Odd-frequency pairing in Non-uniform superconducting systems

Yukio Tanaka
Department of Applied Physics
Nagoya University

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Main Collaborators

A. A. Golubov	Twente University
Y. Asano	Hokkaido University
S. Kashiwaya	NIAIST (Tsukuba)
M. Ueda	Tokyo Institute of Technology
Y. Tanuma	Kanagawa University
T. Yokoyama	Nagoya University
Y. Sawa	Nagoya University
Y. V. Nazarov	Delft University

Other Main Collaborators (2)

K. Miyake Osaka University
S. Onari Nagoya University
K. Yada Nagoya University

Y. Fuseya University of Tokyo
H. Kohno Osaka University

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（5）Odd-pairing order (gap function) in bulk system

What is odd-frequency pairing states?

Conventional Cooper pair

Pair amplitude

$$F_{ss'}(\vec{k}) = \langle \hat{c}_{\vec{k}s} \hat{c}_{-\vec{k}s'} \rangle = \underbrace{\Phi(\vec{k})}_{\text{orbital}} \underbrace{\chi(s,s')}_{\text{spin}}$$

Sign chance due to the exchange of two electrons

$$\vec{k} \rightarrow -\vec{k} \qquad s \leftrightarrow s'$$

Even Parity $\Phi(-\vec{k}) = \Phi(\vec{k})$ → **S=0 spin-singlet**

L = 0, 2, 4, 6...

s-wave, **d-wave, g-wave , i-wave** **Spin-singlet even-parity**

Odd Parity $\Phi(-\vec{k}) = -\Phi(\vec{k})$ → **S=1 spin-triplet**

L = 1 ,3 ,5, ...

p-wave, f-wave, h-wave **Spin-triplet odd-parity**

More general classification of a Cooper pair

- Cooper pair is not necessarily formed between two electrons at the same time.
- Time difference (Frequency dependence)
- **We must consider Fermi-Dirac statistics (Pauli's rule) by taking account of the frequency dependence.**

Symmetry of the pair function

+ symmetric, – anti-symmetric

	Frequency (time)	Spin	Orbital	Total
ESE	+(even)	– (singlet)	+(even)	–
ETO	+(even)	+ (triplet)	–(odd)	–
OTE	–(odd)	+ (triplet)	+(even)	–
OSO	–(odd)	– (singlet)	–(odd)	–

ESE (**Even-frequency spin-singlet even-parity**)
ETO (**Even-frequency spin-triplet odd-parity**)
OTE (**Odd-frequency spin-triplet even-parity**)
OSO (**Odd-frequency spin-singlet odd-parity**)

Odd-frequency pairing state

(1)Odd-freqeuncy pairing order (pair potential, gap function) in uniform bulk system

(odd-frequency superconductor)

(2)Odd-frequency pairing state (not order, pair amplitude) in superconducting junctions

$$\Delta(\boldsymbol{k}, i\omega_n) = -T\sum_{\boldsymbol{k}',m} V(\boldsymbol{k}-\boldsymbol{k}', i\omega_n - i\omega_m)F(\boldsymbol{k}', i\omega_m)$$

Pair potential

Pair amplitude

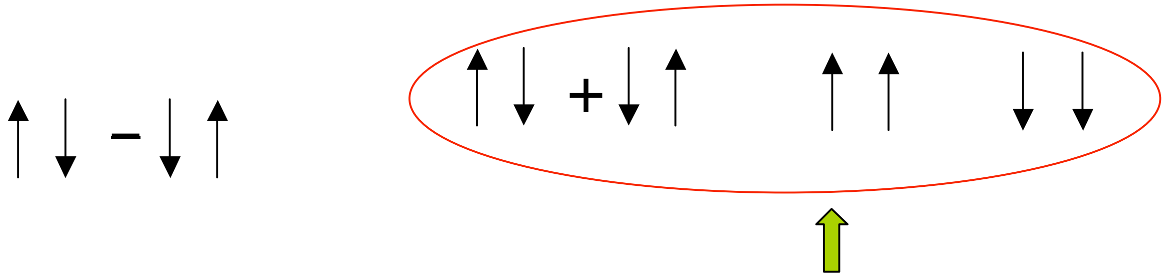
Odd frequency pairing state in bulk system (Odd-frequency superconductor)

1974	**Berezinskii**	Superfluid ^{3}He	s-wave triplet	×experimentally
1992-1993	Balatsky-Abrahams Schrieffer Scalapino	High-Tc	p-wave singlet	phenomenological ×experimentally
1994 1997	Coleman Mirranda Tsvelik	Kondo lattice	p-wave singlet	phenomenological Experimentally ?
1999	Vojta-Dagotto	Triangular lattice κ-(BEDT-TTF)$_2$X	s-wave triplet	RPA?? Experimentally??
2003	Fuseya Kohno Miyake	Ce compound	p-wave singlet	phenomenological Experimentally ?

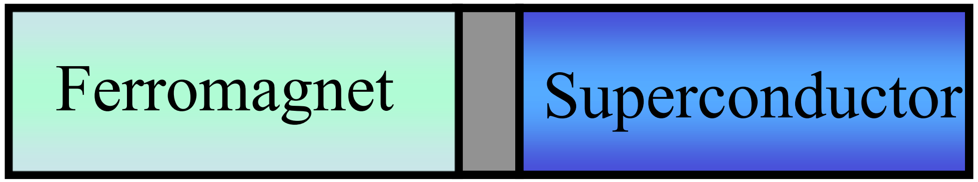
Kirtpatrik(1991)
Zachar et al (1998), Belitz (1998)

$$\Delta(\boldsymbol{k}, i\omega_n)$$

Odd-frequency Pairing state (not pair potential) is generated in ferromagnet junctions



Odd frequency spin-triplet s-wave pair



Bergeret, Efetov, Volkov, (2001)

Eschrig, Buzdin,Golubov, Kadigrobov,Fominov, Radovic…

Generation of the odd-frequency pair amplitude in ferromagnet

Is the existence of ferromagnet a necessary condition for the realization of the odd-frequency pairing state?

No

Ubiquitous presence of the odd frequency pairing state

⇩

Even in the conventional even-frequency superconductors, odd-frequency pairing state can be expected near the interface (surface).

Y. Tanaka, A. Golubov, S. Kashiwaya, and M. Ueda
Phys. Rev. Lett. 99, 037005 (2007)

M. Eschrig, T. Lofwander, Th. Champel, J.C. Cuevas and G. Schon
J. Low Temp. Phys 147 457(2007)

Eilenberger Equation

$$-i\boldsymbol{v}_F \cdot \nabla \hat{g}^R = \left[\hat{H}_{qc} + \hat{\Sigma}^R, \hat{g}^R\right],$$

$$\hat{H}_{qc} = \begin{pmatrix} \varepsilon & \Delta(\hat{\boldsymbol{p}}, \boldsymbol{r}) \\ -\Delta^*(\hat{\boldsymbol{p}}, \boldsymbol{r}) & -\varepsilon \end{pmatrix}, \qquad \hat{\Sigma}^R = \frac{i}{2\tau_{imp}} < \hat{g}^R >$$

$$\hat{g}^R \hat{g}^R = \hat{1}$$

$\hat{\Sigma}^R = 0$ **(ballistic)**

Quasiclassical approximation

(1) Cooper pair is formed by two electrons on the Fermi surface

(2) The effective pair potential is determined by the direction of the motion of electrons.

Kopnin Theory of Nonequilibrium Superconductivity",
Oxford University Press (2001).

Quasi-classical Green's functions

Eilenberger's equation

$$\mp i v_{Fx}\partial_x f_{1\pm} = 2\omega_n f_{2\pm} - 2\bar{\Delta}_{\pm}(x) g_{\pm}$$

$$\mp i v_{Fx}\partial_x g_{\pm} = 2\bar{\Delta}_{\pm}(x) f_{1\pm},$$

$$\mp i v_{Fx}\partial_x f_{2\pm} = -2\omega_n f_{1\pm},$$

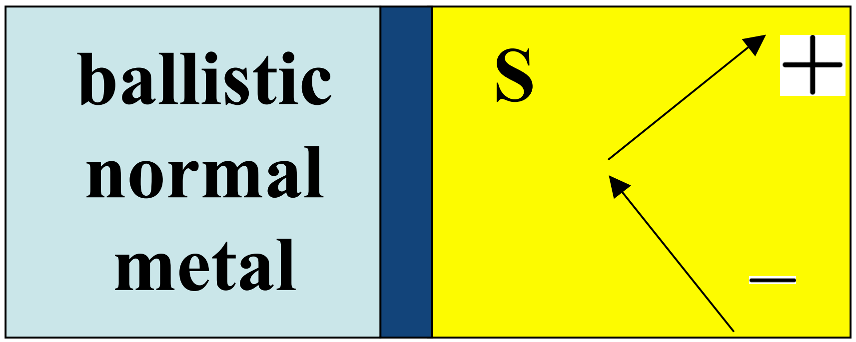
$$f_{1\pm}^2 + f_{2\pm}^2 + g_{\pm}^2 = 1,$$

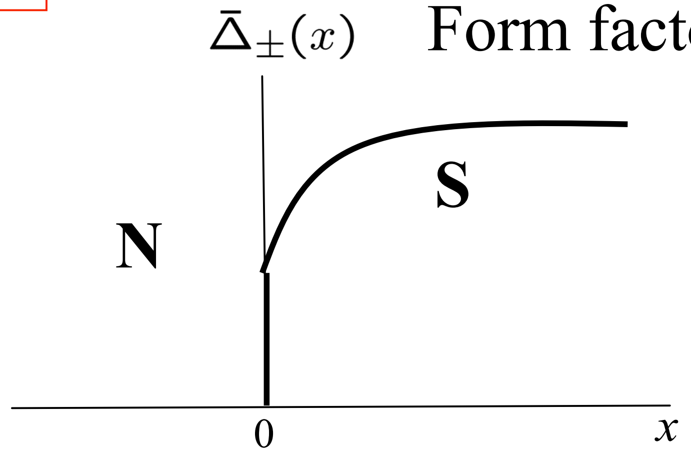
$$\bar{\Delta}_{\pm}(x) = \Delta(x)\Phi_{\pm}(\theta).$$

Quasiparticle function	$g_{\pm}$
Pair amplitude	$f_{1\pm}$, $f_{2\pm}$

$\Phi_{\pm}(\theta)$ Form factor

Bulk state ⇨ $f_{2\pm}$





General properties (frequency)

Pair potential has even-frequency symmetry.

$$f_{2\pm}(\omega_n,\theta) = f_{2\pm}(-\omega_n,\theta)$$

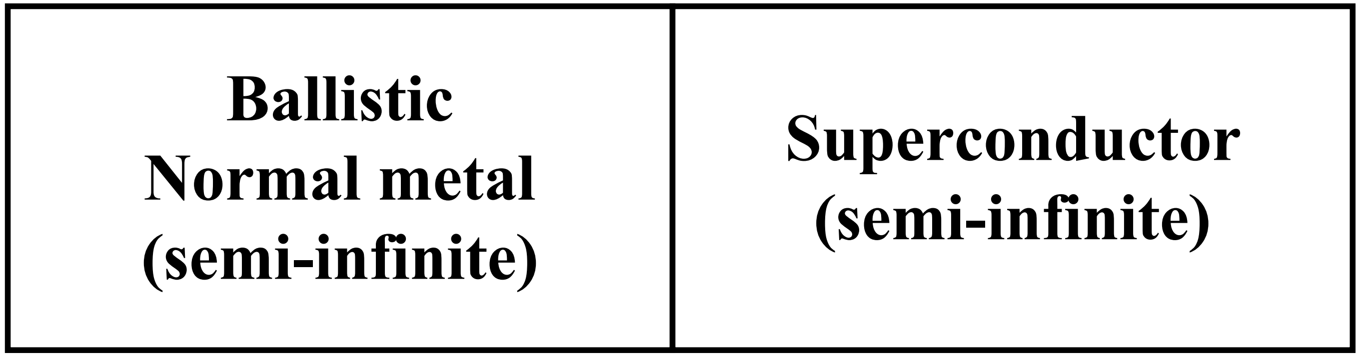
Even-frequency (real), bulk-state

↓ **Spatial change of the pair potential**

$$f_{1\pm}(\omega_n,\theta) = -f_{1\pm}(-\omega_n,\theta)$$

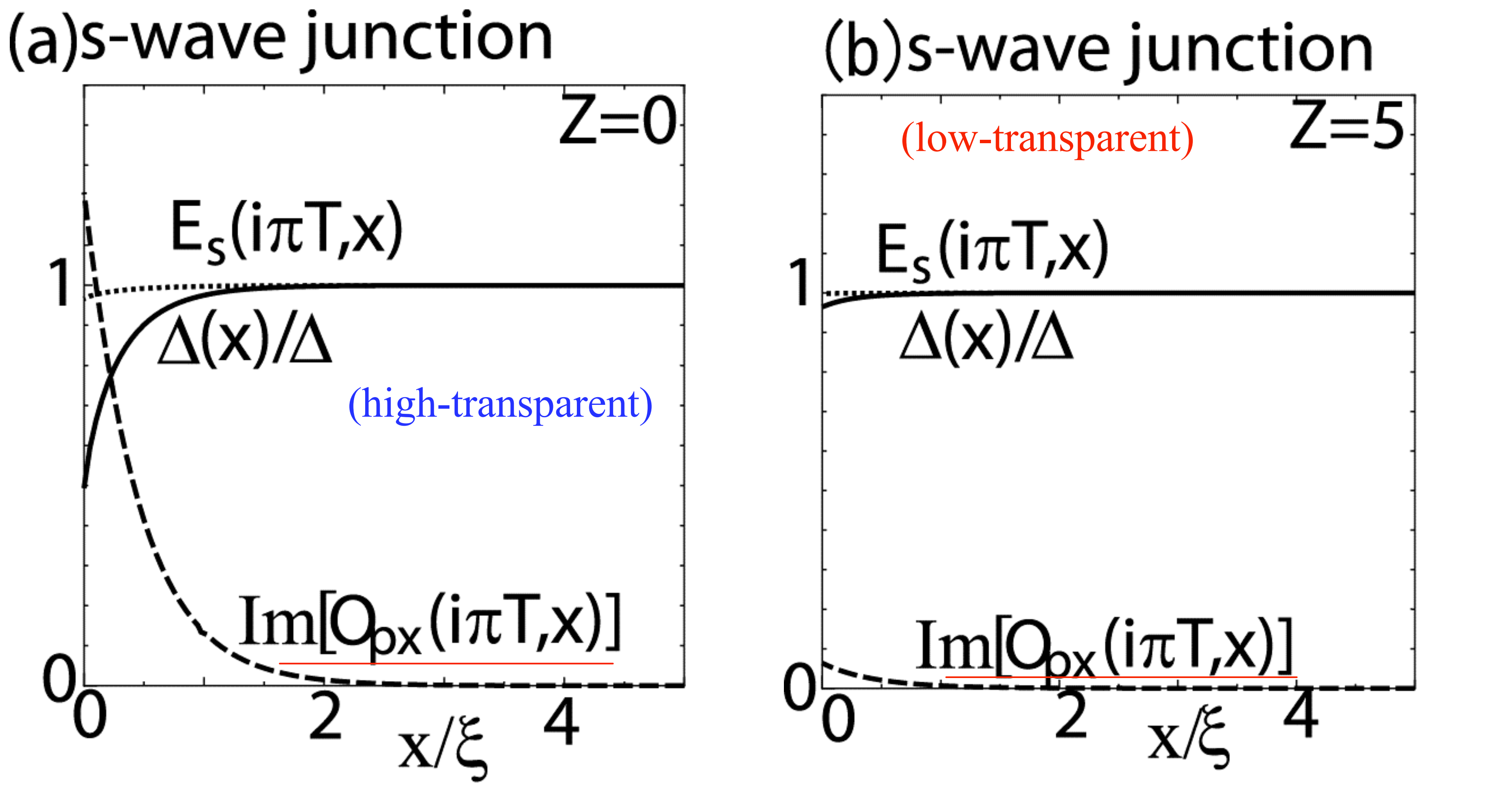
Odd-frequency (imaginary) Interface-induced state

Ballistic junction



Y. Tanaka, A. Golubov, S. Kashiwaya, and M. Ueda
Phys. Rev. Lett. 99 037005 (2007)

Even-frequency Spin-singlet even parity (ESE) superconductor

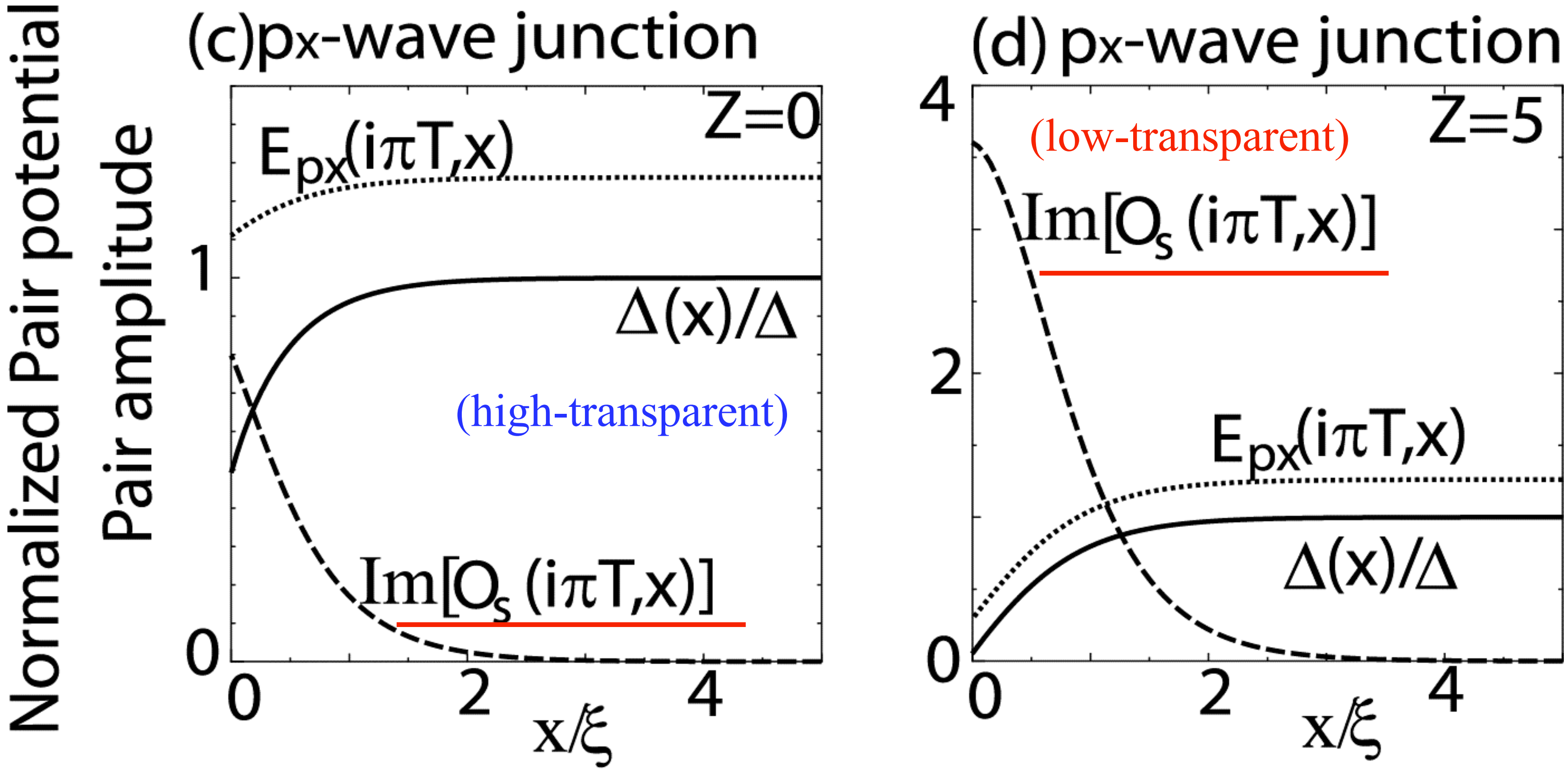


$E_s(i\omega_n, x)$ **even-frequency s-wave component**

$O_{px}(i\omega_n, x)$ **odd-frequency p_x-wave component**

Y. Tanaka, et al PRL 99 037005 (2007)

Even-frequency Spin-triplet odd-parity (ETO) superconductor

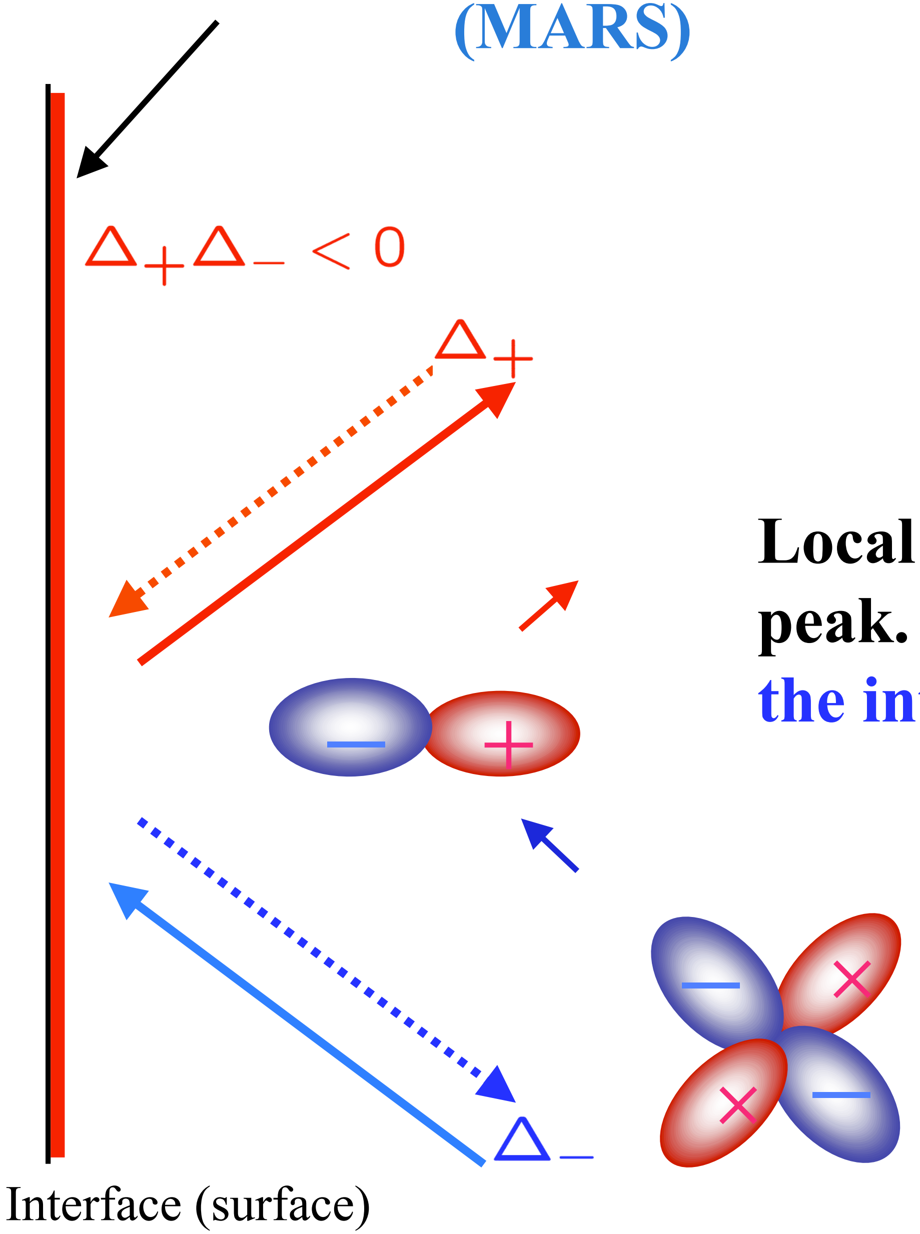


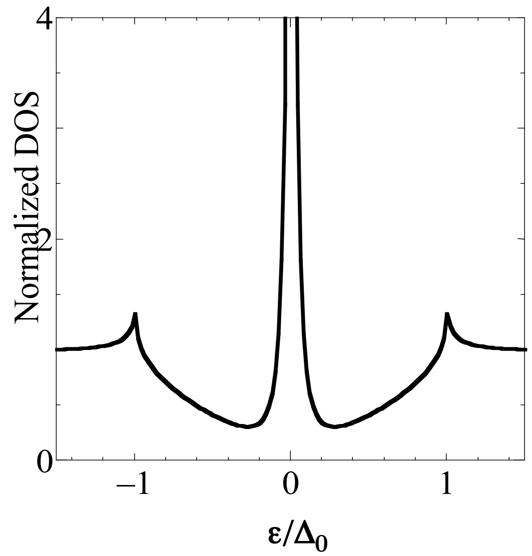
$E_{px}(i\omega_n, x)$ **even-frequency p_x-wave component**

$O_s(i\omega_n, x)$ **odd-frequency s-wave component**

Y. Tanaka, et al PRL 99 037005 (2007)

Mid gap Andreev resonant (bound) state (MARS)

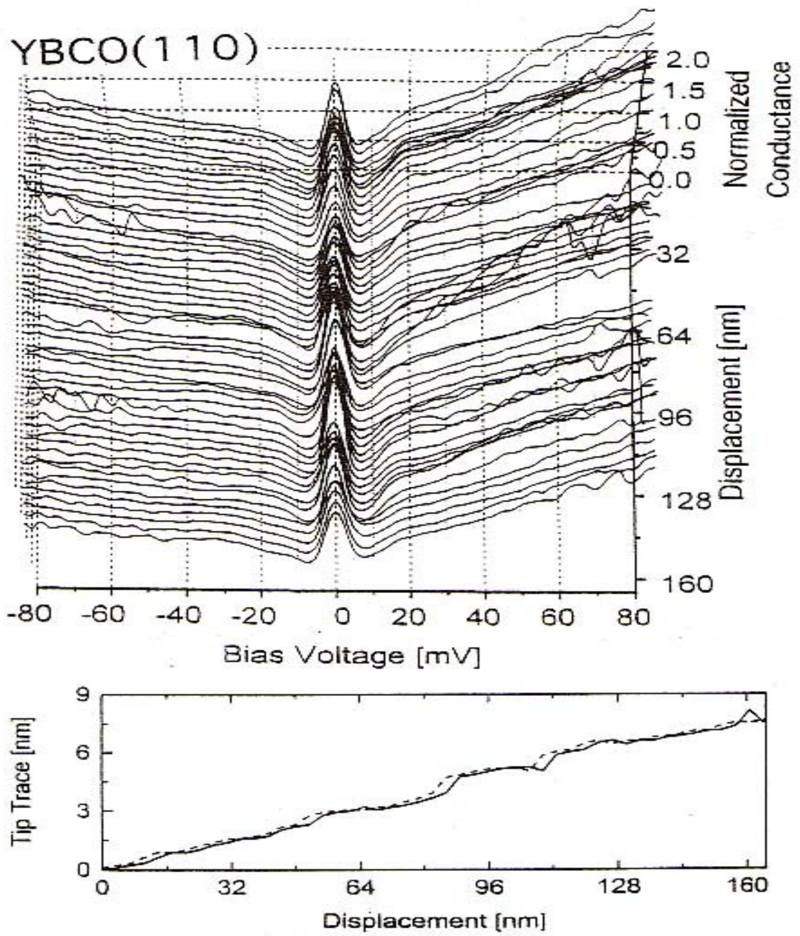


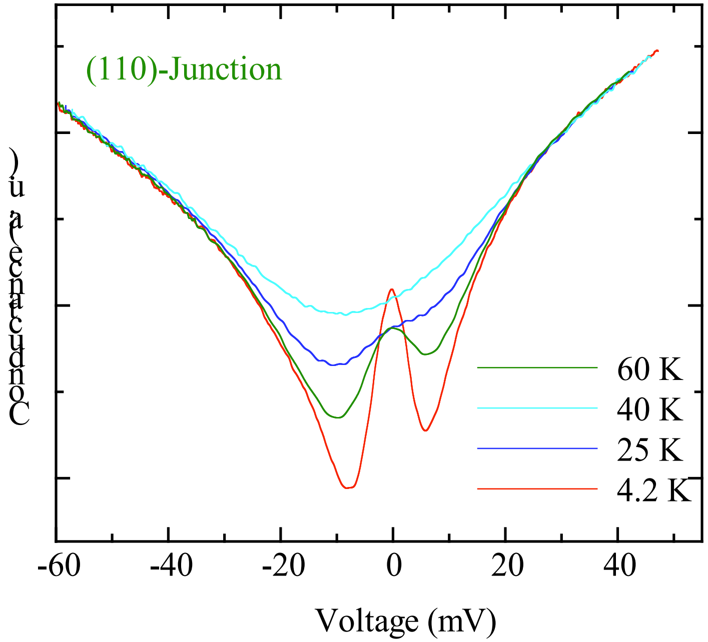


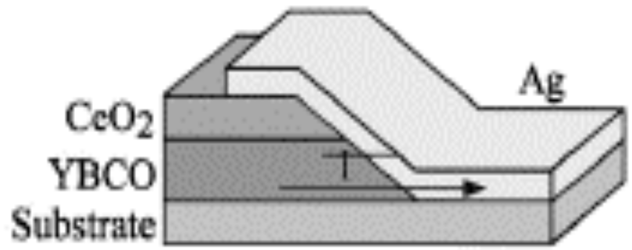
Local density of state has a zero energy peak. Sign change of the pair potential at the interface.

Tanaka Kashiwaya PRL 74 3451 (1995),
Rep. Prog. Phys. 63 1641 (2000)
Buchholz(1981) Hara Nagai(1986)
Hu(1994) Matsumoto Shiba(1995)
Ohashi Takada(1995)
Hatsugai and Ryu (2002)

Zero bias conductance peak (by MARS) observed in cuprate







S. Kashiwaya and Y. Tanaka
Rep. Prog. Phys. 63 1641
(2000)

Iguchi Wang et al.
Phys. Rev. B60, 4272 (1999)

Superconducting Materials where MARS is observed

$YBa_2CuO_{7-\delta}$ (Geerk, Kashiwaya, Iguchi, Greene, Yeh, …)

$Bi_2Sr_2CaCu_2O_y$ (Ng, Suzuki, Greene….)

$La_{2-x}Sr_xCuO_4$ (Iguchi)

$La_{2-x}Ce_xCuO_4$ (Cheska)

$Pr_{2-x}Ce_xCuO_4$ (R.L.Greene)

Sr_2RuO_4 (Mao, Meno, Kawamura,Lube)

κ-(BEDT-TTF)$_2$X, X=Cu[N(CN)$_2$]Br (Ichimura)

UBe_{13} (Ott)

$CeCoIn_5$ (Wei Greene)

$PrOs_4Sb_{12}$ (Wei)

Exact solution (1D)

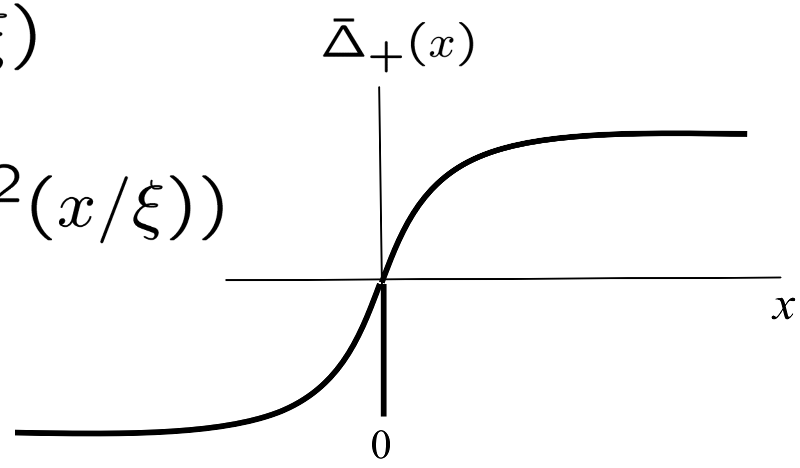
$$-iv_{Fx}\partial_x f_{1+} = 2\omega_n f_{2+} - 2\bar{\Delta}_+(x) g_+$$
$$-iv_{Fx}\partial_x g_+ = 2\bar{\Delta}_+(x) f_{1+},$$
$$-iv_{Fx}\partial_x f_{2+} = -2\omega_n f_{1+},$$

$$\bar{\Delta}_+(x) = \Delta \tanh(x/\xi)$$
$$\xi = v_{Fx}/\Delta$$

$$f_{2+} = \frac{1}{\sqrt{\omega_n^2+\Delta^2}}\Delta \tanh(x/\xi)$$

$$f_{1+} = \frac{i}{\sqrt{\omega_n^2+\Delta^2}}\frac{\Delta^2}{2\omega_n}\mathrm{sech}^2(x/\xi)$$

$$g_+ = \frac{1}{\sqrt{\omega_n^2+\Delta^2}}\left(\omega_n + \frac{\Delta^2}{2\omega_n}\mathrm{sech}^2(x/\xi)\right)$$



Schopohl arXiv:cond-mat/9804064,
Hara, Nagai Prog. Theor. Phys. 74 1237 (1986)

Summary of proximity effect (No spin flip)

	Bulk state	Sign change (MARS)	Interface-induced state (subdominant component)
(1)	ESE (s,d_{x2-y2} -wave)	No	**ESE** + (OSO)
(2)	ESE (d_{xy}-wave)	Yes	**OSO** +(ESE)
(3)	ETO (p_x-wave)	Yes	**OTE** + (ETO)
(4)	ETO (p_y-wave)	No	**ETO** + (OTE)

- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**
- **OSO (Odd-frequency spin-singlet odd-parity**

Y. Tanaka, A. Golubov, S. Kashiwaya, and M. Ueda
Phys. Rev. Lett. 99 037005 (2007)

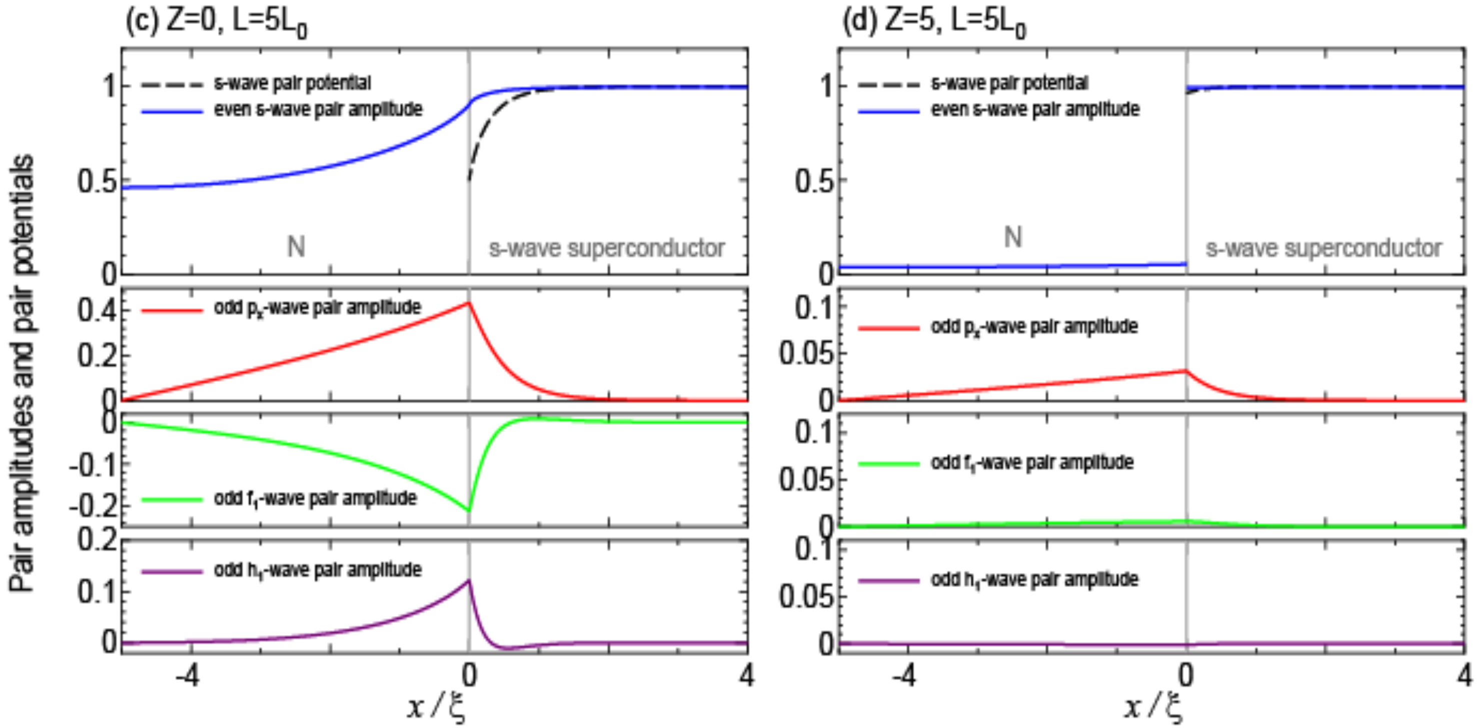
Ballistic junction

Ballistic Normal metal (finite length)	**Superconductor (semi-infinite)**

Y. Tanaka, Y. Tanuma and A.A. Golubov, Phys. Rev. B 76 054522 (2007)

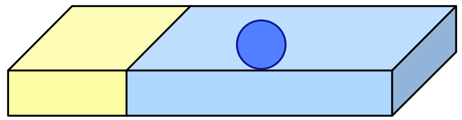
Y. Tanaka, Y. Tanuma and A.A. Golubov, PRB 76 054522 (2007)

Odd-frequency pairing state in spin-singlet s-wave superconductor junctions



High transmissivity

Low transmissivity



OSO (Odd-frequency spin-singlet odd-parity)

ESE (Even-frequency spin-singlet even-parity)

OSO state is generated from superconductor with ESE symmetry

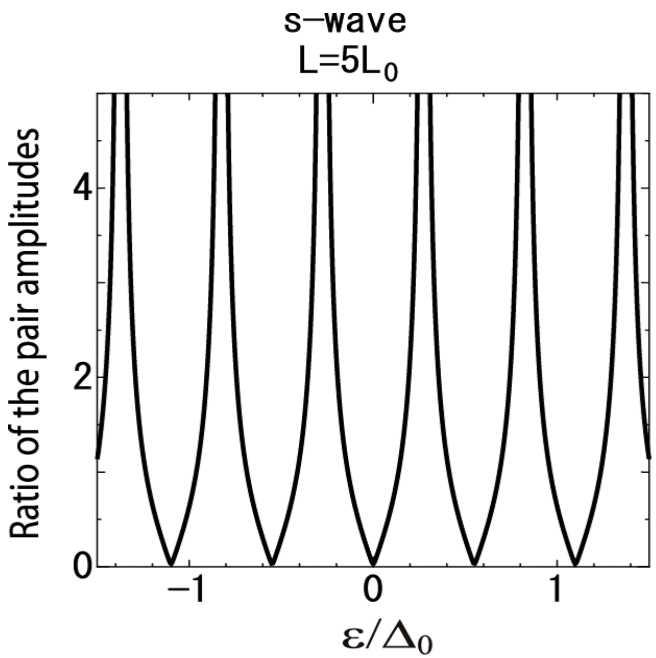
At the N/S interface(S-side), ESE state is dominant for low transmissivity.

Y. Tanaka, Y. Tanuma A.A. Golubov, PRB 76 054522 (2007)

Ratio of the pair amplitude in the N region (odd/even)

$$\frac{\left|f_{1+}^{(N)}(\varepsilon,\theta)\right|}{\left|f_{2+}^{(N)}(\varepsilon,\theta)\right|} = \left|\tan\left(\frac{2\varepsilon}{v_{Fx}}(L+x)\right)\right|.$$

At some energy, odd-frequency component can exceed over even frequency one.



$\theta = 0$

$x = 0$

$f_{1+}^{(N)}(\varepsilon,\theta)$

Odd-frequency pairing

$f_{2+}^{(N)}(\varepsilon,\theta)$

Even-frequency pairing

Ratio at the interface

PRB 76 054522 (2007)

Ratio of the pair amplitude at the N/S interface and the bound state level

$\Delta_0 \gg \varepsilon$ **Bound states condition (Z=0)**

(McMillan Thomas)

$$\varepsilon_n = \frac{\pi v_F x}{2L}(n+1/2), \quad n = 0,1,2,...$$

$$\frac{\left| f_{1+}^{(N)}(\varepsilon,\theta) \right|}{\left| f_{2+}^{(N)}(\varepsilon,\theta) \right|} = \left| \tan\left(\pi/2 + \pi n\right) \right| = \infty.$$

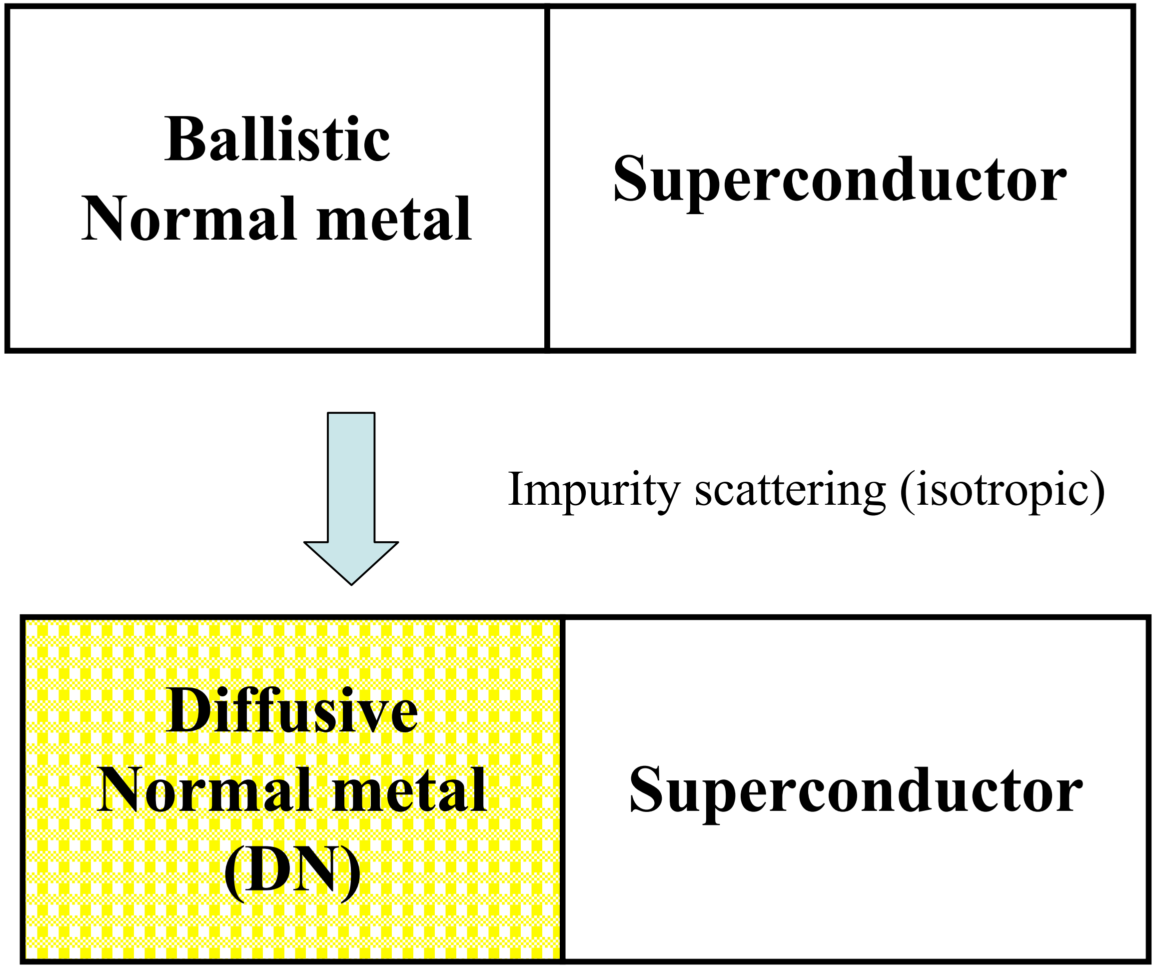
$f_{1+}^{(N)}(\varepsilon,\theta)$ Odd-frequency pairing

$f_{2+}^{(N)}(\varepsilon,\theta)$ Even-frequency pairing

At the N/S interface, only the odd-frequency component exists just at the bound state energy!!

Y. Tanaka, Y. Tanuma and A.A. Golubov, PRB 76 054522 (2007)

Impurity scattering effect



Only s-wave pairing state is possible in DN

Tanaka and Golubov, Phys. Rev. Lett. 98, 037003 (2007)

Usadel equation

Available for diffusive limit $\tau_{imp}T << 1$

$$D\nabla(\hat{g}_0^R\nabla\hat{g}_0^R) + i[\hat{H}_0, \hat{g}_0^R] = 0$$

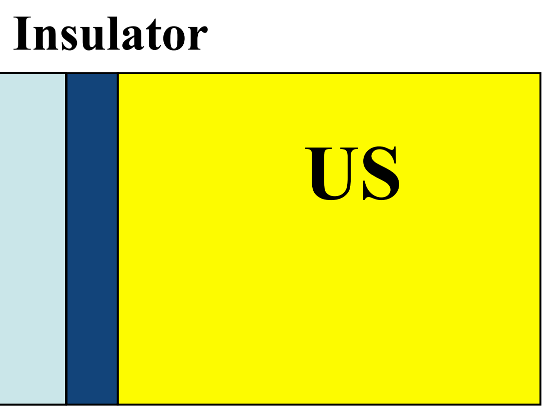
$\hat{g}_0^R \longleftarrow \hat{g}^R$ **Diffusive limit**

Angular average

Diffusive normal metal region attached to superconductor

$$\hat{H}_0 = \varepsilon\hat{\tau}_3$$

Diffusive normal metal (DN)/ unconventional superconductor (US) junctions



We must make a proper boundary condition at the interface of Usadel Green's function in DN. (T=0K)

Tanaka et al,
PRL 90 167003 (2003)
PRB 70 012507 (2004)

Theory

Quasiclassical Keldysh-Usadel equation

$$i\hbar D\hat{\nabla}\left(\hat{\boldsymbol{g}}^X\hat{\nabla}\hat{\boldsymbol{g}}^X\right)-[\hat{\boldsymbol{H}},\hat{\boldsymbol{g}}^X]_-=0,\ \text{for}\ X=R,A,$$

$$i\hbar D\hat{\nabla}\left(\hat{\boldsymbol{g}}^R\hat{\nabla}\hat{\boldsymbol{g}}^K+\hat{\boldsymbol{g}}^K\hat{\nabla}\hat{\boldsymbol{g}}^A\right)-[\hat{\boldsymbol{H}},\hat{\boldsymbol{g}}^K]_-=0,$$

$$\hat{H}=\epsilon\hat{\tau}_3,\quad \hat{g}^K=\hat{g}^R\hat{h}-\hat{h}\hat{g}^A,\quad \hat{h}=f_L+f_T\hat{\tau}_3,$$

$$I=\frac{eN_0D}{4}\int d\epsilon\,\mathrm{Tr}\left[\hat{\tau}_3(\hat{g}^R\nabla\hat{h}+\hat{g}^K\nabla\hat{g}^A)\right],$$

G. Eilenberger, Z. Phys. 214, 195 (1968).

K. Usadel, PRL 25, 507 (1970).

A. I. Larkin and Yu. N. Ovchinnikov, JETP 26, 1200 (1968); 41, 155 (1977)

W. Belzig, et. al., Superlattices and Microstructures 25, 1251 (1999).

A. F. Volkov, A. V. Zaitsev, and T. M. Klapwijk, Physica C 210, 21 (1993).

A. V. Zaitsev, Phys. Lett. A 194, 315 (1994).

Theory

Boundary condition at NS interface

s-wave: A. V. Zaitsev, JETP 59, 1015 (1984)

M. Yu. Kuprianov and V. F. Lukichev JETP 67, 1163 (1988)

Yu. V. Nazarov, PRL 73, 1420 (1994);
Superlattices and Microstructures 25, 1221 (1999).

d-wave: **Y. Tanaka, Yu. V. Nazarov, and S. Kashiwaya, PRL 90, 167003 (2003).**

p-wave: **Y. Tanaka and S. Kashiwaya, PRB 70, 012507 (2004)**

θ-parameterization

$$\hat{g}^R = \cos\phi \sin\theta \hat{\tau}_1 + \sin\phi \sin\theta \hat{\tau}_2 + \cos\theta \hat{\tau}_3$$

$$\phi = \begin{cases} \pi/2 & : \text{singlet} \\ 0 & : \text{triplet} \end{cases}$$

$$\hbar D \nabla^2 \theta + 2i\epsilon \sin(\theta) = 0$$

General theory of Proximity effect

(diffusive normal metal/ superconductor junctions)

	Symmetry of the pair potential	Induced pair amplitude in DN
(1)	**Even-frequency spin-singlet even-parity (ESE)**	**ESE**
(2)	**Even-frequency spin-triplet odd-parity (ETO)**	**OTE**

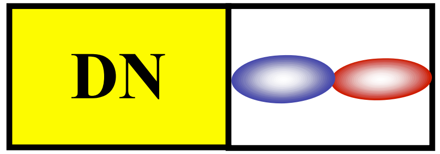
- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**

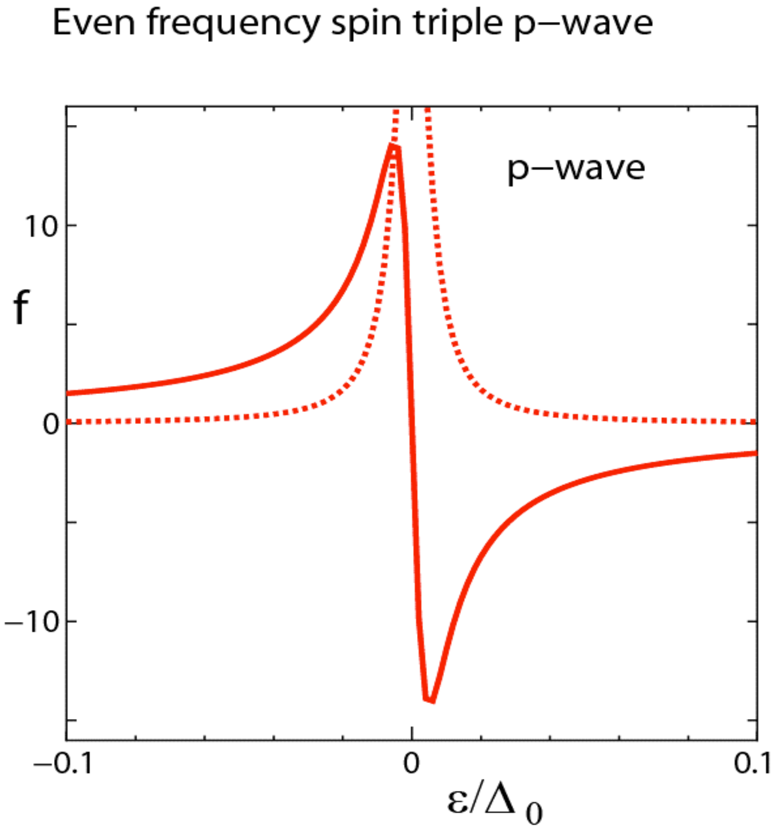
Proximity into DN (Diffusive normal metal)

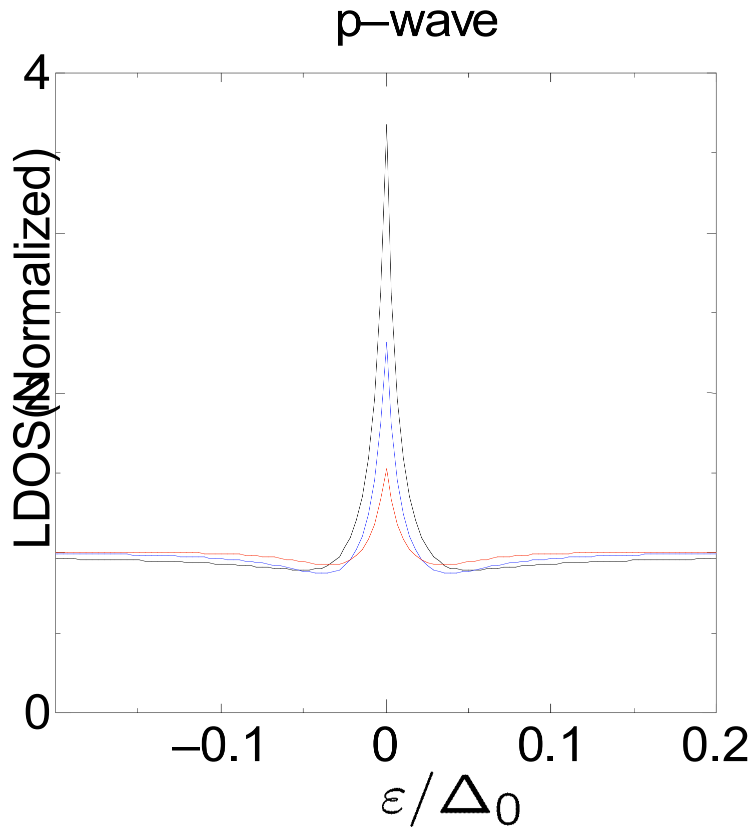
even-parity ○ Odd-parity ×

PRL 98, 037003 (2007)

Unusual proximity effect in spin-triplet p-wave junction







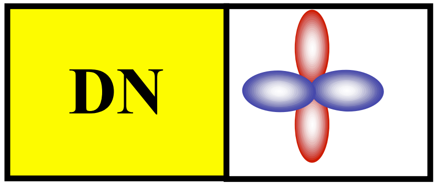
OTE
(Odd-frequency
spin-triplet
even-parity)

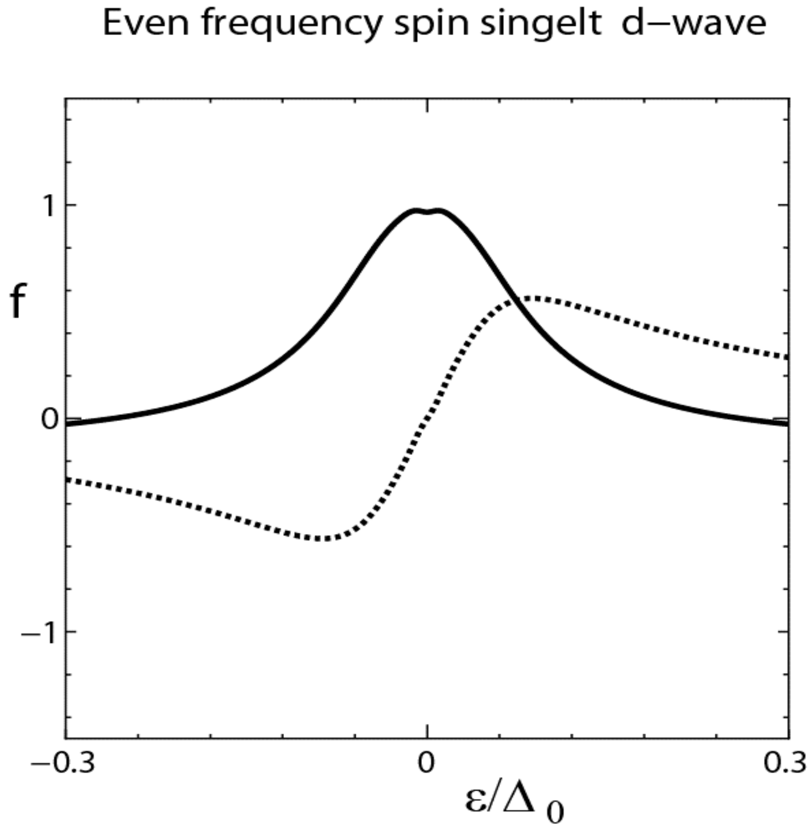
$$f(\varepsilon) = -f^*(-\varepsilon)$$ ⟹ Generation of odd-frequency state in DN

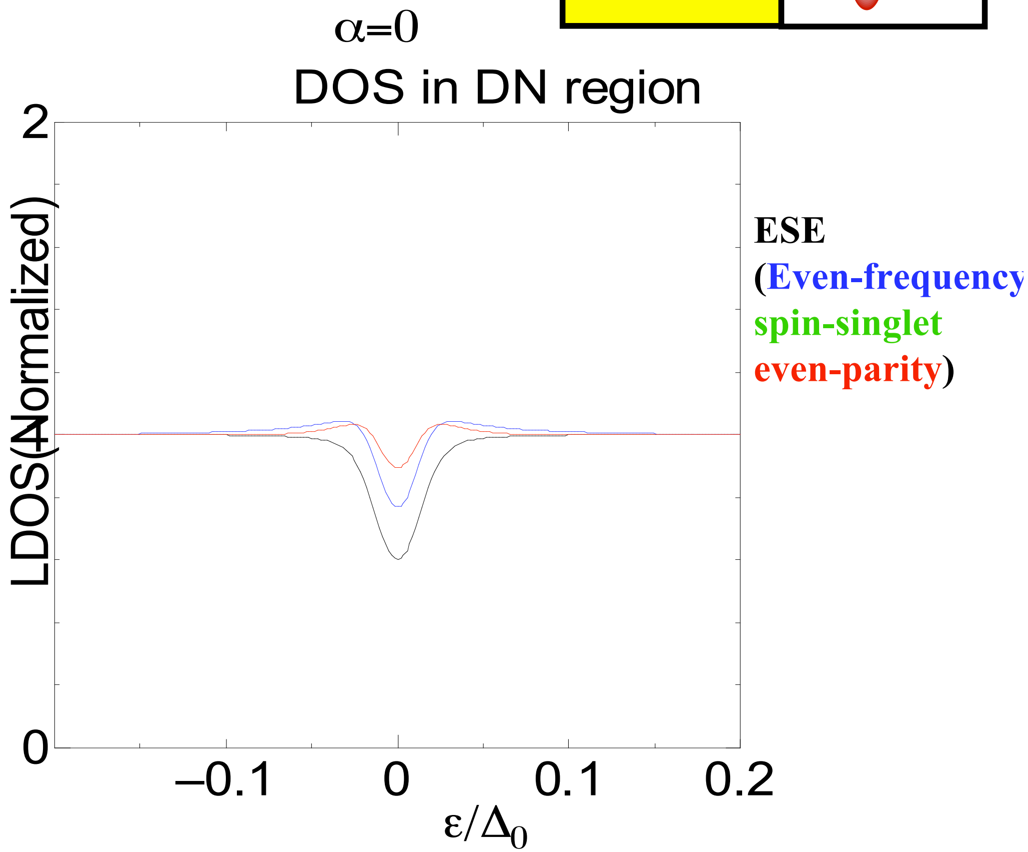
Purely OTE (s-wave) state in DN

PRL 98, 037003 (2007)

Conventional proximity effect in spin-singlet d-wave junction (similar to s-wave)





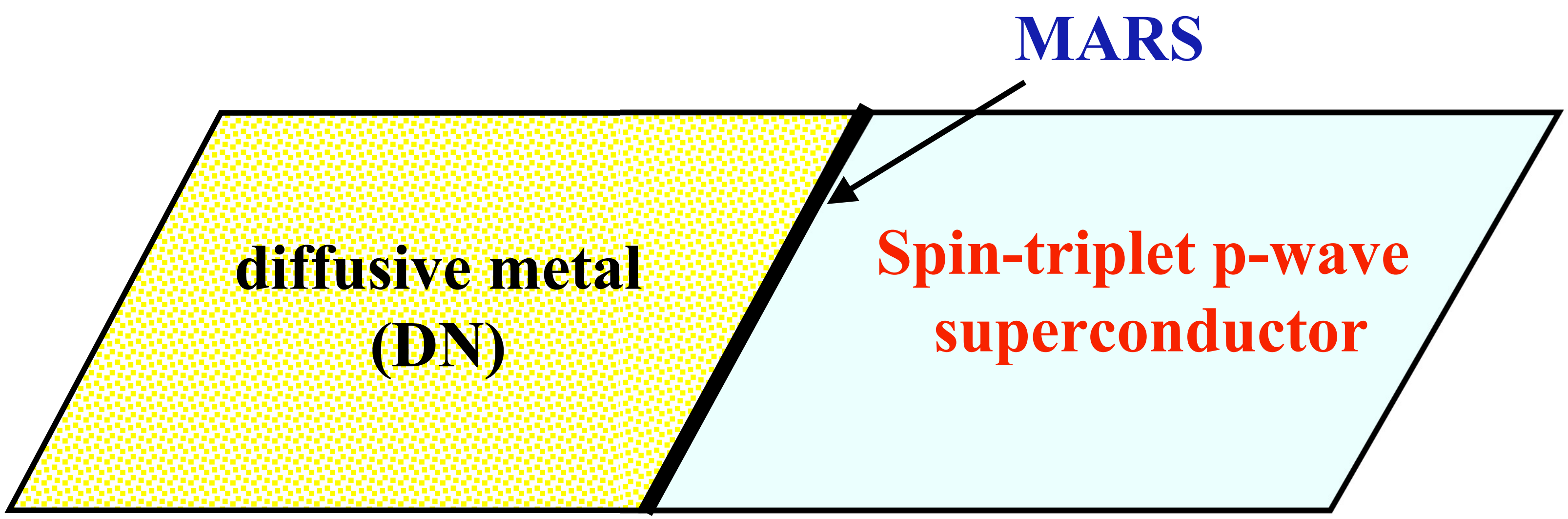


$f(\varepsilon) = f^*(-\varepsilon)$ ⟹ Generation of odd-frequency state in DN

Purely ESE (s-wave) state in DN

PRL 98, 037003 (2007)

Odd-frequency spin-triplet s-wave pairing state is induced in diffusive normal metal



Mid gap Andreev state (MARS) can penetrate into DN as an odd-frequency pairing state.

Summary of proximity effect (No spin flip)

	Bulk state	Sign change	Interface-induced state (subdominant)	Proximity into DN
(1)	ESE(s,dx2-y2 -wave)	No	**ESE** + (OSO)	**ESE**
(2)	ESE (d_{xy}-wave)	Yes	**OSO** +(ESE)	No
(3)	ETO (p_x-wave)	Yes	**OTE** + (ETO)	**OTE**
(4)	ETO (p_y-wave)	No	**ETO** + (OTE)	No

- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**
- **OSO (Odd-frequency spin-singlet odd-parity)**

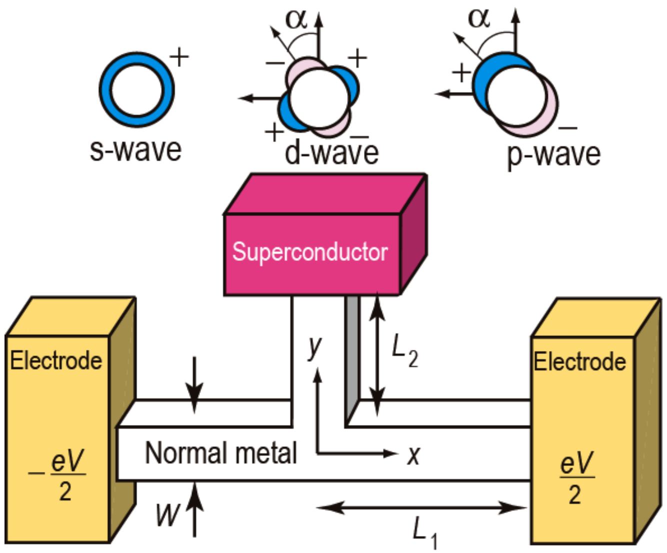
Proximity into DN (Diffusive normal metal)

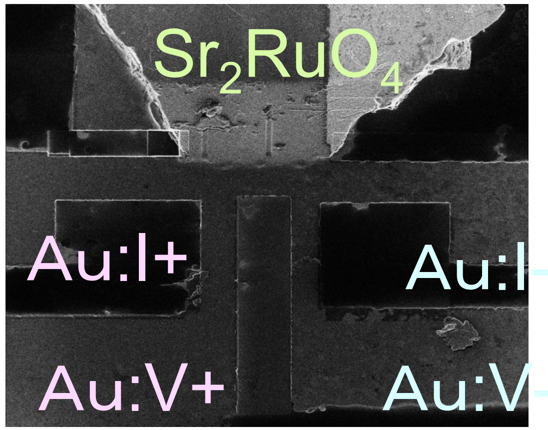
even-parity (s-wave)○ Odd-parity ×

Y. Tanaka, et al Phys. Rev. Lett. 037005 (2007)

Theoretical prediction to detect odd-frequency paring state

Asano Tanaka Golubov Kashiwaya, Phys. Rev. Lett. **99**, 067005 (2007).



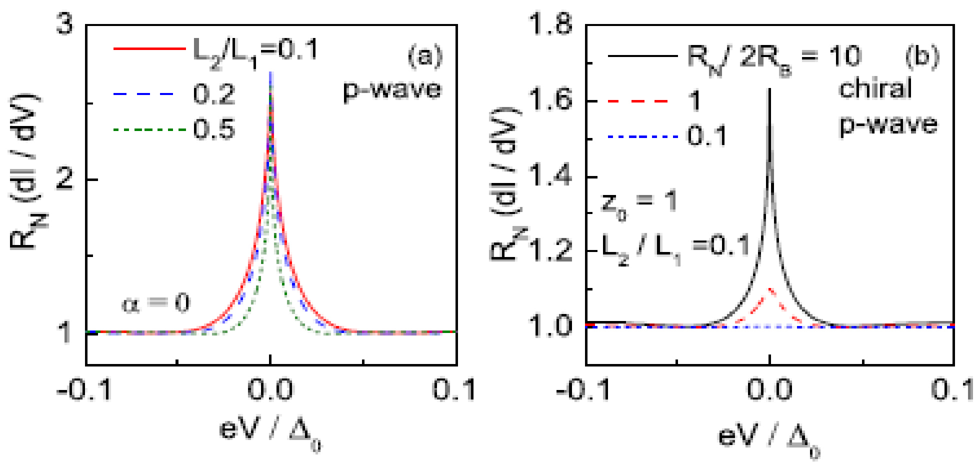


Kashiwaya, Maeno 2007

OTE (Odd-frequency spin-triplet even-parity)

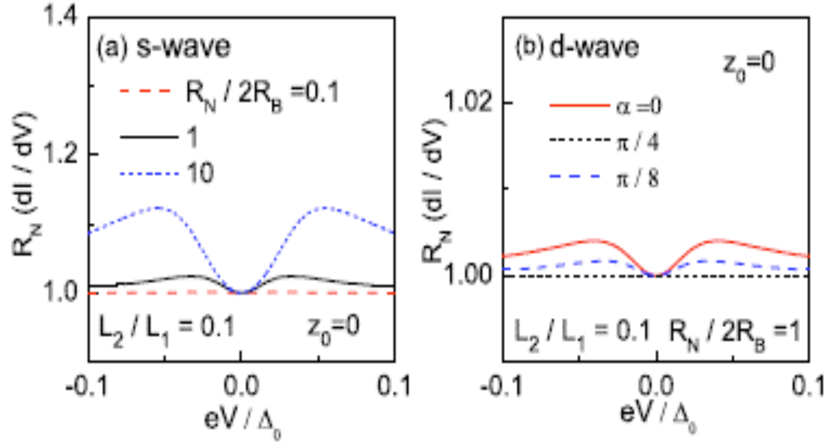
ESE (Even-frequency spin-singlet even-parity)

OTE state



Zero energy peak

ESE state

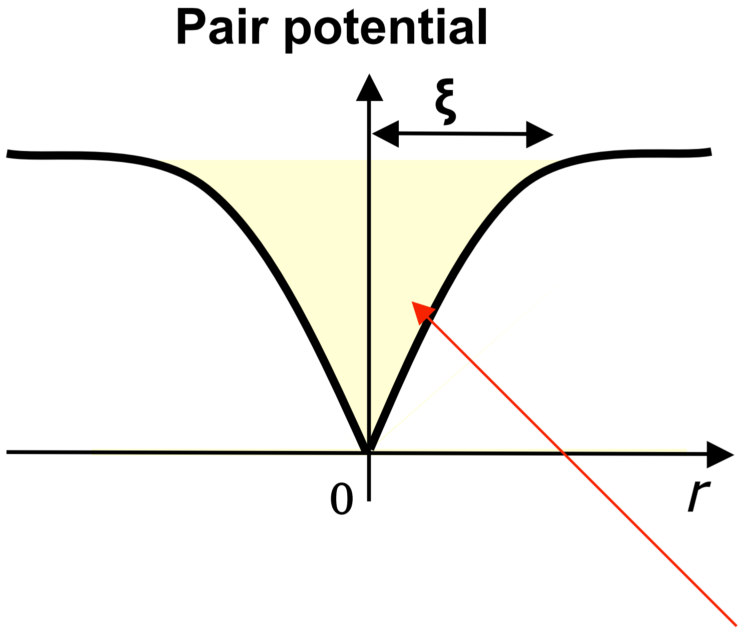


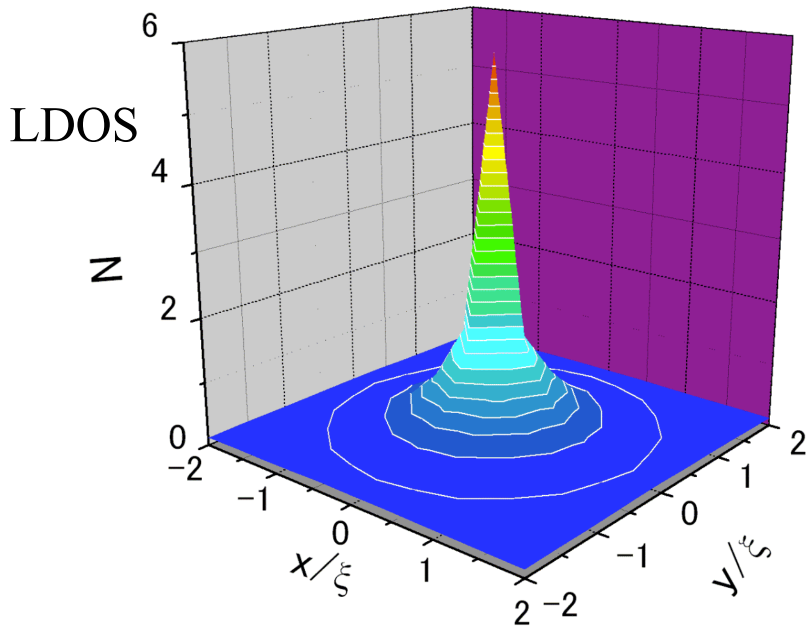
No Zero energy peak

Vortex core state (s-wave)

$$\Delta(\mathbf{r}) = \Delta_0 \tanh\left(\frac{\sqrt{x^2+y^2}}{\xi}\right)\frac{x+iy}{\sqrt{x^2+y^2}}$$

Core center sits at the origin （x＝y＝0）in s-wave superconductor

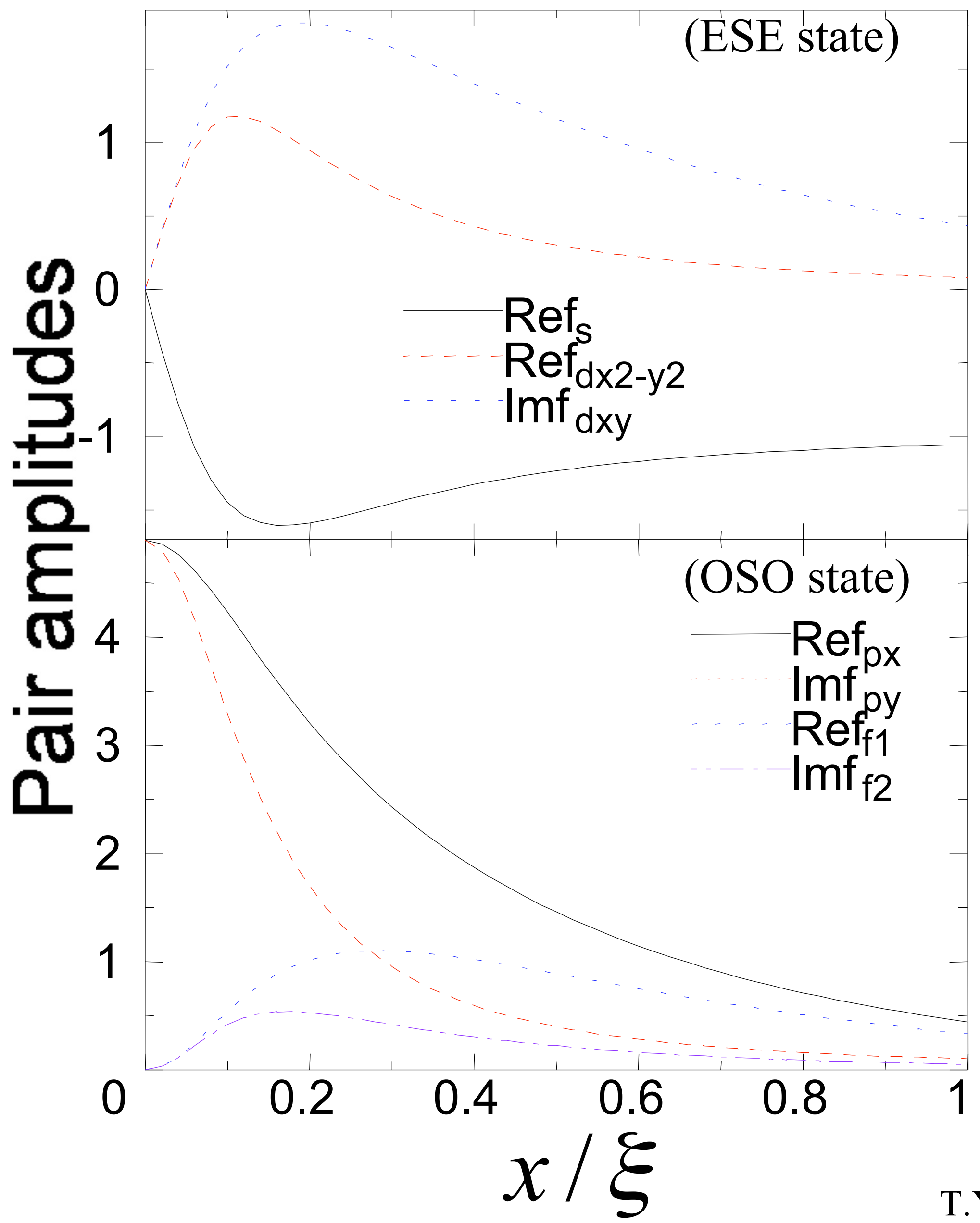




How about the symmetry of the pairing amplitude around the core?

T.Yokoyama, Y. Tanaka, A.A. Golubov **arXiv: 0710.2967**

Spatial dependence of pair amplitude



（y=E=0）

$$f \cong f_s + f_{px}\cos\theta + f_{py}\sin\theta$$
$$+ f_{dx^2-y^2}\cos 2\theta + f_{dxy}\sin 2\theta$$
$$+ f_{f1}\cos 3\theta + f_{f2}\sin 3\theta$$

$$\mathrm{Re}\, f_{px} = \mathrm{Im}\, f_{py}$$
$$\mathrm{Im}\, f_{px} = -\mathrm{Re}\, f_{py}$$

(core center)

At the core center only odd-frequency singlet chiral p-wave pairing amplitude can survive

T.Yokoyama, Y. Tanaka, A.A. Golubov **arXiv: 0710.2967**

Symmetry of the vortex core

$$\Delta(\mathbf{r}) = \Delta_0 \exp(il\varphi) \tanh\left(\frac{\sqrt{x^2+y^2}}{\xi}\right)\left(\frac{x+iy}{\sqrt{x^2+y^2}}\right)^m$$

l angular momentum

m vorticity

l	m	bulk	Center of the vortex core
Even	Even	ESE (s-wave..)	ESE
Even	Odd	ESE (s-wave..)	**OSO**
Odd	Even	ETO (chiral p-wave)	ETO
Odd	Odd	ETO (chiral p-wave)	**OTE**

ESE (Even-frequency spin-singlet even-parity)
ETO (Even-frequency spin-triplet odd-parity)
OTE (Odd-frequency spin-triplet even-parity)
OSO (Odd-frequency spin-singlet odd-parity

Angular momentum at the center of core
l*+*m

Yokoyama Tanaka Golubov(2007)

Possible realization of odd-frequency pair potential (bulk)

- Near the magnetic critical point
- Nesting condition is not good
- d-wave pairing should be suppressed

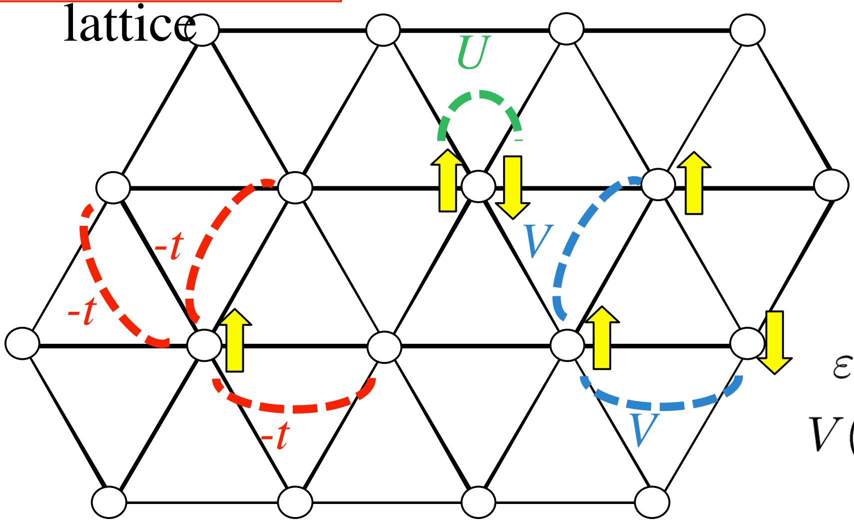
2D Hubbard model

$$H = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{i} U n_{i\uparrow} n_{i\downarrow} + \sum_{\langle i,j \rangle} V_{ij} n_i n_j$$

U: on-site Coulomb interaction

V: off-site Coulomb interaction

Triangular lattice



$$\varepsilon_{\mathbf{k}} = -2t(\cos(k_x) + \cos(k_x) + \cos(k_x + k_y))$$

$$V(\mathbf{q}) = 2V(\cos(q_x) + \cos(q_y) + \cos(q_x + q_y))$$

Formulation ($V \neq 0$)

spin susceptibility: $$\chi^s(q) = \frac{\chi^0(q)}{1 - U\chi^0(q)}$$

charge susceptibility: $$\chi^c(q) = \frac{\chi^0(q)}{1 + (U + 2V(\mathbf{q}))\chi^0(q)}$$

pairing interaction

singlet: $$V_{\sigma-\sigma}(q) = U + V(\mathbf{q}) + \frac{3}{2}U^2\chi^s(q) - \frac{1}{2}(U + 2V(\mathbf{q}))^2\chi^c(q)$$

triplet: $$V_{\sigma\sigma}(q) = V(\mathbf{q}) - \frac{1}{2}U^2\chi^s(q) - \frac{1}{2}(U + 2V(\mathbf{q}))^2\chi^c(q)$$

Eliashberg equation

$$\lambda\Delta_{\sigma\sigma'}(k) = -\frac{T}{N}\sum_{k'} V_{\sigma\sigma'}(k,k')G_{\sigma}(k')G_{\sigma'}(-k')\Delta_{\sigma\sigma'}(k')$$

$\lambda = 1 \text{ at } T = T_{\rm c}$

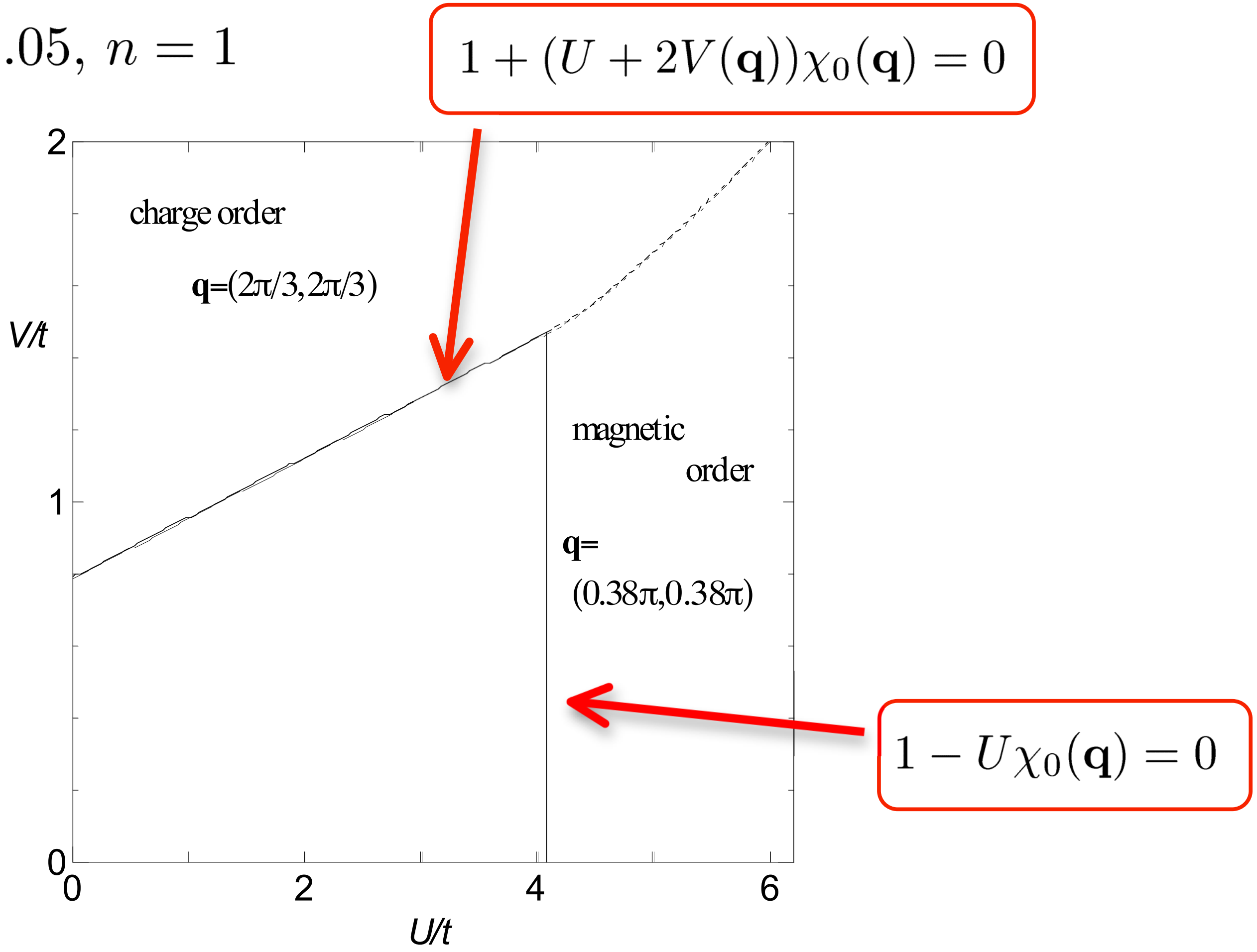
⇨ Pairing symmetry with largest λ is dominant

$$\Delta_{\sigma\sigma'}(\mathbf{k}, i\varepsilon_n) = \begin{cases} \Delta_{\sigma\sigma'}(\mathbf{k}, -i\varepsilon_n) & : \quad \text{even-frequency} \\ -\Delta_{\sigma\sigma'}(\mathbf{k}, -i\varepsilon_n) & : \quad \text{odd-frequency} \end{cases}$$

$$\Delta_{\sigma\sigma'}(\mathbf{k}, i\varepsilon_n) = \begin{cases} -\Delta_{\sigma'\sigma}(\mathbf{k}, i\varepsilon_n) & : \quad \text{spin-singlet} \\ \Delta_{\sigma'\sigma}(\mathbf{k}, i\varepsilon_n) & : \quad \text{spin-triplet} \end{cases}$$

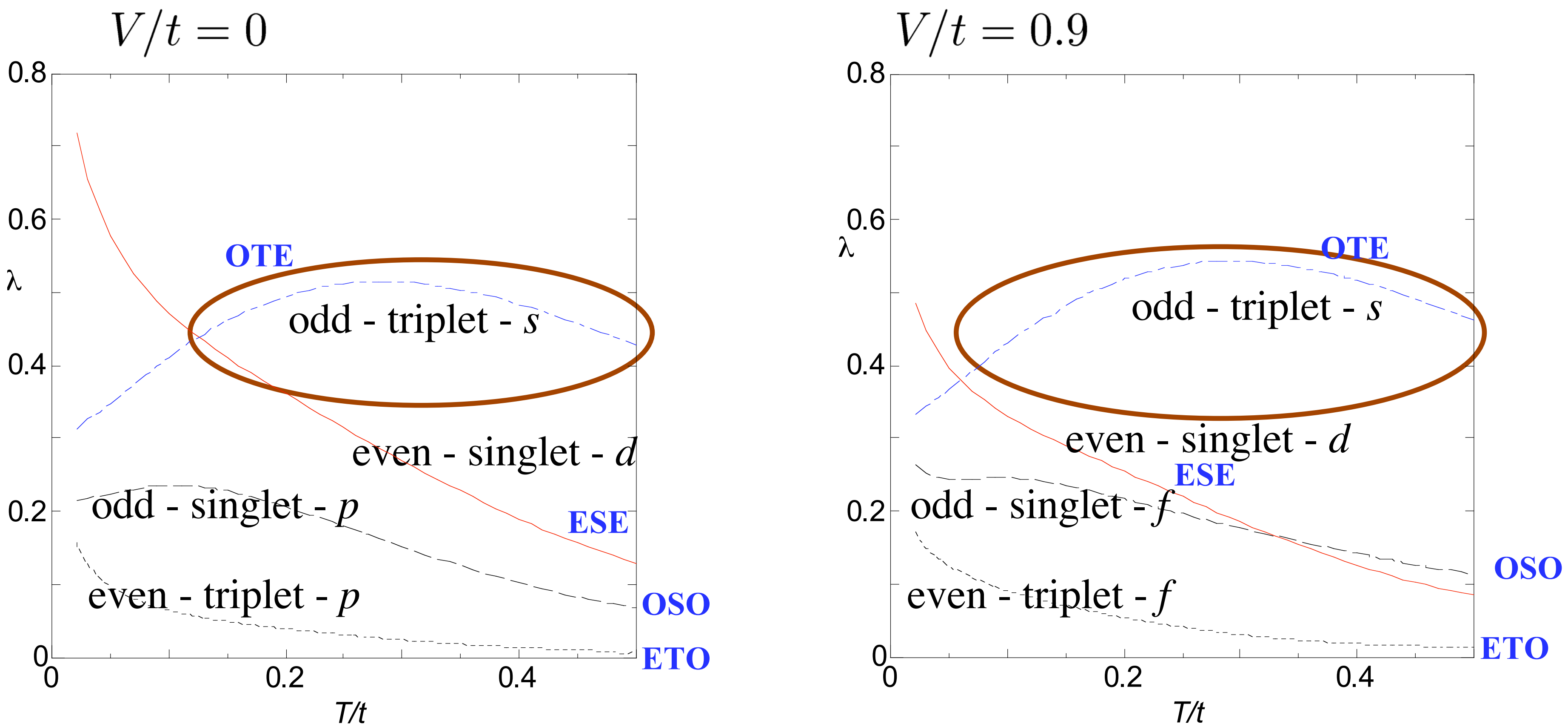
U-V Phase diagram

$T/t = 0.05,\ n = 1$

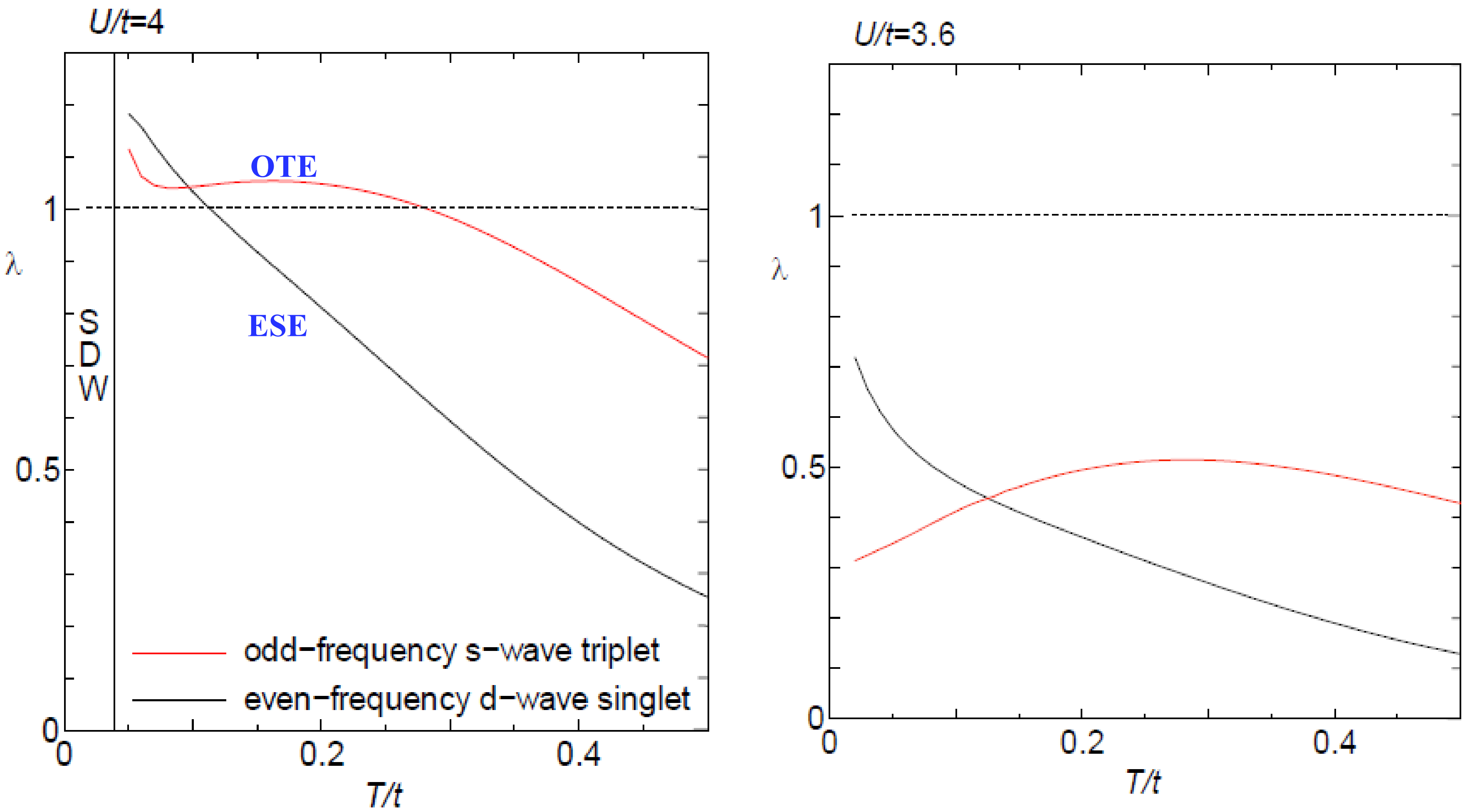


T-dependence of λ

$U/t = 3.6,\ n = 1$



T-dependence of λ



Bulk Odd-pairing state (OTE state) can be stabilized at some parameter region

Extensions to odd-frequency superconductors

Y. Tanaka, A. Golubov, S. Kashiwaya, and M. Ueda
Phys. Rev. Lett. 99 037005 (2007)

Low transparency limit

	bulk state	sign change	interface state
(1)	ESE (s or $d_{x^2-y^2}$-wave)	No	ESE
(2)	ESE (d_{xy}-wave)	Yes	OSO
(3)	ETO (p_y-wave)	No	ETO
(4)	ETO (p_x-wave)	Yes	OTE
(5)	OSO (p_y-wave)	No	OSO
(6)	OSO (p_x-wave)	Yes	ESE
(7)	OTE (s or $d_{x^2-y^2}$-wave)	No	OTE
(8)	OTE (d_{xy}-wave)	Yes	ETO

- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**
- **OSO (Odd-frequency spin-singlet odd-parity**

PRL 99 037005 (2007)

Josephson couplings between even-frequency superconductor and odd-frequency one

	bulk state	sign change	interface state
(1)	ESE (s or $d_{x^2-y^2}$-wave)	No	ESE
(2)	ESE (d_{xy}-wave)	Yes	OSO
(3)	ETO (p_y-wave)	No	ETO
(4)	ETO (p_x-wave)	Yes	OTE
(5)	OSO (p_y-wave)	No	OSO
(6)	OSO (p_x-wave)	Yes	ESE
(7)	OTE (s or $d_{x^2-y^2}$-wave)	No	OTE
(8)	OTE (d_{xy}-wave)	Yes	ETO

1. (1) and (6)
2. (2) and (5)
3. (3) and (8)
4. (4) and (7)

Presence of the Lowest order Josephson coupling

PRL 99 037005 (2007)

Previous theory

Abrahams, Balatsky, Scalapino, and Schrieffer
Phys. Rev. B 52, 1271 - 1278 (1995)

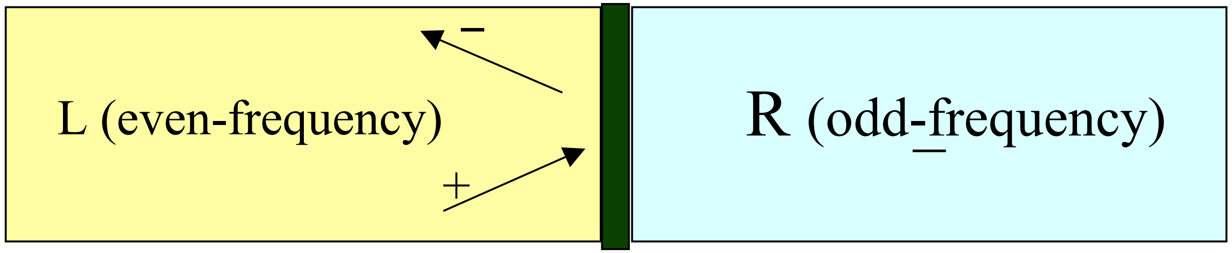
There is no lowest-order Josephson coupling between odd- and even-frequency superconductors.

Interface induced state is neglected!!

Josephson current

$$R_N I(\varphi) = \frac{\pi}{2e}\sum_{\sigma} k_B T \sum_{\omega}\{< f_{1L+}f_{1R+}+f_{2L+}f_{2R+} > \sin\varphi + < f_{1L+}f_{2R+}-f_{2L+}f_{1R+} > \cos\varphi\}$$

(Lowest Order coupling)



φ

(Macroscopic phase difference between two superconductors)

f_{1L+} f_{1R+} **Interface state**

(1)L-side (Even-frequency superconductor)

f_{1L+} Odd function of Matsubara

f_{2L+} Even function of Matsubara

(2)R-side (odd-frequency superconductor)

f_{1R+} Even function of Matsubara

f_{2R+} Odd function of Matsubara

$\cos\varphi$

Anomalous current phase relation

PRL 99 037005 (2007)

Underlying physics (1)

Near the interface, even and odd-parity pairing states can mix due to the breakdown of the translational symmetry.

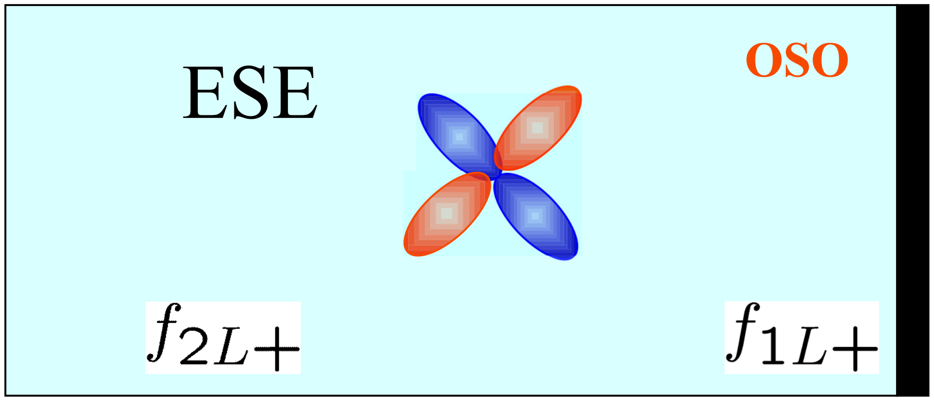
↓

The Fermi-Dirac statistics then dictates that the induced pair amplitude at the interface should be odd (even) in frequency where the bulk pair potential has an even (odd)-frequency component.

PRL 99 037005 (2007)

Underlying physics (2)

To be compatible with the time reversal invariance in each superconductor, the phase of the interface induced pair amplitude undergoes a π/2 shift from that of the bulk one.

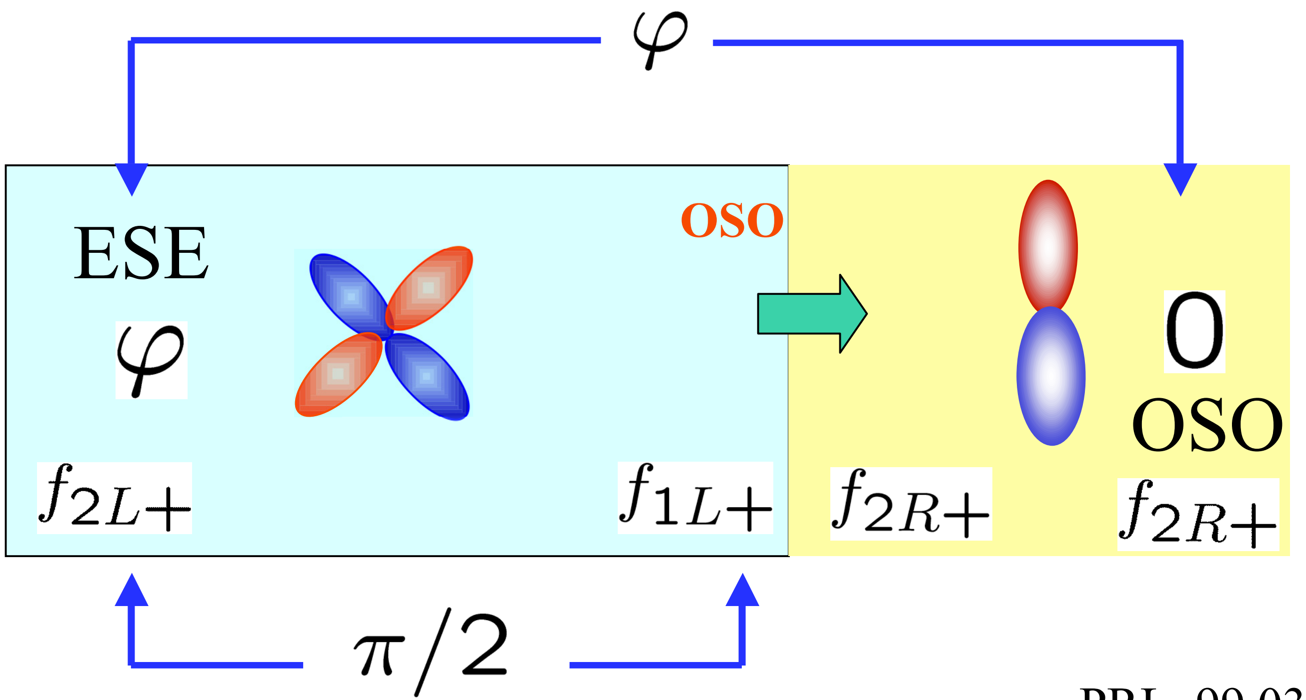


The phase of f_{1L+} has a π/2 shift from that of f_{2L+}.

PRL 99 037005 (2007)

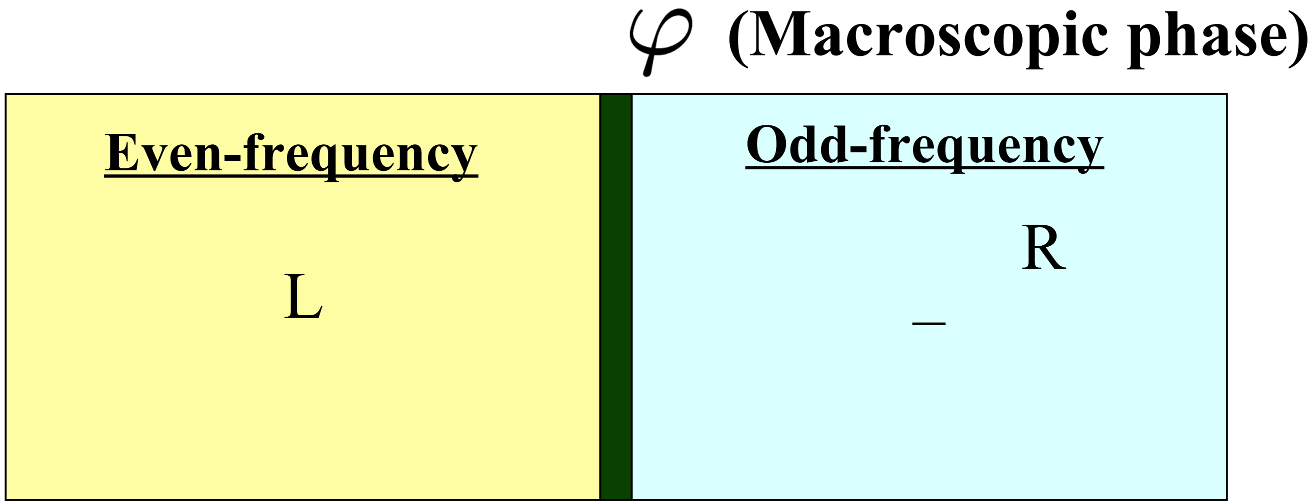
Underlying physics (3)

This twist of the phase of the pair amplitude gives rise to an anomalous Josephson current, where the current phase relation is proportional cosφ.



PRL 99 037005 (2007)

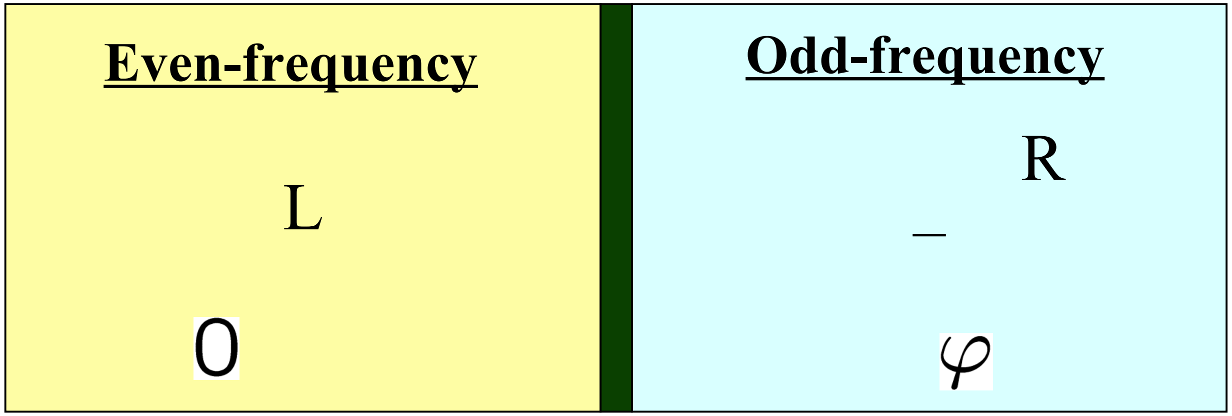
Breakdown of the time reversal symmetry when we make a junction



Although both superconductors **do not break the time reversal symmetry themselves**, the resulting Josephson coupling **breaks the time reversal symmetry** since the parities with respect to frequency dependence in even- and odd-frequency superconductors differ from each other.

PRL 99 037005 (2007)

Anomalous Josephson effect between even- and odd-frequency superconductors



Current-phase relation becomes $\cos\varphi$

Y. Tanaka, A. Golubov, S. Kashiwaya, and M. Ueda
PRL 99 037005 (2007)

Conclusions

(1)Ubiquitous presence of the odd-frequency pairing state

(2)Odd-frequency pairing state is enhanced in the presence of the mid gap Andreev resonant state.

(3)The origin of the **anomalous proximity effect in DN/spin-triplet p-wave junction** is the generation of the odd-frequency pairing state.

(4)Odd-frequency pairing state in a vortex core

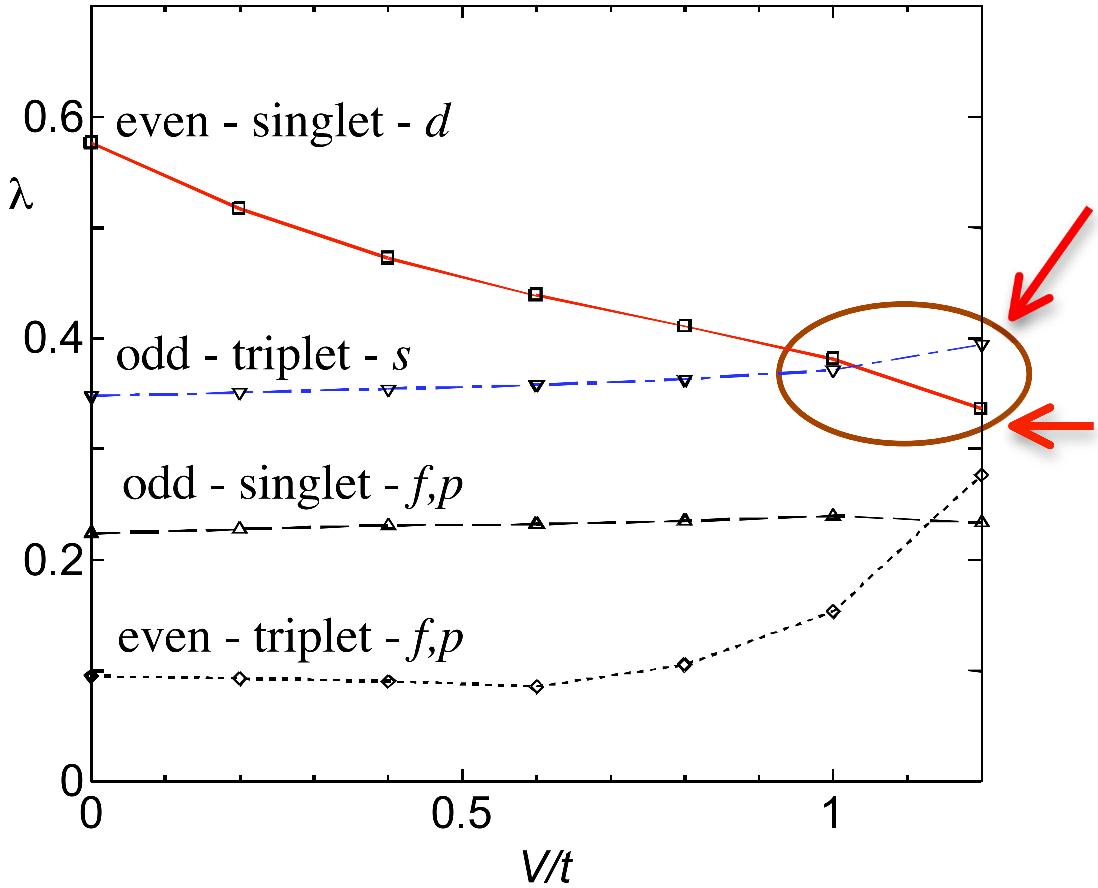
(5)Anomalous Josephson coupling between even and odd-frequency superconductor

Summary

Odd-frequency pairing becomes a key concept in the physics of superconductor junctions and non-uniform superconducting and superfluid systems.

V-dependence of λ

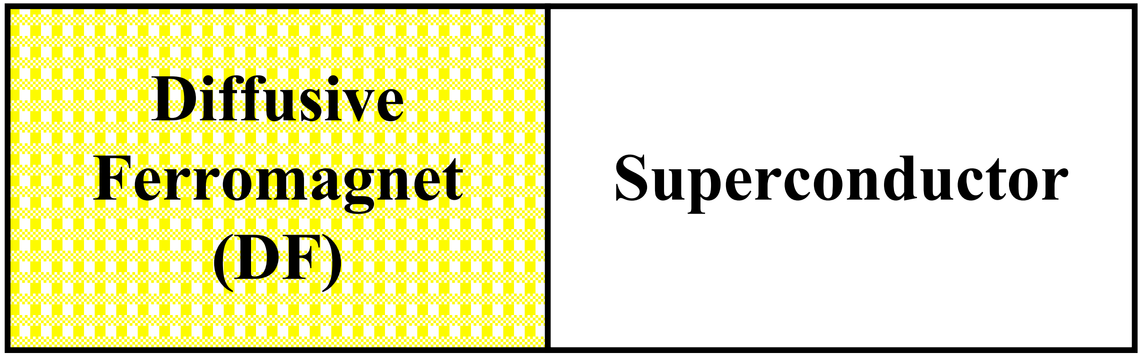
$T/t = 0.05,\ U/t = 3.6,\ n = 1$



奇周波数 スピン三重項
*s*波は*V*の増大に伴って
わずかに増大

偶周波数 スピン一重
項
*d*波は*V*によって抑えら
れる

Ferromagnet (metal)/superconductor junctions



Only s-wave pairing state is possible in DF

（1）Weak spin-polarized ferromanget

T. Yokoyama, Y. Tanaka, and A.A. Golubov PRB 75 134510 (2007)

（2）Fully spin-polarized ferromagnet

Y.Asano, Y.Tanaka and A.A. Golubov PRL 98, 107002 (2007)

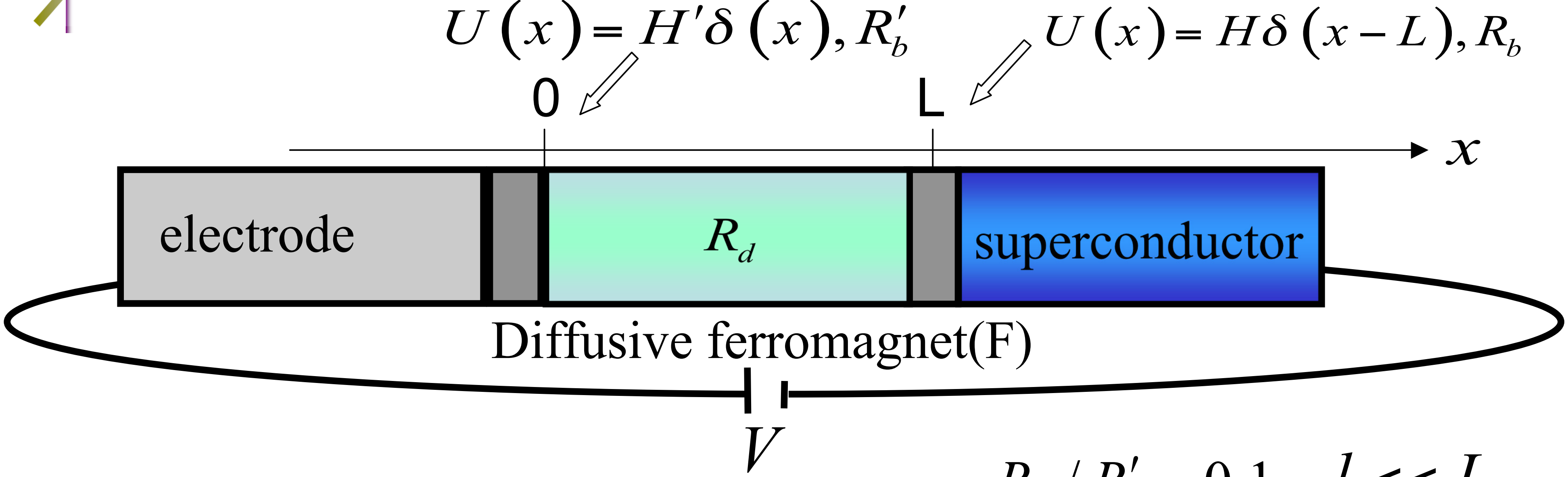
General theory of Proximity effect
(diffusive ferromagnet (metal)/ superconductor junctions)

	Symmetry of the pair potential	Induced pair amplitude in DN
(1)	**Even-frequency spin-singlet even-parity (ESE)**	**ESE + OTE**
(2)	**Even-frequency spin-triplet odd-parity (ETO)**	**OTE + ESE**

- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**

T. Yokoyama, Y. Tanaka, and A.A. Golubov PRB 75 134510 (2007)

Model and calculation



$R_d / R_b' = 0.1 \quad l << L$

$h \ll E_F$

Usadel equation

$$D\frac{\partial}{\partial x}\left(\breve{G}_1 \frac{\partial}{\partial x}\breve{G}_1\right) + i\left[\breve{H}, \breve{G}_1\right] = 0,$$

D	Diffusion constant
ε	quasiparticle energy
h	Exchange field

$$\breve{H} = \begin{pmatrix} (\varepsilon + (-)h)\tau_3 & 0 \\ 0 & (\varepsilon + (-)h)\tau_3 \end{pmatrix}$$

for majority (minority) spin

Parametrization in Usadel Equation

$$D\frac{\partial^2\theta}{\partial x^2} + 2i(\varepsilon - h)\sin\theta = 0 \qquad D\frac{\partial^2\bar{\theta}}{\partial x^2} + 2i(\varepsilon + h)\sin\bar{\theta} = 0$$

$$\sin\theta = f_{\uparrow,\downarrow} = \bar{f}_{\downarrow,\uparrow} \qquad \sin\bar{\theta} = f_{\downarrow,\uparrow} = \bar{f}_{\uparrow,\downarrow}$$

$$\bar{\theta}(\varepsilon) = -\theta^*(-\varepsilon)$$

Singlet component

$$f_0 = \frac{1}{2}(f_{\uparrow,\downarrow} - f_{\downarrow,\uparrow}) \qquad f_0(\varepsilon) = [f_0(-\varepsilon)]^*$$

Triplet component

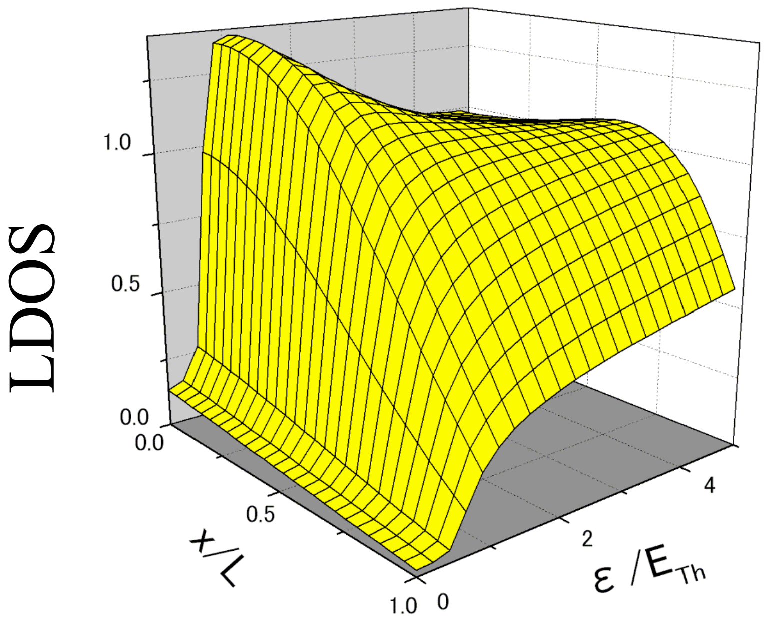
$$f_3 = \frac{1}{2}(f_{\uparrow,\downarrow} + f_{\downarrow,\uparrow}) \qquad f_3(\varepsilon) = -[f_3(-\varepsilon)]^*$$

LDOS（s-wave superconductor）

$R_d / R_b = 5 \quad E_{Th} / \Delta = 0.01$

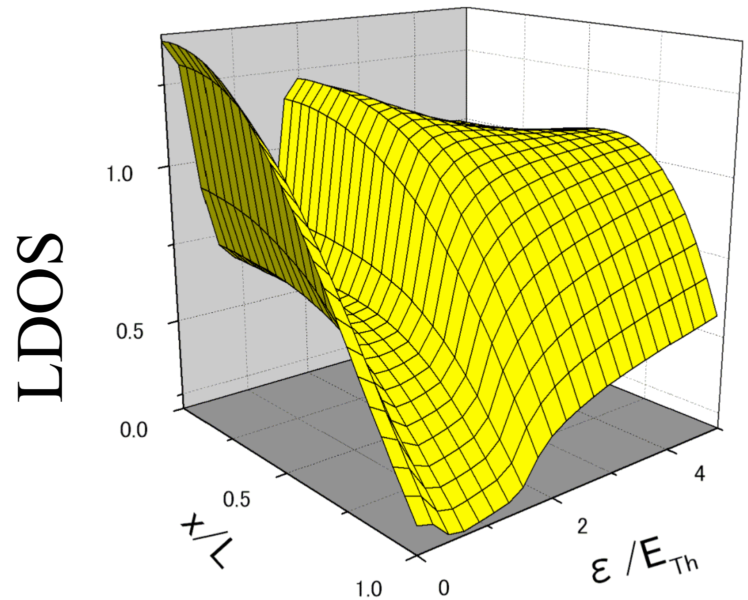
$E_{Th} = \dfrac{\hbar D}{L^2}$ **Thouless energy**

$h / E_{Th} = 0$



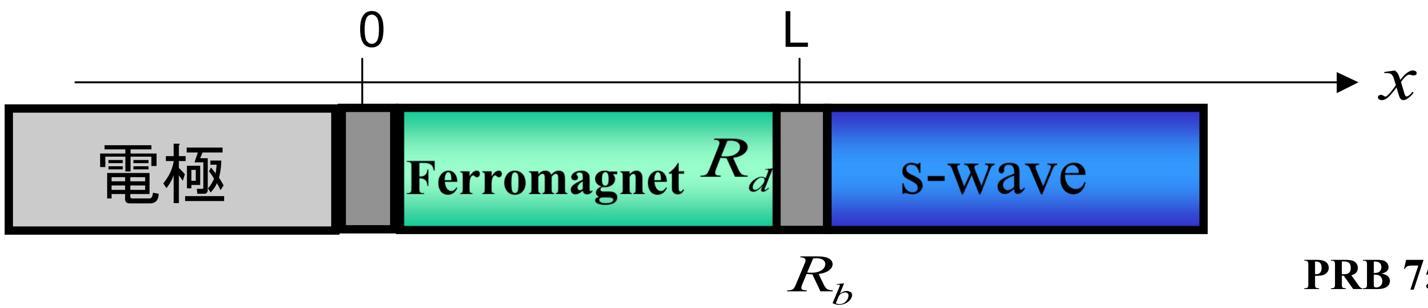
Normal metal

$h / E_{Th} = 1$



Ferromagnet

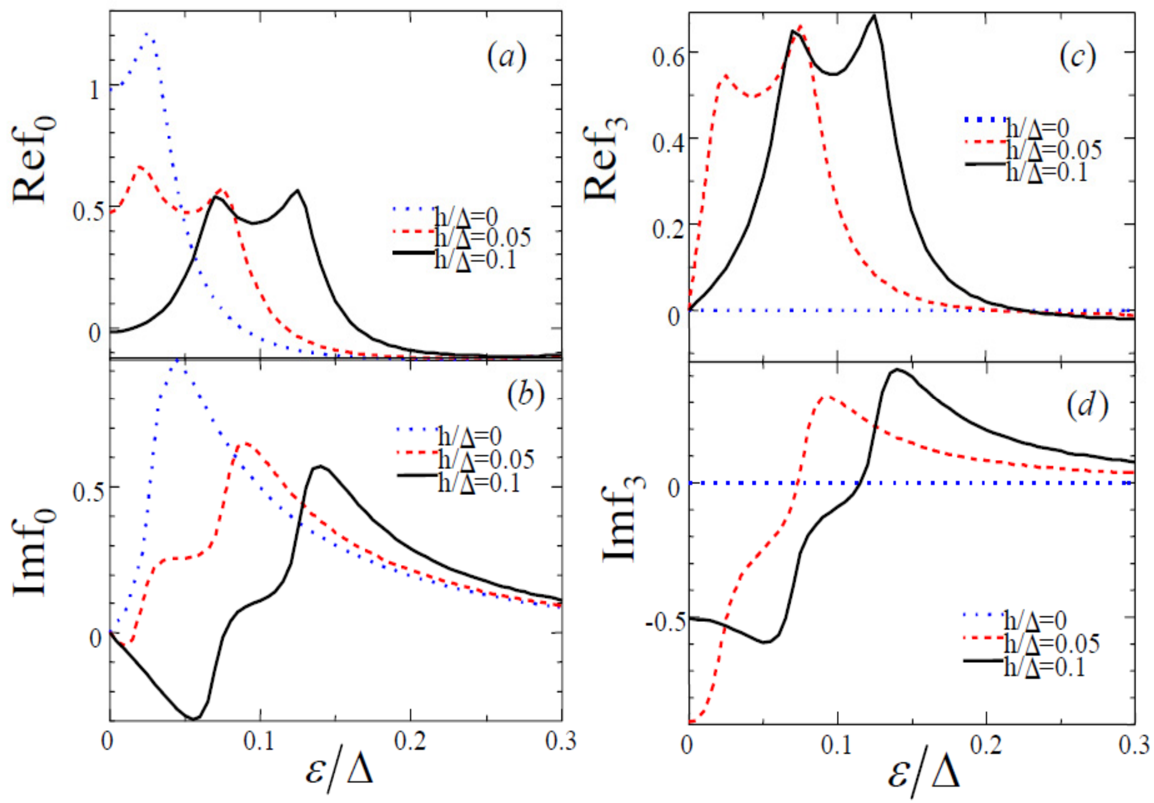
ε : energy



PRB 75 134510 (2007)

Pair amplitude in diffusive ferromagnet

ESE OTE



$$f_0(\varepsilon) = [f_0(-\varepsilon)]^*$$

(ESE state)

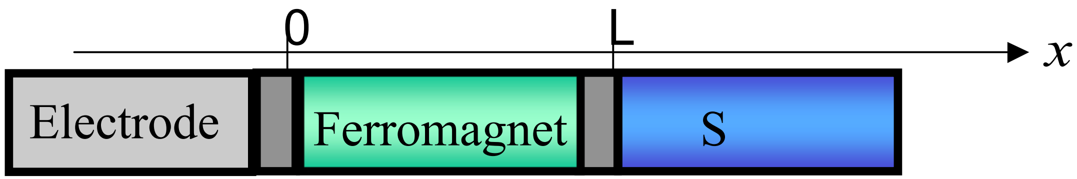
$$f_3(\varepsilon) = -[f_3(-\varepsilon)]^*$$

(OTE state)

ESE (Even-frequency spin-singlet even-parity)

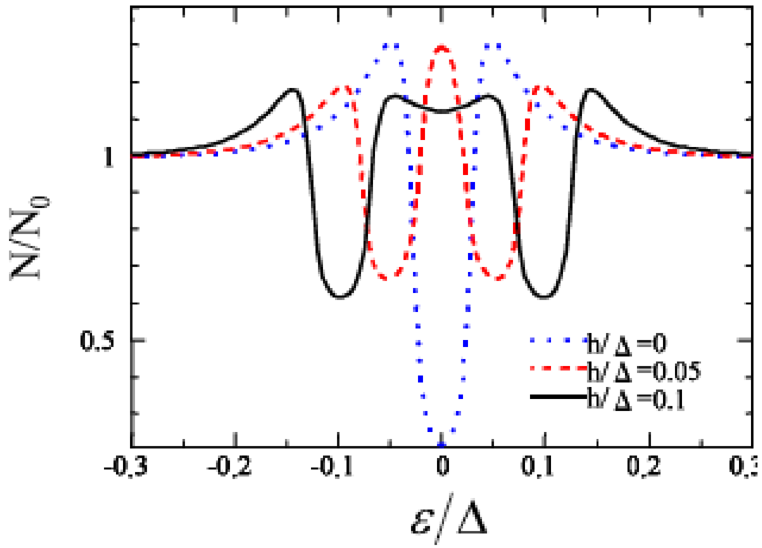
OTE (Odd-frequency spin-triplet even-parity)

$Z = 3 \quad Z' = 3 \quad R_d / R_b = 1 \quad R_d / R_b' = 0.1 \quad E_{Th} / \Delta = 0.1$

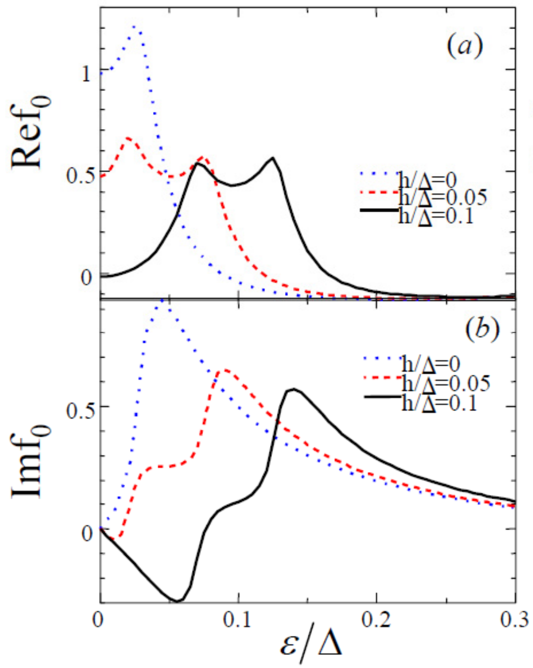


PRB 75 134510 (2007)

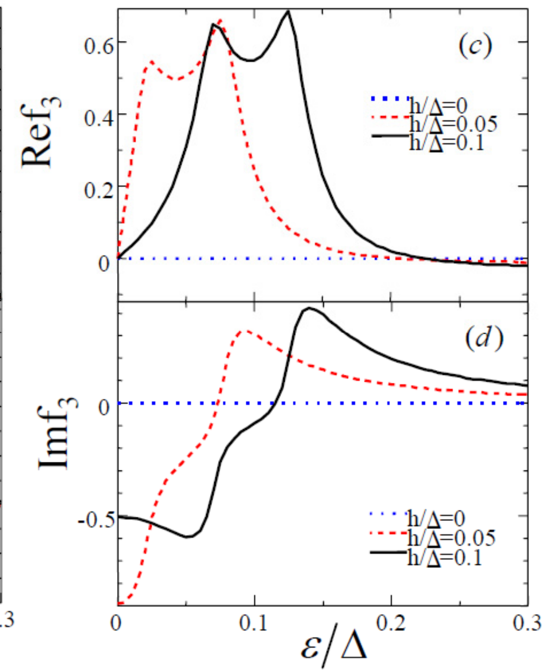
LDOS (x=0)



ESE (x=0)

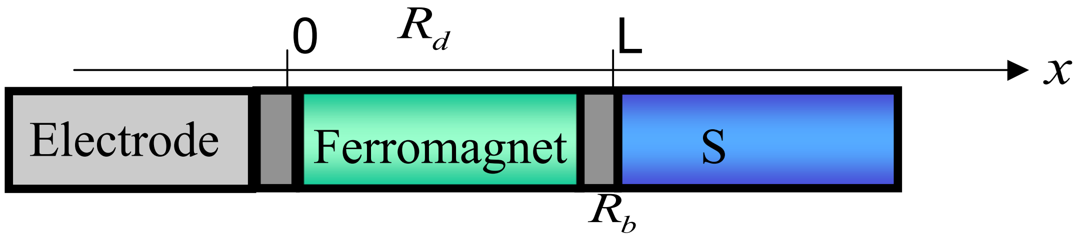


OTE (x=0)



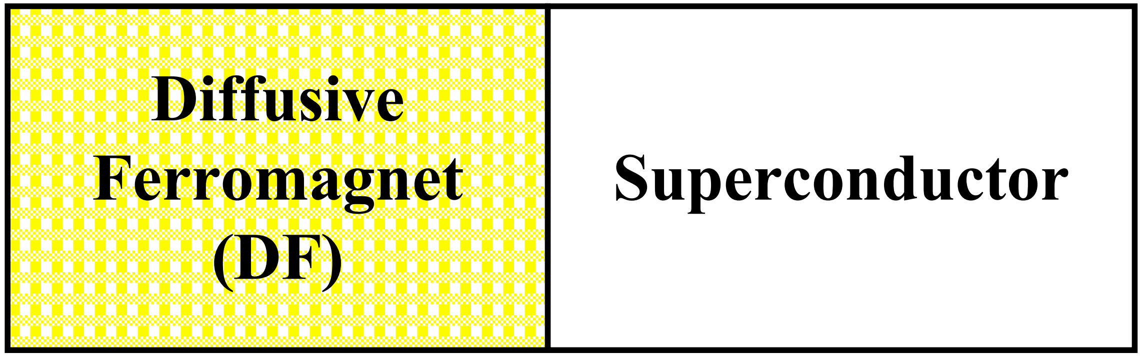
$Z = 3 \quad Z' = 3 \quad R_d / R_b = 1 \quad R_d / R_b' = 0.1 \quad E_{Th} / \Delta = 0.1$

LDOS at ε=0 is enhanced, when the odd frequency component (spin-triplet s-wave) is enhanced.



T. Yokoyama, Y. Tanaka, and A.A. Golubov PRB 75 134510 (2007)

Ferromagnet (metal)/superconductor junctions



Only s-wave pairing state is possible in DF

(1) Weak spin-polarized ferromanget

T. Yokoyama, Y. Tanaka, and A.A. Golubov PRB 75 134510 (2007)

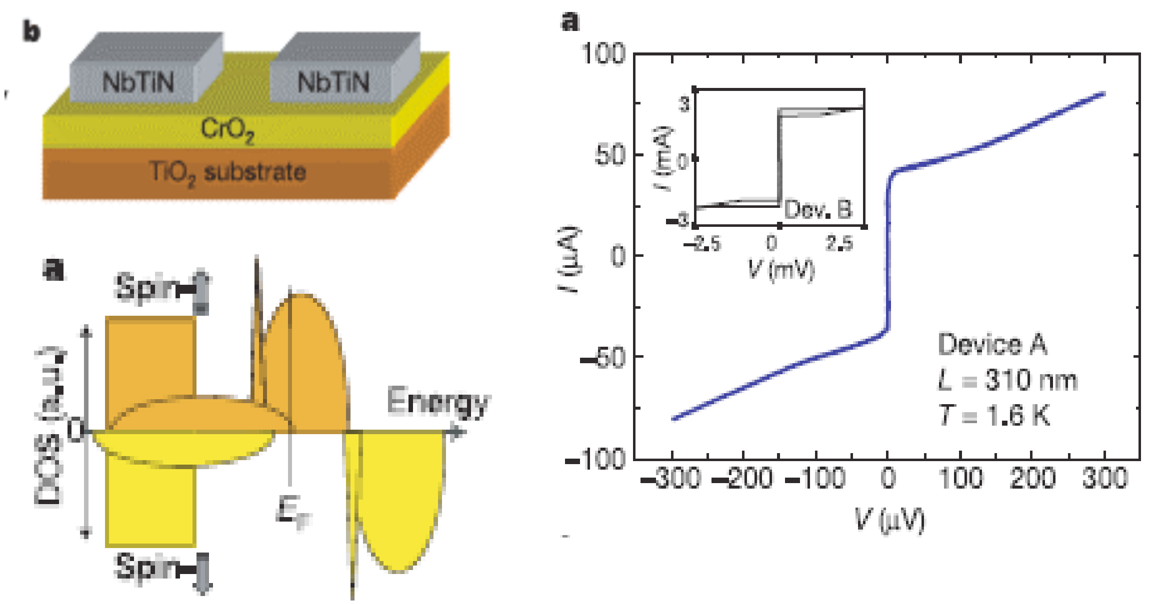
(2) Fully spin-polarized ferromagnet

Y.Asano, Y.Tanaka and A.A. Golubov PRL 98, 107002 (2007)

Only Spin-triplet pairing Is possible

Josephson current in S/HM/S

Half metal : CrO_2 Keizer et.al., Nature ('06)



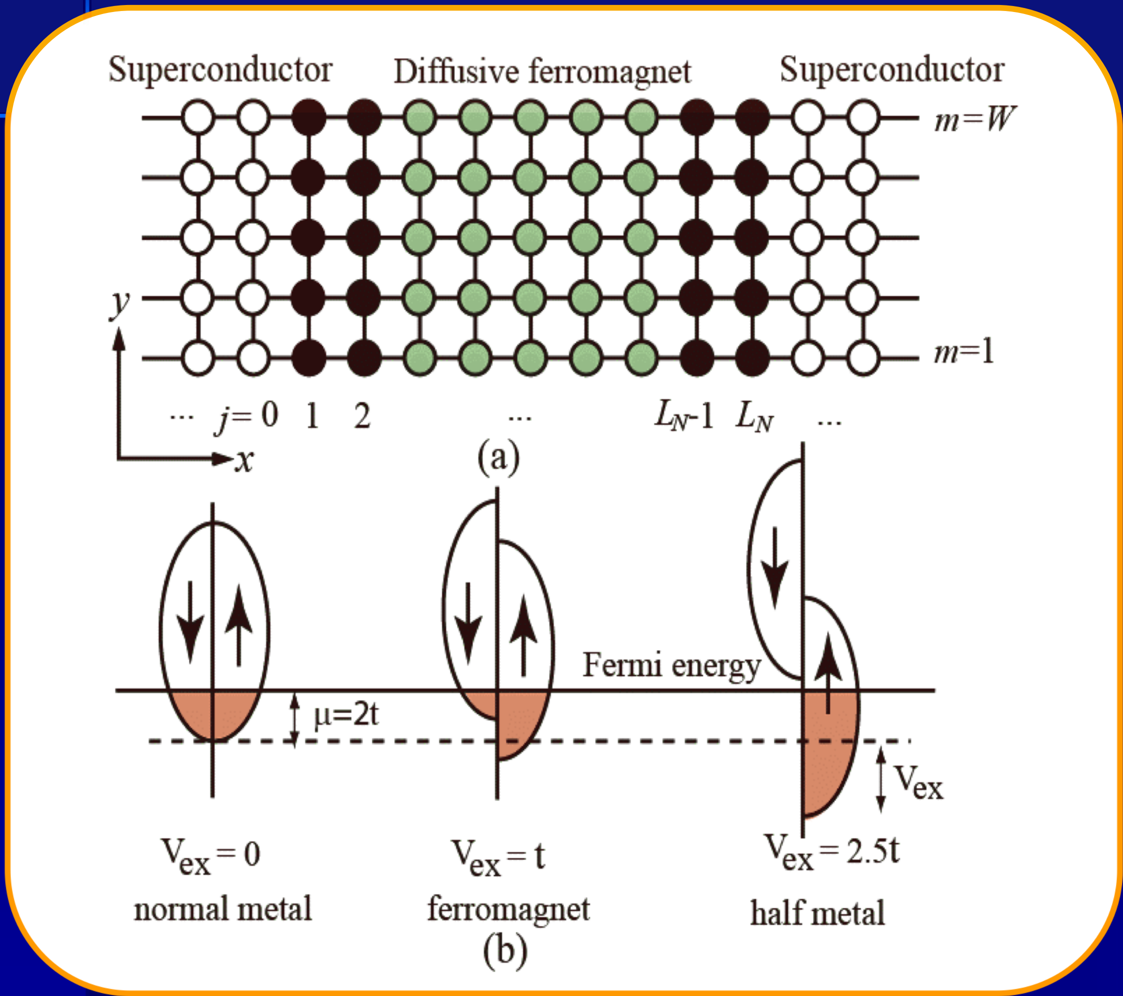
Spin active interface Bergeret et. al., PRL('01), Kadigrobov et. al., Europhys Lett.('01)

Theory in the clean limit Eschrig et. al., PRL('03)

Theory in the diffusive limit Aasno Tanaka Golubov, PRL('07)

Lattice model (numerical)

Furusaki, Physica B('92),
Asano, PRB('01)



Advantages

SNS, SFS, S/HM/S

[illegible]

[illegible]

Parameters

[illegible] : exchange

[illegible] : spin-flip

Usadel equation is not available!!

Y.Asano, Y.Tanaka and A.A. Golubov PRL 98, 107002 (2007)

Definition of Pairing functions

Gor'kov Green Function

ESE (Even-frequency spin-singlet even-parity)

OTE (Odd-frequency spin-triplet even-parity)

Spin states

Singlet (ESE)

hetero spin triplet (OTE)

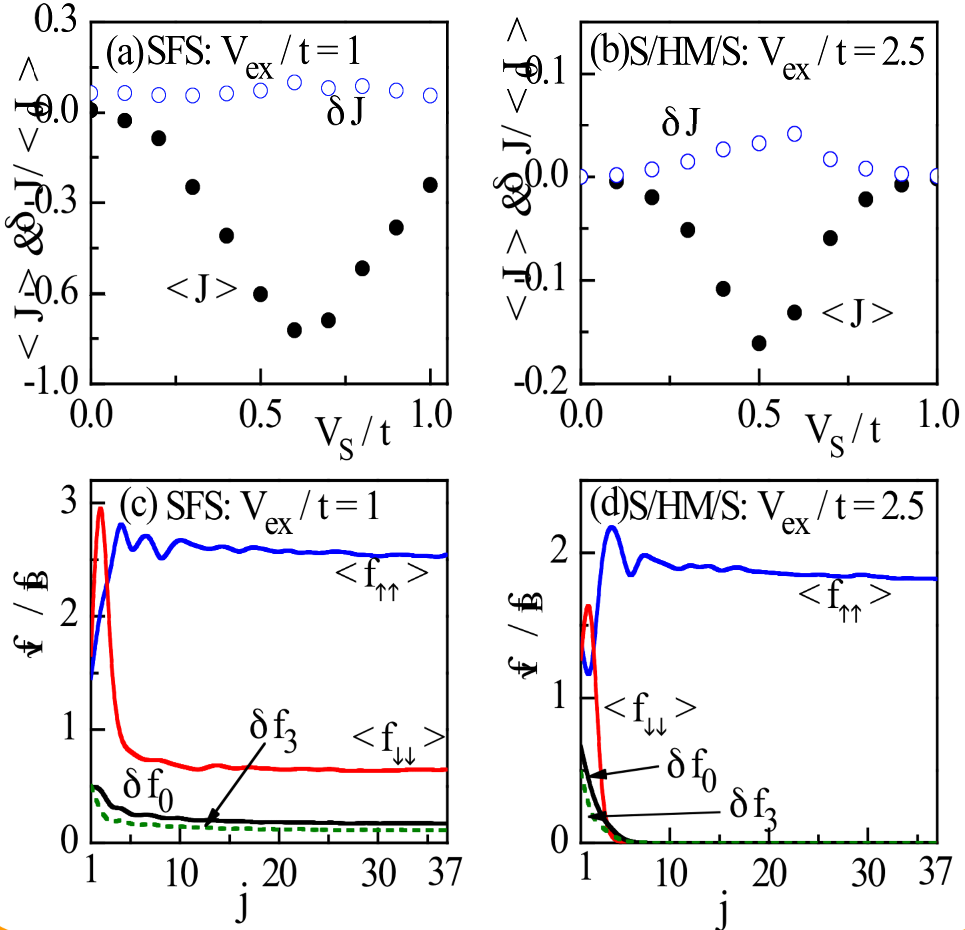
equal spin triplet (OTE)

Y.Asano, Y.Tanaka and A.A. Golubov PRL 98, 107002 (2007)

Spin active interface [redacted]

SFS, S/HM/S

[redacted]



self-averaging

[redacted]

No sign change is needed

OTE **Odd-frequency spin-triplet even-parity**

[redacted]

ESE **Even-frequency spin-singlet even-parity**

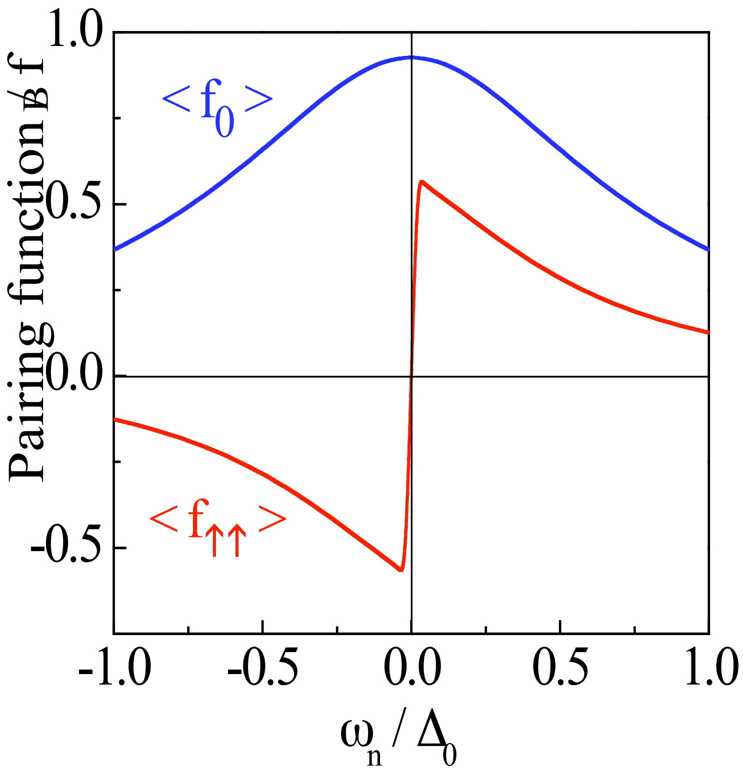
[redacted]

SFS [redacted]

S/HM/S [redacted] only

Y.Asano, Y.Tanaka and A.A. Golubov PRL 98, 107002 (2007)

Typical frequency dependence



$\langle f_0 \rangle$

Even-frequency
spin-singlet
s-wave (ESE)
$V_{ex}=0$

$\langle f_{\uparrow\uparrow} \rangle$

Odd-frequency
spin-triplet
s-wave (OTE)

Y.Asano, Y.Tanaka and A.A. Golubov PRL 98, 107002 (2007)

Quasiparticle DOS in HM

SFS

⟨[redacted]⟩

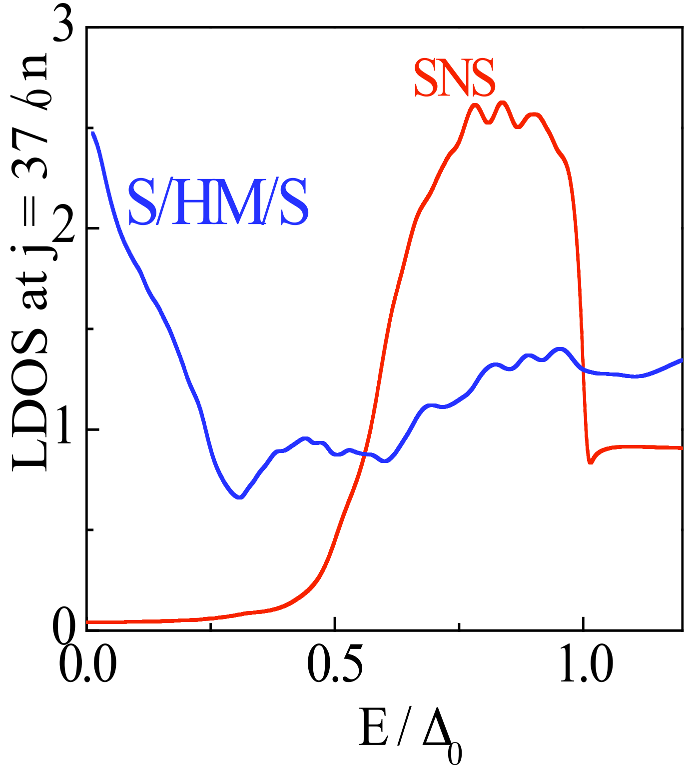
⟨[redacted]⟩, ⟨[redacted]⟩, ⟨[redacted]⟩

even-odd mix

S/HM/S

⟨[redacted]⟩ only

pure odd



ZEP

can be detected by STS

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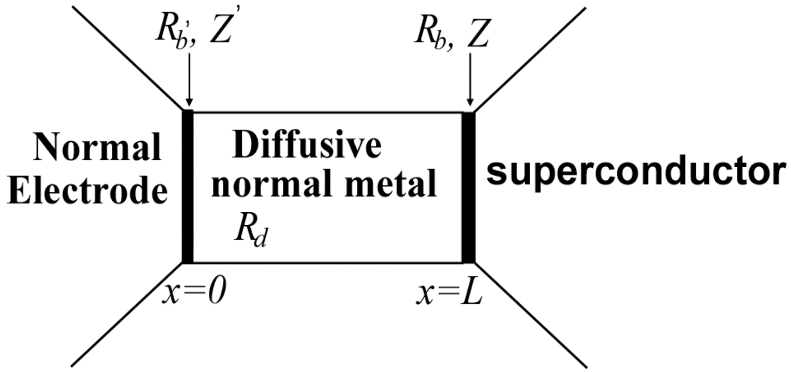
Meissner effect



$$\hat{R}_N(x) = \sin\theta\hat{\tau}_2 + \cos\theta\hat{\tau}_3$$

$$j(x) = \pi e^2 N(0) D T \sum_{\omega_n} \text{Trace}[\hat{\tau}_3 \hat{R}_N(x)[\hat{\tau}_3, \hat{R}_N(x)]] A(x)$$

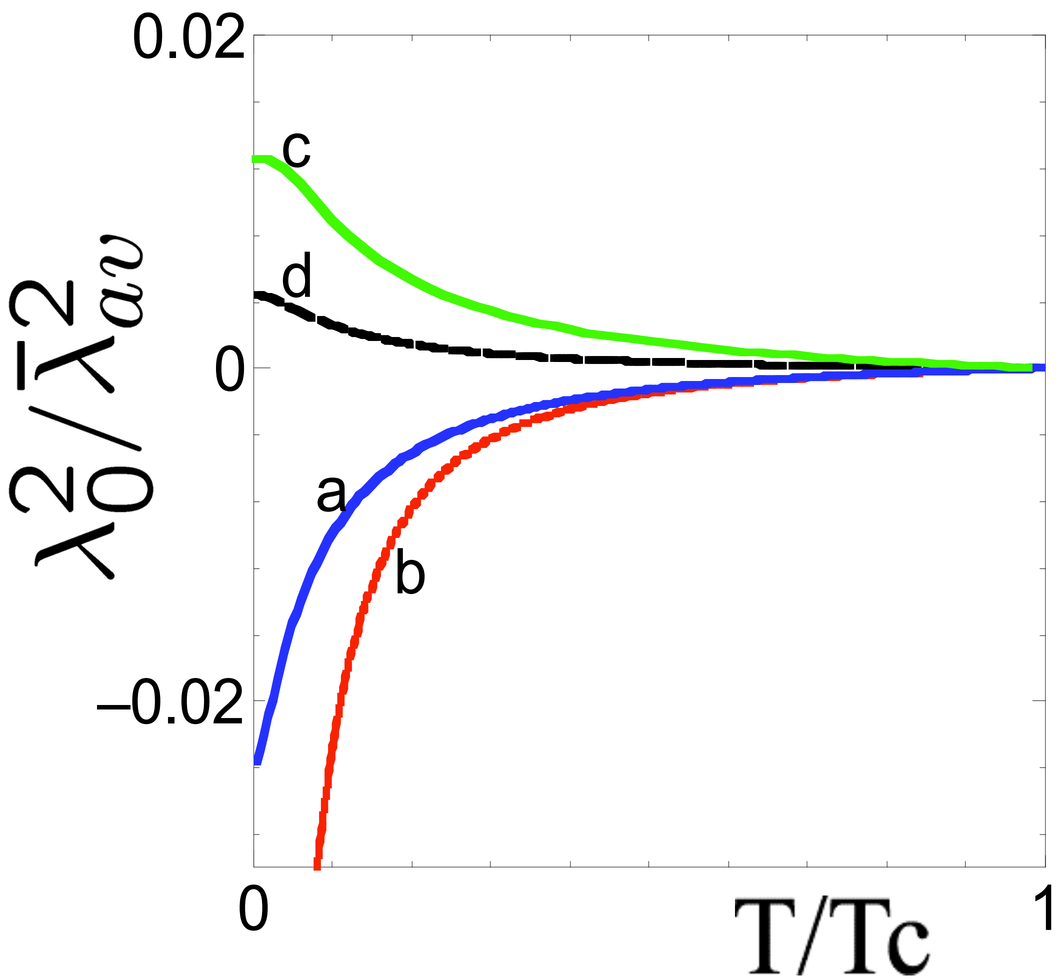
$$H(x) \sim \exp(-x/\lambda(x))$$

$$\frac{1}{\lambda^2(x)} = \frac{T\sum_{\omega_n} \sin^2\theta(\omega_n)}{\lambda_0^2}, \quad \lambda_0^{-2} = 32\pi^2 e^2 N(0) D T_C$$

$$\bar{\lambda}_{av}^2 = L/\int_0^L \frac{dx}{\lambda^2(x)}$$

Narikiyo and Fukuyama, J. Phys. Soc. Jpn. 58, 4557 (1989)
Belzig Bruder PRB 53 5727 (1996)

Temperature dependence of averaged value of local penetration depth



$\bar{\lambda}_{av}$

a purely imaginary number for spin-triplet junctions

a: $p_x + ip_y$-wave
b: p_x-wave
c: s-wave
d: $d_{x^2-y^2} + id_{xy}$-wave

Tanaka, et al, PRB **72**, 140503R 2005