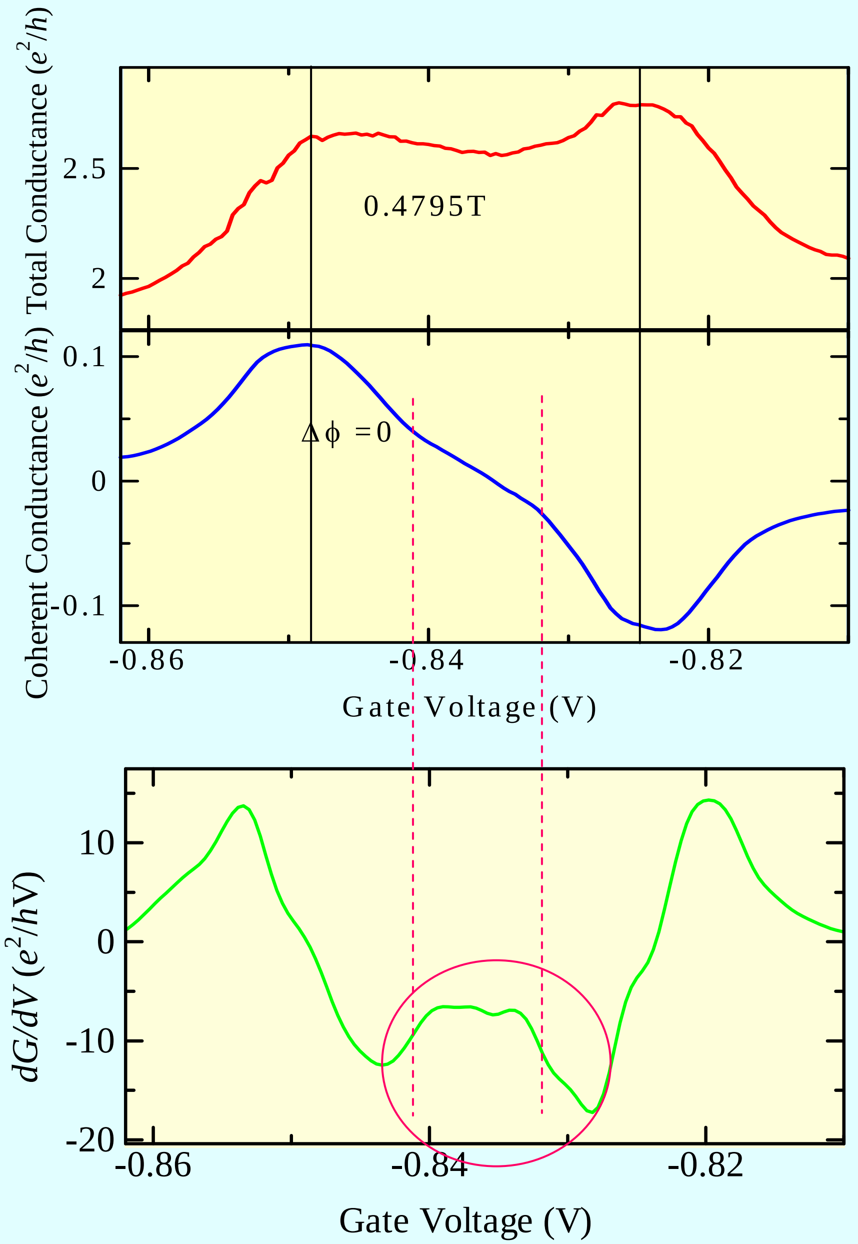


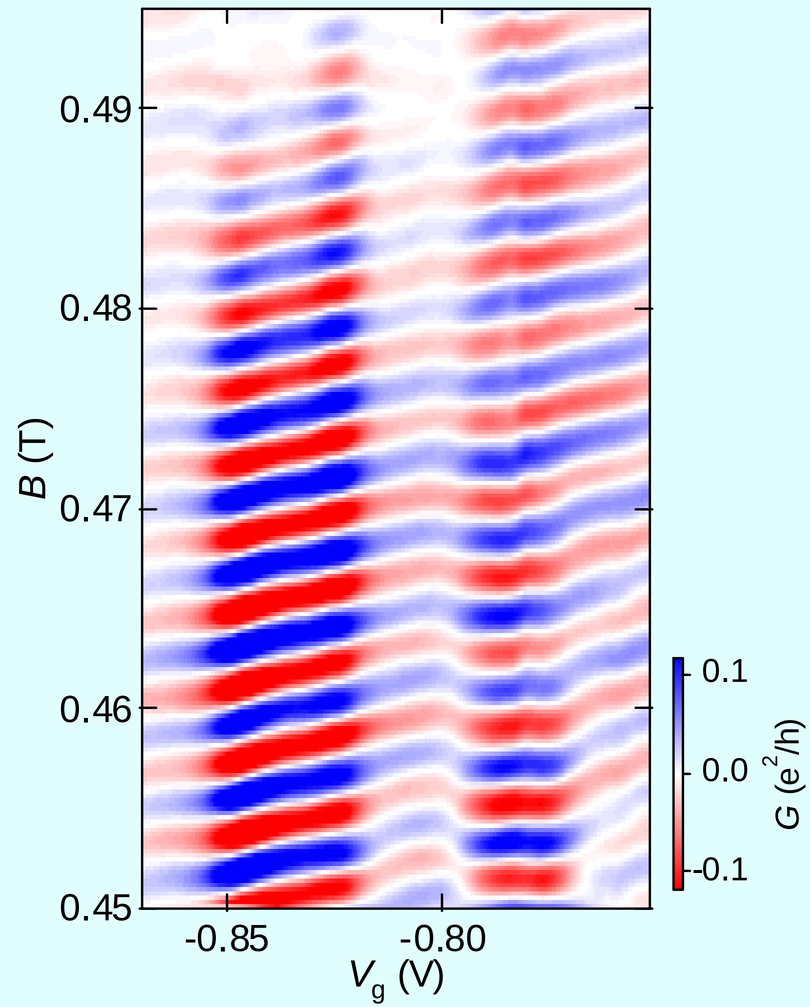
“Coherent” component and the Fano-Kondo Effect



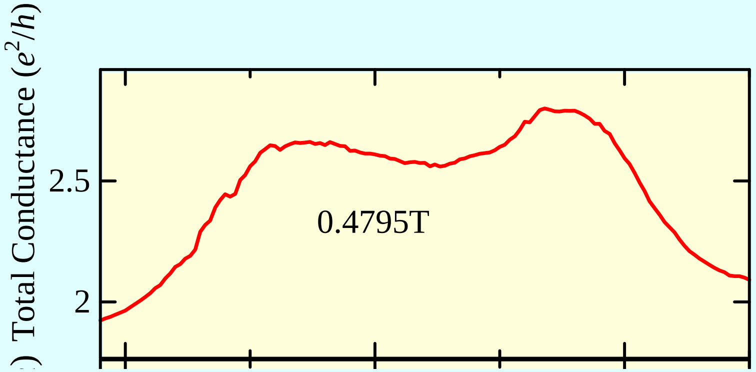


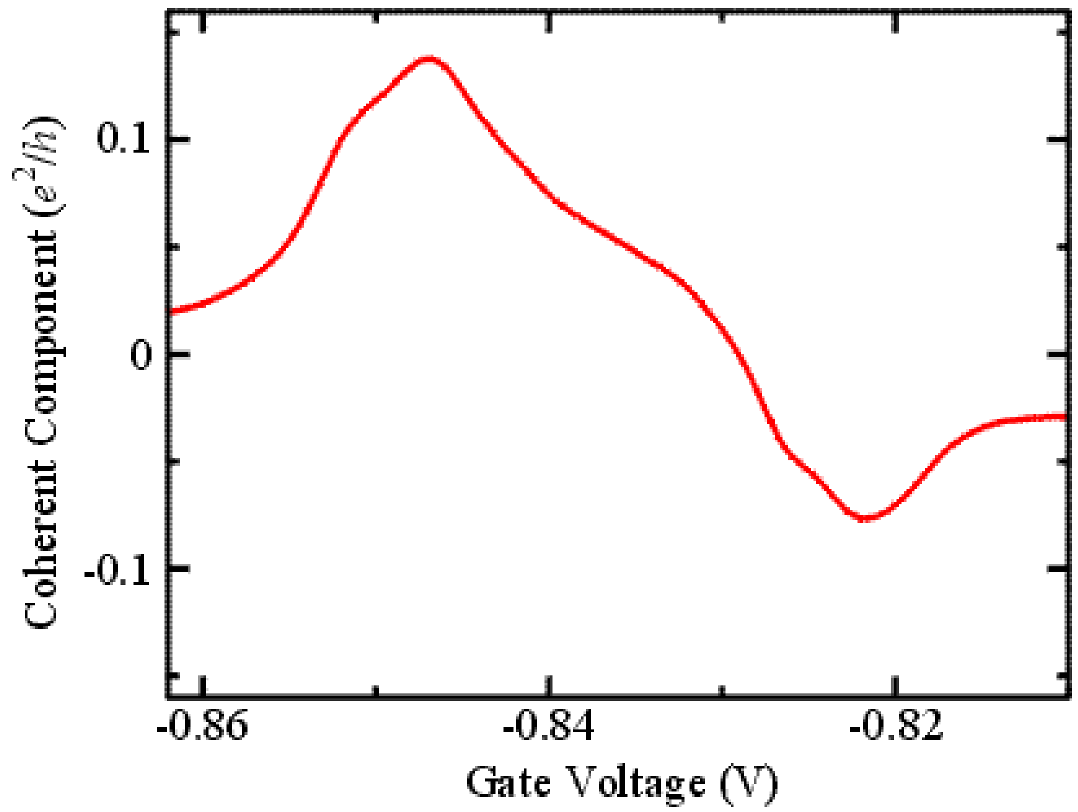
"Coherent" component and the Fano-Kondo Effect

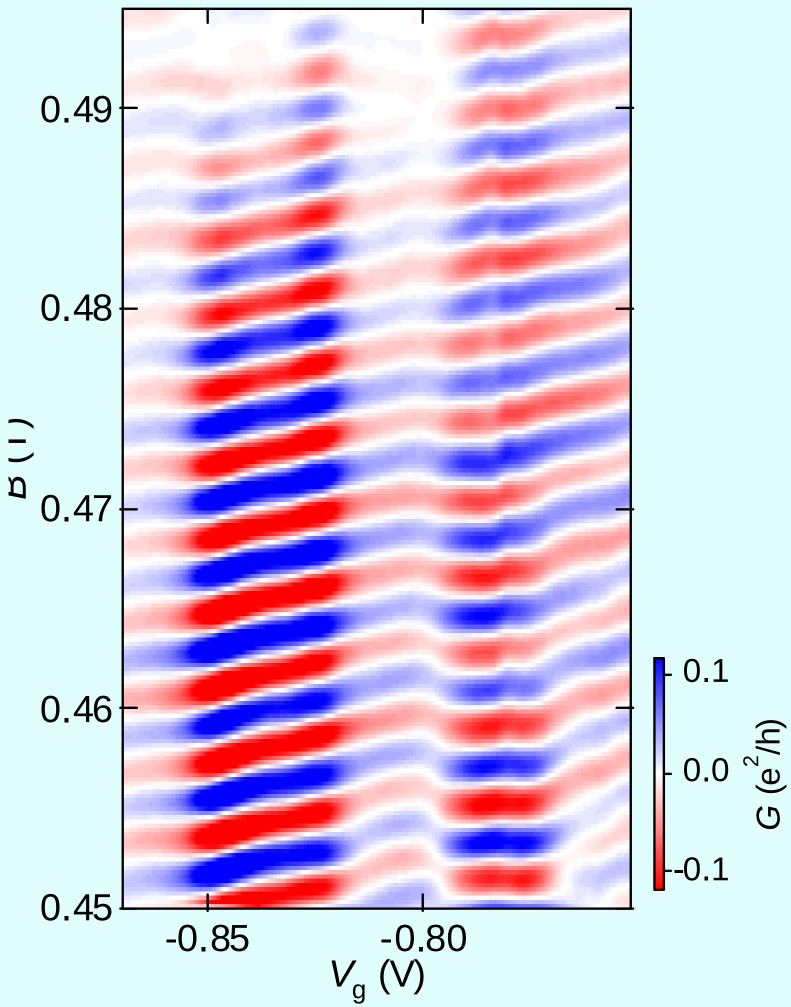




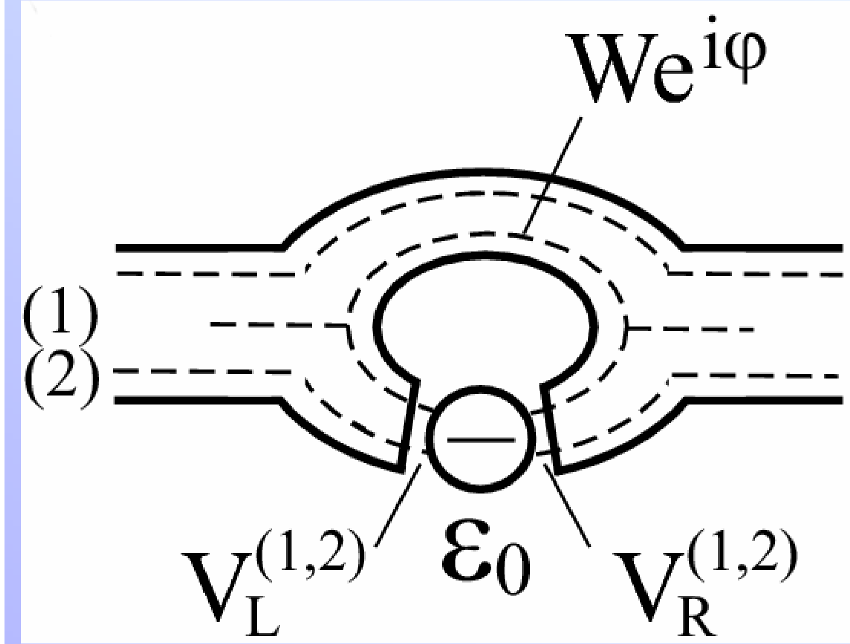
“Coherent” component and the Fano-Kondo Effect







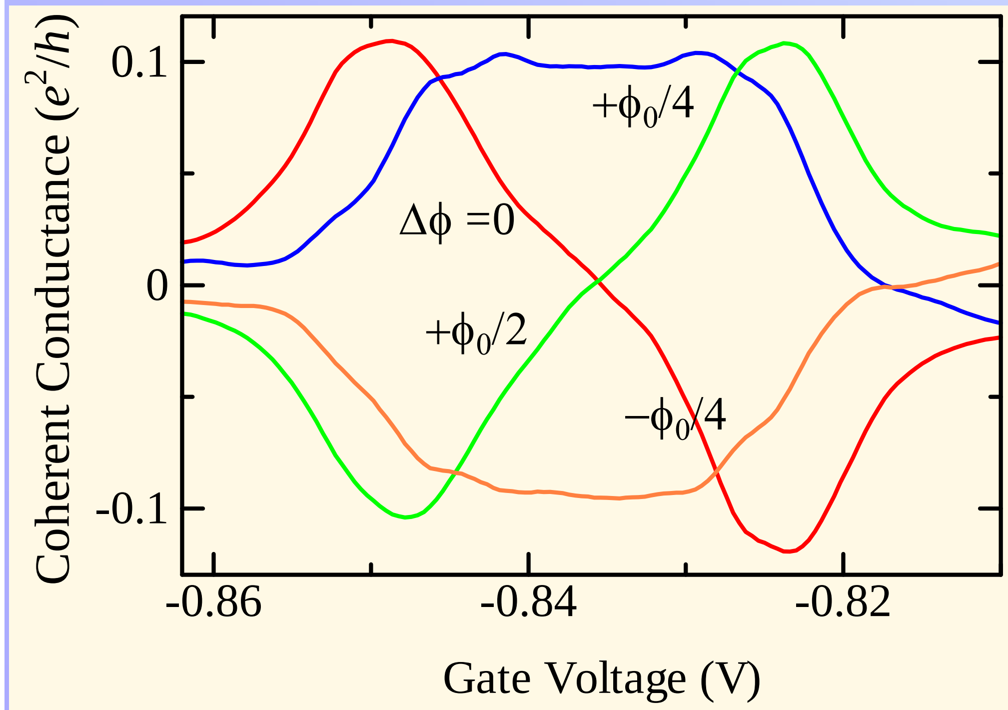
Multi-channel model

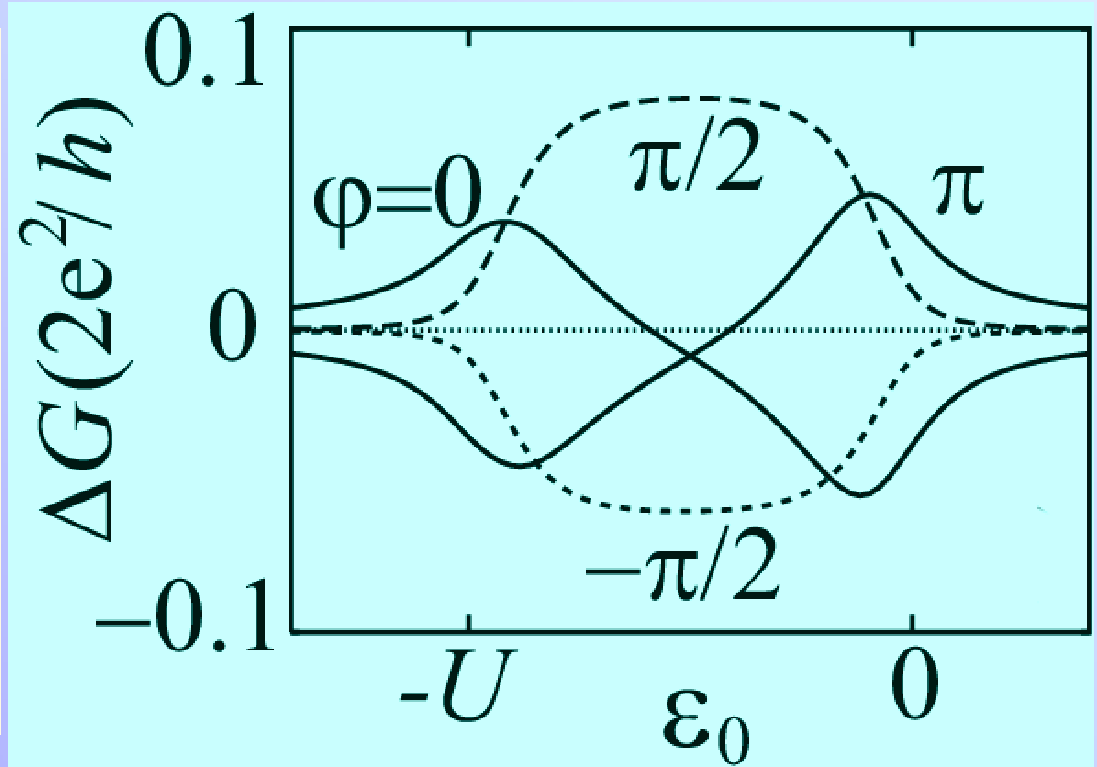


$$H = \sum_{k\sigma\alpha=1,2} \varepsilon_{k\sigma} a^{(\alpha)+}_{rk\sigma} a^{(\alpha)}_{rk\sigma} + \varepsilon_0 \sum_{\sigma} d^+_\sigma d_\sigma + U n_\uparrow n_\downarrow$$
$$+ \sum_{k\sigma r\alpha=1,2} \left(V^{(\alpha)}_r d^+_\sigma a_{rk\sigma} + h.c. \right) + \sum_{k'k\sigma} \left(W e^{i\varphi} a^{(1)+}_{Rk'\sigma} a^{(1)}_{Lk\sigma} + h.c. \right)$$

Kondo effect: finite-U slave boson approx.

SK, M. Eto, et al., phys. stat. sol. (c) 3 (2006) 4208-4215.





Weak entanglement between localized spin and conduction spin?

S. Oh & J. Kim, PRB73-052407(`06)

Yosida's variational ground state

$$|\Psi_s\rangle = \frac{1}{\sqrt{2}}\left(|\phi_\downarrow\rangle |\chi_\uparrow\rangle - |\phi_\uparrow\rangle |\chi_\downarrow\rangle \right)$$

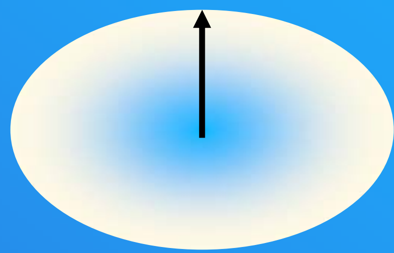
$$|\phi_\downarrow\rangle = \frac{1}{\sqrt{\mathcal{N}}} \sum_{k>k_F} \Gamma_{\mathbf{k}} c^\dagger_{\mathbf{k}\downarrow} |F\rangle$$

Entanglement entropy between electron spins in Kondo cloud and localized spin

$$\rho_{\rm im} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

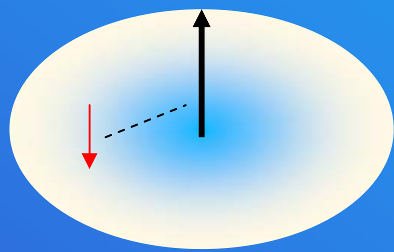
$$S(\rho_{\rm im}) = 1$$

Maximally entangled



Entanglement entropy between an electron spin in Kondo cloud and localized spin

$$S(\rho) \approx O(1/N)$$

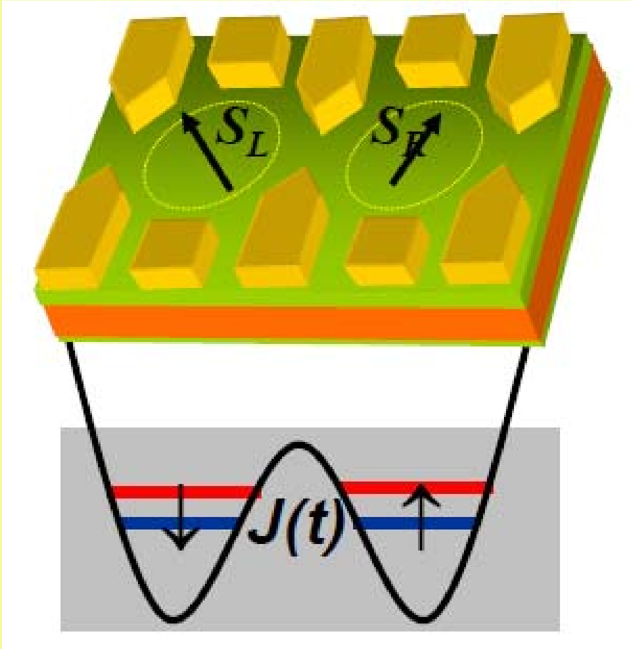


Exchange coupling J(t) in double dot:

$$H_S(t) = J(t) S_L \cdot S_R$$

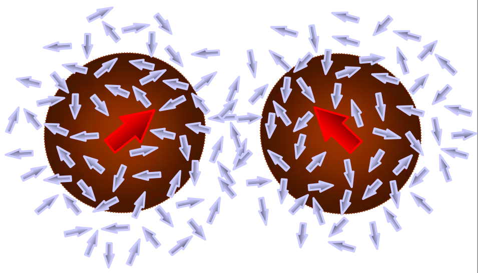
Tunable entanglement

1. Theory for artificial atoms and molecules
   ->exchange J
2. Theory for electrical current through system
   ->measurements



Interaction of a qubit with its environment leads to entanglement of qubit with environment and decoherence.

Two impurity Kondo: A model of entangled qubits and the environment?

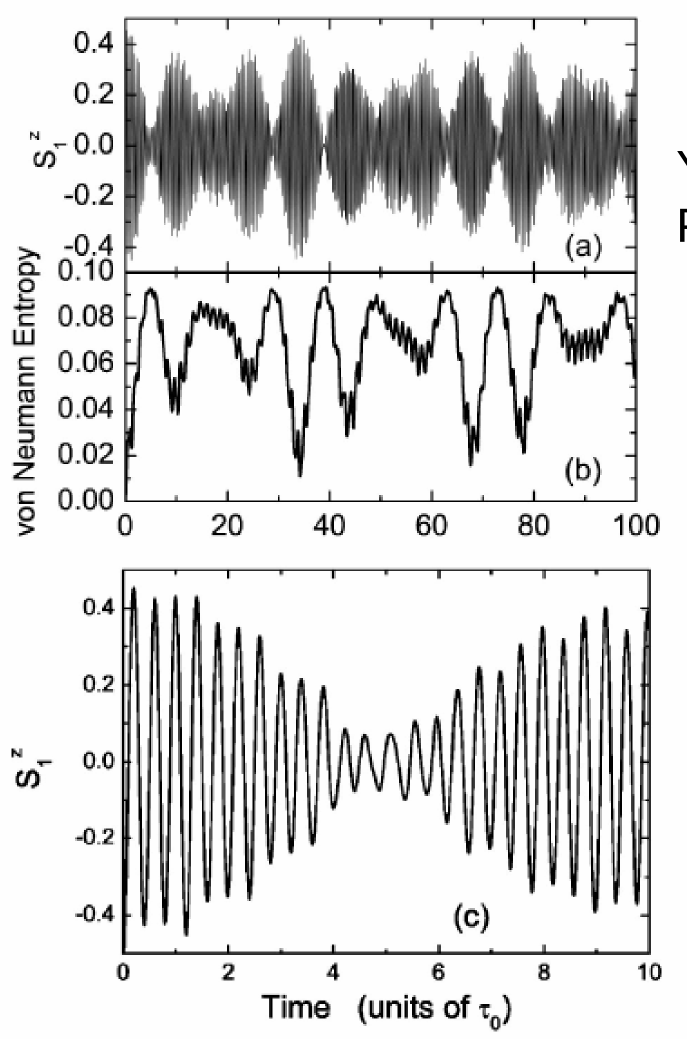


Two impurity Kondo system

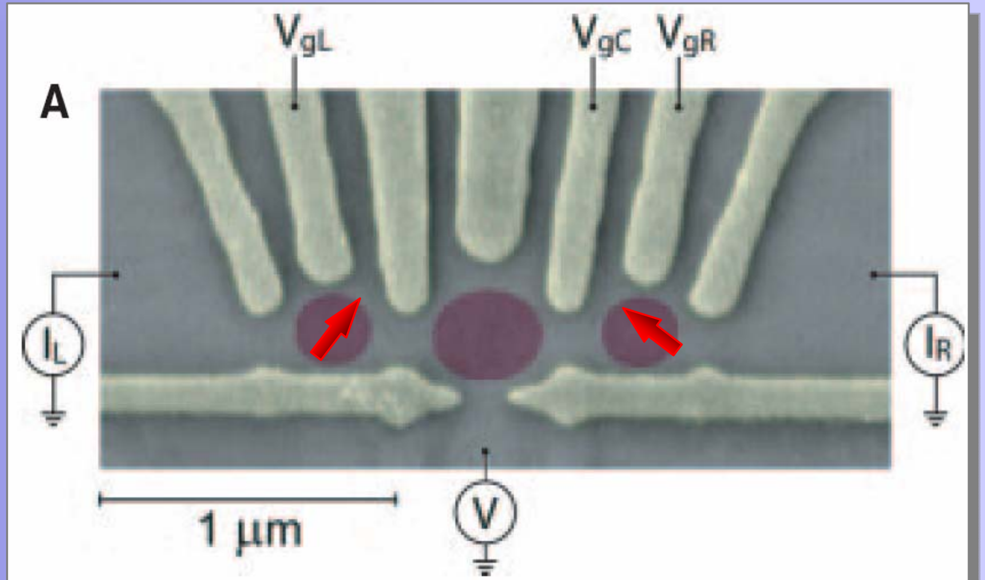
$$H = H_C - J\Big(\mathbf{S}_A \cdot \mathbf{s}_c(A) + \mathbf{S}_B \cdot \mathbf{s}_c(B)\Big)$$

$$H_{RKKY} = I(R)\, \mathbf{S}_A \cdot \mathbf{S}_B$$

S. Y. Cho & R. H. McKenzie, PRA (2006)

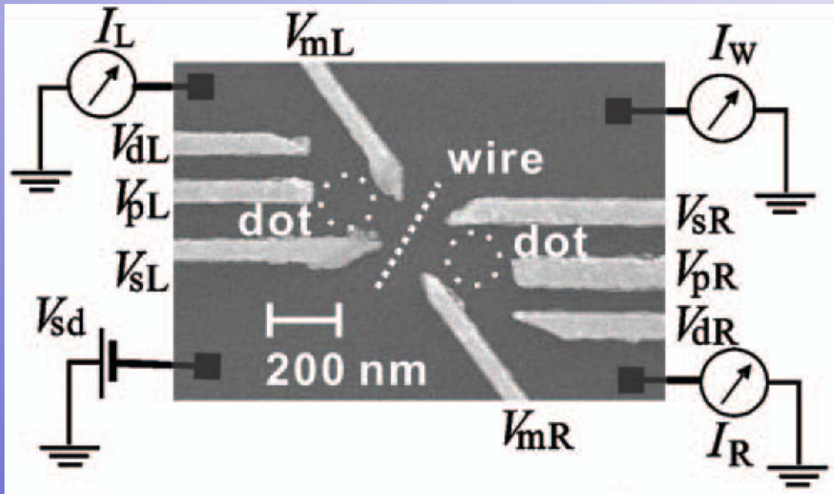


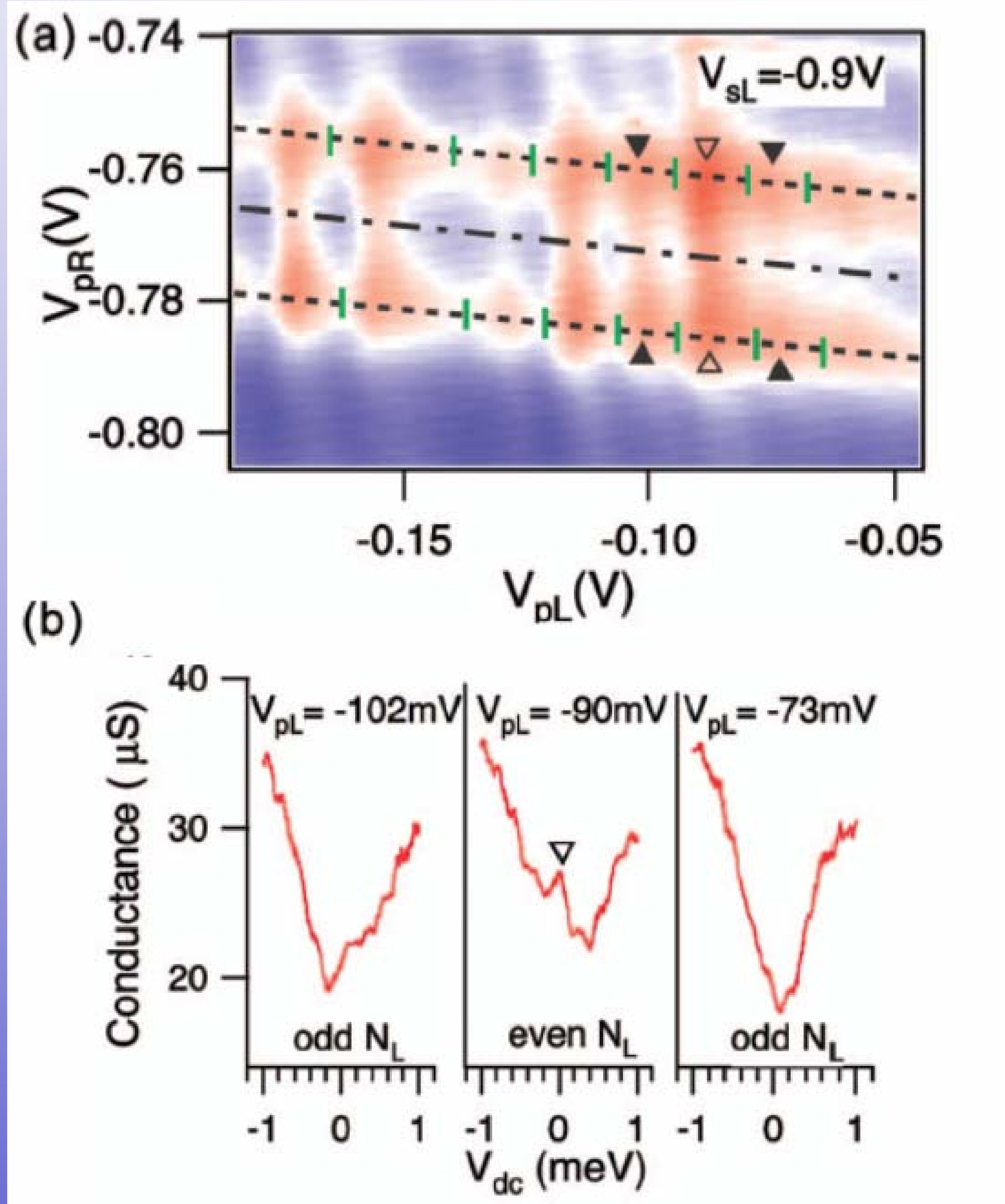
Y. Gao&S-J Xiong, PRA (2005)



N. J. Craig et al., *Science* 304, 565 (2004)

Two impurity Kondo exeperiment?

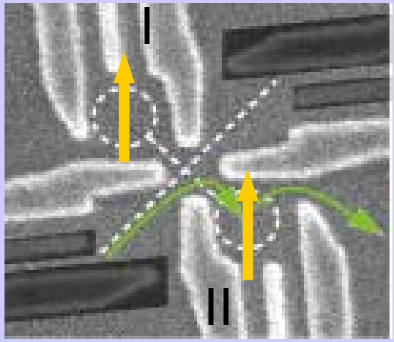


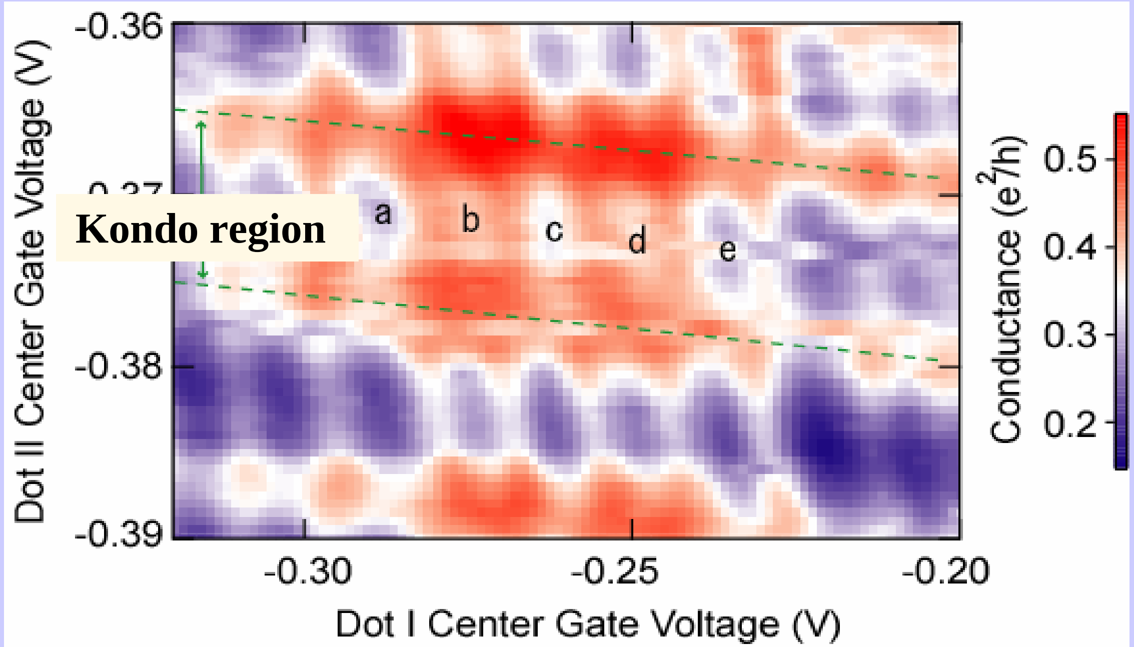


S. Sasaki *et.al*,
Phys. Rev. B 73 161303 (2006)

Reduction of the Kondo effect: fronting alignment

Face to Face configuration





In Kondo region

high conductivity: b, d

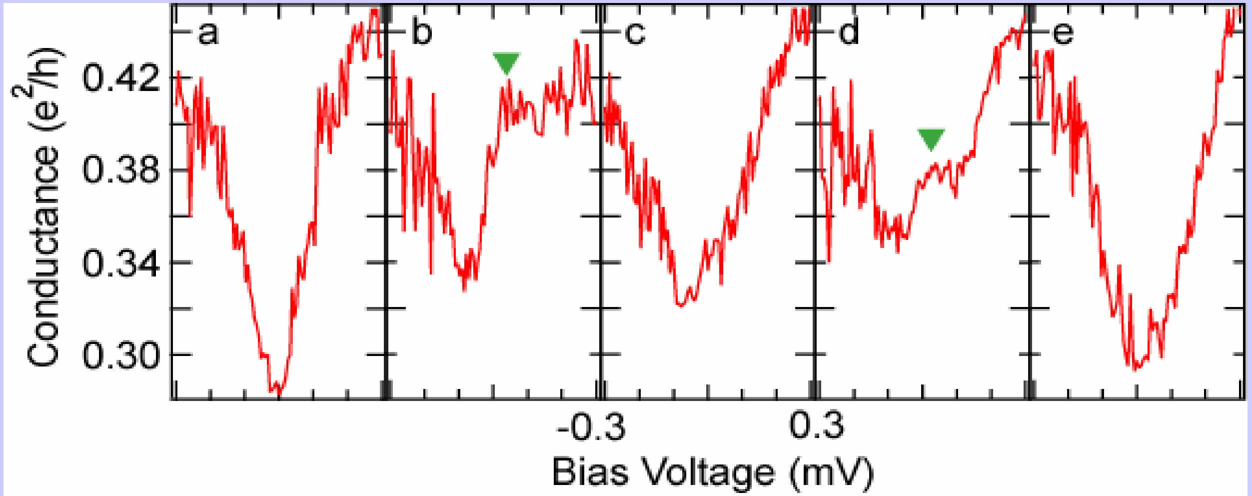
low conductivity: a, c, e

Resonance peak

in I-V characteristics

b, d: Small peaks

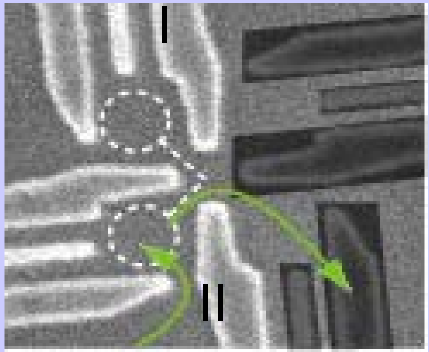
a, c, e: no peak

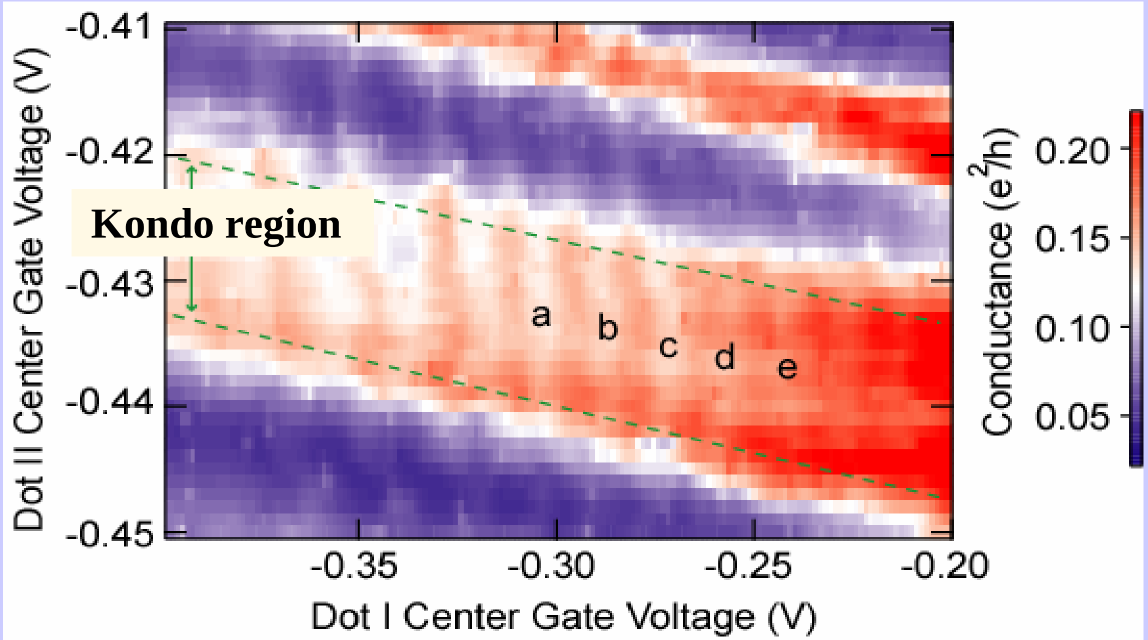


➡ Reduction of the Kondo effect depending on the parity of the other dot

Reduction of the Kondo effect: parallel alignment

Parallel configuration





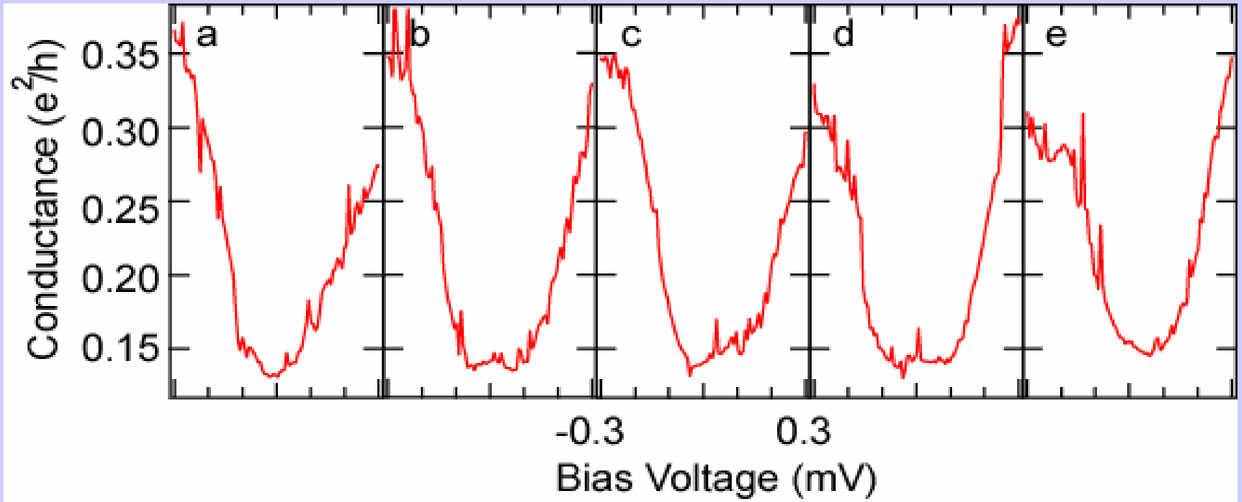
Kondo region

Similar conductivity

for all electron numbers

I-V characteristics

no clear Kondo peak



➡ Need clearer experiments

Conclusion

1. Spin-orbit Berry phase due to spin-orbit interaction
2. Decoherence due to spin-orbit entanglement via quantum dot
3. Observation of the Fano-Kondo effect in T-shaped and AB interferometers with a quantum dotl.
4. Kondo-RKKY competition in two-impurity Kondo model