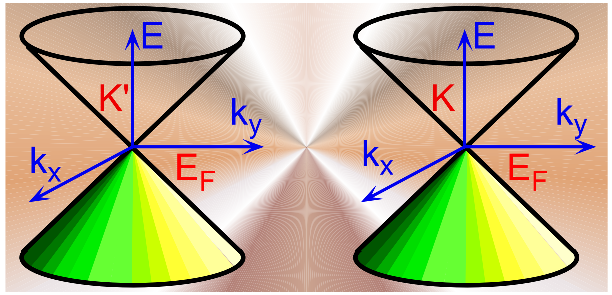
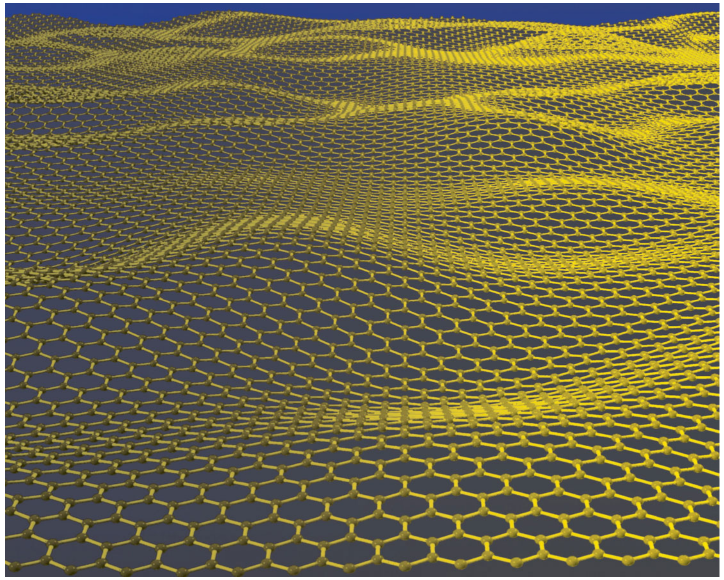
Exotic Transport Properties of Graphene and Nanotube

Tsuneya ANDO

1. **Weyl's equation for neutrino**
2. **Rise of graphene**
   - Fabrication and quantum Hall effect
   - Landau-level spectroscopy
   - Local density of states
   - Optical phonon, Spin transport, $\cdots$
3. **Transport in graphene and nanotube**
   - Berry's phase & topological anomaly
   - Zero mode anomalies
   - Special time reversal symmetry
4. **Multi-layer graphene**
   - Hamiltonian decomposition
5. **Summary**



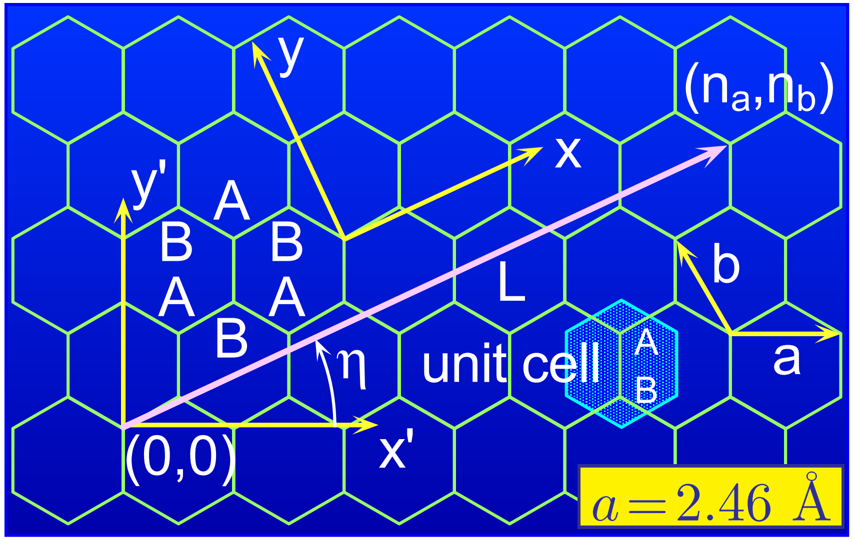


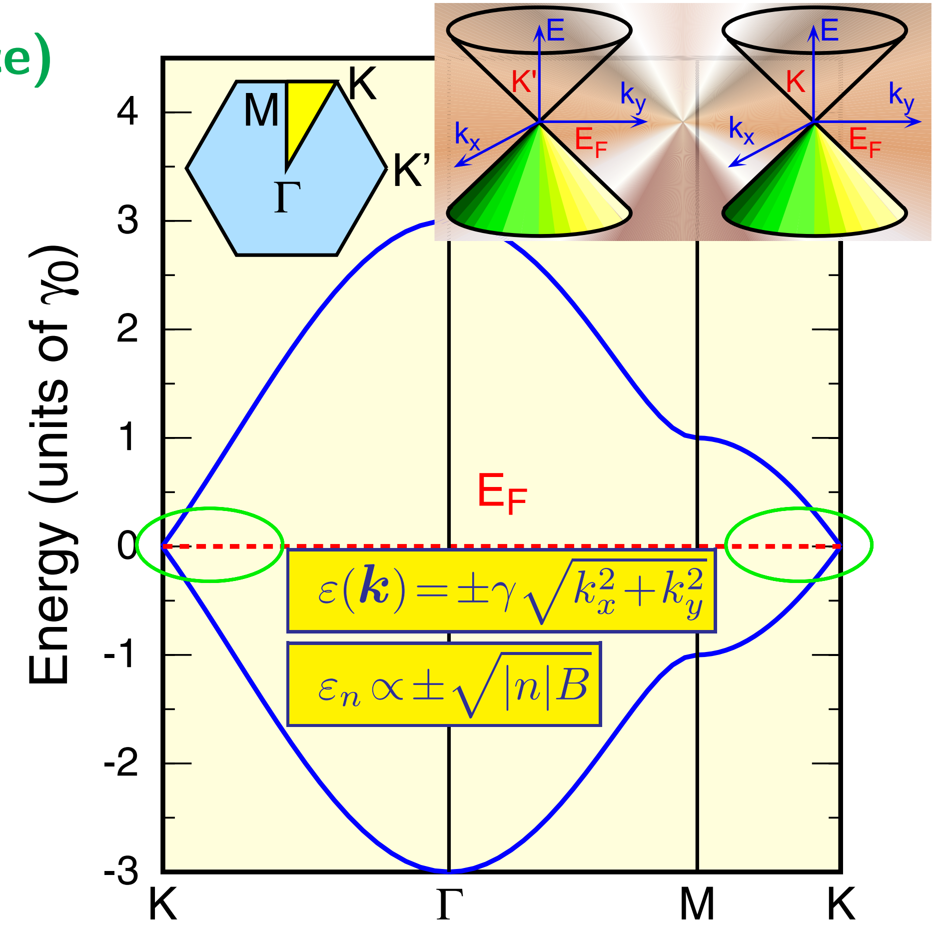
Clock Tower Centennial Hall
Kyoto Univ, Nov 23 (Fri) 2007

Yukawa International Seminar 2007 on Interaction and Nanostructural Effects in Low-Dimensional Systems
November 5–30, 2006 [15:15–16:00 (35+10)]

Effective-Mass Description: Neutrino or Massless Dirac Electron

Graphene (Triangular antidot lattice)





Wave Vector

Weyl's equation for neutrino

$$\Leftrightarrow \gamma(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}})\boldsymbol{F}(\boldsymbol{r}) = \varepsilon\boldsymbol{F}(\boldsymbol{r})$$

$$\Leftrightarrow \gamma(\sigma_x\hat{k}_x + \sigma_y\hat{k}_y)\boldsymbol{F}(\boldsymbol{r}) = \varepsilon\boldsymbol{F}(\boldsymbol{r})$$

$$\begin{pmatrix} 0 & \gamma(\hat{k}_x - i\hat{k}_y) \\ \gamma(\hat{k}_x + i\hat{k}_y) & 0 \end{pmatrix}\begin{pmatrix} F^A(\boldsymbol{r}) \\ F^B(\boldsymbol{r}) \end{pmatrix} = \varepsilon\begin{pmatrix} F^A(\boldsymbol{r}) \\ F^B(\boldsymbol{r}) \end{pmatrix}$$

$\hat{\boldsymbol{k}} = -i\vec{\nabla}$

Velocity: $v_F = \gamma/\hbar$

K': $\boldsymbol{\sigma} \rightarrow \boldsymbol{\sigma}^*$

Massless (Dirac)

$v_F \sim c/300\ (\gamma_0 \sim 3\,\text{eV})$

Constant velocity (~light, cannot stop)

Topological anomaly

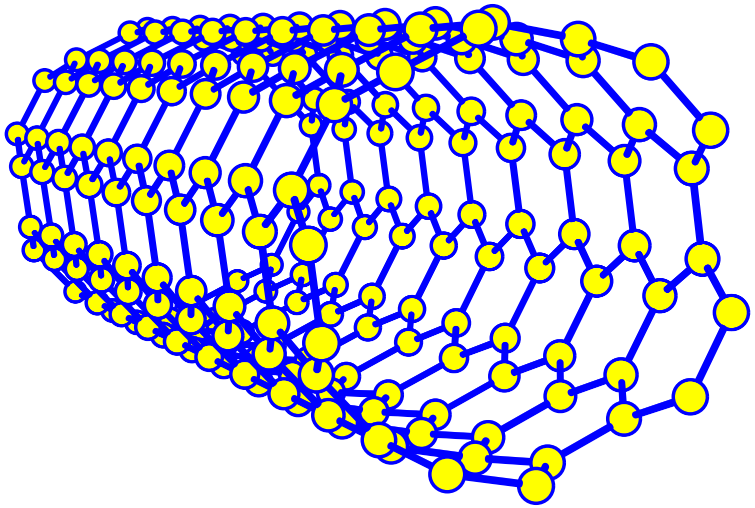
$\gamma = \sqrt{3}\gamma_0 a/2\ \ (\gamma_0:$ **Hopping integral**$)$

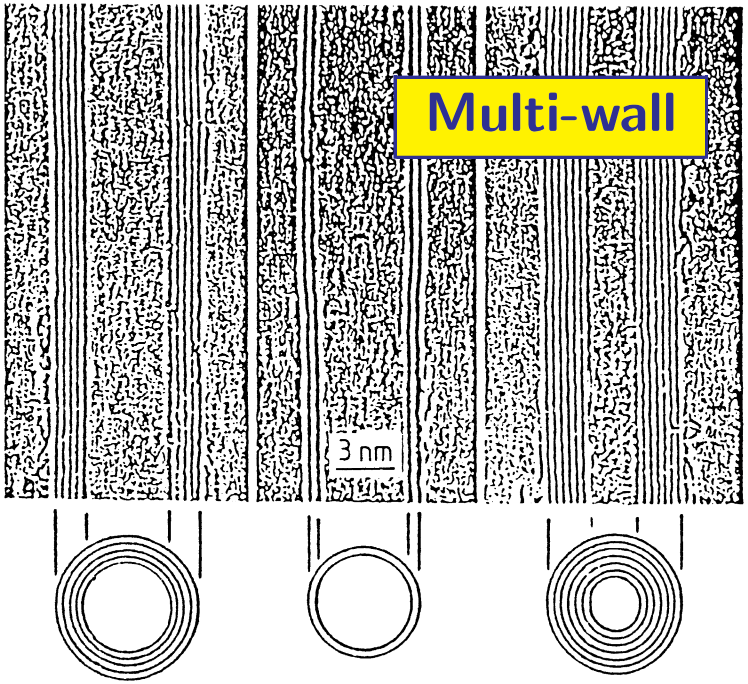
Carbon Nanotubes

Discovery: *S. Iijima, Nature (London) 354, 56 (1991)* **TEM image**

Concentric cylinders 2D graphite

Length: $> 1\ \mu\text{m}$





Diameter:	**67 Å**	**55 Å**	**65 Å**
Walls:	**5**	**2**	**7**

Single-wall nanotubes

S. Iijima and T. Ichihashi, Nature (London) 363, 603 (1993)
D.S. Bethune et al., Nature (London) 363, 605 (1993)

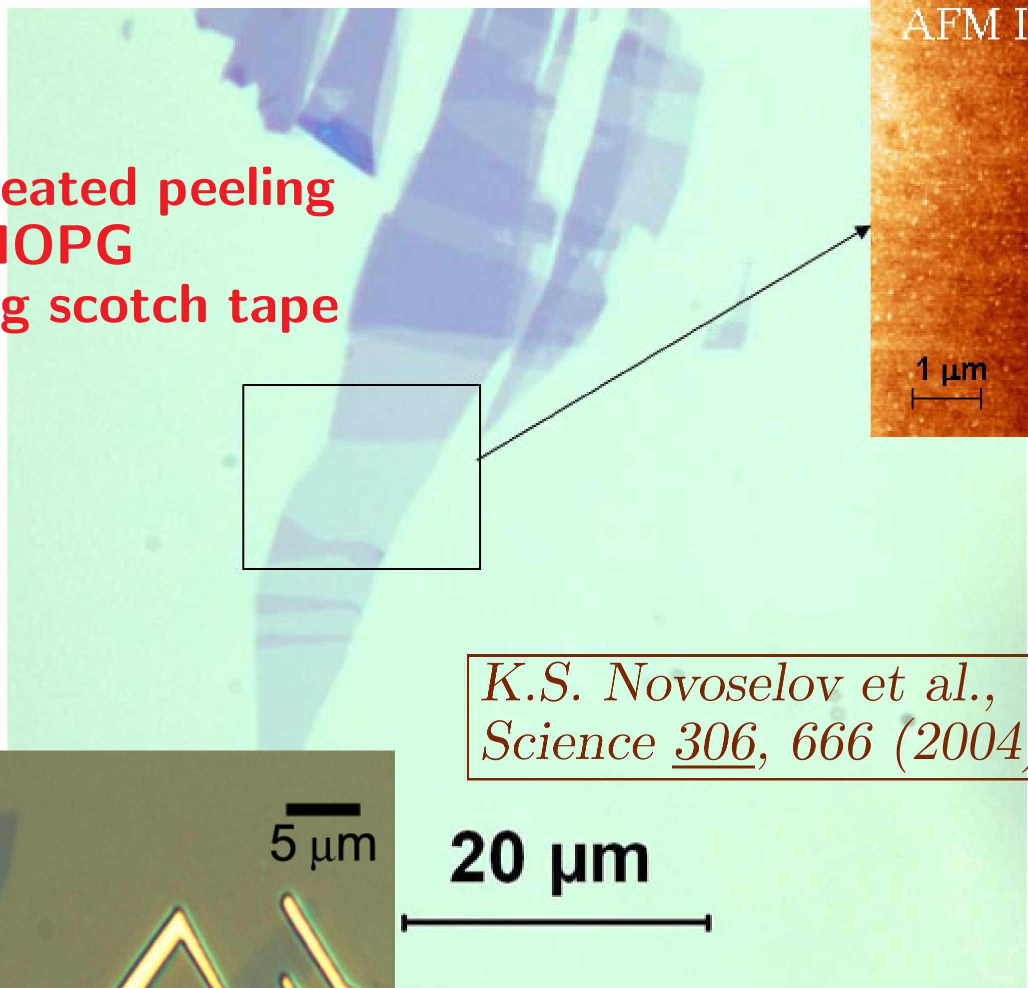
Tight-binding model

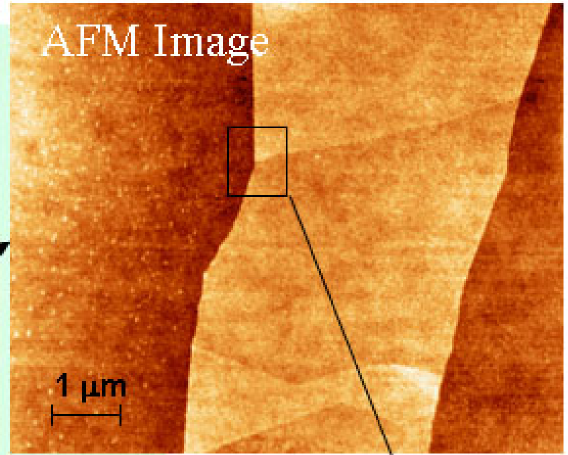
N. Hamada et al, PRL 68, 1579 (1992); R. Saito et al, PRB 46, 1804 (1992); J.W. Mintmire et al, PRL 68, 631 (1992); ...

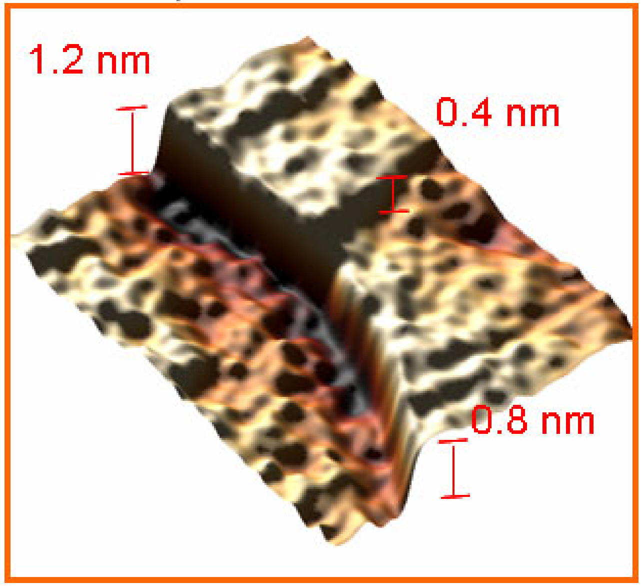
k·p scheme [*H. Ajiki and T. Ando, JPSJ 62, 1255 (1993)*]

Fabrication of Monolayer Graphene on SiO_2/Si Substrate

Repeated peeling of HOPG using scotch tape

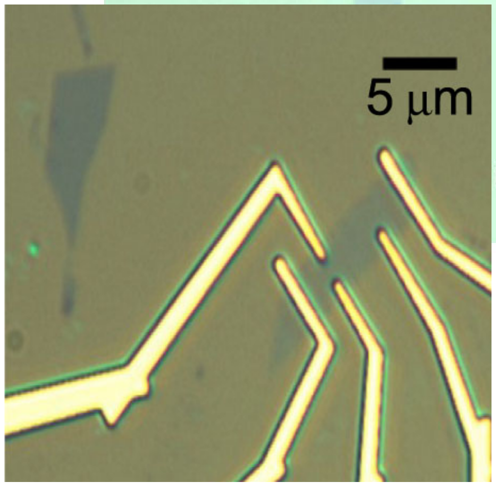






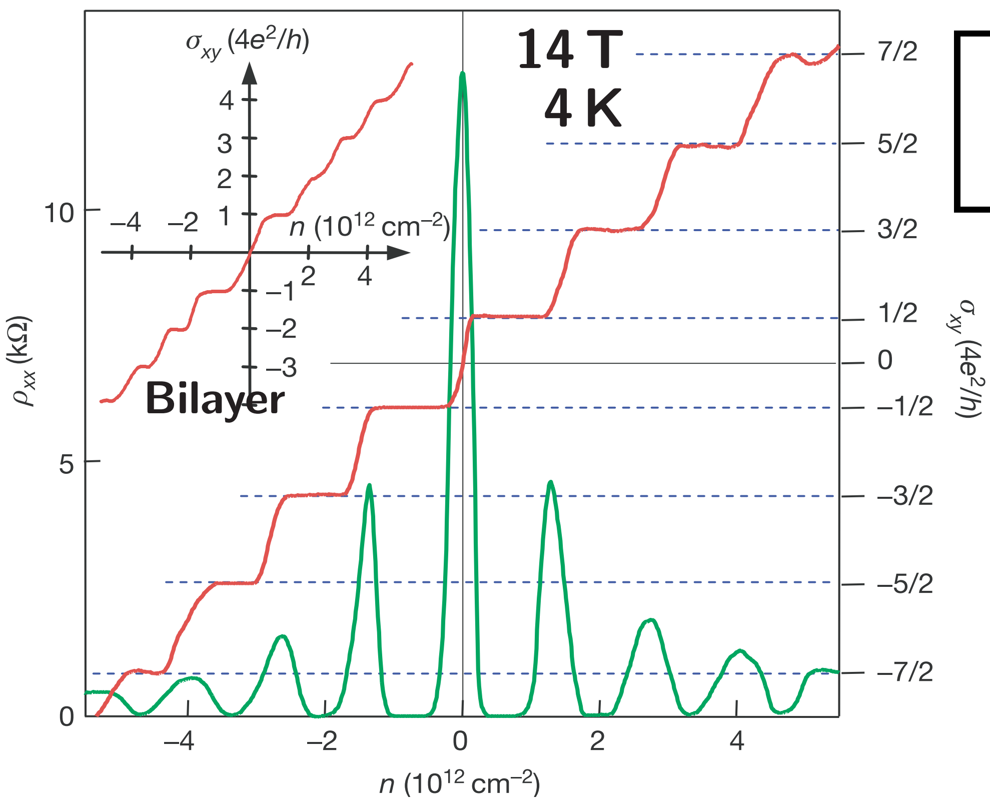
K.S. Novoselov et al., Science 306, 666 (2004)

Optical microscope image



FET: Field Effect Transistor $(n_s < 5 \times 10^{13}\,\mathrm{cm}^{-2})$

[MOSFET $(10^{13}\,\mathrm{cm}^{-2})$ **HEMT** $(10^{12}\,\mathrm{cm}^{-2})$**]**



Quantum Hall Effect in Monolayer Graphene

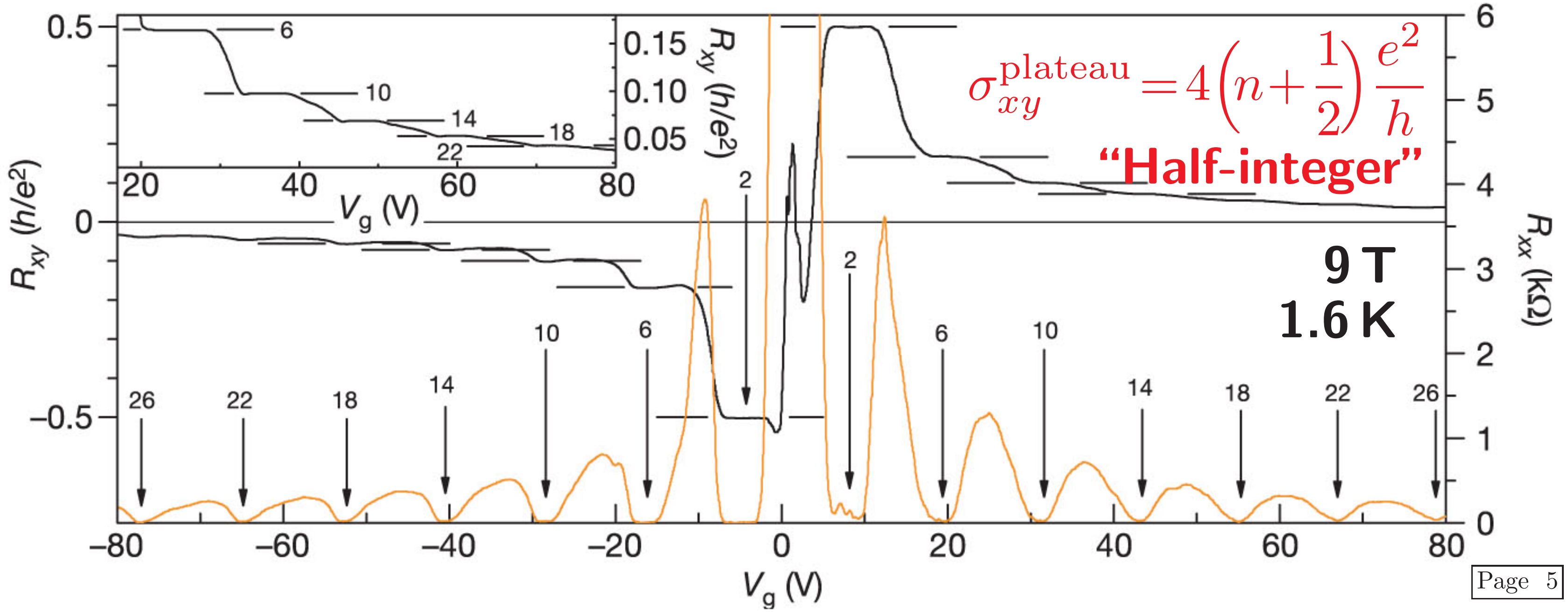
Theory

Y. Zheng and T. Ando, PRB <u>*65*</u>*, 245420 (2002)*

Experiments

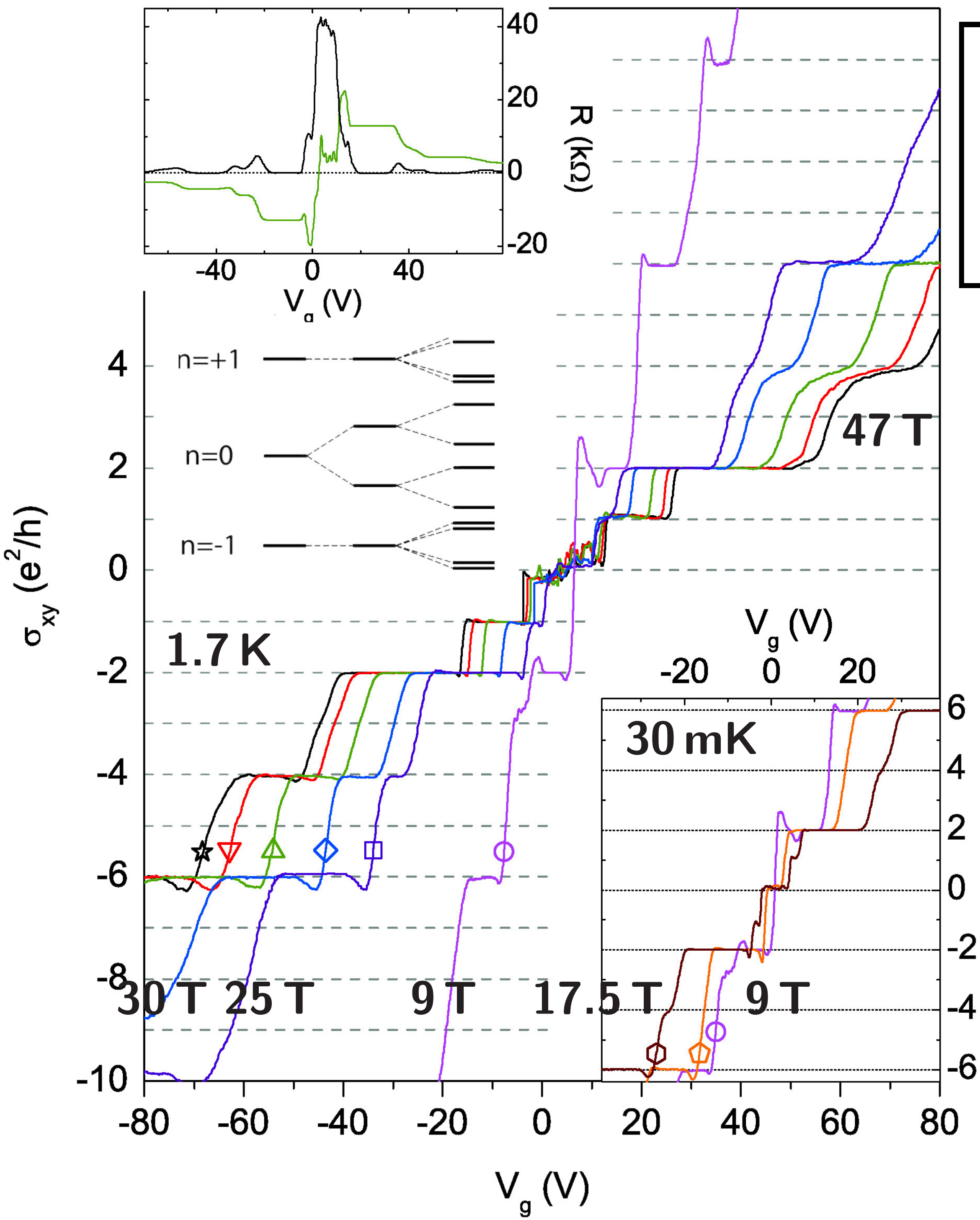
⇐ *K.S. Novoselov et al., Nature* <u>*438*</u>*, 197 (2005)*

⇓ *Y. Zhang et al., Nature* <u>*438*</u>*, 201 (2005)*



$$\sigma_{xy}^{\text{plateau}} = 4\left(n+\frac{1}{2}\right)\frac{e^2}{h}$$

"Half-integer"



Splitting of Landau Levels and Tilted Field
[*Y. Zhang et al., PRL 96, 136806 (2006)*]

Quantum Hall ferromagnet

K. Nomura & A.H. MacDonald, PRL 96, 256602 (2006)

Lattice instability

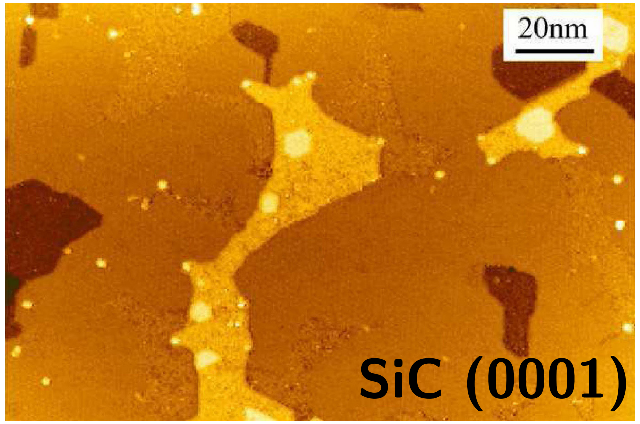
H. Ajiki & T. Ando, JPSJ 65, 2976 (1996)

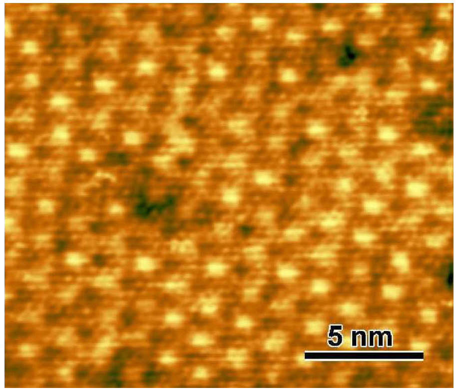
K-K' splitting

M. Koshino & T. Ando, PRB 75, 033412 (2007)

Tilted magnetic field

No g factor enhancement

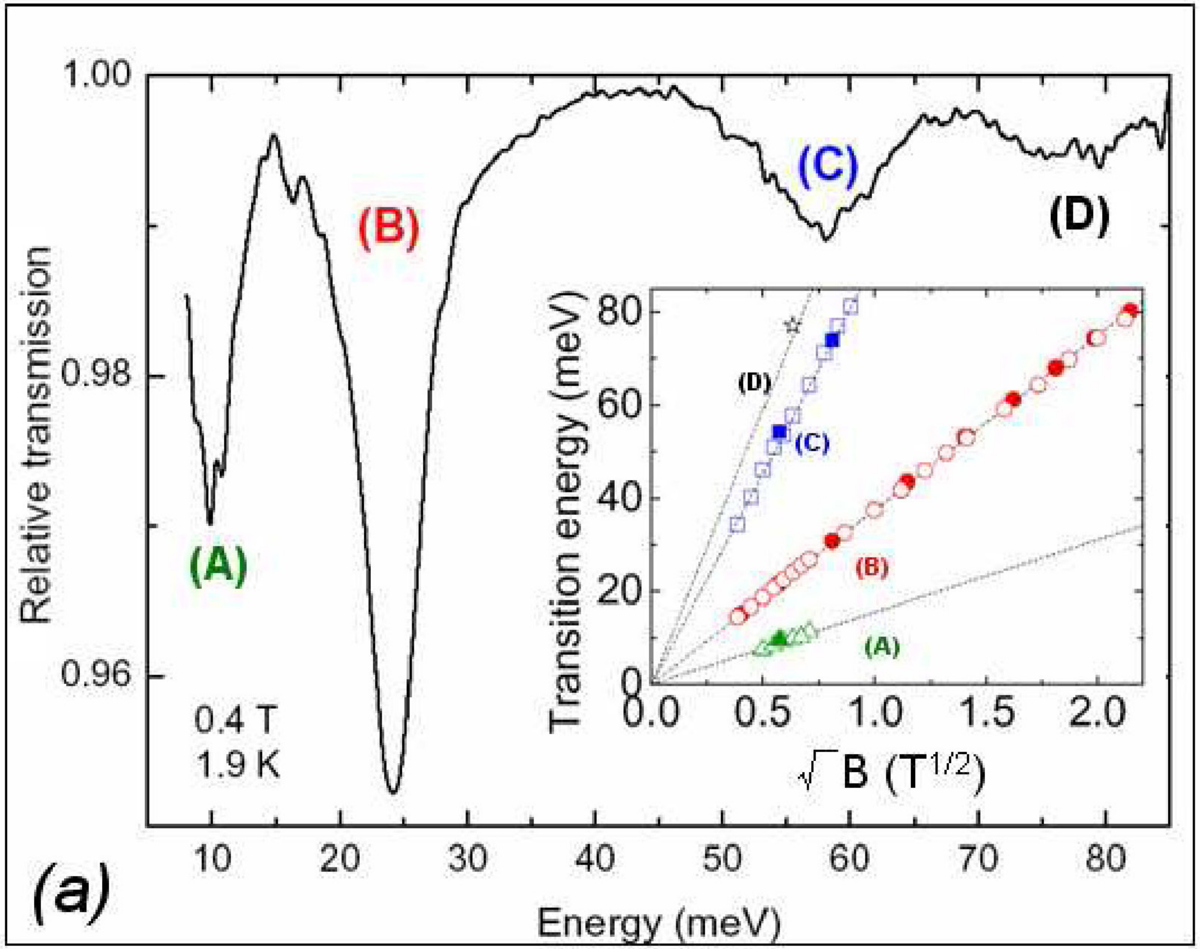


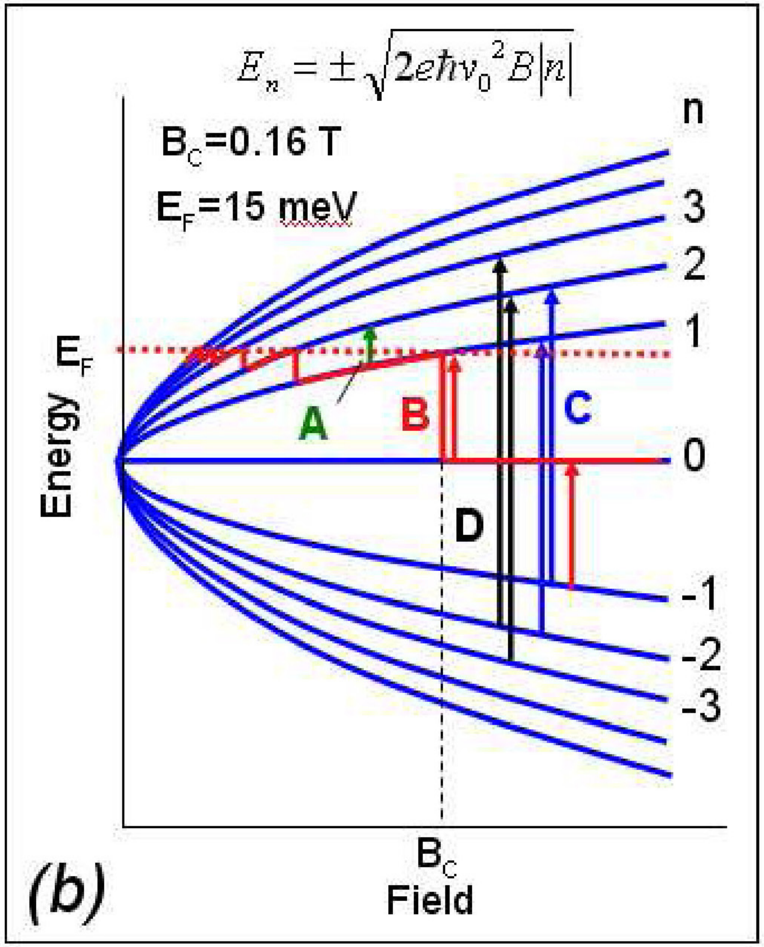


Landau-Level Spectroscopy of Epitaxial Graphene

[M.L. Sadowski et al., PRL 97, 266405 (2006)]

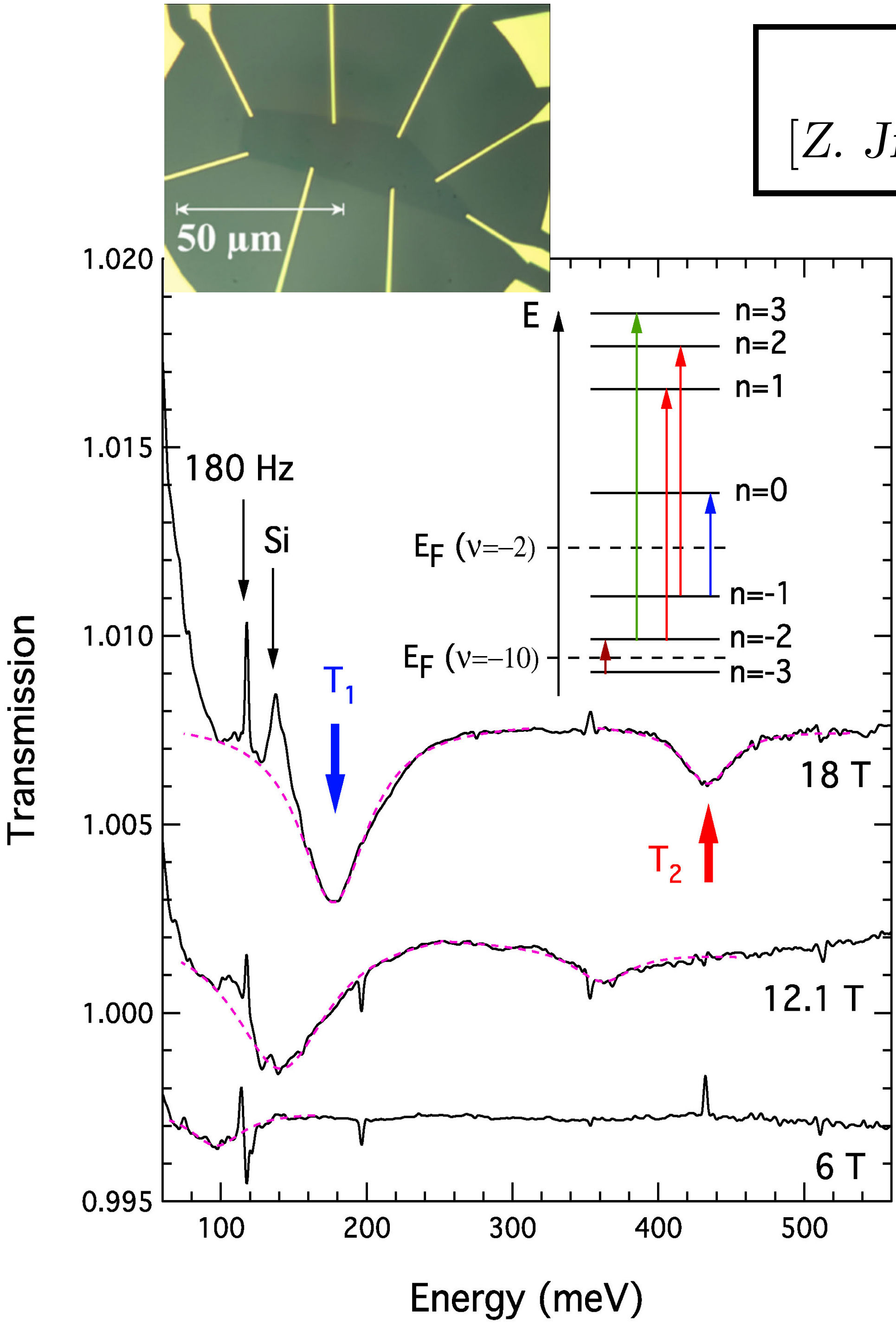
C. Berger et al., JPCB 108, 19912 (2004)

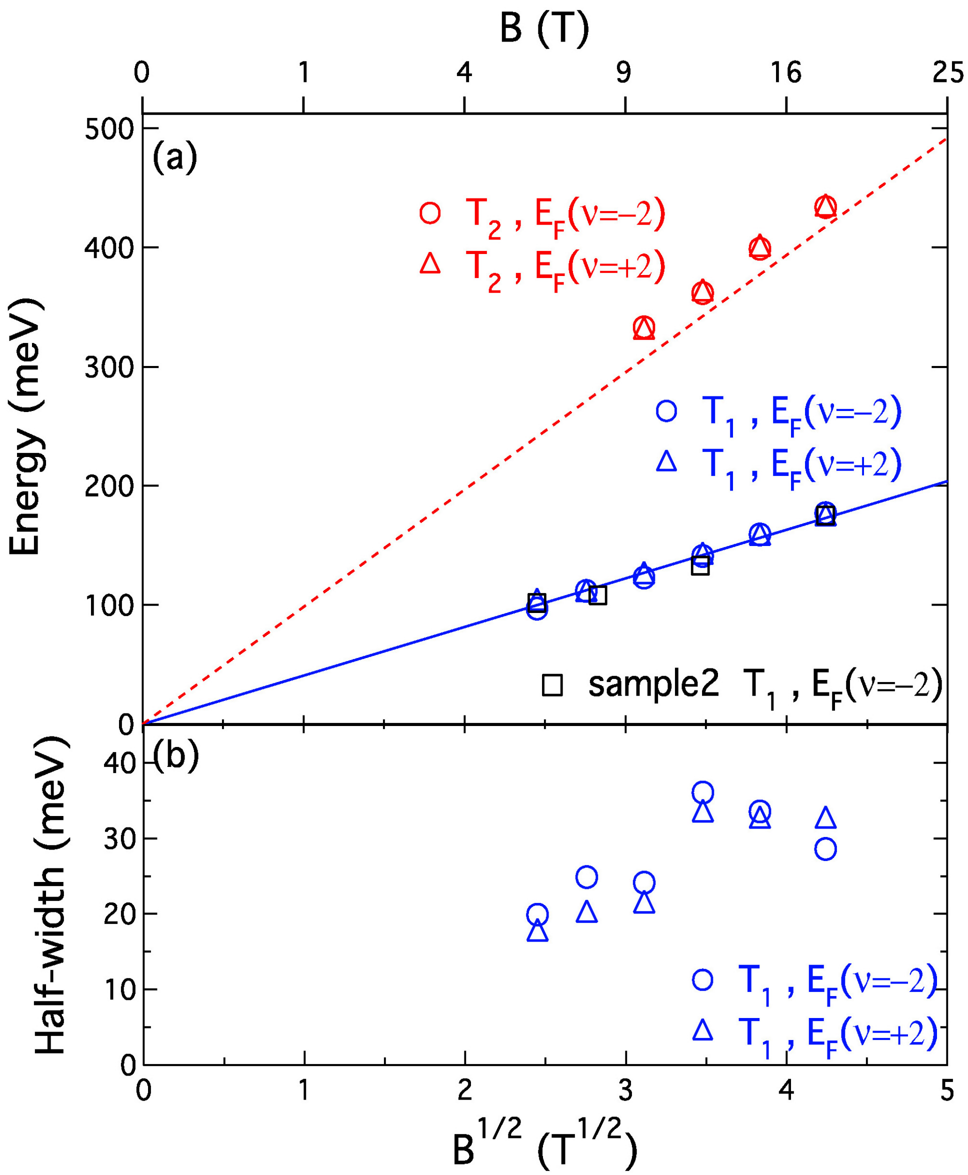




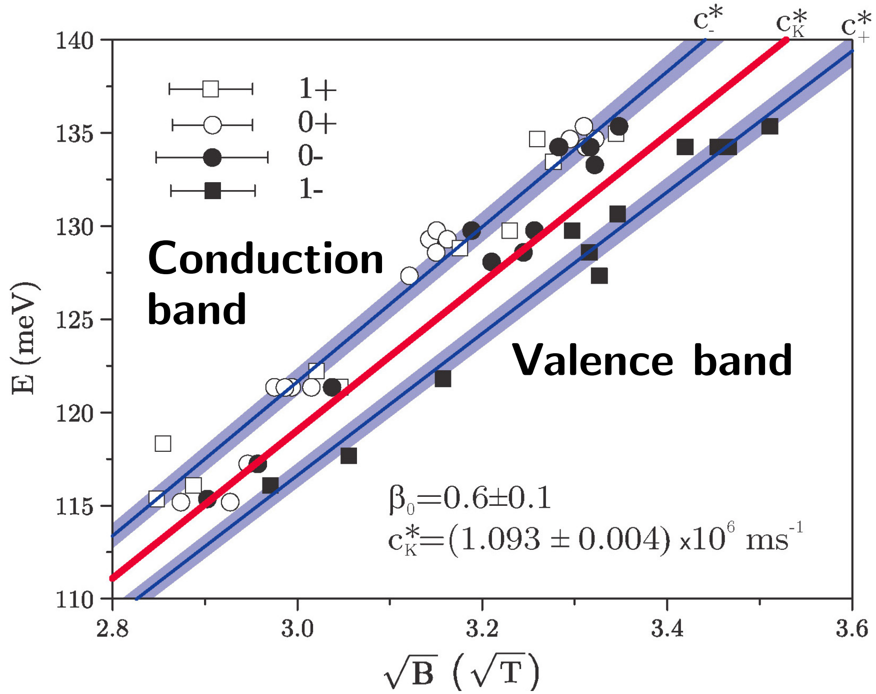
Cyclotron Resonance

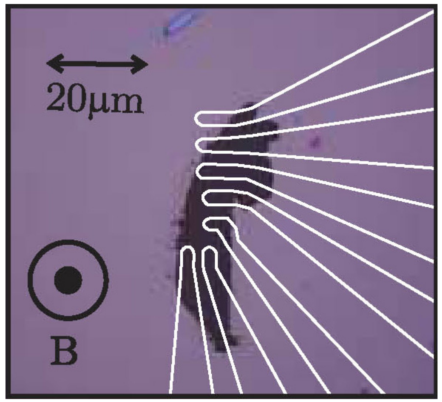
[*Z. Jiang et al., PRL* <u>*98*</u>*, 197403 (2007)*]

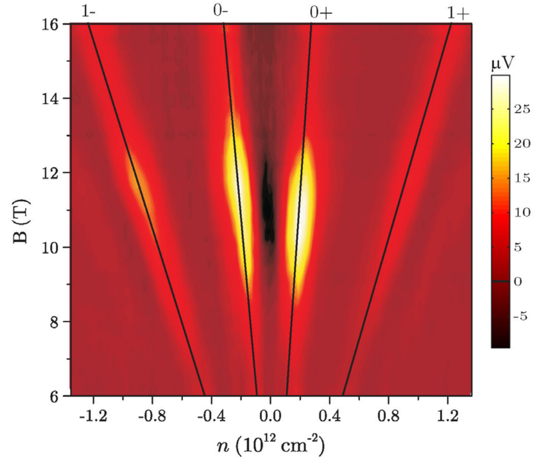


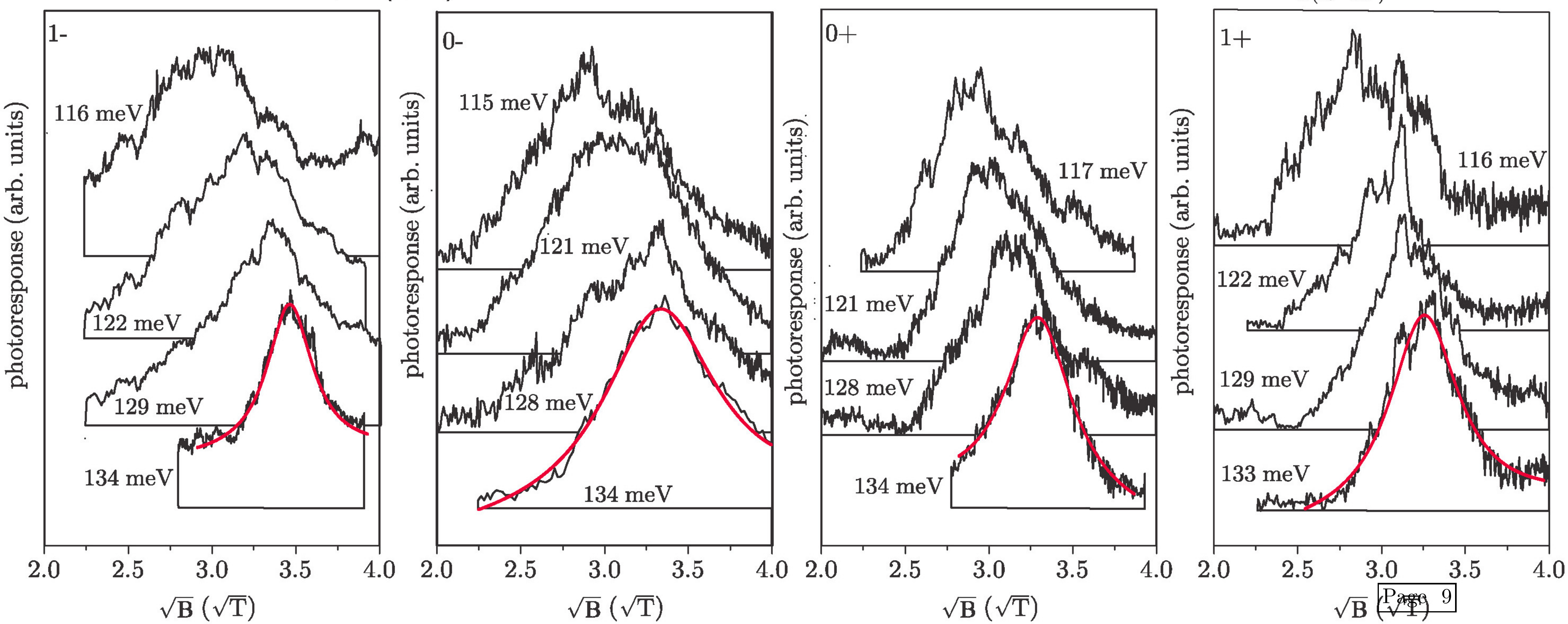


Cyclotron Resonance [*R.S. Deacon et al., PRB 76, 081406 (2007)*]





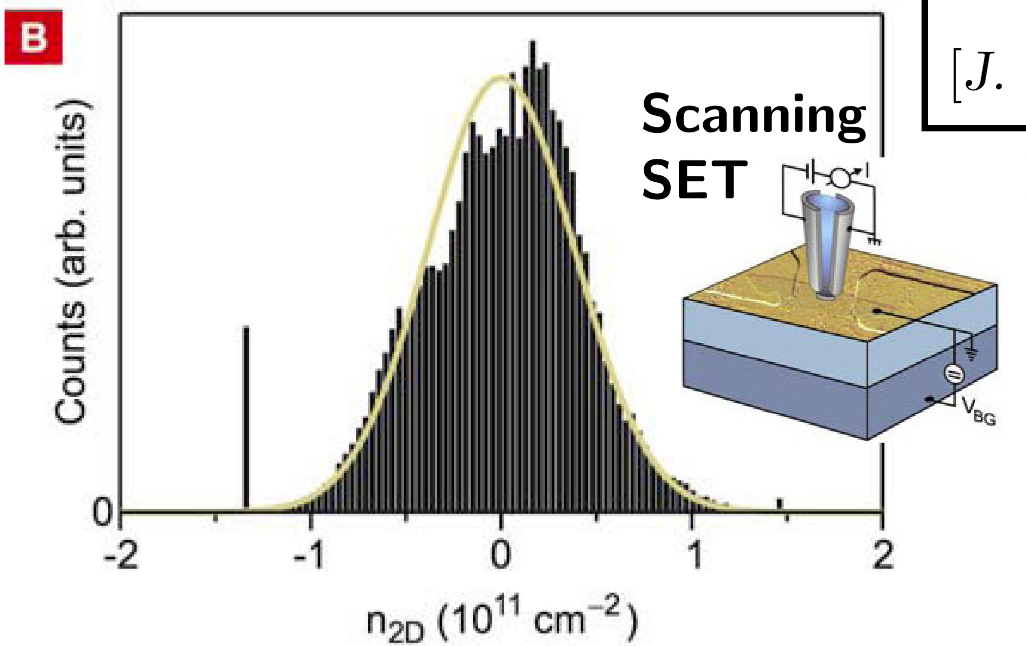


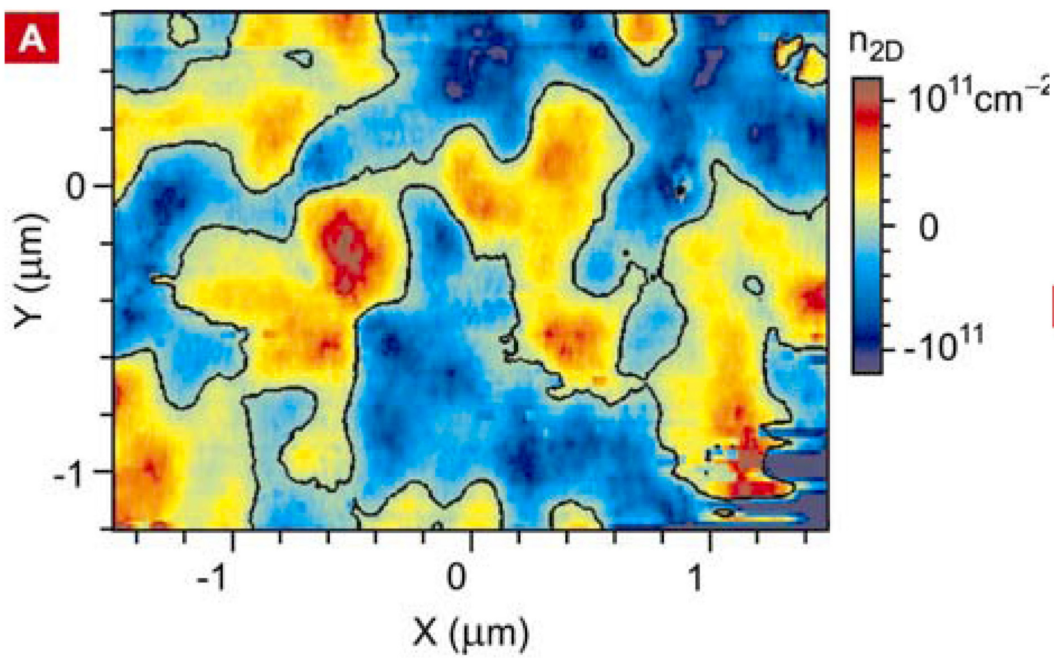


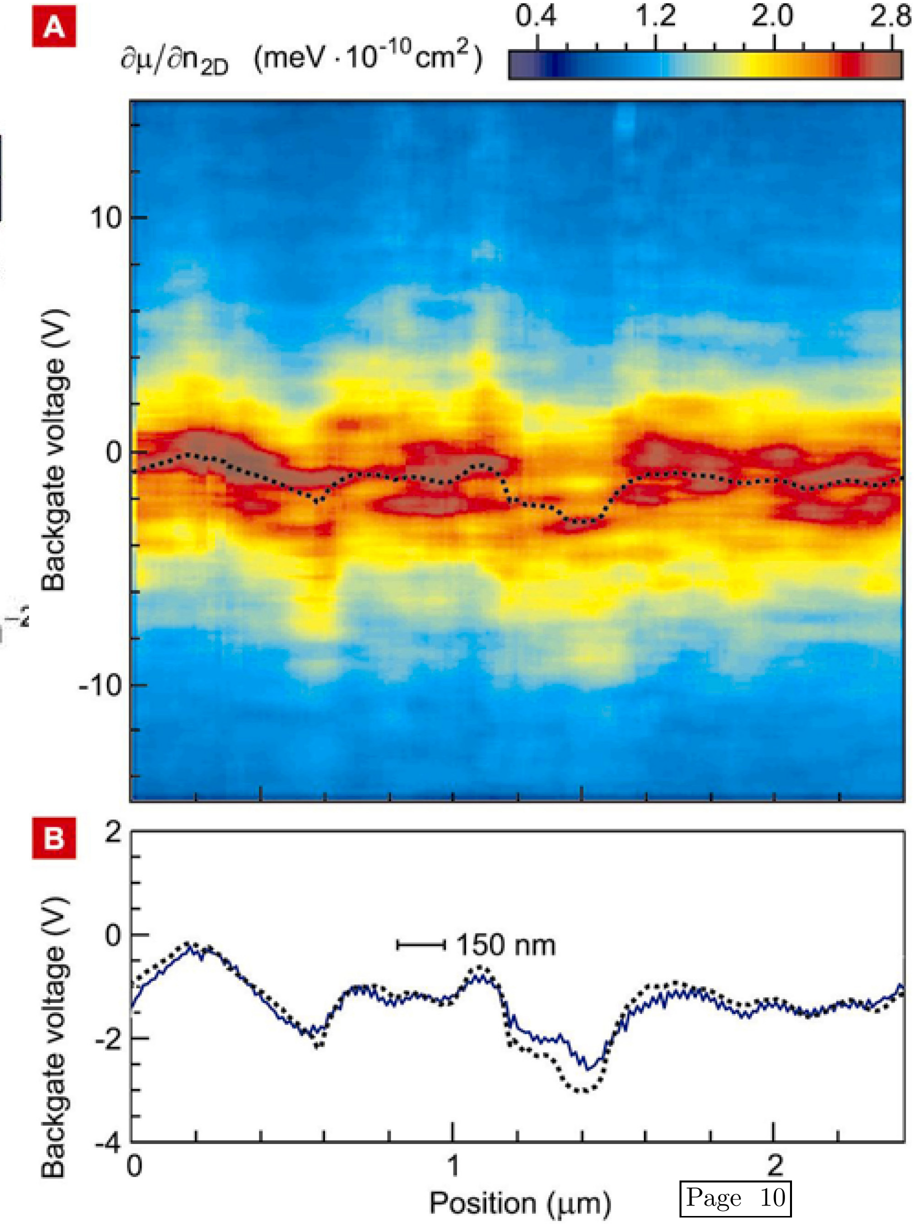
Electron-Hole Puddles

[*J. Martin et al., arXiv:0705.2180*]

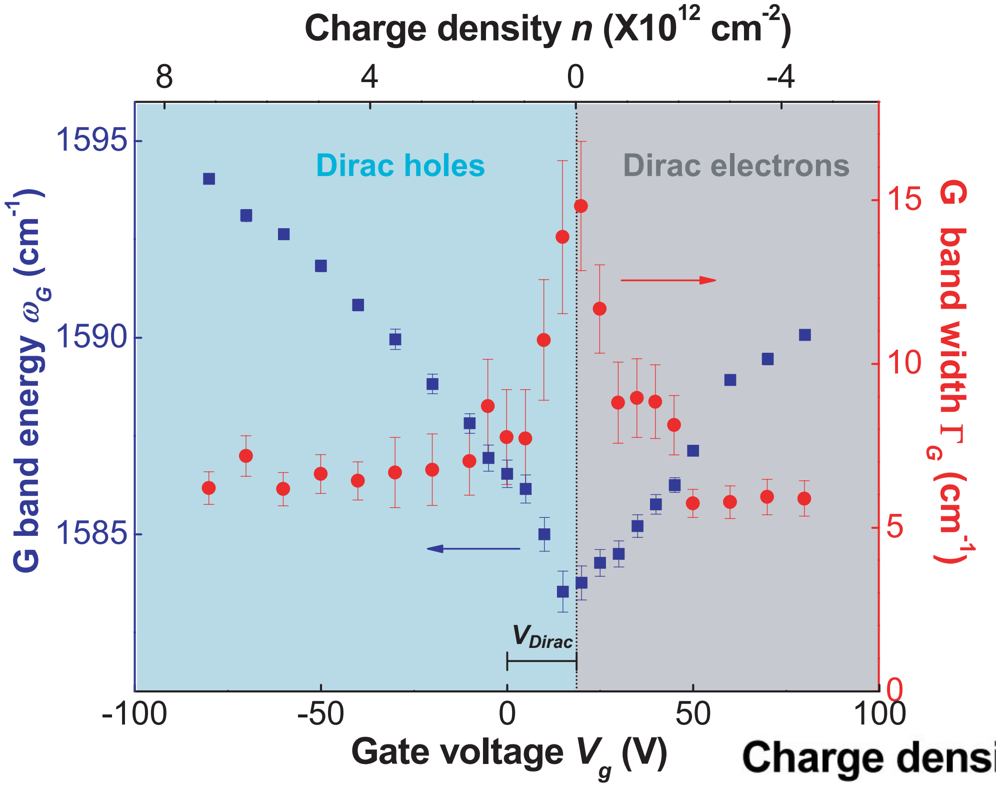
Carrier Distribution



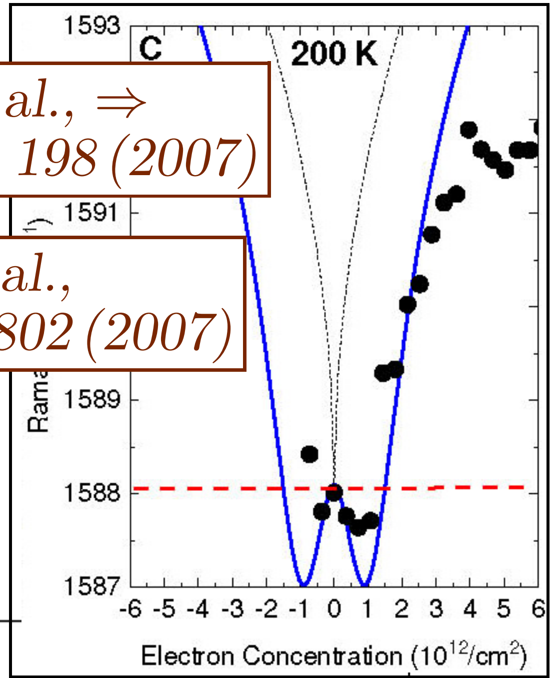




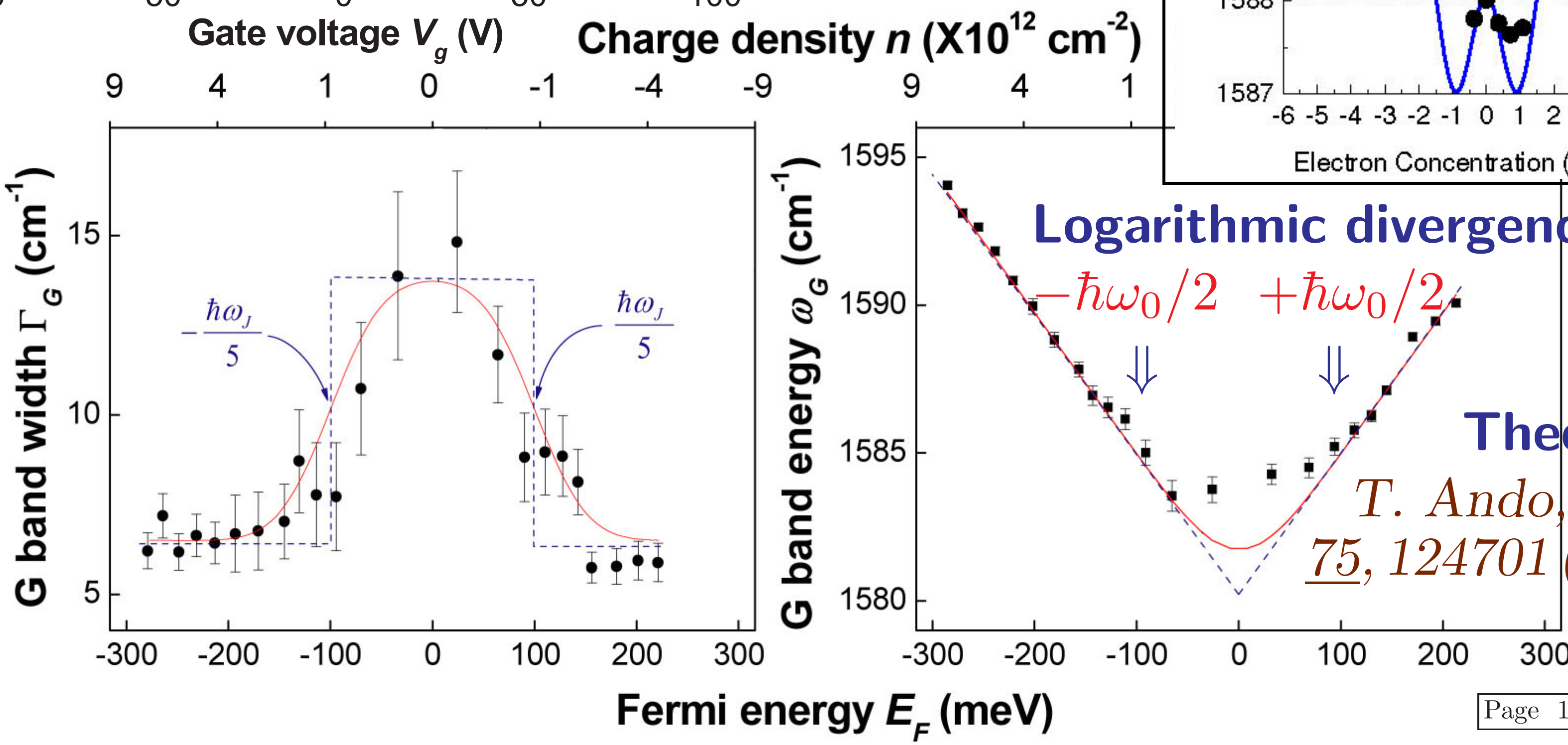
Optical Phonon: Raman Experiments (G Band)



S. Pisana et al., $\Rightarrow$ *Nat. Phys.* *6, 198 (2007)*



$\Leftarrow$ *J. Yan et al., PRL 98, 166802 (2007)*



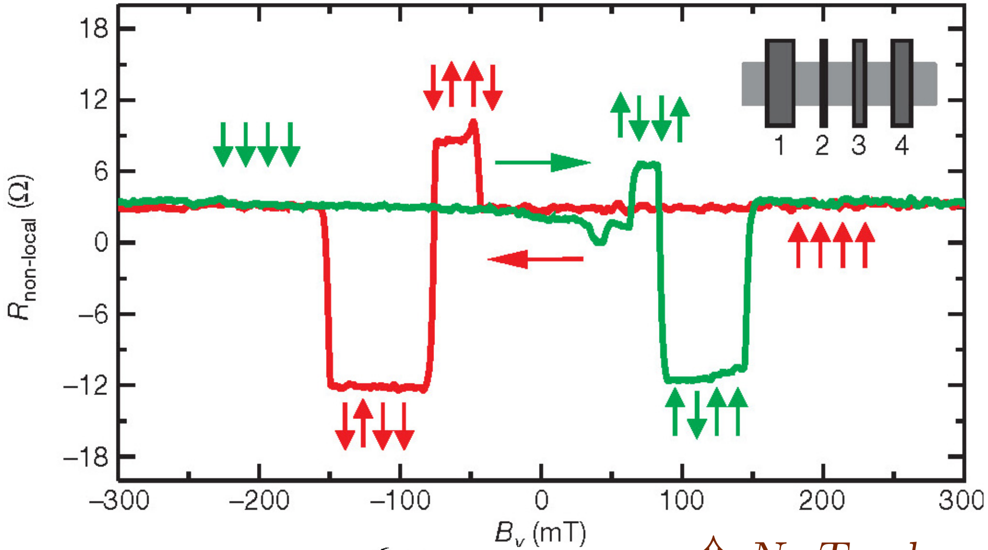
Logarithmic divergence

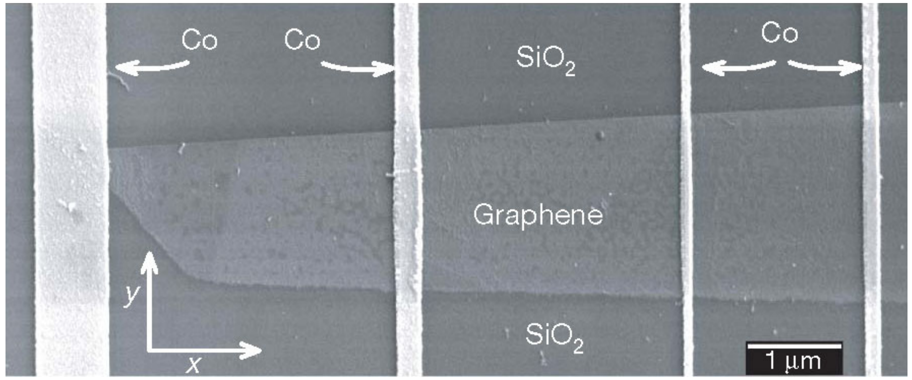
$-\hbar\omega_0/2 \quad +\hbar\omega_0/2$

$\Downarrow \quad \Downarrow$

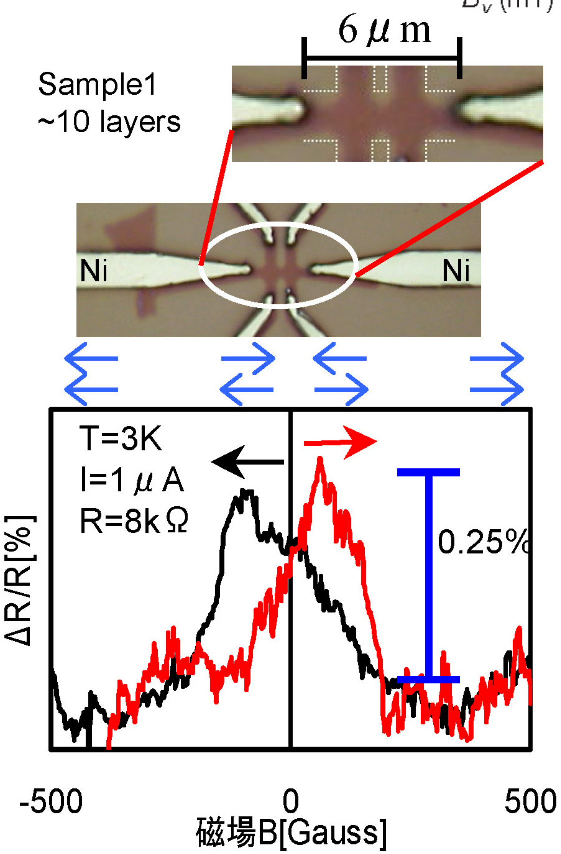
Theory $\Rightarrow$ *T. Ando, JPSJ 75, 124701 (2006)*

Spin Transport

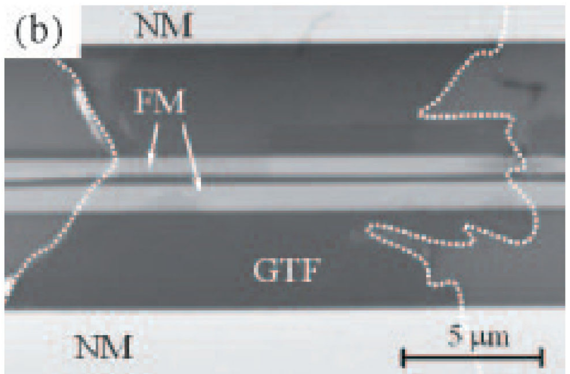


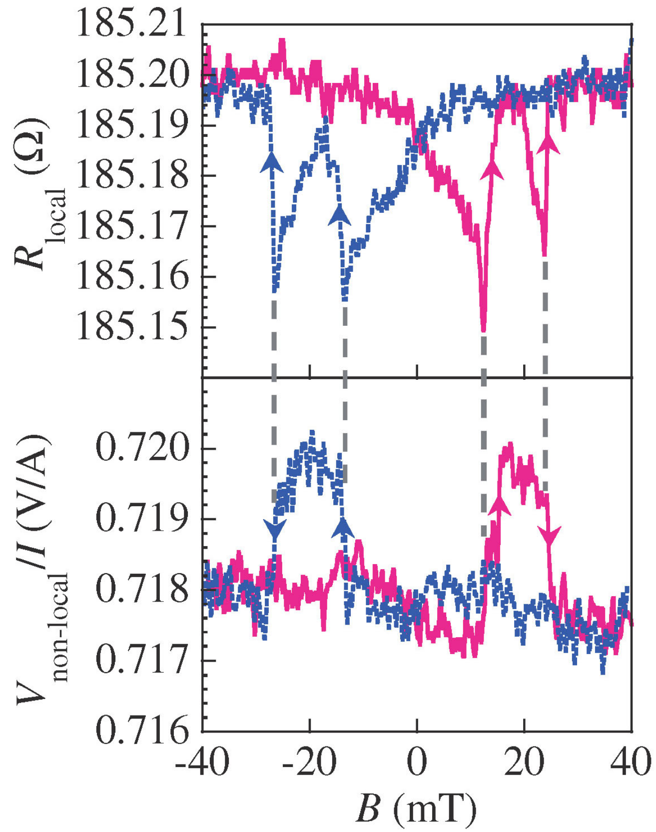


⇑ *N. Tombros et al., Nature 448, 571 (2007)*



Konishi et al (2007 Fall JSAP Meeting)

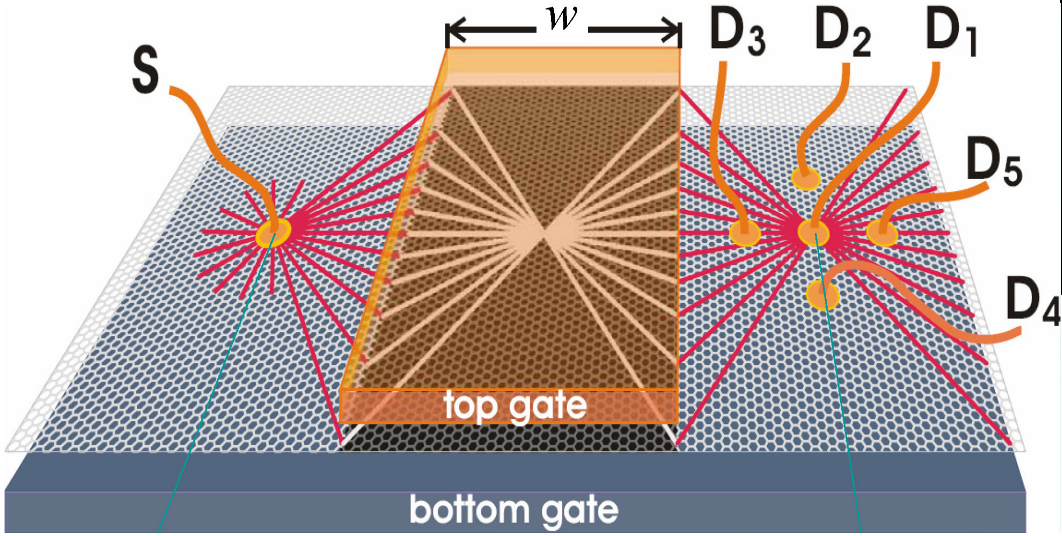


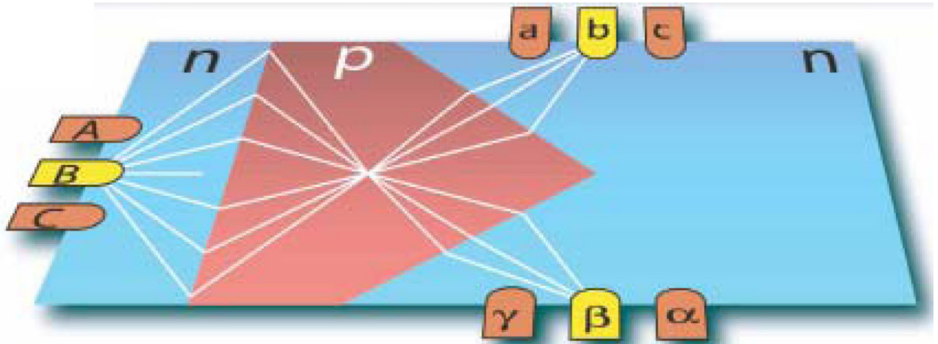


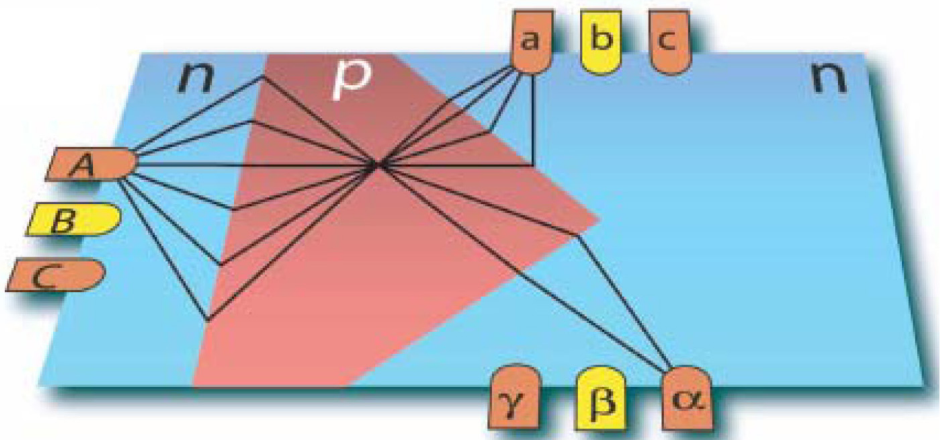
M. Ohishi et al., JJAP 46, L405 (2007)

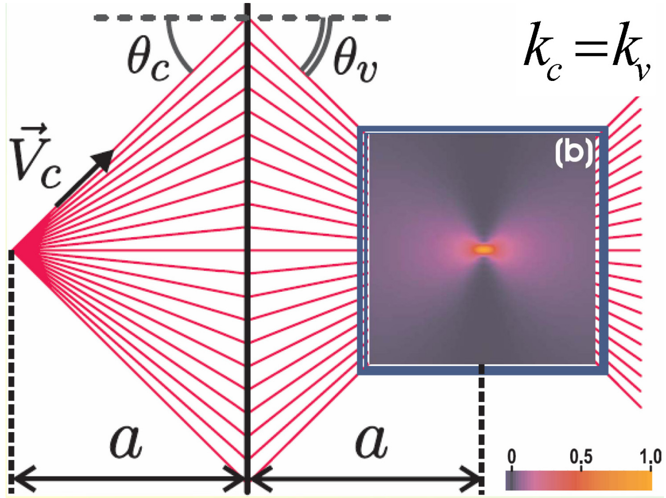
Negative Refractive Index and Veselago Lens

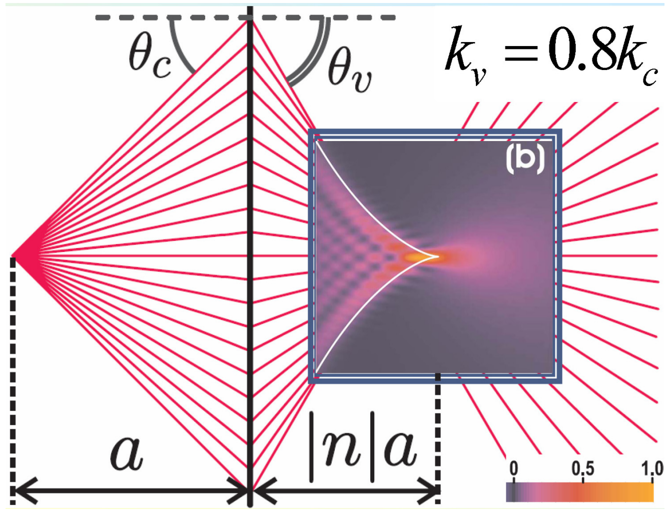
V.V. Cheianov et al., Science 315, 1252 (2007)





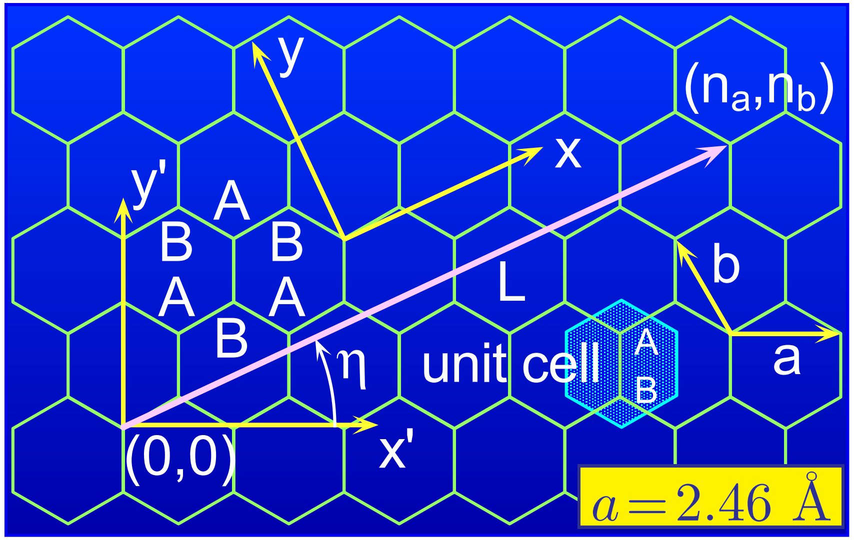


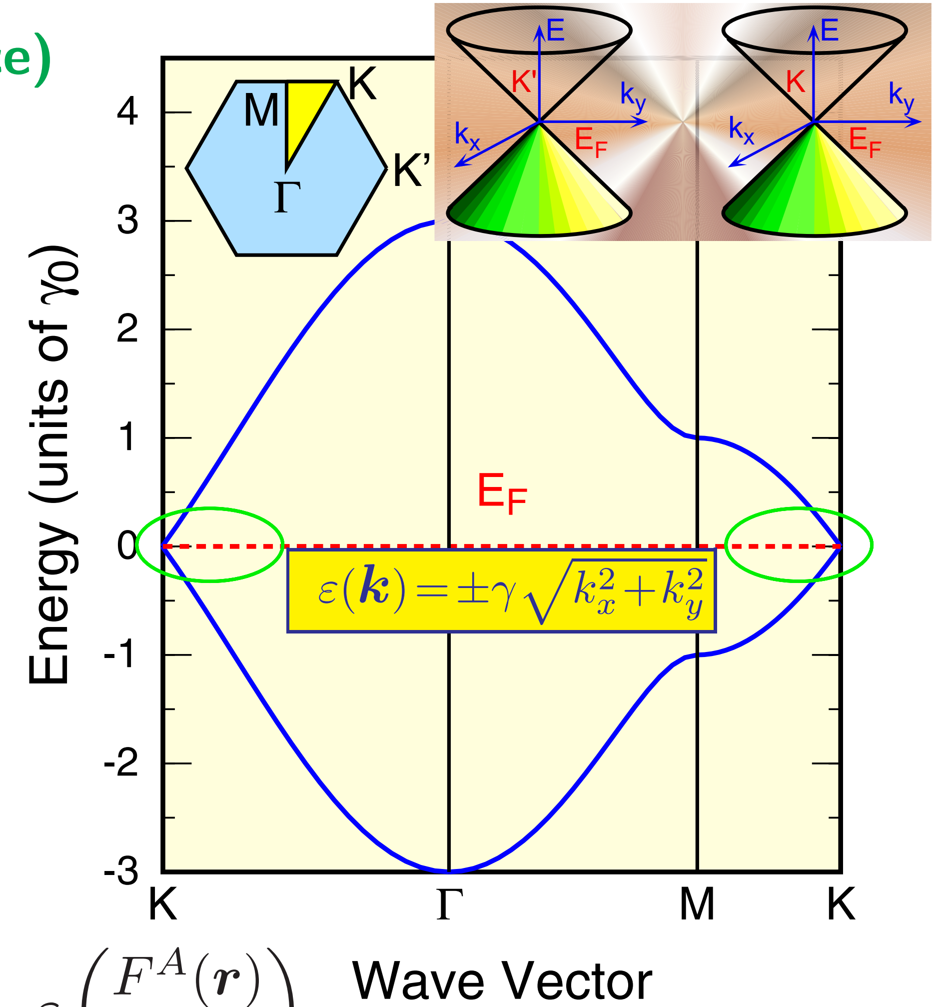




Effective-Mass Description: Neutrino or Massless Dirac Electron

Graphene (Triangular antidot lattice)





Weyl's equation for neutrino

$$\Leftrightarrow\ \gamma(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}})\boldsymbol{F}(\boldsymbol{r})=\varepsilon\boldsymbol{F}(\boldsymbol{r})$$

$$\Leftrightarrow\ \gamma(\sigma_x\hat{k}_x+\sigma_y\hat{k}_y)\boldsymbol{F}(\boldsymbol{r})=\varepsilon\boldsymbol{F}(\boldsymbol{r})$$

$$\begin{pmatrix} 0 & \gamma(\hat{k}_x-i\hat{k}_y) \\ \gamma(\hat{k}_x+i\hat{k}_y) & 0 \end{pmatrix}\begin{pmatrix} F^A(\boldsymbol{r}) \\ F^B(\boldsymbol{r}) \end{pmatrix}=\varepsilon\begin{pmatrix} F^A(\boldsymbol{r}) \\ F^B(\boldsymbol{r}) \end{pmatrix}$$

$\boldsymbol{\hat{k}}=-i\vec{\nabla}$

Velocity: $v_F=\gamma/\hbar$

K': $\boldsymbol{\sigma}\rightarrow\boldsymbol{\sigma}^*$

Massless (Dirac)

$v_F\sim c/300\ \ (\gamma_0\sim 3\,\mathrm{eV})$

Constant velocity (∼light, cannot stop)

Topological anomaly

$\gamma=\sqrt{3}\gamma_0 a/2$ (γ_0: **Hopping integral**)

Topological Anomaly and Berry's Phase

Weyl's equation : Neutrino $\Leftrightarrow$ Helicity ($\boldsymbol{\sigma} \leftrightarrow \boldsymbol{k}$)

$$\gamma(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}})\,\boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r}) = \varepsilon_s(\boldsymbol{k})\,\boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r}) \qquad \boldsymbol{F}_{s\boldsymbol{k}}(\boldsymbol{r}) = \frac{1}{\sqrt{L^2}}\exp(i\boldsymbol{k}\cdot\boldsymbol{r})\,R^{-1}[\theta(\boldsymbol{k})]\big|s\big)$$

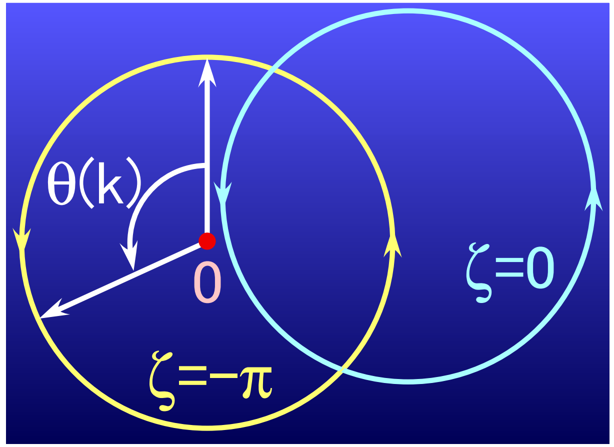
$$R(\theta\pm 2\pi) = -R(\theta) \qquad R(-\pi) = -R(+\pi)$$

$$\varepsilon_s(\boldsymbol{k}) = s\,\gamma\,|\boldsymbol{k}| \qquad s=\pm 1$$

$$\boxed{\begin{aligned}&R(\theta+2\pi)\\ &\quad = e^{-i\zeta}R(\theta)\end{aligned}}$$

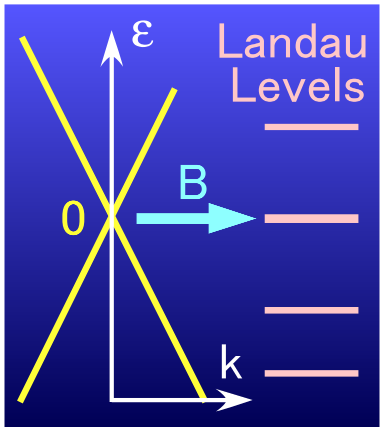
Pseudo spin $\Rightarrow$ Berry's phase

$$\zeta = -i\int_0^T dt\left\langle s\boldsymbol{k}(t)\middle|\frac{d}{dt}\middle|s\boldsymbol{k}(t)\right\rangle = -\pi$$



Landau levels at $\varepsilon=0$ [*J.W. McClure, PR 104, 666 (1956)*]

$$\chi = -\frac{g_{\rm v}g_{\rm s}\gamma^2}{6\pi}\left(\frac{e}{c\hbar}\right)^2\int\left(-\frac{\partial f(\varepsilon)}{\partial\varepsilon}\right)\delta(\varepsilon)\,d\varepsilon$$

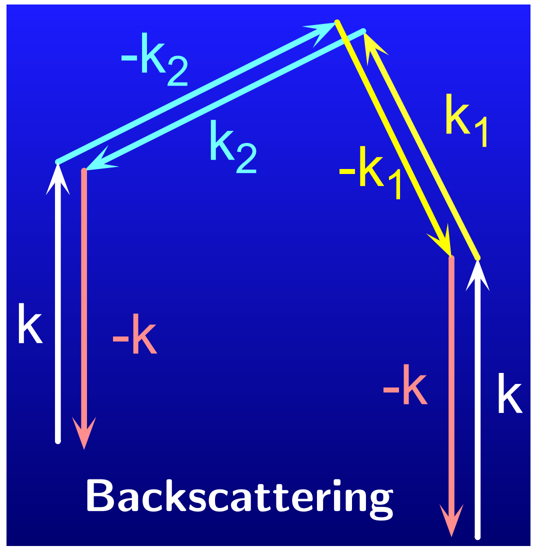


Absence of backscattering

Metallic CN with scatterers

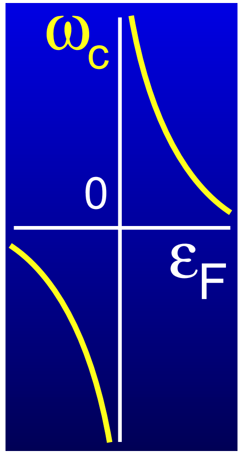
$\Rightarrow$ Perfect conductor

T. Ando & T. Nakanishi, JPSJ 67, 1704 (1998)



Zero-Mode Anomaly: Boltzmann Conductivity

Equation of motion: $\hbar\frac{d\boldsymbol{k}}{dt}=-\frac{e}{c}\boldsymbol{v}\times\boldsymbol{B} \Rightarrow \omega_c=\frac{eBv^2}{c\,\varepsilon_F}$ $\boxed{\begin{aligned}m_c&\propto\varepsilon_F\\&\propto\sqrt{n_s}\end{aligned}}$



Semiclassical: $\varepsilon_n=\pm\sqrt{|n|+\frac{1}{2}}\,\hbar\omega_B \Leftarrow \oint k_x dk_y=\pm\frac{2\pi}{l^2}\Big(|n|+\frac{1}{2}\Big)$

Full quantum: $\varepsilon_n=\pm\sqrt{|n|}\,\hbar\omega_B$ $\quad \hbar\omega_B=\sqrt{2}\,\frac{\gamma}{l}$ $\quad l=\sqrt{\frac{c\hbar}{eB}}$

Density of states: $D(\varepsilon)=\frac{|\varepsilon|}{2\pi\gamma^2}$ $\Rightarrow$ **Zero-gap semiconductor**

Boltzmann conductivity

$$\sigma(\varepsilon_F)=e^2D^*D(\varepsilon_F)=\frac{e^2}{\pi^2\hbar}\frac{1}{4W}$$

Einstein relation

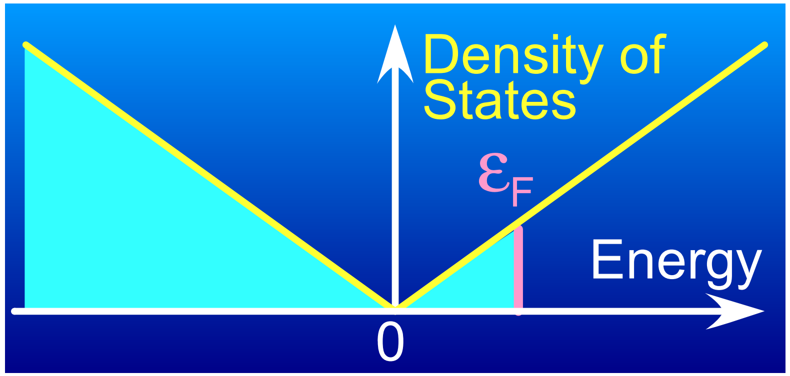
$$D^*=v_F^2\tau=\frac{\gamma^2}{\hbar^2}\tau \qquad W=\frac{n_{\rm i}u^2}{4\pi\gamma^2}$$

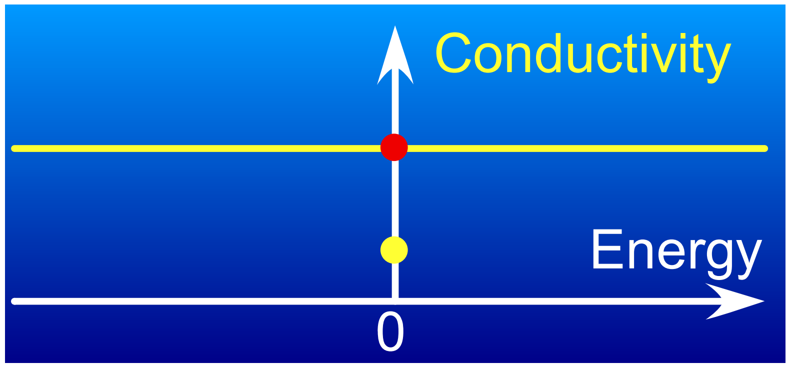
$$\frac{\hbar}{\tau}=2\pi n_{\rm i}u^2D(\varepsilon_F) \qquad \tau\propto D(\varepsilon_F)^{-1}$$

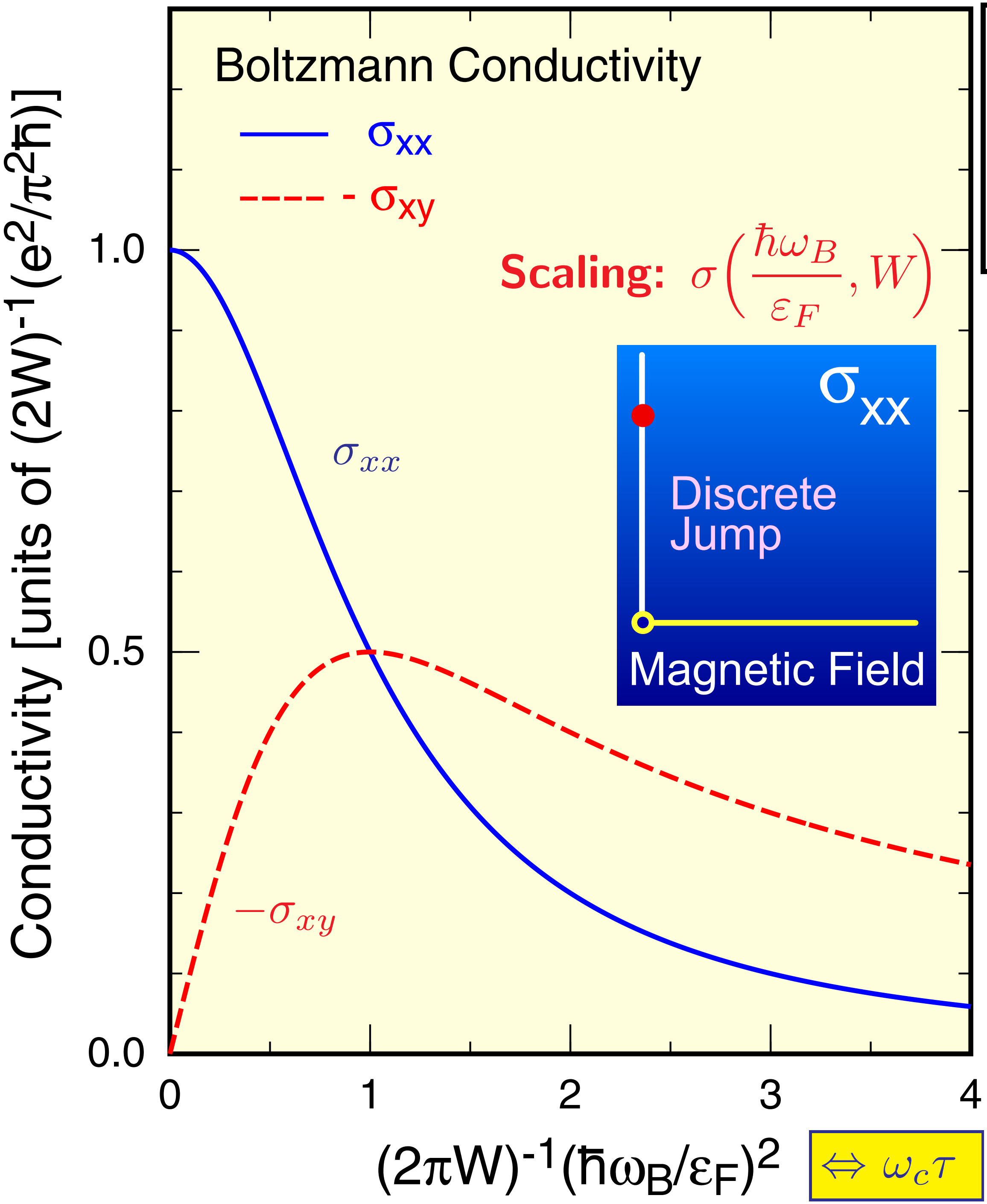
u **Impurity strength**
$n_{\rm i}$ **Impurity density**

$\Rightarrow$ **Independent of ε_F (Metal!)**

$\Rightarrow$ $\sigma(0)$ for $D(0)=0$?







Conductivity Tensor in Magnetic Fields

Y. Zheng & T. Ando, PRB 65, 245420 (2002)

Cyclotron frequency

$$\omega_c = \frac{eBv^2}{c\,\varepsilon_F}$$

$$\frac{\hbar}{\tau} = 2\pi|\varepsilon_F|W$$

$$\omega_c\tau \propto \left(\frac{\hbar\omega_B}{\varepsilon_F}\right)^2$$

$$\hbar\omega_B = \sqrt{2}\,\frac{\gamma}{l}$$

Magneto-conductivity

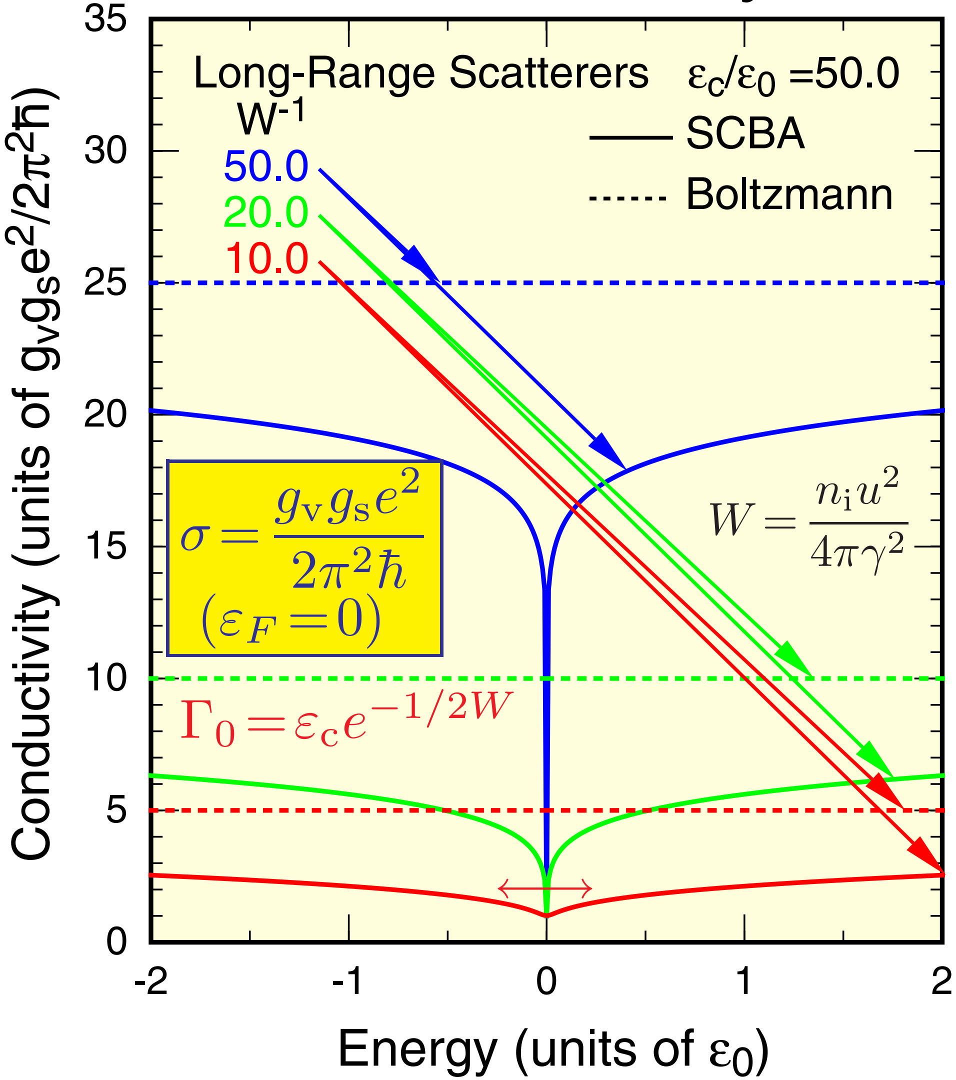
$$\sigma_{xx} = \frac{\sigma_0}{1+\omega_c^2\tau^2}$$

$$\sigma_{xy} = -\frac{\sigma_0\omega_c\tau}{1+\omega_c^2\tau^2}$$

$$\sigma_0(W) = \frac{e^2}{\pi^2\hbar}\,\frac{1}{2W}$$

Zero-Mode Anomalies (Self-Consistent Born Approximation)

Static conductivity



ε_0: **Arbitrary energy**
ε_c: **Cutoff energy (π-band width)**

Singularity at the Dirac point $(\varepsilon_F = 0)$

$\Leftrightarrow$ **Fermi energy scaling**

Magnetoconductivity

$$\sigma_{xx}(B) = \sigma_{xx}\left(\frac{\hbar\omega_B}{\varepsilon_F}\right), \text{ etc.}$$

Dynamical conductivity

$$\sigma(\omega) = \sigma\left(\frac{\hbar\omega}{\varepsilon_F}\right)$$

Diagonal conductivity σ_{xx}

N.H. Shon and T. Ando, JPSJ 67, 2421 (1998) $\dfrac{g_v g_s e^2}{2\pi^2\hbar}$

Quantum Hall effect σ_{xy}

Y. Zheng and T. Ando, PRB 65, 245420 (2002) $4\left(n+\dfrac{1}{2}\right)\dfrac{e^2}{h}$

Dynamical conductivity $\sigma(\omega)$

T. Ando, Y. Zheng, & H. Suzuura, JPSJ 71, 1318 (2002)

Diamagnetic susceptibility $\chi(\varepsilon_F)$

M. Koshino and T. Ando, PRB 75, 235333 (2007)

Diamagnetic Susceptibility: Disorder Effects

Singular diamagnetism

J.W. McClure, Phys. Rev. 104, 666 (1956)

S.A. Safran & F.J. DiSalvo, PRB 20, 4889 (1979)

$$\chi = -\frac{g_{\mathrm{v}} g_{\mathrm{s}} \gamma^2}{6\pi}\left(\frac{e}{c\hbar}\right)^2 \delta(\varepsilon_F)$$

Constant broadening Γ

H. Fukuyama, JPSJ 76, 043711 (2007)

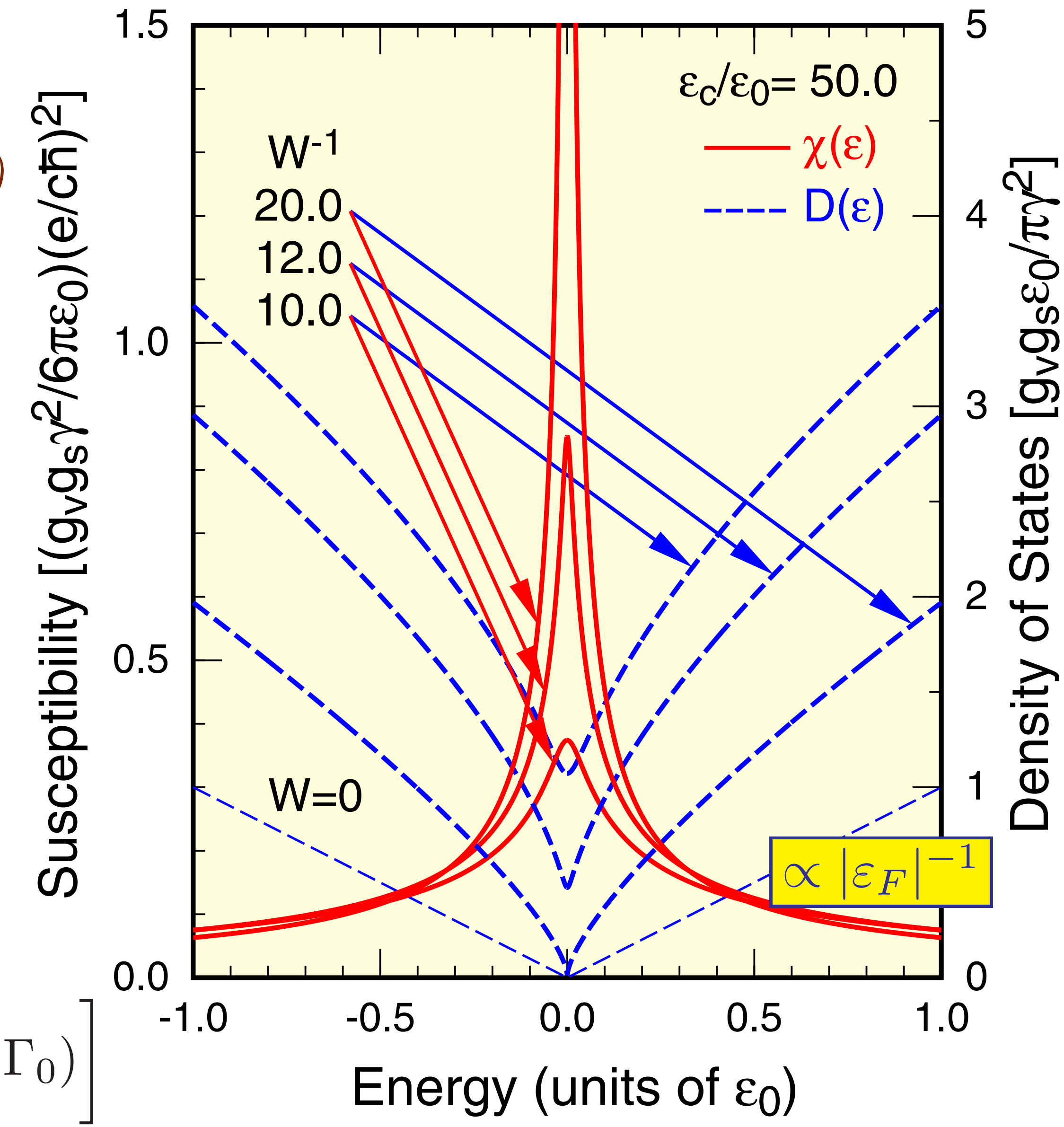
$$\delta(\varepsilon_F) \rightarrow \frac{\Gamma}{\pi(\varepsilon_F^2+\Gamma^2)}$$

Self-consistent Born approximation

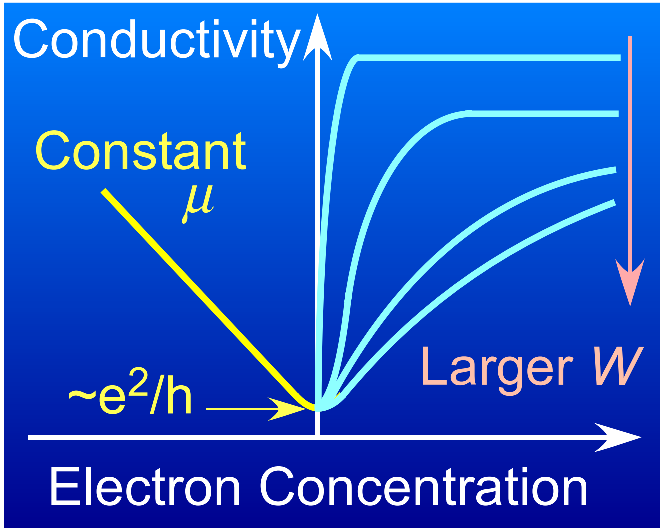
M. Koshino and T. Ando, PRB 75, 235333 (2007)

$$\delta(\varepsilon_F) \rightarrow \frac{W}{2|\varepsilon_F|} \quad \left[\frac{2W}{\pi\Gamma_0} \quad (|\varepsilon_F|<\Gamma_0)\right]$$

Cutoff energy: $\Gamma_0 = \varepsilon_c \, e^{-1/2W}$



Sharp peak and long tail



Conductivity vs Concentration

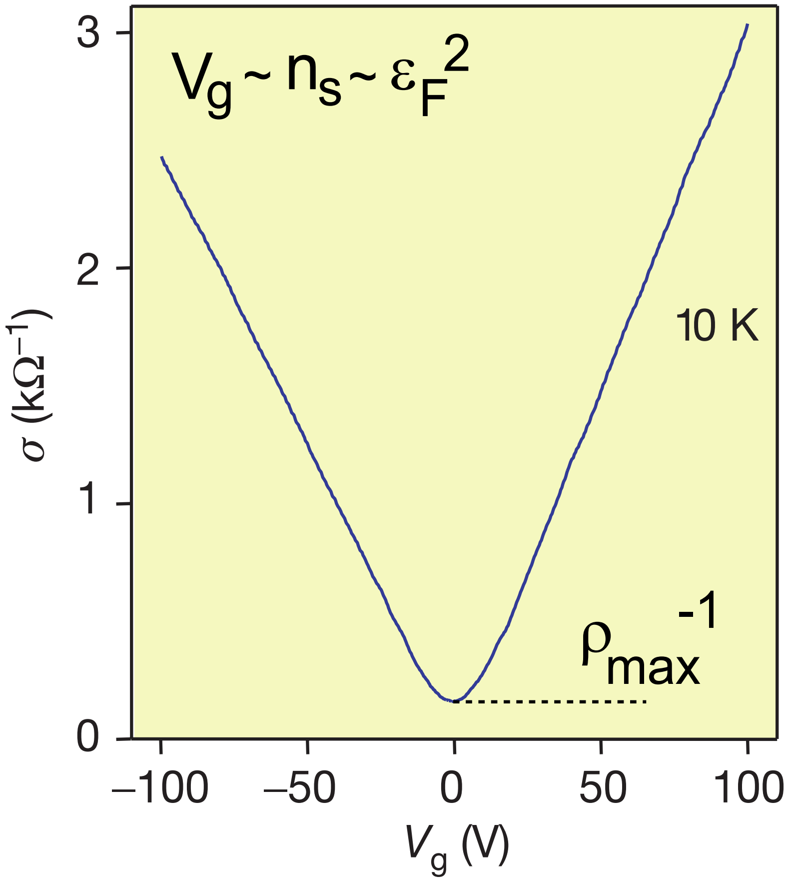
K.S. Novoselov et al., Nature 438, 197 (2005)

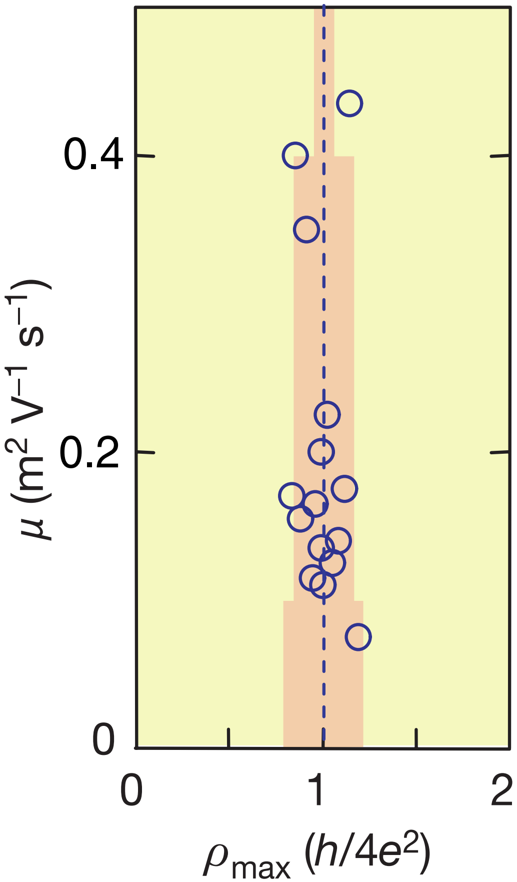
Scattering mechanisms

Boltzmann conductivity $\sigma(\varepsilon_F) = \dfrac{e^2}{\pi^2\hbar}\dfrac{1}{4W}$

Constant mobility $\Leftrightarrow W \propto n_s^{-1}$

Charged impurity with screening?





- *T. Ando, JPSJ 75, 074716 (2006)*
- *K. Nomura & A.H. MacDonald, PRL 96, 256602 (2006)*
- ...

Minimum conductivity

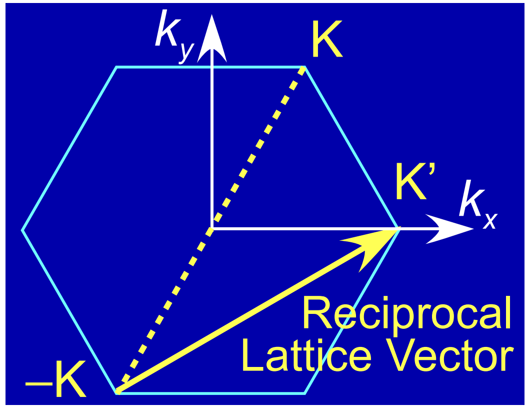
Universal?

- *H. Kumazaki & D.S. Hirashima, JPSJ 75, 053707 (2006)*
- ...

Special Time Reversal Symmetry and Universality Class

Real time reversal (K↔K'): T $\quad \boldsymbol{F}_K^T = \sigma_z \boldsymbol{F}_{K'}^*, \quad \boldsymbol{F}_{K'}^T = \sigma_z \boldsymbol{F}_K^* \quad T^2 = 1$

Special time reversal (within K and K'): S

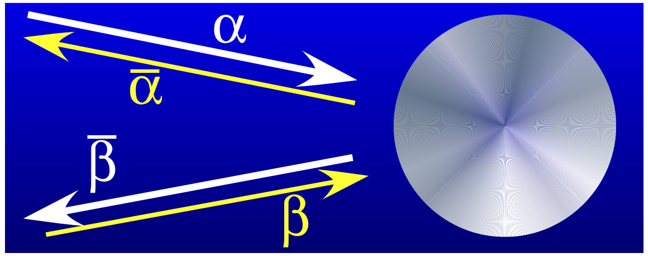


$$\boldsymbol{F}^S = K\boldsymbol{F}^* \quad K = -i\sigma_y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad K^2 = -1$$

Time reversal of P $\quad \Rightarrow S^2 = -1$

$$P^S = K\,{}^tPK^{-1} \Rightarrow (\boldsymbol{F}_\alpha^S, P^S\boldsymbol{F}_\beta^S) = (\boldsymbol{F}_\beta, P\boldsymbol{F}_\alpha)$$

Time reversal		Symmetry	Matrix
Real	$T^2 = +1$	Orthogonal	Real
Special	$S^2 = -1$	Symplectic	Quaternion
None		Unitary	Complex



Reflection coefficient: $r_{\bar\beta\alpha} = (\boldsymbol{F}_{\bar\beta}, T\boldsymbol{F}_\alpha) = (\boldsymbol{F}_\beta^S, T\boldsymbol{F}_\alpha) \;\Leftrightarrow\; r_{\bar\alpha\beta}$

T matrix: $T = V + V\dfrac{1}{E-\mathcal{H}_0+i0}V + V\dfrac{1}{E-\mathcal{H}_0+i0}V\dfrac{1}{E-\mathcal{H}_0+i0}V + \cdots$

Real : $r_{\bar\alpha\beta} = (\boldsymbol{F}_\alpha^T, T\boldsymbol{F}_\beta) = (\boldsymbol{F}_\beta^T, T(\boldsymbol{F}_\alpha^T)^T) = +(\boldsymbol{F}_\beta^T, T\boldsymbol{F}_\alpha) = +r_{\bar\beta\alpha}$

Special: $r_{\bar\alpha\beta} = (\boldsymbol{F}_\alpha^S, T\boldsymbol{F}_\beta) = (\boldsymbol{F}_\beta^S, T(\boldsymbol{F}_\alpha^S)^S) = -(\boldsymbol{F}_\beta^S, T\boldsymbol{F}_\alpha) = -r_{\bar\beta\alpha}$

Absence of backward scattering: $r_{\bar\alpha\alpha} = 0$ **(⇐ Berry's phase)**

Presence of perfect channel (Odd channel numbers)

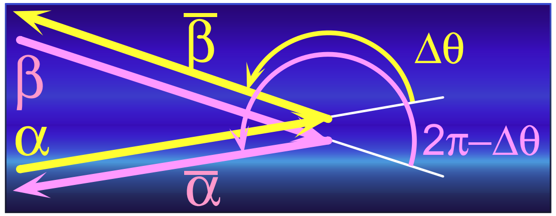
Metallic Nanotubes: Perfect Channel without Backscattering

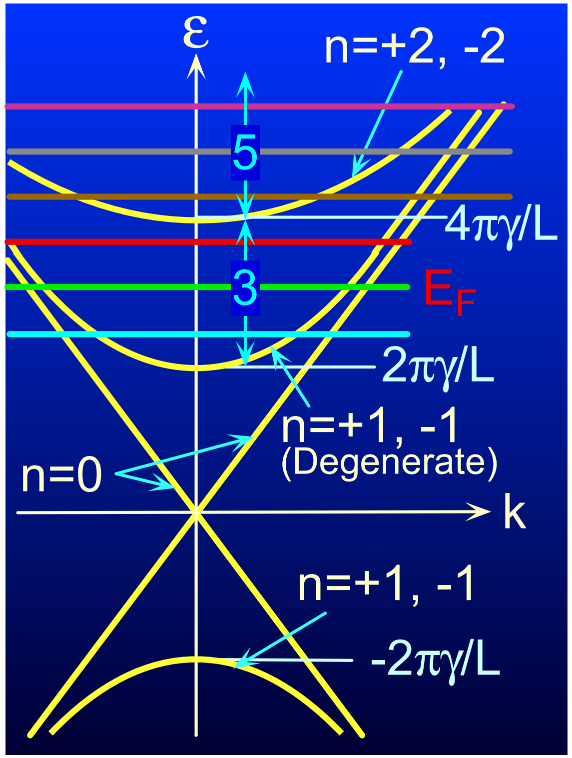
T. Ando and H. Suzuura, J. Phys. Soc. Jpn. 71, 2753 (2002)

Time reversal processes: $\boxed{\alpha \to \bar{\beta}} \Leftrightarrow \boxed{\beta \to \bar{\alpha}} \quad \Rightarrow \quad r_{\bar{\beta}\alpha} = -r_{\bar{\alpha}\beta}$

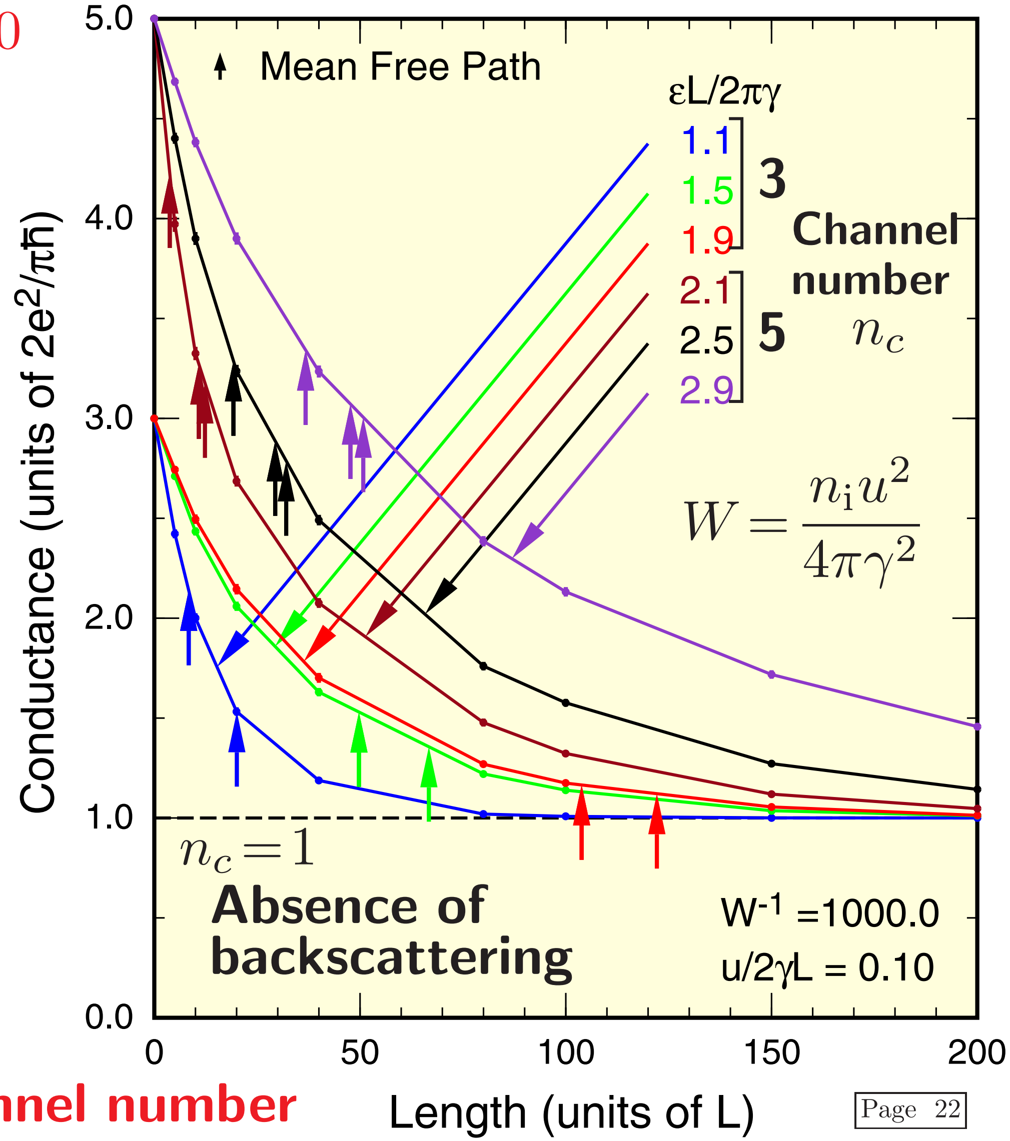
Reflection matrix $\Rightarrow \det(r) = 0$

$\Rightarrow$ **Perfect channel**





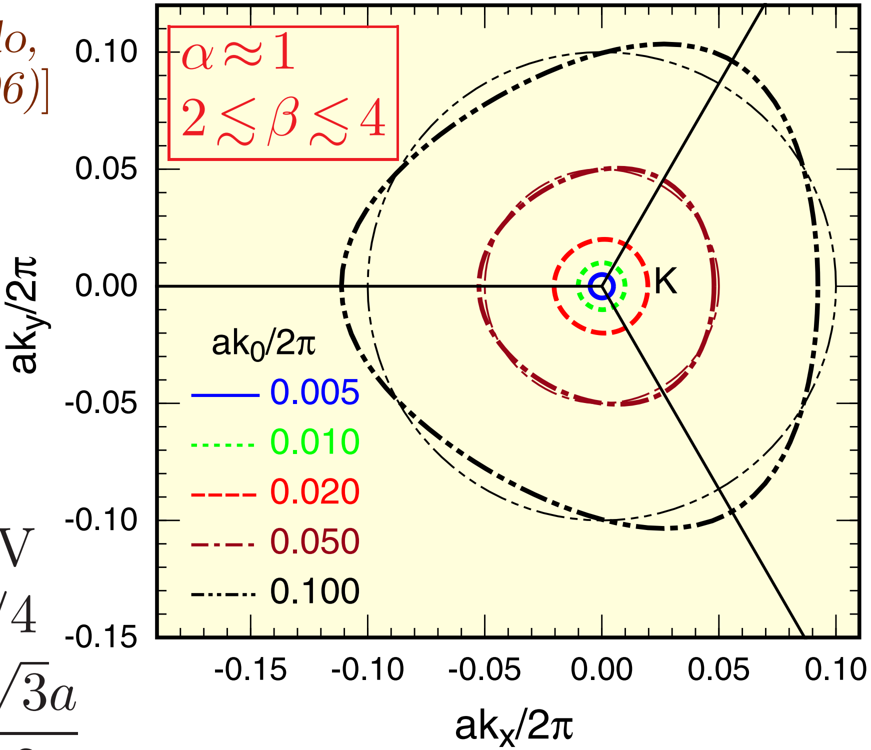
Odd channel number



Symmetry Breaking Effects: Symplectic ⇒ Unitary

Trigonal warping (S) [H. Ajiki & T. Ando, JPSJ 65, 505 (1996)]

$$\mathcal{H}' = \alpha \frac{\gamma a}{4\sqrt{3}} \begin{pmatrix} 0 & (\hat{k}_x + i\hat{k}_y)^2 \\ (\hat{k}_x - i\hat{k}_y)^2 & 0 \end{pmatrix}$$



Lattice distortion [H. Suzuura & T. Ando, PRB 65, 235412 (2002)]

$$\mathcal{H}' = g_1(u_{xx} + u_{yy}) + g_2[(u_{xx} - u_{yy})\sigma_x - 2u_{xy}\sigma_y]$$

Deformation potential : $g_1 \sim 16$ eV

Bond-length (b) change: $g_2 \approx \beta\gamma_0/4$

$$\beta = -\frac{d\ln\gamma_0}{d\ln b}, \quad \gamma = \frac{\sqrt{3}\gamma_0 a}{2}, \quad b = \frac{\sqrt{3}a}{2}$$

$$u_{xx} = \frac{\partial u_x}{\partial x} + \frac{u_z}{R} \quad u_{yy} = \frac{\partial u_y}{\partial y} \quad u_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

$$p = 1 - \frac{3}{8}\frac{\gamma'}{\gamma}$$

Curvature: $\mathcal{H}' = p\frac{\gamma a}{4\sqrt{3}}\left[\left(\frac{\partial^2 u_z}{\partial x^2} - \frac{\partial^2 u_z}{\partial y^2}\right)\sigma_x - 2\frac{\partial^2 u_z}{\partial x \partial y}\sigma_y\right]$

$$\gamma = -\frac{\sqrt{3}}{2}V_{pp}^{\pi}a$$

$$\gamma' = \frac{\sqrt{3}}{2}(V_{pp}^{\sigma} - V_{pp}^{\pi})a$$

Optical phonon: $\mathcal{H}' = -\frac{\beta\gamma}{b^2}\boldsymbol{\sigma}\times[\boldsymbol{u}_A - \boldsymbol{u}_B]$ [T. Ando, JPSJ 69, 1757 (2000)]

[K. Ishikawa & T. Ando, JPSJ 75, 084713 (2006)]

Symmetry Breaking Effects and Crossover

Short-range scatterers $(d/a < 1)$

Symplectic $\Rightarrow$ Orthogonal

Intervalley (K$\leftrightarrow$K')

'Spin' dependent potential

Metallic nanotubes

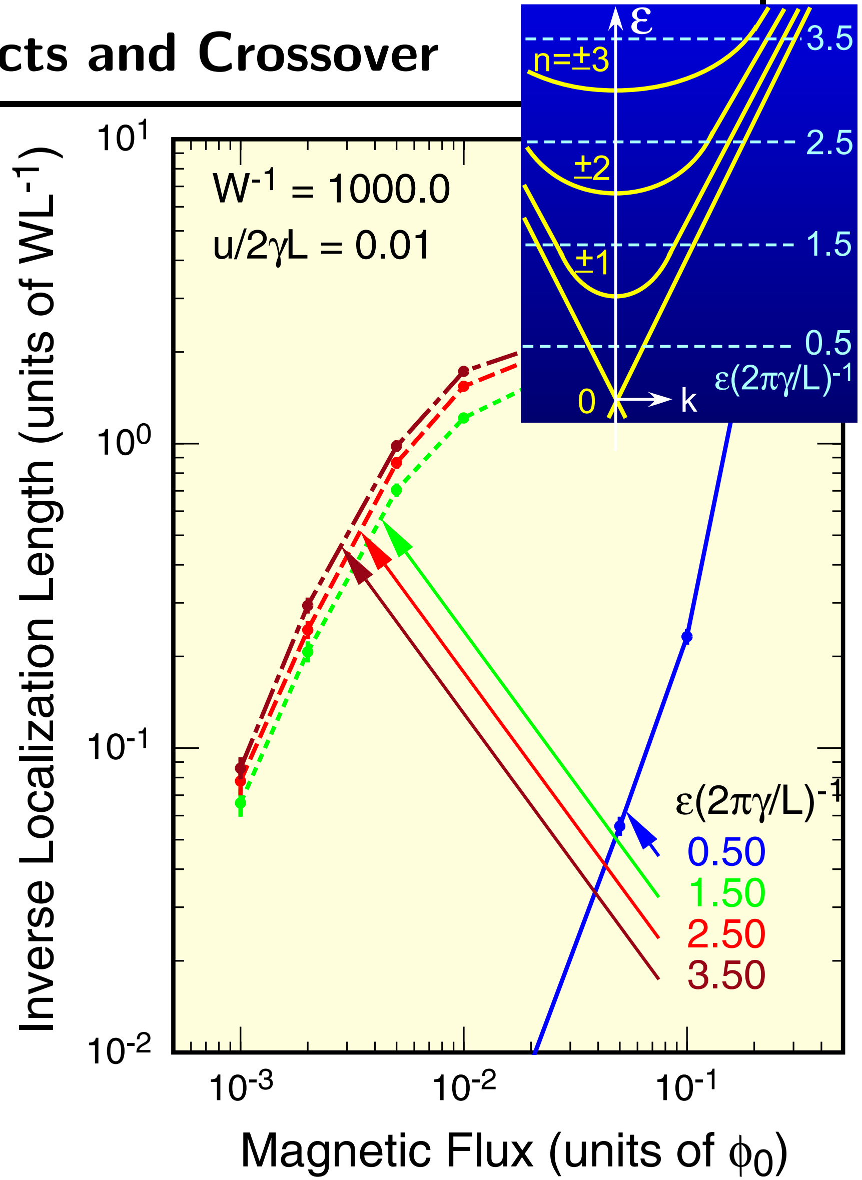
Absence of backscattering: Robust

Perfect channel : Fragile

T. Ando, JPSJ 73, 1273 (2004)
TA & K. Akimoto, JPSJ 73, 2895 (2004)
K. Akimoto & TA, JPSJ 73, 2194 (2004)
T. Ando, JPSJ 75, 054701 (2006)

Quantum correction to conductivity

		Magnetoresistance
Orthogonal	$\Delta\sigma < 0$	Negative
Symplectic	$\Delta\sigma > 0$	Positive
Unitary	$\Delta\sigma = 0$	No

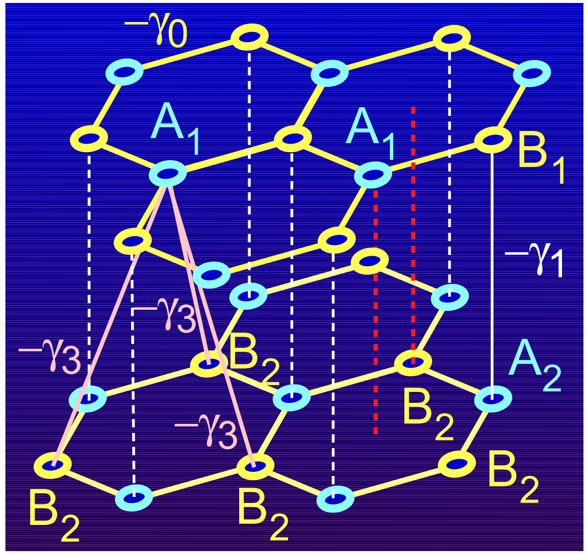


Crossover: *H. Suzuura and T. Ando, PRL 81, 266603 (2002)*

Experiments: *S.V. Morozov et al., PRL 97, 016801 (2006)* $(\Delta\sigma \approx 0)$
X.-S. Wu et al., PRL 98, 136801 (2007) $(\Delta\sigma > 0)$

Theory: *E. McCann et al., PRL 97, 146805 (2006)*

Bi-Layer Graphene



Quantum Hall effect in bilayer graphene

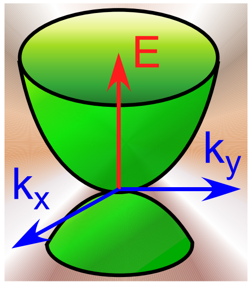
K.S. Novoselov et al., Nature 438, 197 (2005)
K.S. Novoselov et al., Nat. Phys. 2, 177 (2006)

ARPES [*T. Ohta et al., PRL 98, 206802 (2007)*]

Effective Hamiltonian in bilayer graphene

$$\mathcal{H}=\begin{array}{cc} & \begin{array}{cccc} A_1 & B_1 & A_2 & B_2 \end{array} \\ & \begin{pmatrix} 0 & \gamma\hat{k}_- & 0 & 0 \\ \gamma\hat{k}_+ & 0 & \Delta & 0 \\ 0 & \Delta & 0 & \gamma\hat{k}_- \\ 0 & 0 & \gamma\hat{k}_+ & 0 \end{pmatrix} \end{array}$$

$$\hat{k}_\pm=\hat{k}_x\pm i\hat{k}_y \qquad \Delta=\gamma_1$$



$$\mathcal{H}\approx\frac{\hbar^2}{2m^*}\begin{pmatrix} 0 & \hat{k}_-^2 \\ \hat{k}_+^2 & 0 \end{pmatrix} \qquad m^*=\frac{\hbar^2\Delta}{2\gamma^2}$$

$$\varepsilon_n=\pm\hbar\omega_c\sqrt{n(n+1)} \qquad (n=0,1,\ldots)$$

$\Rightarrow$ **Two Landau levels at $\varepsilon=0$**

- *E. McCann and V.I. Falko, PRL 96, 086805 (2006)*
- *M. Koshino and T. Ando, PRB 73, 245403 (2006)*

Tight-binding models

- *S. Latil and L. Henrard, PRL 97, 036803 (2006)*
- *F. Guinea et al., PRB 73, 245426 (2006), $\cdots$*

Susceptibility $\Rightarrow \chi(\varepsilon)=-\dfrac{g_v g_s}{4\pi}\dfrac{e^2\gamma}{c^2\hbar^2}\ln\left|\dfrac{\Delta}{\varepsilon}\right|$ [*S.A. Safran, PRB 30, 421 (1984)*]

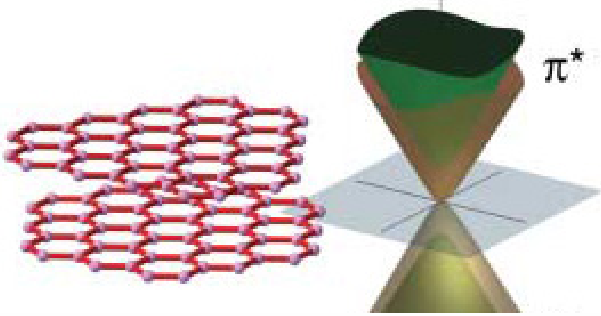
Multi-Layer Graphene [*M. Koshino & T. Ando, PRB* <u>*76*</u>*, 085425 (2007)*]

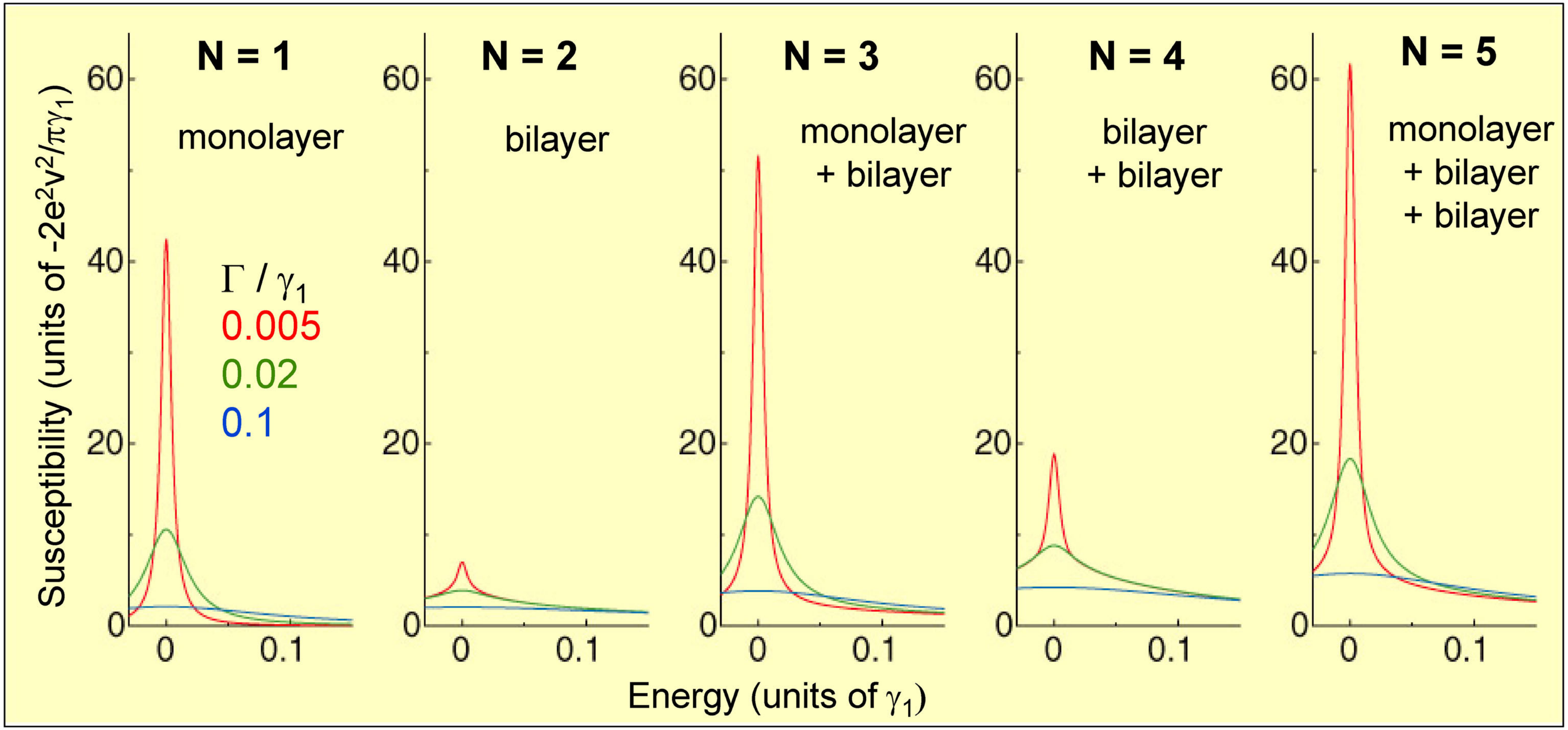
Exact decomposition of effective Hamiltonian

$2M+1$ **Layers** $=$ $1\times$ **Monolayer** $+$ $M\times$ **Bilayers**

$2M$ **Layers** $=$ $0\times$ **Monolayer** $+$ $M\times$ **Bilayers**

Three parameters: γ_0, γ_1, γ_3 **(trigonal warping)**





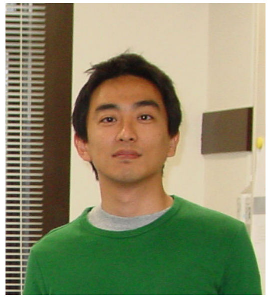
Diamagnetic susceptibility

Summary: Exotic Transport Properties of Graphene and Nanotube

Collaborators

N.H. Shon
(Vietnam)

Y. Zheng
(China)



M. Koshino
(Titech)

T. Nakanishi
(AIST)

H. Suzuura
(Hokkaido Univ)

1. Weyl's equation for neutrino
2. Rise of graphene
   - Fabrication and quantum Hall effect
   - Landau-level spectroscopy
   - Local density of states
   - Optical phonon, Spin transport, $\cdots$
3. Transport in graphene and nanotube
   - Berry's phase & topological anomaly
   - Zero mode anomalies
   - Special time reversal symmetry
4. Multi-layer graphene
   - Hamiltonian decomposition

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www.stat.phys.titech.ac.jp/ando/
www.stat.phys.titech.ac.jp/~ando/reprint/graphene/reprints.htm