The Quantum Spin Hall Effect

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Science*, 318, *766 (2007)

XL Qi, T. Hughes, SCZ
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The quantum Hall state, a topologically non-trivial state of matter

$$\sigma_{xy} = n\frac{e^2}{h}$$

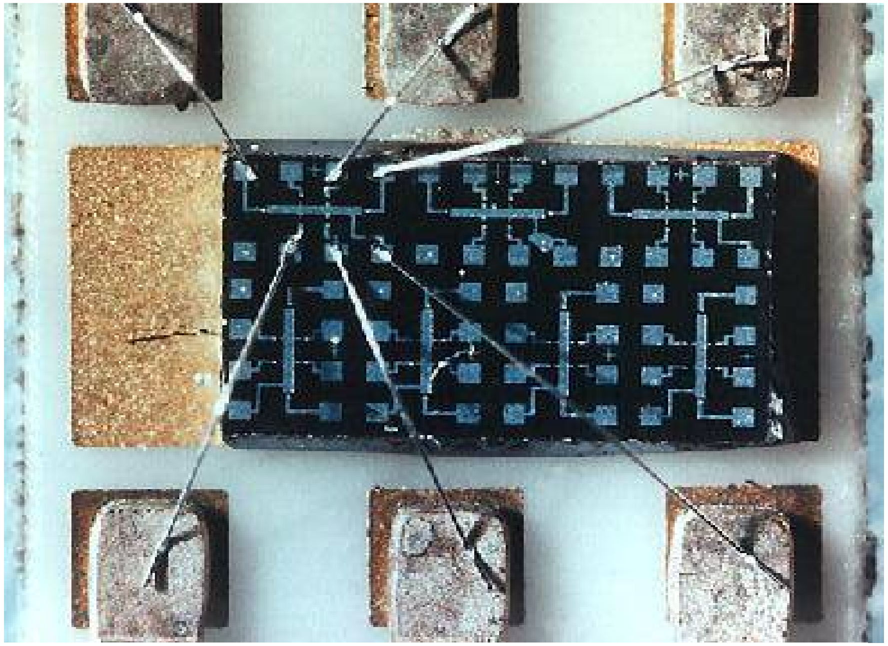
- Following Laughlin’s gauge argument, TKNN showed that n is a topological integer, called the first Chern number.

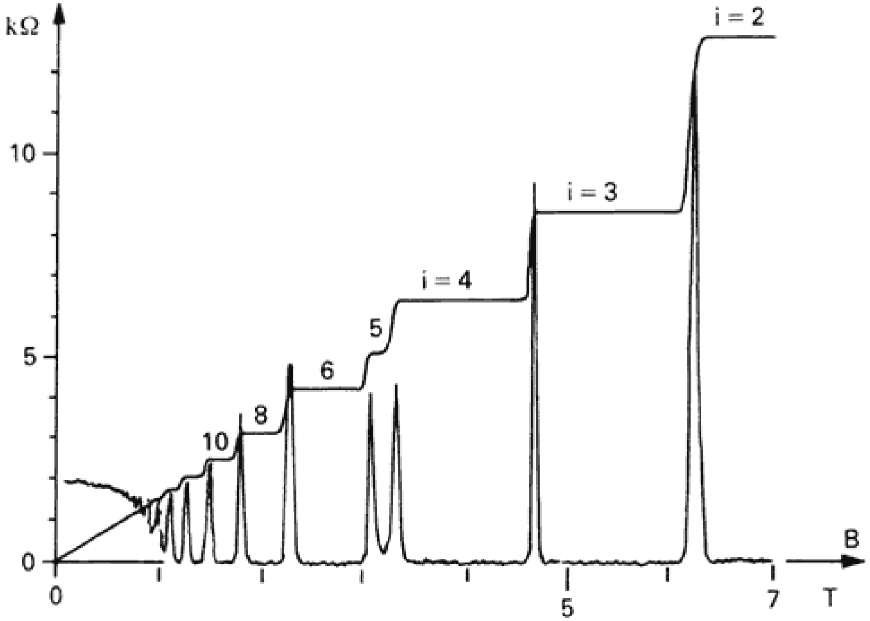
$$n = \int \frac{d^2k}{(2\pi)^2}\varepsilon^{\mu\nu}F_{\mu\nu}(k)$$

- A simple example of a topological integer:

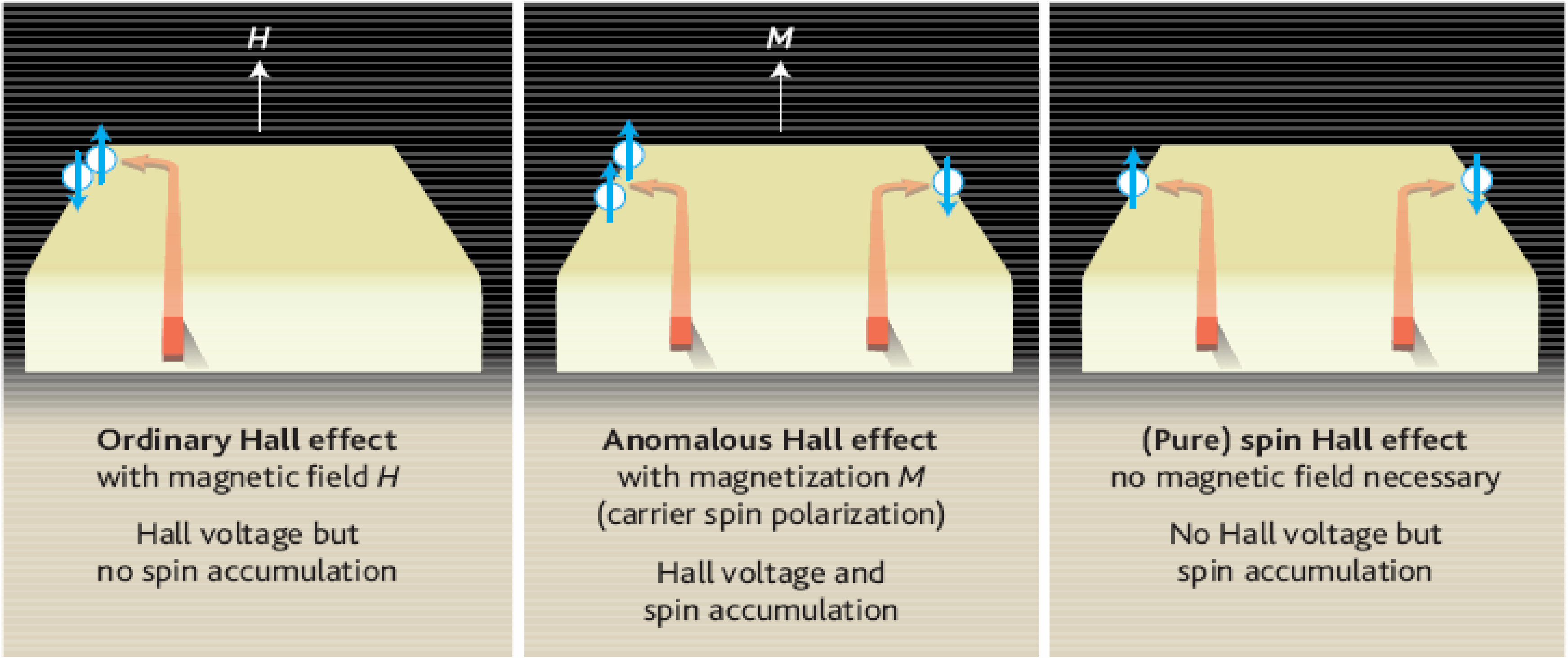
$$n = \int \frac{dx}{2\pi}\partial_x\theta(x)$$

$$e^{i\theta(x)} = 1,\ x = 0,2\pi$$





The Generalizations of the Hall Effect



Ordinary Hall effect
with magnetic field *H*

Hall voltage but
no spin accumulation

Anomalous Hall effect
with magnetization *M*
(carrier spin polarization)

Hall voltage and
spin accumulation

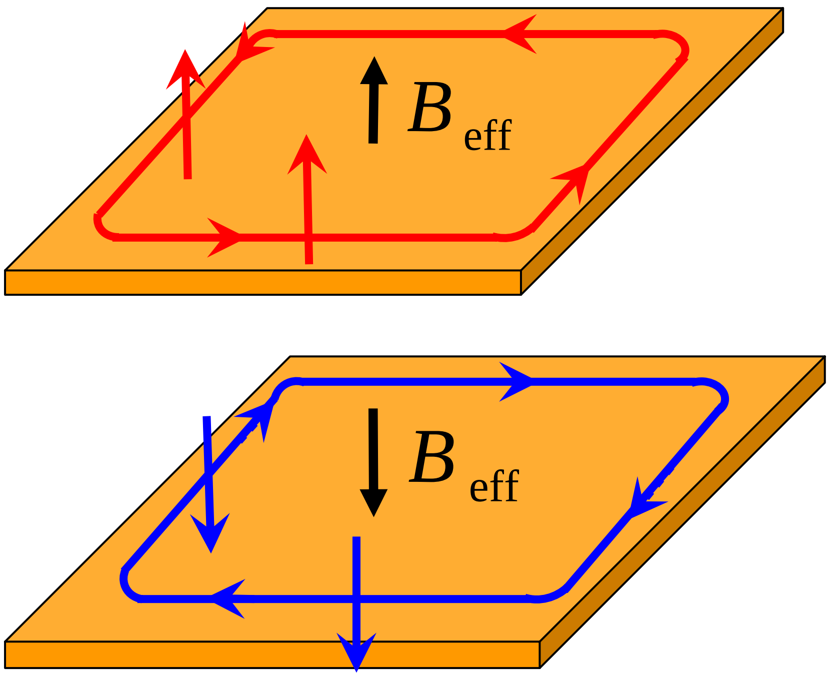
(Pure) spin Hall effect
no magnetic field necessary

No Hall voltage but
spin accumulation

- Theoretical predictions of the intrinsic spin Hall effect (Science 2003, PRL 2004).
- The spin Hall effect has now been experimentally observed. (Science 2004, PRL 2004)

What about the quantum spin Hall effect?

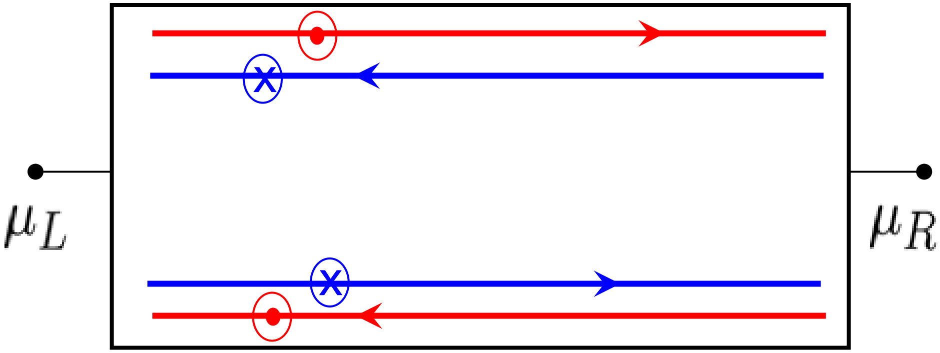
Quantum Spin Hall Effect



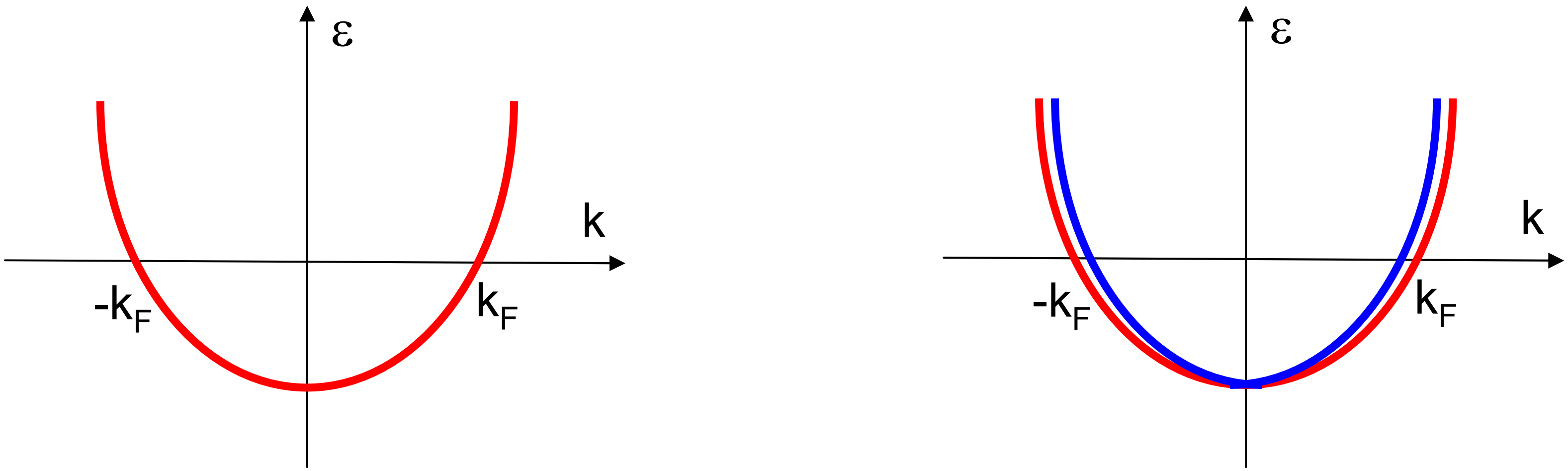
- The QSH state can be thought of as two copies of QH states, one for each spin component, each seeing the opposite magnetic field. (Bernevig and Zhang, PRL, 2006)
- The QSH state does not break the time reversal symmetry, and can exist without any external magnetic field.

$$H_{so} = \lambda_{so}\vec{\sigma}(\vec{p}\times\vec{E})$$

- Insulating gap in the bulk.
- Helical edge states: Two states with opposite spins counter-propagate at a given edge.

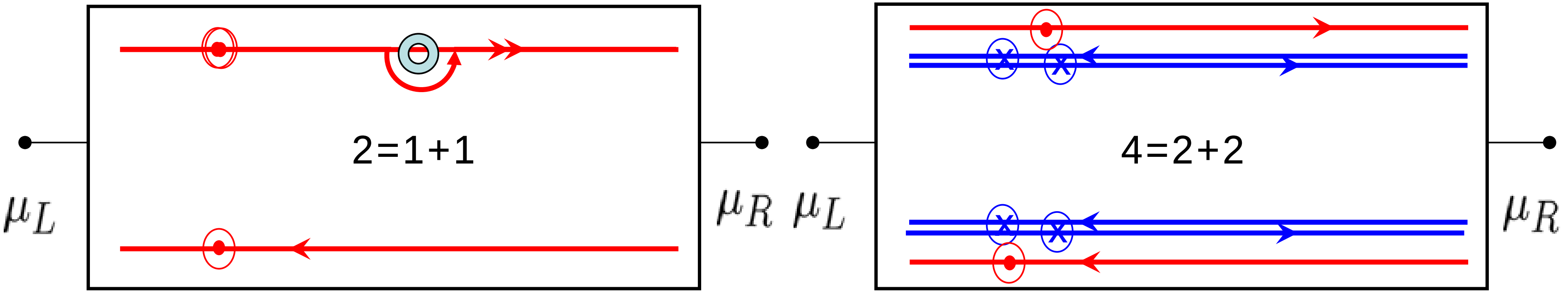


Chiral (QHE) and helical (QSHE) liquids in D=1



The QHE state spatially separates the two chiral states of a spinless 1D liquid

The QSHE state spatially separates the four chiral states of a spinful 1D liquid

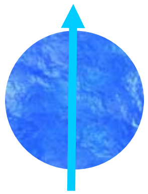


No go theorems: chiral and helical states can never be constructed microscopically from a purely 1D model (Wu, Bernevig, Zhang, 2006; Nielsen, Ninomiya, 1981)

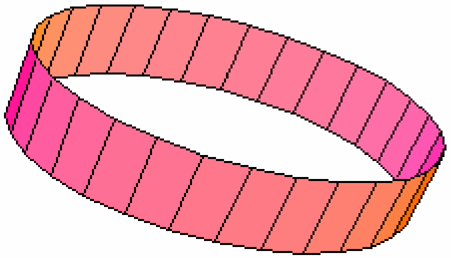
Time reversal symmetry in quantum mechanics

- Wave function of a particle with integer spin changes by 1 under 2π rotation.
- Wave function of a half-integer spin changes by -1 under 2π rotation.
- Kramers theorem, in a time reversal invariant system with half-integer spins, $T^2=-1$, all states for degenerate doublets.
- Application in condensed matter physics: Anderson's theorem. BCS pair=(k,↑)+(-k,↓). General pairing between Kramers doublets.

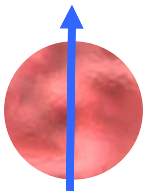
Spin=1



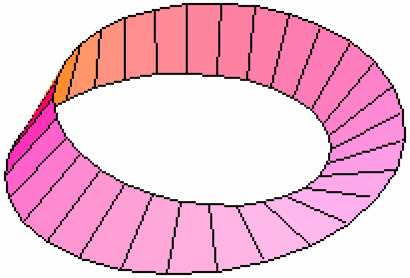
$\boxed{\psi \Longrightarrow \psi}$



Spin=1/2



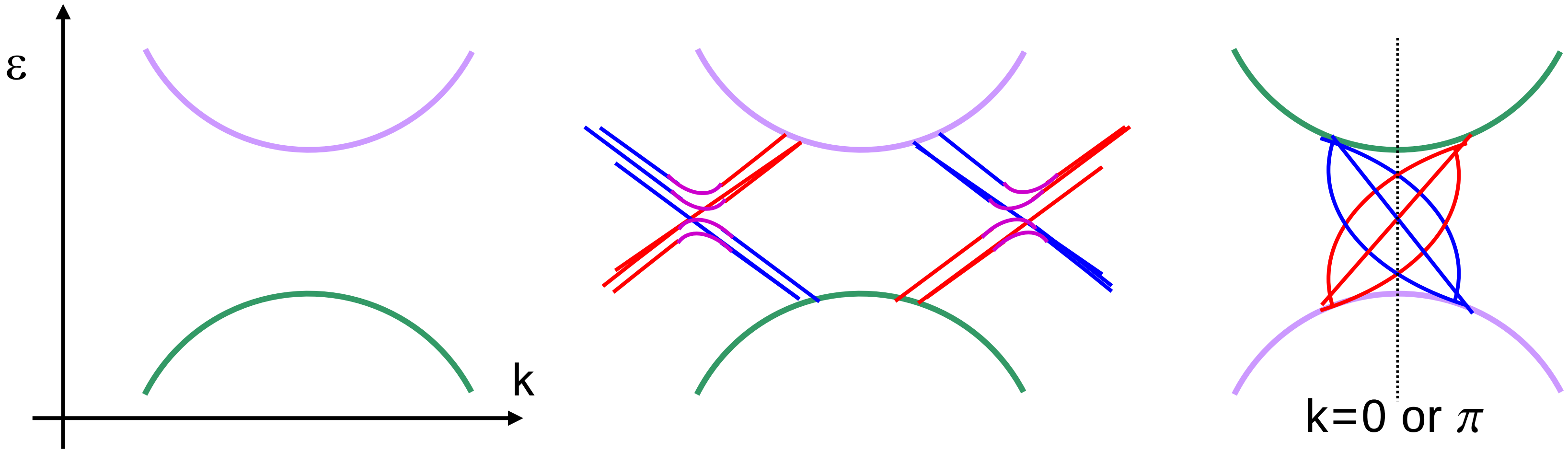
$\boxed{\psi \Longrightarrow -\psi}$



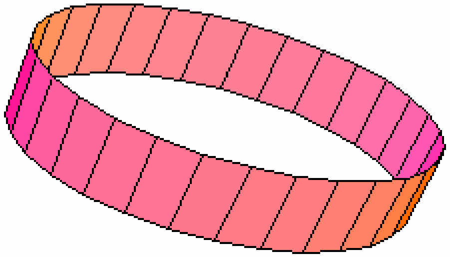
The topological distinction between a conventional insulator and a QSH insulator

Kane and Mele PRL, (2005); Wu, Bernevig and Zhang, PRL (2006); Xu and Moore, PRB (2006)

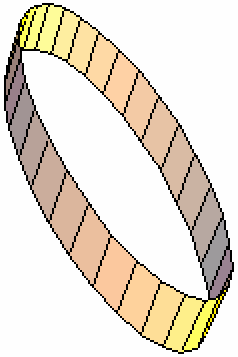
- Band diagram of a conventional insulator, a conventional insulator with accidental surface states (with animation), a QSH insulator (with animation). Blue and red color code for up and down spins.



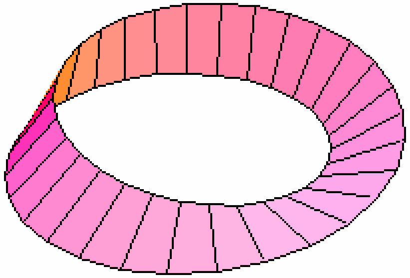
Trivial



Trivial



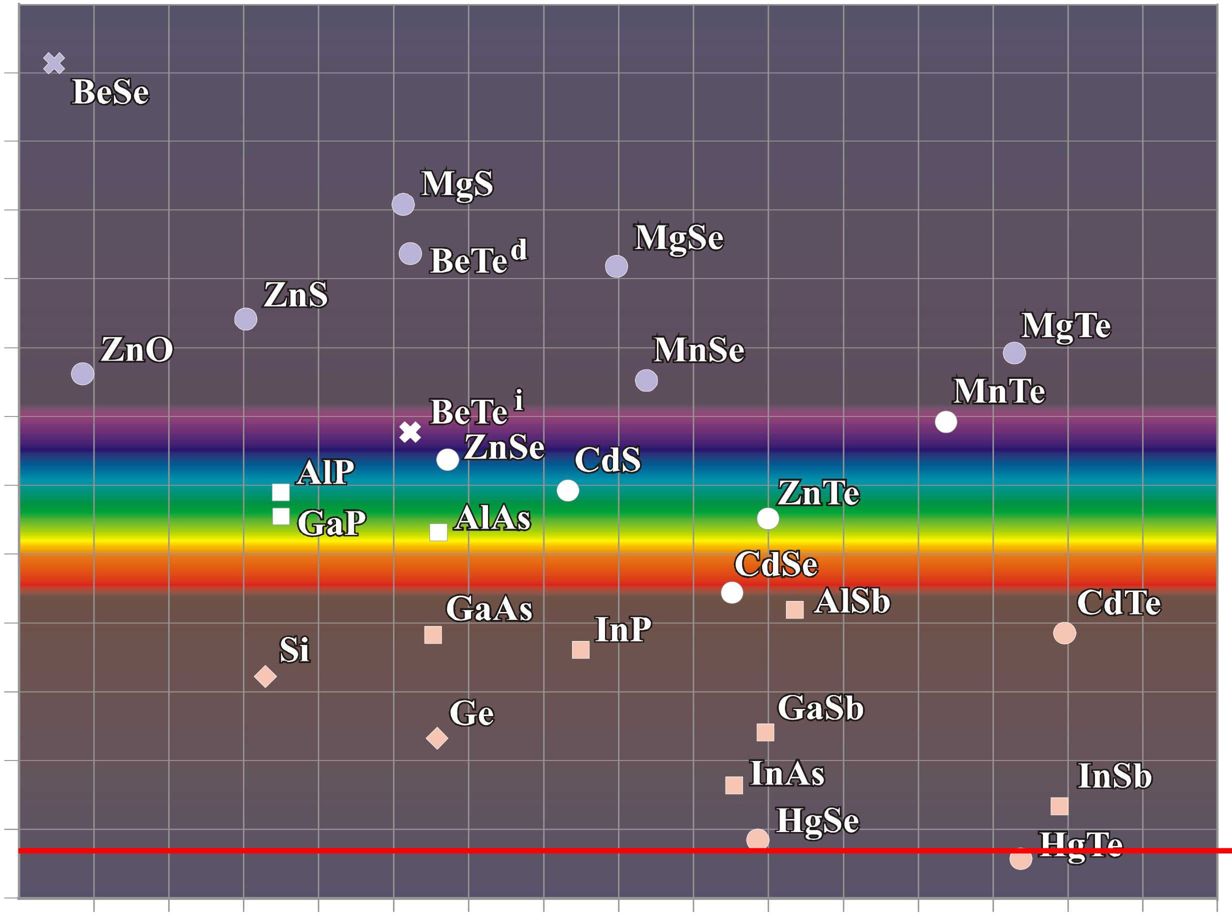
Non-trivial



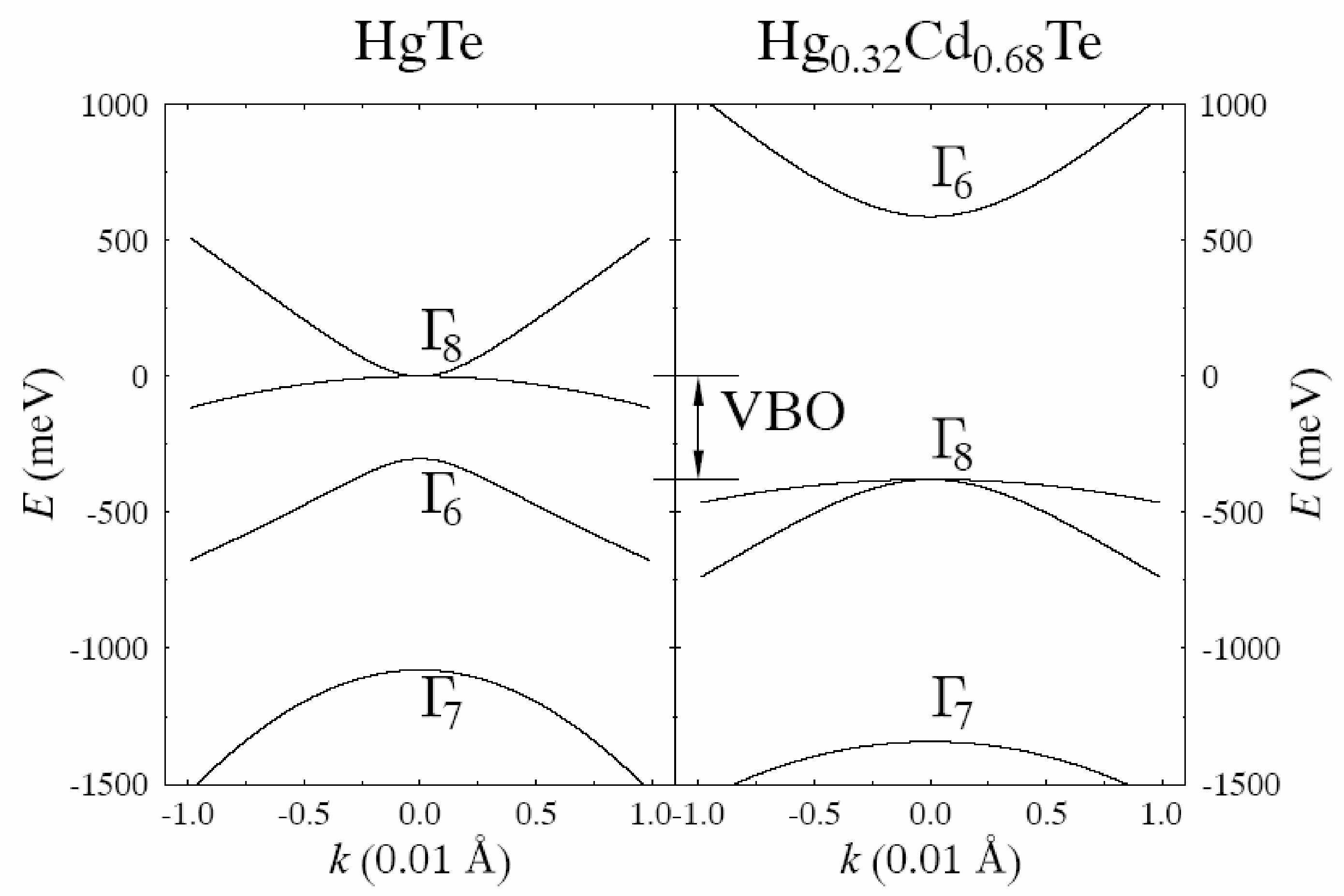
From topology to chemistry: the search for the QSH state

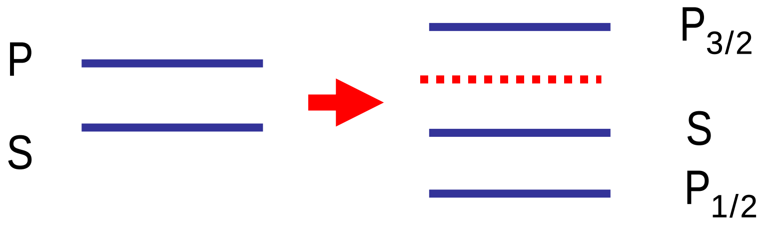
- Graphene – spin-orbit coupling only about 10^{-3}meV. Not realizable in experiments. (Kane and Mele, 2005, Yao et al, 2006, MacDonald group 2006)
- Quantum spin Hall with Landau levels – spin-orbit coupling in GaAs too small. (Bernevig and Zhang, PRL, 2006)
- QSH in Bi? (Murakami, 2006)

- Type III quantum wells work. HgTe has a negative band gap! (Bernevig, Hughes and Zhang, Science 2006)
- Tuning the thickness of the HgTe/CdTe quantum well leads to a topological quantum phase transition into the QSH state.



Band Structure of HgTe





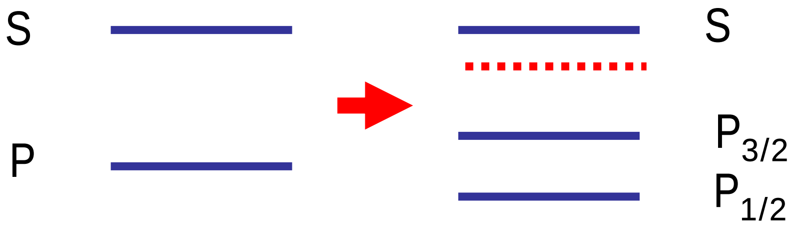
P

S

$P_{3/2}$

S

$P_{1/2}$



S

P

S

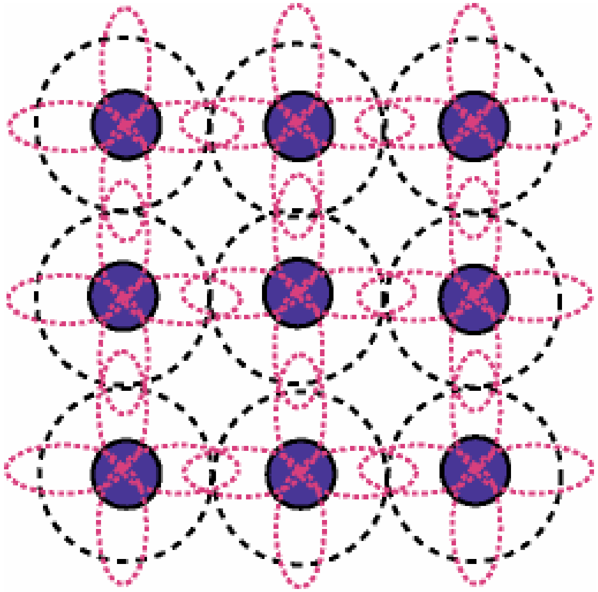
$P_{3/2}$

$P_{1/2}$

Effective tight-binding model

Square lattice with 4-orbitals per site:

$$\left|s,\uparrow\right\rangle, \left|s,\downarrow\right\rangle, \left|(p_x+ip_y,\uparrow\right\rangle, \left|-(p_x-ip_y),\downarrow\right\rangle$$



Nearest neighbor hopping integrals. Mixing matrix elements between the s and the p states must be odd in k.

$$H_{eff}(k_x,k_y)=\begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix}$$

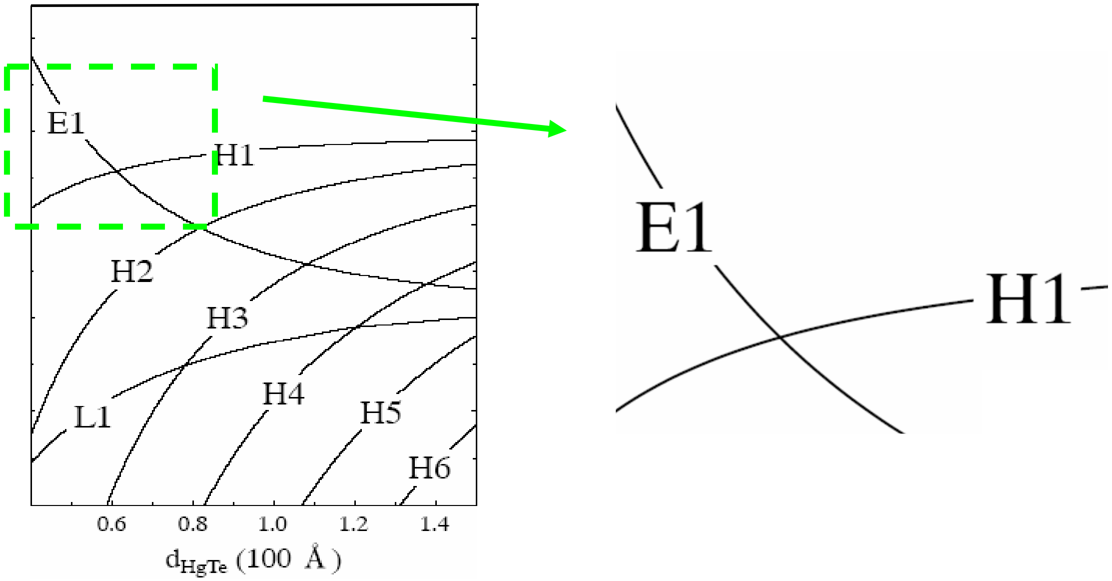
$$h(k)=\begin{pmatrix} m(k) & A(\sin k_x - i\sin k_y) \\ A(\sin k_x + i\sin k_y) & -m(k) \end{pmatrix} \equiv d_a(k)\tau^a$$

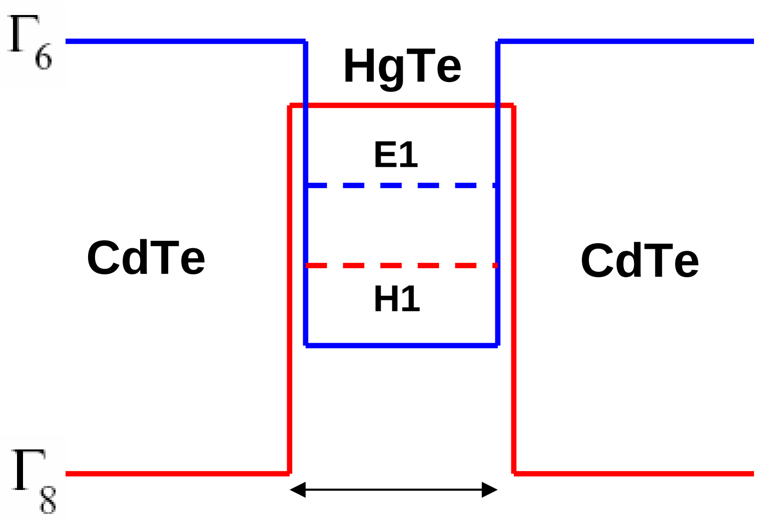
$$\Rightarrow \begin{pmatrix} m & A(k_x-ik_y) \\ A(k_x+ik_y) & -m \end{pmatrix} \qquad \Delta\sigma^{\uparrow}{}_{xy}=\frac{1}{2}\Delta\, sign(m) \qquad \Delta\sigma^{\downarrow}{}_{xy}=-\Delta\sigma^{\uparrow}{}_{xy}$$

Relativistic Dirac equation in 2+1 dimensions, with a tunable mass term!

Quantum Well Sub-bands

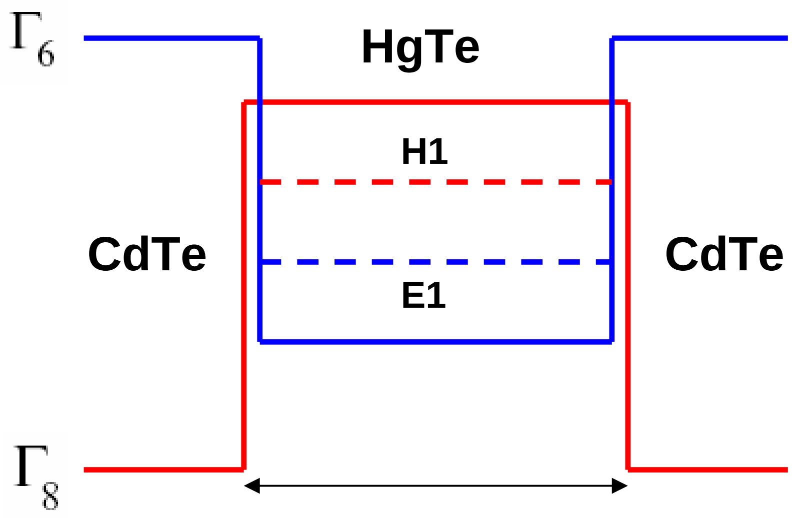
Let us focus on E1, H1 bands close to crossing point





$d < d_c$

normal

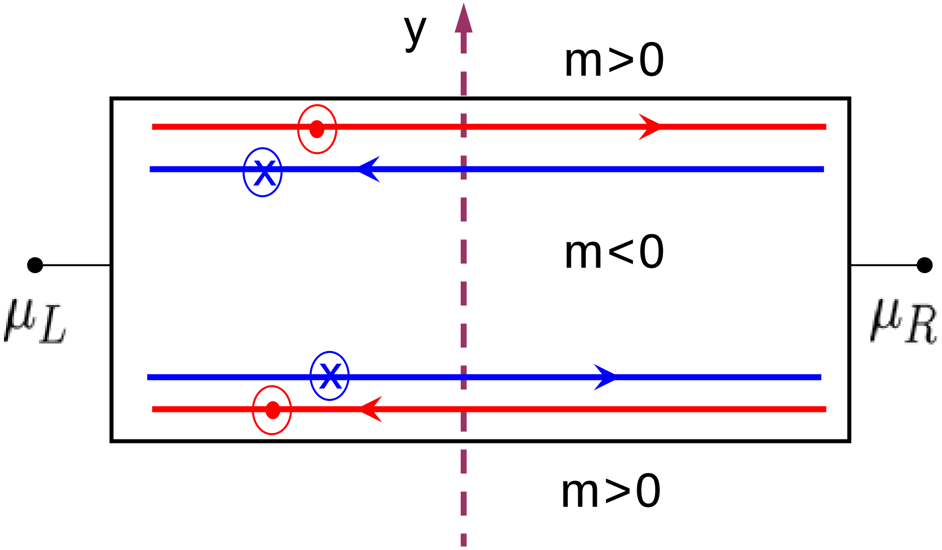


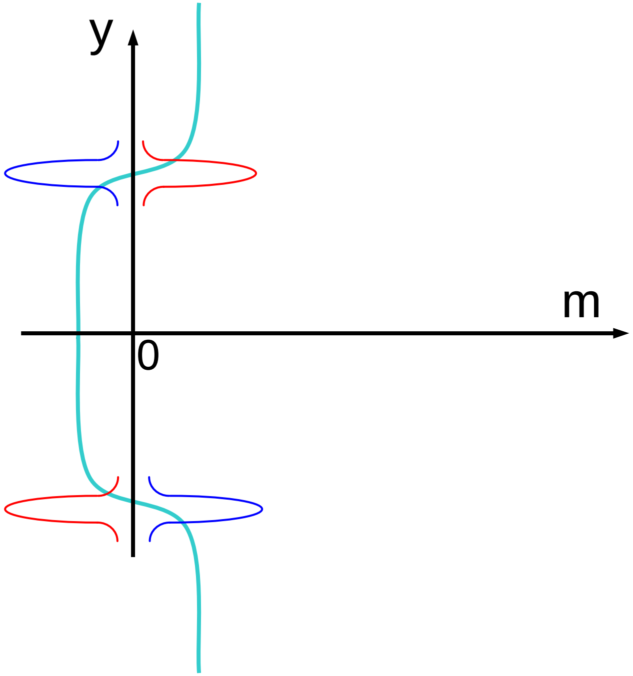
$d > d_c$

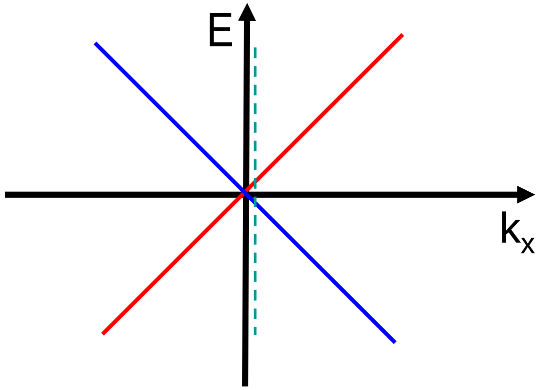
inverted

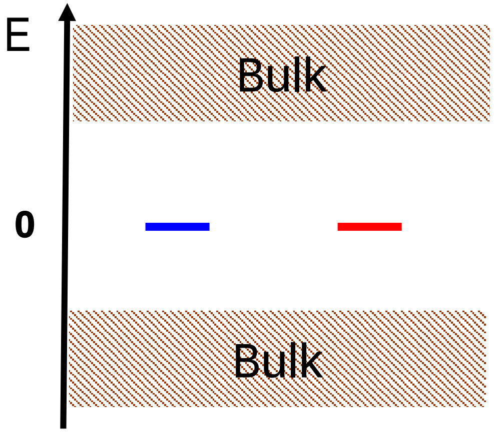
Mass domain wall

Cutting the Hall bar along the y-direction we see a domain-wall structure in the band structure mass term. This leads to states localized on the domain wall which still disperse along the x-direction.



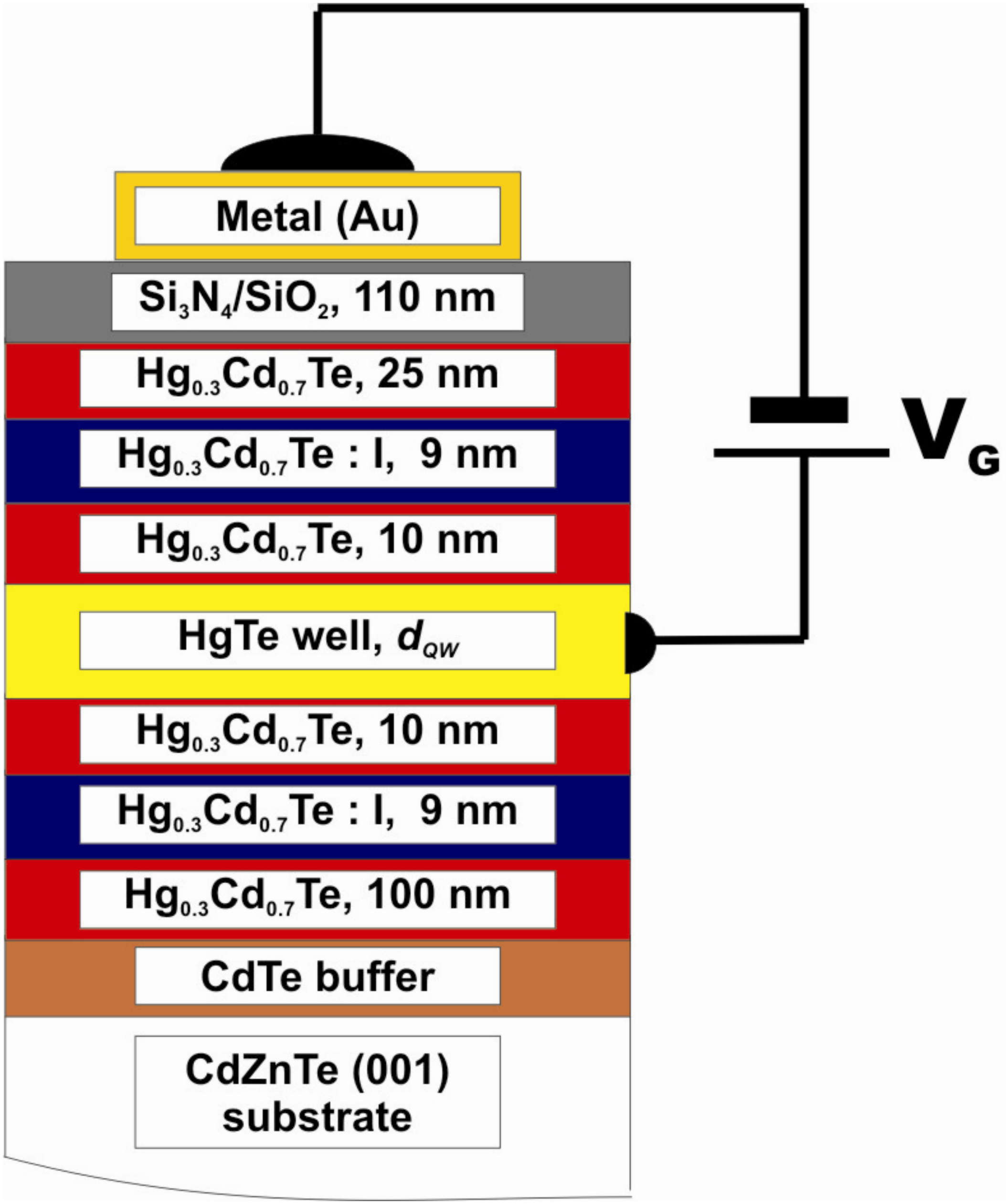






Experimental setup

- High mobility samples of HgTe/CdTe quantum wells have been fabricated.
- Because of the small band gap, about several meV, one can gate dope this system from n to p doped regimes.
- Two tuning parameters, the thickness d of the quantum well, and the gate voltage.



Metal (Au)

Si_3N_4/SiO_2, 110 nm

$Hg_{0.3}Cd_{0.7}Te$, 25 nm

$Hg_{0.3}Cd_{0.7}Te$: I, 9 nm

$Hg_{0.3}Cd_{0.7}Te$, 10 nm

HgTe well, d_{QW}

$Hg_{0.3}Cd_{0.7}Te$, 10 nm

$Hg_{0.3}Cd_{0.7}Te$: I, 9 nm

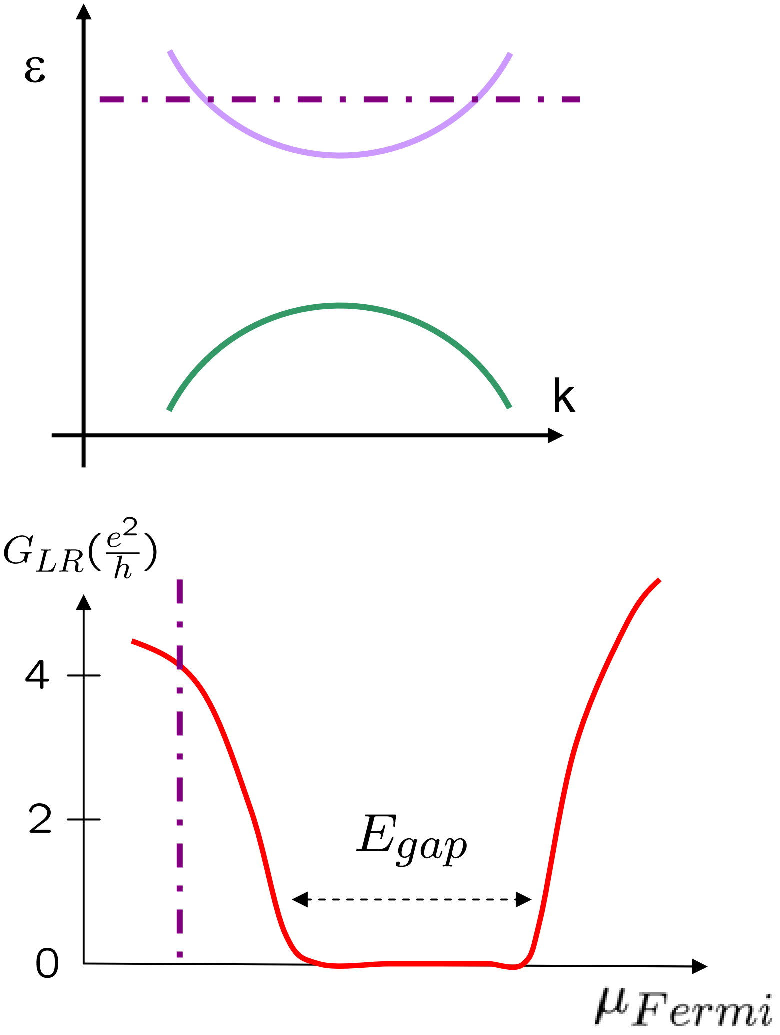
$Hg_{0.3}Cd_{0.7}Te$, 100 nm

CdTe buffer

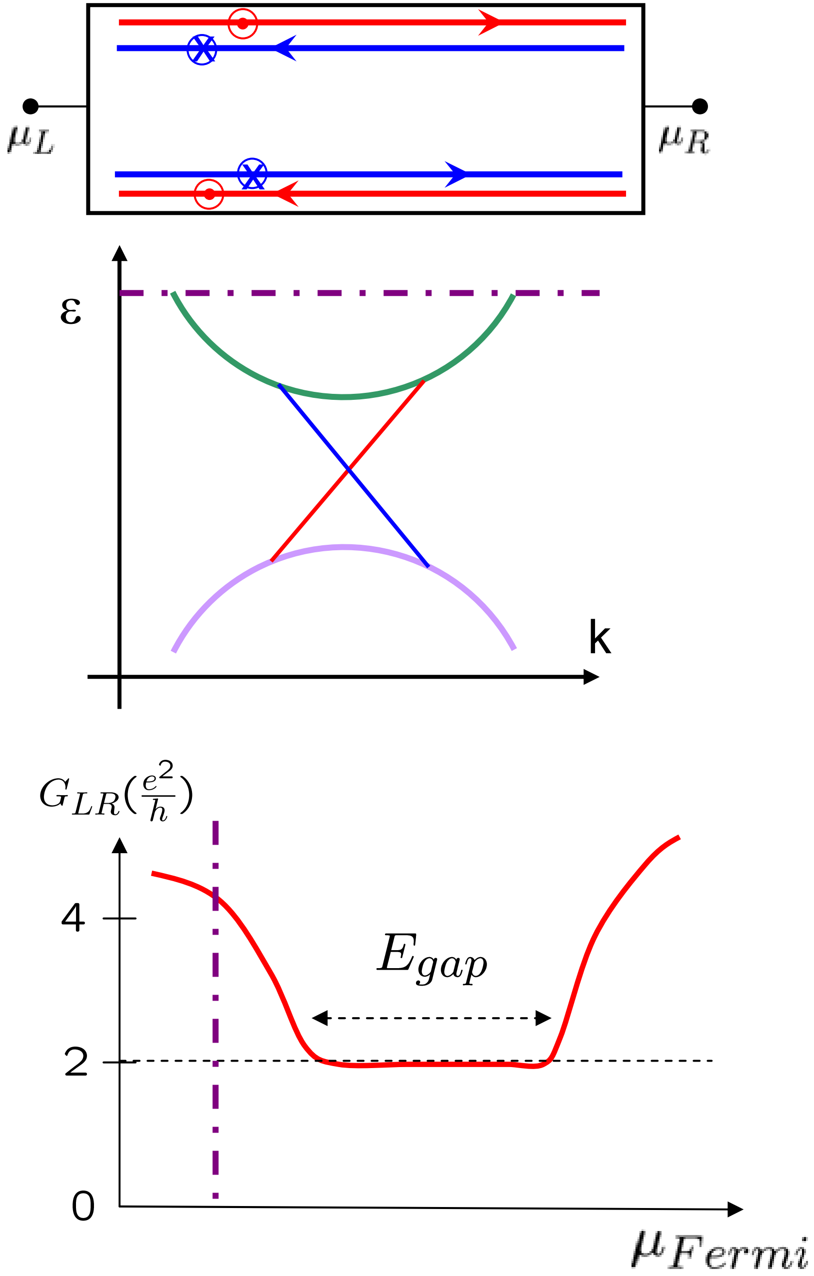
CdZnTe (001)
substrate

V_G

Experimental Predictions



$d < d_c$, normal regime

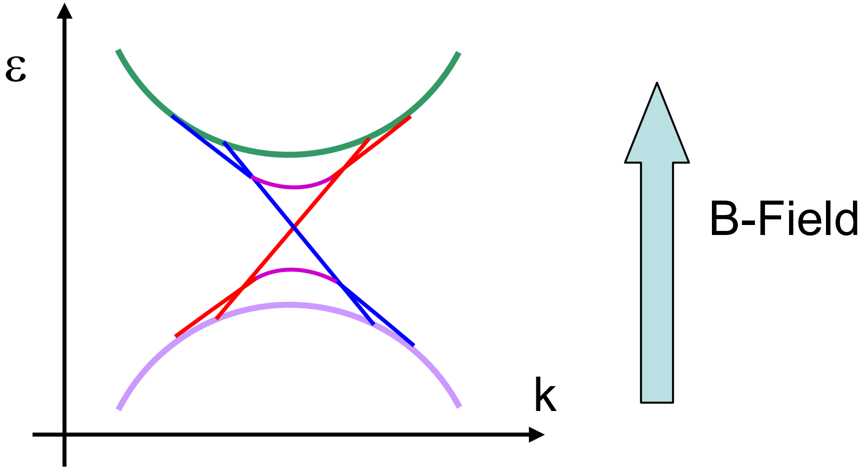


$d > d_c$, inverted regime

Smoking gun for the helical edge state: Magneto-Conductance

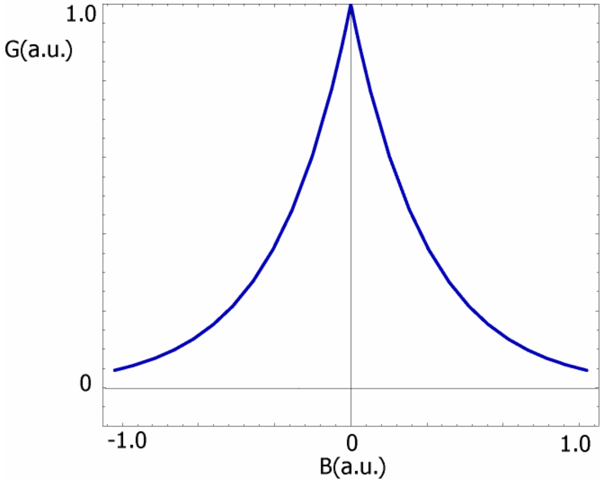
The crossing of the helical edge states is protected by the TR symmetry. TR breaking term such as the Zeeman magnetic field causes a singular perturbation and will open up a full insulating gap:

$$E_g \propto g|B|$$

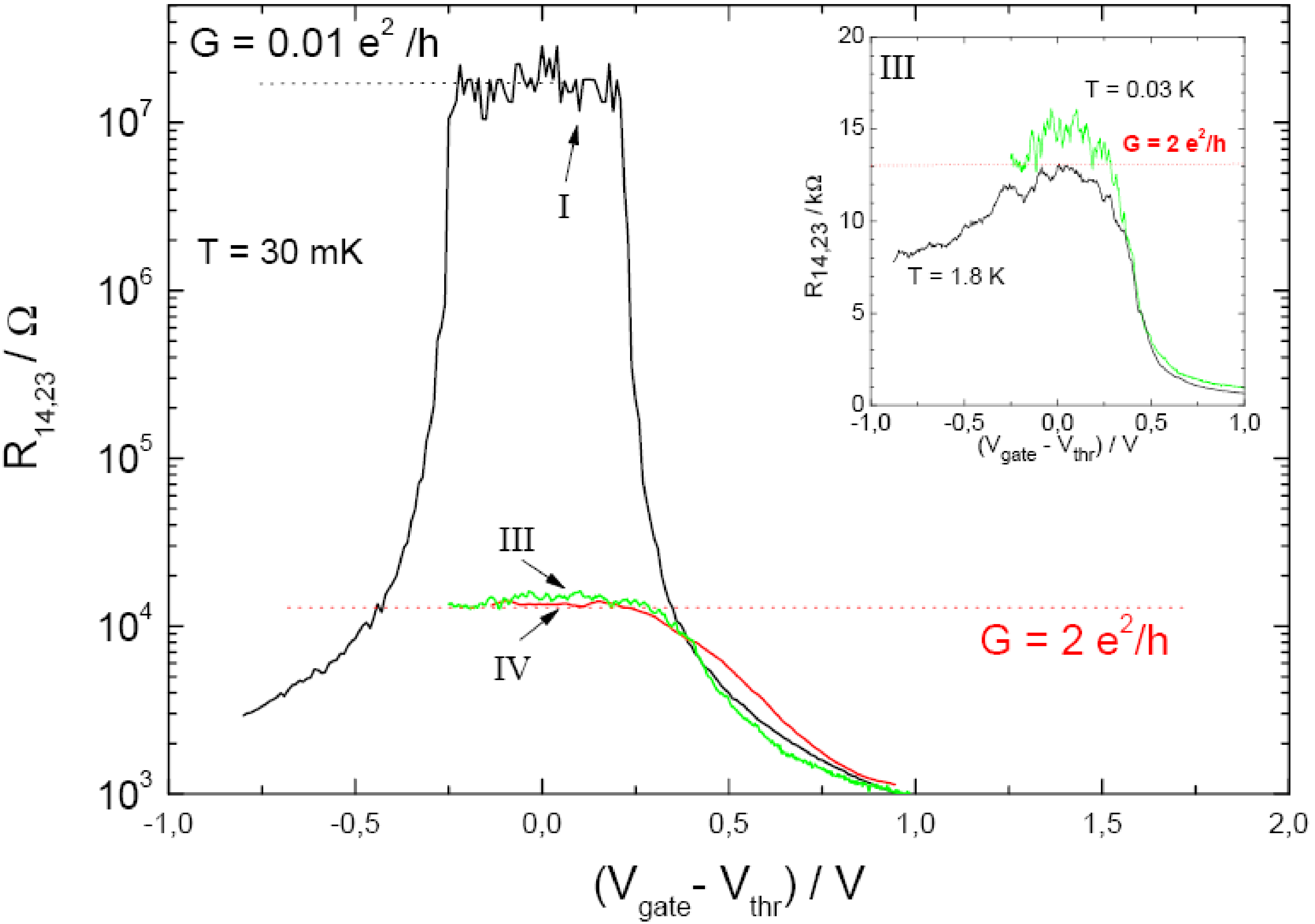


Conductance now takes the activated form:

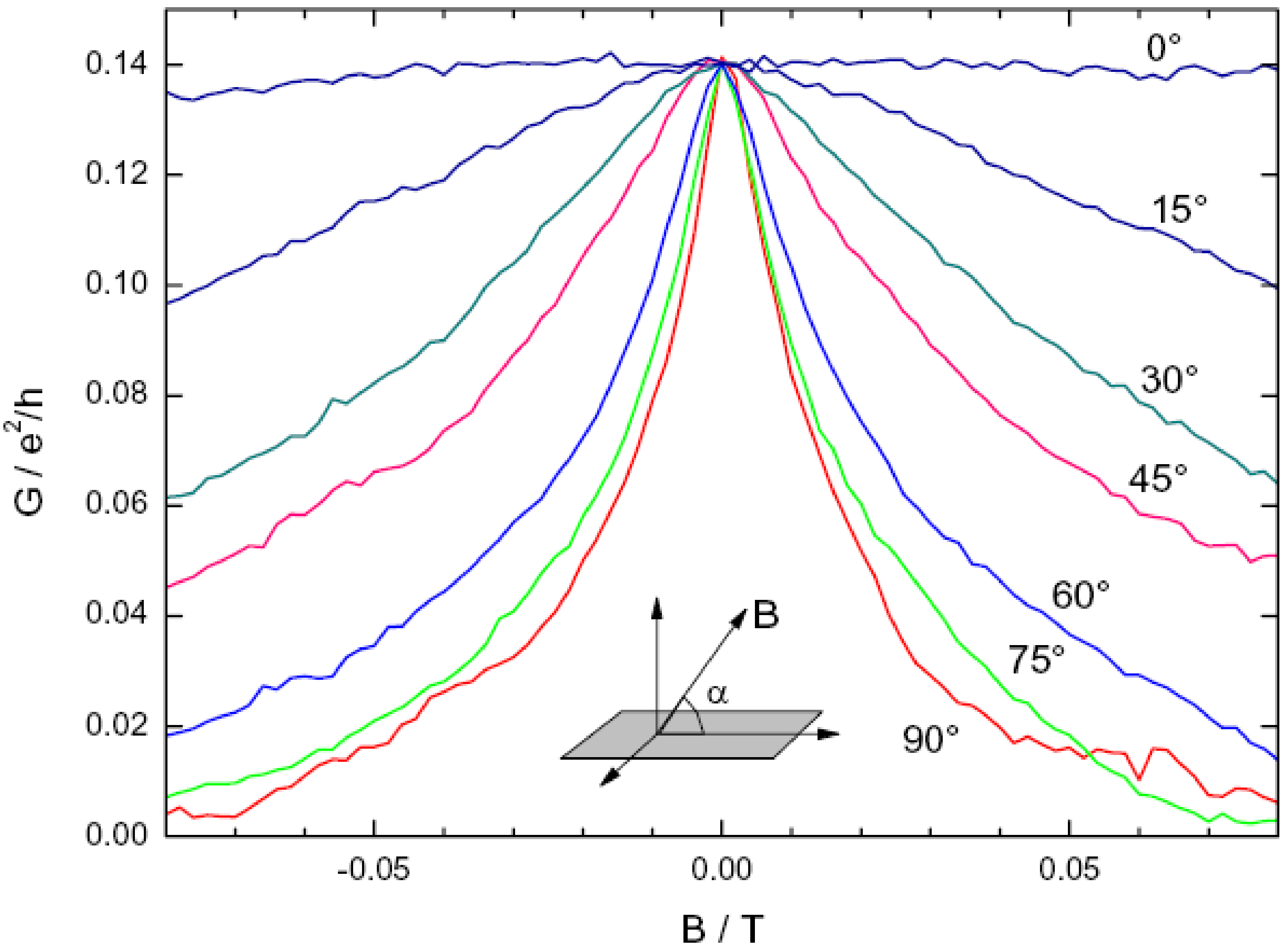
$$\sigma \propto f(T)e^{-g|B|/kT}$$



Experimental evidence for the QSH state in HgTe

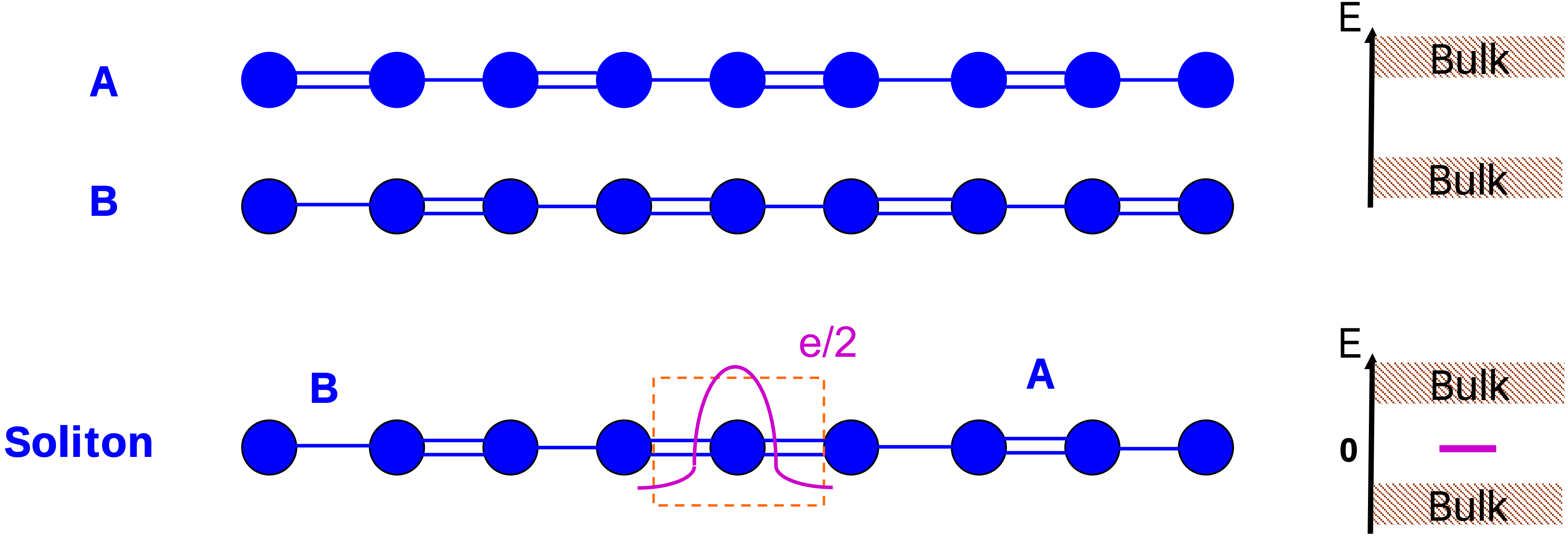


Magnetic field dependence of the residual conductance



A brief history of fractional charge

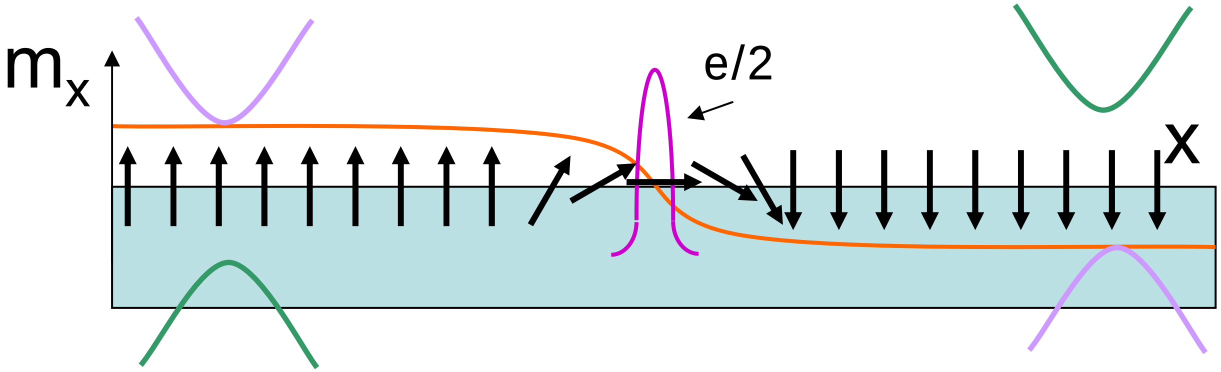
- Jackiw & Rebbi (PRD (1976)) predicted that a fractional charge e/2 is carried by the mass domain wall (soliton) of 1-d Dirac model.
- Su, Schrieffer and Heeger (PRB (1979)) presented a model of polyacetylene with two-fold degenerate ground states. A domain wall defect carries fractional charge e/2.



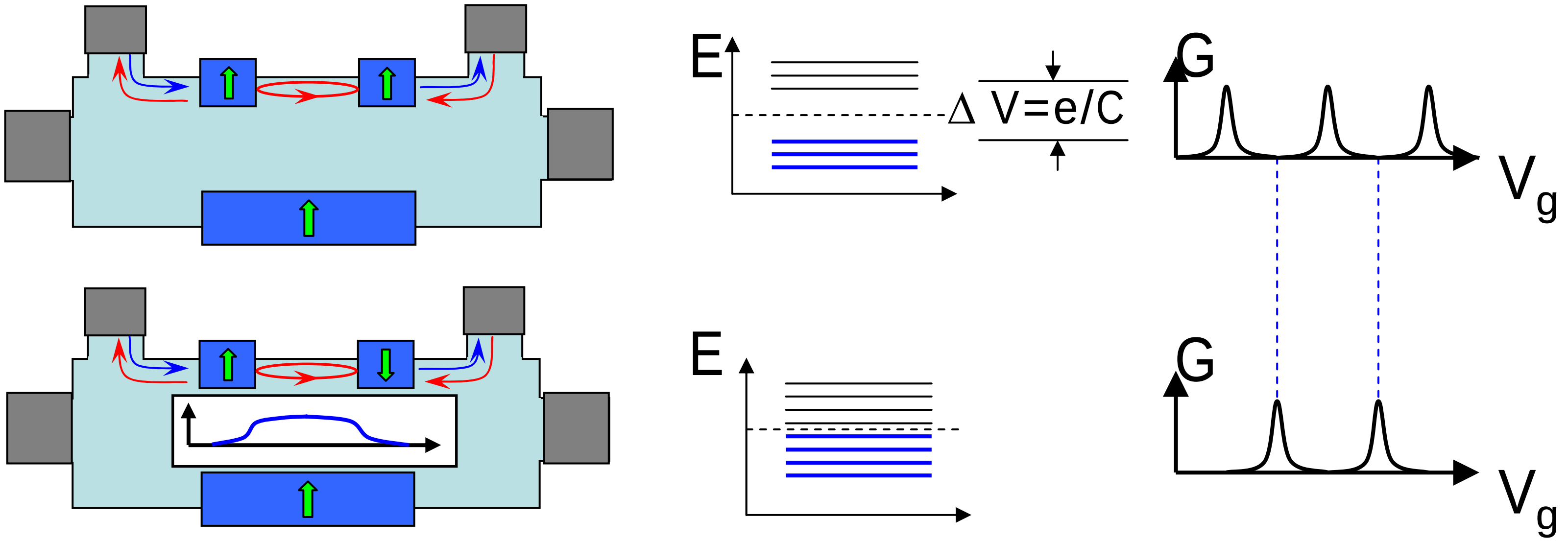
- Because of both up and down spin components carry fractional charge e/2, the net system only carries integer charge. Fractional charge has never been observed in any 1D system!

Fractional charge in the QSH state

- Since the mass is proportional to the magnetization, a magnetization domain wall leads to a mass domain wall on the edge.

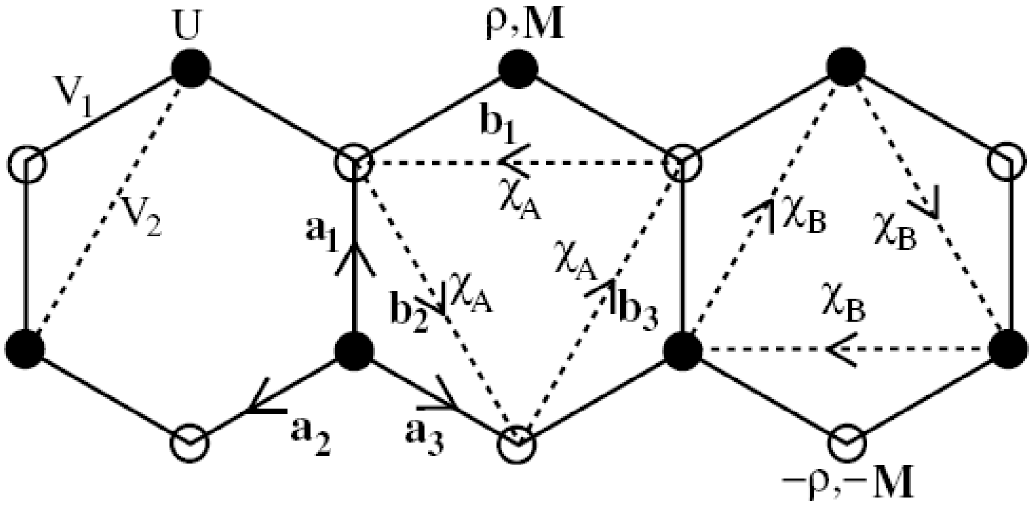


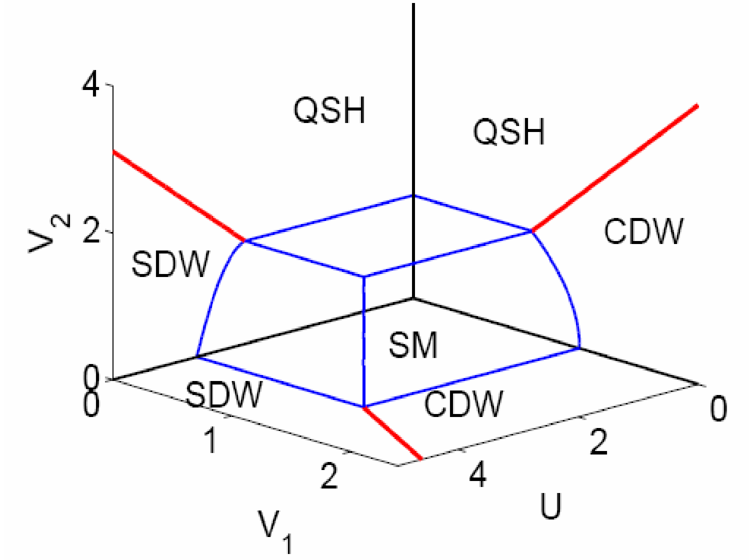
- The fractional charge e/2 can be measured by a Coulomb blockade experiment, one at the time!



Topological Mott insulators

- So far, the QSH insulator is a topologically non-trivial band insulator. Can we have a topological Mott insulator, where the topologically non-trivial gap arises from interactions, not from band structure?
- Yes, on a honeycomb lattice with U, V1 and V2, one can obtain a TMI phase in the limit of V2>>U, V1. (Raghu et al, arXiv:0710.0030)
- This model provides an example of dynamic generation of spin-orbit coupling. (Wu+Zhang, PRL 2004).





Conclusions

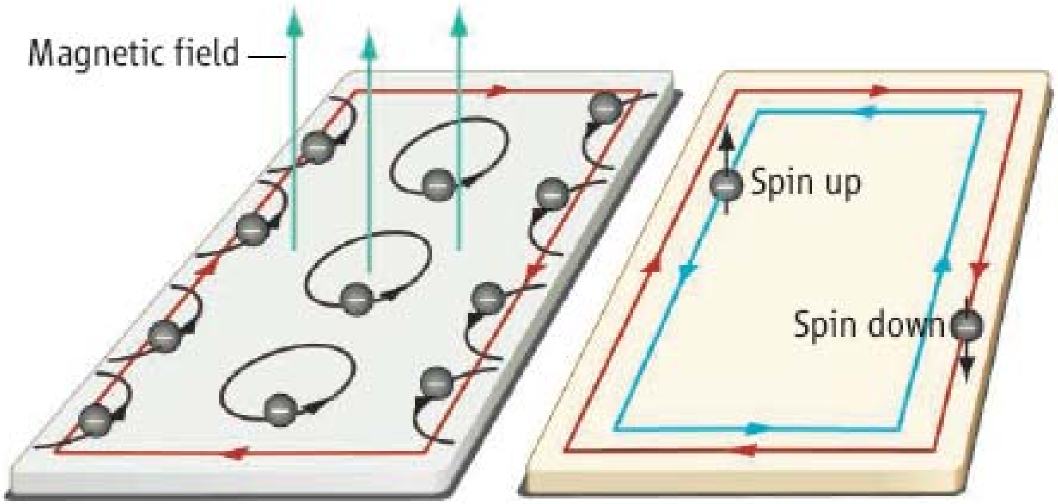
PHYSICS

A New State of Quantum Matter

Experiments show that electron spins can flow without dissipation in a novel electrical insulator.

Naoto Nagaosa

- QSH state is a new state of matter, topologically distinct from the conventional insulators.
- It is predicted to exist in HgTe quantum wells, in the “inverted” regime, with $d>d_c$.
- Theoretical predictions confirmed by experiments.
- Topological Mott insulators.



Quantum Hall system

Quantum spin Hall system