Looking for Supersolid Phases in Frustrated

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Collaborators

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O. Tchernyshyov, D. Clarke (Baltimore)
K. Penc (Budapest)

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C. Berthier, M. Horvatic (Grenoble)

Scope

- Supersolid phases in lattice bosonic models
- From quantum magnets in a field to bosons
- Magnetization plateaux in $SrCu_2(BO_3)_2$

  → Boson Mott insulator
- Broken translation above 1/8 plateau

  → Supersolid?
- Conclusions/Perspectives

Hubbard boson models

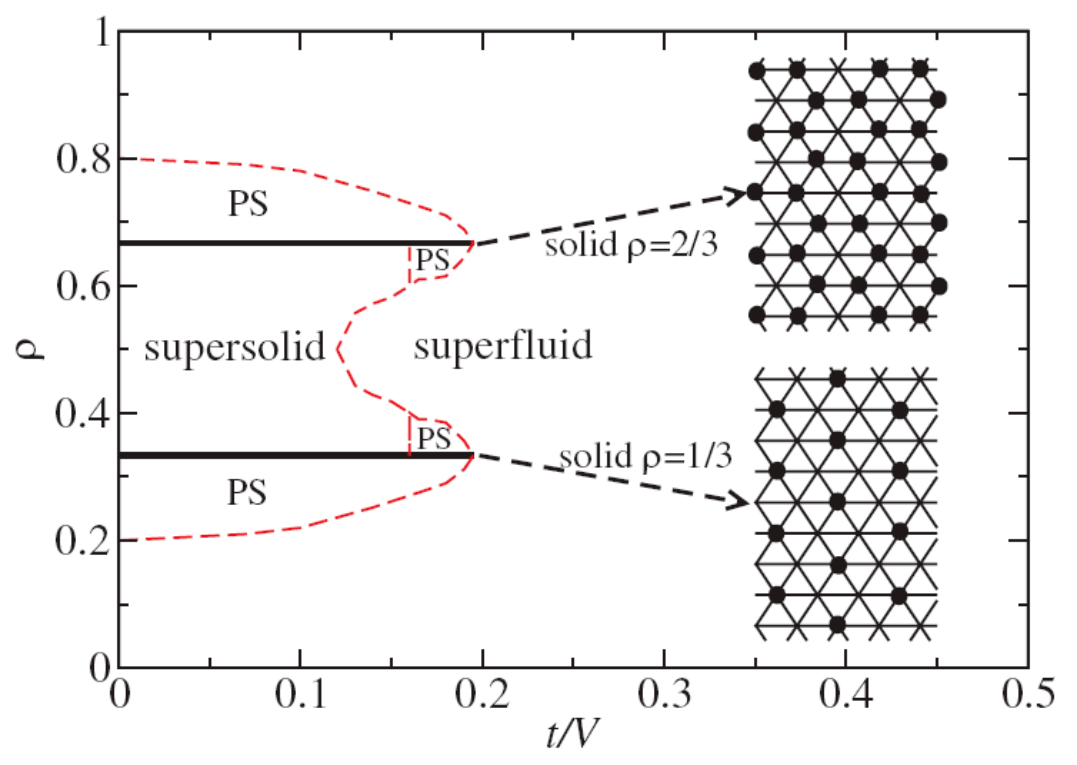
Hard-core bosons

$$H = -t\sum_{\langle i,j\rangle}(a_i^\dagger a_j + a_j^\dagger a_i) - \mu\sum_i n_i + V\sum_{\langle i,j\rangle} n_i n_j$$

Soft-core bosons

$$H \to H + \frac{U}{2}\sum_i n_i(n_i - 1)$$

Insulating and supersolid phases



Hard-core bosons on triangular lattice

Wessel and Troyer, '05
Heidarian et al, '05
Melko et al, '05

NB: Not really generic!
No supersolid for same model on square or kagome

From quantum magnets to hard-core bosons

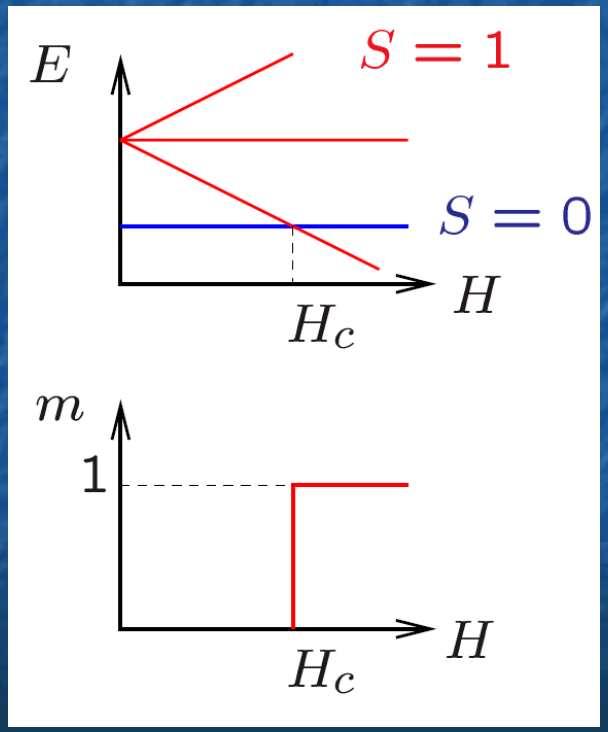
$$\mathcal{H} = J\vec{S}_1 \cdot \vec{S}_2 - g\mu_B H(S_1^z + S_2^z)$$

$$|\downarrow\downarrow\rangle$$

$$(|\uparrow\downarrow + \downarrow\uparrow)/\sqrt{2}\rangle$$

$$|\uparrow\uparrow\rangle$$

$$(|\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}\rangle$$



Isolated dimer

From quantum magnets to hard-core bosons

$$\mathcal{H} = J\vec{S}_1 \cdot \vec{S}_2 - g\mu_B H(S_1^z + S_2^z)$$

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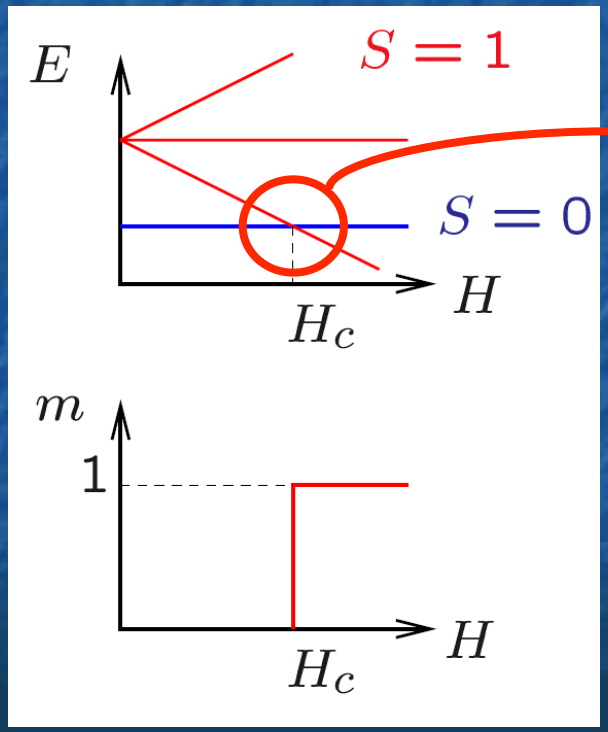
Isolated dimer

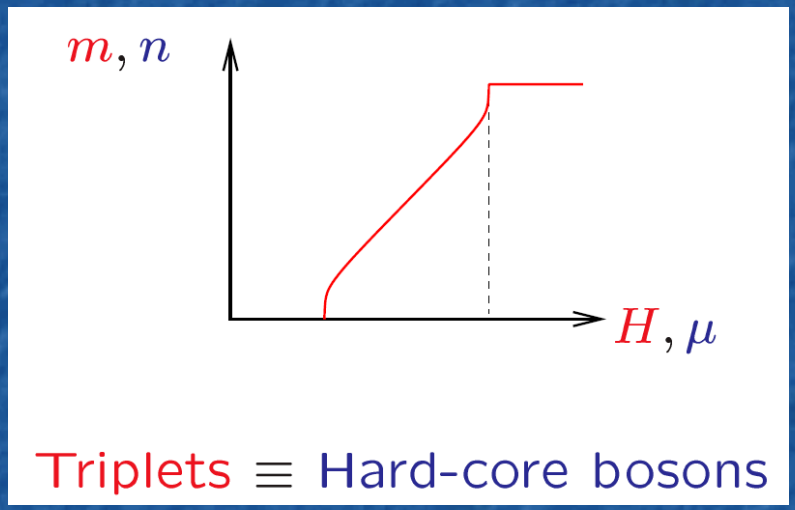


S=0: empty site

S_z=1: boson

From quantum dimers to hard-core bosons

Coupled dimers



Modulation of S_z <-> CDW

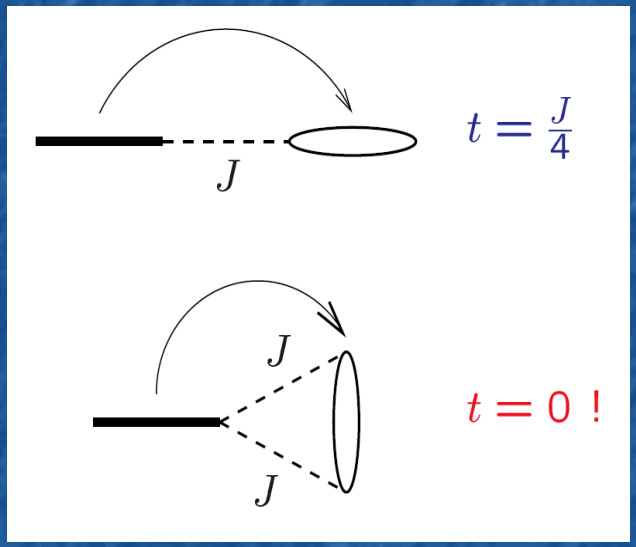
Ordering of $S_{x,y}$ <-> BEC

Program

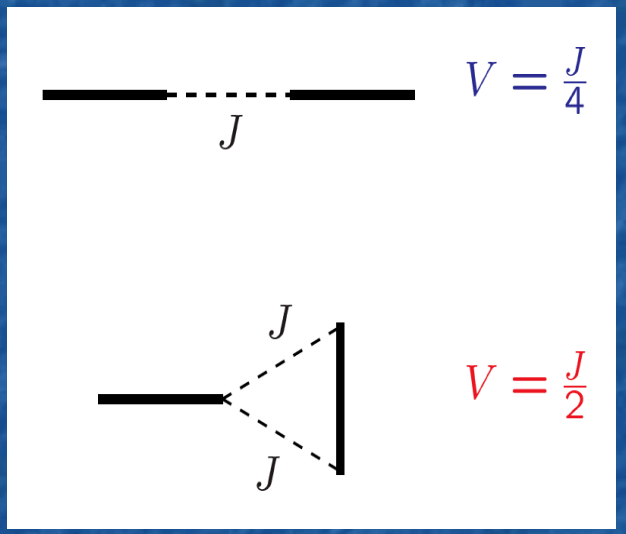
- How to reduce kinetic energy ($t/V<0.2$)?
  → Frustration
- Supersolid with square geometry?
  → Correlated hopping (second order)
- Experimental signature?
  → 2 phase transitions
- Experimental realization?
  → $SrCu_2(BO_3)_2$
  → Not completely sorted out yet…

Frustrated Coupled Dimers

Triplet Hopping

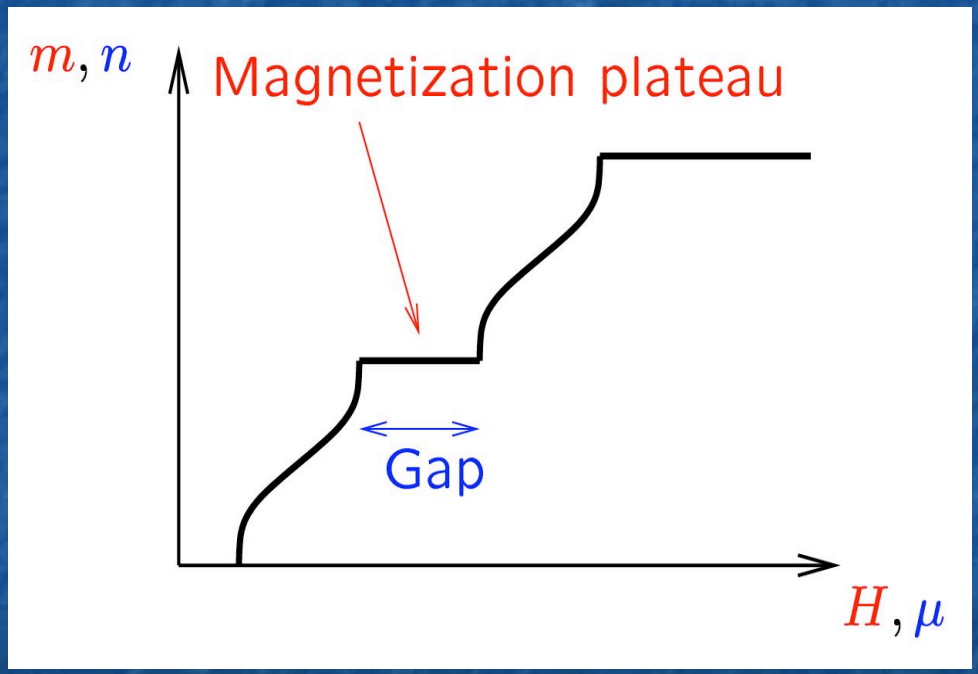


Triplet Repulsion



Frustration → Kinetic energy ↘ / Repulsion ↗ → **Metal-insulator transition** → **Magnetization plateau**

Frustration → plateaux



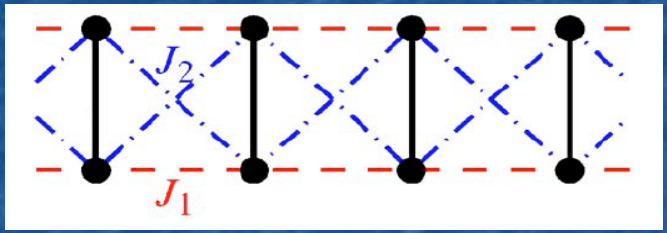
Plateaux in 1D:

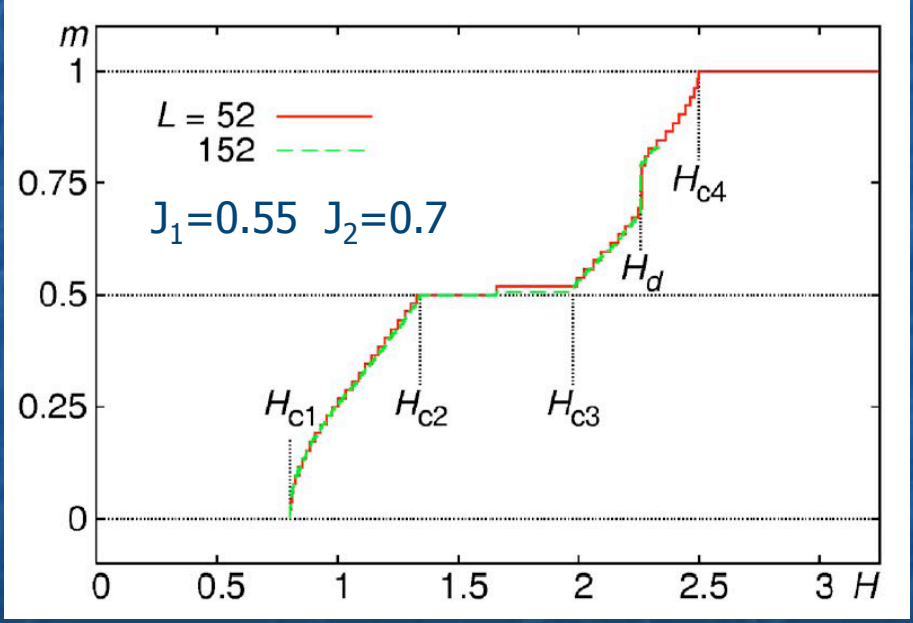
Hida, JPSP '94
Cabra et al, PRL '97
Oshikawa, Yamanaka, Affleck, PRL '97

Frustration induced Plateaux:

K. Totsuka, PRB '98
F. Mila, EPJB '98
T. Tonegawa et al, PRB '99

Frustrated ladder





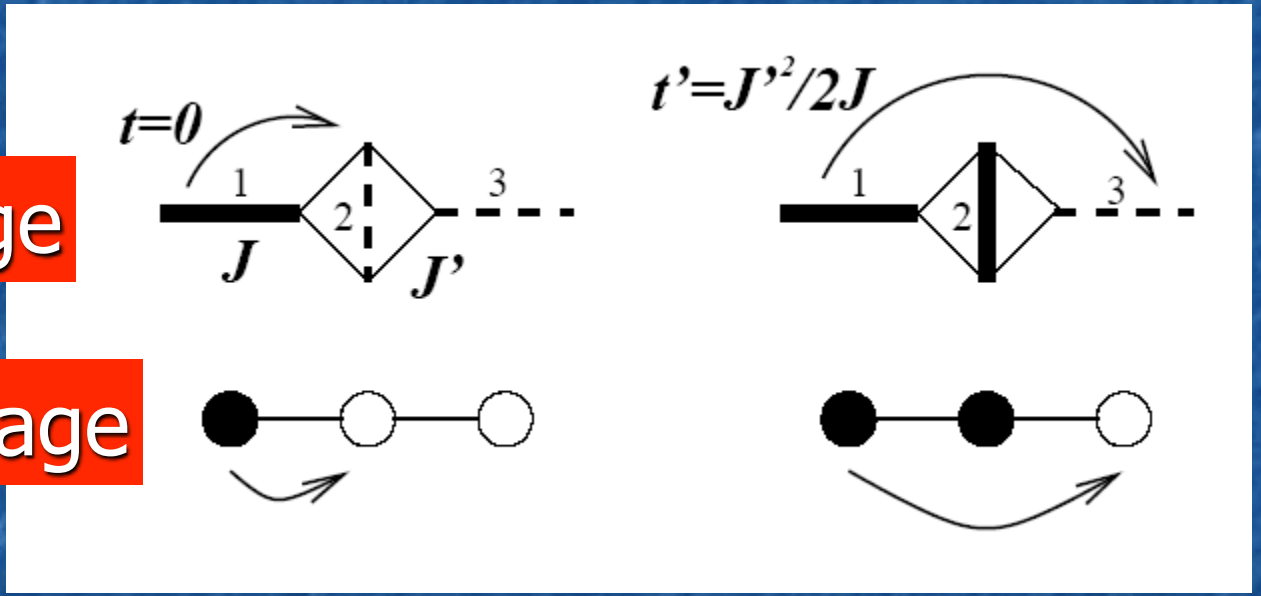
Translation symmetry NOT broken outside plateau but can be broken by DM interaction (Penc, Fouet, Miyahara, Tchernyshyov, Mila, PRL'07)

DMRG results: Fouet, Mila, Clarke, Youk, Tchenyshyov, Fendley, Noack, PRB '05

Correlated hopping

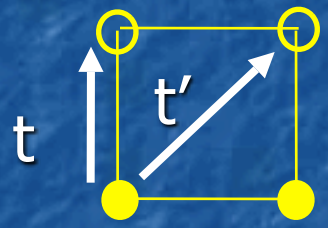
Spin language

Boson language



Supersolid on square lattice

$$H = -t\sum_i \sum_{\delta=\pm x,\pm y} b^\dagger_{i+\delta} b_i - \mu \sum_i n_i$$
$$-t'\sum_i \sum_{\delta=\pm x;\delta'=\pm y} n_i \left[b^\dagger_{i+\delta} b_{i+\delta'} + h.c. \right]$$
$$+V\sum_i \sum_{\delta=+x,+y} n_{i+\delta} n_i$$



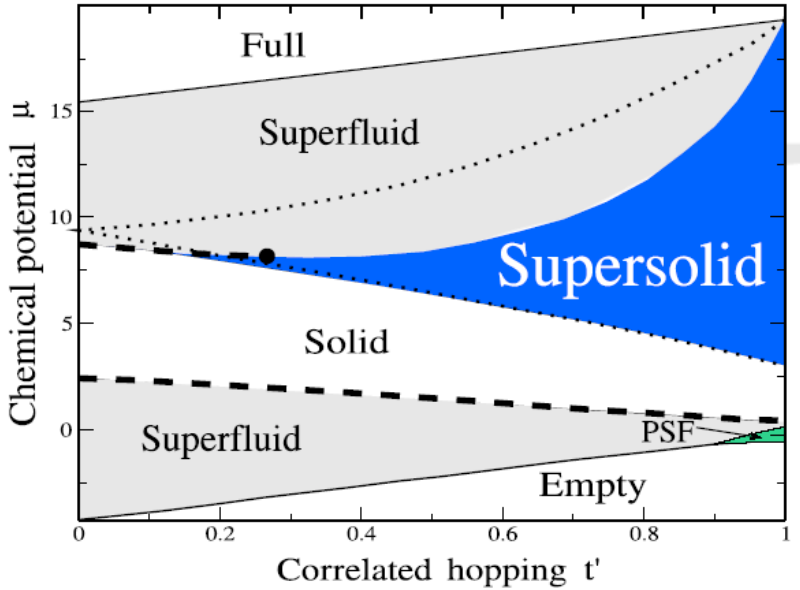
$SrCu_2(BO_3)_2$

$t \propto J^6/J_\perp^5$

$t' \propto J^2/J_\perp$

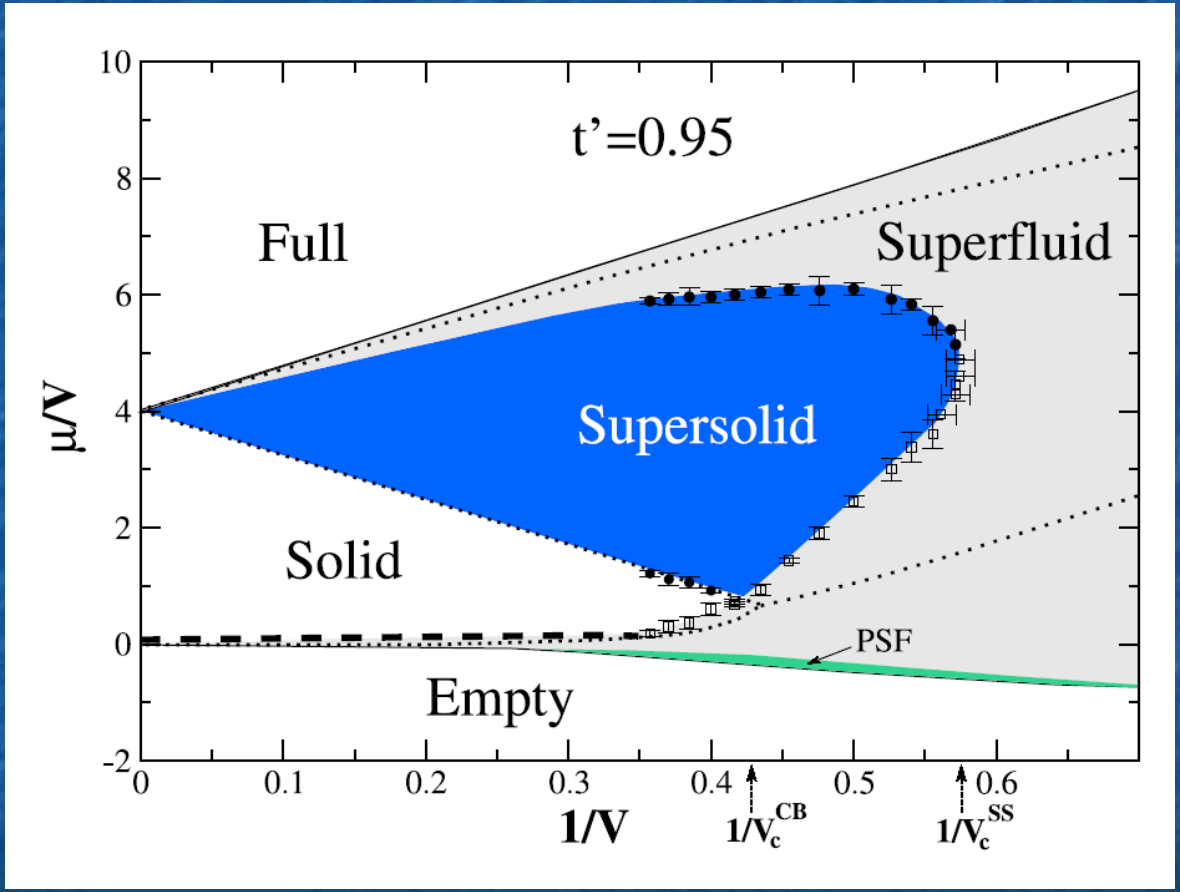
V=2.8 t+t'=1

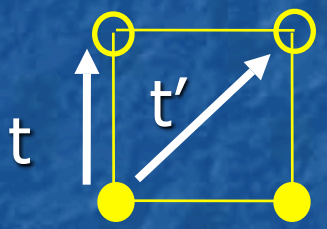
QMC + mean-field



K. Schmidt, J. Dorier, A. Läuchli, F. Mila, arXiv:0706.1517

Supersolid without solid

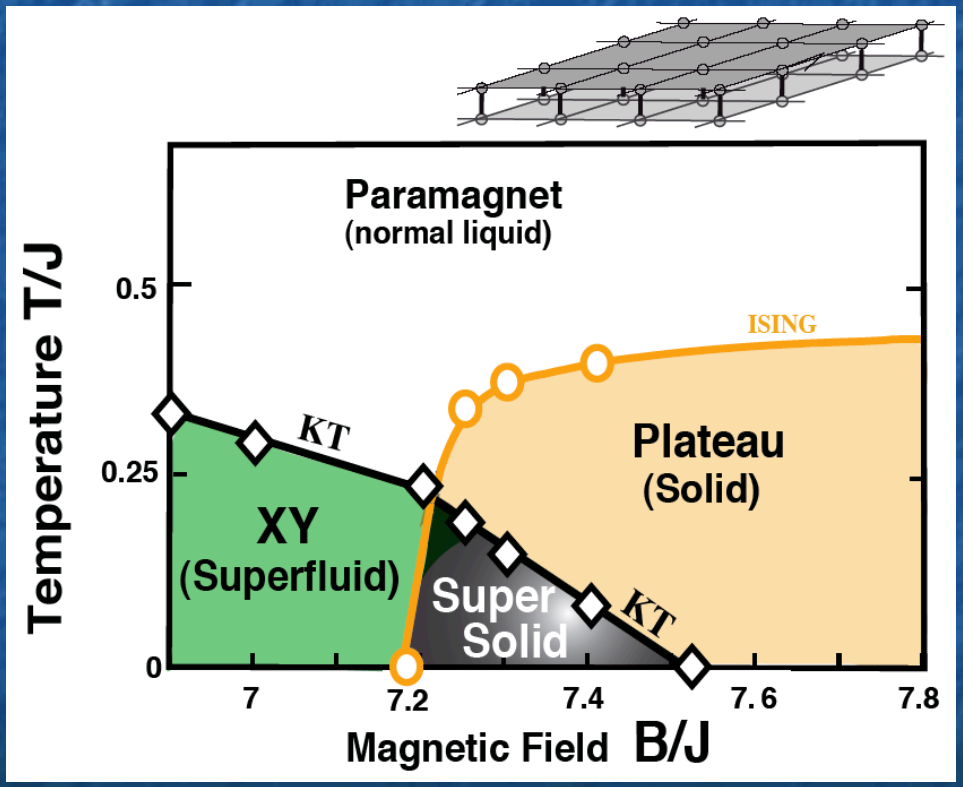




K. Schmidt, J. Dorier, A. Läuchli, F. Mila, arXiv:0706.1517

Temperature phase diagram

2 phase transitions



N. Laflorencie, F. Mila, PRL 2007

Inter-dimer coupling

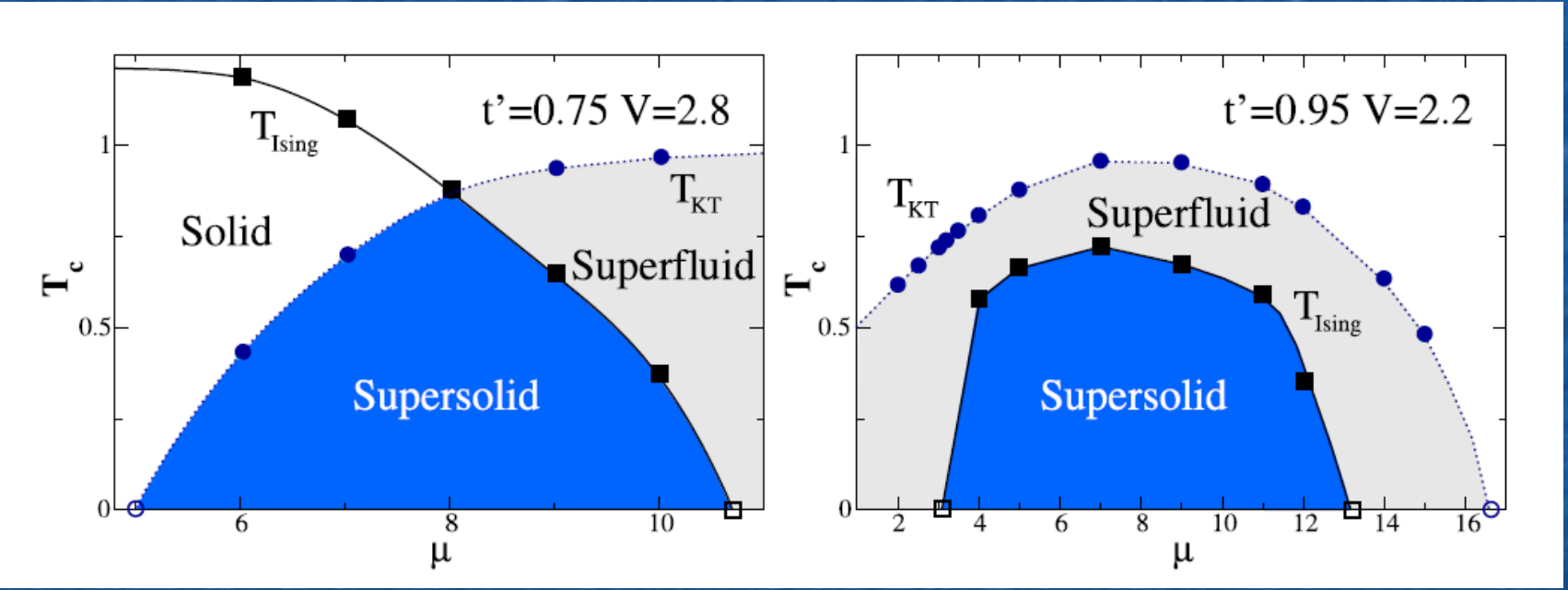
$J_z \gg J_{xy}$

Reduced kinetic energy

Supersolid at T=0

Ng and Lee, PRL 2006

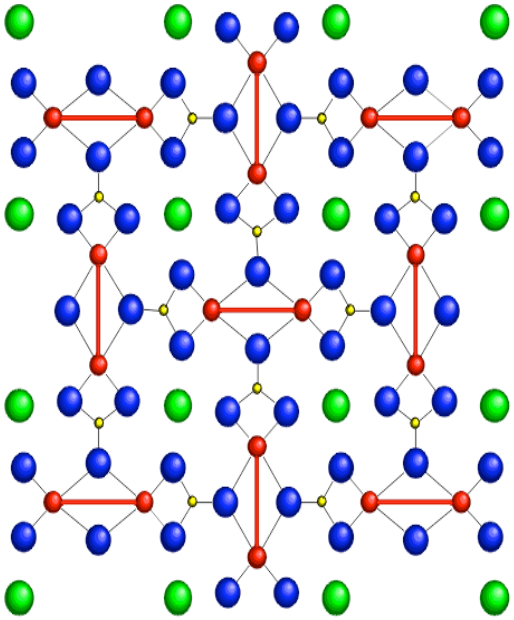
Temperature phase diagram with correlated-hopping



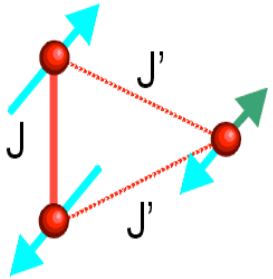
K. Schmidt, J. Dorier, A. Läuchli, F. Mila, arXiv:0706.1517

$SrCu_2(BO_3)_2$

Kageyama et al, PRL '99



Cu
O
Sr
B

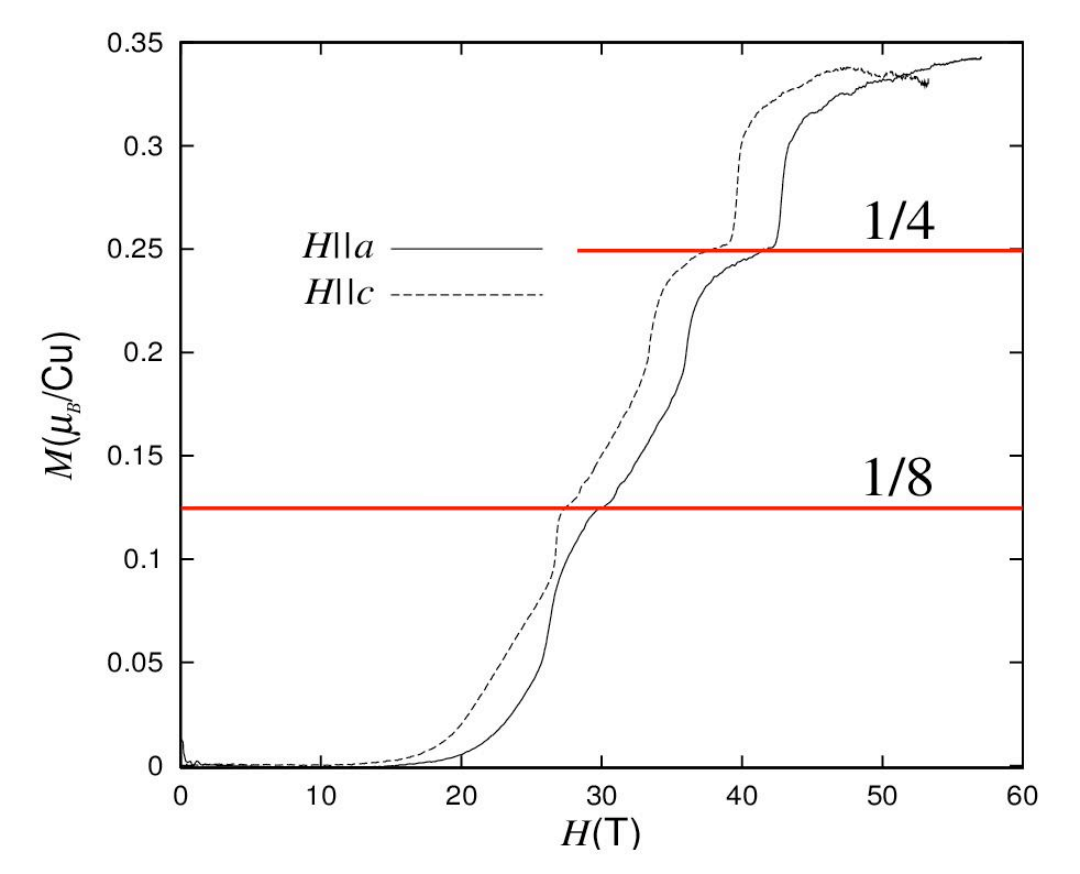


Cu^{2+} -> Spin 1/2

$J \simeq 85$ K

$J'/J \simeq 0.63$

Magnetization of $SrCu_2(BO_3)_2$

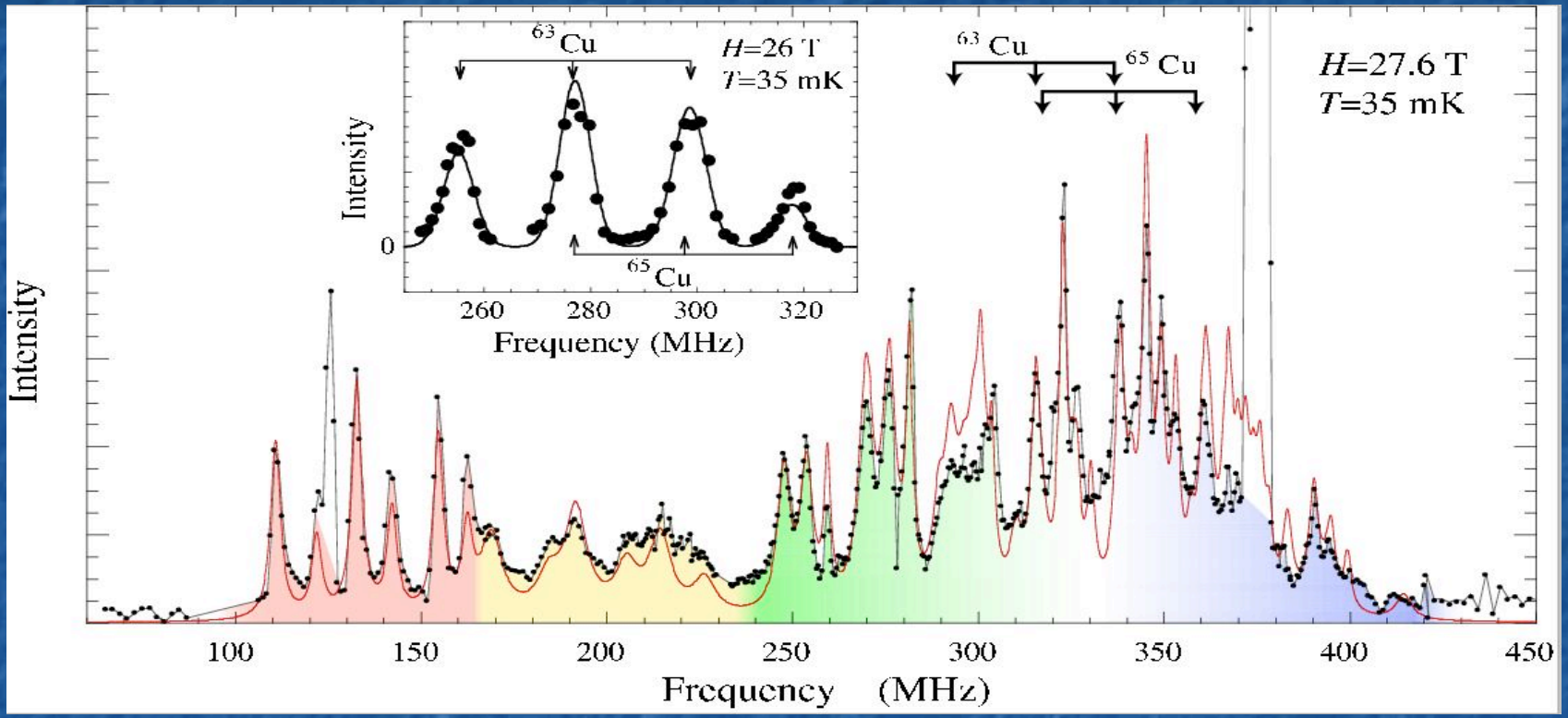


Kageyama et al PRL '99

Plateaux

- M=0
- M=1/8
- M=1/4
- M=1/3

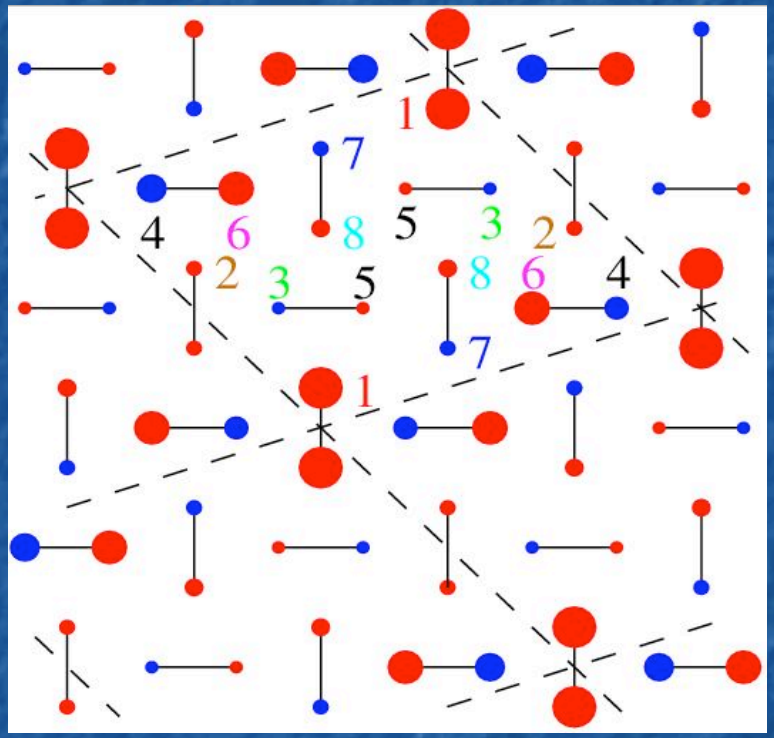
NMR at 1/8-plateau



At least 11 different sites!

K. Kodama, M. Takigawa, M. Horvatic, C. Berthier, H. Kageyama, Y. Ueda, S. Miyahara, F. Becca, F. Mila, Science '02

Magnetization profile at 1/8



Magnetization opposite to field

Magnetization in field direction

Symmetry breaking

16 sites/unit cell

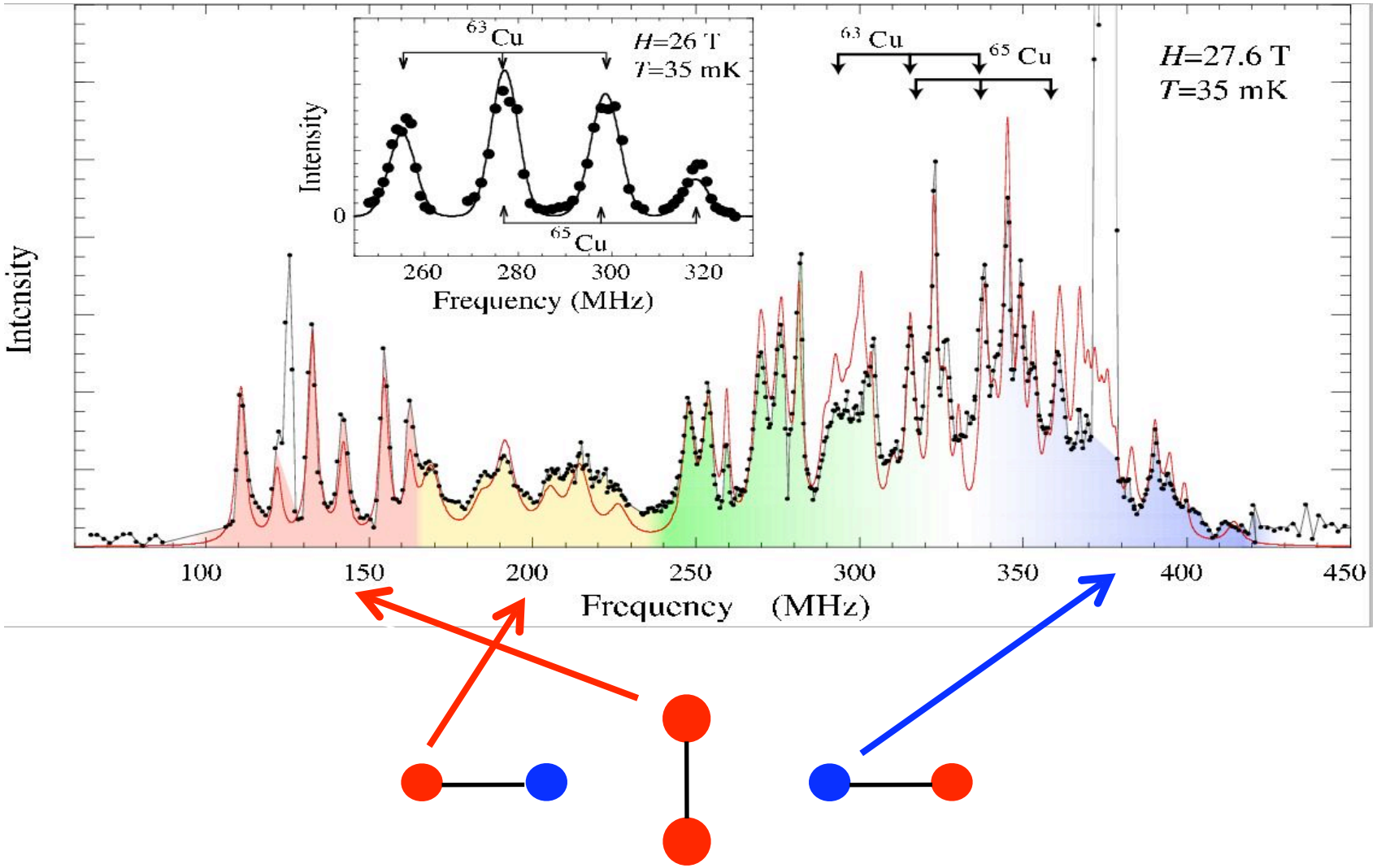
8-fold degenerate GS

Lattice distortion

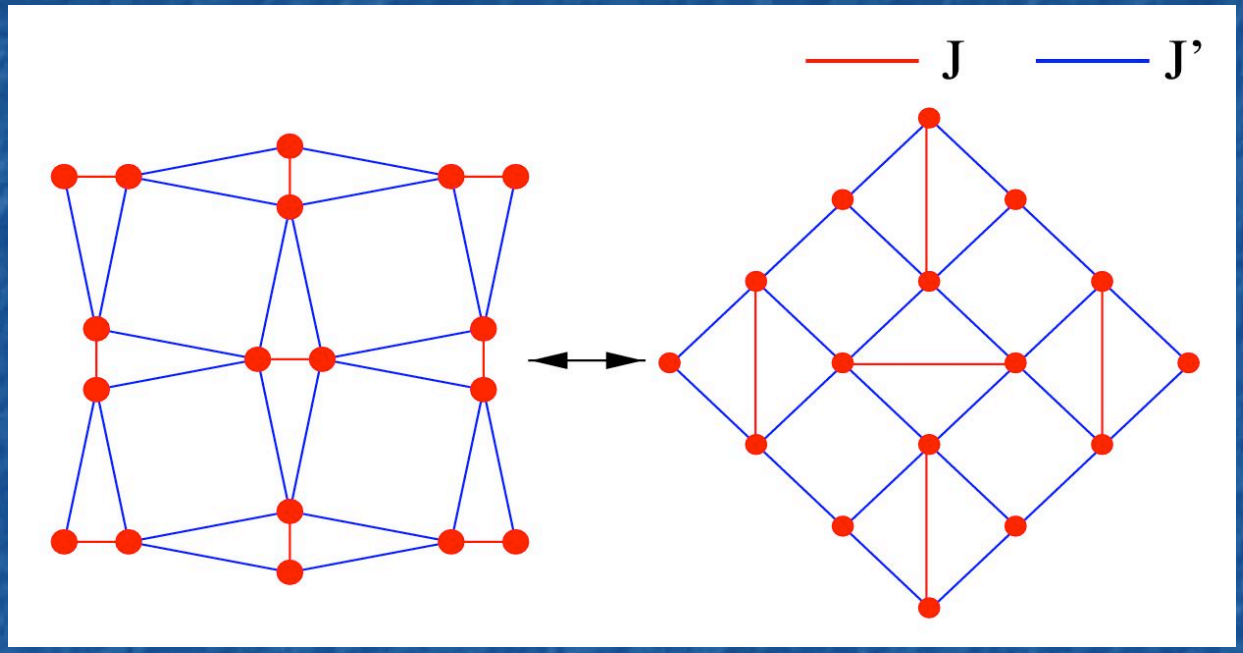
Sound-velocity
(Wolf et al, PRL '01)

Selection of one GS
with Friedel-like oscillations

Interpretation



Shastry-Sutherland model



$J'/J \simeq .63$

$t \propto J'^6/J^5$

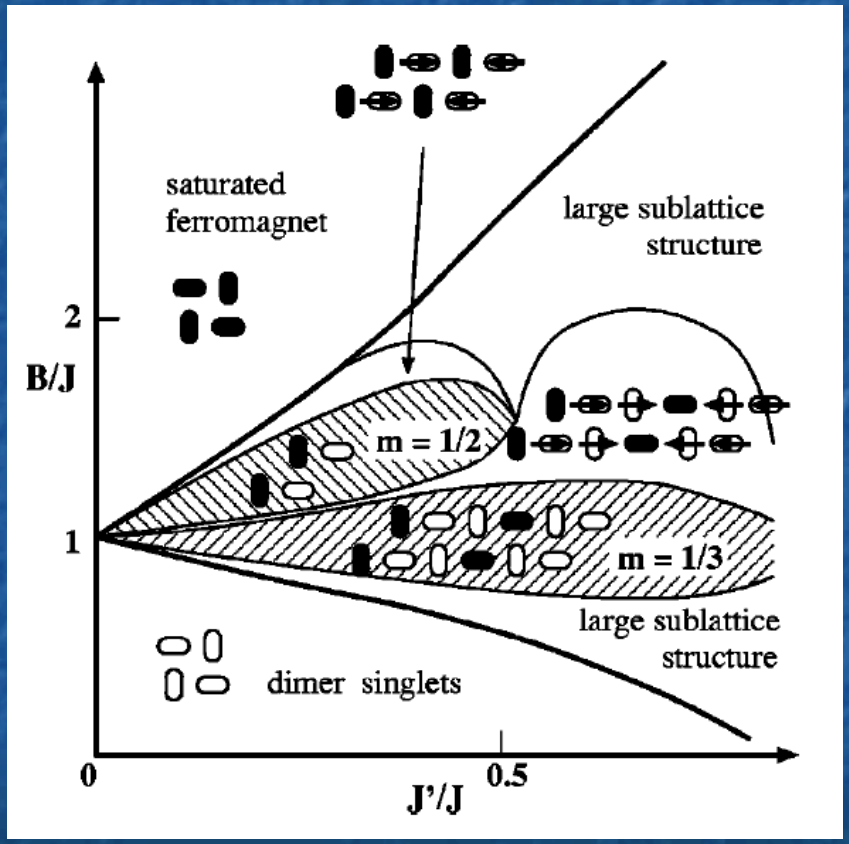
$t' \propto J'^2/J$

Ground-state Product of singlets on J-bonds (Shastry, Sutherland, '81)

Triplets Almost immobile and repulsive (Miyahara et al, '99)

→ Plateaux (Miyahara et al, '00)

Supersolids in $SrCu_2(BO_3)_2$?



Effective model
up to 3rd order

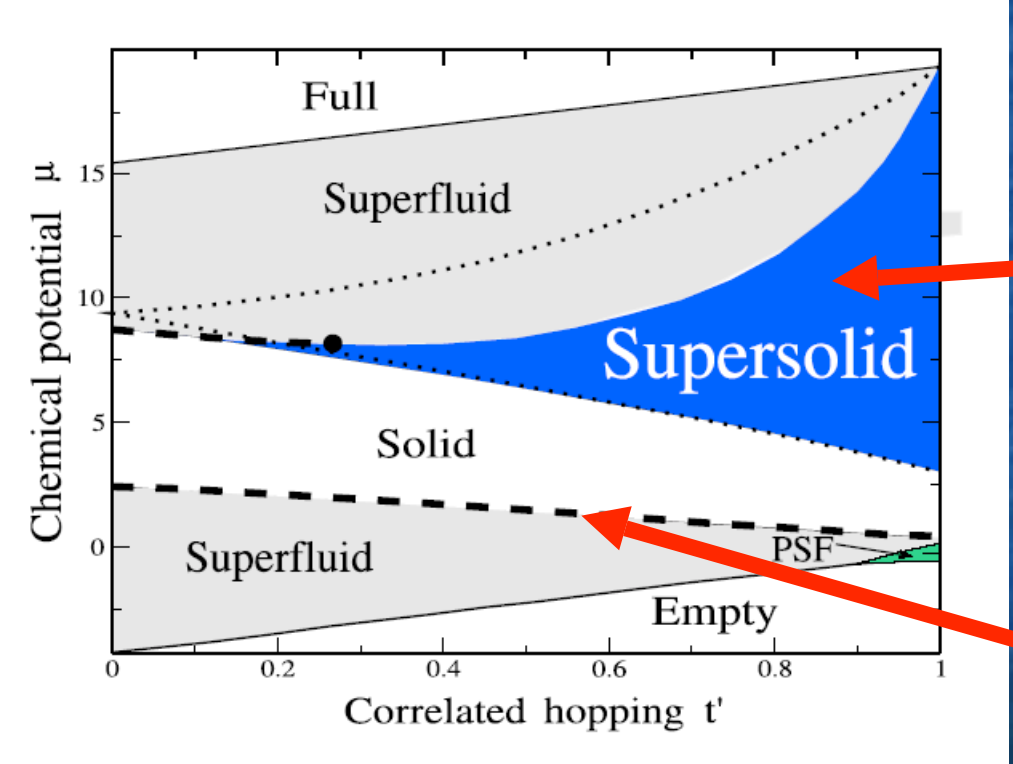
Supersolids above
1/3 and 1/2 plateaux

Nb: not accessible yet!

Momoi and Totsuka, PRB (2000)

Solid → Superfluid transition

Landau theory: two possibilities



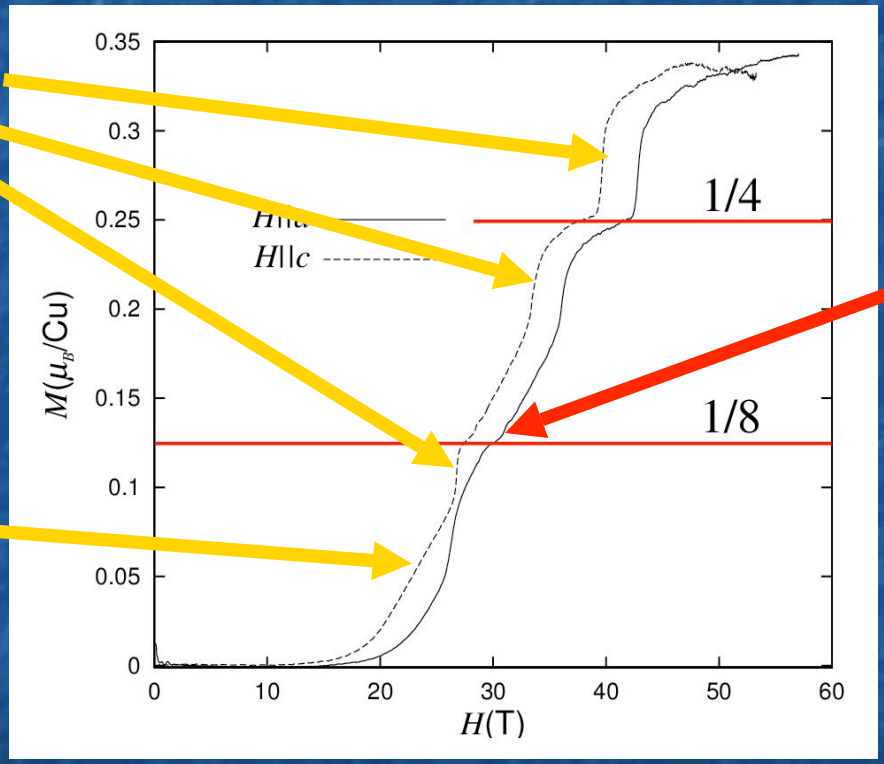
Through a supersolid

First order

Between the plateaux

Magnetization jumps

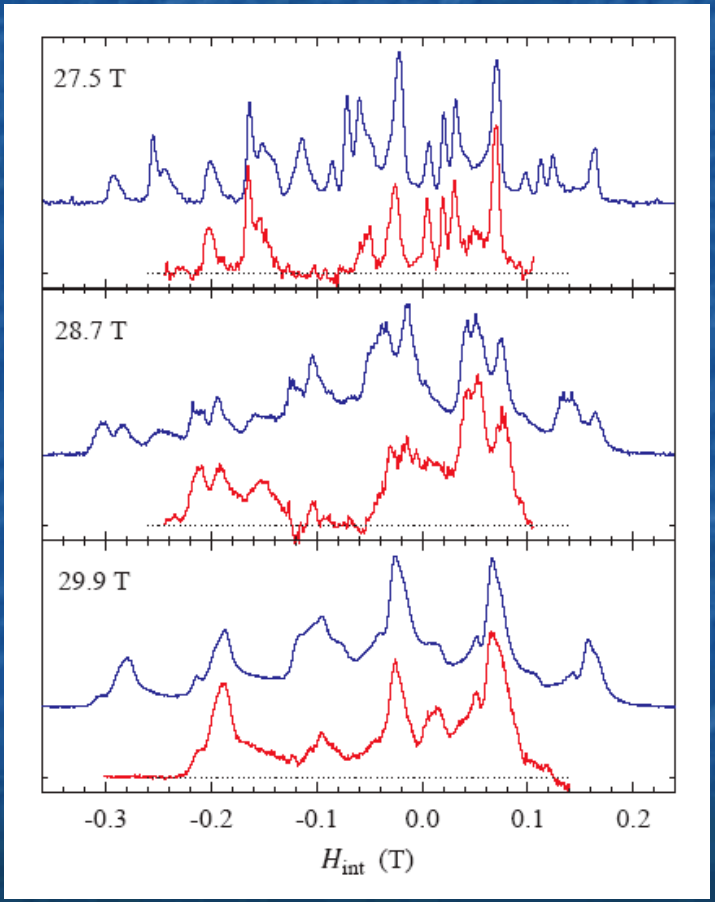
Uniform state (NMR)



No jump!

Broken translational symmetry above 1/8?

Boron NMR above 1/8 plateau



Takigawa et al, 2007

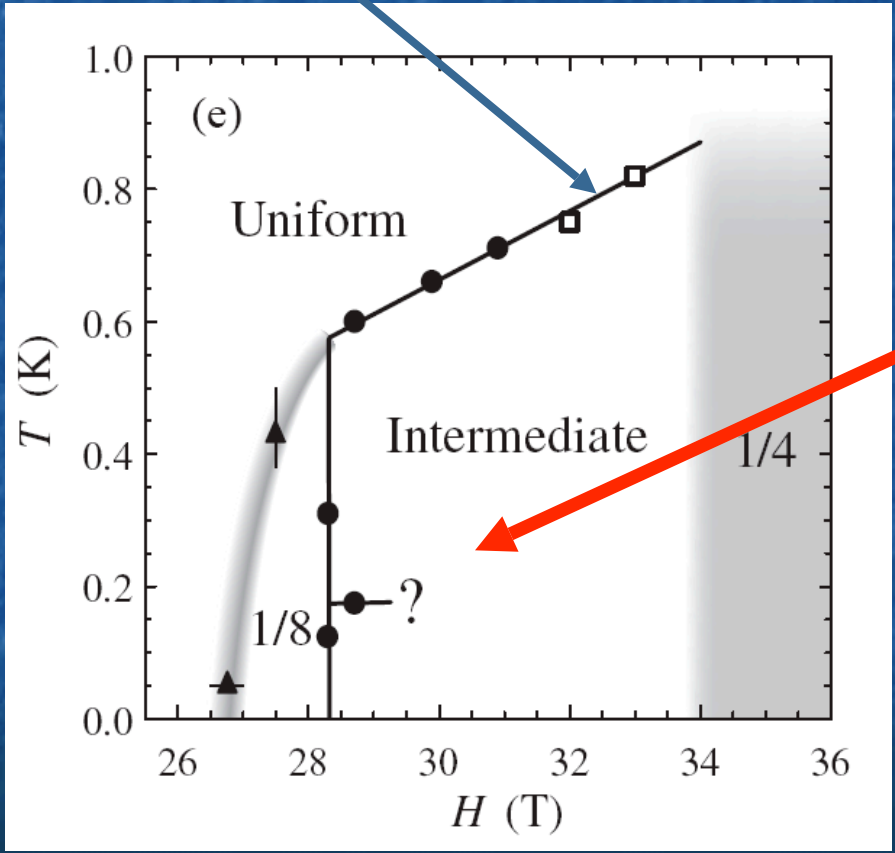
Blue curve: raw data

Red curve: deconvoluted spectrum (spin 3/2 → quadrupolar splitting → 3 lines per boron site)

Translation symmetry still broken!

Tentative phase diagram

Specific heat: Tsujii et al, '03



Translation symmetry still fully broken

Supersolid?

NMR: Takigawa et al, 2006, 2007

Open issues

- Only one phase transition
  → Dzyaloshinskii-Moriya interaction
- Magnetization profile above 1/8 plateau?
  → Interpretation of NMR under way
- Quantitative theory of $SrCu_2(BO_3)_2$?
  (plateaux at 1/8, 1/4, 1/3, supersolid,…)
  → High order effective bosonic model