The Kondo Screening Cloud: What it is and how to observe it in Quantum Dots

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Outline

- Kondo effect and screening cloud
- Non-observation of screening cloud
  in conventional experiments
- Numerical results
- Possible experiments on mesoscopic
systems
- Including Coulomb interactions

Kondo Effect & Screening Cloud

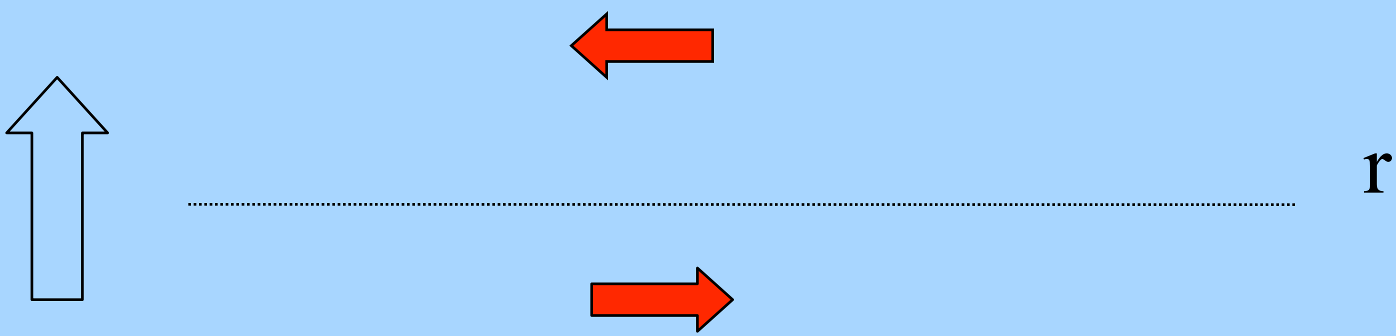
- a single impurity spin in a metal is described by the Kondo (or s-d) model:

$$H = \sum_{k\alpha} \psi^{+}_{\vec{k}\alpha} \psi_{\vec{k}\alpha} \varepsilon_{k} + J\vec{S}_{imp} \bullet \vec{S}_{el}(r=0)$$

- here $\vec{S}_{imp}$ is the impurity spin operator (S=1/2) and $\vec{S}_{el}(\vec{r})$ is the electron spin density at position $\vec{r}$: $\vec{S}_{el}(\vec{r}) \equiv \psi^{+}(\vec{r}) \frac{\vec{\sigma}}{2} \psi(\vec{r}),\ \psi(\vec{r}) \equiv \frac{1}{\sqrt{V}} \sum_{k} e^{i\vec{k}\cdot\vec{r}} \psi(\vec{k})$

(sum over spin indices implied)

• after expanding the electron field $\psi(\vec{r})$ in spherical harmonics, keeping only the s-wave, and linearizing the dispersion relation we obtain a relativistic quantum field theory, defined on a 1/2-line with the impurity at the origin:



$$H = iv_F \int_0^\infty dx \left[\psi_L^+ \frac{d}{dx} \psi_L - \psi_R^+ \frac{d}{dx} \psi_R \right] + 2\pi v_F \lambda \vec{S}_{imp} \bullet \vec{S}_{el}(0)$$

$$\psi_L(0) = -\psi_R(0)$$

- here λ is the dimensionless Kondo coupling, $J\nu$, where ν is the density of states
- Kondo physics is fundamentally 1-dimensional

• to study the problem at low energies, we may apply the renormalization group, integrating out high energy Fourier modes of the electron operators, reducing the band-width, D:

$$\frac{d\lambda}{d\ln D} \approx -\lambda^2 + \bullet\bullet\bullet$$

$$\lambda_{eff}(D) \approx \frac{\lambda_0}{1-\lambda_0 \ln(D_0 / D)} + \bullet\bullet\bullet$$

• effective coupling becomes O(1) at energy scale T_K:

$$T_K = D_0 \exp(-1/\lambda_0)$$

(D_0 is of order the Fermi energy)

• effective Hamiltonian has *wave-vector* cutoff:

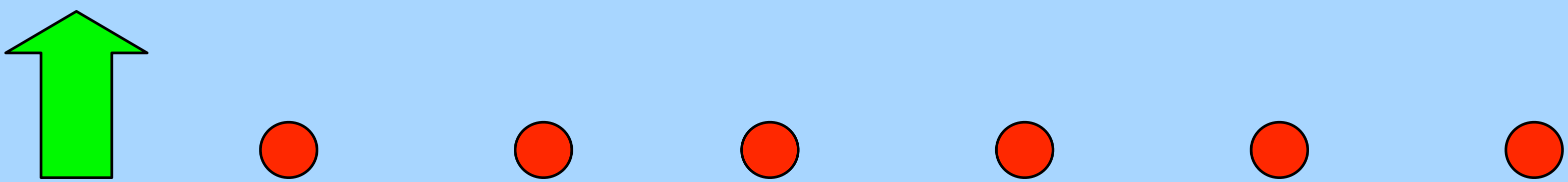
$$|k-k_F| < T_K/v_F \equiv 1/\xi_K$$

• this defines a characteristic *length scale* for the Kondo effect - typically around .1 to 1 micron

- $\lambda_{eff} \rightarrow \infty$ at low energies ($<<T_K$)

- strong coupling physics is easiest to understand in tight-binding model:

$$H = -t\sum_{j=0}^{\infty}(\psi_j^{+}\psi_{j+1} + \psi_{j+1}^{+}\psi_j) + J\vec{S}_{imp} \bullet \vec{S}_{el}(0)$$

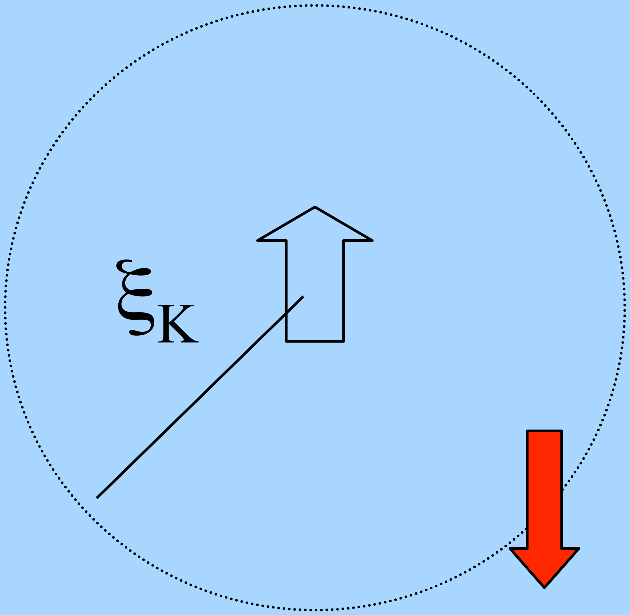


- for J>>t, we simply find ground state of last term:
- 1 electron at j=0 forms spin singlet with impurity: $|\phi_0\rangle = (|\Uparrow\downarrow\rangle - |\Downarrow\uparrow\rangle)/\sqrt{2}$

- other electrons are free except that they must not go to j=0 since they would break the singlet

- effectively an infinite repulsion at j=0, corresponding to $\pi/2$ phase shift

• for finite (small) λ_0, this description only holds at low energies and small $|k-k_F|$



• only long wavelength probes see simple $\pi/2$ phase shift- local Fermi liquid theory

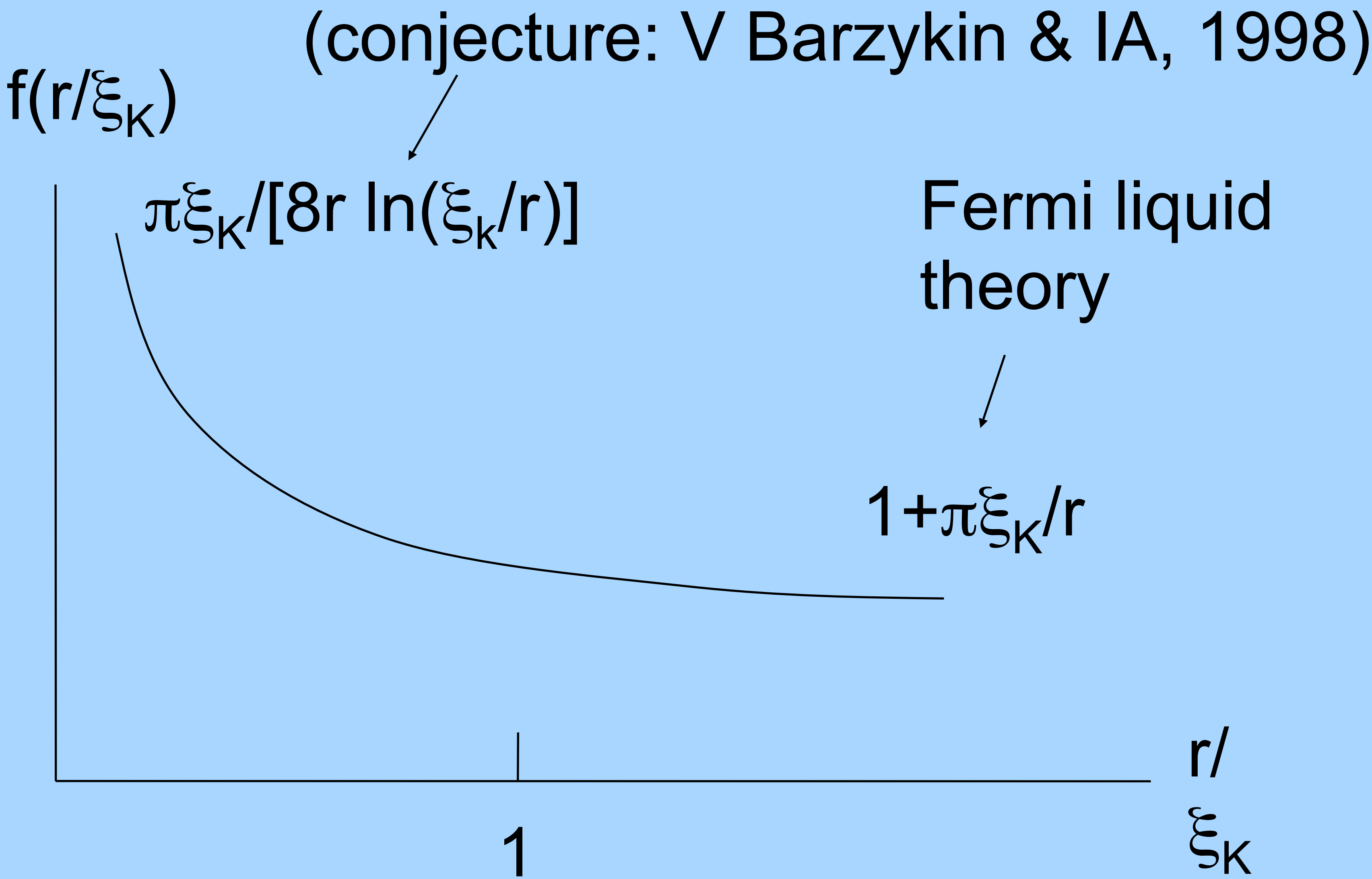
Non-Observation of Screening Cloud in Experiments

- the Knight shift as a function of distance from an impurity can be measured by NMR
- at T=0 this takes the form:

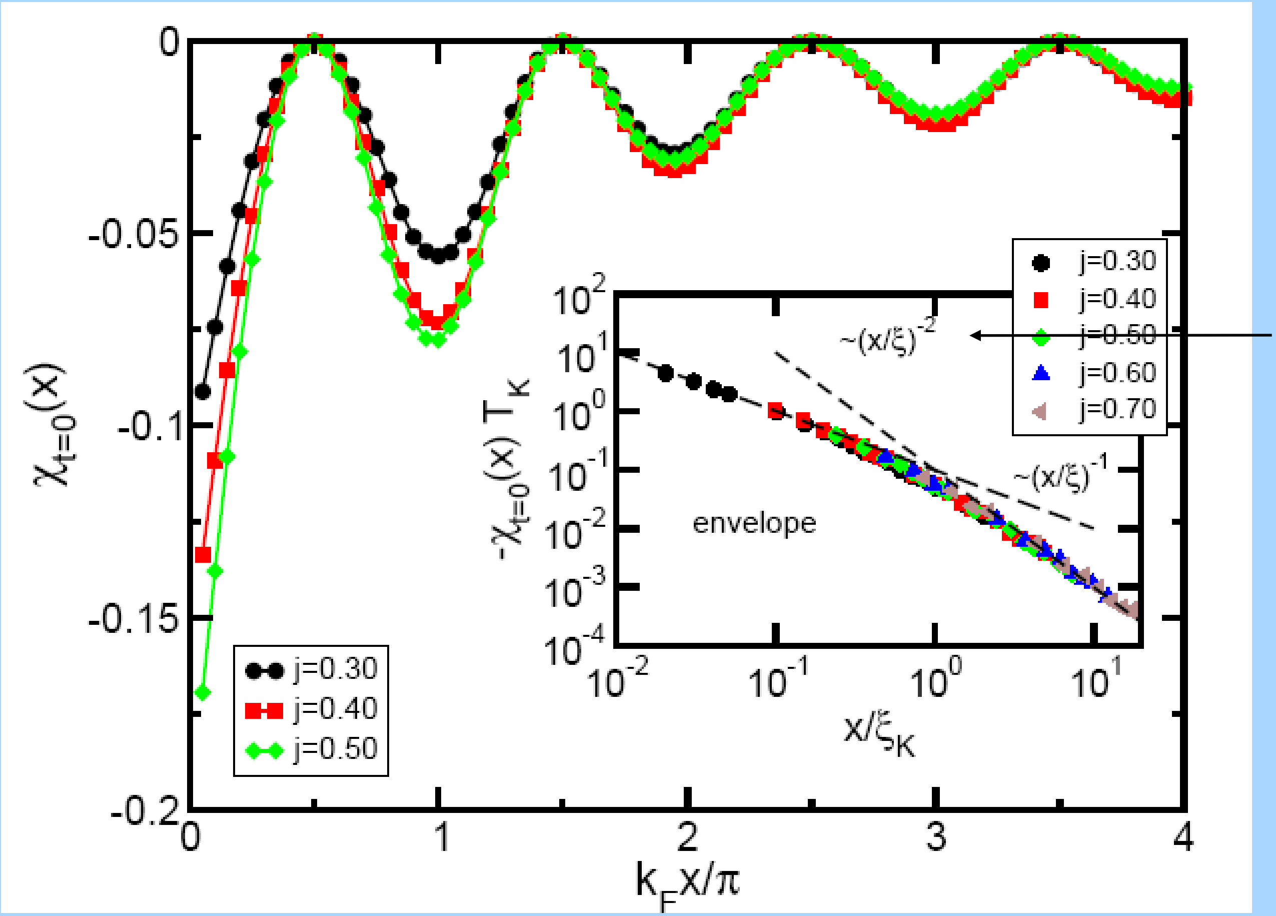
$$\chi(r) = \chi_0 + \frac{\cos(2k_F r)}{r^2} f\left(\frac{r}{\xi_K}\right)$$

• rapid oscillations and power-law pre-factor makes this difficult to observe (Boyce-Slichter, 1974)

Schematic sketch of scaling function



Numerical renormalization group confirmation: $\langle 0|\mathbf{S}_{imp}(t)\cdot\mathbf{S}_{el}(x,t)|0\rangle$



L. Borda 2006

Fermi Liquid theory

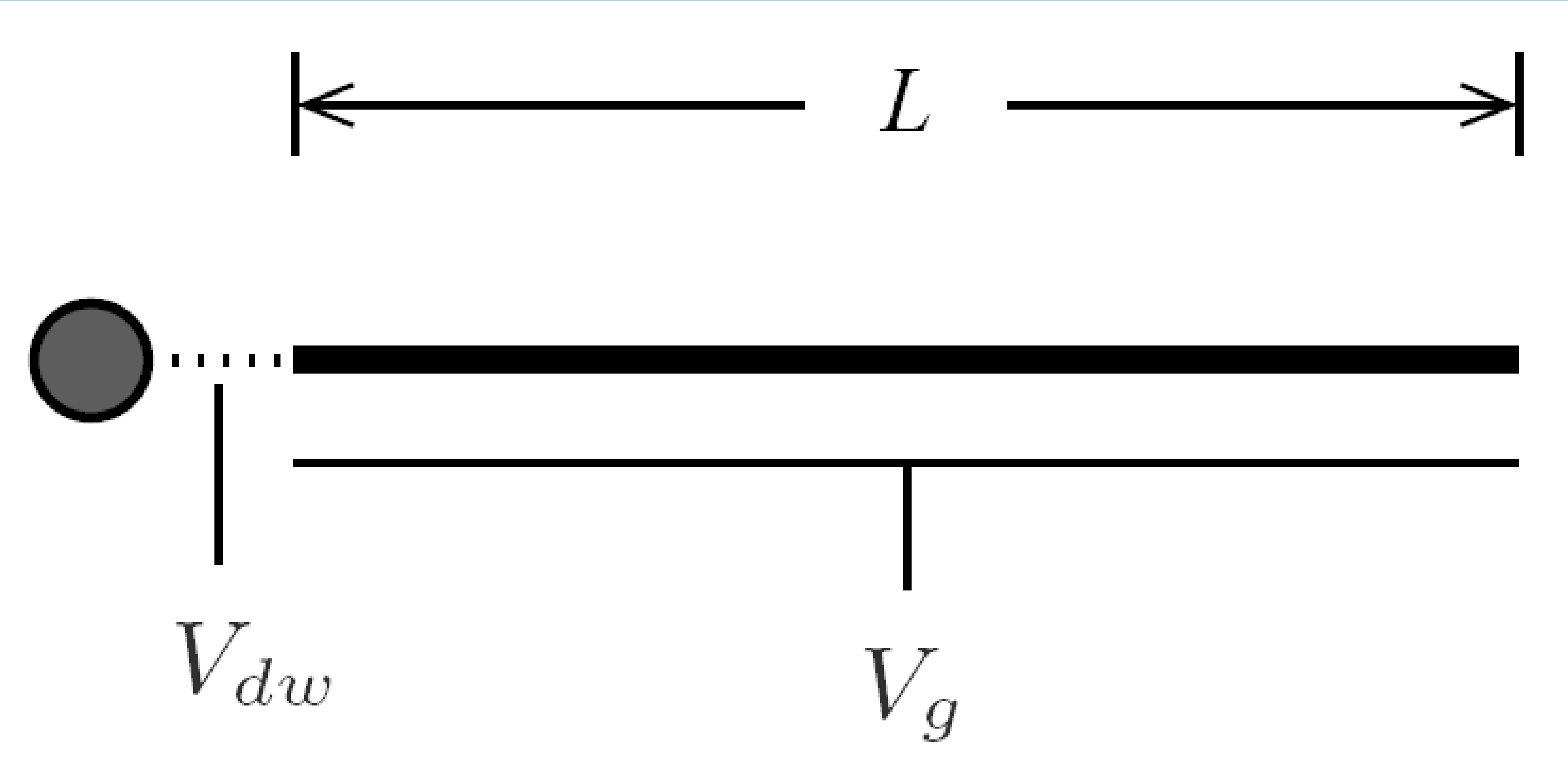
Possible Experiments on Mesoscopic Systems

- If conduction electrons are confined to a region of size $L < \xi_K$, Kondo effect is suppressed (at least in simplest models)

$$\lambda_{eff}(L) \approx \frac{\lambda_0}{1-\lambda_0 \ln(L/a)} + \bullet\bullet\bullet \approx \frac{1}{\ln(\xi_K / L)}$$

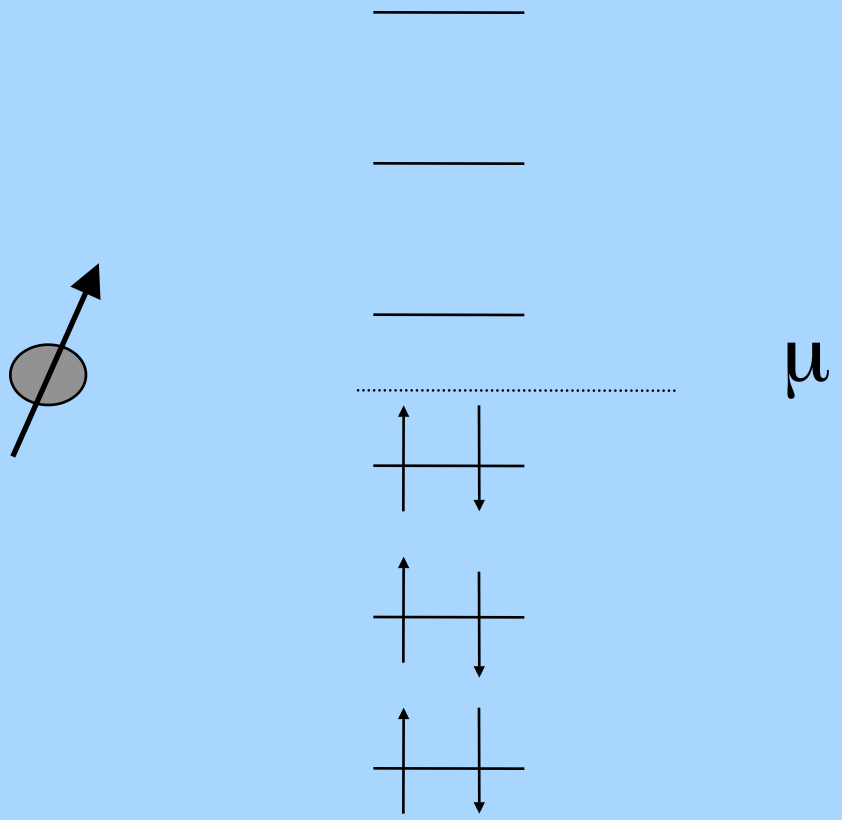
•finite system size cuts off infrared divergences of perturbation theory
•Screening cloud doesn’t “fit” into system

One possibility is to consider a quantum wire coupled to a quantum dot which acts as a Kondo impurity



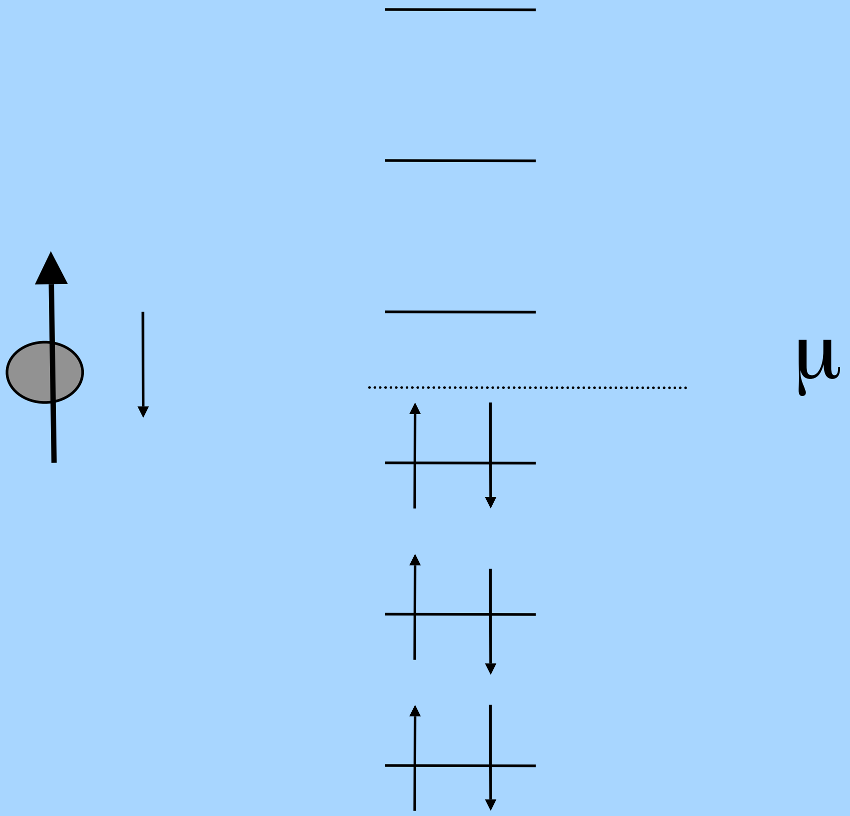
- A gate voltage, V_G is applied to the wire while maintaining the dot in the Kondo regime
- The wire also is very weakly coupled to an electron reservoir (not shown)
- As V_G (or effectively, μ, the chemical potential) is varied at T=0, electrons enter or leave the wire in single steps
- Ratios of plateau widths with odd or even electron number depends on L/ξ_K
- Could be measured from transport experiment

- ignore, for now, any other interaction effects apart from Kondo interaction (see below)
- at $L \ll \xi_K$, λ_{eff} is small and we get nearly free electron behaviour
- for nearly all values of μ, N is odd
- i.e. even number on wire, odd number on dot since single electron levels of wire are doubly occupied (or empty)

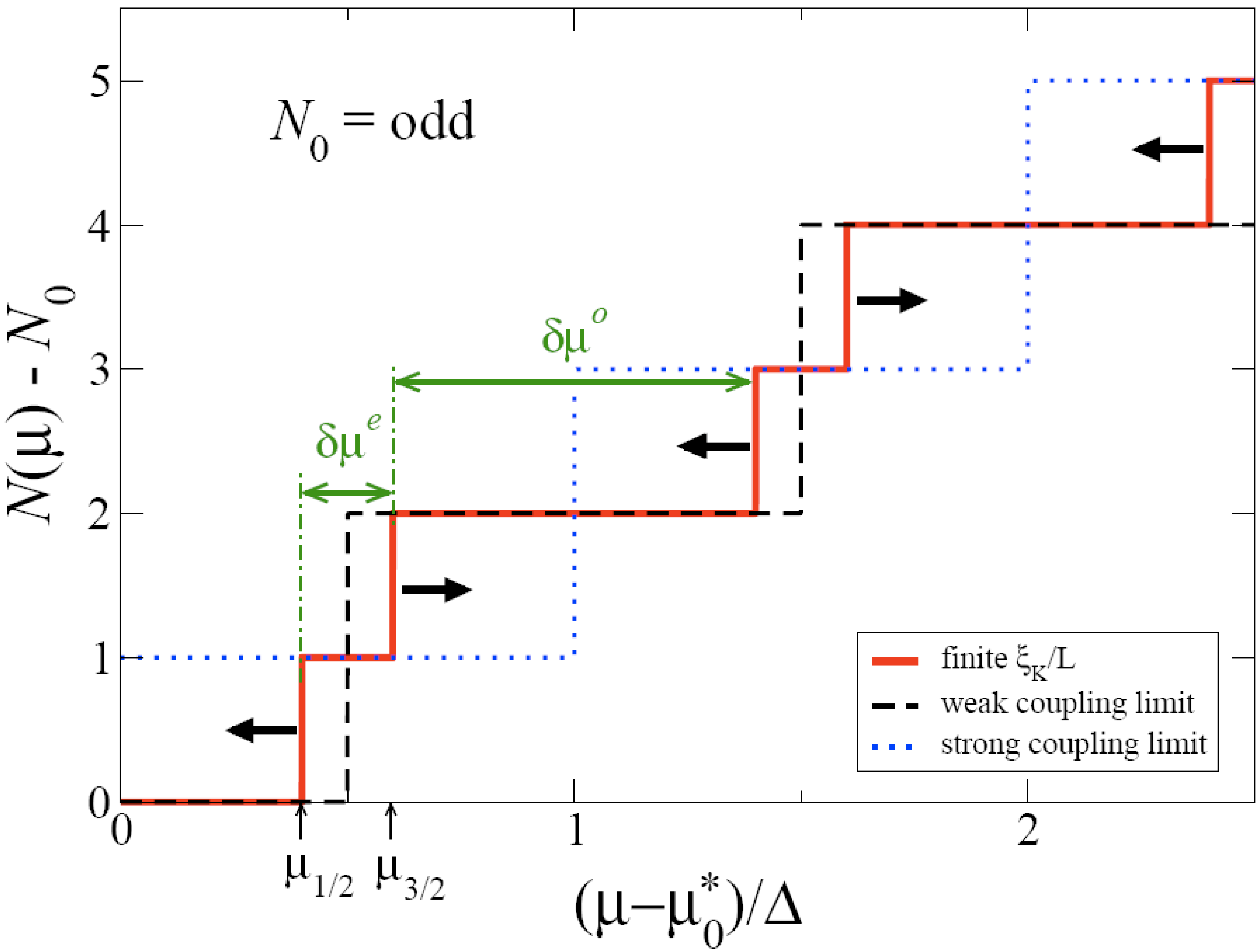


N odd

- As we increase L/ξ_K, i.e. increase λ_{eff}, even N plateaus get wider
- For L>> ξ_K, we enter strong coupling regime
- now N is even for almost all μ because one electron from wire forms singlet with the quantum dot spin while the rest doubly occupy single electron levels



N even



N_0 = odd
$N(\mu) - N_0$
$(\mu-\mu_0^*)/\Delta$
$\delta\mu^o$
$\delta\mu^e$
$\mu_{1/2}$
$\mu_{3/2}$
0
1
2
3
4
5
0
1
2
finite ξ_K/L
weak coupling limit
strong coupling limit

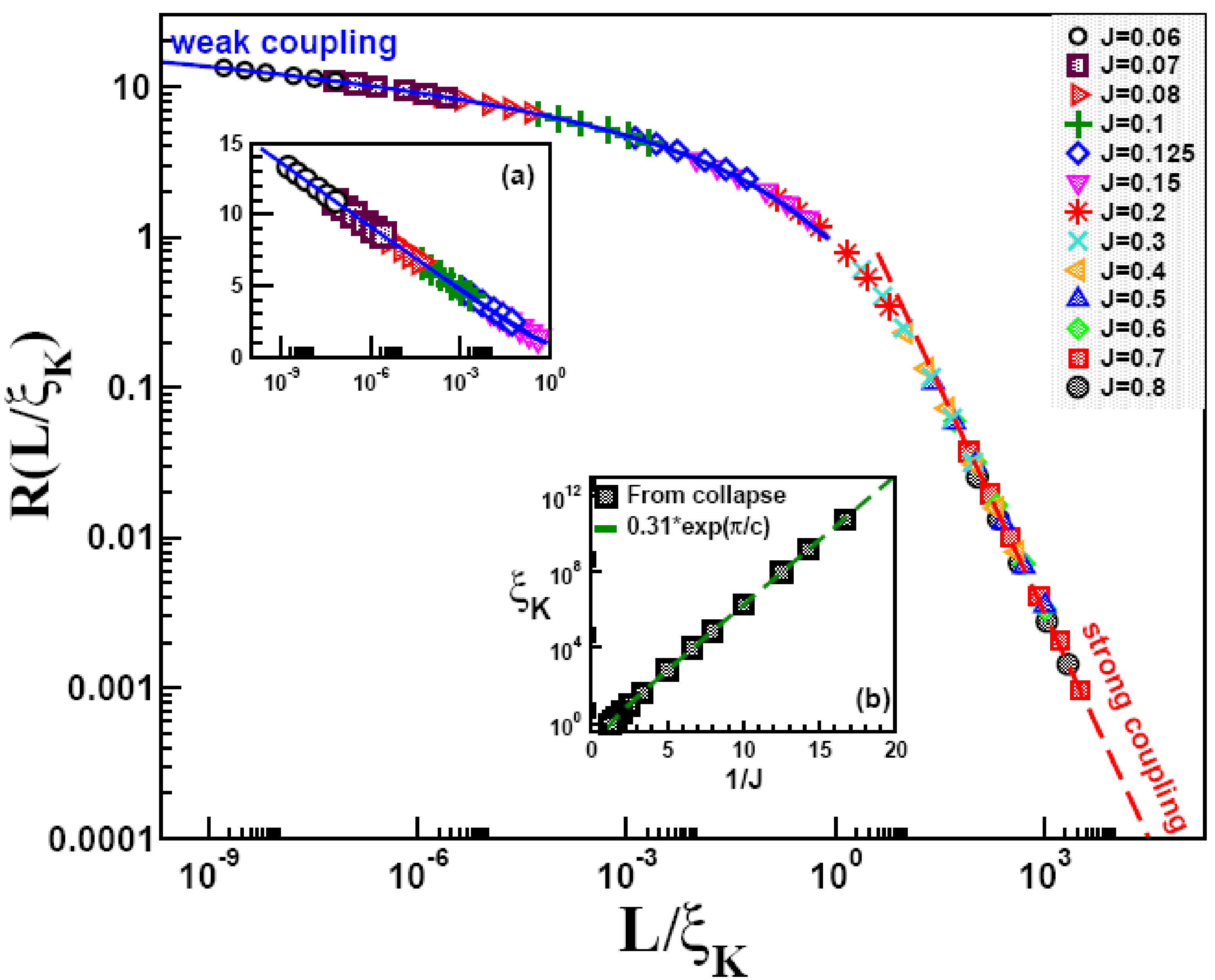
- We consider $R=\delta\mu^0/\delta\mu^e$
- At weak coupling, $\xi_K >> L$ from perturbation theory:

$R \cong 1/[(3/2)\lambda_{eff}] \cong (2/3) \ln (\xi_K/L) >> 1$

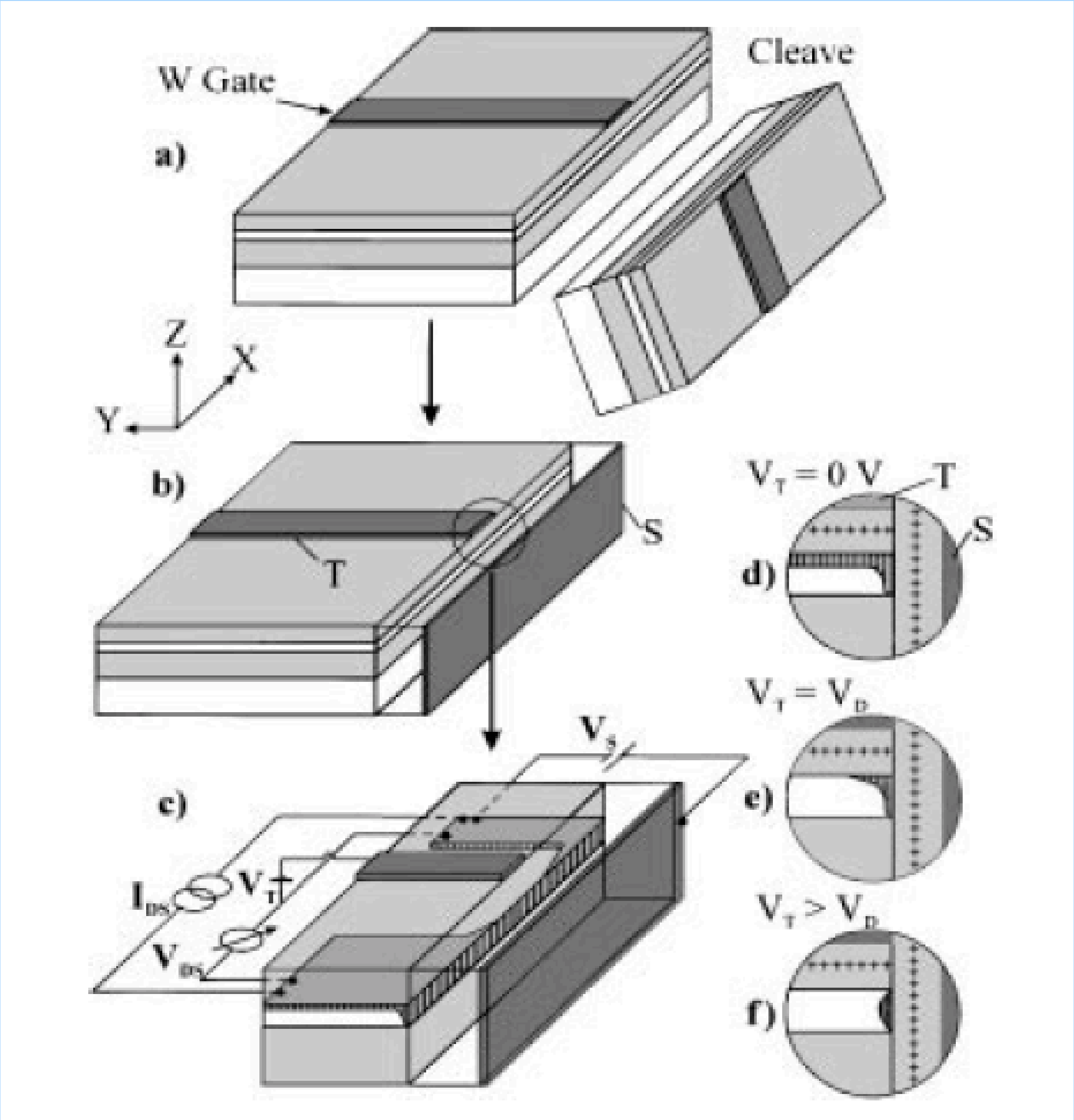
- At strong coupling, $\xi_K << L$ we can use local Fermi liquid theory:

$R \cong (\pi\xi_K)/(4L) << 1$

- In between, R is given by a universal scaling function of ξ_K/L
- we calculated this using Bethe ansatz solution of Kondo problem at finite L

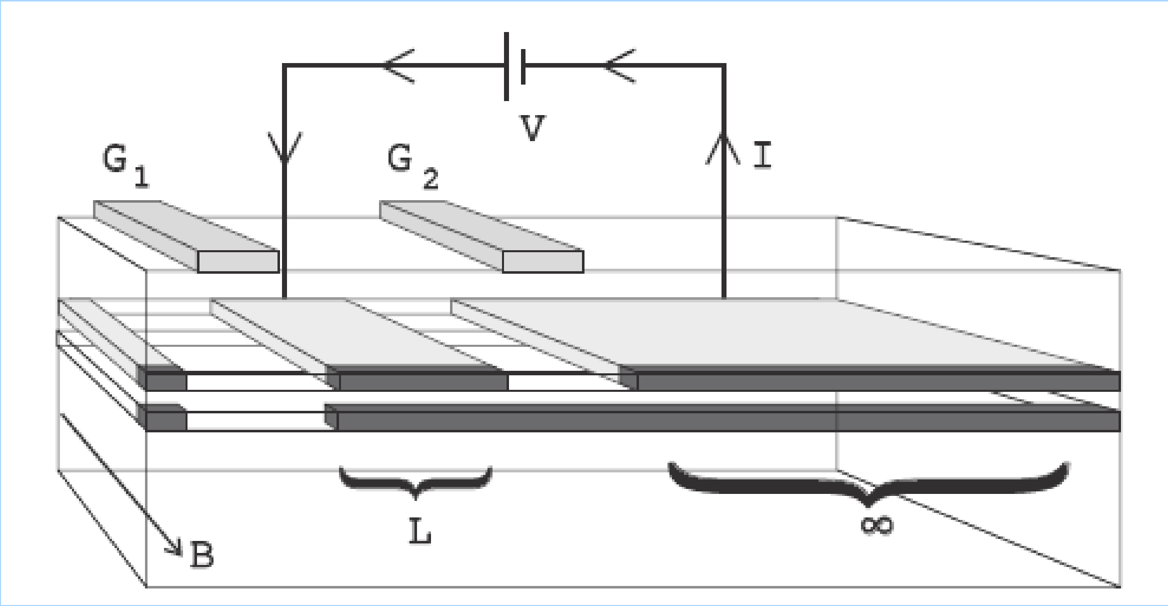

weak coupling
strong coupling
J=0.06
J=0.07
J=0.08
J=0.1
J=0.125
J=0.15
J=0.2
J=0.3
J=0.4
J=0.5
J=0.6
J=0.7
J=0.8
(a)
(b)
From collapse
0.31*exp(π/c)
ξ_K
1/J
$R(L/\xi_K)$
L/ξ_K

Cleaved overgrowth quantum wires
Yacoby et al.



l~10 μ
L~ μ's

3 or fewer channels



- Coulomb interaction screened (to $1/r^3$) by second nearby wire, a distance d away
- Luttinger model seems to work
- $v_c \sim 1.5 v_F$, $v_s \sim v_F$, $K \sim .7$

Including Coulomb Interactions

With Kondo impurity coupled at end of wire,
spin-charge separation applies

$$E(N+n) = \frac{\pi v_c}{4KL} n^2 + \frac{\pi v_s}{4L} f_s(\lambda, g, L) - \mu n$$

- $V_{c/s}$ are charge/spin velocities
- K is Luttinger parameter
- f_s is dimensionless spin energy:

s=1/2 or 0 for n even or odd

- g is marginal coupling constant in spin sector, deriving from Coulomb interactions

•charge step ratio depends only on $f(\lambda,g,L)=f_{1/2}-f_0$ and $u=(v_s K/v_c)\sim.5$

$$R=\frac{1+uf(\lambda,g,L)}{1-uf(\lambda,g,L)}$$

•Coulomb interactions lead to u<1- suppressing somewhat even-odd effect in charge steps

•Spin-charge separation: $H=H_c+H_s$

•Only important interaction is in spin sector $H_s=H_0+H_{int}$

$$H_{\text{int}} = 2\pi\, v_s \left[-g \int_0^L dx\, \vec{J}_L \times \vec{J}_R + \lambda\, \vec{J}_L(0) \times \vec{S} \right]$$

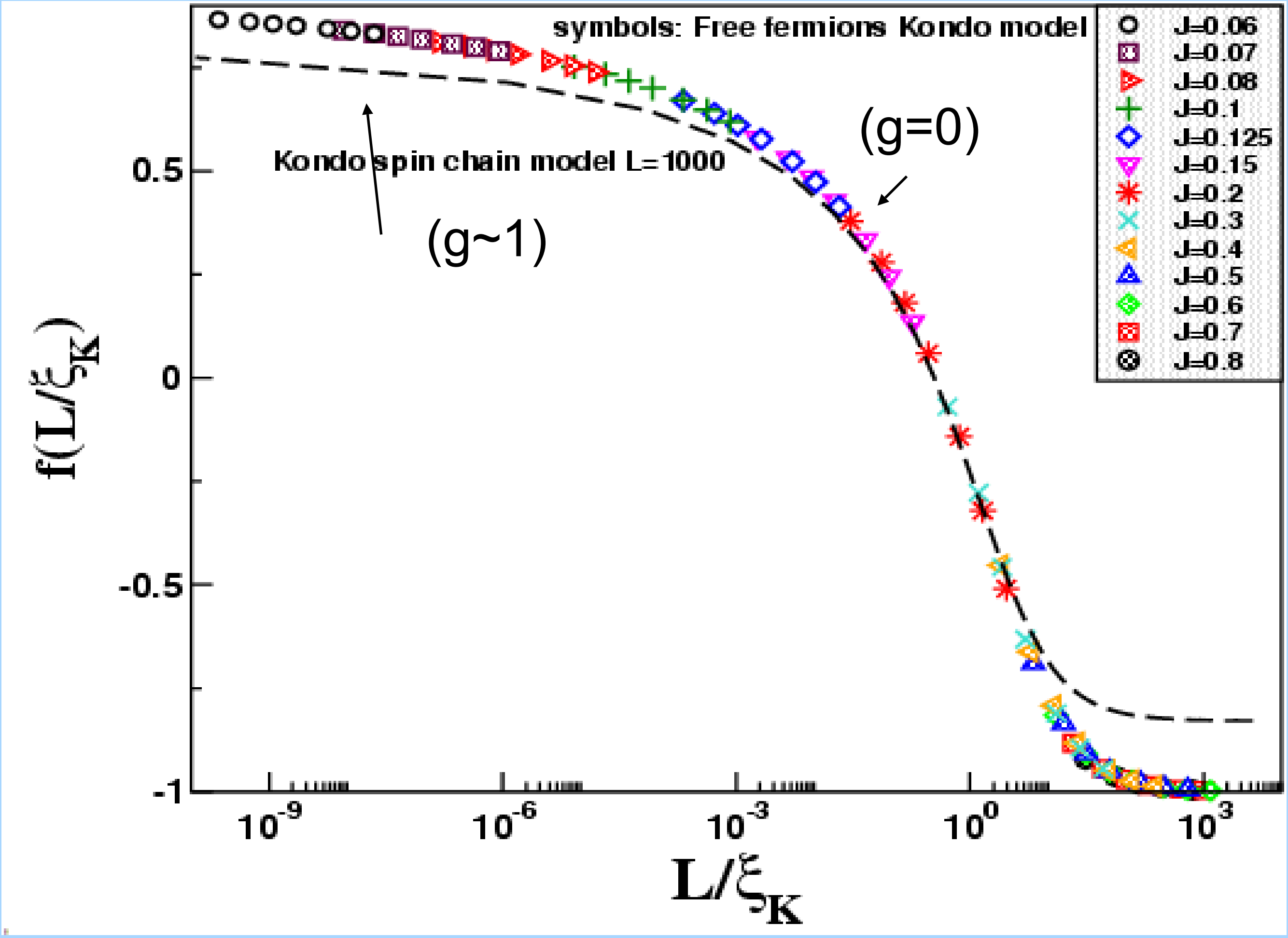
•Coulomb interaction, g, is marginally *irrelevant*:
$g(L) \sim 1/\ln(L/a)$, $L >> a$, where $a \sim 1/k_F$

•Kondo interaction, λ, is marginally *relevant*:
$\lambda(L) \sim 1/\ln(\xi_K/L)$, $L << \xi_K$

•Coulomb interactions modifies $f(\xi_K/L)$
but effect gets weaker and weaker for larger L

•non-interacting case discussed previously is:

$$R_0 = \frac{1 + f(0, \lambda, L)}{1 - f(0, \lambda, L)}$$



- some further reduction of even/odd effect from marginal interaction
- goes away as L increases

Conclusions

- Physical quantities change at large length scale $\xi_K = v_F/T_K \leq 1\mu$, in ideal model
- This effect not yet seen experimentally
- Mesoscopic systems with $L \sim \xi_K$ provide new possibilities
- Interactions modify situation somewhat

More Realistic Models

-more channels may couple to impurity
-system may be disordered (eg. 1 magnetic impurity and many non-magnetic ones)
-interactions among conduction electrons may be important
-interactions between magnetic impurities (eg. RKKY) may be important

Including more channels:

Thimm, Kroha & Van Delft (1999) considered a 3-dimensional disordered “box” (volume V) containing a single impurity spin

They assumed:

- All single electron states in box couple with equal strength to spin
- Energy levels are equally spaced $\Delta \propto 1/V$

-Now suppression of Kondo effect occurs when $T_K \sim \Delta \rightarrow V \sim \xi_K a^2$ $(a \sim 1/k_F)$

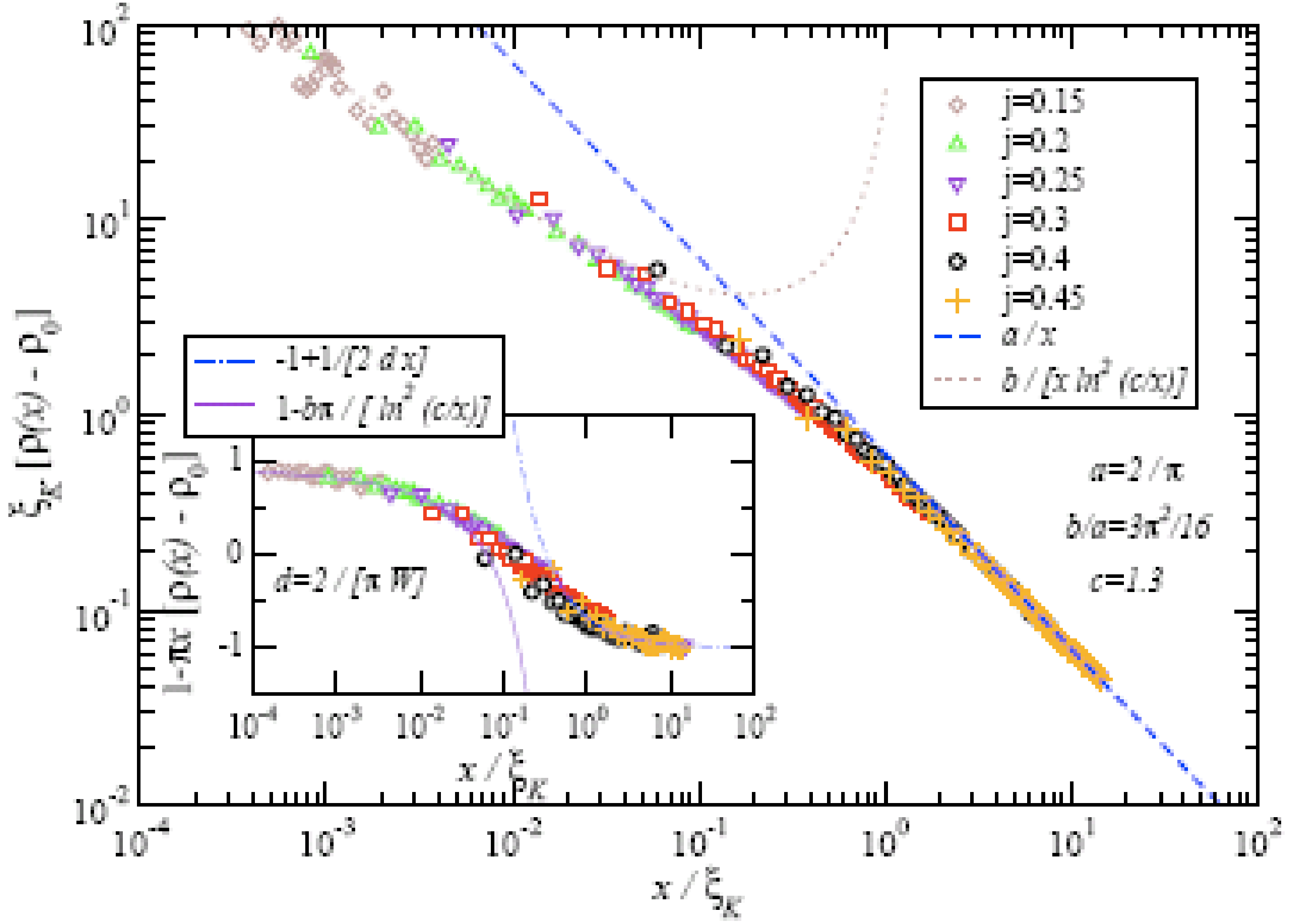
-a much smaller system would be needed to see this effect

- In quantum wires with N channels, typically require $L \cong \xi_K/N$ to see screening cloud effects (P. Simon & IA, 2003)
- This can make it much more difficult to see these effects

Friedel (charge) oscillations around a Kondo impurity

(with Laszlo Borda and Hubert Saleur)

$$\rho(r) - \rho_0 \to \frac{C_D}{r^D}[\sin(2k_F r + 2\delta_P)F(r/\xi_K) - \sin(2k_F r)]$$



j=0.15
j=0.2
j=0.25
j=0.3
j=0.4
j=0.45
a / x
b / [x ln² (c/x)]
-1+1/[2 d x]
1-bπ / [ln² (c/x)]
a=2 / π
b/a=3π²/16
c=1.3
d=2 / [π W]
ξK [ρ(x) - ρ0]
1-πx [ρ(x) - ρ0]
x / ξK
x / ξK