The Kondo Screening Cloud: What it is and how to observe it in Quantum Dots

Collaborators: Erik Sorensen, Victor Barzykin, Pascal Simon, Rodrigo Pereira, Nicolas Laflorencie, Bertrand Halperin
Outline

• Kondo effect and screening cloud
• Non-observation of screening cloud in conventional experiments
• Numerical results
• Possible experiments on mesoscopic systems
• Including Coulomb interactions
Kondo Effect & Screening Cloud

• a single impurity spin in a metal is described by the Kondo (or s-d) model:

\[ H = \sum_{k\alpha} \psi_{k\alpha}^+ \psi_{k\alpha} \varepsilon_k + J \hat{S}_{\text{imp}} \cdot \hat{S}_{el} (r = 0) \]

• here \( \hat{S}_{\text{imp}} \) is the impurity spin operator \((S=1/2)\) and \( \hat{S}_{el} (\vec{r}) \) is the electron spin density at position \( \vec{r} \): \[ \hat{S}_{el} (\vec{r}) \equiv \psi^+(\vec{r}) \frac{\hat{\sigma}}{2} \psi (\vec{r}), \quad \psi (\vec{r}) \equiv \frac{1}{\sqrt{V}} \sum_k e^{i \vec{k} \cdot \vec{r}} \psi (\vec{k}) \]

(sum over spin indices implied)
after expanding the electron field $\psi(\vec{r})$ in spherical harmonics, keeping only the s-wave, and linearizing the dispersion relation we obtain a relativistic quantum field theory, defined on a 1/2-line with the impurity at the origin:
\[ H = i \nu_F \int_0^\infty dx \left[ \psi_L^+ \frac{d}{dx} \psi_L - \psi_R^+ \frac{d}{dx} \psi_R \right] + 2\pi \nu_F \lambda \vec{S}_{\text{imp}} \cdot \vec{S}_{\text{el}}(0) \]

\[ \psi_L(0) = -\psi_R(0) \]

- here \( \lambda \) is the dimensionless Kondo coupling, \( J \nu \), where \( \nu \) is the density of states
- Kondo physics is fundamentally 1-dimensional
• to study the problem at low energies, we may apply the renormalization group, integrating out high energy Fourier modes of the electron operators, reducing the band-width, $D$:

\[
\frac{d\lambda}{d \ln D} \approx -\lambda^2 + \cdots \\
\lambda_{\text{eff}} (D) \approx \frac{\lambda_0}{1 - \lambda_0 \ln(D_0 / D)} + \cdots
\]
• effective coupling becomes $O(1)$ at energy scale $T_K$:

$$T_K = D_0 \exp\left(-\frac{1}{\lambda_0}\right)$$

($D_0$ is of order the Fermi energy)

• effective Hamiltonian has wave-vector cutoff:

$$|k-k_F| < T_K/v_F = 1/\xi_K$$

• this defines a characteristic length scale for the Kondo effect - typically around .1 to 1 micron
• $\lambda_{\text{eff}} \rightarrow \infty$ at low energies ($<< T_K$)

• strong coupling physics is easiest to understand in tight-binding model:

$$H = -t \sum_{j=0}^{\infty} \left( \psi_j^+ \psi_{j+1} + \psi_{j+1}^+ \psi_j \right) + J \vec{S}_{\text{imp}} \cdot \vec{S}_{\text{el}}(0)$$
• for $J \gg t$, we simply find ground state of last term:
• 1 electron at $j=0$ forms spin singlet with impurity: $|\phi_0> = (|\uparrow \downarrow> - |\downarrow \uparrow>) / \sqrt{2}$

• other electrons are free except that they must not go to $j=0$ since they would break the singlet

• effectively an infinite repulsion at $j=0$, corresponding to $\pi/2$ phase shift
• for finite (small) $\lambda_0$, this description only holds at low energies and small $|k-k_F|$

- only long wavelength probes see simple $\pi/2$ phase shift - local Fermi liquid theory
Non-Observation of Screening Cloud in Experiments

- the Knight shift as a function of distance from an impurity can be measured by NMR
- at T=0 this takes the form:

$$\chi(r) = \chi_0 + \frac{\cos(2k_F r)}{r^2} f\left(\frac{r}{\xi_K}\right)$$

- rapid oscillations and power-law pre-factor makes this difficult to observe (Boyce-Slichter, 1974)
Schematic sketch of scaling function

\[ f(r/\xi_K) \]

(\text{conjecture: V Barzykin & IA, 1998})

\[ \pi \xi_K / [8r \ln(\xi_K/r)] \]

Fermi liquid theory

\[ 1 + \pi \xi_K / r \]
Numerical renormalization group confirmation: $<0|\mathbf{S}_{\text{imp}}(t) \cdot \mathbf{S}_{\text{el}}(x,t)|0>$

L. Borda
2006

Fermi Liquid theory
Possible Experiments on Mesoscopic Systems

• If conduction electrons are confined to a region of size $L < \xi_K$, Kondo effect is suppressed (at least in simplest models)

$$\lambda_{\text{eff}}(L) \approx \frac{\lambda_0}{1 - \lambda_0 \ln(L/a)} + \cdots \approx \frac{1}{\ln(\xi_K/L)}$$

• Finite system size cuts off infrared divergences of perturbation theory
• Screening cloud doesn’t “fit” into system
One possibility is to consider a quantum wire coupled to a quantum dot which acts as a Kondo impurity.
• A gate voltage, $V_G$ is applied to the wire while maintaining the dot in the Kondo regime.
• The wire also is very weakly coupled to an electron reservoir (not shown).
• As $V_G$ (or effectively, $\mu$, the chemical potential) is varied at $T=0$, electrons enter or leave the wire in single steps.
• Ratios of plateau widths with odd or even electron number depends on $L/\xi_K$.
• Could be measured from transport experiment.
• ignore, for now, any other interaction effects apart from Kondo interaction (see below)
• at $L \ll \xi_K$, $\lambda_{\text{eff}}$ is small and we get nearly free electron behaviour
• for nearly all values of $\mu$, $N$ is odd
• i.e. even number on wire, odd number on dot since single electron levels of wire are doubly occupied (or empty)
\( \mu \)

\( N \text{ odd} \)
• As we increase $L/\xi_K$, i.e. increase $\lambda_{\text{eff}}$, even $N$ plateaus get wider
• For $L \gg \xi_K$, we enter strong coupling regime
• now $N$ is even for almost all $\mu$ because one electron from wire forms singlet with the quantum dot spin while the rest doubly occupy single electron levels
N even
$N_0 = \text{odd}$

Graph showing the relationship between $N(\mu)$ and $N_0$, with steps indicating changes in $\delta\mu^e$ and $\delta\mu^o$ as a function of $(\mu - \mu^*)/\Delta$. The graph includes legends for finite $\xi_k/L$, weak coupling limit, and strong coupling limit.
• We consider $R = \delta \mu^0 / \delta \mu^e$

• At weak coupling, $\xi_K \gg L$ from perturbation theory:
  $R \approx 1 / [(3/2) \lambda_{\text{eff}}] \approx (2/3) \ln (\xi_K / L) >> 1$

• At strong coupling, $\xi_K << L$ we can use local Fermi liquid theory:
  $R \approx (\pi \xi_K) / (4L) << 1$

• In between, $R$ is given by a universal scaling function of $\xi_K / L$

• We calculated this using Bethe ansatz solution of Kondo problem at finite $L$
Cleaved overgrowth quantum wires
Yacoby et al.

I \sim 10 \mu
L \sim \mu's

3 or fewer channels
• Coulomb interaction screened (to $1/r^3$) by second nearby wire, a distance $d$ away
• Luttinger model seems to work
• $v_c \sim 1.5v_F$, $v_s \sim v_F$, $K \sim .7$
Including Coulomb Interactions

With Kondo impurity coupled at end of wire, spin-charge separation applies

$$E(N + n) = \frac{\pi}{4} v_{c/s} n^2 + \frac{1}{4L} f_s(\lambda, g, L) - \mu n$$

- $V_{c/s}$ are charge/spin velocities
- $K$ is Luttinger parameter
- $f_s$ is dimensionless spin energy:
  - $s = 1/2$ or 0 for $n$ even or odd
- $g$ is marginal coupling constant in spin sector, deriving from Coulomb interactions
• charge step ratio depends only on \( f(\lambda, g, L) = f_{1/2} - f_0 \) and \( u = (v_s K / v_c)^{\sim 0.5} \)

\[
R = \frac{1 + uf(\lambda, g, L)}{1 - uf(\lambda, g, L)}
\]
• Coulomb interactions lead to $u<1$- suppressing somewhat even-odd effect in charge steps

• Spin-charge separation: $H = H_c + H_s$

• Only important interaction is in spin sector $H_s = H_0 + H_{\text{int}}$

\[
H_{\text{int}} = 2\pi \nu_s \left[ - g \int_{0}^{L} dx \, \vec{J}_L \times \vec{J}_R + \lambda \, \vec{J}_L(0) \times \vec{S} \right]
\]
• Coulomb interaction, $g$, is marginally irrelevant:
  
  \[ g(L) \sim \frac{1}{\ln \left( \frac{L}{a} \right)}, \quad L \gg a, \quad \text{where} \quad a \sim 1/k_F \]

• Kondo interaction, $\lambda$, is marginally relevant:
  
  \[ \lambda(L) \sim \frac{1}{\ln \left( \frac{\xi_K}{L} \right)}, \quad L \ll \xi_K \]

• Coulomb interactions modifies $f(\xi_K/L)$ but effect gets weaker and weaker for larger $L$

• Non-interacting case discussed previously is:
  
  \[ R_0 = \frac{1 + f(0, \lambda, L)}{1 - f(0, \lambda, L)} \]
• some further reduction of even/odd effect from marginal interaction
• goes away as $L$ increases
Conclusions

• Physical quantities change at large length scale $\xi_K = \frac{v_F}{T_K} \leq 1 \mu$, in ideal model
• This effect not yet seen experimentally
• Mesoscopic systems with $L \sim \xi_K$ provide new possibilities
• Interactions modify situation somewhat
More Realistic Models

- more channels may couple to impurity
- system may be disordered (e.g., 1 magnetic impurity and many non-magnetic ones)
- interactions among conduction electrons may be important
- interactions between magnetic impurities (e.g., RKKY) may be important
Including more channels: Thimm, Kroha & Van Delft (1999) considered a 3-dimensional disordered “box” (volume V) containing a single impurity spin. They assumed:

• All single electron states in box couple with equal strength to spin
• Energy levels are equally spaced $\Delta \propto 1/V$

- Now suppression of Kondo effect occurs when $T_K \sim \Delta \rightarrow V \sim \xi_K a^2 \quad (a \sim 1/k_F)$

- A much smaller system would be needed to see this effect
• In quantum wires with N channels, typically require $L \approx \xi_K / N$ to see screening cloud effects (P. Simon & IA, 2003)
• This can make it much more difficult to see these effects
Friedel (charge) oscillations around a Kondo impurity

(with Laszlo Borda and Hubert Saleur)

\[ \rho(r) - \rho_0 \rightarrow \frac{C_D}{r_D} \left[ \sin(2k_F r + 2\delta_P)F(r/\xi_K) - \sin(2k_F r) \right] \]