Center for
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University of Augsburg

Superfluid Helium-3:

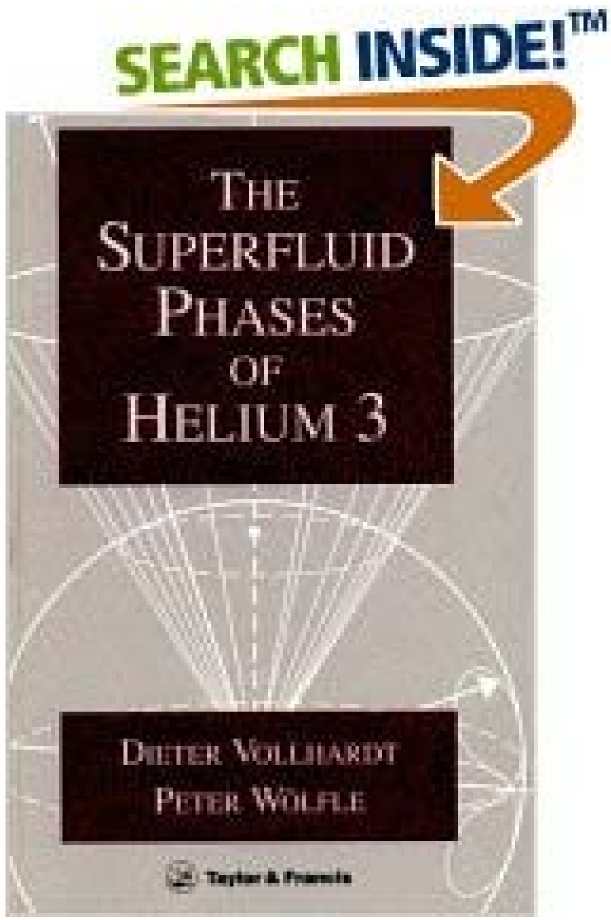
From very low Temperatures to the Big Bang

Dieter Vollhardt

Yukawa Institute, Kyoto; November 27, 2007

Contents:

- The quantum liquids ^3He and ^4He
- Superfluid phases of ^3He
- Broken symmetries and long-range order
- Topological defects
- Big Bang simulation in the low temperature lab



The Superfluid Phases of Helium 3
Dieter Vollhardt and Peter Wölfle
(Taylor and Francis, 1990)

Helium

Two stable Helium isotopes:

Janssen/Lockyer (1868)

^{4}He: air, oil wells, … Ramsay (1895)

^{3}He: $^6_3Li + ^1_0n \rightarrow ^3_1H + \alpha$ (1939)

$$\downarrow$$

$$^3_2He + e^-$$

$$\frac{\text{He}}{\text{Luft}} \approx 5 \times 10^{-6}, \quad \left.\frac{^3\text{He}}{^4\text{He}}\right|_{\text{Luft}} \approx 1 \times 10^{-6}$$

Research on macroscopic samples of ^{3}He since 1947

Helium

Atoms: spherical, hard core diameter ~ 2.5 Å

Interaction:
- hard sphere repulsion
- van der Waals dipole (+ ...) attraction

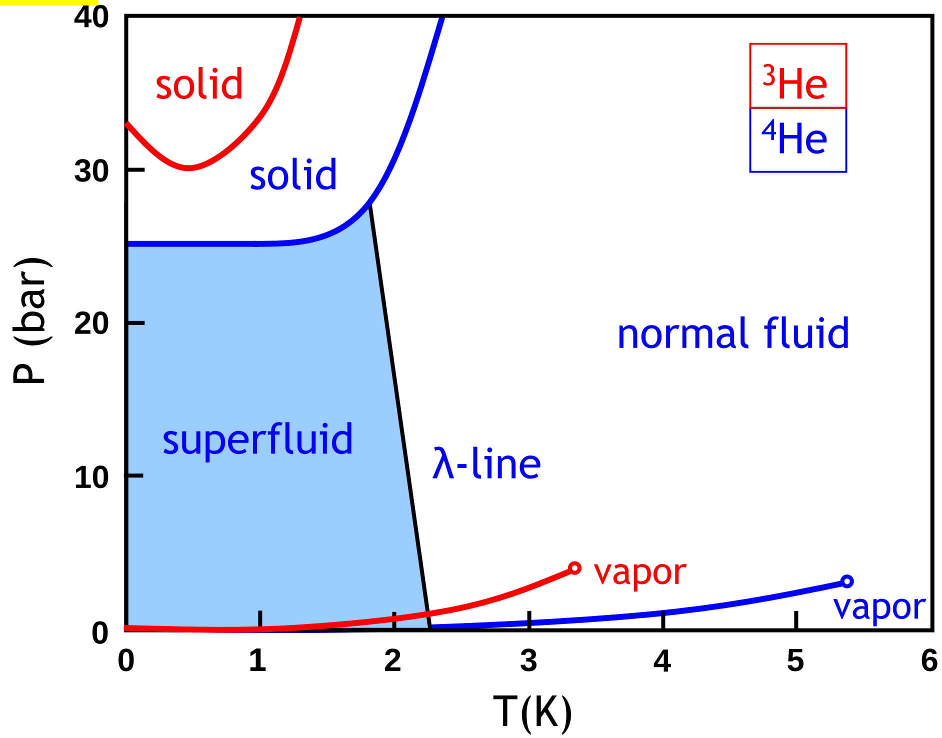
Boiling point: 4.2 K, ^4He Kamerlingh Onnes (1908)

Nobel Prize 1913

3.2 K, ^3He Sydoriak, *et al.* (1949)

Dense, simple liquid
- isotropic
- short-range interactions
- extremely pure
- nuclear spin S=1/2

Helium

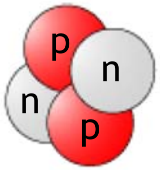
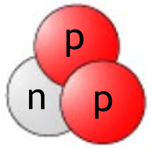


Atoms:
- spherical shape → weak attraction
- light mass → strong zero-point motion

T→0, P $\lesssim$ 30 bar: Helium remains liquid

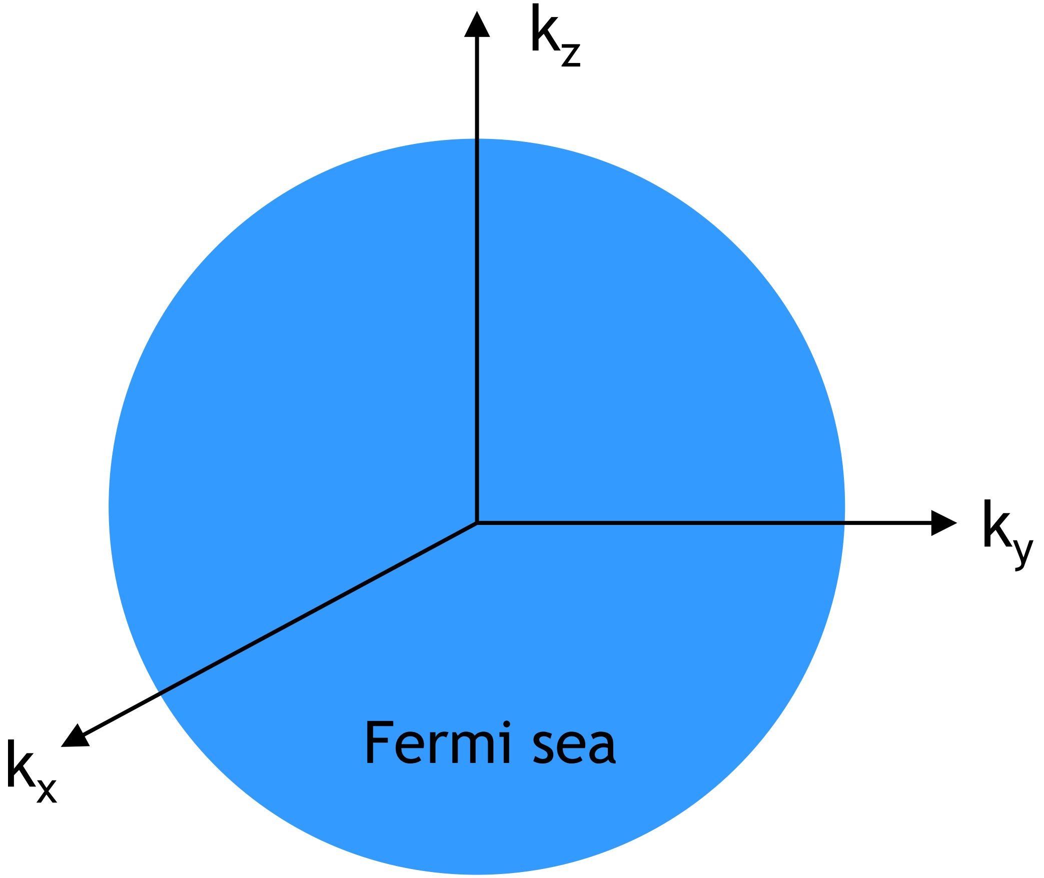
$$\lambda \propto \frac{\hbar}{\sqrt{k_B T}} \xrightarrow{T\to 0}$$ Macroscopic quantum phenomena

Helium

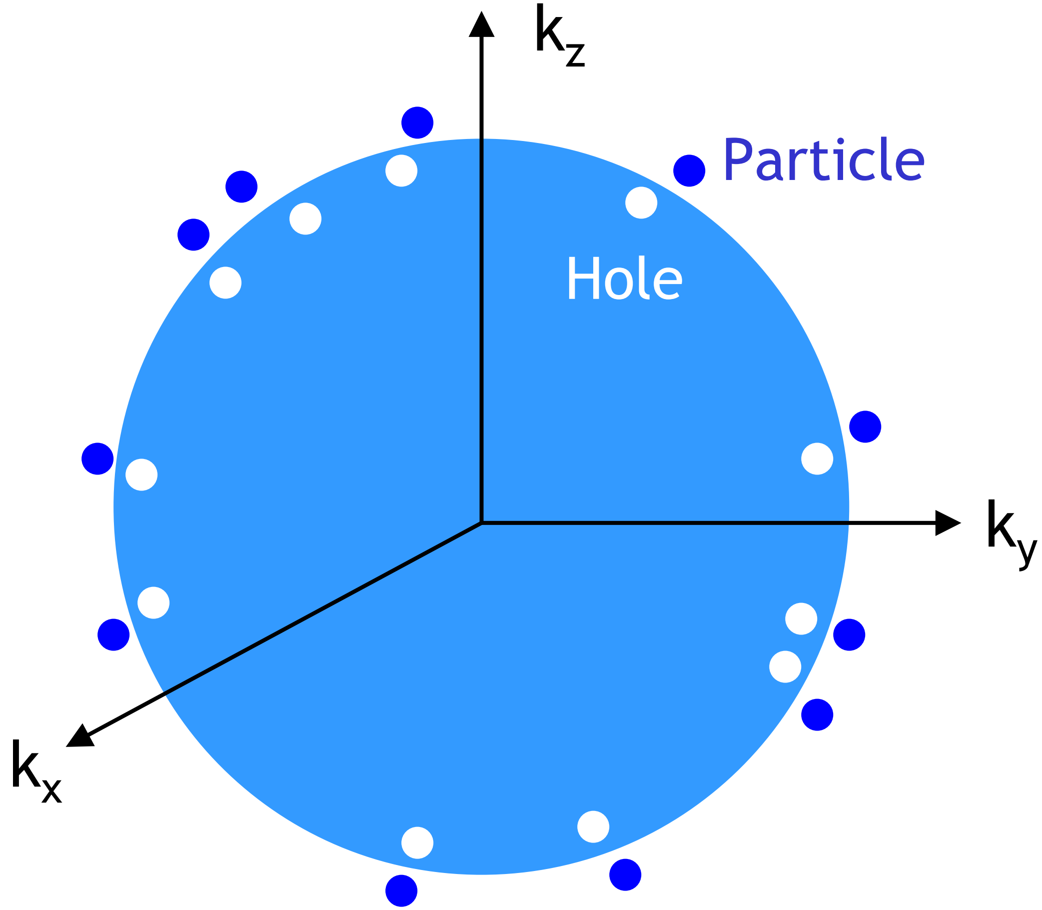
	^4He	^3He
Electron shell:	$2\ e^-$, $S = 0$	
Nucleus:		
	$S = 0$	$S = \frac{1}{2}\hbar$
Atom(!) is a	Boson	Fermion → Quantum liquids
Phase transition	$T_\lambda = 2.2$ K ("BEC")	T_c = ???

Fermi liquid theory

Fermi gas: **Ground state**



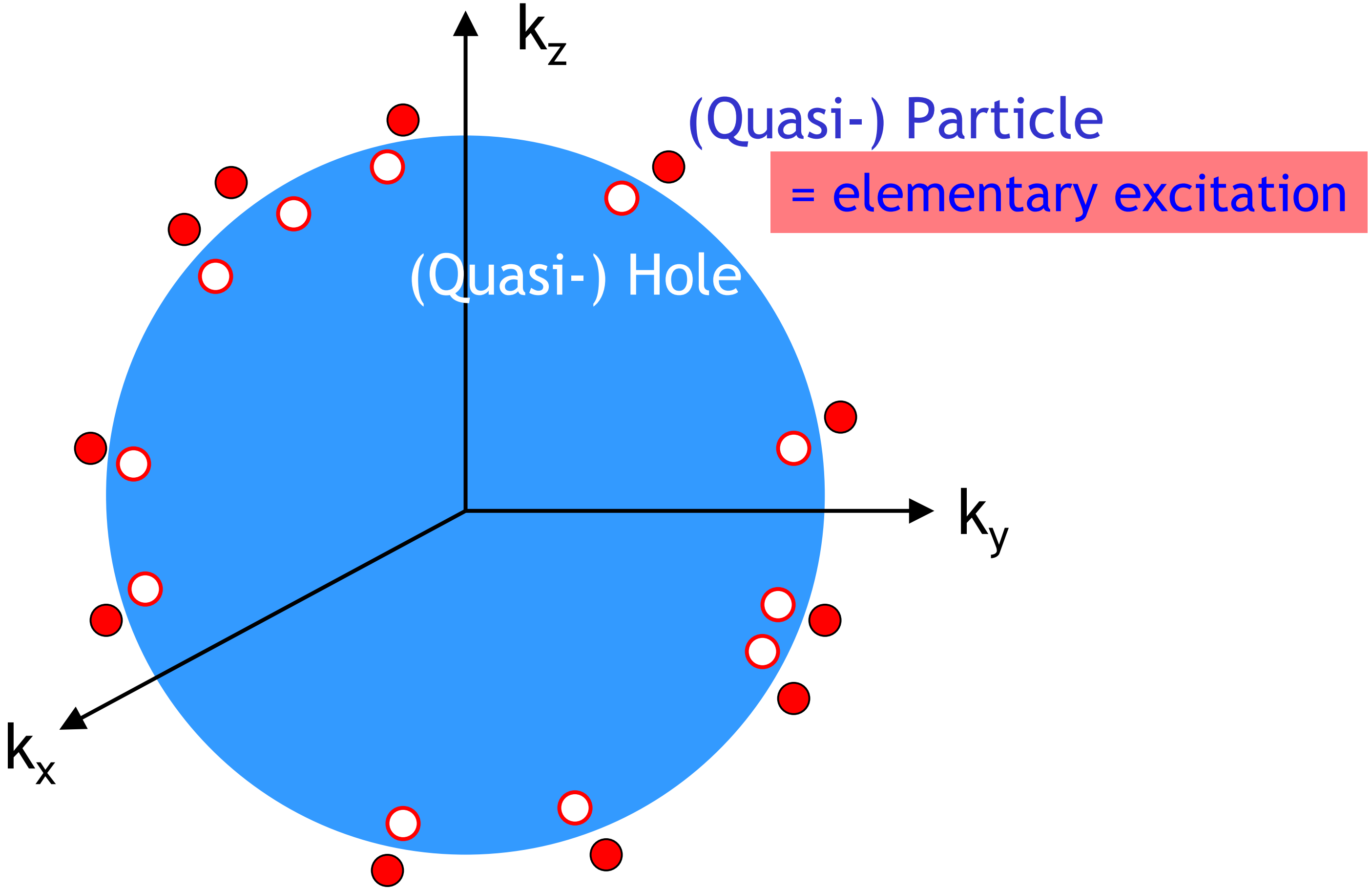
Fermi gas: **Excited states (T>0)**



Switch on interaction adiabatically

Landau Fermi liquid

1-1 correspondence
between states

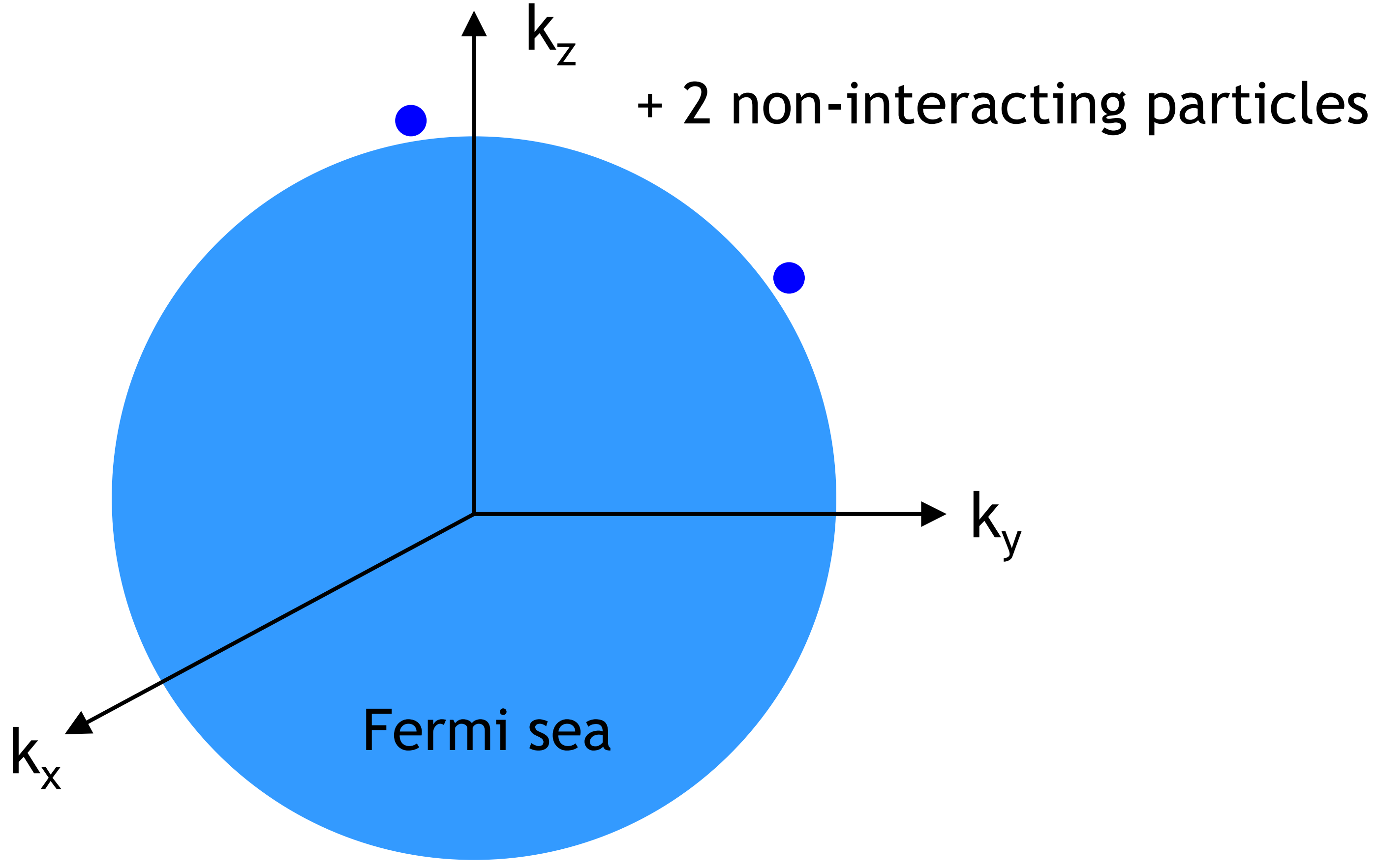


Prototype: Helium-3

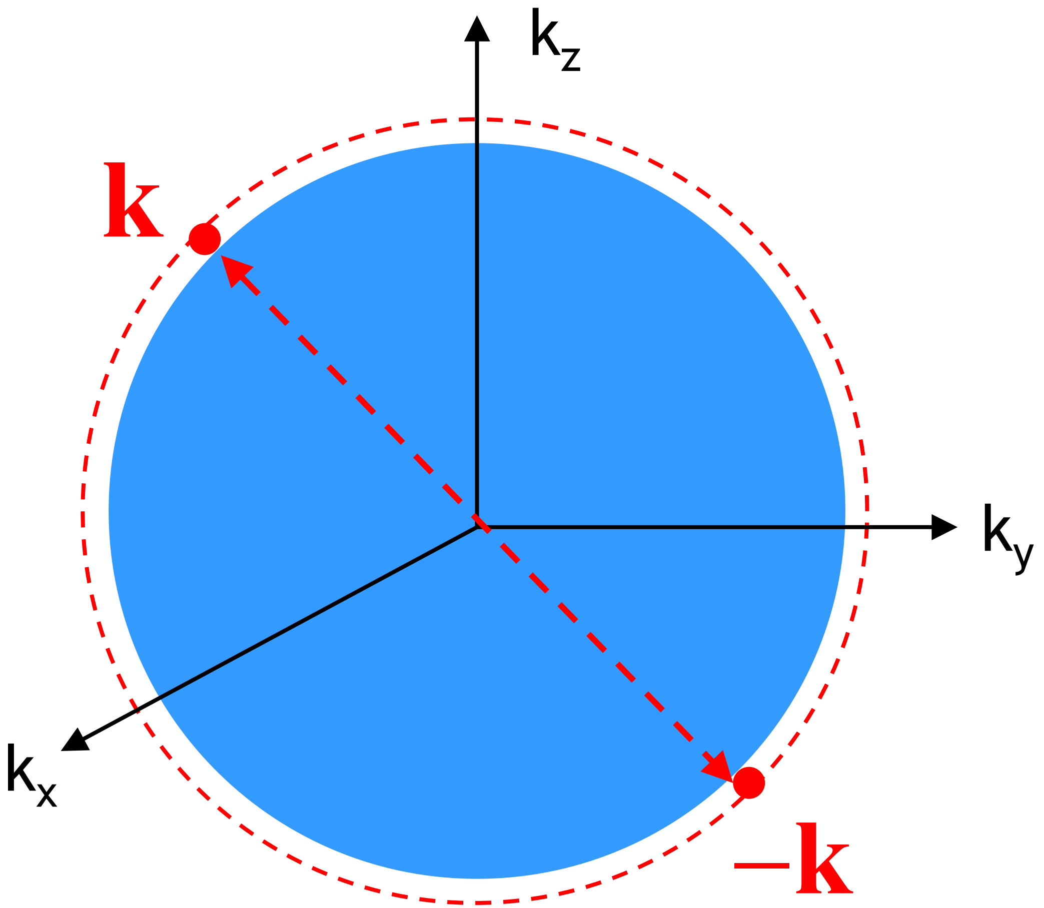
- Large effective mass
- Strongly enhanced spin susceptibility
- Strongly reduced compressibility

Instability ?

Instability of Landau Fermi liquid

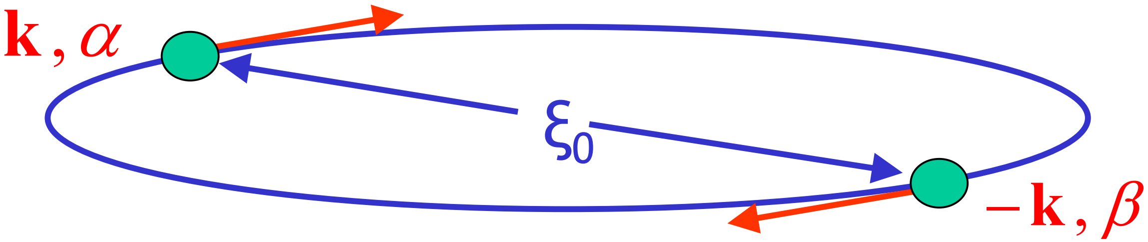


Arbitrarily weak attraction ⟹ Cooper instability



Universal fermionic property

Arbitrarily weak attraction $\Rightarrow$ Cooper pair $(\mathbf{k}, \alpha; -\mathbf{k}, \beta)$



$$\Psi_{L=0,2,4,...} = \psi(\mathbf{r}) \left| \uparrow\downarrow - \downarrow\uparrow \right\rangle$$

S=0 (singlet)

$$\Psi_{L=1,3,5,...} = \psi_+(\mathbf{r}) \left| \uparrow\uparrow \right\rangle + \psi_0(\mathbf{r}) \left| \uparrow\downarrow + \downarrow\uparrow \right\rangle + \psi_-(\mathbf{r}) \left| \downarrow\downarrow \right\rangle$$

S=1 (triplet)

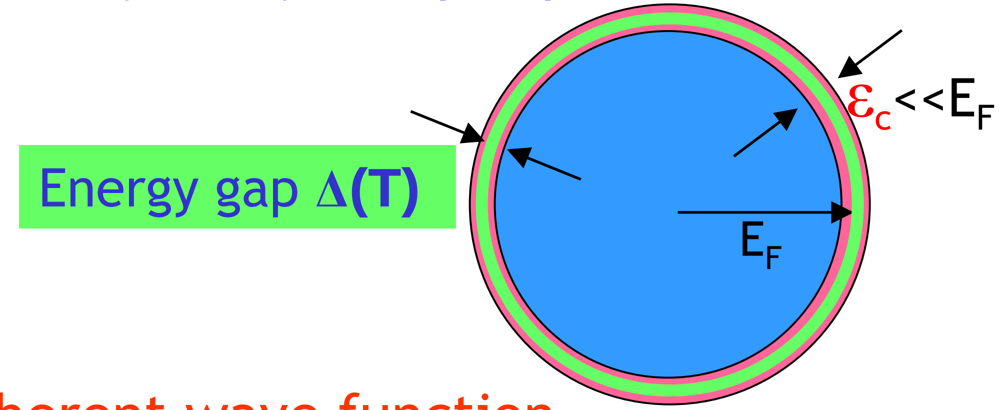
L = 0: isotropic wave function
L > 0: anisotropic wave function

Helium-3: Strongly repulsive interaction → L > 0 expected

BCS theory

Bardeen, Cooper, Schrieffer (1957)

Generalization to macroscopically many Cooper pairs



→ "Pair condensate"
with macroscopically coherent wave function

Transition temperature $$T_c = 1.13\, \varepsilon_c \exp(-1/N(0)|V_L|)$$

"weak coupling theory"

ε_c, V_L: Magnitude ? Origin ? → T_c ?

Thanksgiving 1971: Transition in ^{3}He at $T_c = 0.0026$ K

Osheroff, Richardson, Lee (1972)

The Nobel Prize in Physics 1996

"for their discovery of superfluidity in helium-3"

David M. Lee
USA, Cornell

b. 1931

Douglas D. Osheroff USA, Stanford

b. 1945

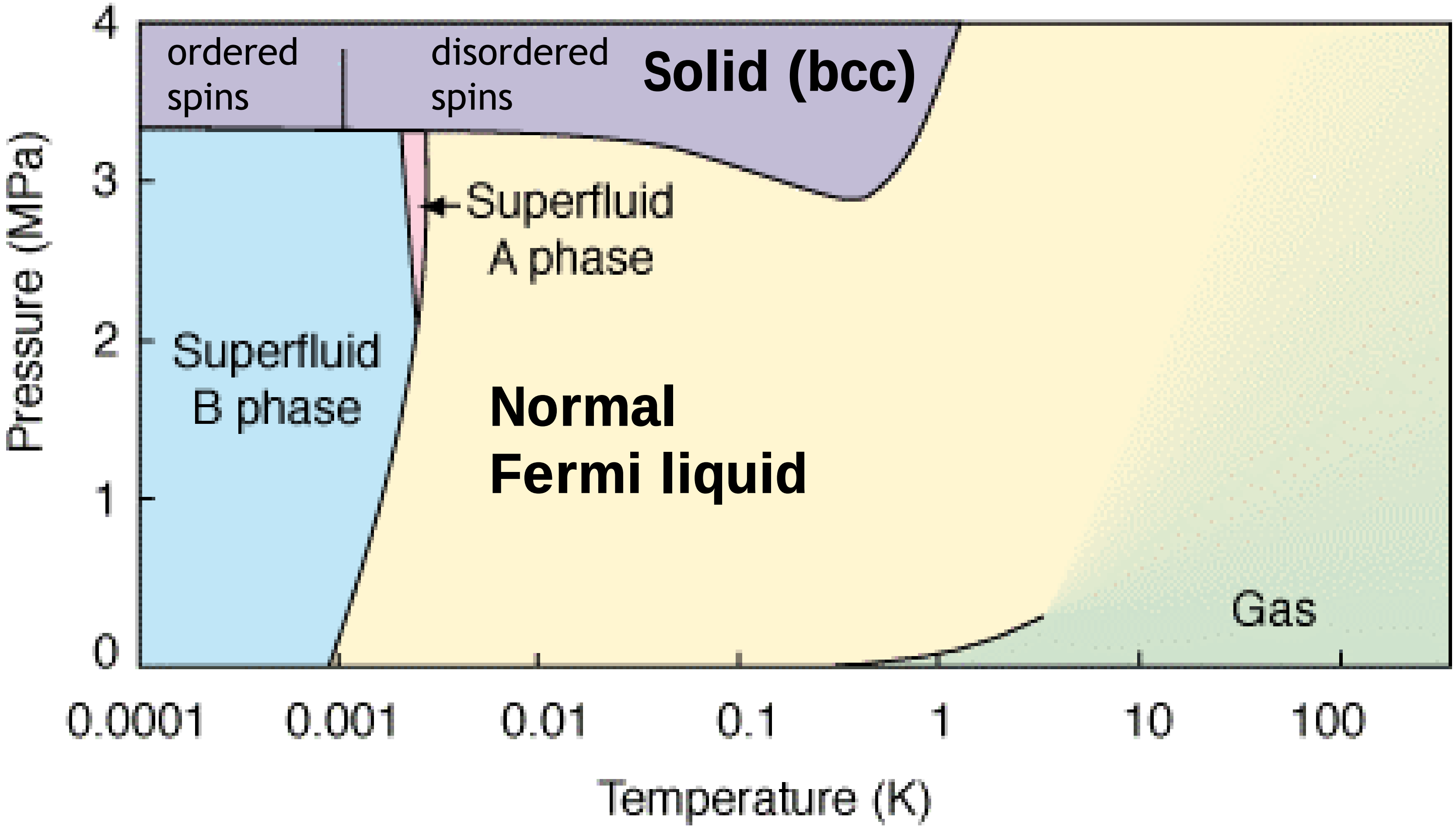
Robert C. Richardson
USA, Cornell

b. 1937

Phase diagram of Helium-3

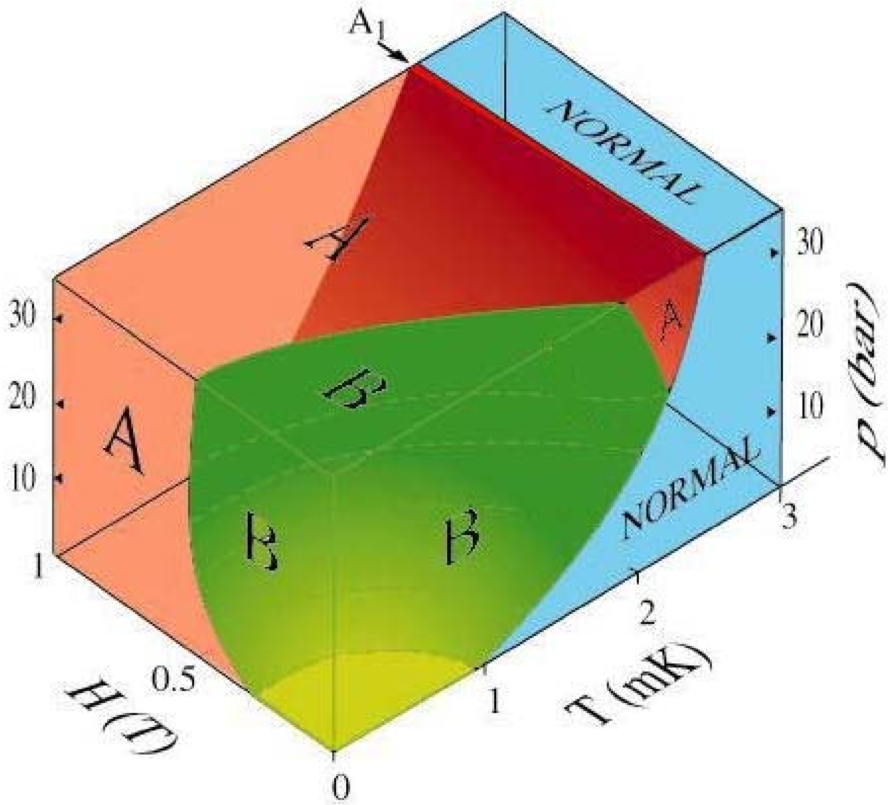
P-T phase diagram

Dense, simple liquid
- isotropic
- short-range interactions
- extremely pure
- nuclear spin S=1/2



Phase diagram of Helium-3

P-T-H phase diagram

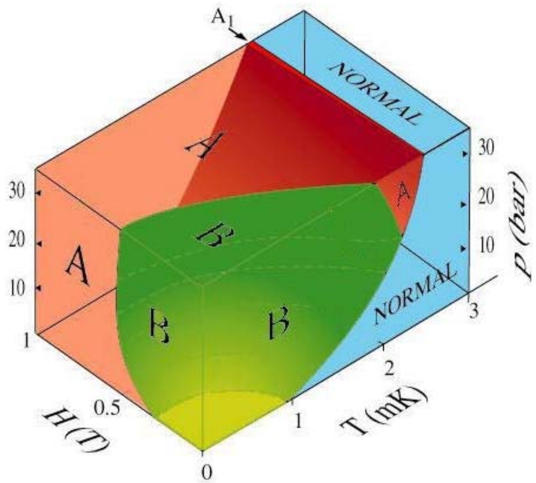


"Very low temperatures": T << $T_{boiling}$ ~ 3-4 K
<< $T_{backgr.\ rad.}$ ~ 3 K

Superfluid phases of ^3He

Theory + experiment: L=1, S=1 in all phases

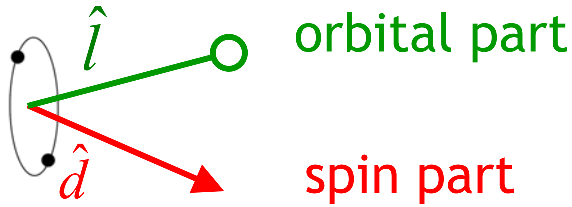
Leggett
Wölfle
Mermin, ...



Pairing mechanism: Spin fluctuations

Anderson, Brinkman (1973)

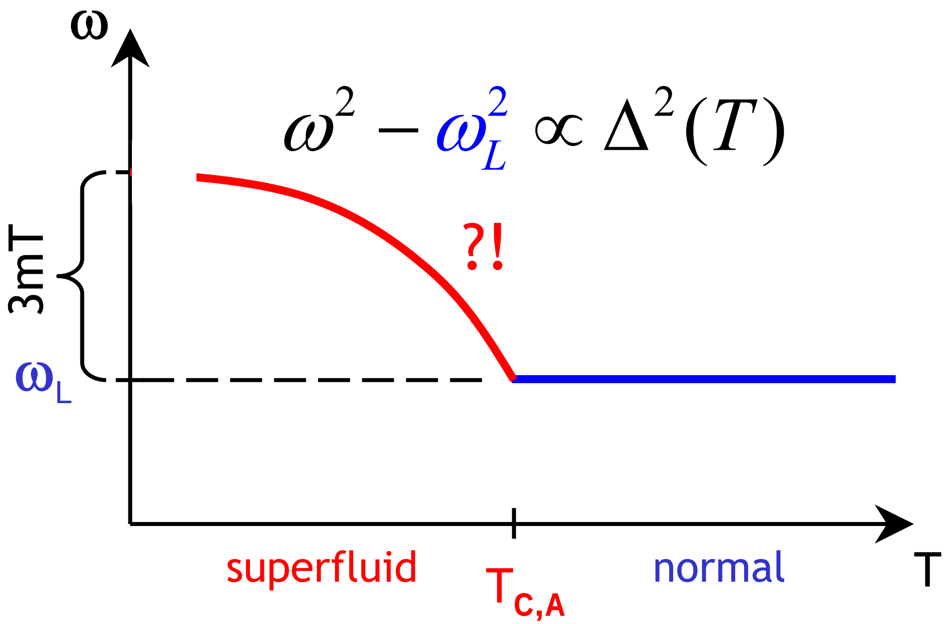
→ anisotropy directions in a ^3He Cooper pair



… and a mystery!

NMR experiment on nuclear spins $I = \frac{1}{2}\hbar$

Osheroff *et al.* (1972)



← Larmor frequency: $\omega_L = \gamma H$

Shift of ω_L ⇔ spin-nonconserving interactions

→ nuclear dipole interaction $g_D \sim 10^{-7} K \ll T_C$

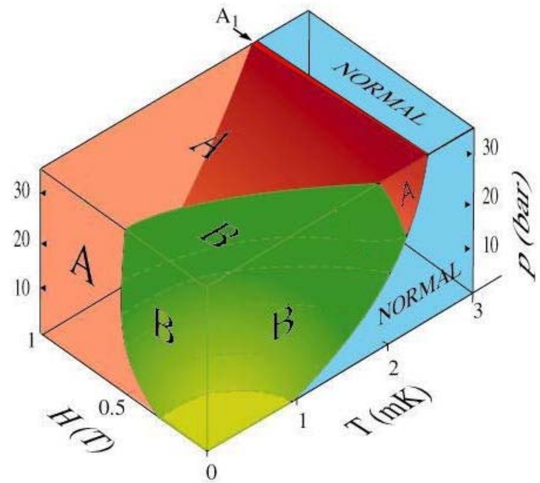
Origin of frequency shift ?!

Leggett (1973)

The superfluid phases of ^3He

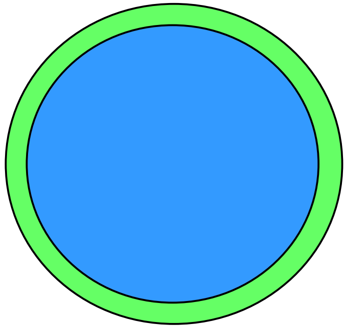
B-phase

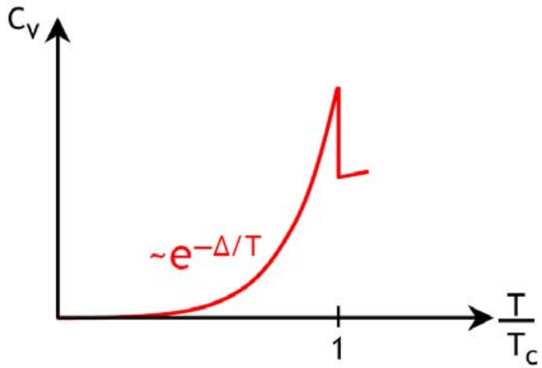
$$\Psi = |\uparrow\uparrow\rangle + |\uparrow\downarrow + \downarrow\uparrow\rangle + |\downarrow\downarrow\rangle$$



$$\Delta(\mathbf{k}) = \Delta_0$$

Balian, Werthamer (1963)
Vdovin (1963)





(pseudo-) isotropic state ↔ s-wave superconductor

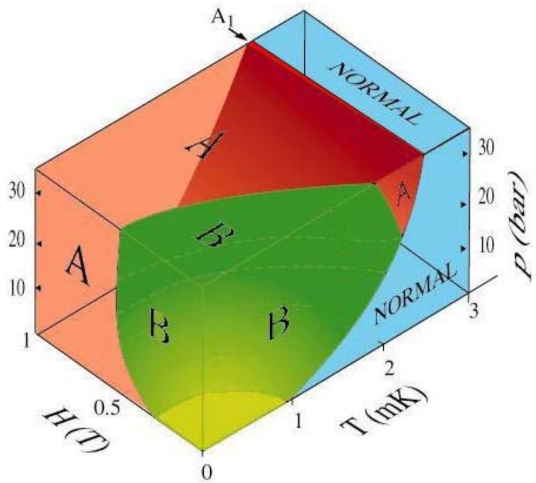
Weak-coupling theory: stable for all $T<T_c$

A-phase

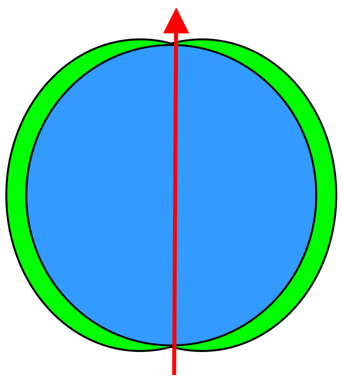
$\Psi = |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$ → strong anisotropy

$$\Delta(\hat{k}) = \Delta_0 \sin(\hat{k}, \hat{l})$$

Anderson, Morel (1961)



$\hat{l}$ Cooper pair orbital angular momentum



Axial state has point nodes

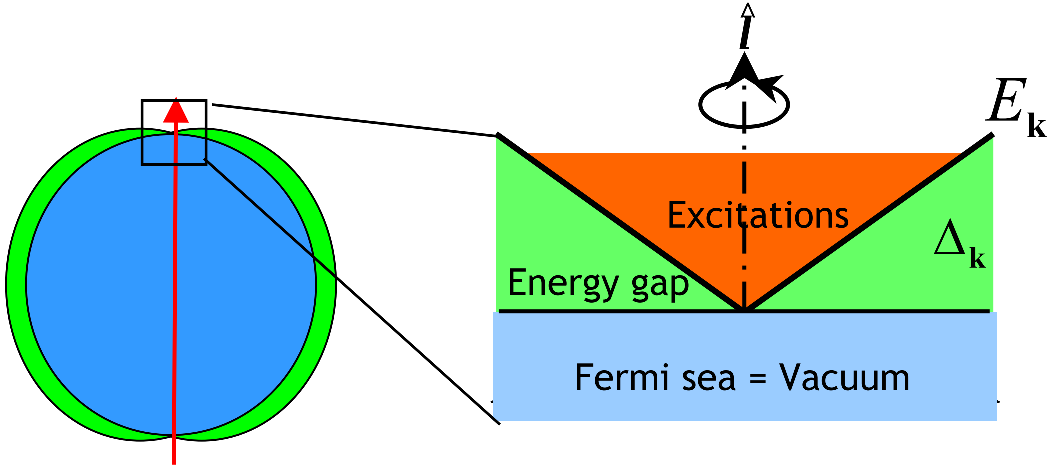
→ "unconventional" pairing in

- heavy fermion/high-T_c superconductors
- Sr_2RuO_4

Strong-coupling effect

^{3}He-A: Spectrum near poles

Volovik (1987)



$$E_{\mathbf{k}}^2 = \mathrm{v}_{\mathrm{F}}^2 \left(k - k_F\right)^2 + \Delta_0^2 \sin^2(\hat{k},\hat{l}) = \mathrm{g}^{\mathrm{ij}} p_i p_j$$

$$e = \begin{cases} +1 & \hat{k} \parallel +\hat{l} \\ -1 & \hat{k} \parallel -\hat{l} \end{cases}$$

2 chiralities

$$\mathrm{g}^{\mathrm{ij}} = \mathrm{v}_{\mathrm{F}}^2 l_i l_j + \left(\frac{\Delta}{k_F}\right)^2 (\delta_{ij} - l_i l_j)$$

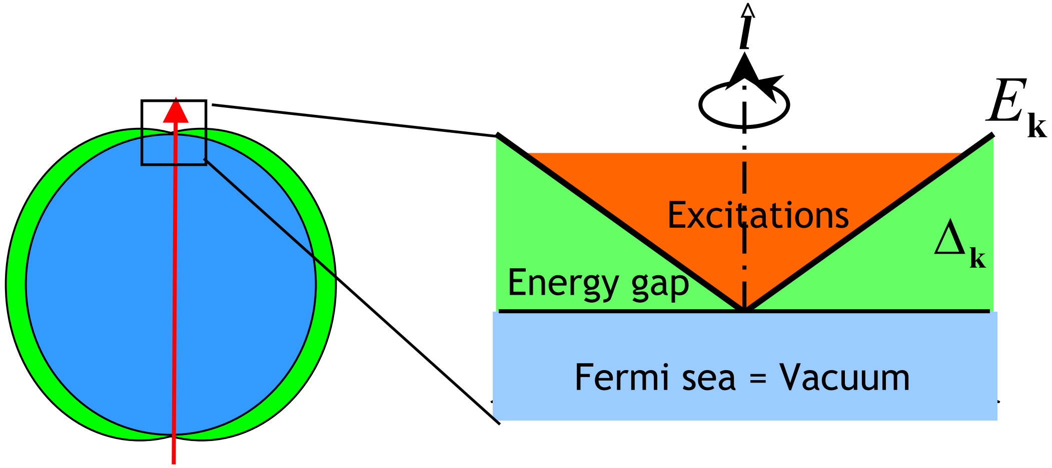
$$\mathbf{A} = k_F \hat{l}$$

$$\mathbf{p} = \mathbf{k} - e\mathbf{A}$$

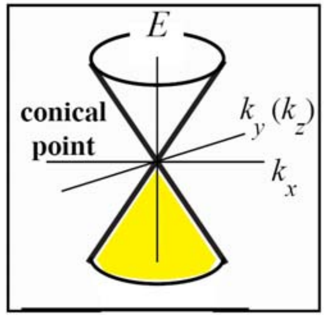
Lorentz invariance:
Symmetry enhancement
at low energies

^{3}He-A: Spectrum near poles

Volovik (1987)



⇔



Fermi point:
spectral flow

$$E_{\mathbf{k}}^2 = \mathrm{v}_F^2 \left(k - k_F\right)^2 + \Delta_0^2 \sin^2(\hat{k}, \hat{l}) = g^{ij} p_i p_j$$

$$e = \begin{cases} +1 & \hat{k} \parallel +\hat{l} \\ -1 & \hat{k} \parallel -\hat{l} \end{cases}$$

2 chiralities

$$g^{ij} = \mathrm{v}_F^2 l_i l_j + \left(\frac{\Delta}{k_F}\right)^2 (\delta_{ij} - l_i l_j)$$

⇔ Massless, chiral leptons, e.g., neutrino $E(\mathbf{p}) = cp$

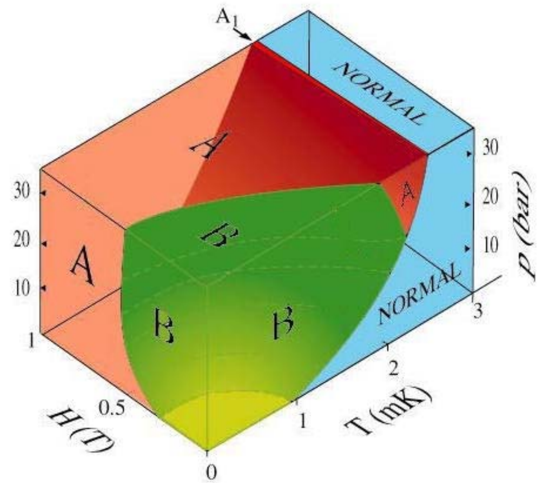
→ Chiral anomaly of standard model

The Universe in a Helium Droplet, Volovik (2003)

A_1-phase

finite magnetic field

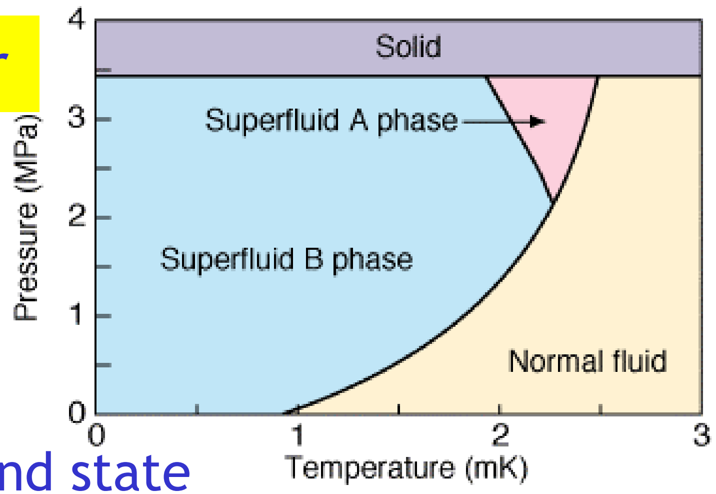
$$\Psi = |\uparrow\uparrow\rangle$$



Long-range ordered magnetic liquid

Broken Symmetries, Long Range Order

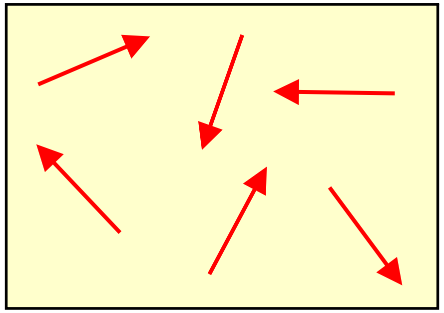
Broken Symmetries, Long Range Order



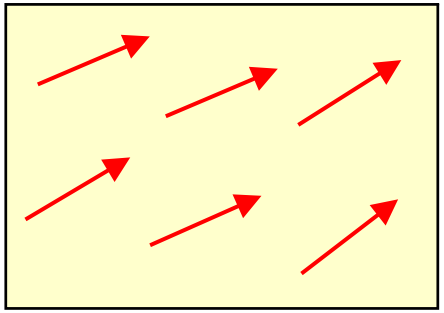
Normal ^{3}He $\leftrightarrow$ ^{3}He-A, ^{3}He-B:
2. order phase transition

T<T_c: higher order, lower symmetry of ground state

I. Ferromagnet



T>T_c



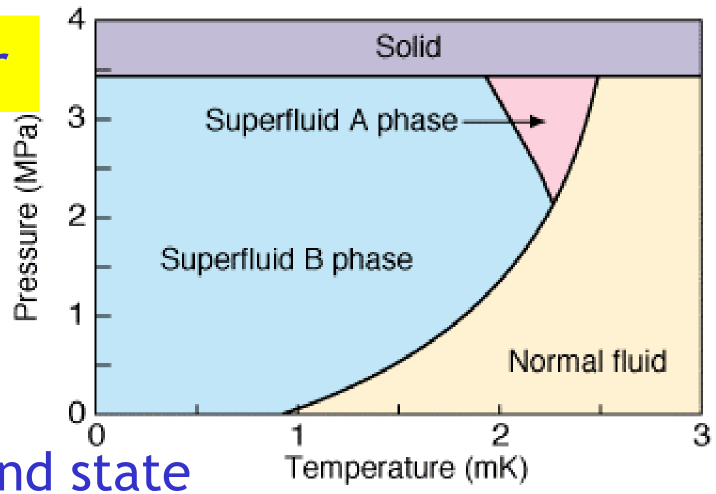
T<T_c

Average magnetization: $\langle \mathbf{M} \rangle = 0$ $\langle \mathbf{M} \rangle \neq 0$ Order parameter

Symmetry group: SO(3) U(1)$\subset$ SO(3)

T<T_C: SO(3) rotation symmetry in spin space spontaneously broken

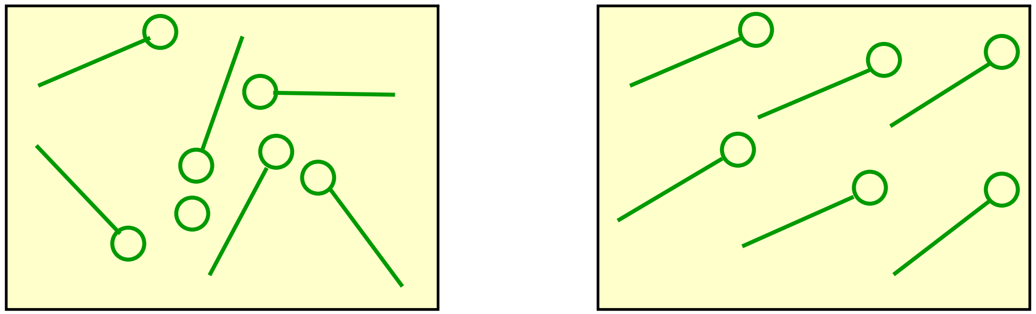
Broken Symmetries, Long Range Order



2. order phase transition

$T<T_c$: higher order, lower symmetry of ground state

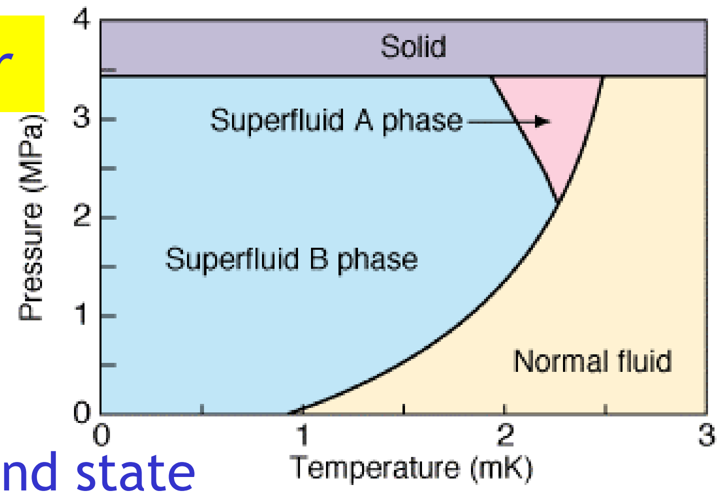
II. Liquid crystal



	$T>T_c$	$T<T_c$
Symmetry group:	SO(3)	$U(1) \subset SO(3)$

$T<T_C$: SO(3) rotation symmetry in real space spontaneously broken

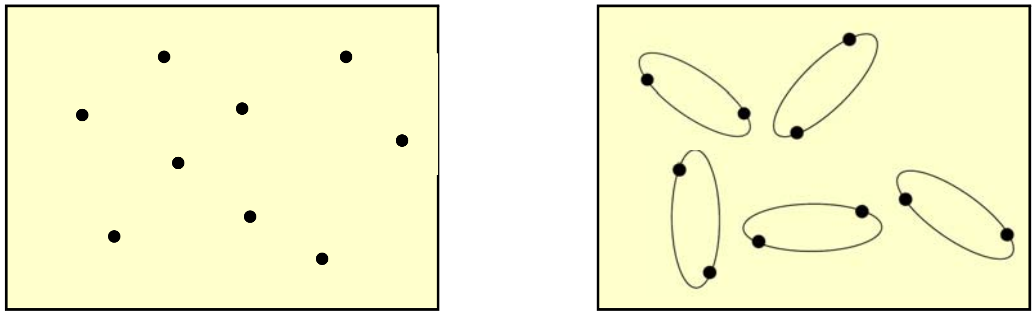
Broken Symmetries, Long Range Order



2. order phase transition

T<T_c: higher order, lower symmetry of ground state

III. Conventional superconductor



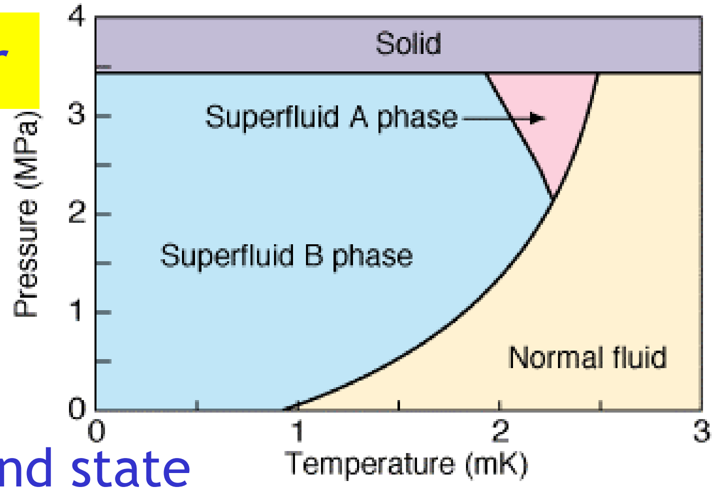
T>T_c T<T_c

Pair amplitude $\left\langle c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \right\rangle =$ 0 $\Delta e^{i\phi}$ Order parameter

Gauge transf. $c^\dagger_{\mathbf{k}\sigma} \to c^\dagger_{\mathbf{k}\sigma} e^{i\varphi}$

gauge invariant not gauge invariant

Broken Symmetries, Long Range Order



2. order phase transition

$T<T_c$: higher order, lower symmetry of ground state

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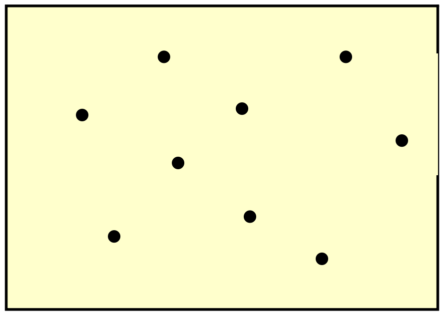
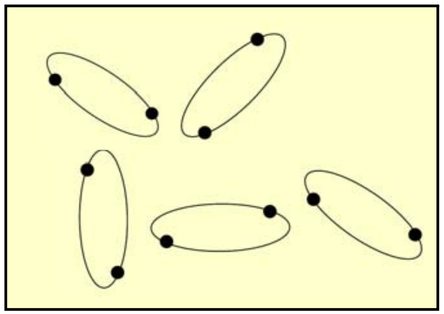


$T>T_c$



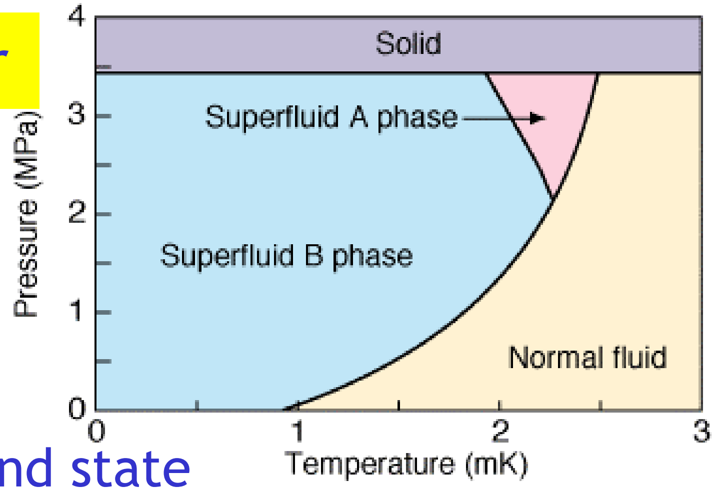
$T<T_c$

Pair amplitude $\left\langle c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} \right\rangle =$ 0 $\quad \Delta e^{i\phi}$ Order parameter

Gauge transf. $\quad c^{\dagger}_{\mathbf{k}\sigma} \rightarrow c^{\dagger}_{\mathbf{k}\sigma} e^{i\varphi}$

Symmetry group: U(1) ---

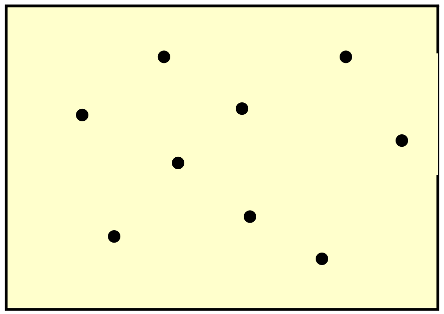
Broken Symmetries, Long Range Order



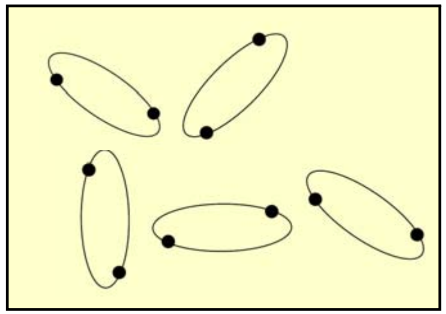
2. order phase transition

$T<T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor



$T>T_c$



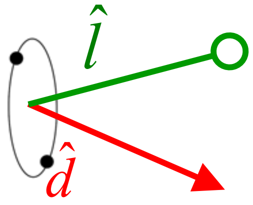
$T<T_c$

$T<T_C$: U(1) “gauge symmetry“ spontaneously broken

Broken symmetries in superfluid ^{3}He

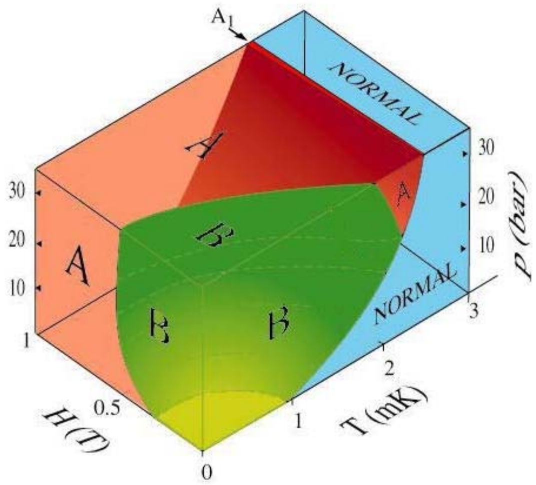
L=1, S=1 in all phases

Cooper pair:



orbital part

spin part



Quantum coherence in	phase anisotropy direction for spin anisotropy direction in real space	Superfluid, magnetic liquid crystal

Characterized by $2\times(2L+1)\times(2S+1)$ = 18 real numbers

3x3 order parameter matrix $A_{i\mu}$

$SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry spontaneously broken

Leggett (1975)

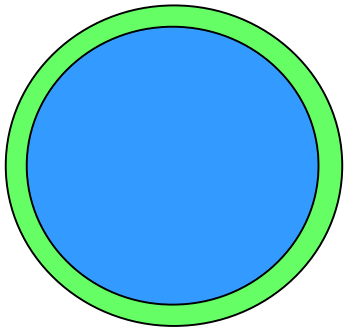
Broken symmetries in superfluid ^{3}He

Mineev (1980)
Bruder, DV (1986)

3He-B $SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry broken

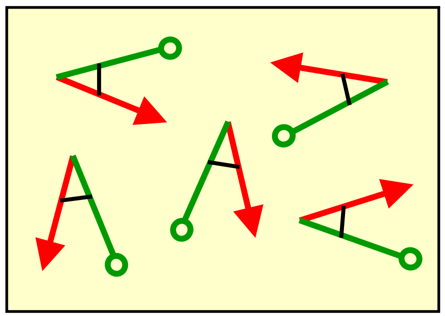
$SO(3)_{S+L}$ –

„Unconventional" pairing



“Spontaneously broken spin-orbit Symmetry“ Leggett (1972)

Cooper pairs



Fixed relative orientation

Broken symmetries in superfluid ^{3}He

Mineev (1980)
Bruder, DV (1986)

3He-B $SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry broken

$SO(3)_{S+L}$ — „Unconventional" pairing

Relation to high energy physics

Isodoublet $\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} u \\ d \end{pmatrix}_R$ chiral invariance

Global symmetry $SU(2)_L \times SU(2)_R$

$q\bar{q}$ condensation ("Cooper pairing")

$SU(2)_{L+R}$

Goldstone bosons 3 pions

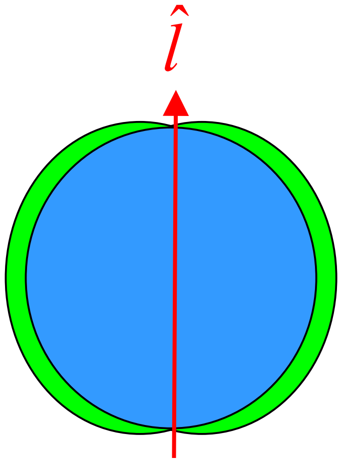
Broken symmetries in superfluid ^{3}He

Mineev (1980)
Bruder, DV (1986)

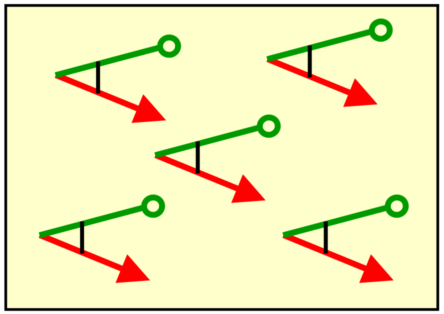
3He-A $SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry broken

$U(1)_{S_z} \times U(1)_{L_z - \varphi}$

„Unconventional" pairing



Cooper pairs

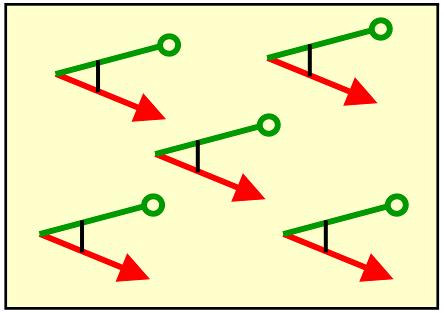


Fixed absolute orientation

Resolution of the NMR puzzle: Macroscopic quantum amplification

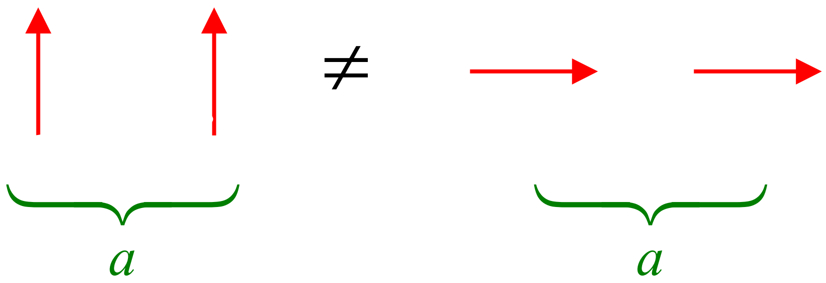
Superfluid ^{3}He as quantum amplifier

Cooper pairs in ^{3}He-A



Coupling of $\hat{d}$, $\hat{l}$?

→ Nuclear dipole interaction



anisotropic!

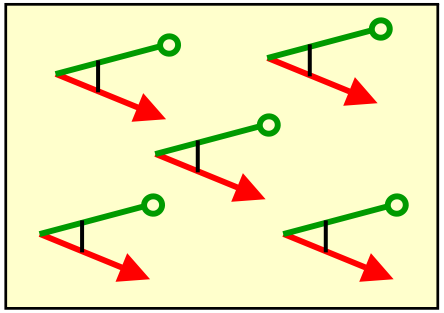
→ spin-orbit coupling

Dipole interaction energy of ^{3}He nuclei: $g_D \sim \frac{\mu^2}{a^3} \approx 10^{-7} K \ll T_C$

Unimportant ?!

Superfluid ^{3}He as quantum amplifier

Cooper pairs in ^{3}He-A



- Long-range order in $\hat{d}, \hat{l}$
- $g_D \sim 10^{-7} K$ lifts degeneracy of relative orientation

Quantum $\Downarrow$ coherence

$\hat{d}, \hat{l}$ locked in all Cooper pairs

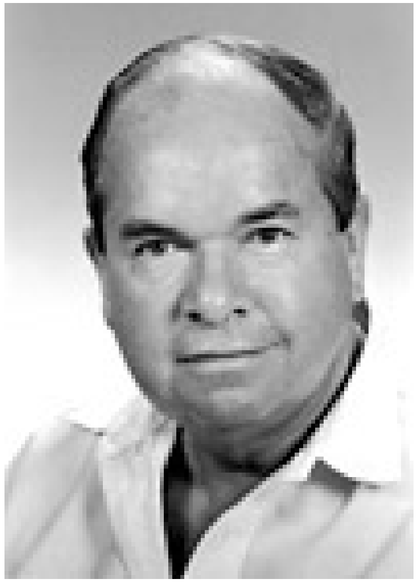
$\Downarrow$

NMR frequency increases: $\omega^2 = (\gamma H)^2 + g_D \Delta^2(T)$ Leggett (1973)

→ Nuclear dipole interaction macroscopically measurable

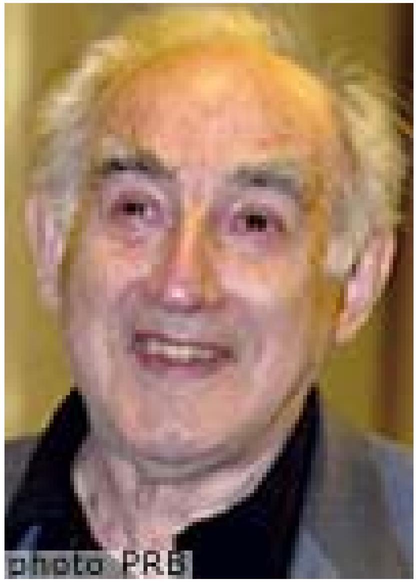
The Nobel Prize in Physics 2003

"for pioneering contributions to the theory of superconductors and superfluids"



Alexei A. Abrikosov, USA and Russia

b. 1928



Vitaly L. Ginzburg, Russia

b. 1916

Anthony J. Leggett, UK and USA

b. 1938

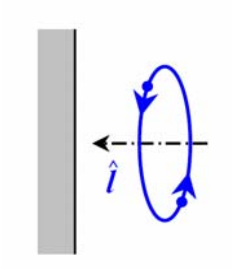
Order parameter textures and topological defects

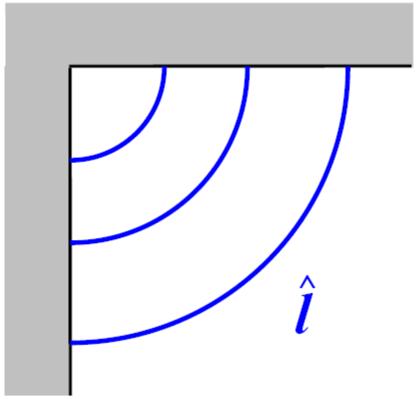
Order parameter textures

Orientation of anisotropy directions $\hat{d}, \hat{l}$ in ^{3}He-A:

Magnetic field → $\hat{d}$

Walls → $\hat{l}$



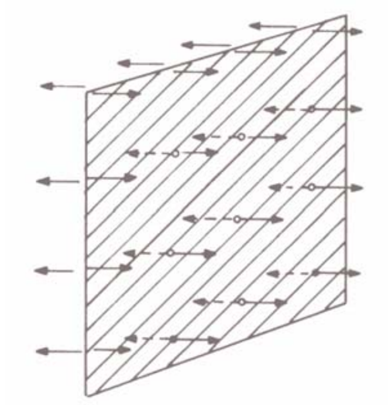


"Textures" in $\hat{d}, \hat{l}$ ↔ liquid crystals

→Topologically stable defects

Order parameter textures and topological defects

D=2: domain walls in $\color{red}{\hat{d}}$ or $\color{green}{\hat{l}}$

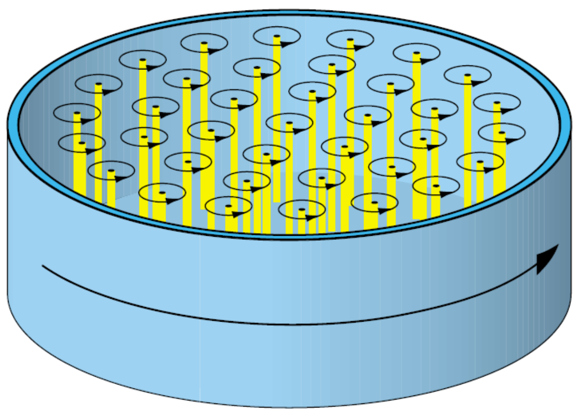


Single domain wall

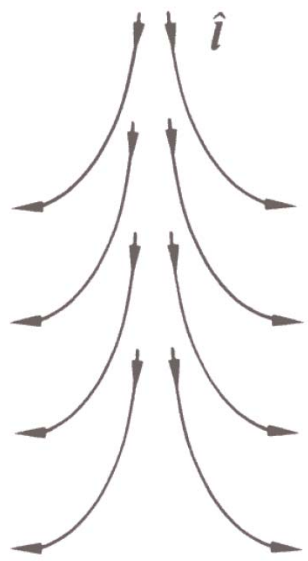
Domain wall lattice

Order parameter textures and topological defects

D=1: Vortices



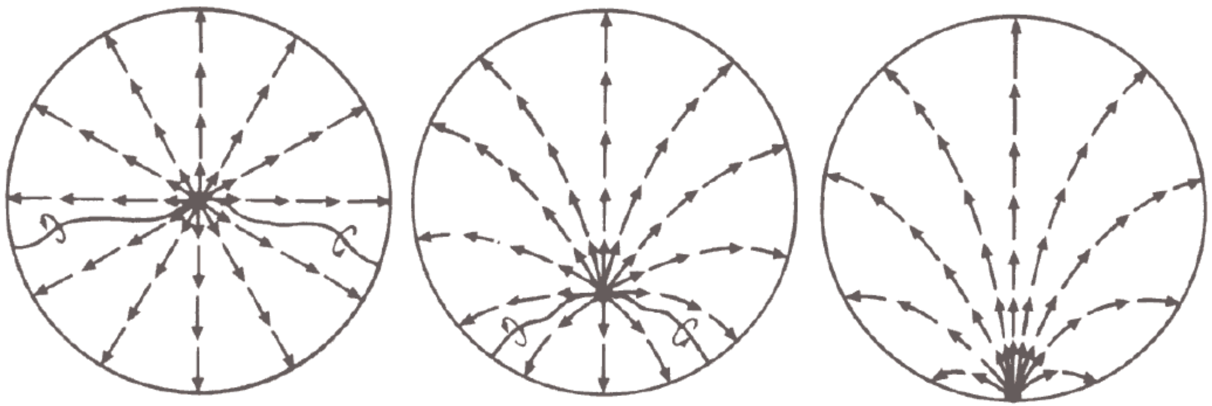
Vortex formation in rotation experiments



e.g., Mermin-Ho vortex
(non-singular)

Order parameter textures and topological defects

D=0: Monopoles



e.g., “boojum” in $\hat{l}$-texture of ^{3}He-A

Defect formation by, e.g.,
- geometric constraints
- rotation
- rapid crossing through phase transition

Universality in continuous phase transitions

High symmetry, short-range order $T>T_c$

Spins: para-magnetic

Helium: normal liquid

Universe: Unified forces and fields

$T=T_c$ Phase transition

Broken symmetry, long-range order

ferromagnetic

superfluid

elementary particles, fundamental interactions

Defects: domain walls

vortices, etc.

cosmic strings, etc. Kibble (1976)

$T<T_c$

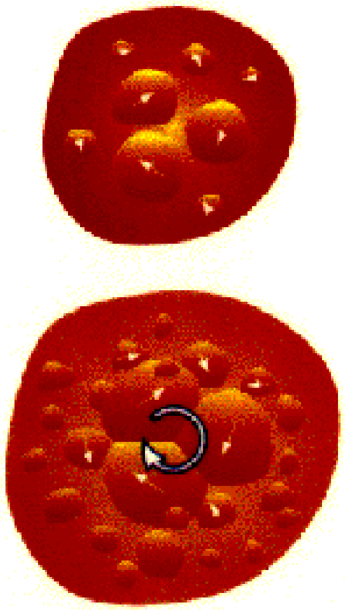
nucleation of galaxies?

Rapid thermal quench through 2. order phase transition — Kibble (1976)

1.

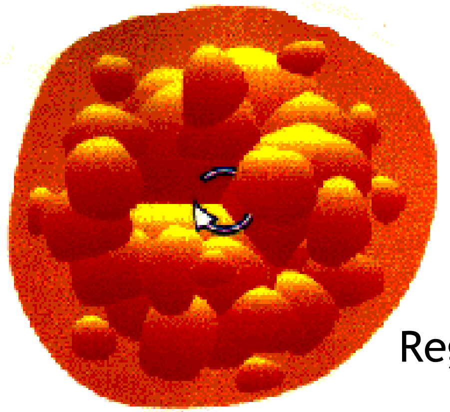
Local temperature $T \gg T_C$

2.



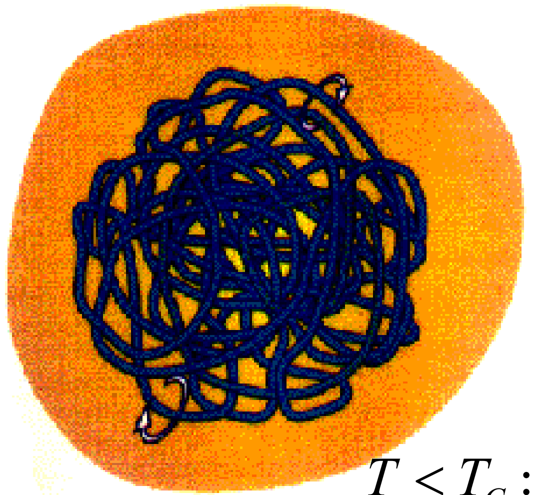
Expansion + rapid cooling
→
Nucleation of independently ordered regions

3.



Regions overlap

4.



$T < T_C$: Vortex tangle

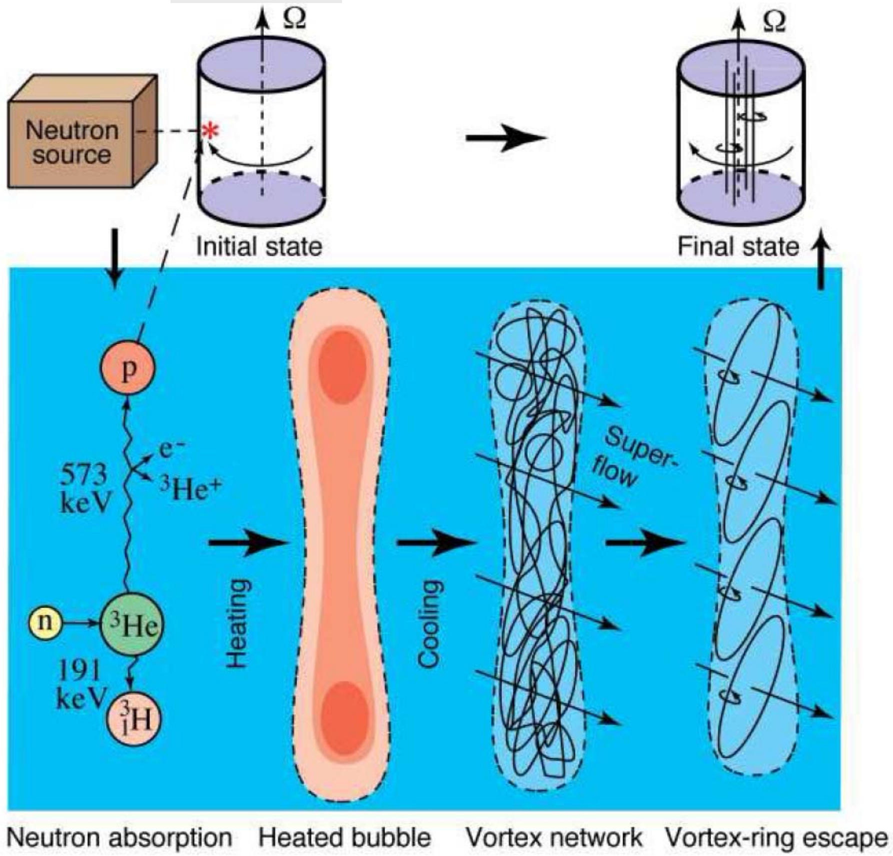
Density of defects — Zurek (1985)

"Kibble-Zurek mechanism": How to test?

Big Bang simulation in the low temperature laboratory

Grenoble: Bäuerle *et al.* (1996), Helsinki: Ruutu *et al.* (1996)

^{3}He-B



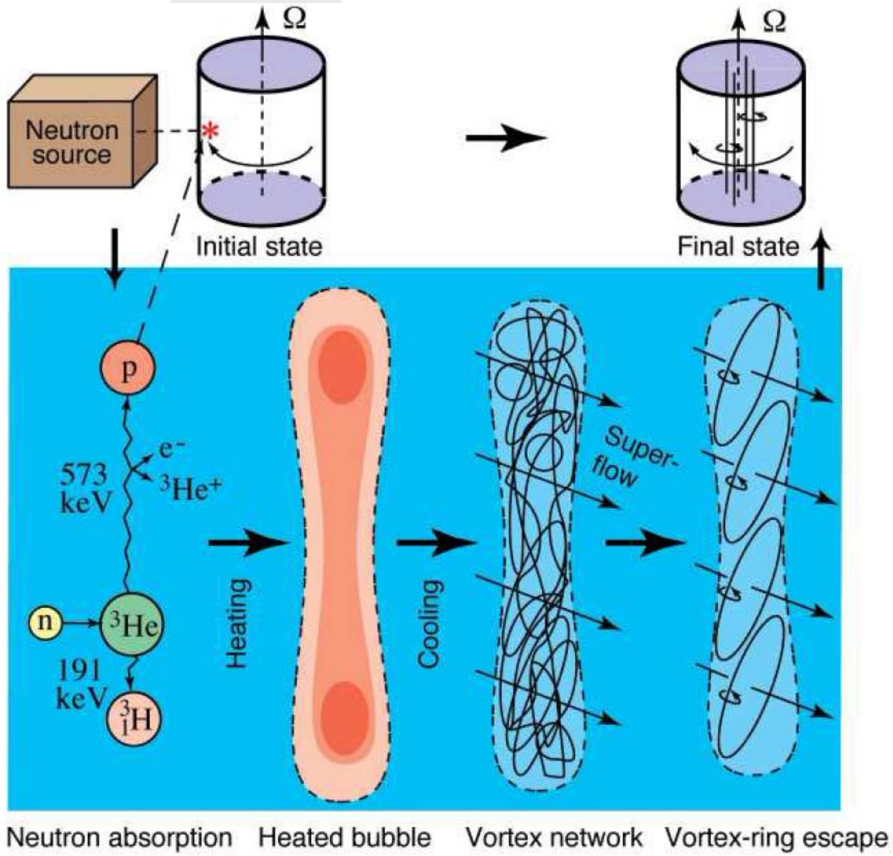
Measured vortex tangle density:

Quantitative support for Kibble-Zurek mechanism

Big Bang simulation in the low temperature laboratory

Grenoble: Bäuerle *et al.* (1996), Helsinki: Ruutu *et al.* (1996)

^{3}He-B



Cosmology in the Laboratory (COSLAB)

2001-2006

Present research on superfluid ^{3}He:

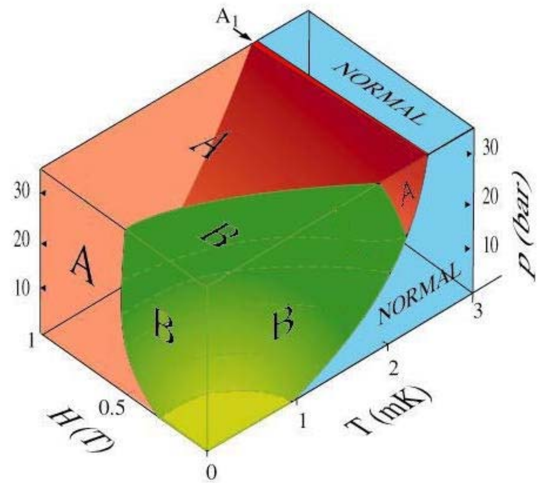
Quantum Turbulence

= Turbulence in the absence of viscous dissipation

Test system: ^{3}He-B

Vinen, Donnelly: **Physics Today** (April, 2007)

Conclusion



Superfluid Helium-3:

- Anisotropic superfluid
  - 3 different bulk phases
  - Cooper pairs with internal structure

- Large symmetry group broken
  - Close connections to particle theory
  - Zoo of topological defects
  - Kibble-Zurek mechanism quantitatively verified