Non-Fermi liquid behavior from critical Fermi surface fluctuations

W. Metzner, *MPI for Solid State Research*

1. Pomeranchuk instability
   (symmetry-breaking Fermi surface deformations)
2. Soft Fermi surface and non-Fermi liquid behavior
3. Transport life-time and electrical resistivity

Collaborators:

D. Rohe, S. Andergassen, L. Dell'Anna, H. Yamase (Stuttgart)

1. Pomeranchuk instability

Stability criterion for isotropic 3D Fermi liquids (Pomeranchuk 1958):

Landau's excitation energy functional

$$\delta E[\delta n] = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} \, \delta n_{\mathbf{k}\sigma} + \frac{1}{2V} \sum_{\mathbf{k}\mathbf{k}'} \sum_{\sigma\sigma'} f_{\mathbf{k}\mathbf{k}'}^{\sigma\sigma'} \delta n_{\mathbf{k}\sigma} \delta n_{\mathbf{k}'\sigma'}$$

positive for *any* choice of $\delta n_{\mathbf{k}\sigma}$ only if *all* Landau parameters satisfy

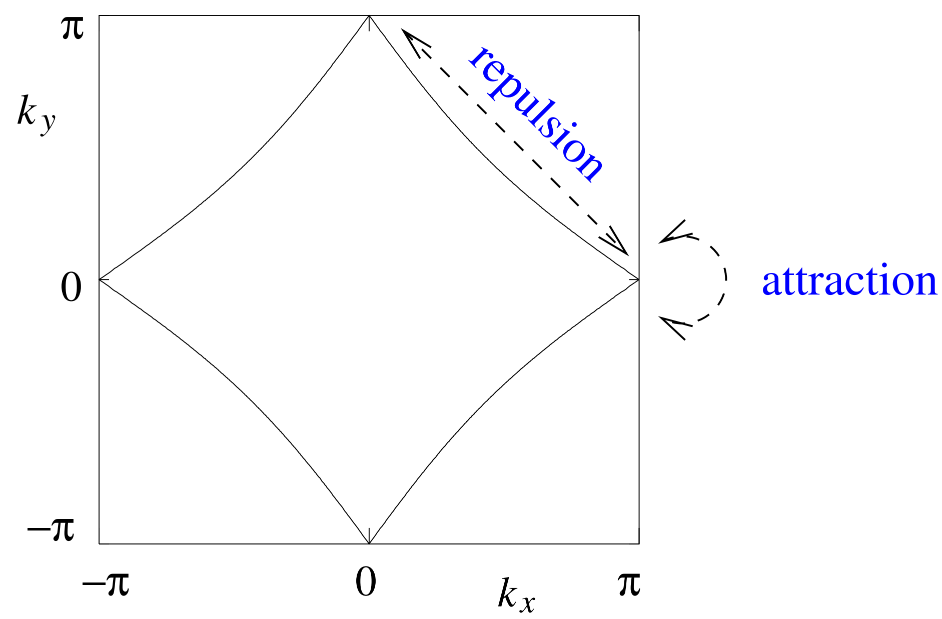
$$F_l^c > -(2l+1) \quad \text{and} \quad F_l^s > -(2l+1) \quad \text{for all} \quad l = 0, 1, 2, \ldots$$

Otherwise negative excitation energy for suitable choice of $\delta n_{\mathbf{k}\sigma}$

$\Rightarrow$ instability

2D Hubbard and tJ model:

Forward scattering interaction
in charge channel $f^c_{\mathbf{k}_F\mathbf{k}'_F}$
has attractive d-wave component

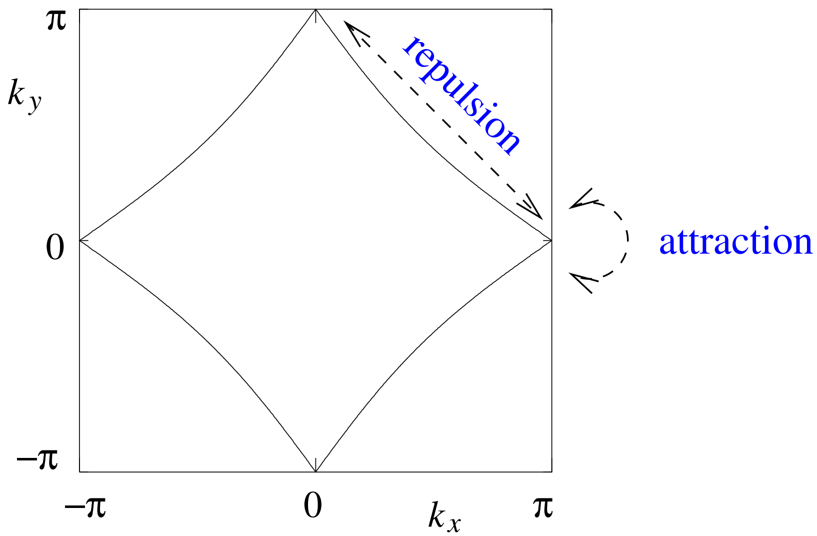


Furthermore: small Fermi velocity $\mathbf{v}_{\mathbf{k}_F}$ near saddle points of $\epsilon_{\mathbf{k}}$

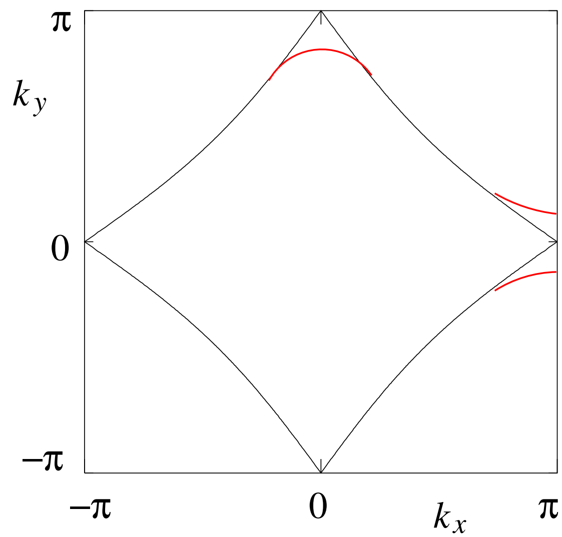
$\Rightarrow$ d-wave Fermi surface deformations easy (low energy cost)

(Halboth, wm 2000; Yamase, Kohno 2000)

Effective interaction $f^c_{\mathbf{k}_F \mathbf{k}'_F}$



Deformation of Fermi surface

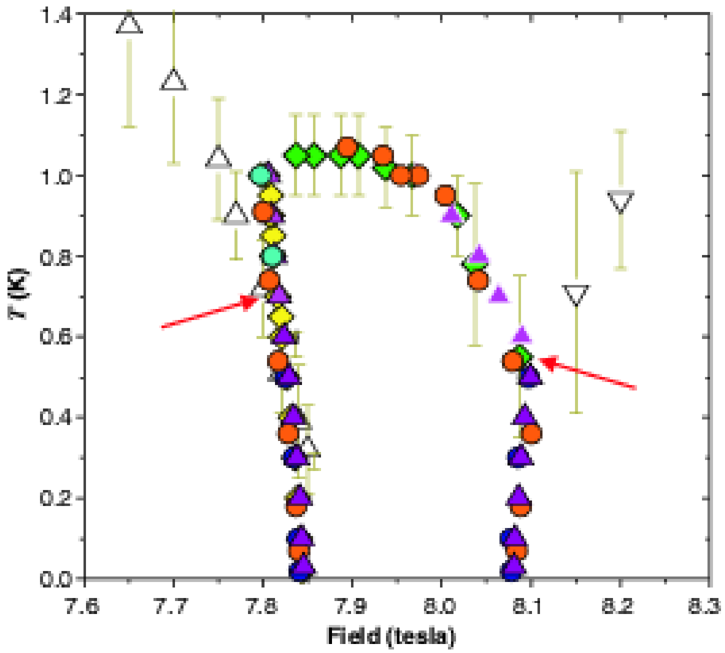


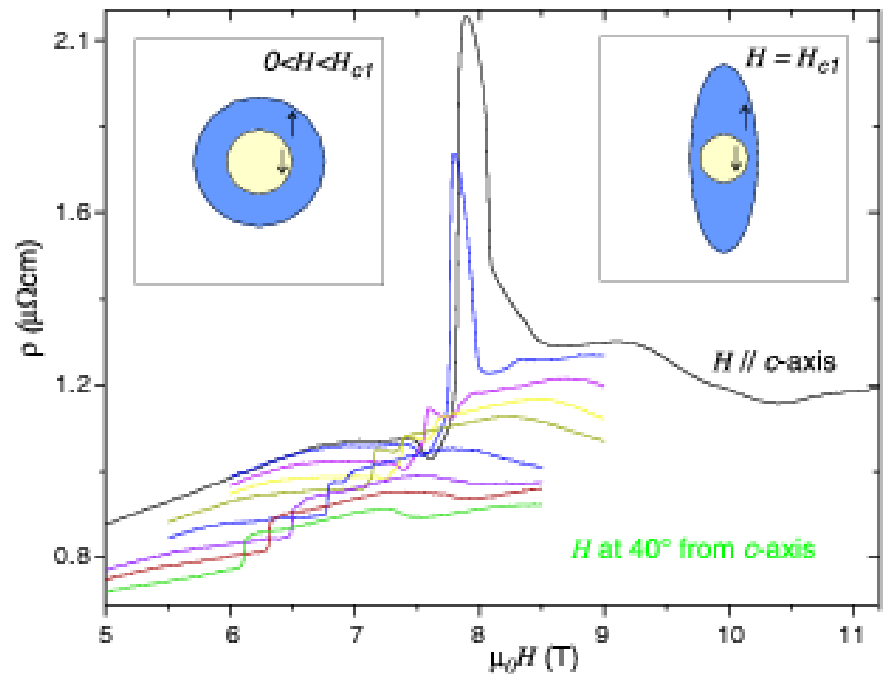
Spontaneous breaking of tetragonal symmetry ("Pomeranchuk instability") for sufficiently strong attractive d-wave component

Order parameter $n_d = \sum_{\mathbf{k}} d_{\mathbf{k}} \langle n_{\mathbf{k}} \rangle$ where $d_{\mathbf{k}} = \cos k_x - \cos k_y$

Realization of "nematic" electron liquid (→ Kivelson et al. 1998)

Nematic electron liquid with d-wave Fermi surface deformation
possibly realized in $Sr_3Ru_2O_7$ at high magnetic fields





Experiment: Grigera et al. 2004; Borzi et al. 2007

Theory: Fradkin et al. 2007; Yamase + Katanin 2007

Phenomenological 2D lattice model:

$$H = H_{\mathrm{kin}} + \frac{1}{2V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} f_{\mathbf{k}\mathbf{k}'}(\mathbf{q})\, n_{\mathbf{k}}(\mathbf{q})\, n_{\mathbf{k}'}(-\mathbf{q})$$

where $n_{\mathbf{k}}(\mathbf{q}) = \sum_\sigma c^\dagger_{\mathbf{k}-\mathbf{q}/2,\sigma}\, c_{\mathbf{k}+\mathbf{q}/2,\sigma}$

and only small momentum transfers $\mathbf{q}$ contribute (forward scattering)

Interaction with uniform repulsion and d-wave attraction:

$$f_{\mathbf{k}\mathbf{k}'}(\mathbf{q}) = u(\mathbf{q}) + g(\mathbf{q})\, d_{\mathbf{k}}\, d_{\mathbf{k}'}$$

with $d_{\mathbf{k}} = \cos k_x - \cos k_y$ and $u(\mathbf{q}) \geq 0$, $g(\mathbf{q}) < 0$

(*qualitatively* as from RG)

yields Pomeranchuk instability

Mean-field theory: Yamase, Oganesyan, wm 2005

2. Soft Fermi surface and non-FL behavior

Soft Fermi surface (near Pomeranchuk instability) $\Rightarrow$

- large response to anisotropic (d-wave) perturbations
- large Fermi surface fluctuations

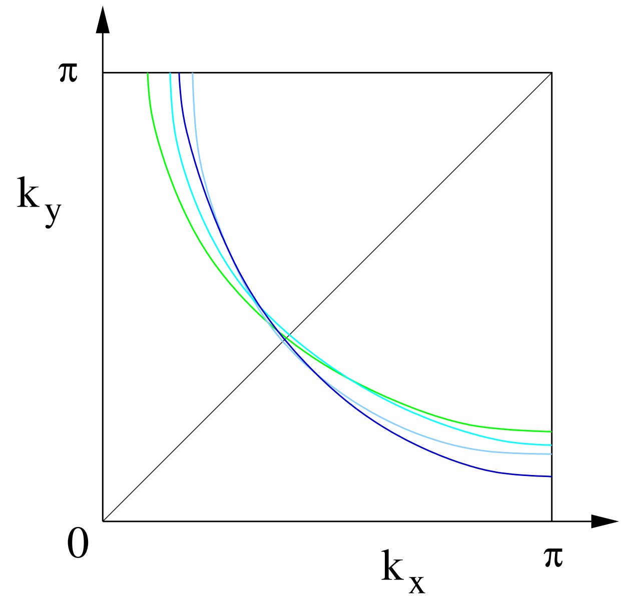
Non-Fermi liquid behavior in quantum critical regime

wm, Rohe, Andergassen, PRL **91**, 066402 (2003)
Dell'Anna, wm, PRB **73**, 045127 (2006)

Origin of non-FL behavior:

Electrons see fluctuating Fermi surface

$\Rightarrow$ enhanced and anisotropic decay rates

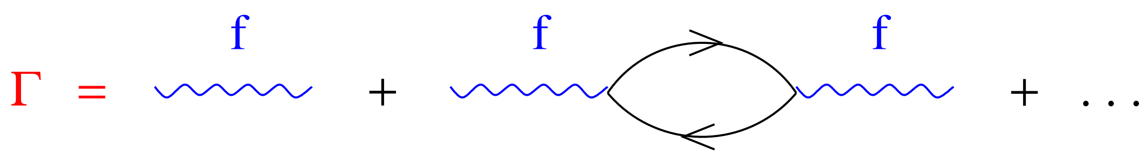


Fluctuations collective and overdamped;

not to be confused with:

- usual thermal smearing
- zero sound (propagating Fermi surface oscillation)

Dynamical effective interaction:



Singular part near Pomeranchuk instability for small $\mathbf{q}$ and small $\omega/|\mathbf{q}|$

$$\Gamma_{kk'}(\mathbf{q},\omega) \sim \frac{g(\mathbf{0})\, d_{\mathbf{k}}\, d_{\mathbf{k}'}}{(\xi_0/\xi)^2 + {\xi_0}^2\, |\mathbf{q}|^2 - i\frac{\omega}{u|\mathbf{q}|}}$$

Parameters:

Velocity $u > 0$ (related to $\mathrm{Im}\Pi_d$)

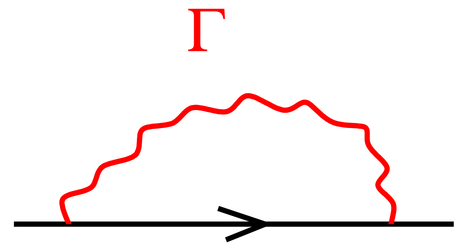
microscopic length scale ξ_0, correlation length ξ

Temperature dependence of ξ determined by interaction of critical fluctuations (Millis '93);

in quantum critical regime: $\xi(T) \propto \dfrac{1}{\sqrt{T \log T}}$

Electron self-energy:

Leading order (RPA)

$\Sigma =$ 

At quantum critical point ($T = 0$, $\xi = \infty$):

$$\mathrm{Im}\Sigma(\mathbf{k}_F, \omega) = \frac{g\, d_{\mathbf{k}_F}^2}{4\sqrt{3}\pi v_{\mathbf{k}_F}} \frac{u^{1/3}}{\xi_0^{4/3}} |\omega|^{2/3} \quad \text{for} \quad \omega \to 0$$

- large anisotropic imaginary part
- maximal near van Hove points,
  minimal near diagonal in Brillouin zone: "cold spots"

$\Rightarrow$ no quasi-particles away from Brillouin zone diagonal

Cf. non-Fermi liquid behavior in isotropic d-wave forward scattering model by Oganesyan et al. '01:

Isotropic decay $\propto |\omega|^{2/3}$ at quantum critical point

Decay $\propto |\omega|^{2/3}$ in 2D also at quantum critical point for

- phase separation (Castellani, Di Castro, Grilli '95)
- ferromagnetism (Chubukov '05)

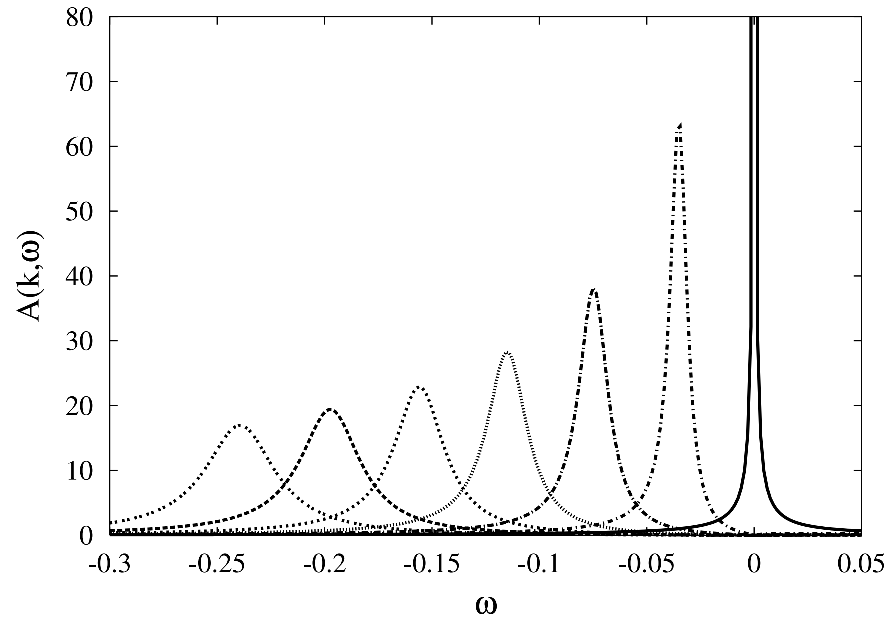
EDC of spectral function $A(\mathbf{k},\omega)$ at QCP

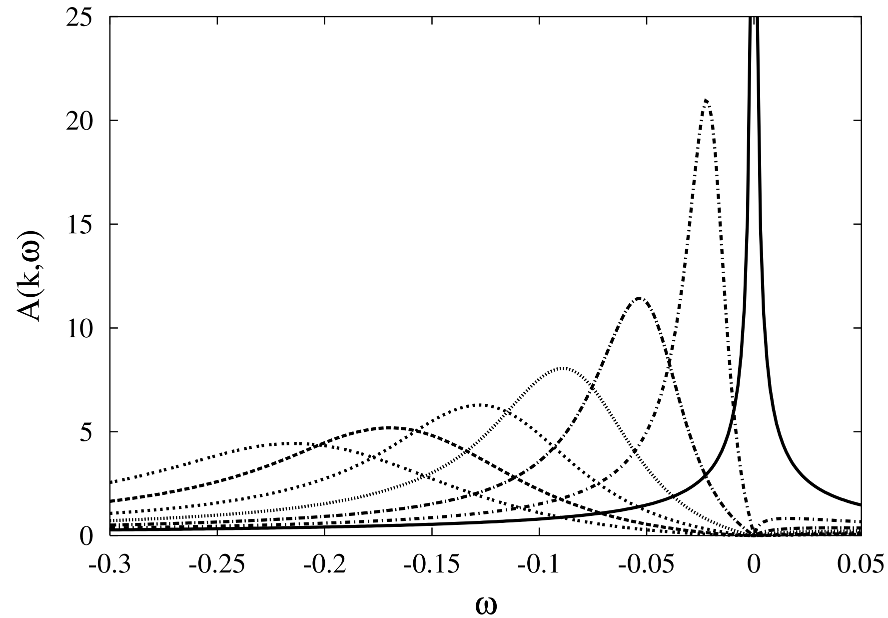
upper panel: $gd^2_{\mathbf{k}_F} = 1$
lower panel: $gd^2_{\mathbf{k}_F} = 4$

Energy scale $\omega^c_{\mathbf{k}_F} \propto \frac{d^6_{\mathbf{k}_F}}{v^3_{\mathbf{k}_F}}$

For $v_{\mathbf{k}_F}|\mathbf{k}-\mathbf{k}_F| < \omega^c_{\mathbf{k}_F}$
flat renormalized dispersion

$\bar{\xi}_{\mathbf{k}} \propto |\mathbf{k}-\mathbf{k}_F|^{3/2}$

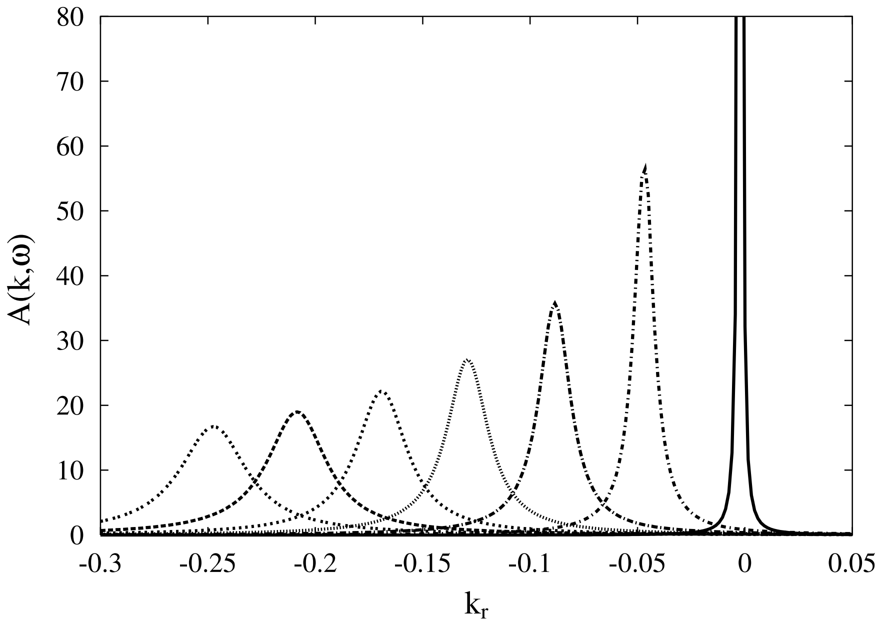


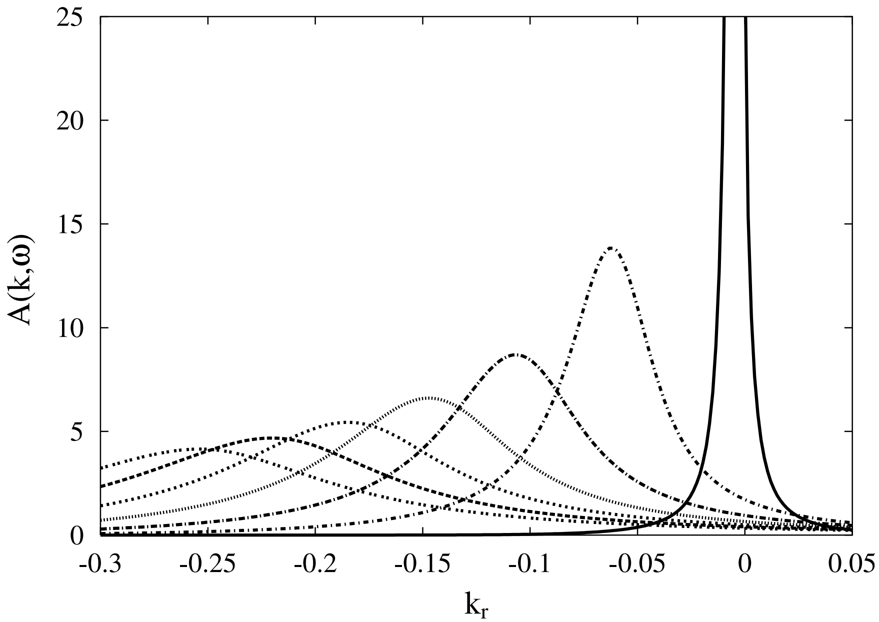


MDC of spectral function $A(\mathbf{k},\omega)$ at QCP

upper panel: $gd^2_{\mathbf{k}_F} = 1$

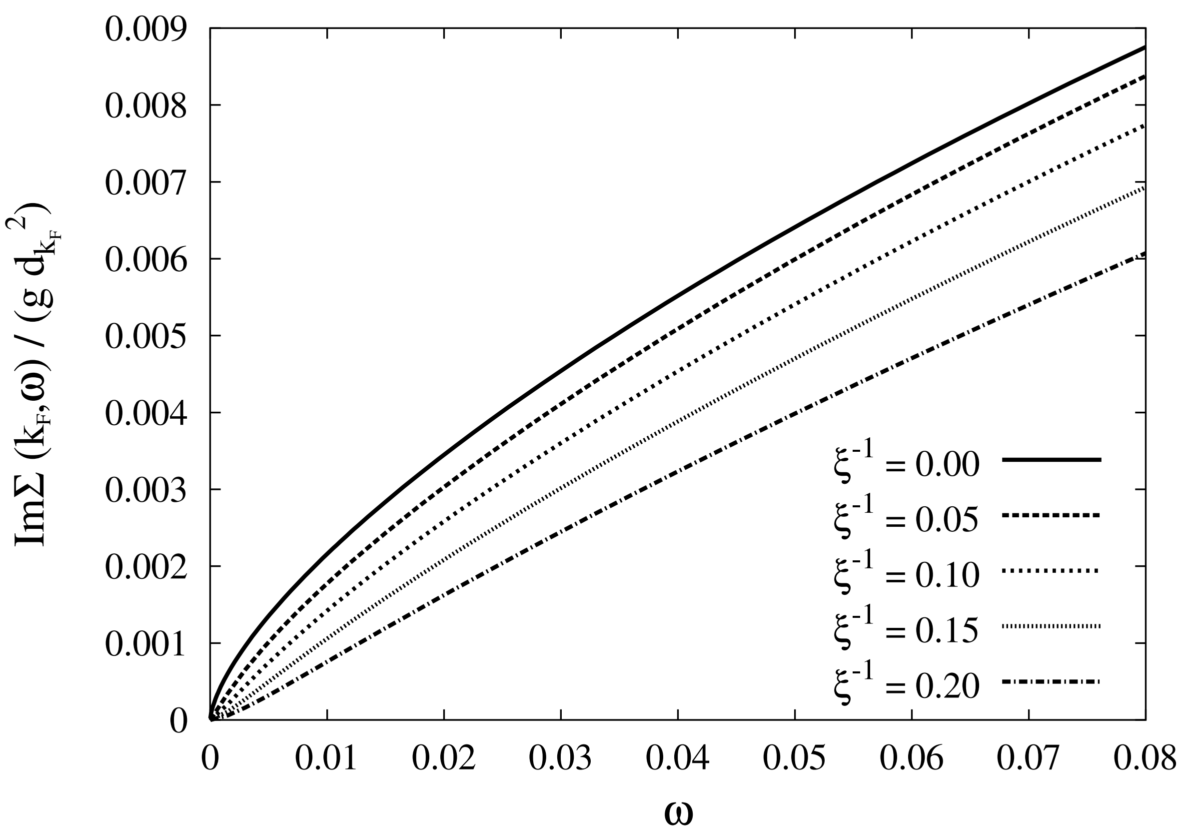
lower panel: $gd^2_{\mathbf{k}_F} = 4$





Lorentzian shape due to weak momentum dependence of $\Sigma(\mathbf{k},\omega)$ *perpendicular* to Fermi surface

$\mathrm{Im}\Sigma(\mathbf{k}_F, \omega)$ in symmetric phase at $T = 0$ for $\xi \geq 0$ (near QCP):



$\xi_0 = 1$

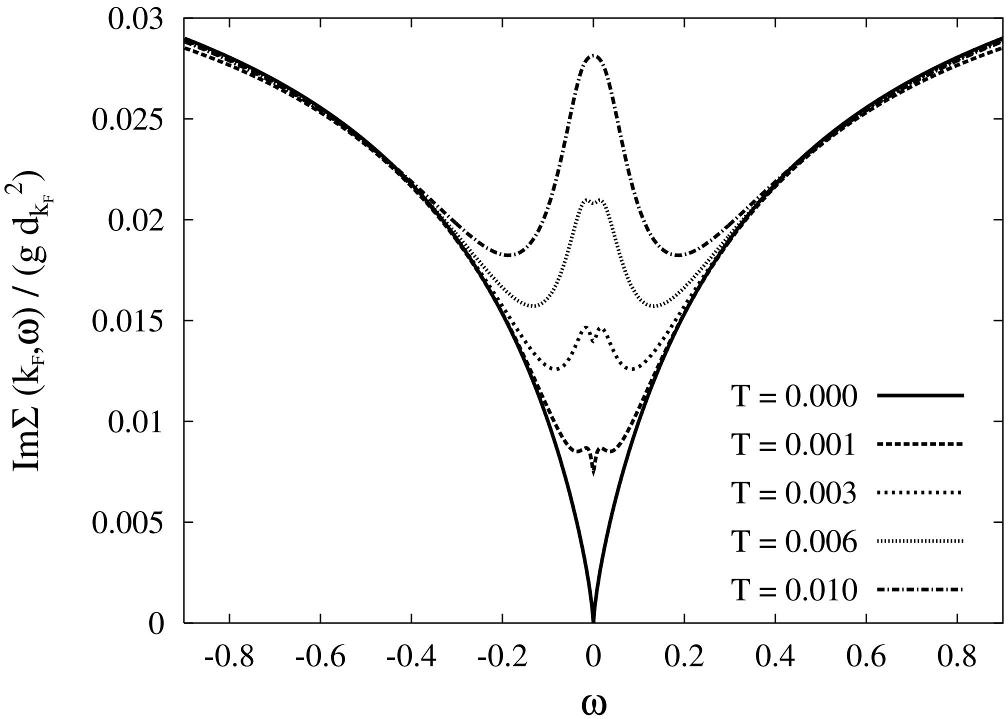
$v_{\mathbf{k}_F} = 1$

$u_{\mathbf{k}_F} = 1$

$\mathrm{Im}\Sigma(\mathbf{k}_F, \omega) \propto \omega^2 \log|\omega|$ for $|\omega| \ll \omega_\xi \propto \xi^{-3}$

$\mathrm{Im}\Sigma(\mathbf{k}_F, \omega)$ looks linear in wide energy range near and above ω_ξ

$\mathrm{Im}\Sigma(\mathbf{k}_F, \omega)$ above quantum critical point ($T > 0$):



$$\mathrm{Im}\Sigma(\mathbf{k}_F, 0) \;\to\; \frac{g\, d^2_{\mathbf{k}_F}}{4 v_{\mathbf{k}_F} \xi_0^2}\, T\, \xi(T) \;\propto\; (T/\log T)^{1/2} \quad \text{for} \;\; T \to 0$$

Classical fluctuations (Matsubara frequency $= 0$) dominate at $\omega = 0$

Selfconsistency (G instead of G_0 in RPA diagram)
yields only minor changes at $T > 0$ and none at $T = 0$.

At least at $T = 0$ results for Σ also stable against vertex corrections
(cf. fermions coupled to gauge field, in particular Altshuler et al. '94)

Vertex corrections due to classical fluctuations at $T > 0$ also finite

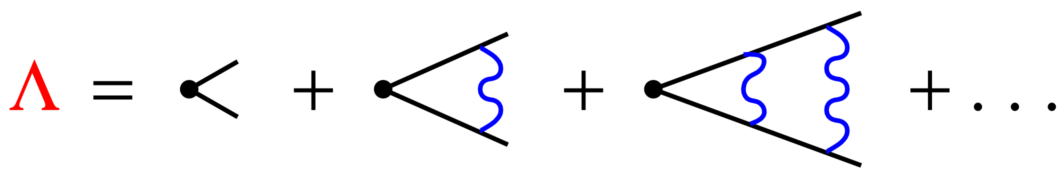
3. Transport life-time and electrical resistivity

DC conductivity from current-current correlation (Kubo formula):

$$\sigma_{jj'} = -\frac{e^2}{\pi} \int \frac{d^2k}{(2\pi)^2} \Lambda_j^0(\mathbf{k}) |G(\mathbf{k},\omega)|^2 \Lambda_{j'}(\mathbf{k},\omega)$$

Bare current vertex $\mathbf{\Lambda}^0(\mathbf{k}) = \mathbf{v}_\mathbf{k}$

Interacting current vertex $\mathbf{\Lambda}(\mathbf{k},\omega)$ from sum over particle-hole ladders



Conserving approximation corresponding to RPA self-energy

under the *assumption* that

order parameter fluctuations remain in equilibrium (no drag)

Again: Classical fluctuations dominate! $\Rightarrow$

Formal equivalence to Born approximation for disordered systems with long-ranged correlator $\Gamma_{\mathbf{kk}}(\mathbf{q},0)$

Integral equation for current vertex can be solved asymptotically

$$\mathbf{\Lambda}(\mathbf{k}_F,0) = \frac{\gamma_{\mathbf{k}_F}}{\gamma^{\rm tr}_{\mathbf{k}_F}}\,\mathbf{v}_{\mathbf{k}_F}$$

with single-particle decay rate $\gamma_{\mathbf{k}_F} = -\mathrm{Im}\Sigma(\mathbf{k}_F,0)$

and transport decay rate

$$\gamma^{\rm tr}_{\mathbf{k}_F} = -\pi T \int \frac{d^2q}{(2\pi)^2}\,\Gamma_{\mathbf{k}_F\mathbf{k}_F}(\mathbf{q},0)\,A(\mathbf{k}_F+\mathbf{q},0)\left(1 - \frac{\mathbf{v}_{\mathbf{k}_F}\cdot\mathbf{v}_{\mathbf{k}_F+\mathbf{q}}}{v^2_{\mathbf{k}_F}}\right)$$

$\Gamma_{\mathbf{k}_F\mathbf{k}_F}(\mathbf{q},0)$ static fluctuation propagator, $A(\mathbf{k},\omega)$ spectral function

$\gamma^{\rm tr}_{\mathbf{k}_F} \ll \gamma_{\mathbf{k}_F}$ for nearly forward scattering, huge vertex renormalization

Explicit result after $\mathbf{q}$-integration: $\gamma^{\rm tr}_{\mathbf{k}_F} \propto d^2_{\mathbf{k}_F} T$

- linear in temperature
- cold spots on Brillouin zone diagonal

Conductivity $\sigma = \frac{e^2}{8\pi^2} \int d\Omega_{\mathbf{k}_F} \frac{v_{\mathbf{k}_F}}{\gamma^{\rm tr}_{\mathbf{k}_F}}$ diverges due to cold spots

Adding Fermi liquid contribution due to regular interactions yields

$$\gamma^{\rm tr}_{\mathbf{k}_F} = a_{\mathbf{k}_F} T^2 + b_{\mathbf{k}_F} d^2_{\mathbf{k}_F} T$$

where $a_{\mathbf{k}_F}$ and $b_{\mathbf{k}_F}$ are finite for all $\mathbf{k}_F$

$\Rightarrow$ resistivity $\rho(T) = \sigma^{-1}(T) \propto T^{3/2}$

Dell'Anna, wm, PRL **98**, 136402 (2007)

Relation to DC transport in overdoped cuprates ?

- $\rho(T) \propto T^{3/2}$ observed in overdoped $\mathrm{La_{2-x}Sr_xCuO_4}$ (Takagi et al. 1992)
- Measurement of $\mathbf{k}$-resolved transport life time via angular magneto-resistance oscillations in overdoped $\mathrm{Tl_2Ba_2CuO_{6+\delta}}$ yields

  $$\gamma^{\mathrm{tr}}_{\mathbf{k}_F} = \text{conventional terms} + b_{\mathbf{k}_F} d^2_{\mathbf{k}_F} T$$

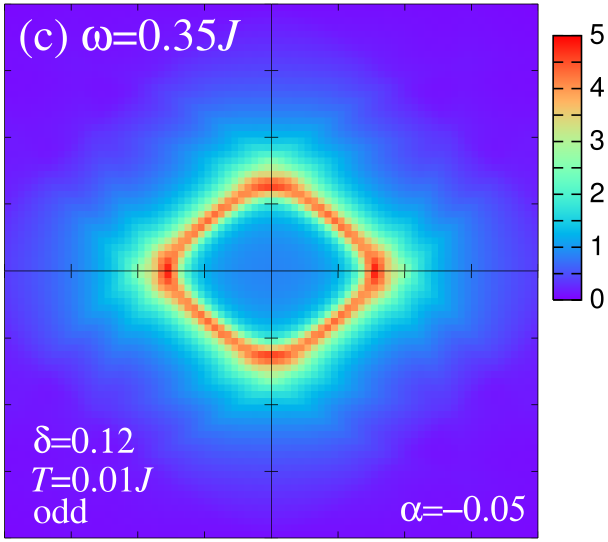
  (Abdel-Jawad,...,Hussey 2006)

Several other indications for d-wave Pomeranchuk/nematic physics in cuprates . . .

For example:

Large response to anisotropic (d-wave) perturbations

natural explanation (Yamase + wm 2006)
of relatively strong in-plane anisotropy
observed for magnetic excitations
in YBCO (Hinkov et al. 2004)



Conclusions:

- Microscopic models for cuprates exhibit attraction in d-wave forward scattering channel, favoring thus a d-wave Pomeranchuk instability

- Near Pomeranchuk instability singular forward scattering and soft Fermi surface, leading to non-Fermi liquid behavior.

- Transport decay rate is linear in T except at cold spots on the zone diagonal, leading to $\rho(T) \propto T^{3/2}$ in pure systems.