

Quantum dynamics in low dimensional spin systems

University of Tokyo

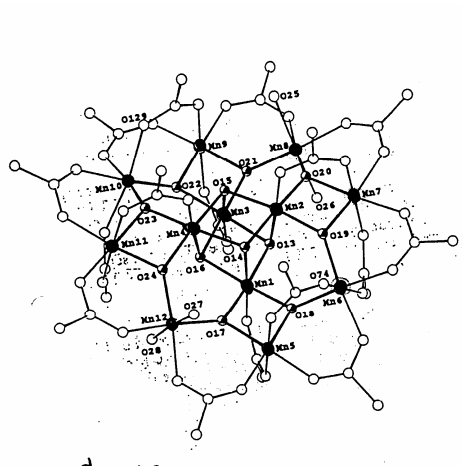
Seiji MIYASHITA

Topics

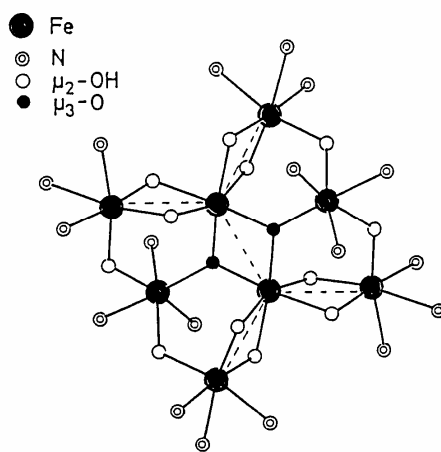
- **Quantum dynamics under time-dependent fields**
 - Quantum hysteresis in single molecular magnets
 - Landau-Zener process + Magnetic Foehn effects (Sweep)
 - Nontrivial Resonance and Coherent Destruction of Tunneling (AC)
 - Quantum mechanical reentrant phenomena
 - Quantum annealing
- **Quantum dynamics between macroscopic states**
 - Quantum spinodal phenomena of quantum phase transition
 - Nagaoka magnetism
- **Quantum Response**
 - ESR in pure quantum dynamic
 - ESR in dissipative dynamics
- **Related topics**
 - Origin of the energy gap and Gap control
 - Potential trap

Quantum dynamics of magnetization

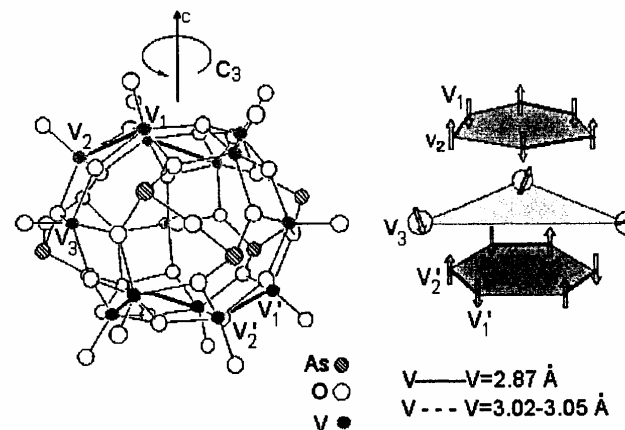
Molecular magnets



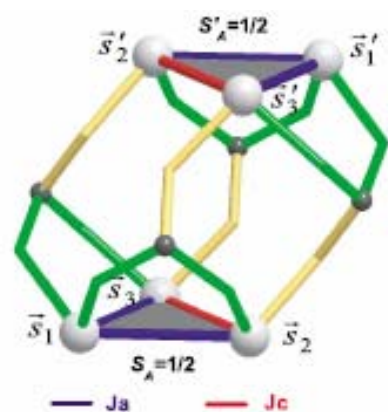
Mn12



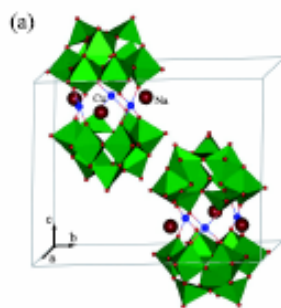
Fe8



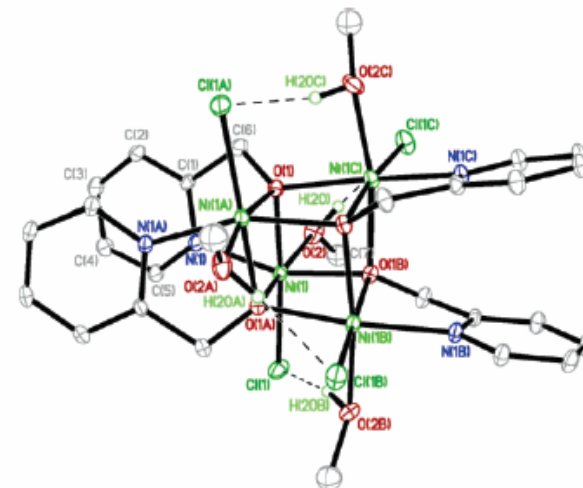
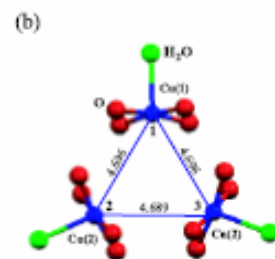
V15



V6

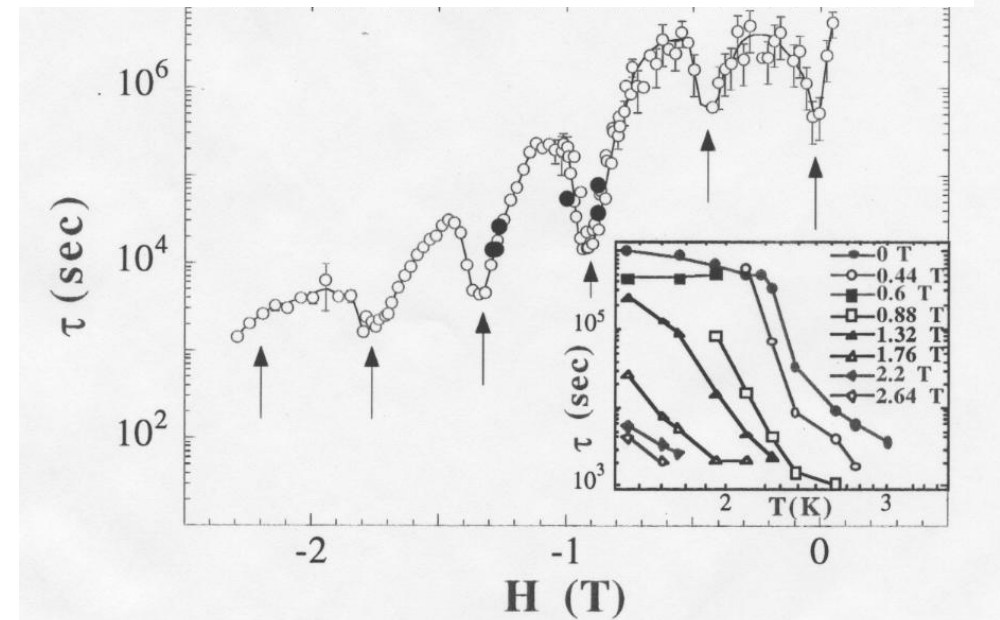
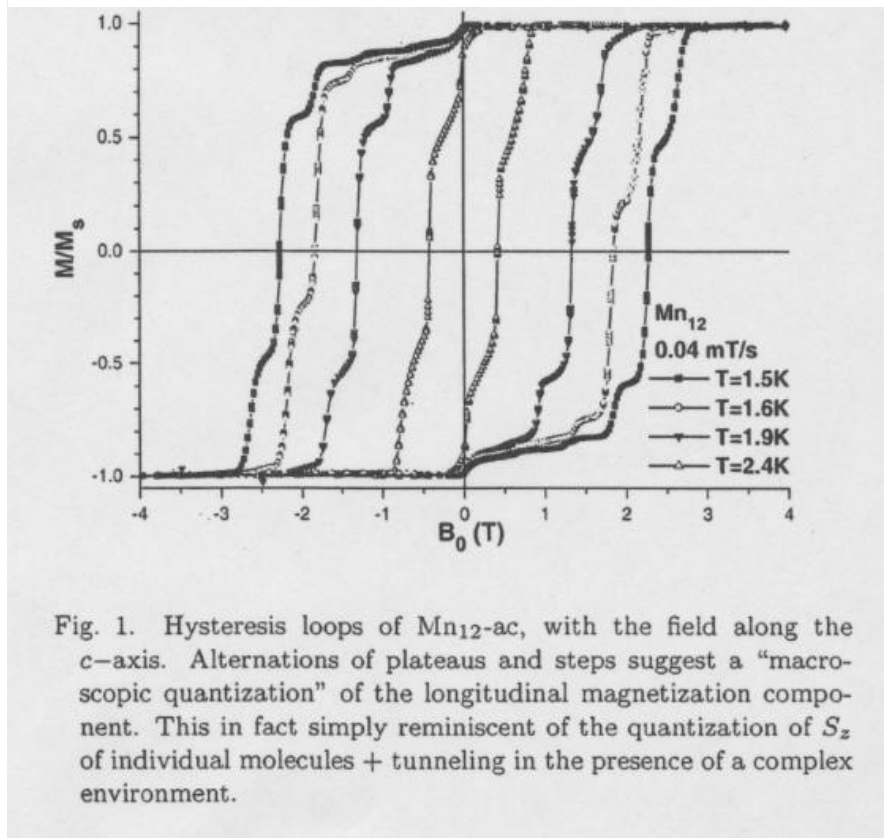


Cu3



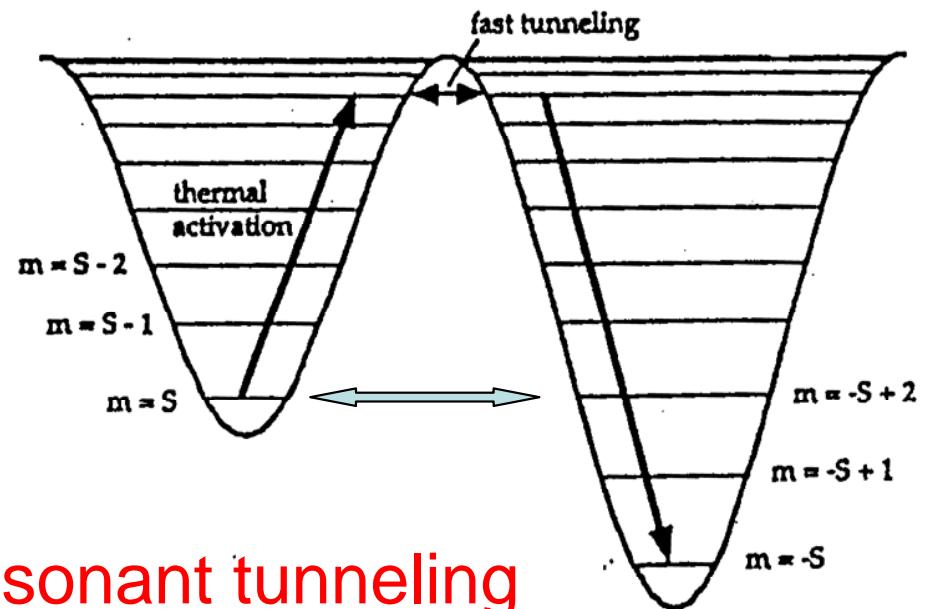
Ni4

Temperature dependence



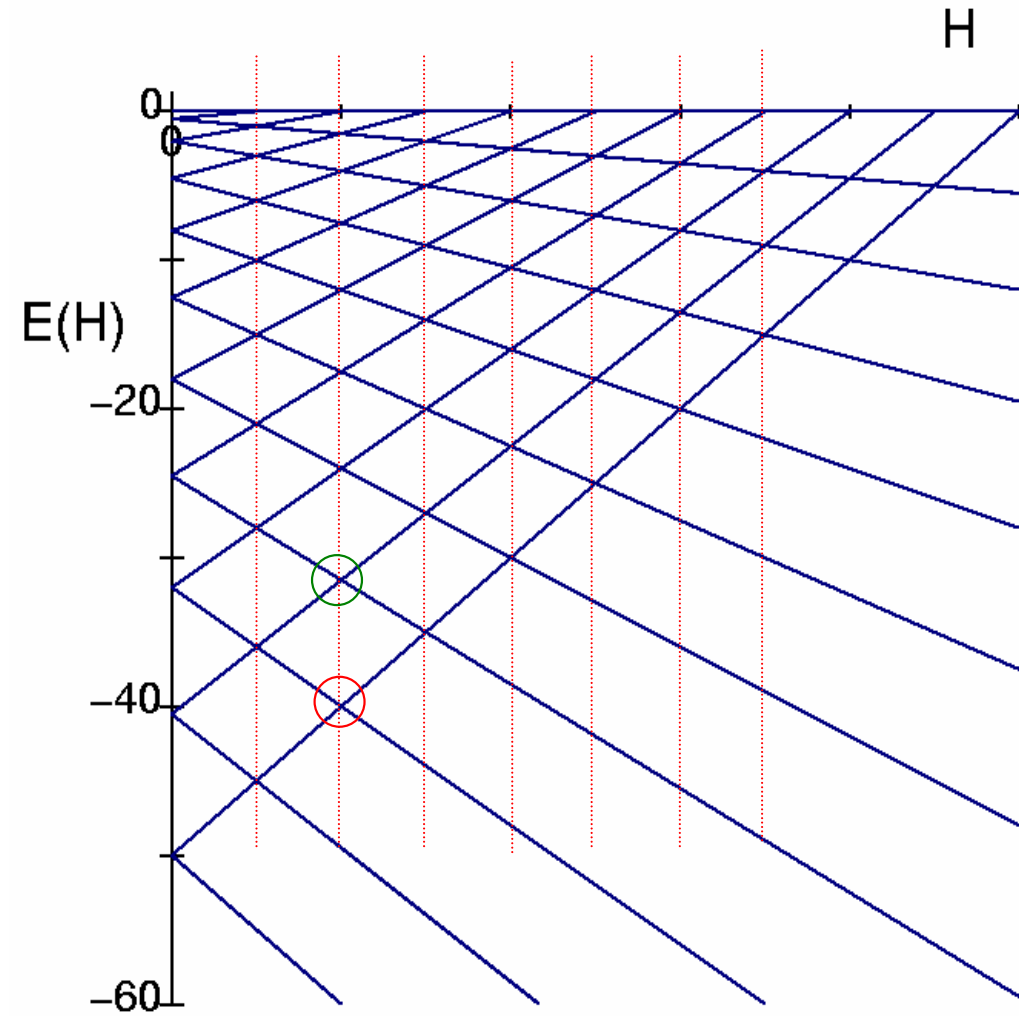
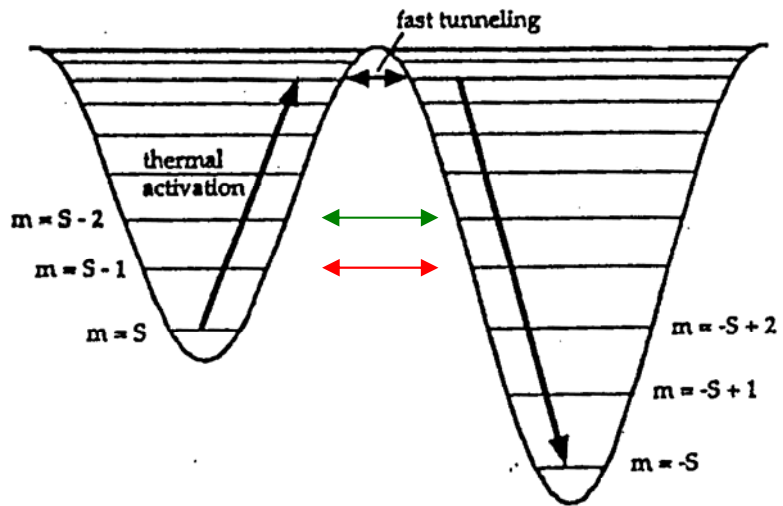
Quantum tunneling
+
Thermal effects

L. Thomas, et al.
Nature 383 (1996) 167.



Resonant tunneling

Resonance tunneling

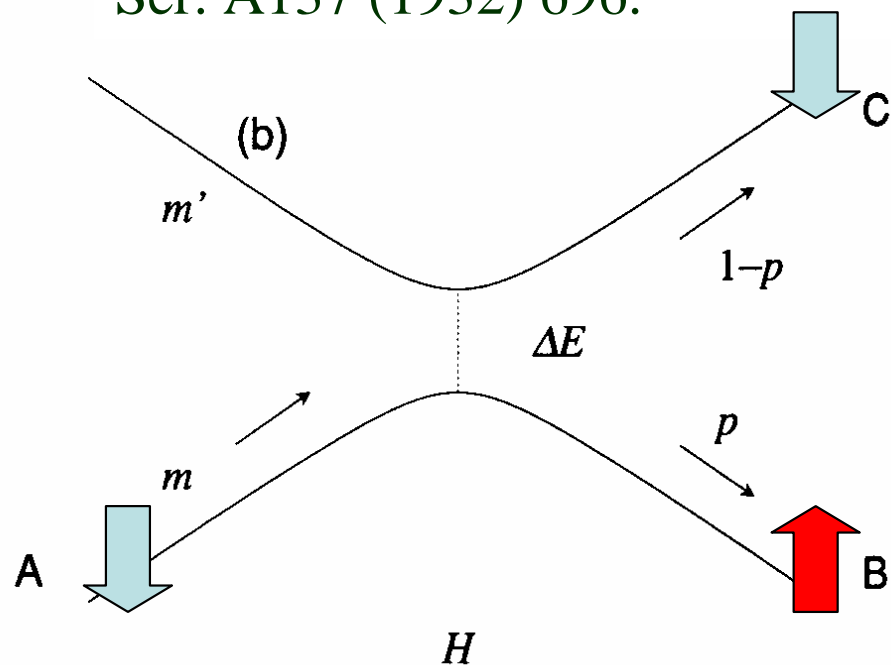


Control of quantum states in Discrete energy structure

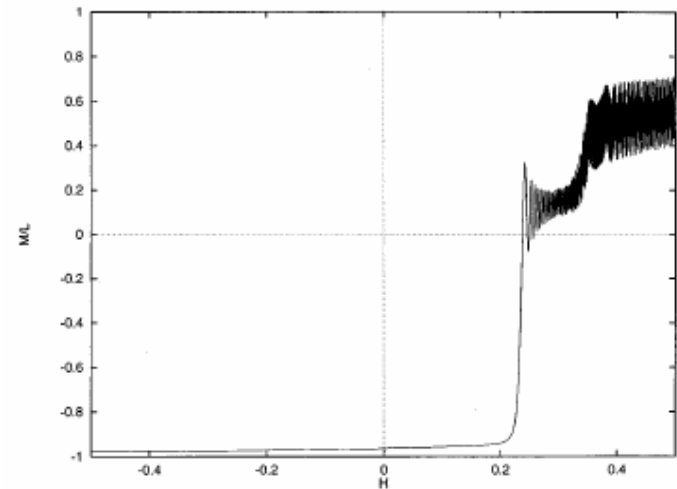
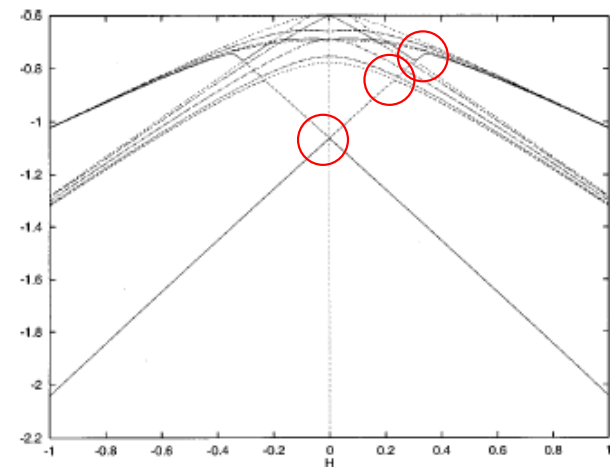
(Non)-adiabatic transition

Landau-Zener-Stueckelberg Mechanism

C. Zener, Proc. R. Soc. (London)
Ser. A137 (1932) 696.

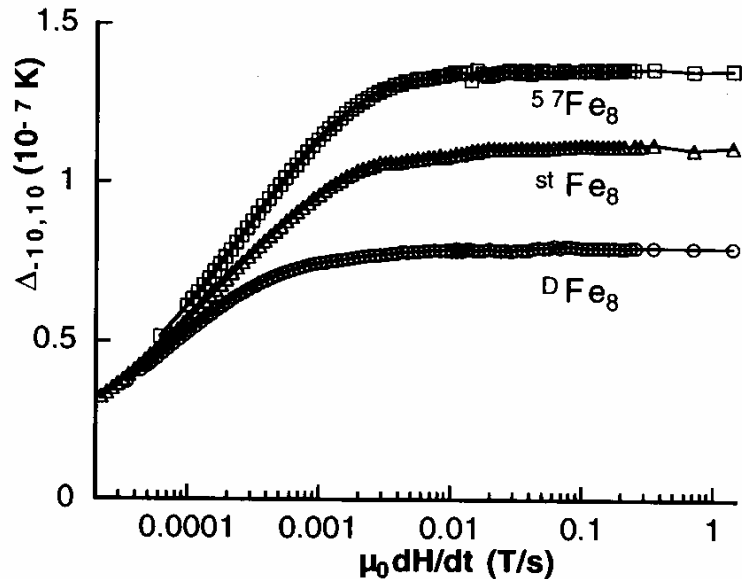


$$p = 1 - \exp\left(-\frac{\pi(\Delta E)^2}{2|M_{\text{in}} - M_{\text{out}}|v}\right)$$



SM, JPSJ 64(1995) 3207, 65(1996) 2734.
H. De Raedt et al, PRB56 (1997) 2734

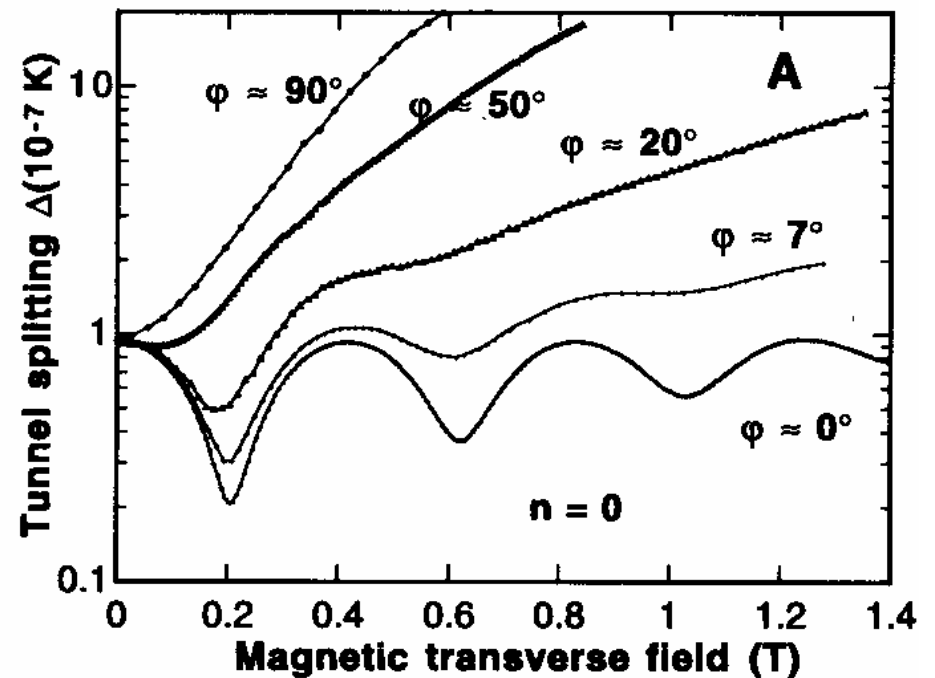
Sweeping velocity dependence



W. Wernsdorfer et al. EPL 50 (2000) 552
JPSJ 69 Suppl. 375.

Quantum interference
Berry phase

W. Wernsdorfer & R. Sessoli:
Science 284 (1999) 133



NMR (H) measurement on Fe8 Effect on the resonant tunneling

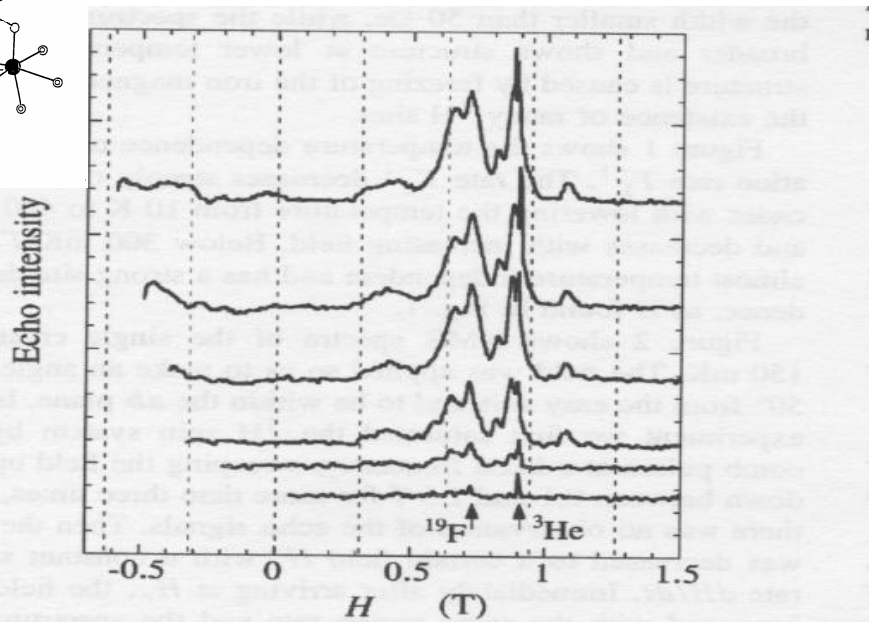
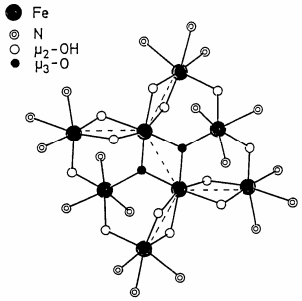
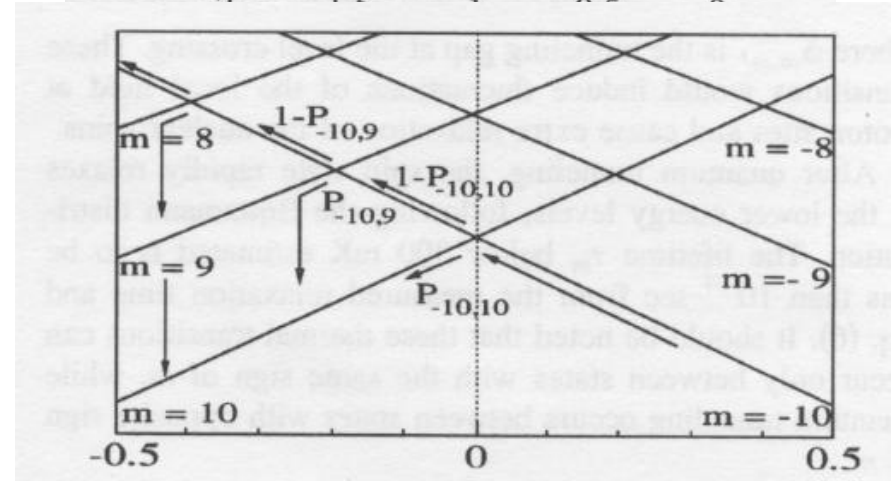
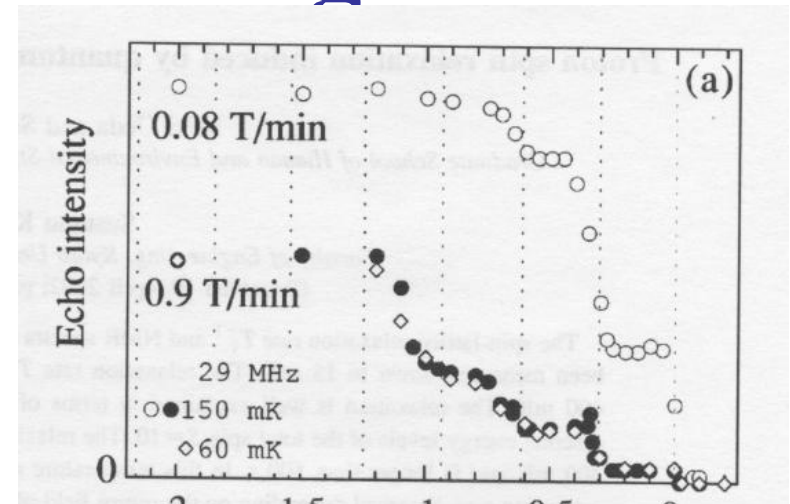
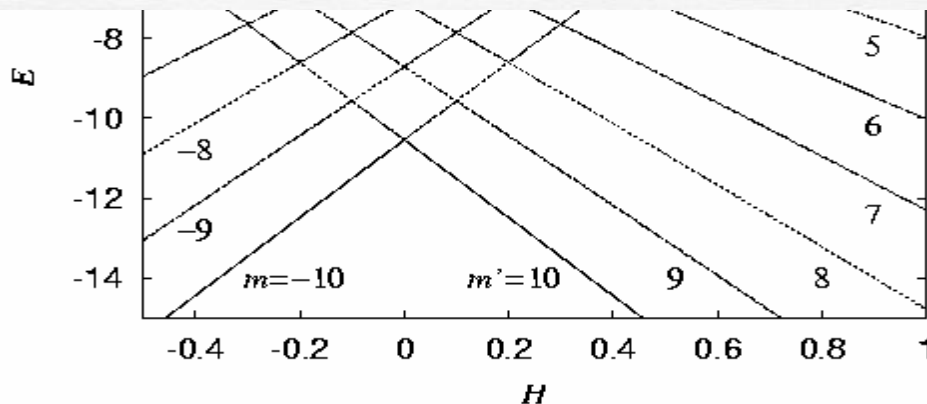


FIG. 2. NMR spectra of single crystal Fe8, which were taken with increasing the field from H_r after the saturation. $T=150$ mK, $f=29$ MHz, $dH/dt=0.9$ T/min, and $\theta=50^\circ$. Broken lines show the level crossing fields.



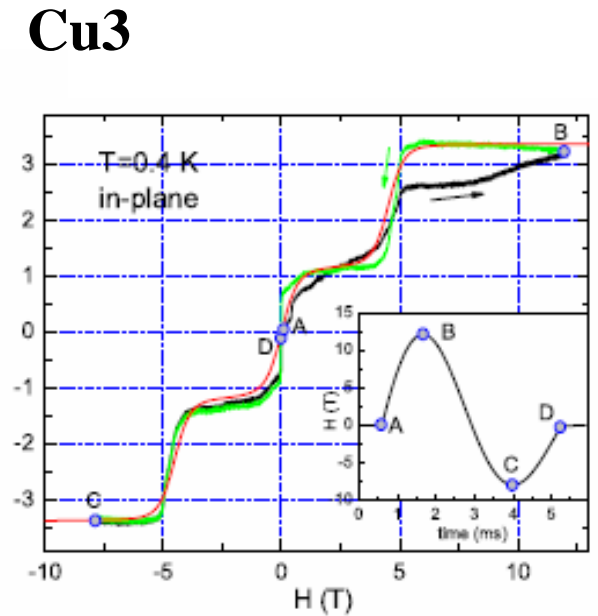
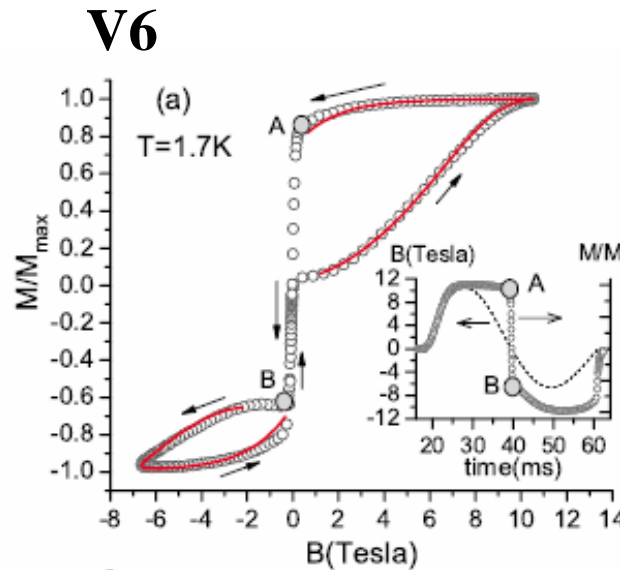
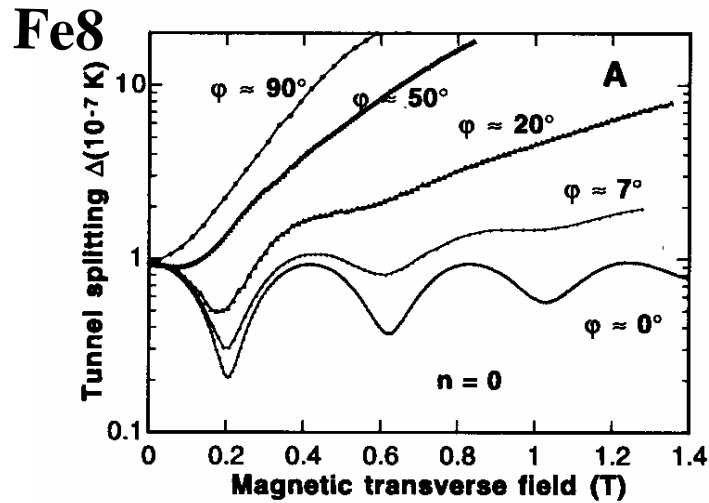
$$p = 1 - \exp \left(- \frac{\pi (\Delta E)^2}{2 |M_{in} - M_{out}| v} \right)$$

$$\Delta_{-10,10} = 3.52 \times 10^{-7} \text{ K}$$

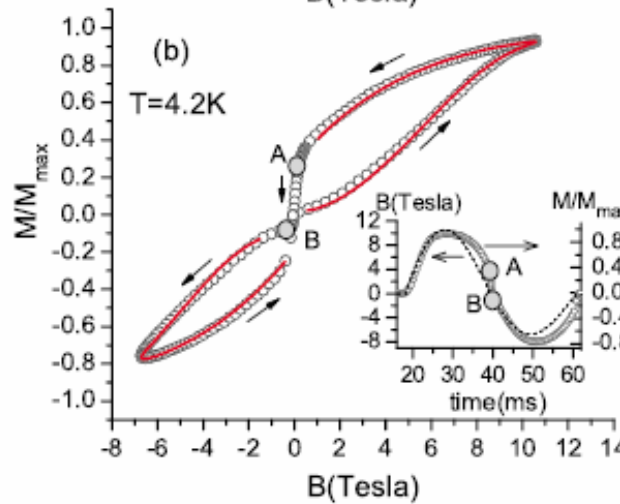
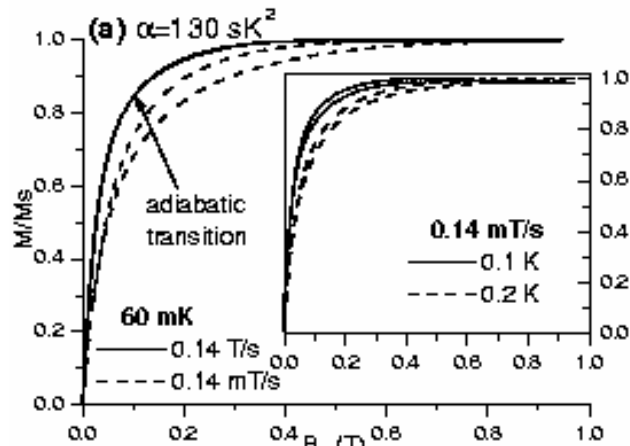
$$\Delta_{-10,9} = 9.66 \times 10^{-7} \text{ K}$$

M. Ueda, S. Maegawa and S. Kitagawa:
Phys. Rev. B66 (2002) 073309

Landau-Zener transitions in magnetization process



V15 **W. Wernsdorfer & R. Sessoli:**
Science 284 (1999) 133



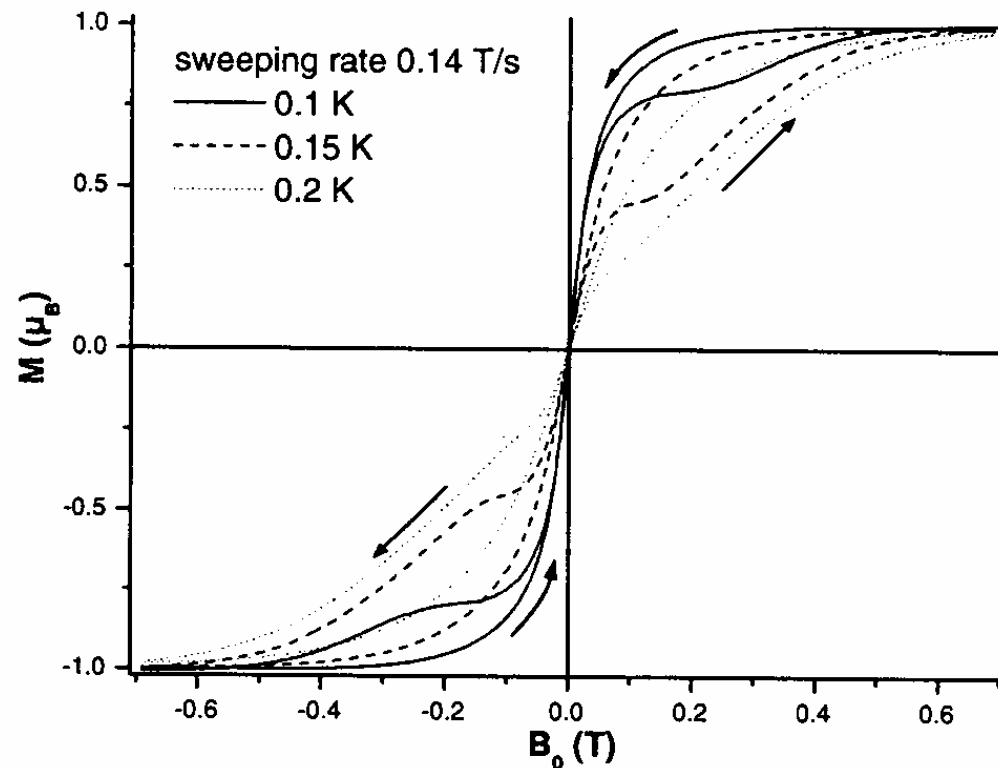
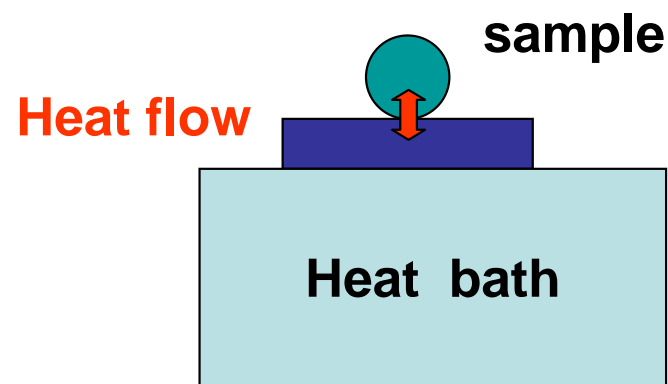
K.Y. Choi, et al.
PRL 96 (2006) 107202

I. Chiorescu, et al
Phys. Rev. Lett. 84 (2000) 3454.

I. Rousouchazakis, et al.
PRL 94 (2005) 147204

Phonon Bottleneck phenomena in V15

Plateau induced by thermal effect



Chiorescu, W. Wernsdorfer,
A. Mueller, H. Boegge, B. Barbara,
Phys. Rev. Lett. 84 (2000) 3454.

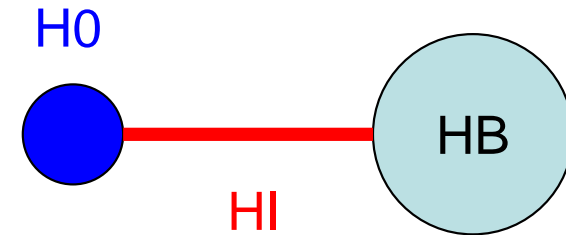
Quantum Master Equation

$$\frac{\partial}{\partial t} \rho = iL\rho = \frac{1}{i\eta} [H_0 + H_I + H_B, \rho]$$

$$H = H_0 + H_I + H_B,$$

$$H_I = \sum_k \lambda_k (b_k^+ + b_k) X,$$

$$H_B = \sum_k \omega_k b_k^+ b_k$$



Reduction of environment

$$\sigma = p\rho = \rho_{\text{eq}} \text{Tr}_B \rho, \quad \rho_{\text{eq}} = e^{-\beta H_0} / \text{Tr}_B e^{-\beta H_0}$$

$$\frac{\partial}{\partial t} \sigma = ipL\sigma + piL \int_0^t e^{(t-s)(1-p)iL} (1-p)iLp\rho(t)ds + piLe^{(t-s)(1-p)iL} (1-p)\rho(0)$$

e.g. Photon dissipation and pumping :

(SM., H. Ezaki, and E. Hanamura PRA 57 (1998) 2046)

$$\frac{\partial \sigma}{\partial t} = \frac{1}{i\eta} [H_0, \sigma] - \kappa (b^+ b \sigma - 2b \sigma b^+ + \sigma b^+ b)$$

Lindblad form \rightarrow Stochastic Schrodinger Equation (antibunching, squeezing photo emission)

General formulation

$$\frac{d\rho}{dt} = \frac{1}{i\eta} [H, \rho] - \frac{\lambda^2}{\eta^2} \int_0^t ds \int_{-\infty}^{\infty} d\omega e^{i\omega t} \Phi(\omega) \left\{ XX(-s)\rho(t) - e^{\beta\eta\omega} X\rho(t)X(-s) + e^{\beta\eta\omega} \rho(t)X(-s)X - X(-s)\rho(t)X \right\}$$

time correlation function of the reservoir's operators $\Phi(t)$

$$\Phi(\omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t} \Phi(t) = \eta\gamma(\omega)^2 \frac{D(\omega) - D(-\omega)}{e^{\beta\eta\omega} - 1}$$

$I(\omega) = \gamma(\omega)^2 D(\omega) = I_0 \omega^\alpha \quad \omega > 0$: the spectral density

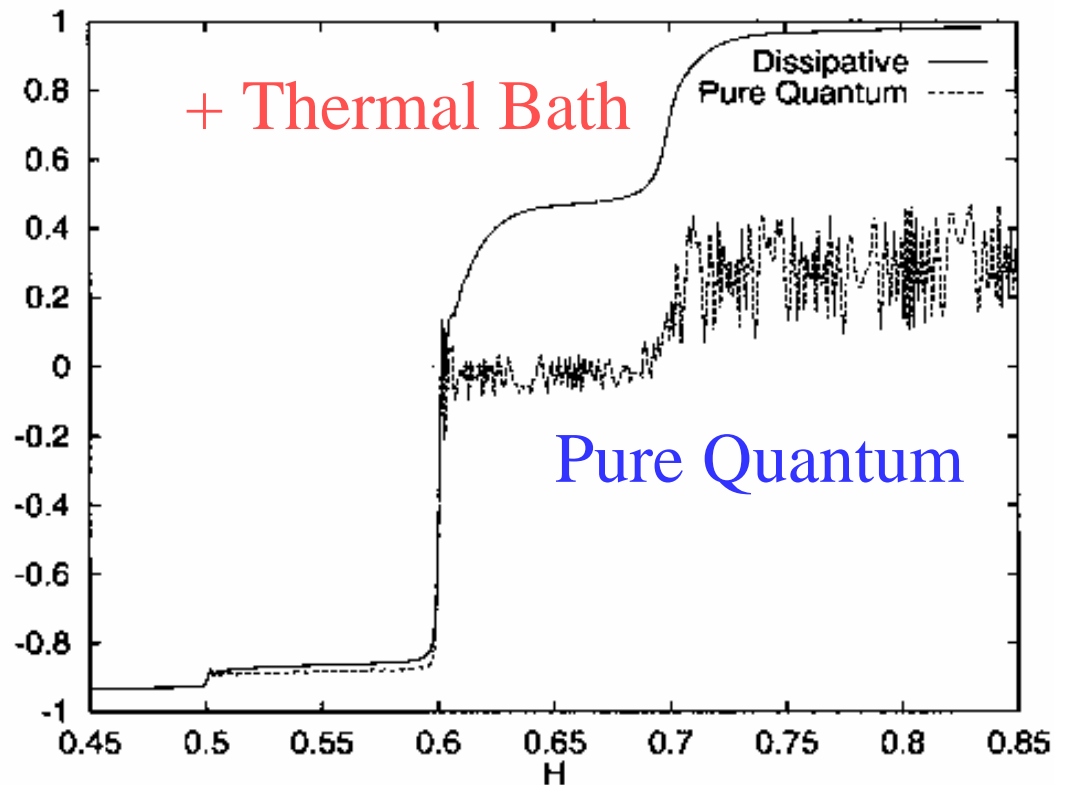
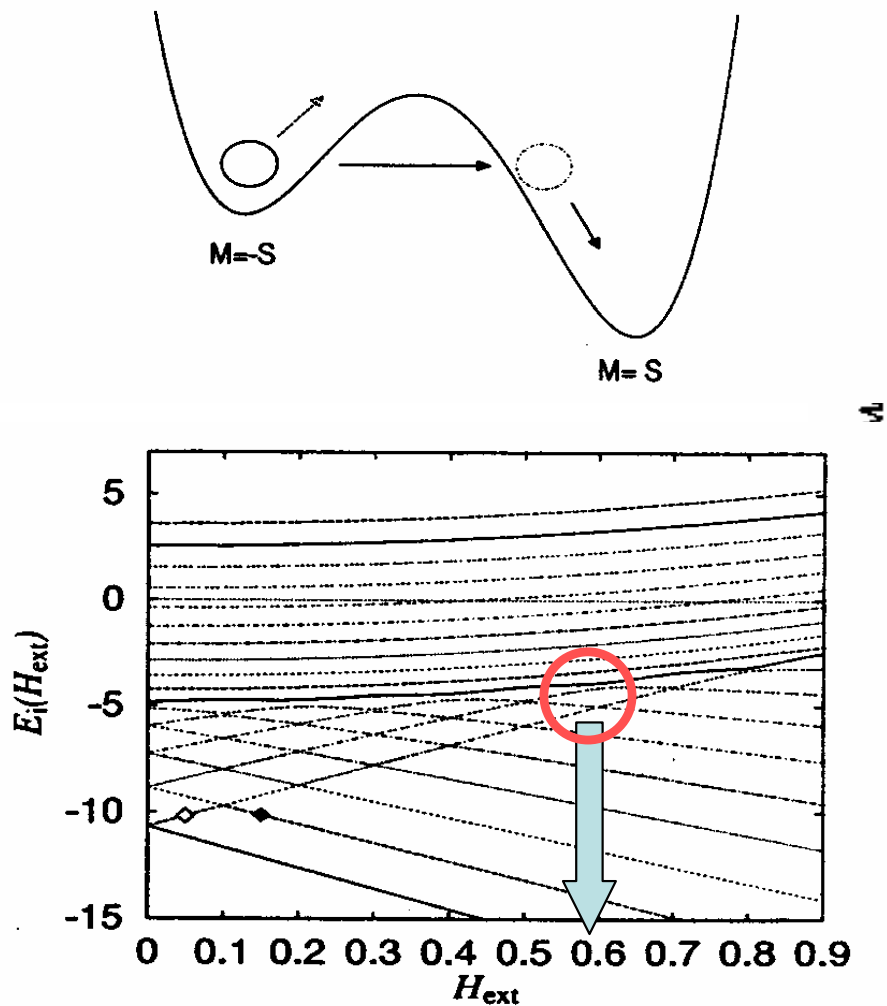
$$\langle k|R|m \rangle = \zeta \left(\frac{E_k - E_m}{\eta} \right) n_\beta(E_k - E_m) \langle k|X|m \rangle,$$

$$\zeta(\omega) = I(\omega) - I(-\omega)$$

$$\boxed{\frac{d\rho}{dt} = -i[H, \rho] - \lambda \left([X, R\rho] + [X, R\rho]^\dagger \right)}$$

Adiabatic transition and Relaxation

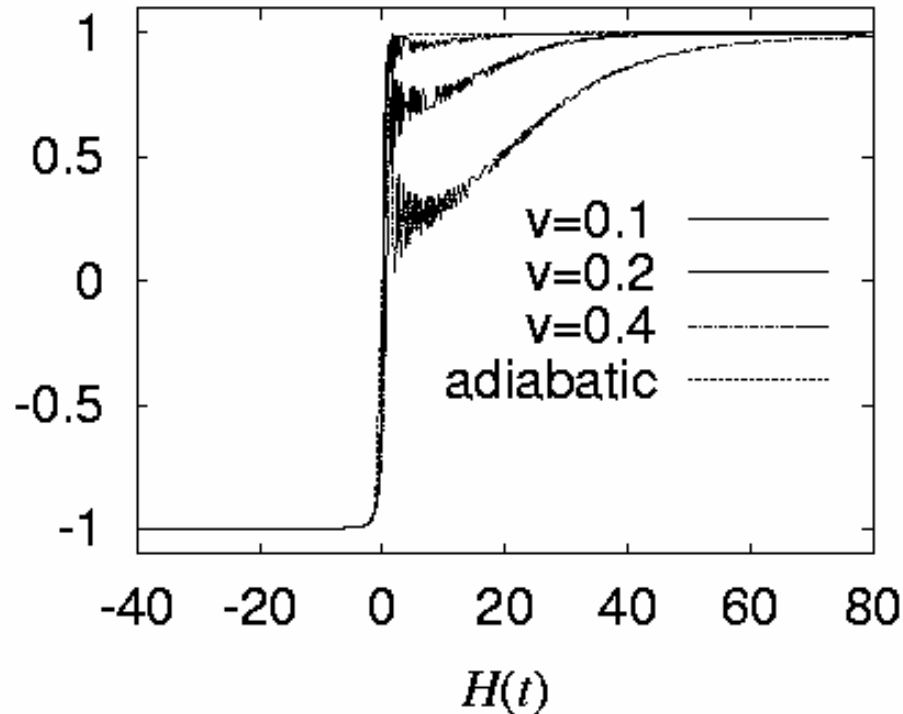
$$T \rightarrow 0$$



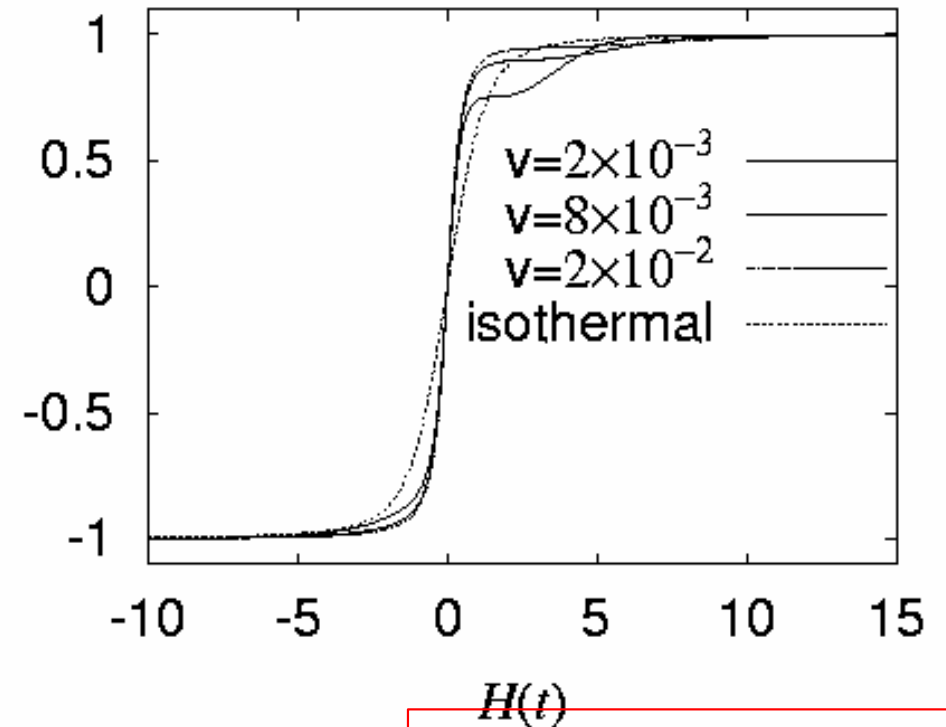
K. Saito, SM, H.de Raedt,
Phys. Rev. B60 (1999) 14553

Field sweeping with thermal bath

Fast sweeping

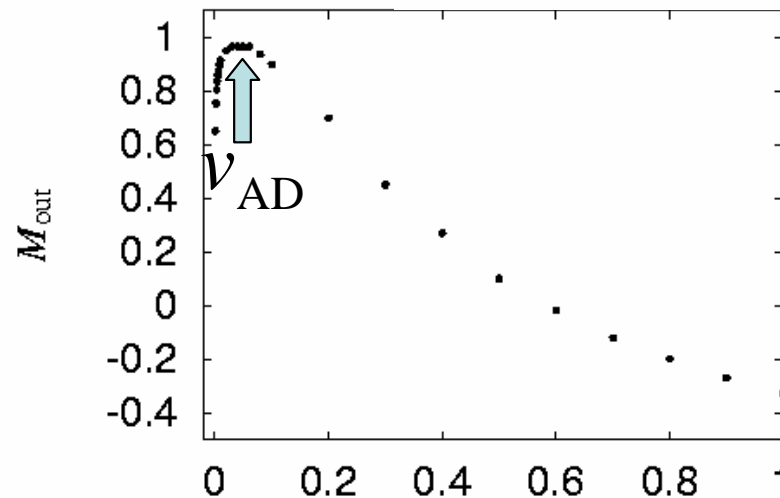


Slow sweeping



$$v_{AD} < v$$

LZS

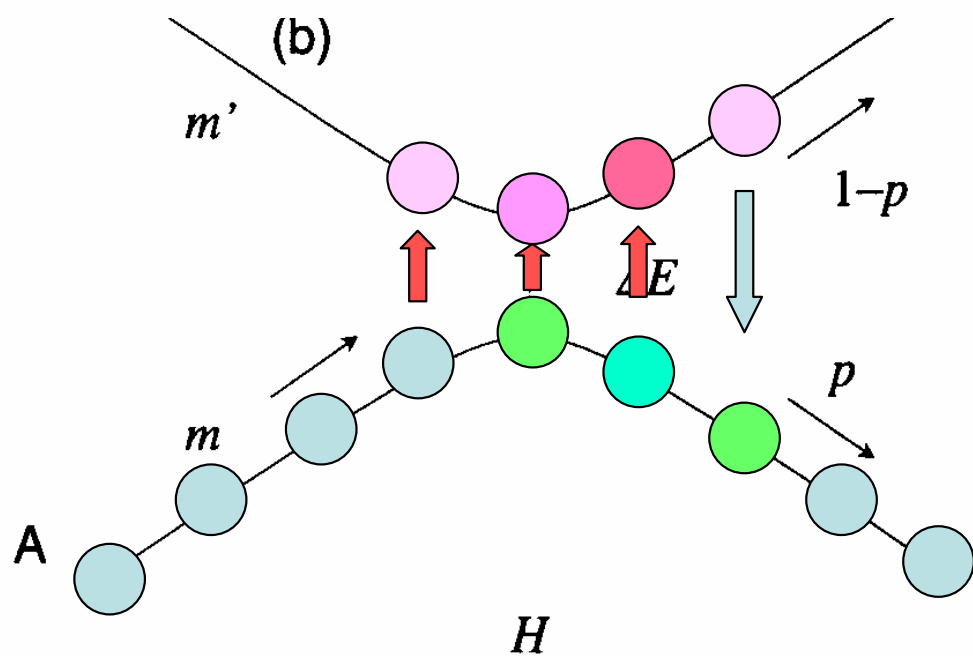
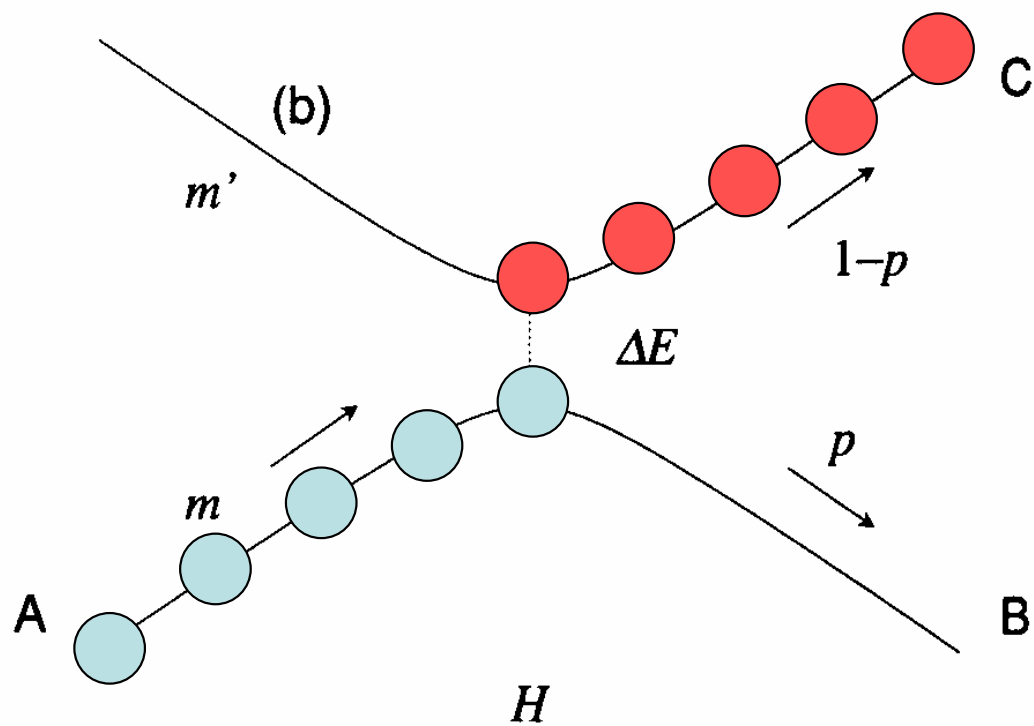


$$v_{TH} < v < v_{AD}$$

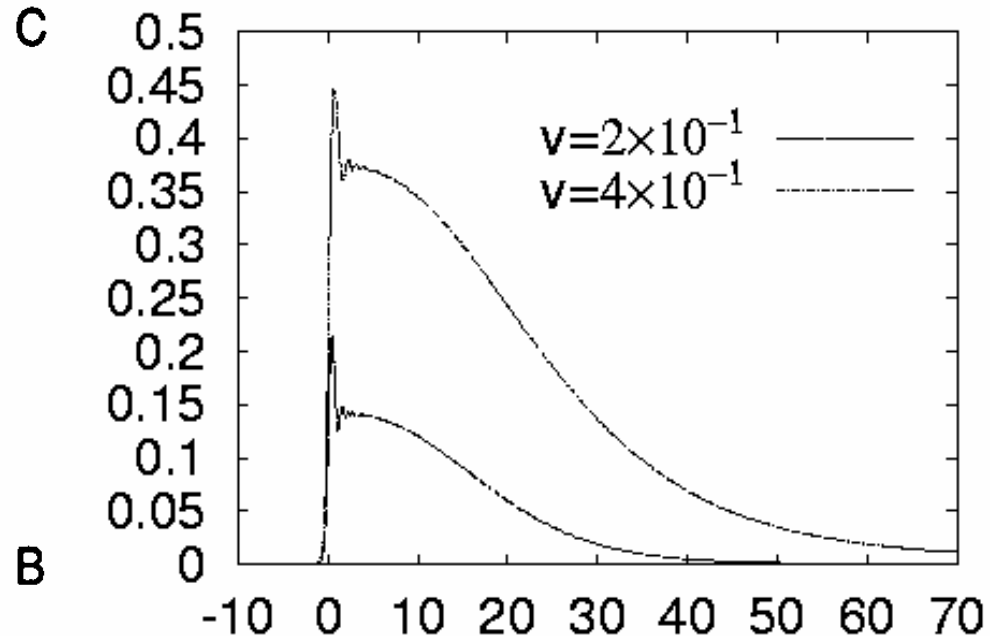
Magnetic
Foehn Effect

K. Saito & SM.
JPSJ (2001) 3385.

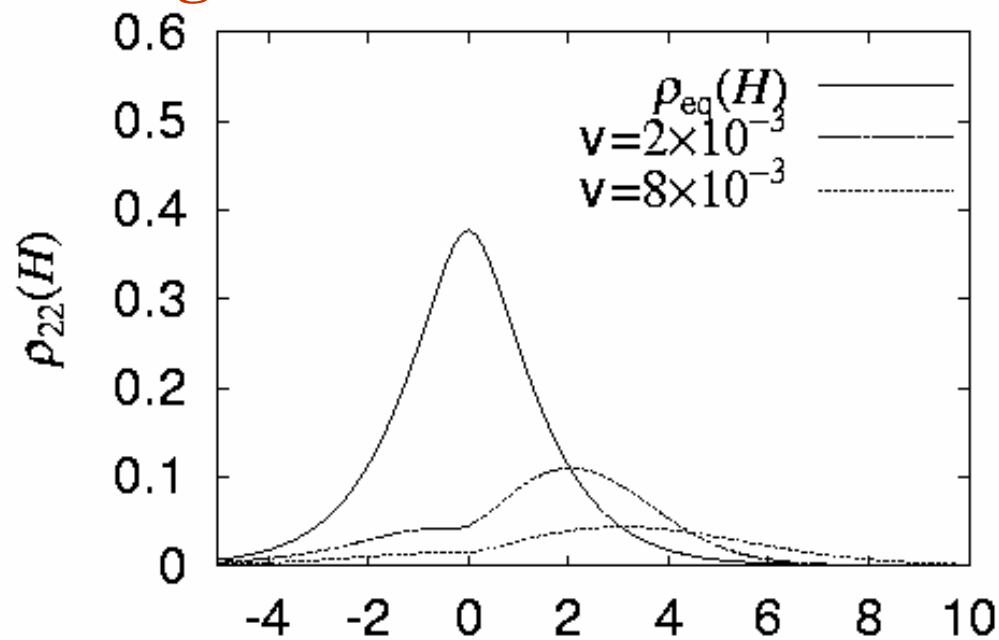
Nonadiabatic Tr. & Heat-inflow



LZ transition



Magnetic Foehn Effect

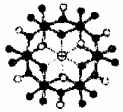


Fe2

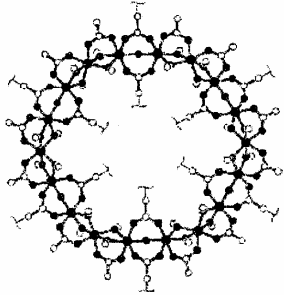
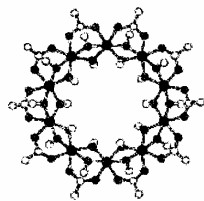
Y. Shapira, et al PRB59 (1999) 1046

Fe-rings

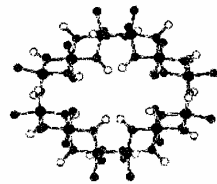
$n = 6$



$n = 10$



$n = 18$



$n = 12$

Y. Ajiro & Y. Inagaki

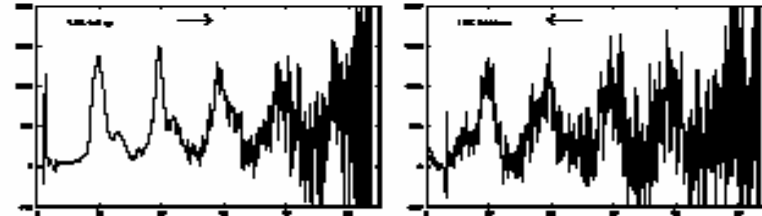
Y. Narumi & K. Kindo

H. Nakano & SM, JPSJ 70(2001) 2151

The second peak is the highest.

$T = 1.3 \text{ K}$

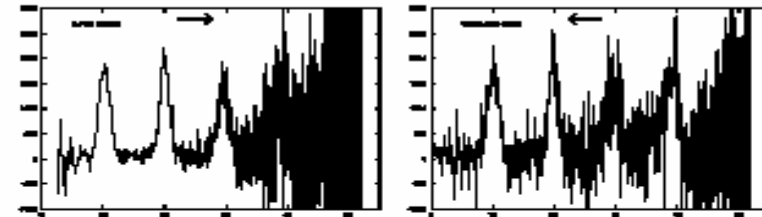
$$\frac{dM}{dH}$$



Satellite peaks appear.

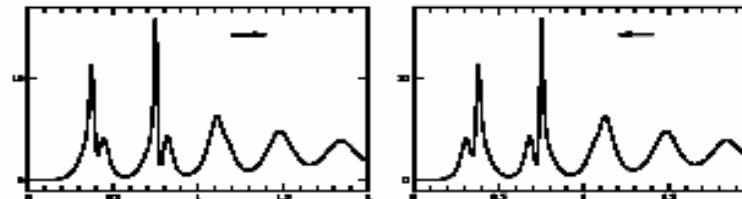
$T = 0.09 \text{ K}$

$$\frac{dM}{dH}$$

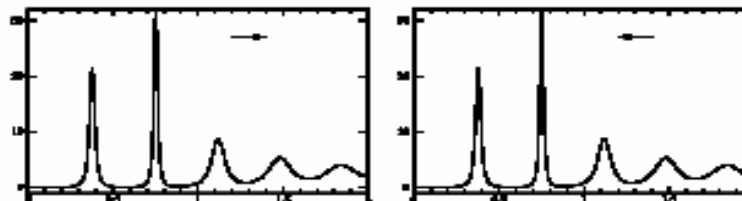


Satellite peaks disappear.

$T = 1.3 \text{ K}$

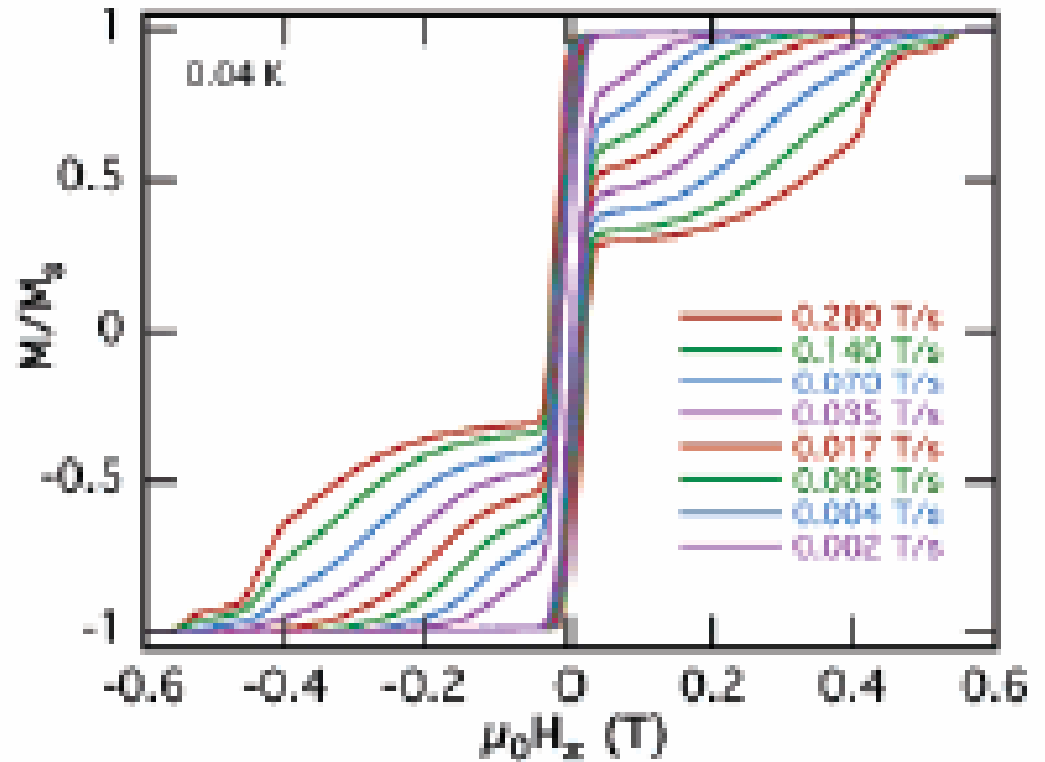
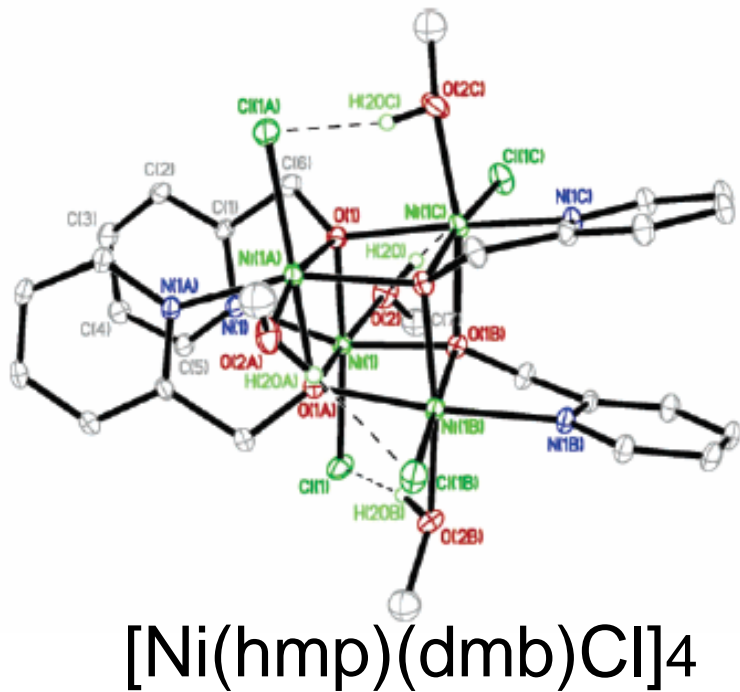


$T = 0.09 \text{ K}$



Fast Magnetization Tunneling in Tetranicke(II) SMM

En-Che Yang, et al: *Inorg. Chem.* 45 (2006) 529



$V=0.002, \dots, 0.28\text{T/s}$

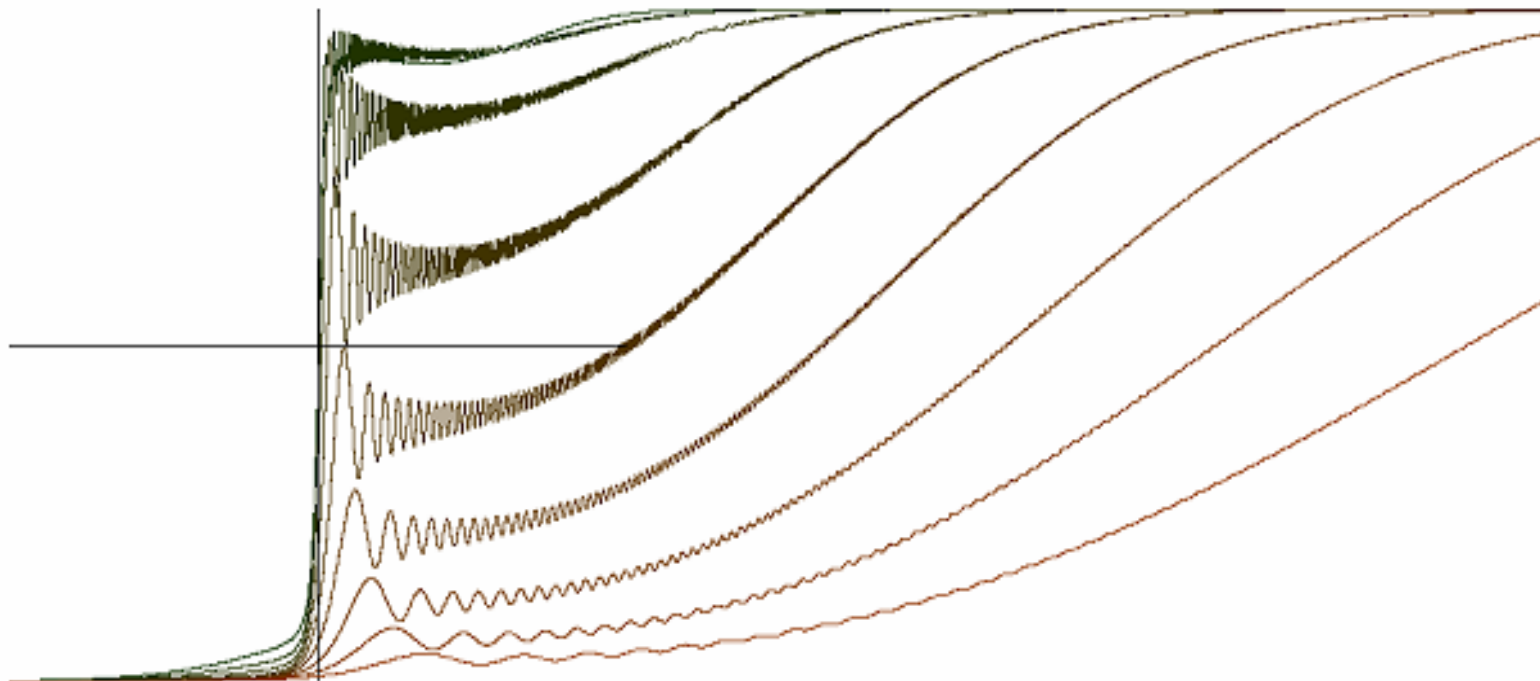
LZ transition + Thermal relaxation + MFE

$$\frac{d\rho}{dt} = -i[H, \rho] - z \left([X, R\rho] + [X, R\rho]^+ \right)$$

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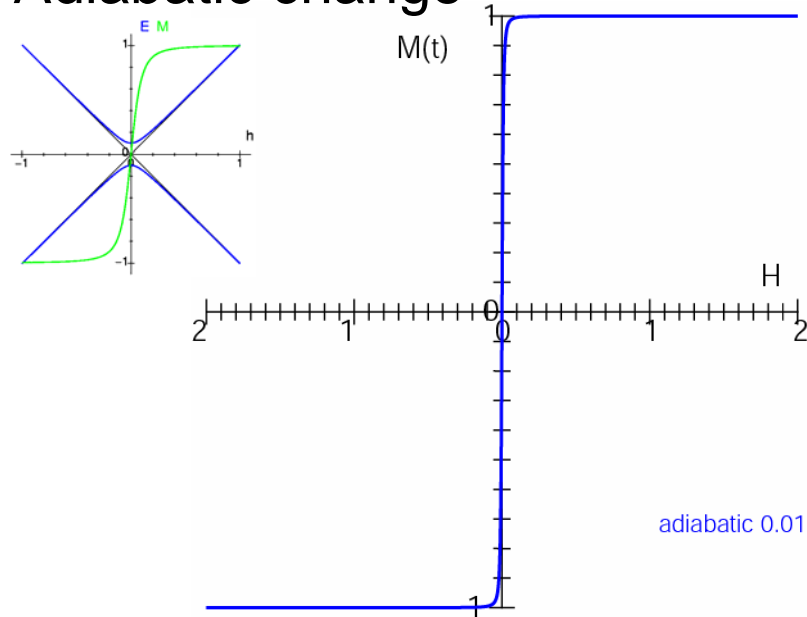
g,vh,zi0,temp= 0.02200 0.00020 0.00010 0.30000
g,vh,zi0,temp= 0.02200 0.00040 0.00010 0.30000
g,vh,zi0,temp= 0.02200 0.00080 0.00010 0.30000
g,vh,zi0,temp= 0.02200 0.00160 0.00010 0.30000
g,vh,zi0,temp= 0.02200 0.00320 0.00010 0.30000
g,vh,zi0,temp= 0.02200 0.00640 0.00010 0.30000
g,vh,zi0,temp= 0.02200 0.01280 0.00010 0.30000
g,vh,zi0,temp= 0.02200 0.02560 0.00010 0.30000
g,vh,zi0,temp= 0.02200 0.05120 0.00010 0.30000
    
```

v=0.0512, ..., 0.0002

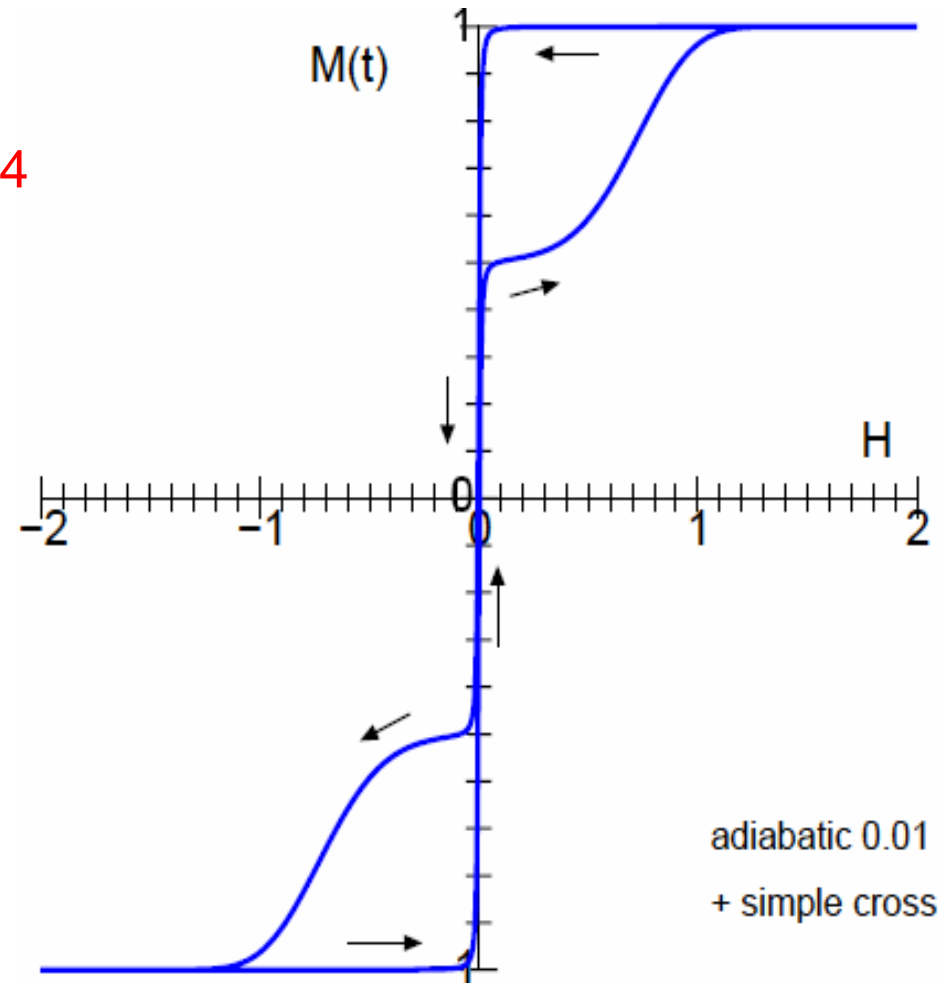


Two different types of sites ?

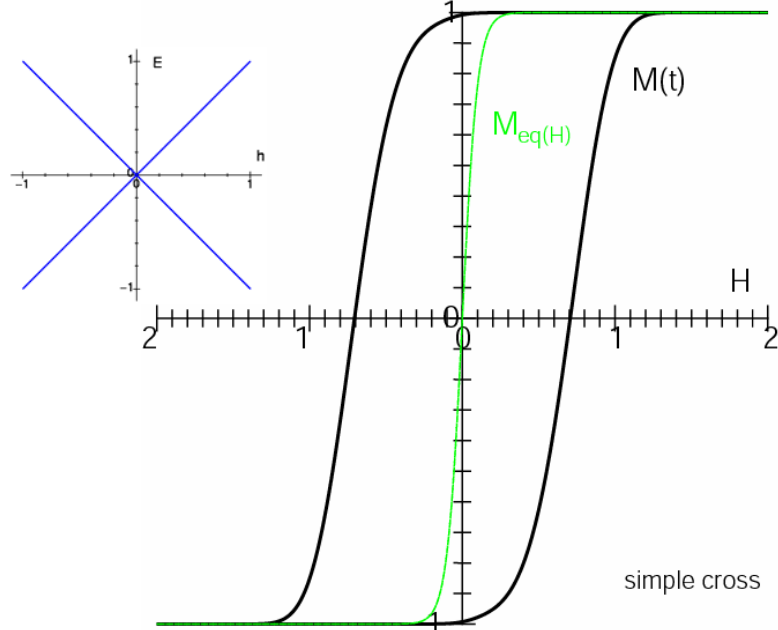
Adiabatic change



x 3/4

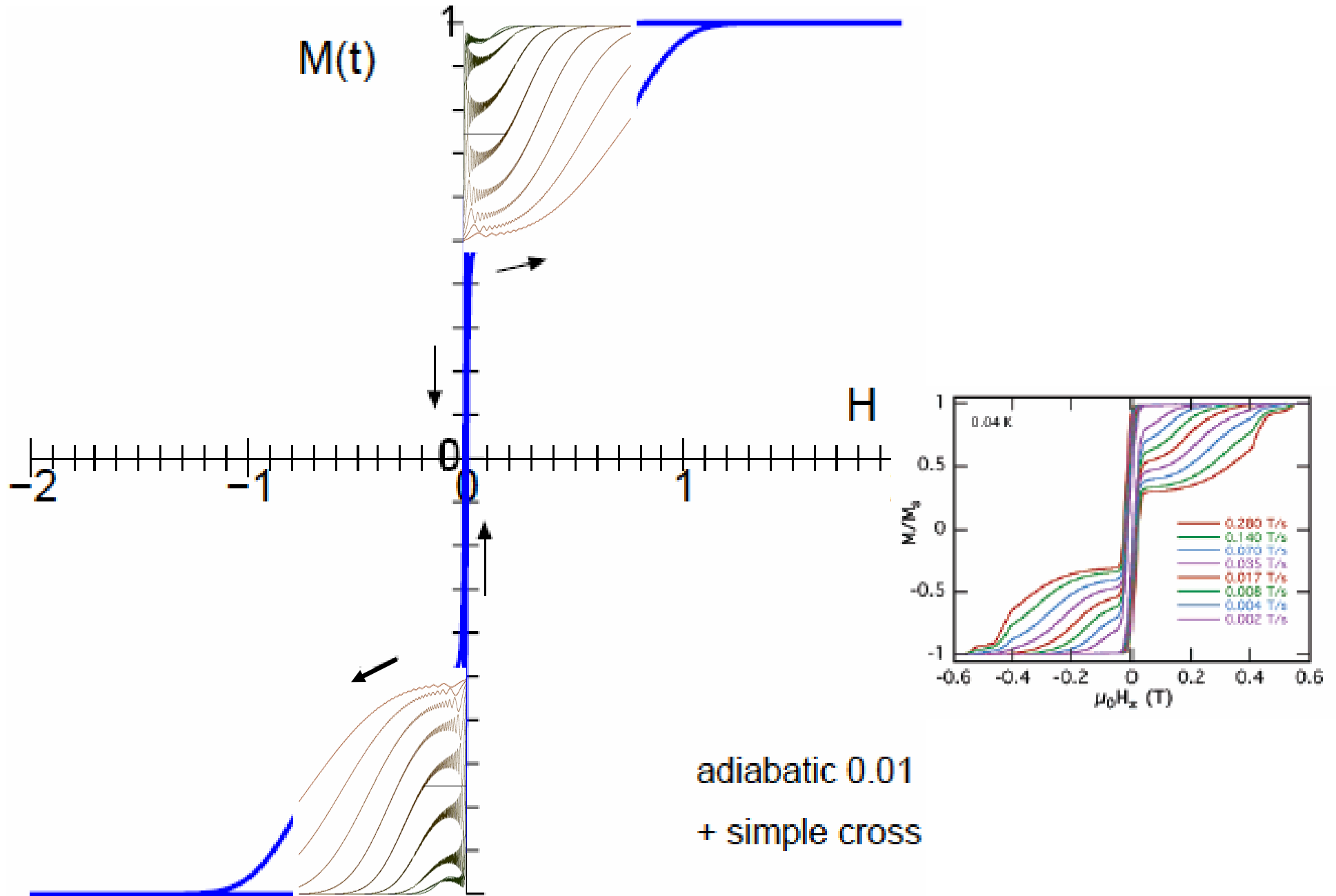


Thermal relaxation



x 1/4

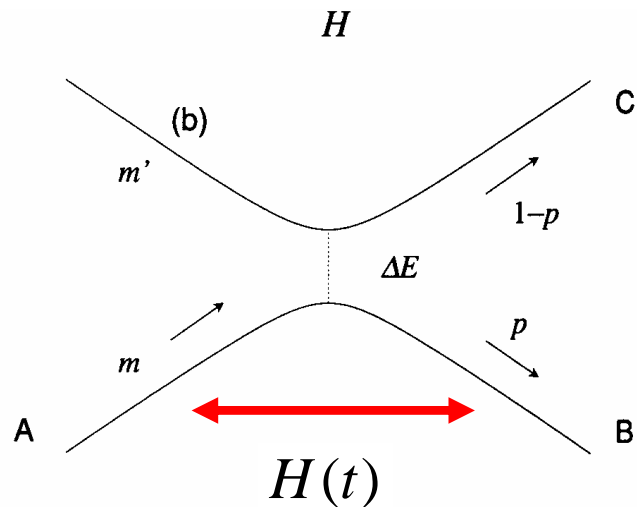
Possible magnetic process



Quantum dynamics under an AC field

Non-trivial Resonance

$$H(t) = -h_W \cos(\omega t) \sum_i S_i^z$$

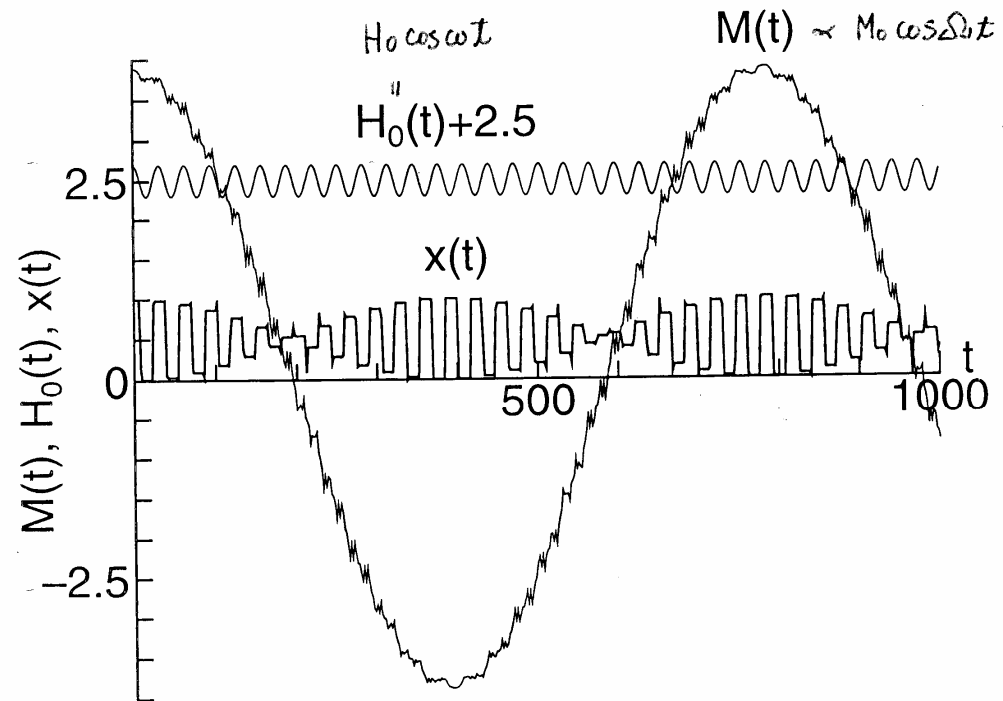


$$M(t) = M_0 \cos(\Omega t + \delta),$$

$$\Omega = \frac{\omega}{\pi} \sqrt{2p(1-\cos\alpha)}$$

$$p = 1 - \exp\left(-\frac{\pi(\Delta E)^2}{4c\Delta M}\right)$$

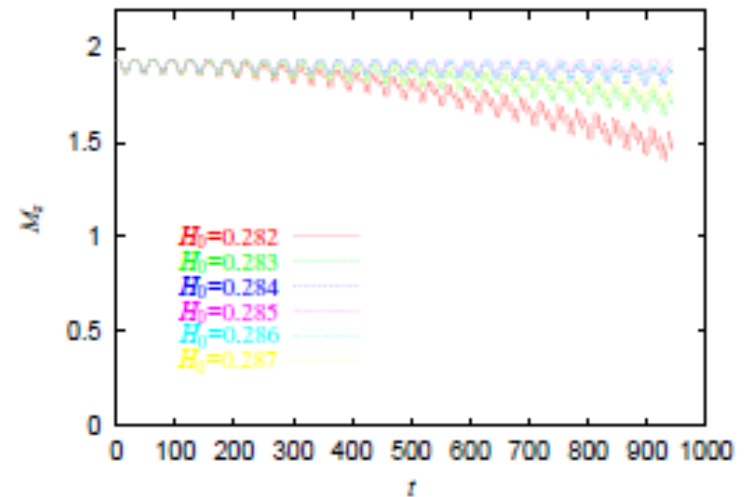
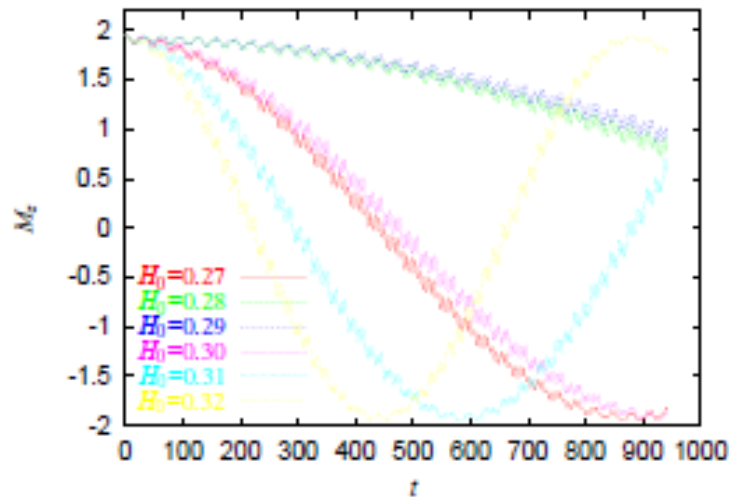
$$M(t) \quad \Gamma=0.5, \omega=0.2, H_0=0.2$$



$$T = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} = \begin{pmatrix} e^{i\theta} \sqrt{p} & e^{i\phi} \sqrt{1-p} \\ \sqrt{1-p} & -e^{i(-\theta+\phi)} \sqrt{p} \end{pmatrix}$$

SM, K. Saito, H. De Daedt,
Phys. Rev. Lett. 80 (1998) 1525.

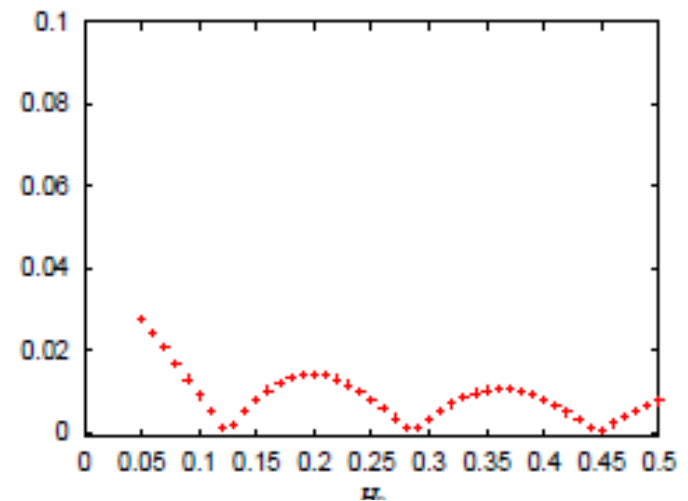
Amplitude dependence (Coherent destruction of tunneling)



Fitting function of magnetization

$$M_z = A \cos(\Omega t)$$

$$\Omega \Rightarrow \alpha$$



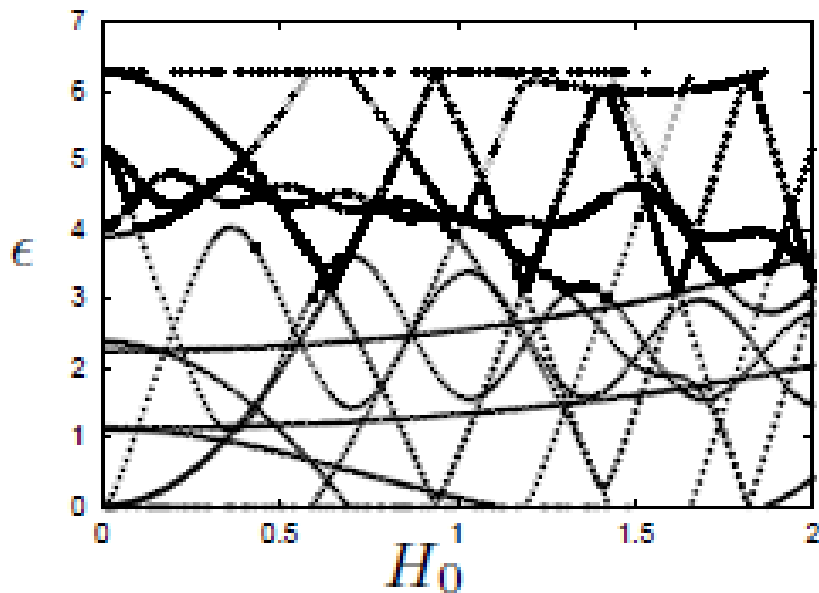
F. Grossman, et al. Phys. Rev. Lett. 67 (1991) 516.

Y. Kayamuma, PRB 47 (1993) 9940.

SM, K. Saito, H. De Daedt, Phys. Rev. Lett. 80 (1998) 1525.

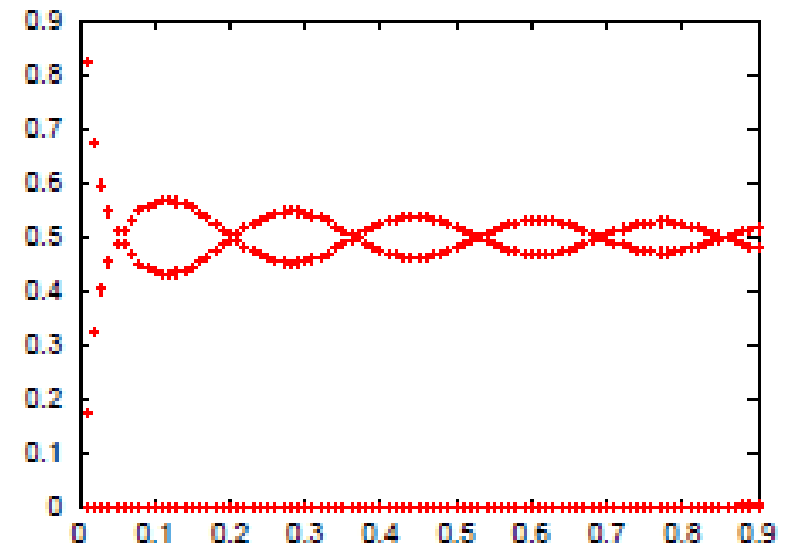
Eigenstates of Floquet operator and Hamiltonian

Floquet energy



Fractions

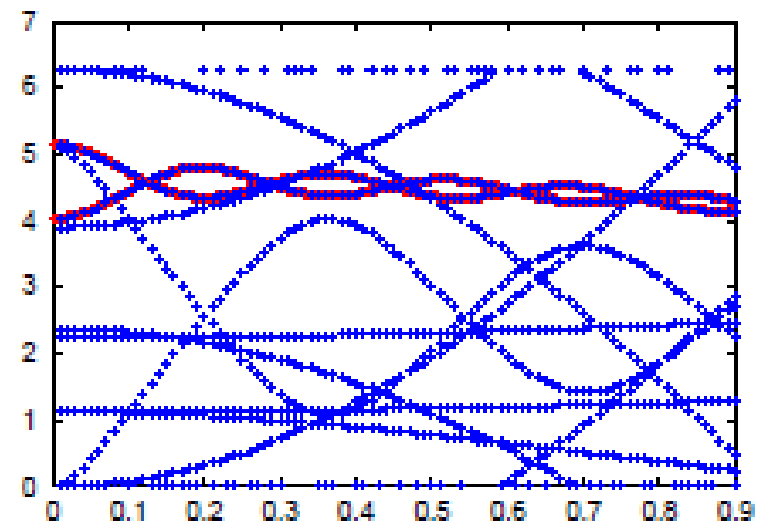
$$|\langle \phi_i | \Psi(T) \rangle|^2$$



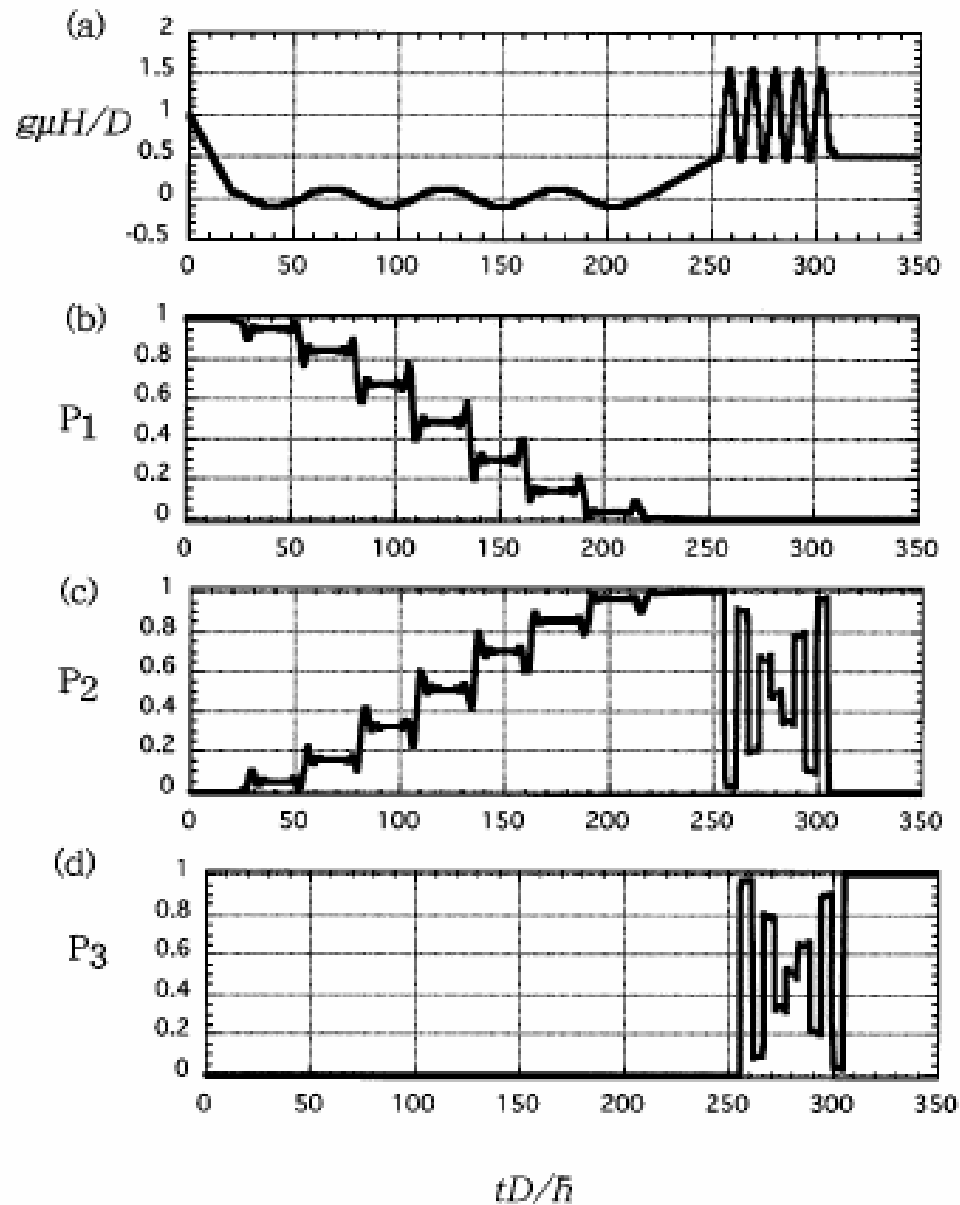
$$F = \text{T exp} \left(-i \int_0^{2\pi/\omega} H(s) ds \right)$$

$$F |\Psi_j(T)\rangle = e^{-i\varepsilon_j} |\Psi_j(T)\rangle$$

$$H |\phi_j\rangle = E_j |\phi_j\rangle$$

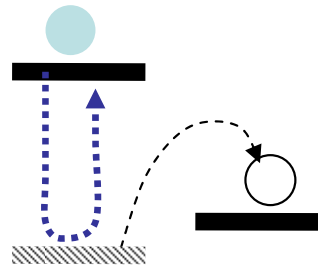


Switching by AC field



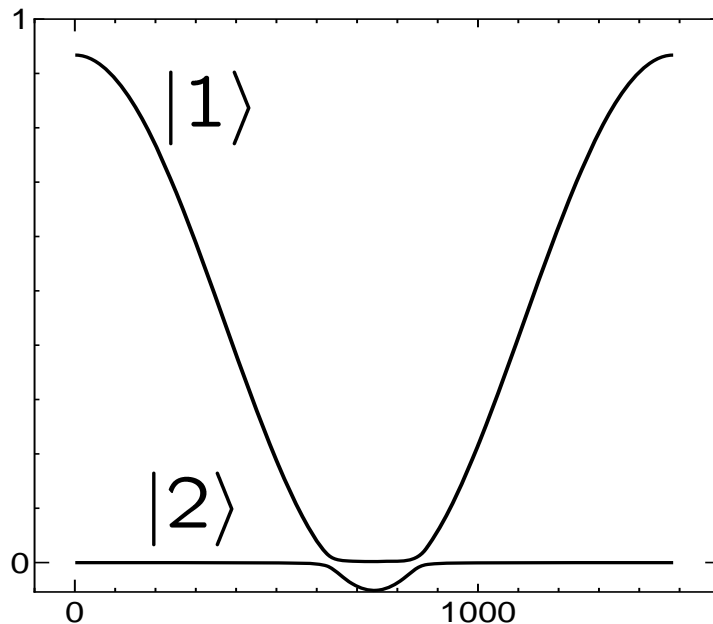
**Y. Teranishi and H. Nakamura:
PRL81(1998) 2032**

With appropriate oscillation,
We may change the state by
a single operation.

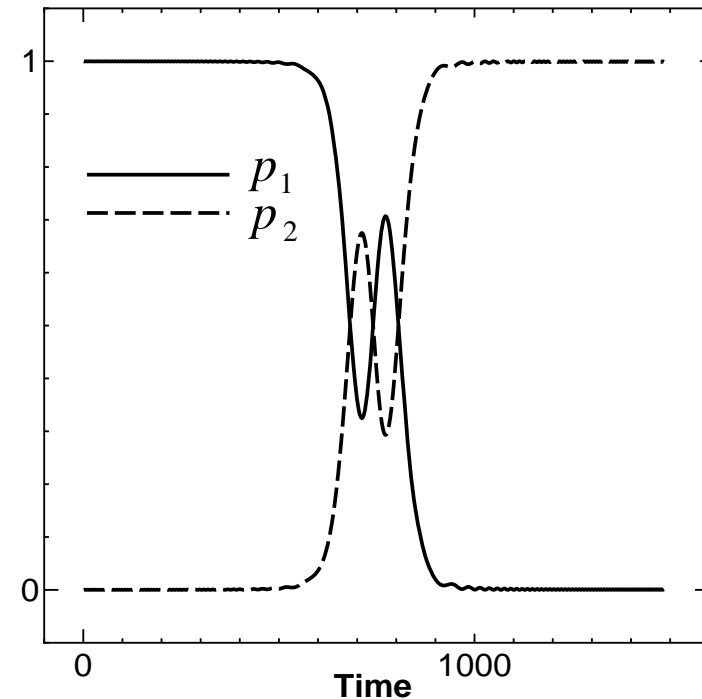


K.Saito and Y. Kayanuma
PRB 70 201304(R) (2004)

Energy level as a function of time



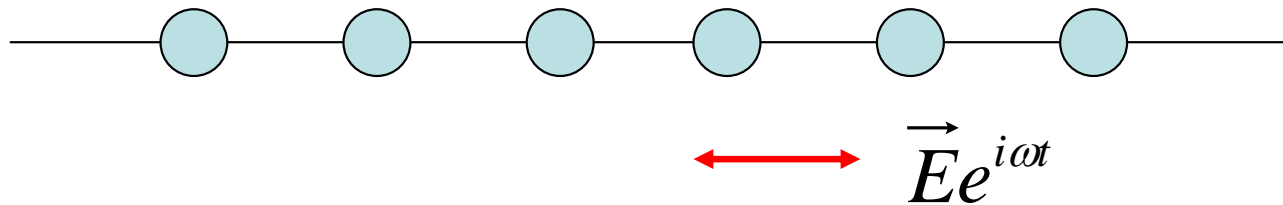
Probability of the Level k P_k



AC field trap by Coherent Destruction of Tunneling (CDT)

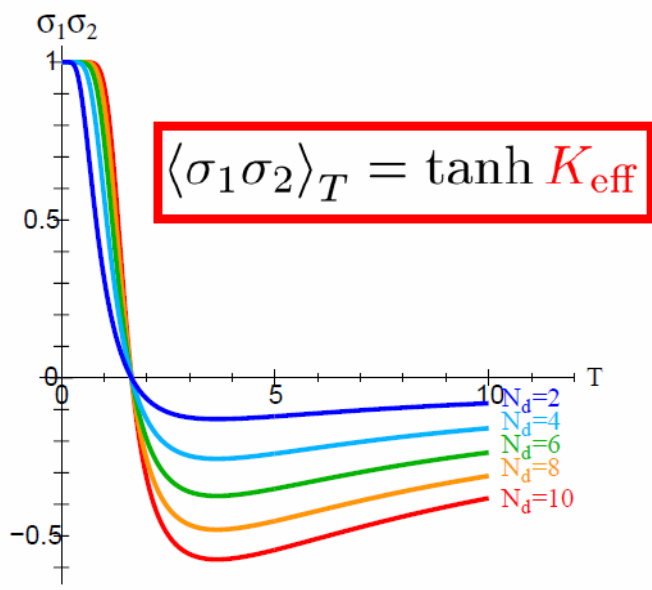
$$H = -t \sum_{ij} c_i^+ c_j - E e^{i\omega t} \sum_i x_i n_i$$

E=0 diffusion
CDT localization

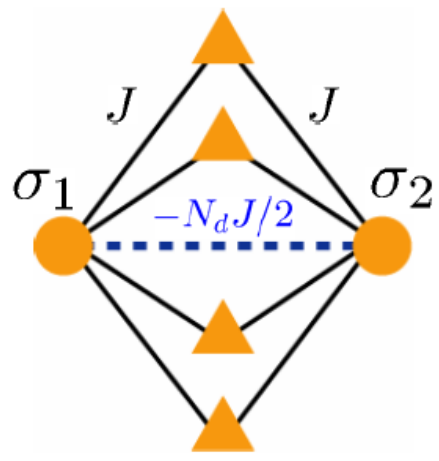


Y. Kayanuma and K. Saito: arXiv:0708.3570

Reentrant behavior in quantum fluctuation

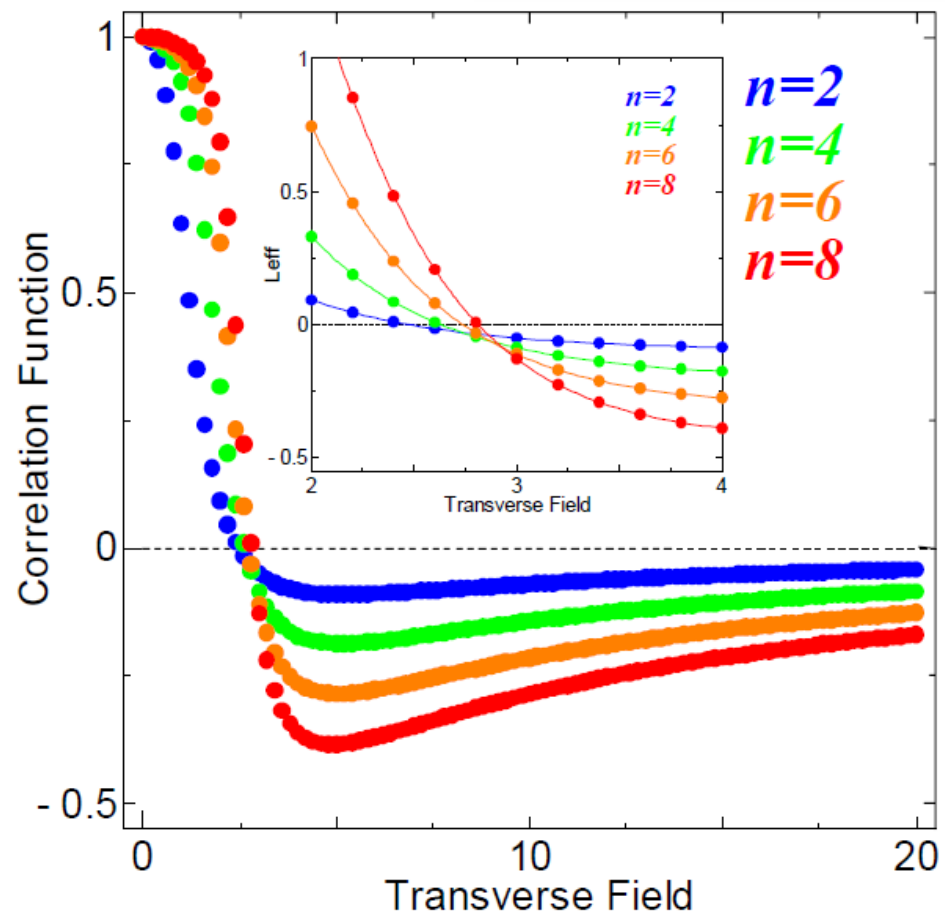


decorated bond system $\mathcal{H} = \mathcal{H}_{\text{classical}} - \Gamma \sum_i \sigma_i^x$



● system spin σ_1, σ_2
▲ decorated spin s_i

number of ▲ spins: N_d



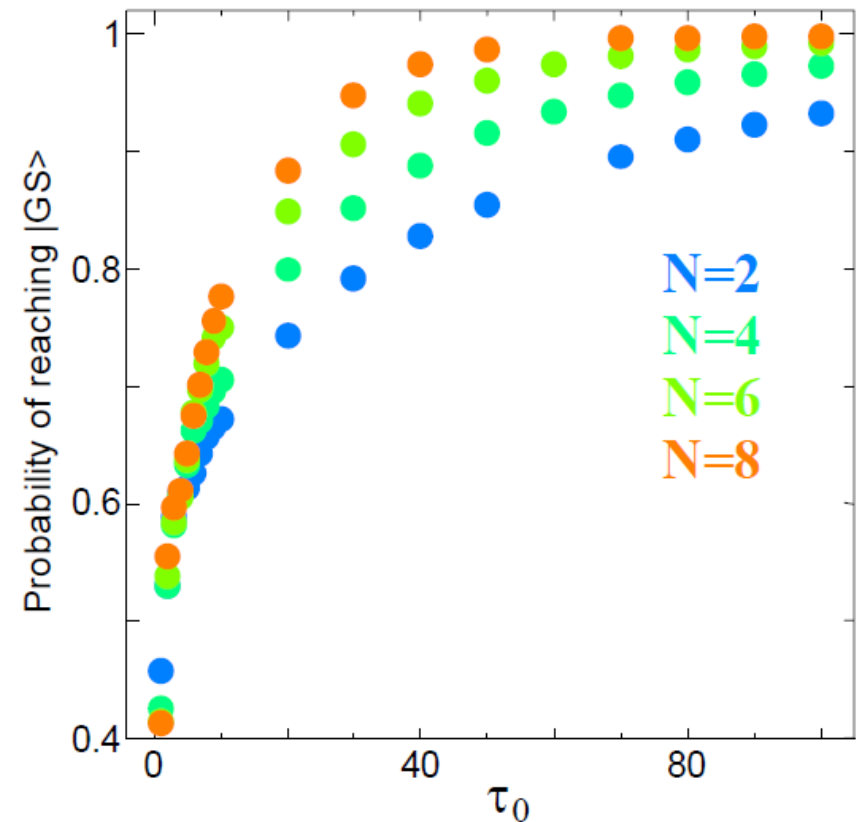
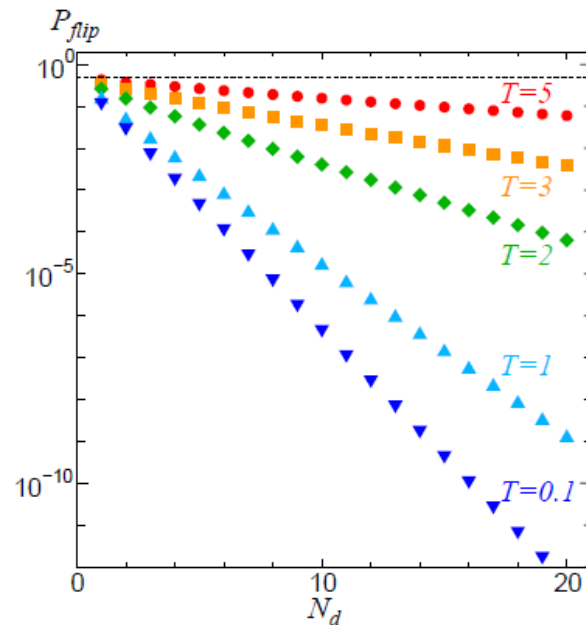
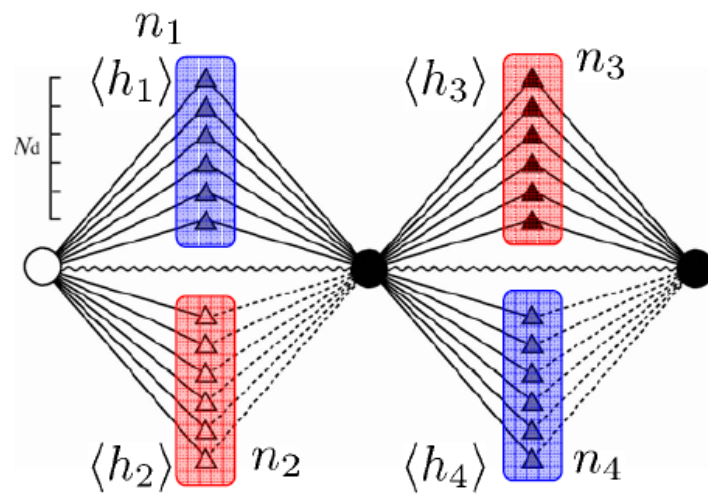
Ground state search: Quantum annealing

$$\mathcal{H}_0 = -J'\sigma_1^z\sigma_2^z - J\sigma_1 \sum_{i=1}^N S_i^z - J\sigma_2 \left(\sum_{i=1}^{N/2} S_i^z - \sum_{i=N/2+1}^N S_i^z \right)$$

$$\mathcal{H}_t(t) = -\Gamma(t) \left(\sigma_1^x + \sigma_2^x + \sum_{i=1}^N S_i^x \right)$$

$$\Gamma(t) = \Gamma_0 (1 - t/\tau_0)$$

$|GS\rangle$: ground state at $\Gamma(\tau_0) = 0$



Quantum dynamics in TI model

$$H = -J \sum_i \sigma_i^z \sigma_{i+1}^z - h(t) \sum_i \sigma_i^z - \Gamma(t) \sum_i \sigma_i^x$$

$$i\eta \frac{d\psi}{dt} = H(t)\psi$$

$$[\sigma_i^z, \sigma_i^x] \neq 0, \quad [M, H] \neq 0,$$

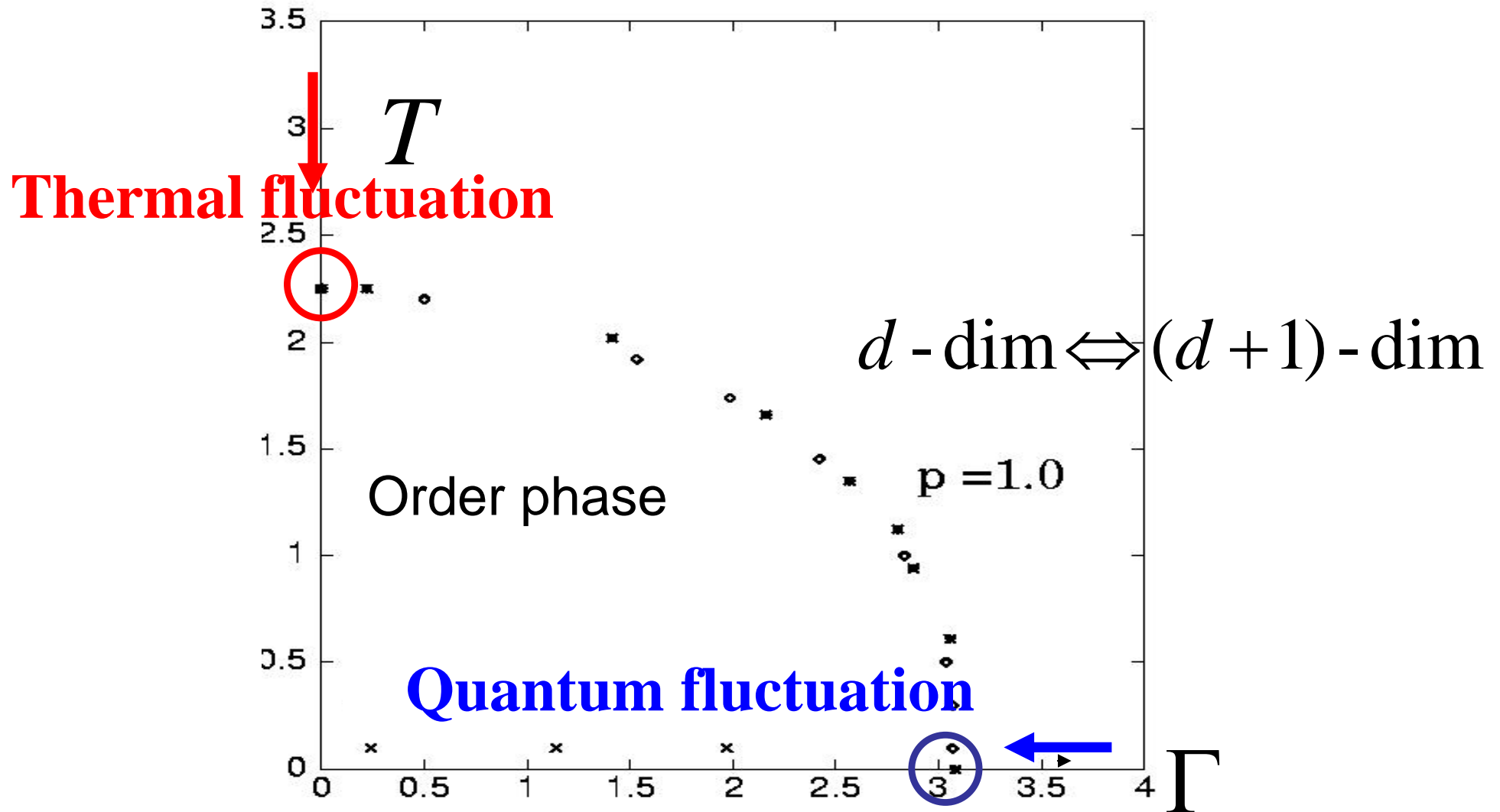
$$\Gamma = H_x$$

Quantum fluctuation

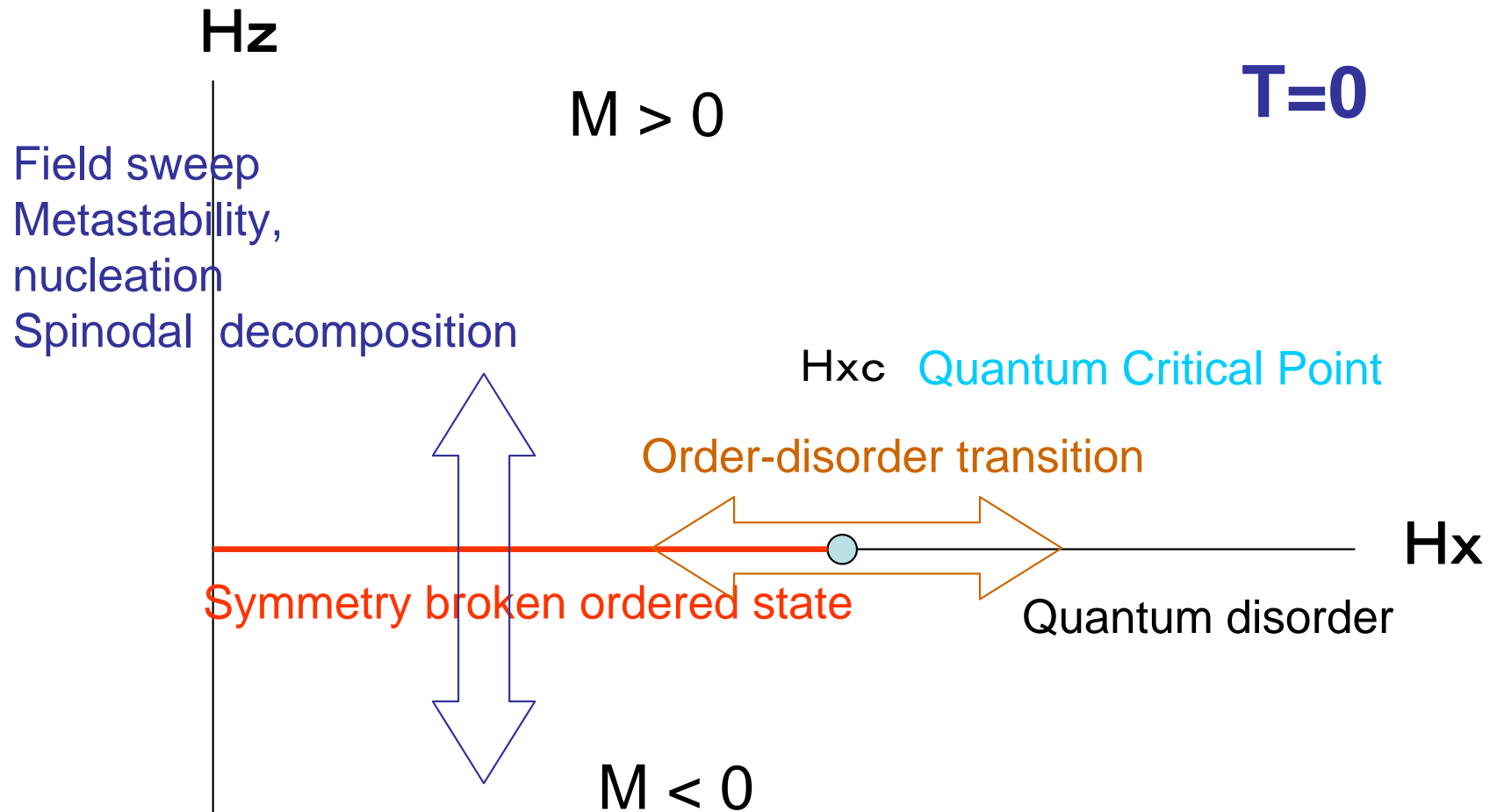
quantum tunneling: LZ in TI model
quantum nucleation
quantum spinodal decomposition?
collective motion?

Ground state Phase transition

$$H = -J \sum \sigma_i^z \sigma_j^z - \Gamma \sum \sigma_i^x$$



Phase diagram of Transverse Ising model



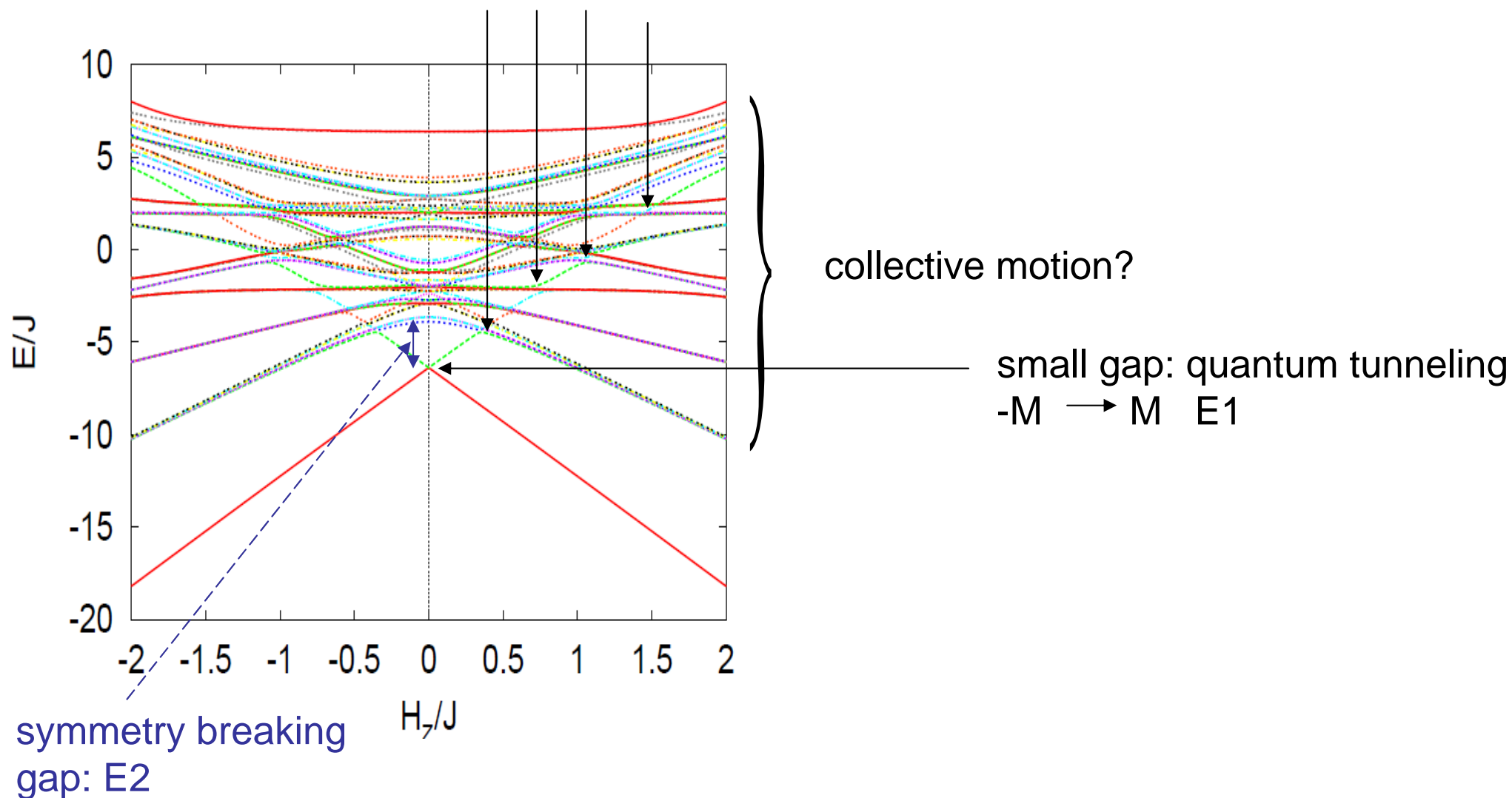
J. Dziarmaga: PRL 95 (2005) 245701

Remaining DWs after quench

$$n = \frac{1}{2\pi\sqrt{2J\tau/\eta}}$$

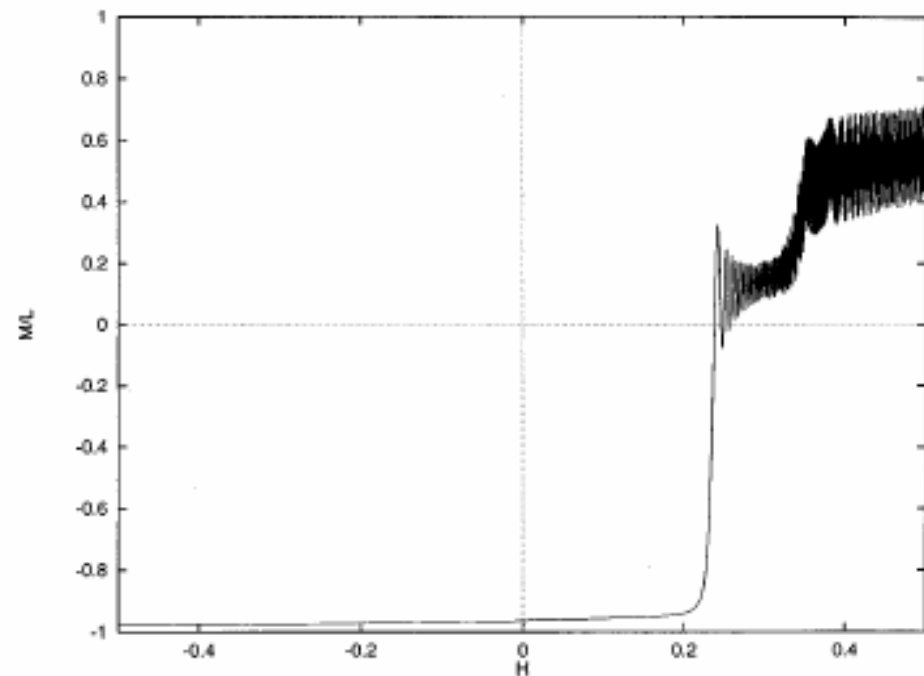
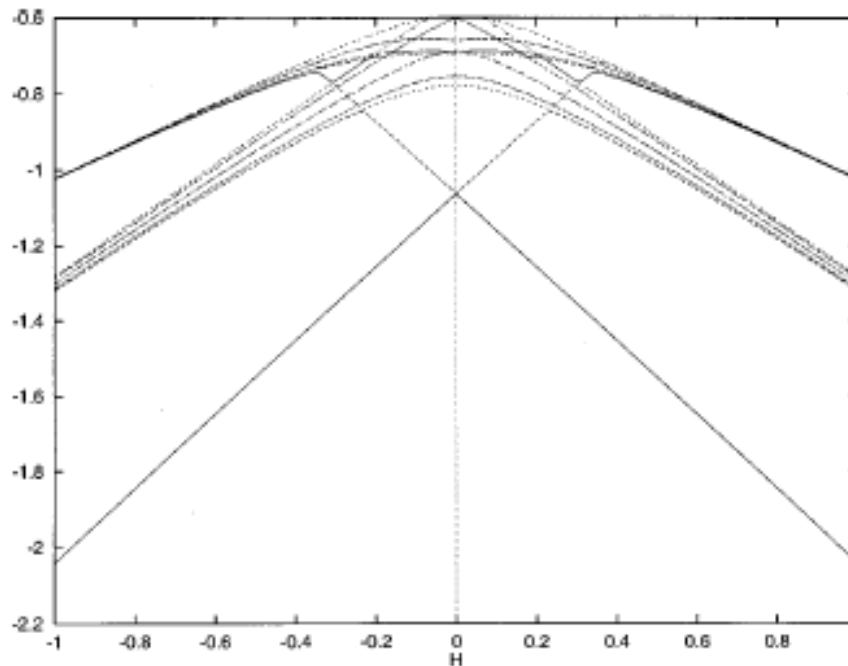
Critical phenomena in Energy spectrum $E(H)$

level crossings: nucleation



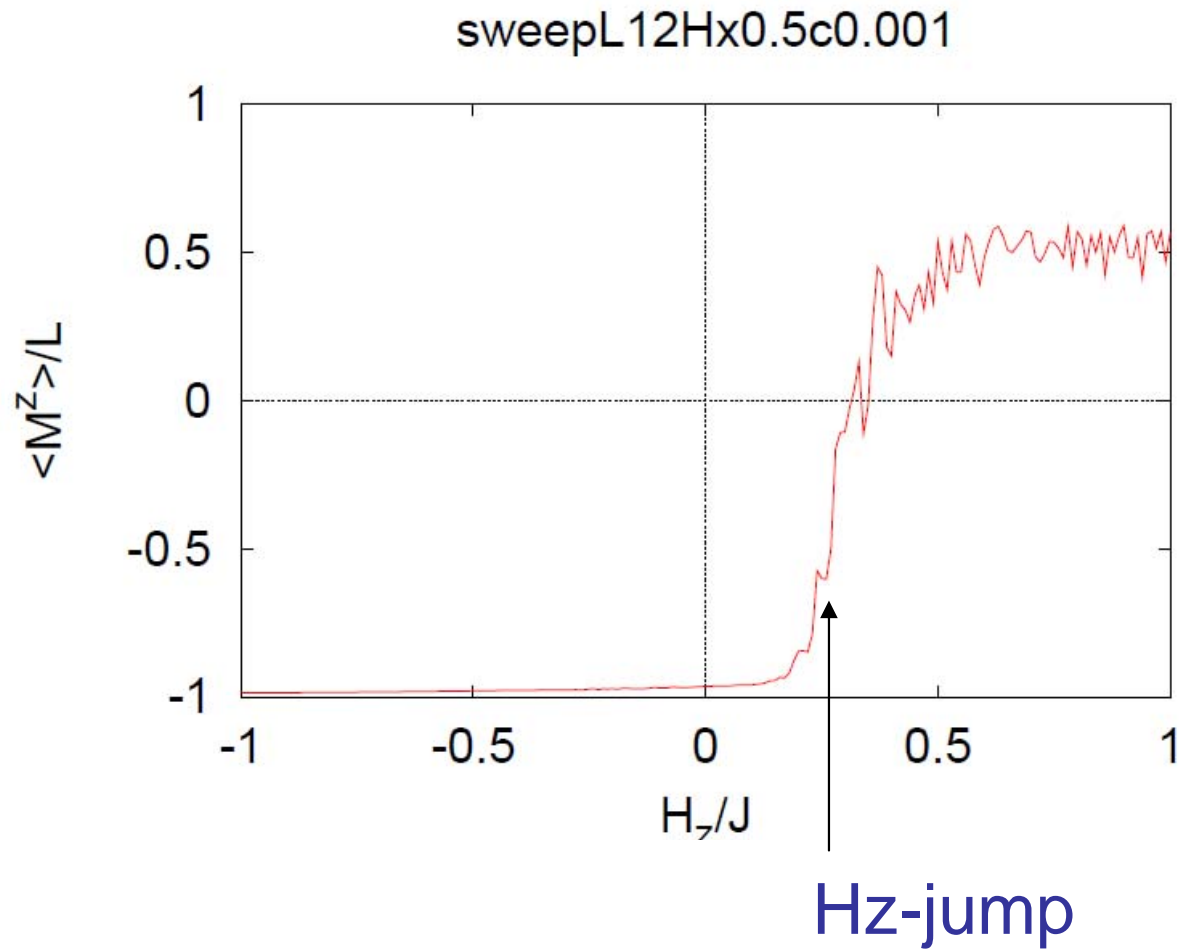
Landau-Zener-Stueckerberg scattering at each crossings

Non-adiabatic transition at the avoided level crossing points

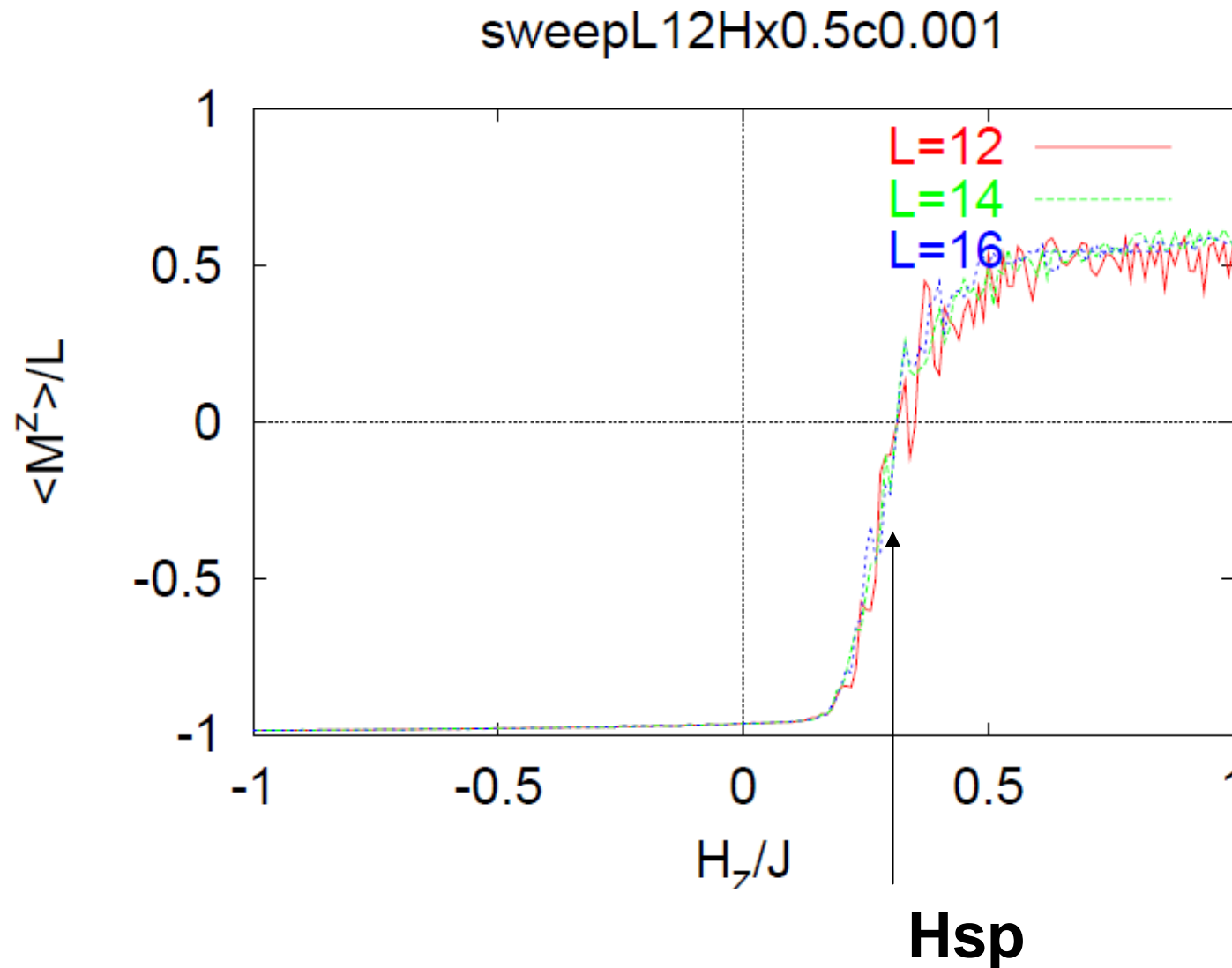


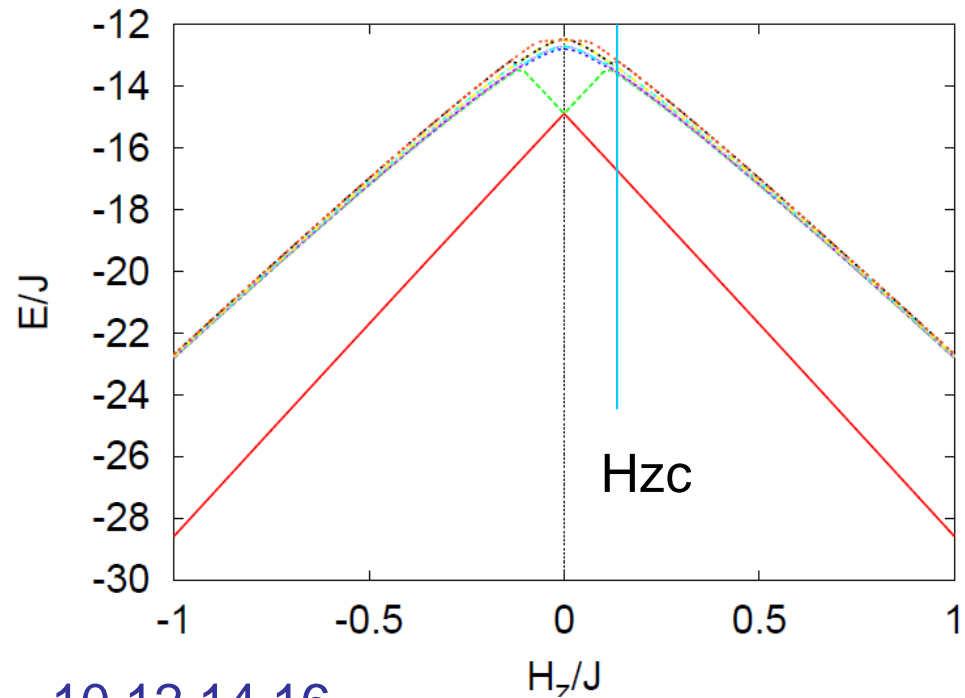
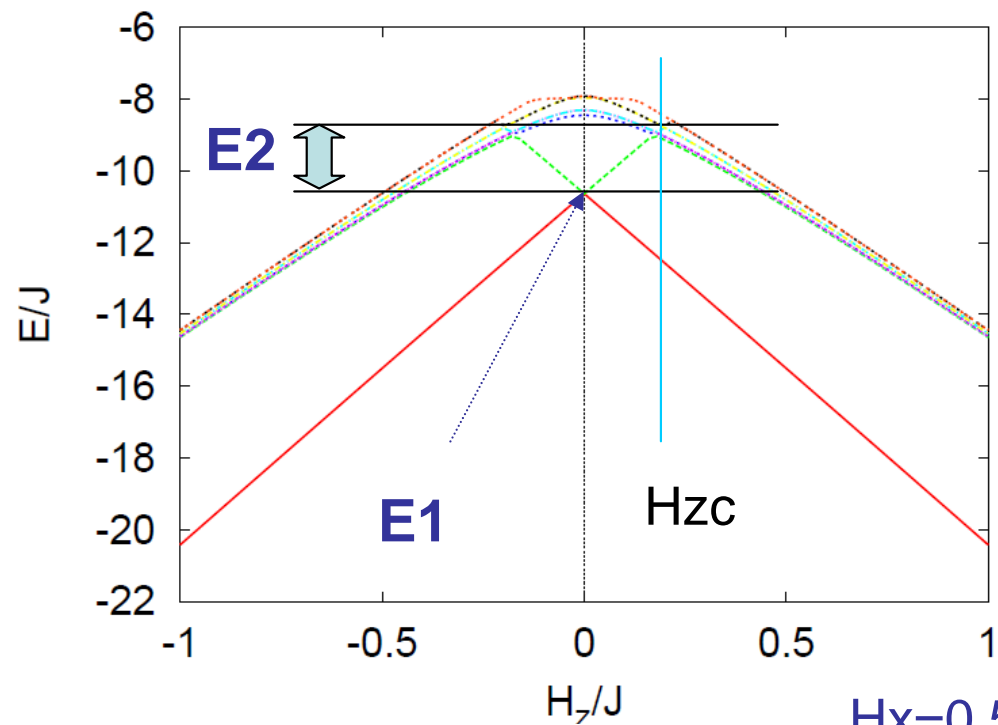
H. De Raedt, S. Miyashita, K. Saito, D. Garcia-Pablos and N. Garcia:
Phys. Rev. B56 (1997) 11761

Field sweep

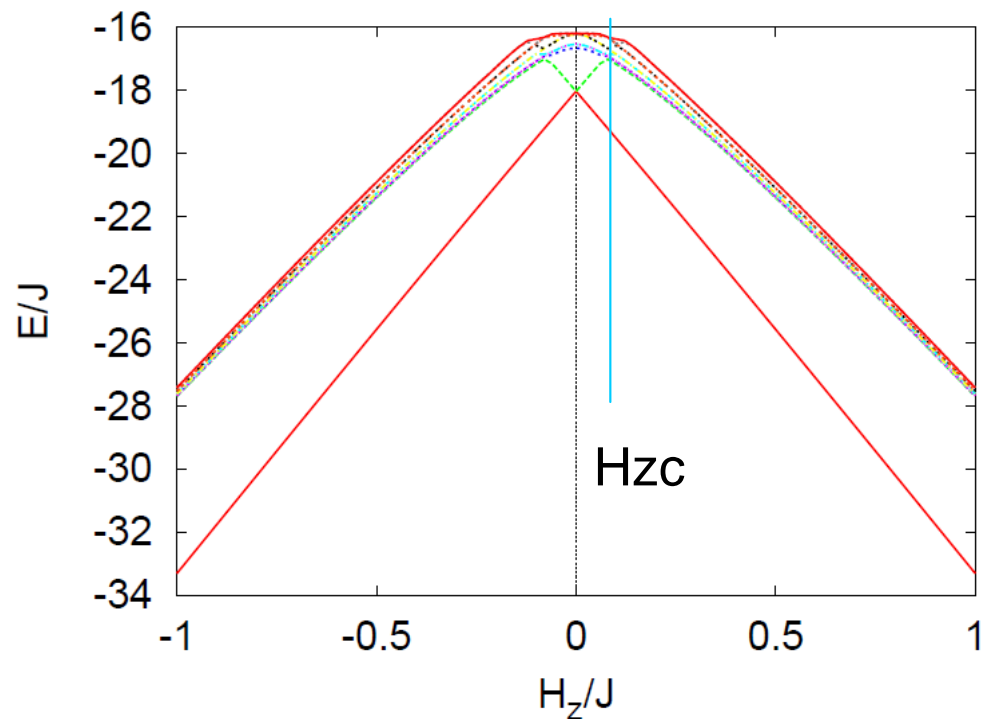
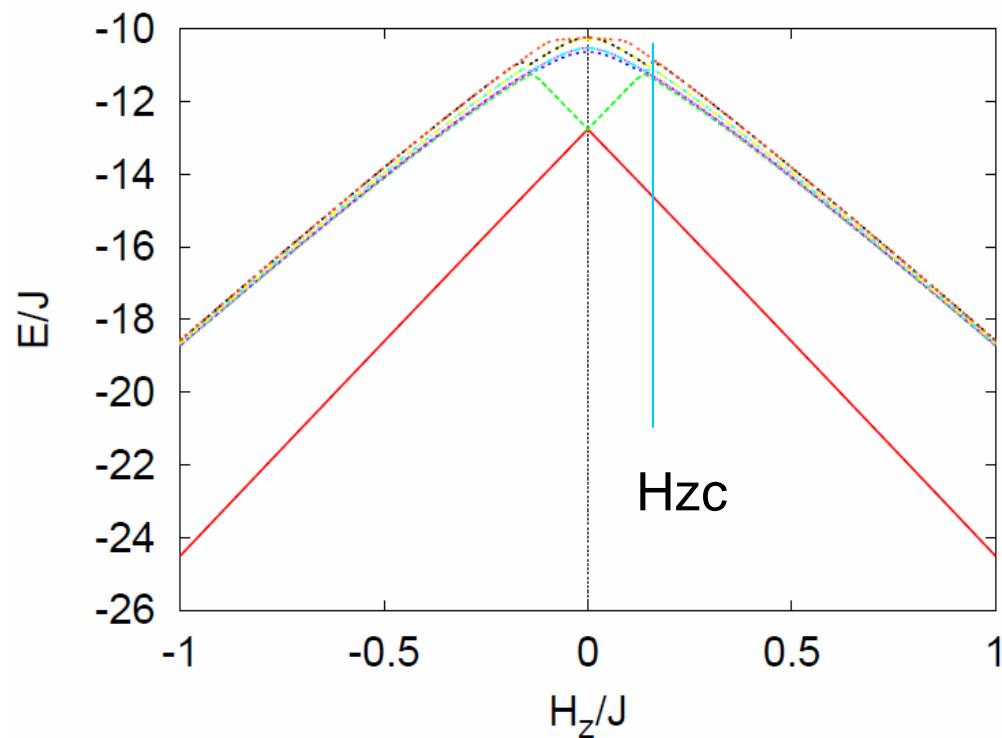


Size-independent phenomena a kind of collective motion(?)

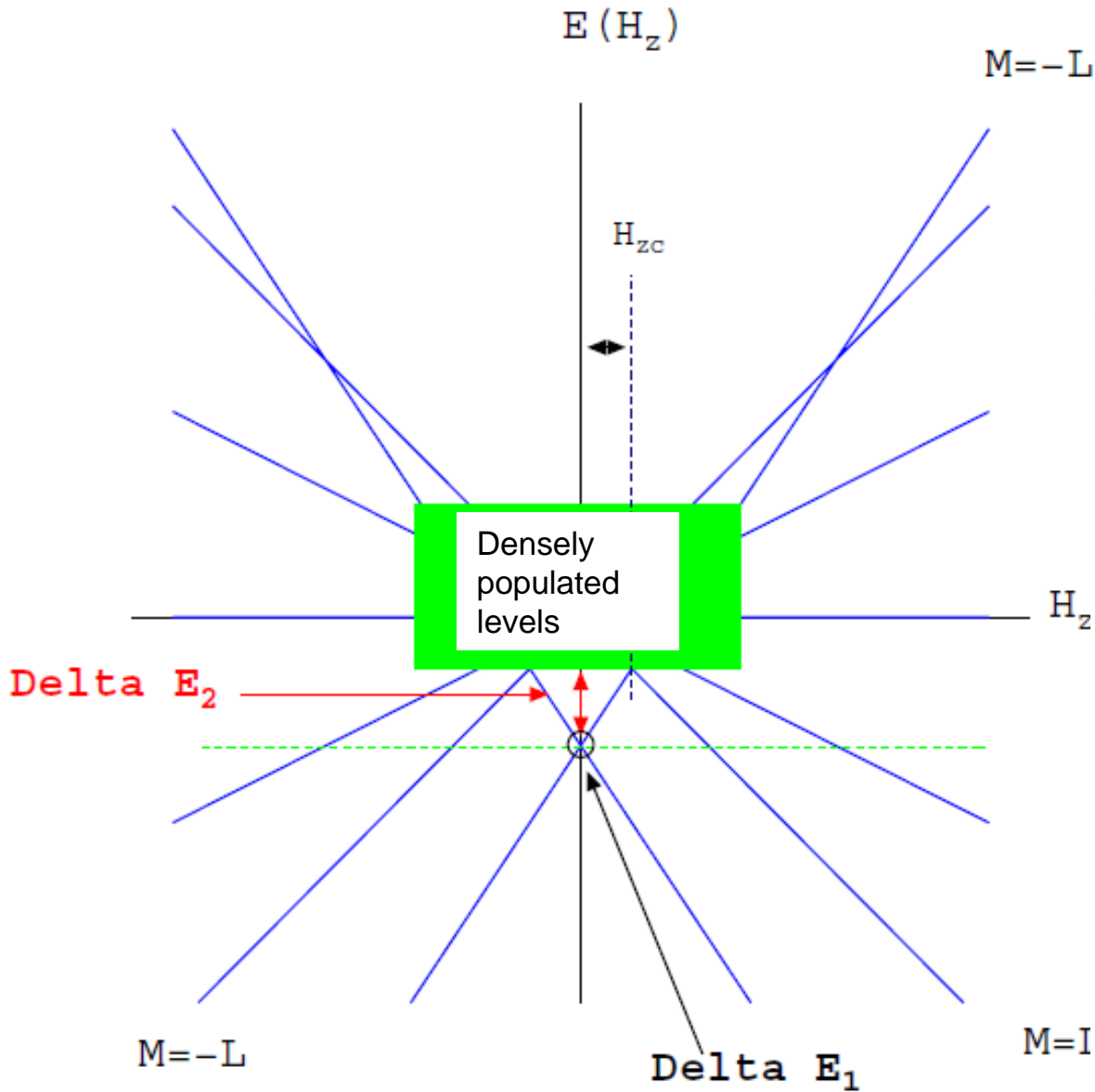




$H_x=0.5$ $L=10,12,14,16$



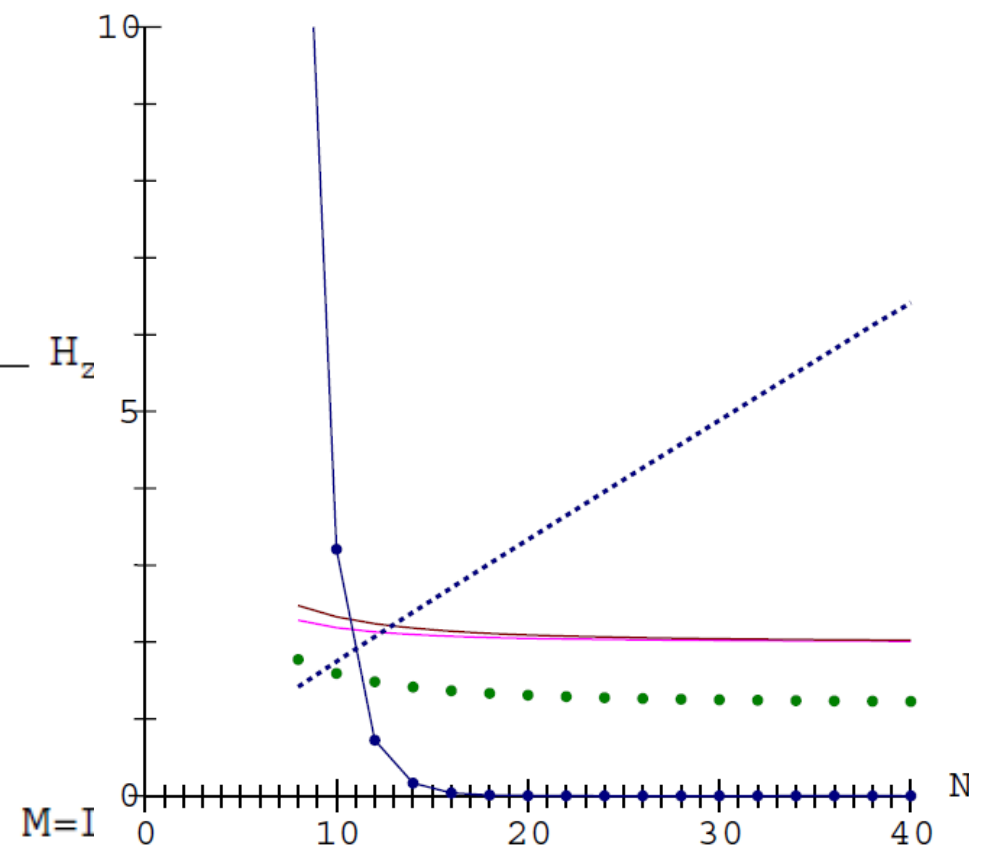
General structure



$$H_{zc} = \Delta E_2 / L$$

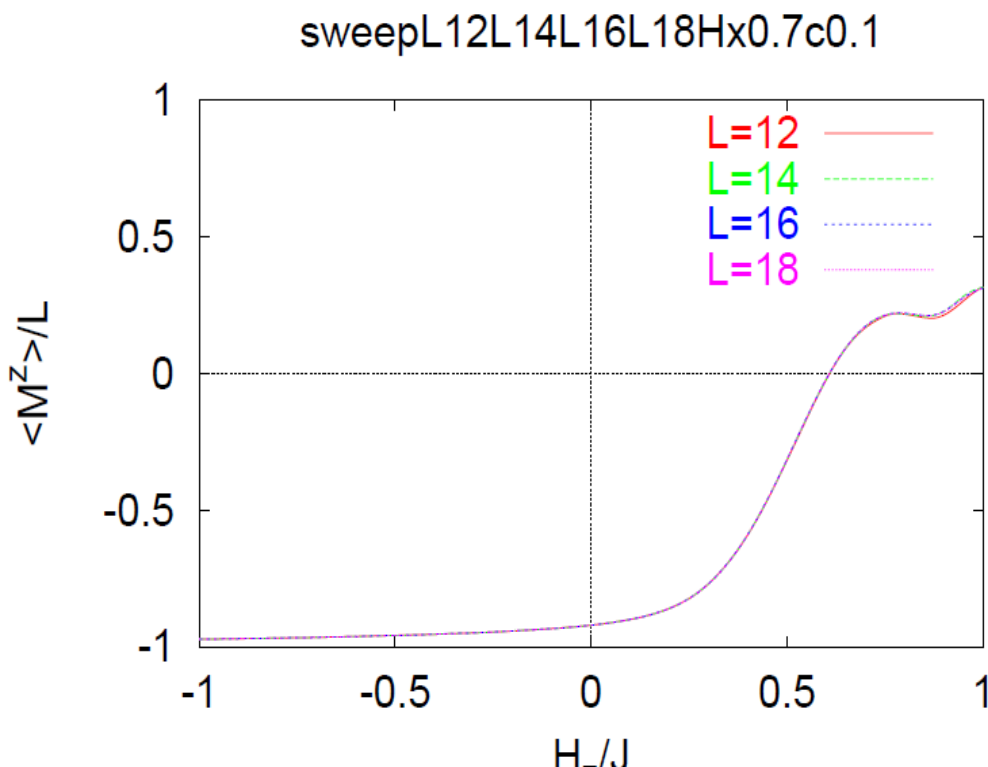
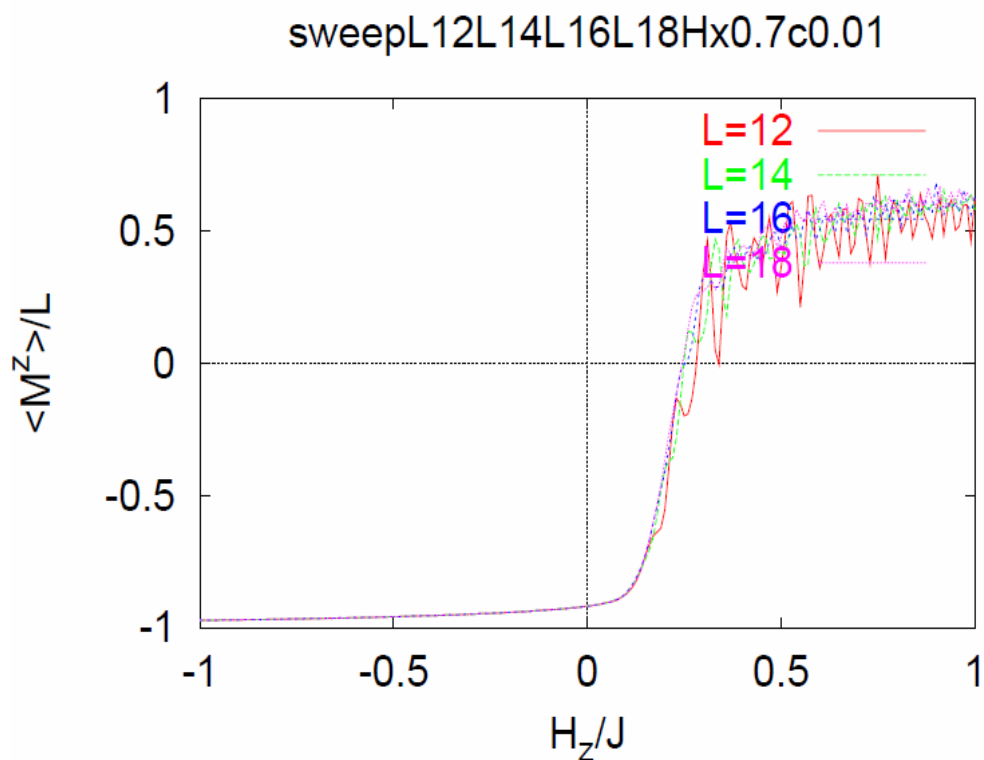
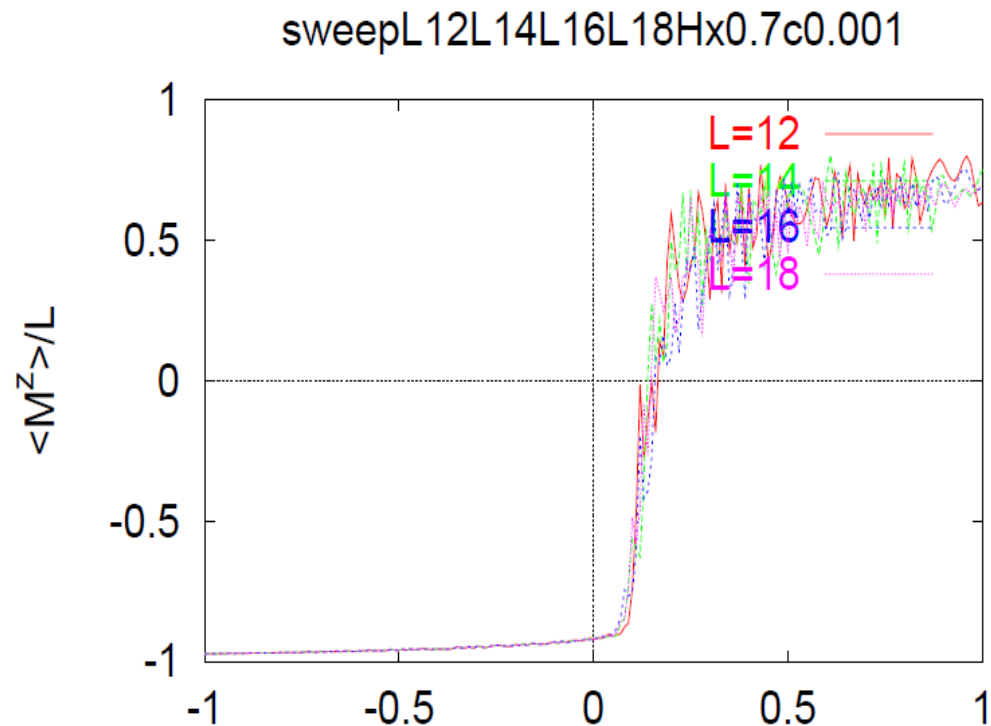
Transverse-Ising model $H_x=0.5$ $H_z=0$

$(E(2)-E(1)) * 10000, \log(E(2)-E(1))$
 $E(3)-E(1)$
 $E(4)-E(1)$

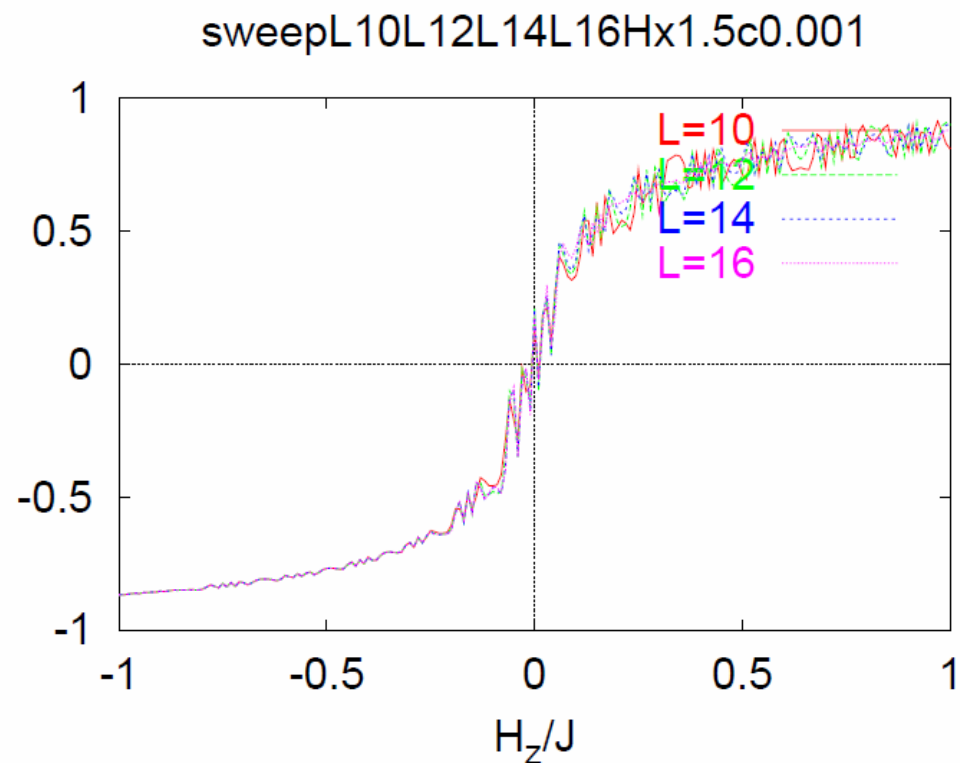
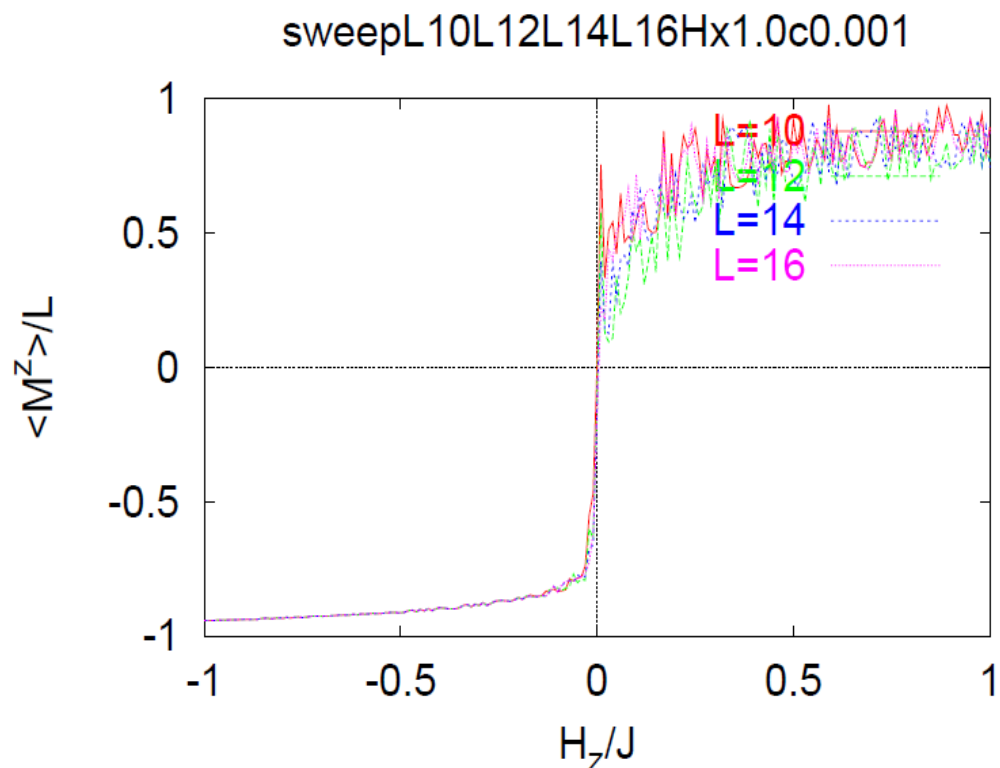
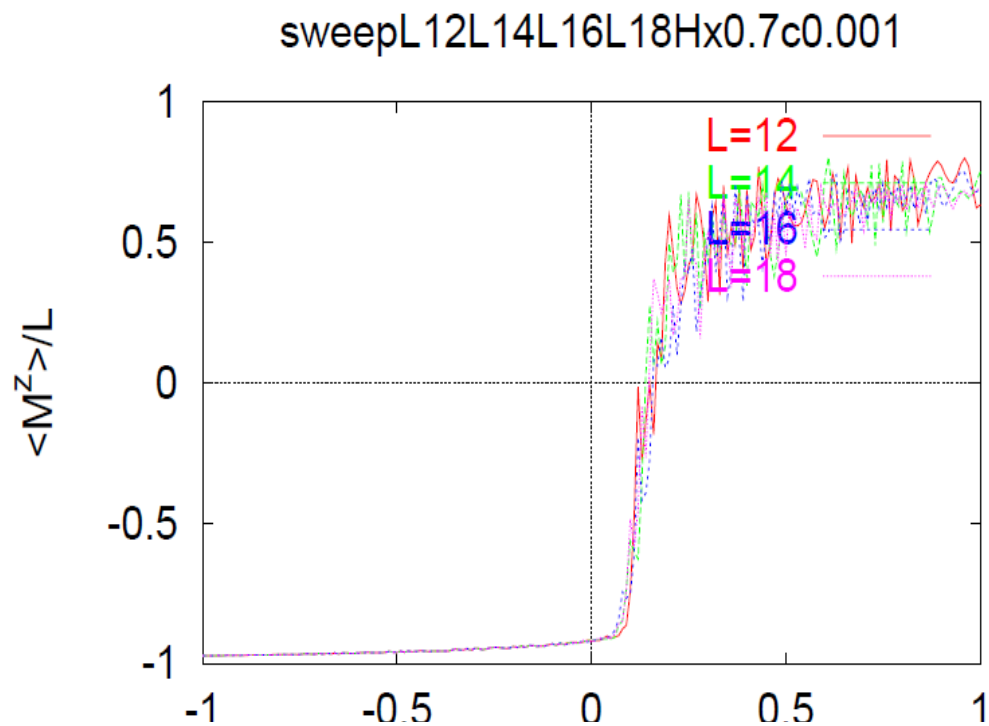


$E(3)-E(1)$ $H_x=0.7$ $H_z=0$

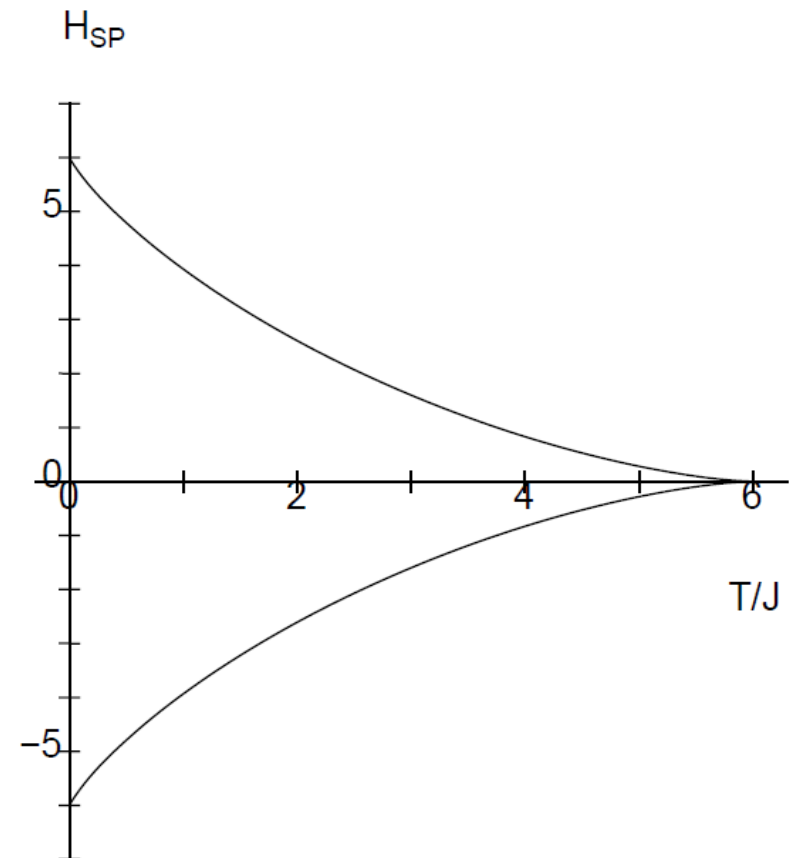
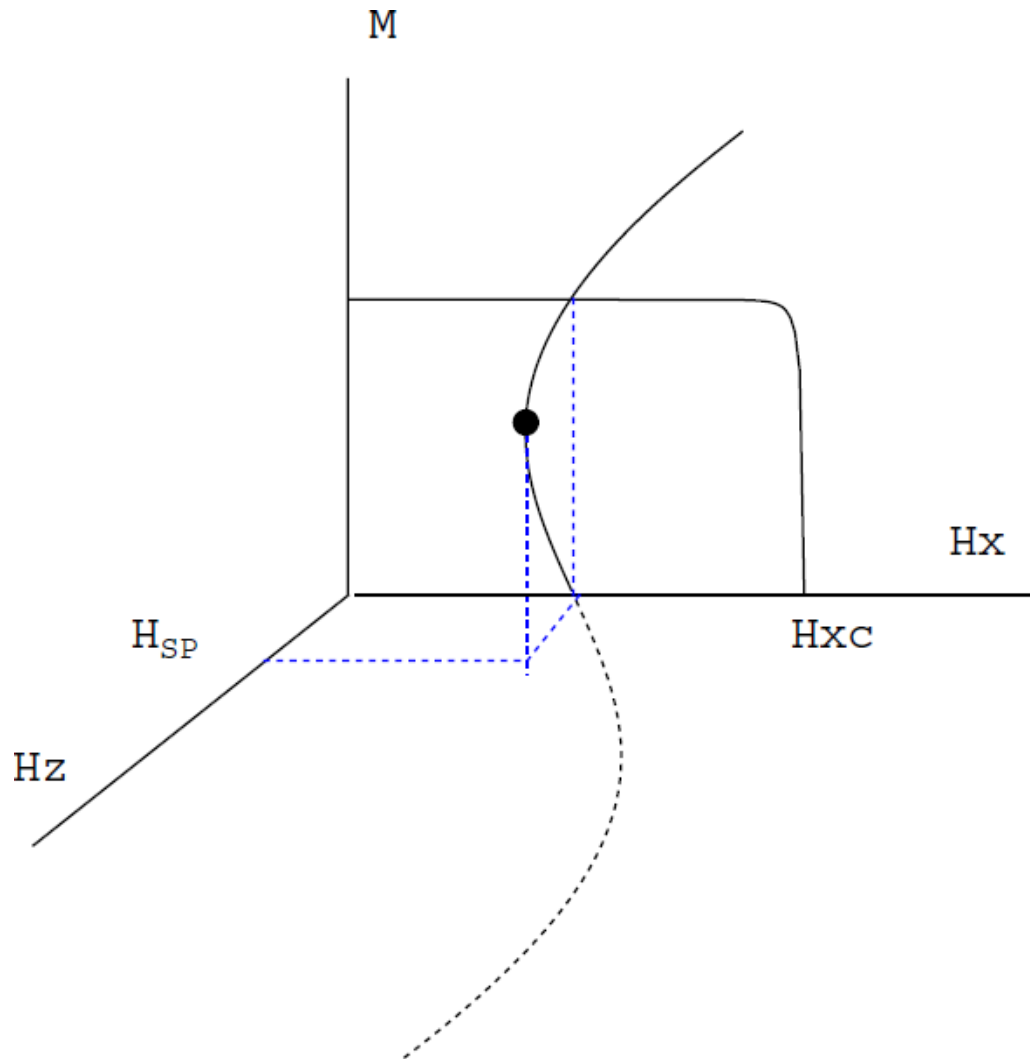
Sweep velocity dependence



Dependence on H_x



Metastability and Spinodal decomposition



Mean field theory : classical spin

Quantum spinodal decomposition

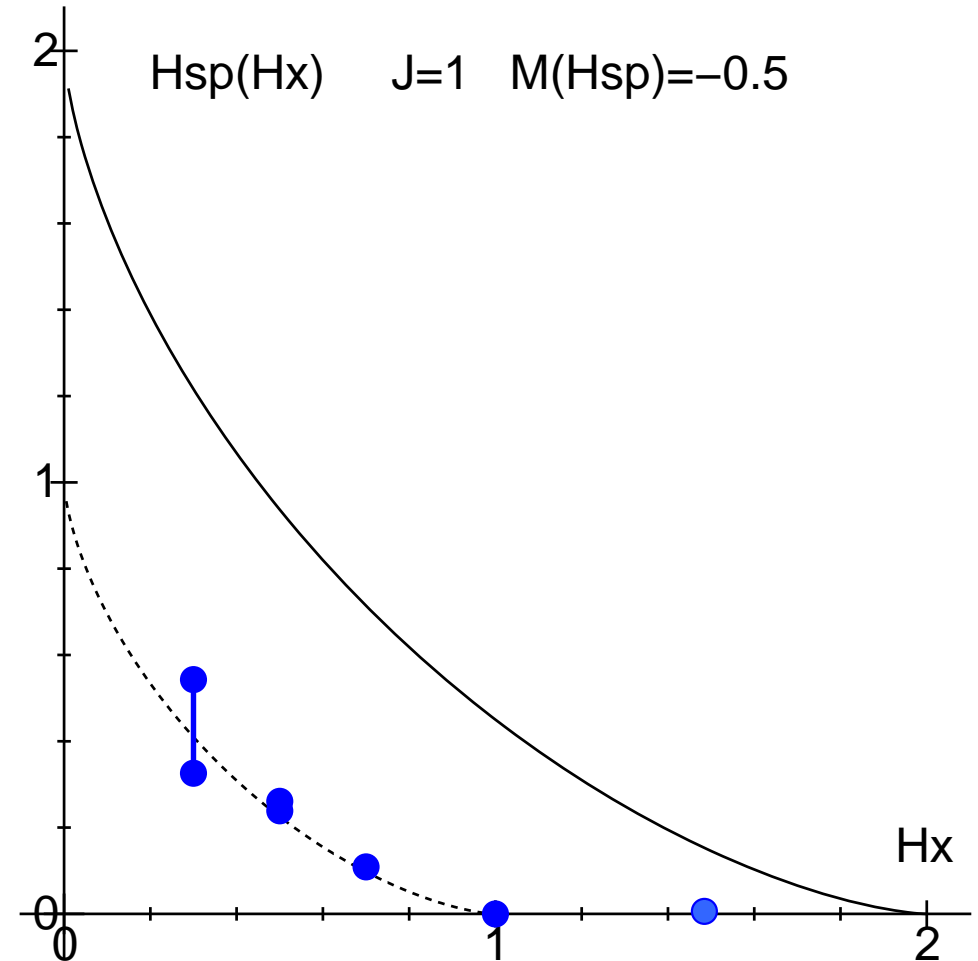
$$E = -J\sigma^2 - \Gamma\sqrt{1-\sigma^2} + H\sigma$$

$$\frac{dE}{d\sigma} = -J\sigma + \frac{\Gamma\sigma}{\sqrt{1-\sigma^2}} + H = 0$$

$$\frac{dH}{d\sigma} = 0 \Rightarrow \sigma = \left(1 - \gamma^{2/3}\right)^{1/2},$$

$$\gamma = \frac{\Gamma}{2J}$$

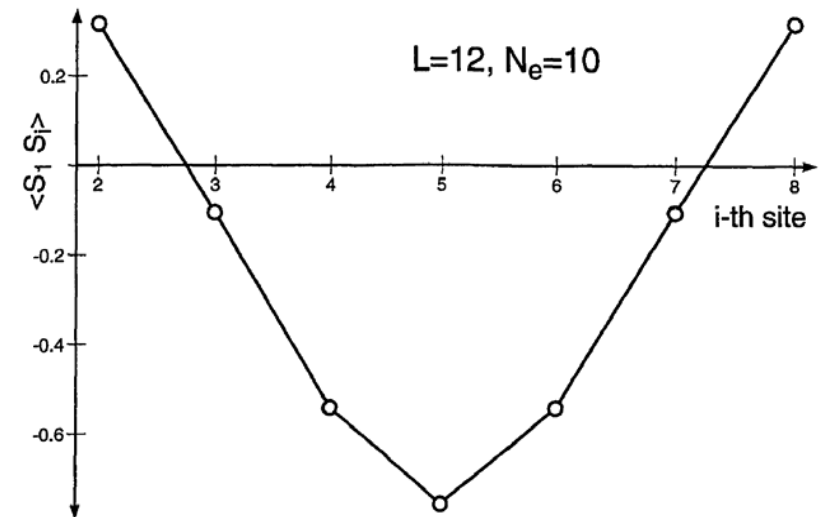
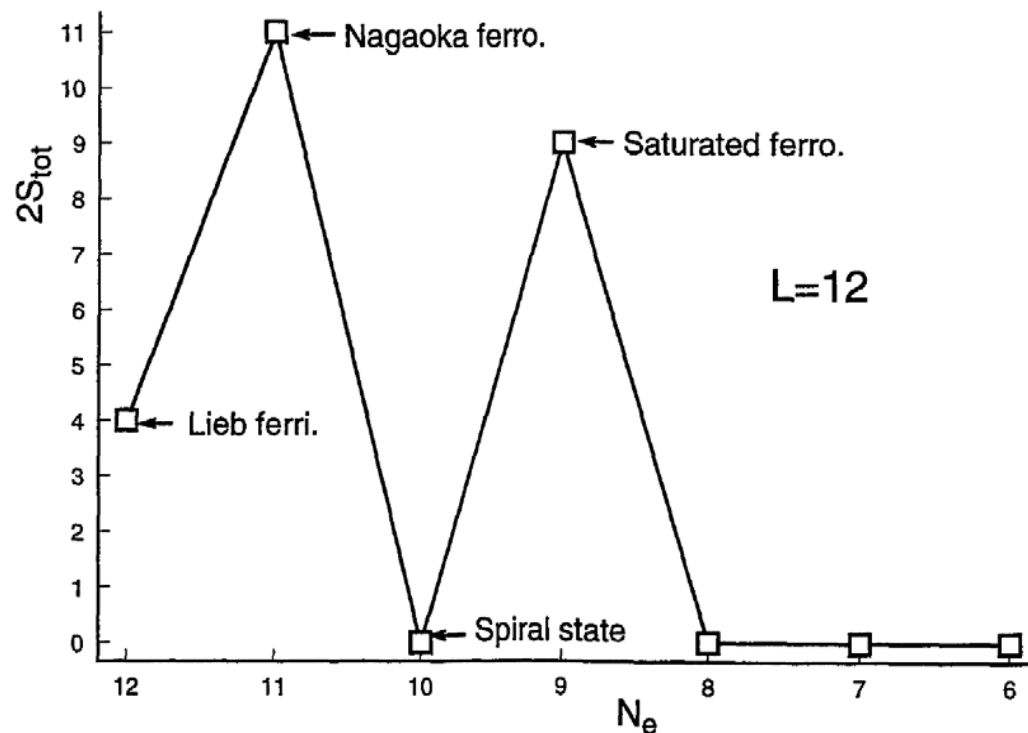
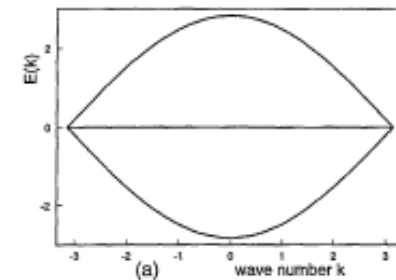
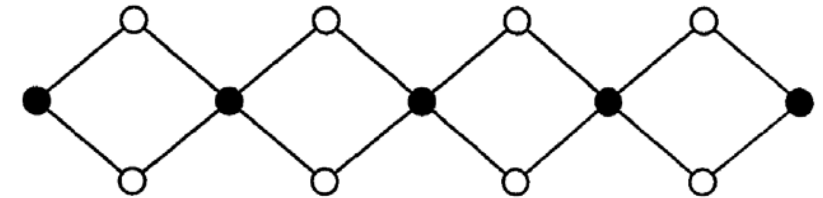
$$\frac{H_{SP}}{2J} = \left(1 - \gamma^{2/3}\right)^{3/2}$$



Itinerant ferromagnetism and its dynamics

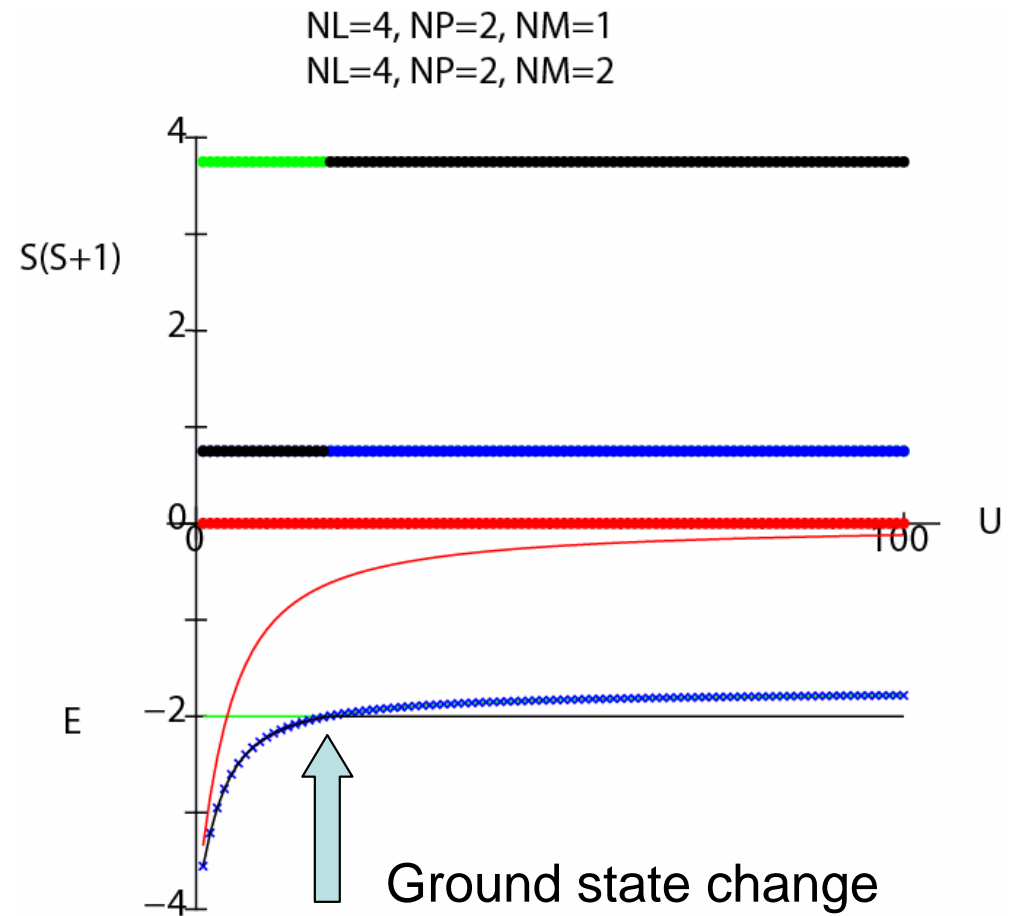
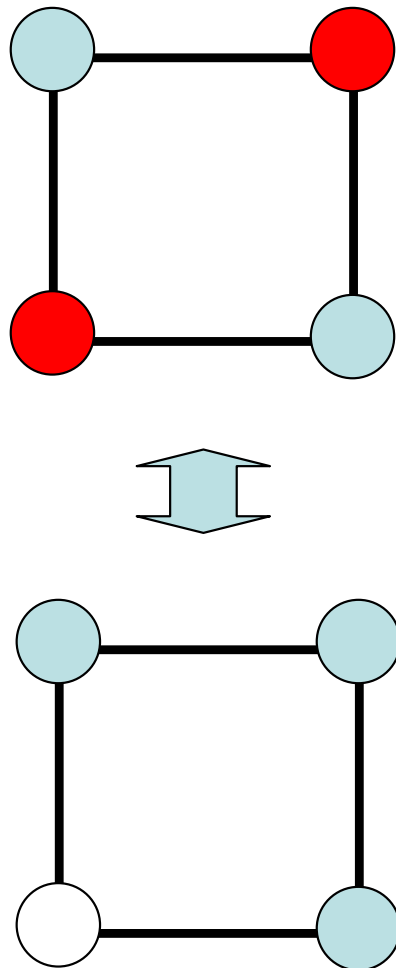
$$H = H_{\text{hop}} + H_{\text{int}}$$

$$= -t \sum_{\langle i,j \rangle \in \Lambda} \sum_{\sigma=\uparrow,\downarrow} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_{i \in \Lambda} n_{i,\uparrow} n_{i,\downarrow}$$



Y. Watanabe and SM: JPSJ 68 (1999) 3086.
66 (1997) 2123,

Transition between AF and Nagaoka-Ferromagnetic state



Dynamics after decimation

$$S^2 |G : (2 + 2)\rangle = 0$$

⇓

$$\sum_i (c_{i\uparrow} + c_{i\downarrow}) |G : (2 + 2)\rangle = |initial \rangle$$

$$|t\rangle = e^{-iHt} |initial \rangle$$

$$S^2 |t\rangle = \frac{3}{2}$$

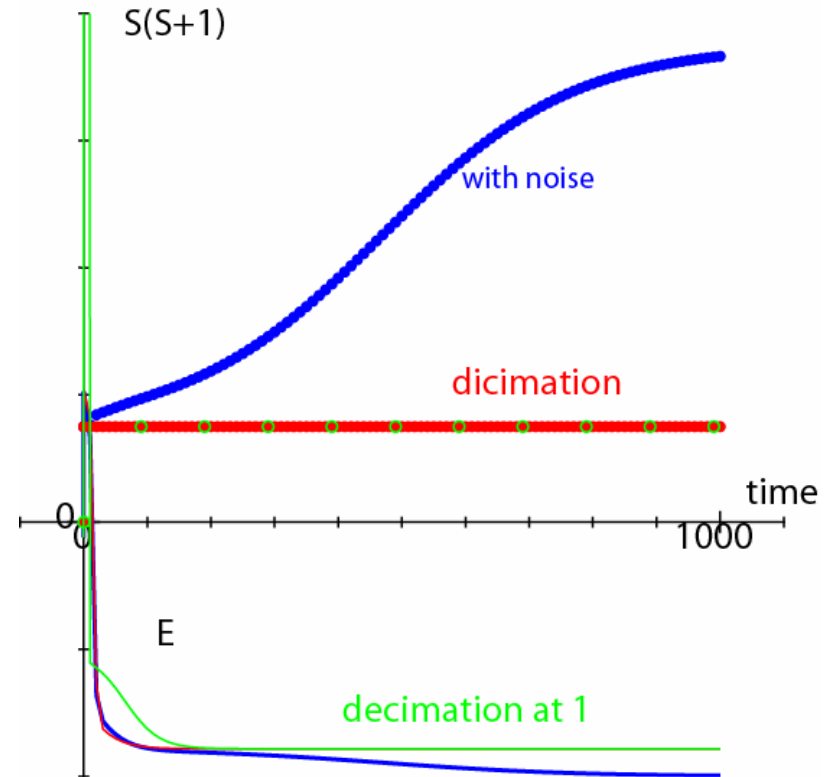
$$S^2 |G : (2 + 2)\rangle = 0$$

⇓

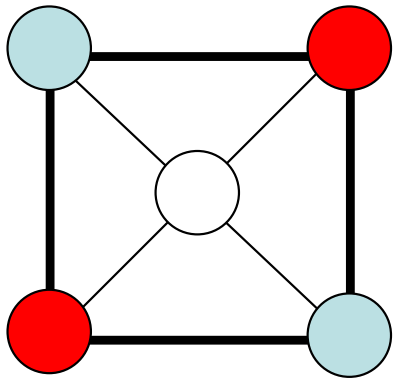
$$\sum_i ((1 + \delta_i) c_{i\uparrow} + (1 + \delta'_i) c_{i\downarrow}) |G : (2 + 2)\rangle = |initial' \rangle$$

$$|t\rangle = e^{-iHt} |initial' \rangle$$

NL=4, (2,2) => (2,1)

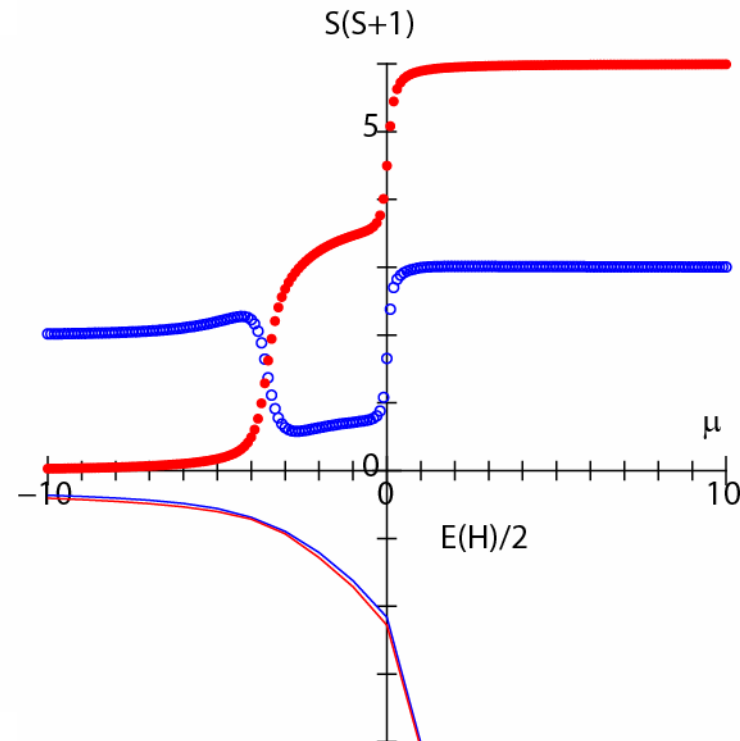
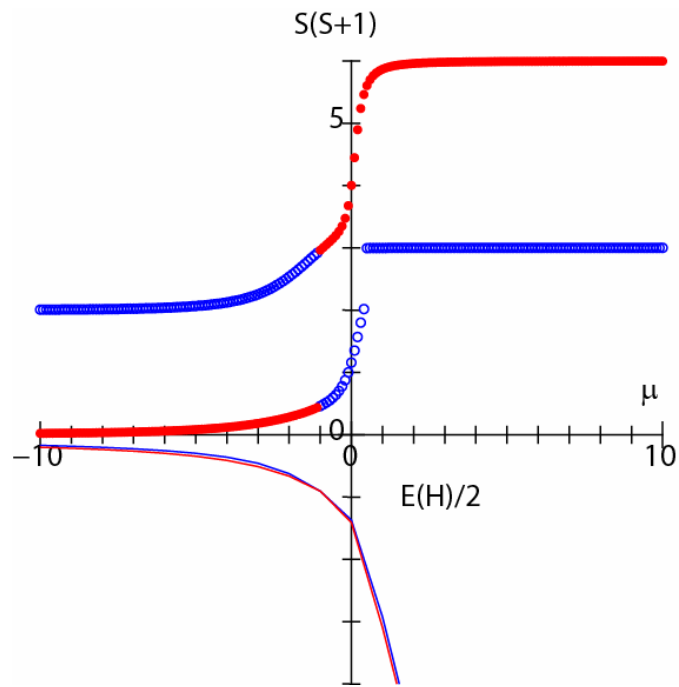


Adiabatic decimation



$$H = -t_{ij} \sum_{ij\sigma} (c_{i\sigma}^+ c_{j\sigma} + c_{j\sigma}^+ c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu (n_{5\uparrow} + n_{5\downarrow})$$

Non-uniform lattice (t=2,3,1.5,1)

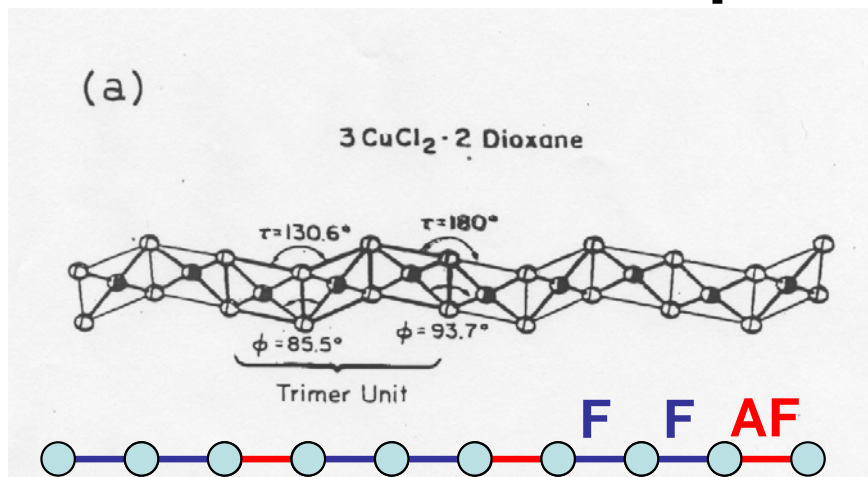


Quantum response in pure quantum and dissipative environments

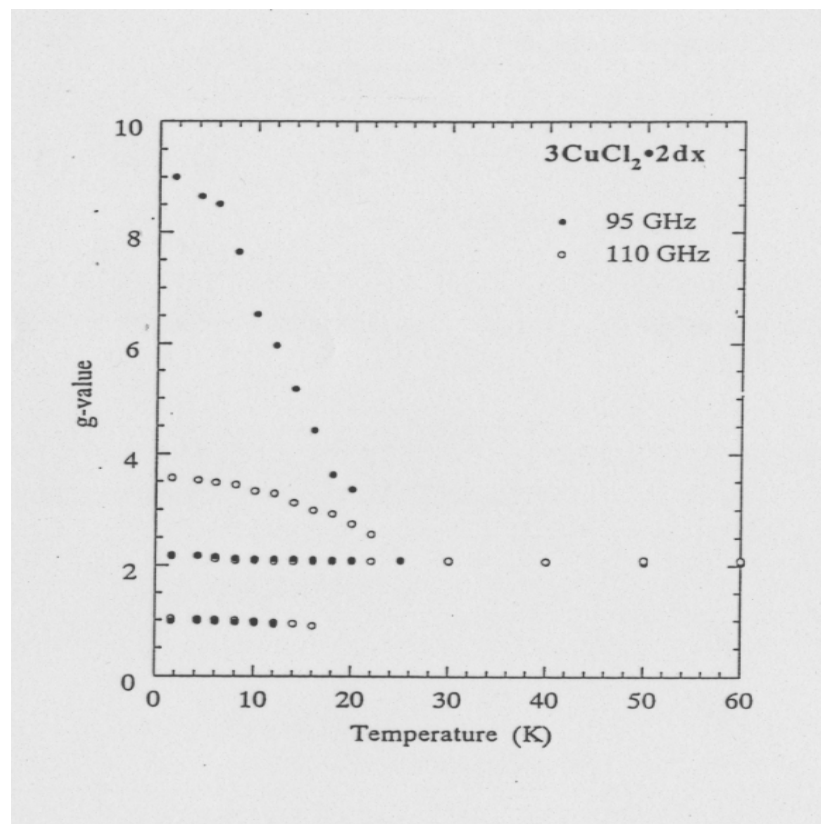
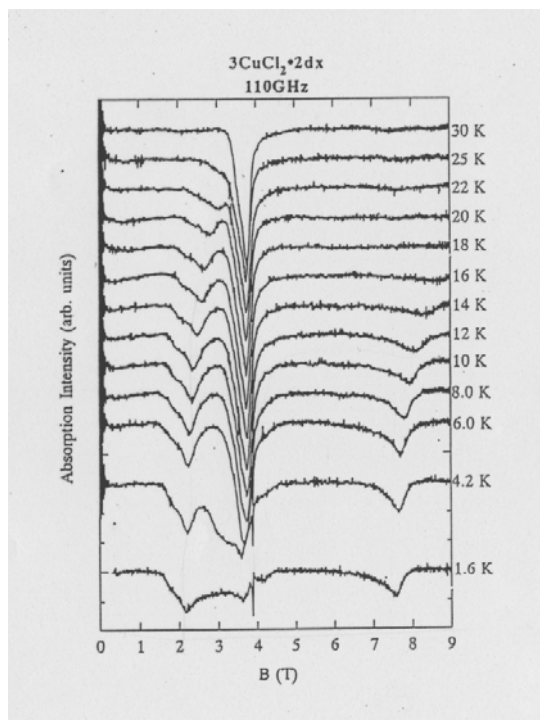
collaborators: Akira Ogasahara, Keiji Saito,
Chikako Uchiyama, and Mizuhiko Saeki

ESR line shape in strongly interacting spin systems

Temperature-dependence of the shift and width in low-dimensional quantum spin systems



Spin trimer: $3\text{CuCl}_2 \cdot 2\text{Dioxane}$



Y. Ajiro, et al: JPSJ 63 (1994) 859.

Microscopic expression of the line shape from Hamiltonian

Kubo Formula

R. Kubo & K. Tomita JPSJ (1954) 888

R. Kubo: JPSJ 12 (1957) 570

$$\chi''_{xx}(\omega) = \frac{1}{2} (1 - e^{-\beta\omega}) \int_{-\infty}^{\infty} \langle M^x(0) M^x(t) \rangle e^{-i\omega t} dt$$

Pure quantum dynamics

$$M(t) = e^{iL(t)} M(0) \Rightarrow e^{iHt/\eta} M e^{-iHt/\eta}$$

$$H |m\rangle = E_m |m\rangle$$

$$\chi''(\omega) = \sum_{mn} D(\omega_{mn}) \delta(\omega - (E_n - E_m))$$

$$D(\omega_{mn}) = \pi \left(e^{-\beta E_m} - e^{-\beta E_n} \right) \frac{|\langle m | M^x | n \rangle|^2}{Z}, \quad (\omega = E_n - E_m)$$

Shift from the PMR

Isotropic models

$$H = -2 \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - H_0 \sum_i S_i^z - H_1 \cos(\omega t) \sum_i S_i^x$$

Paramagnetic Resonance

$$\omega_R = \gamma H, \quad \gamma = \frac{1}{2\eta} g \mu_B$$

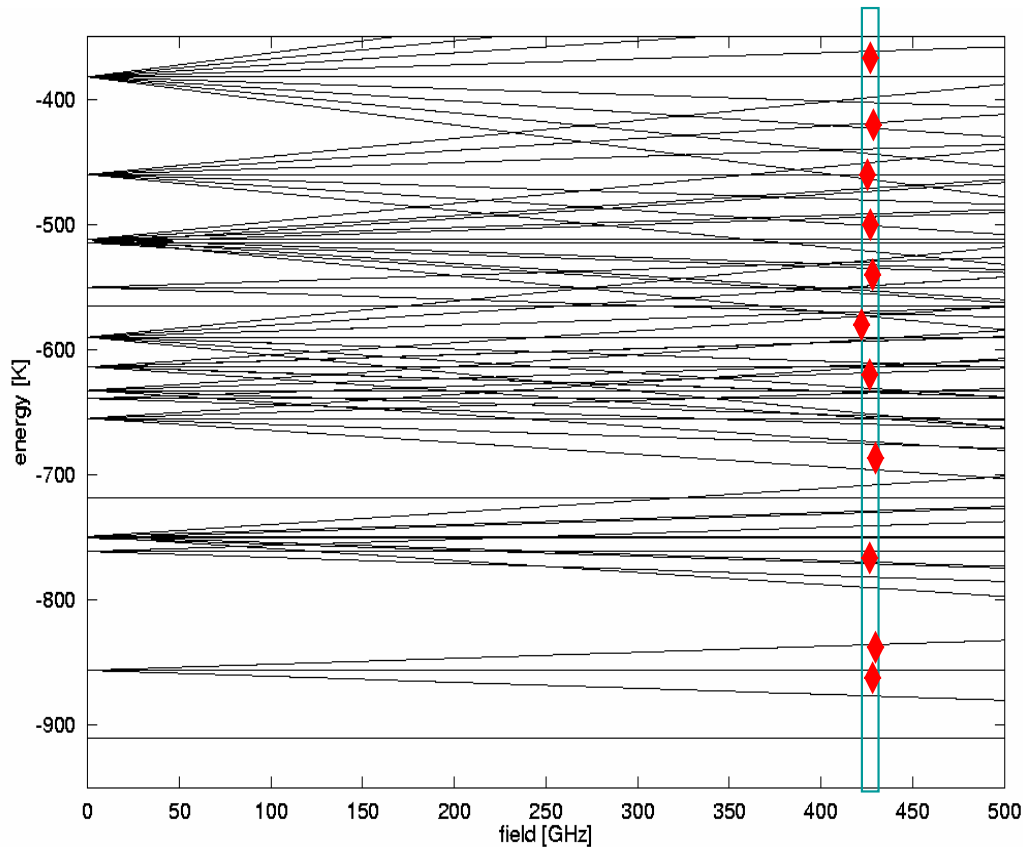
Perturbation

$$H_{\text{perturbation}} = -2 \sum_{\langle ij \rangle} [J (S_i^x S_j^x + S_i^y S_j^y) + J_z S_i^z S_j^z] + D \sum_{\langle mn \rangle} \left(\frac{\mathbf{S}_m \cdot \mathbf{S}_n}{r_{mn}^3} - \frac{3(\mathbf{S}_m \cdot \mathbf{r}_{mn})(\mathbf{S}_n \cdot \mathbf{r}_{mn})}{r_{mn}^5} \right) + \Lambda + \mathbf{D} \cdot \sum_{\langle ij \rangle} \mathbf{S}_i \times \mathbf{S}_j + \Lambda$$

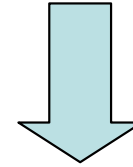
Studies on the line shape

- **F. Bloch: PR 70 (1946) 460. Nuclear Induction (Bloch equation)**
- **J. H. Van Vleck: PR 74 (1948) 1168.**
Dipolar broadening, and exchange narrowing
- **N. Bloembergen, E. M. Purcell and R. V. Pound: PR 73 (1948) 679.**
Relaxation Effects in Nuclear Magnetic Resonance Absorption.
- **I. Solomon: PR 99 (1955) 559.**
Relaxation processes in a system of two spins
- **F. Bloch: PR 105 (1957) 1206. General theory of relaxation**
- **A. Abragam: The principles of Nuclear Magnetism,
Oxford Univ. Press (1978)**

Shift & Width

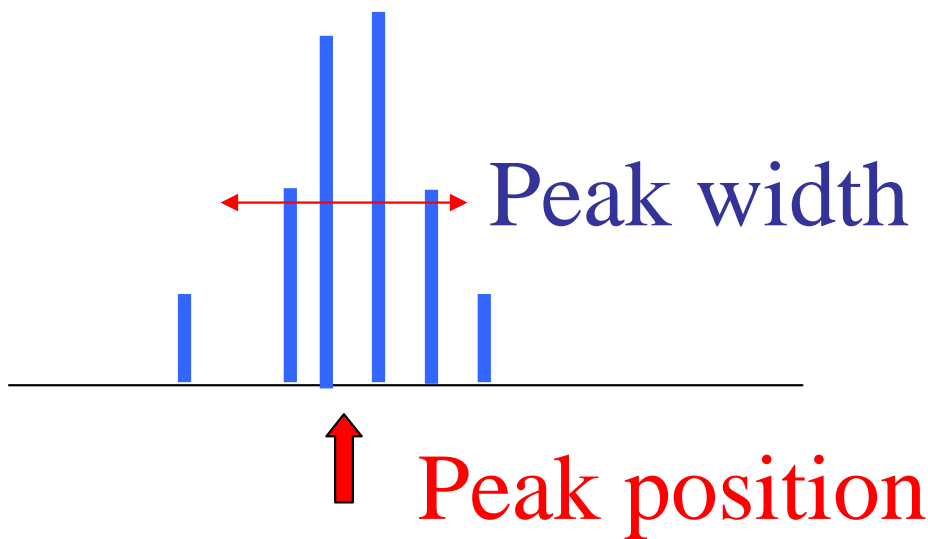


$$\langle M^x(0)M^x(t) \rangle \cong m^2 e^{-i\omega_0 t - t/\tau}$$



$$\chi''(\omega) \propto \frac{\Gamma}{(\omega - \omega_0)^2 + \Gamma^2}$$

$$\omega_0 = \omega_R + \delta\omega$$

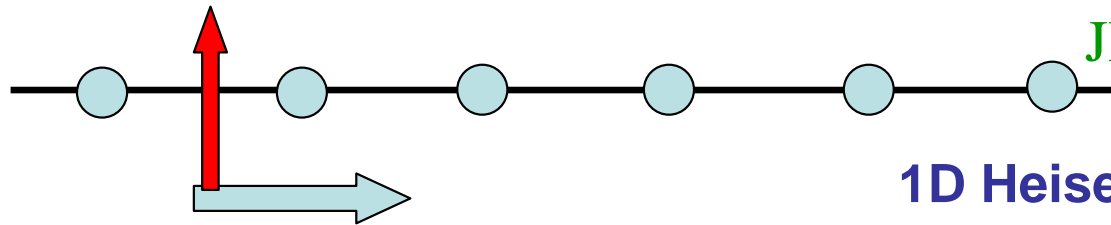


$$D(\omega_{mn}) = \pi \left(e^{-\beta E_m} - e^{-\beta E_n} \right) \frac{\left| \langle m | M^x | n \rangle \right|^2}{Z},$$

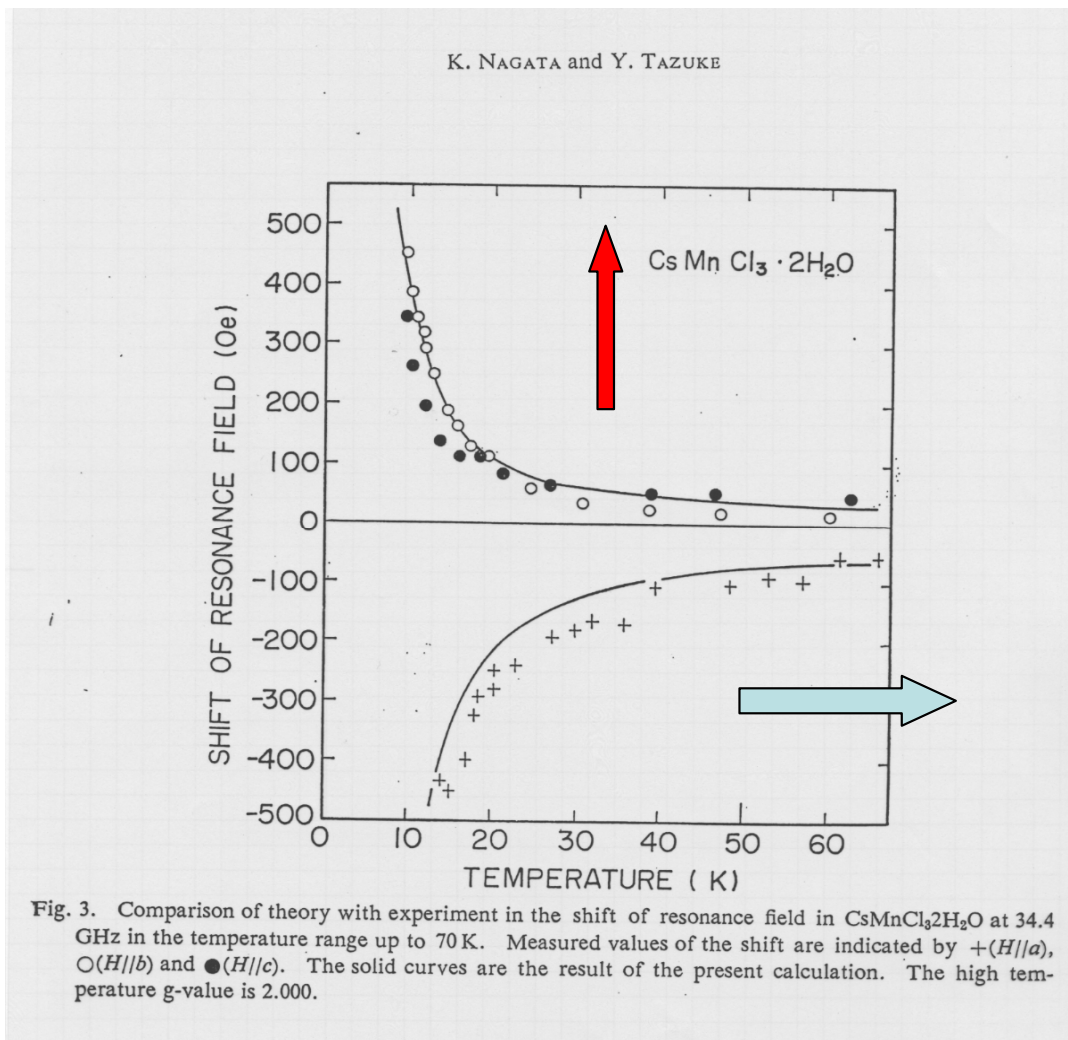
($\omega = E_n - E_m$)

Nagata-Tazuke Dependence

K. Nagata and Y. Tazuke:
JPSJ 32 (1972) 337



1D Heisenberg model with
Dipole-dipole interaction

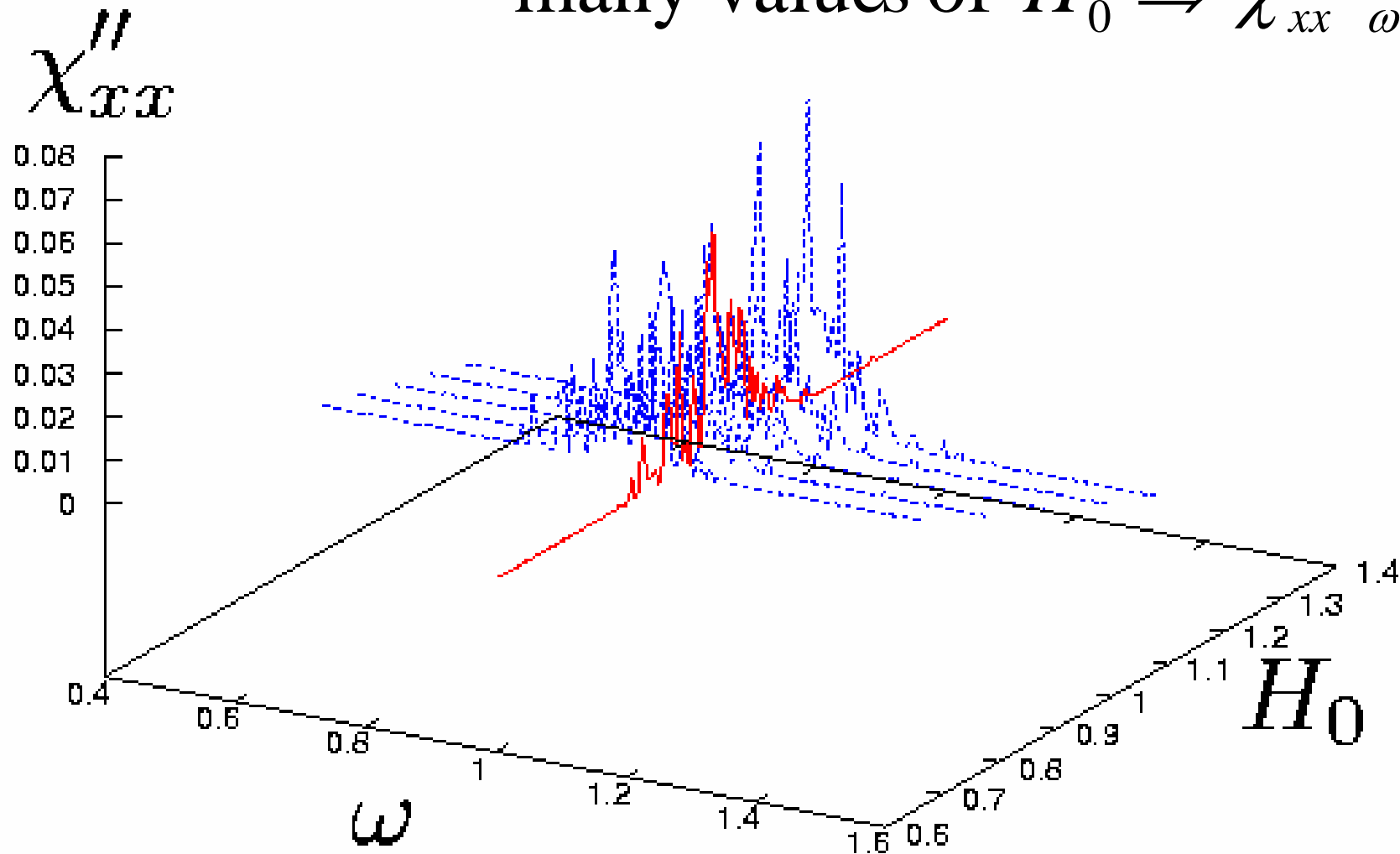


(J. Kanamori & M. Tachiki
JPSJ 48 (1962) 50)

Frequency sweep and Field sweep

$$\chi''_{xx}(\omega, H_0): H_0 \text{ given} \Rightarrow \chi''_{xx, H_0}(\omega)$$

$$\text{many values of } H_0 \Rightarrow \chi''_{xx, \omega}(H_0)$$

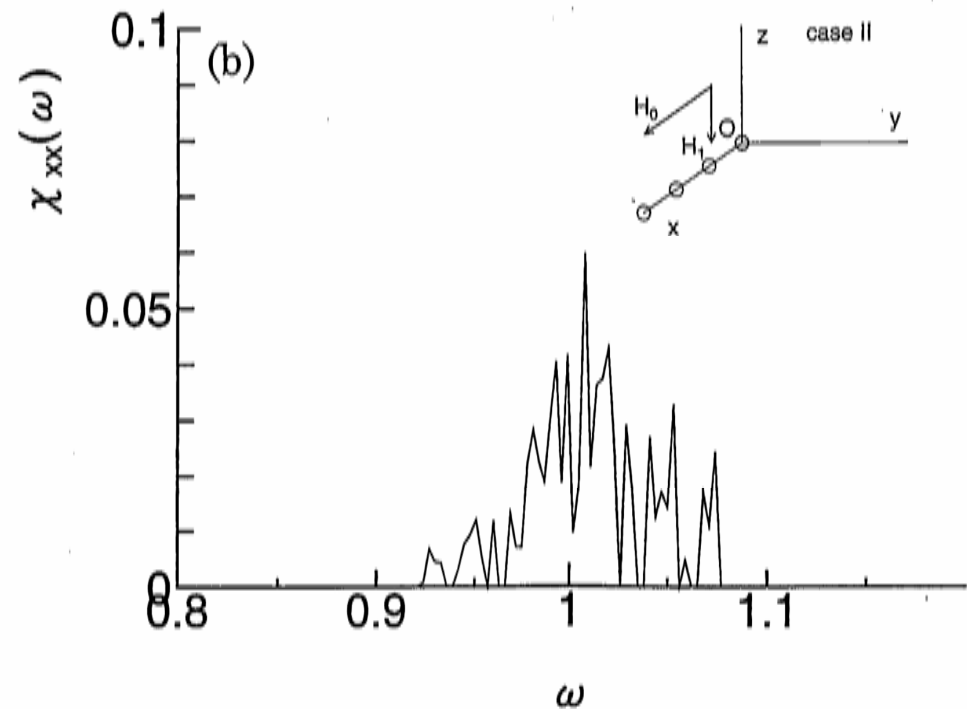
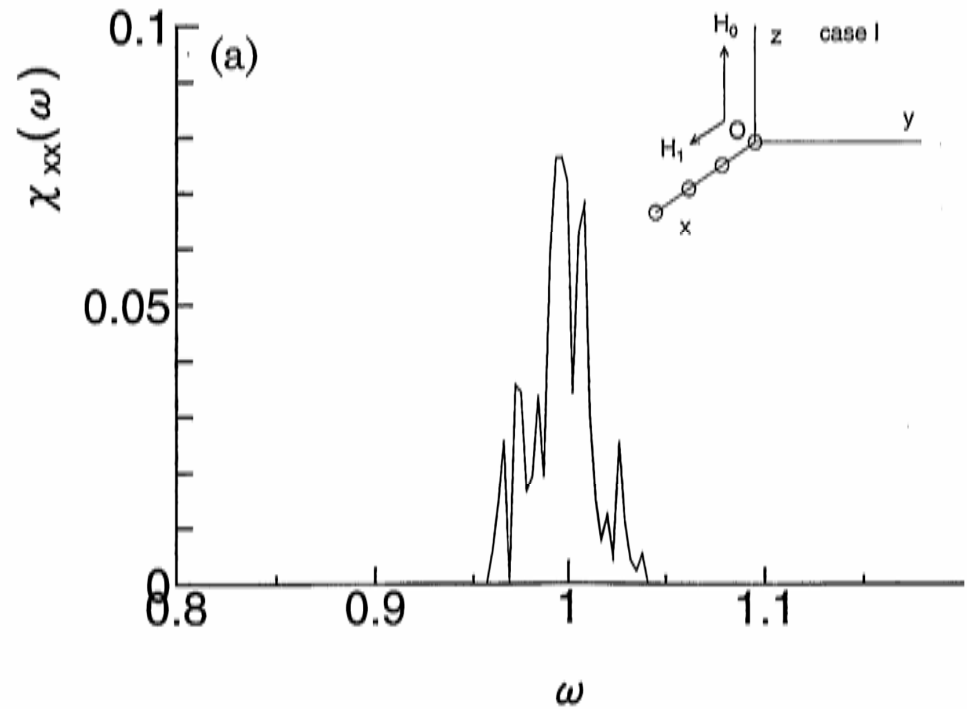


Line shape as an ensemble of delta-function

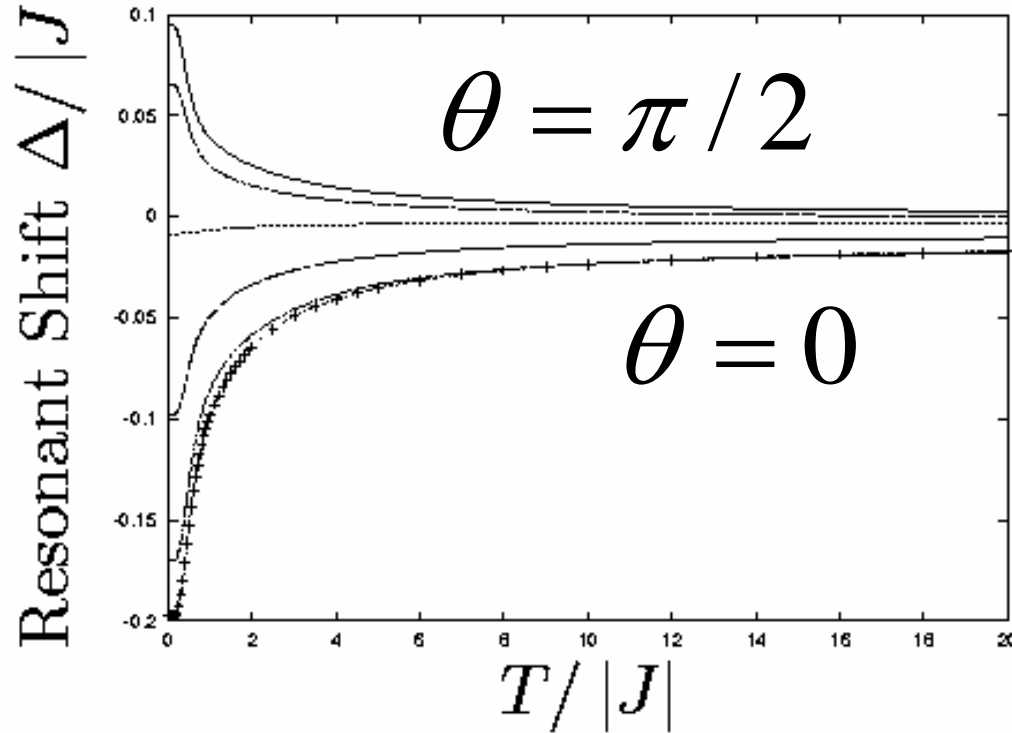
$$D(\omega_{mn}) = \pi \left(e^{-\beta E_m} - e^{-\beta E_n} \right) \frac{|\langle m | M^x | n \rangle|^2}{Z},$$

$$(\omega = E_n - E_m)$$

N=8



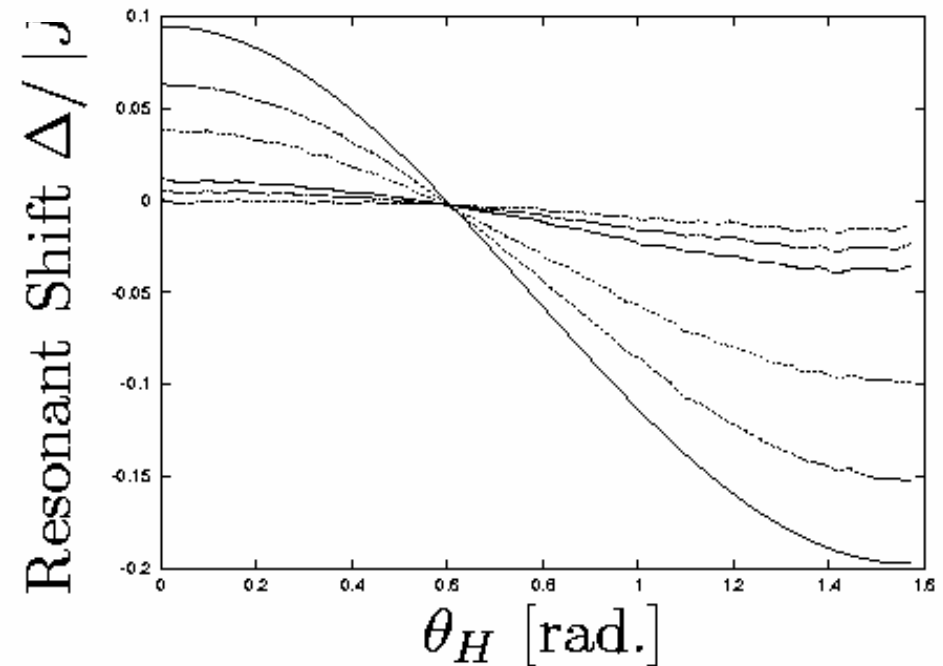
Temperature Dependence



Shift

1D Heisenberg AF

Angle Dependence

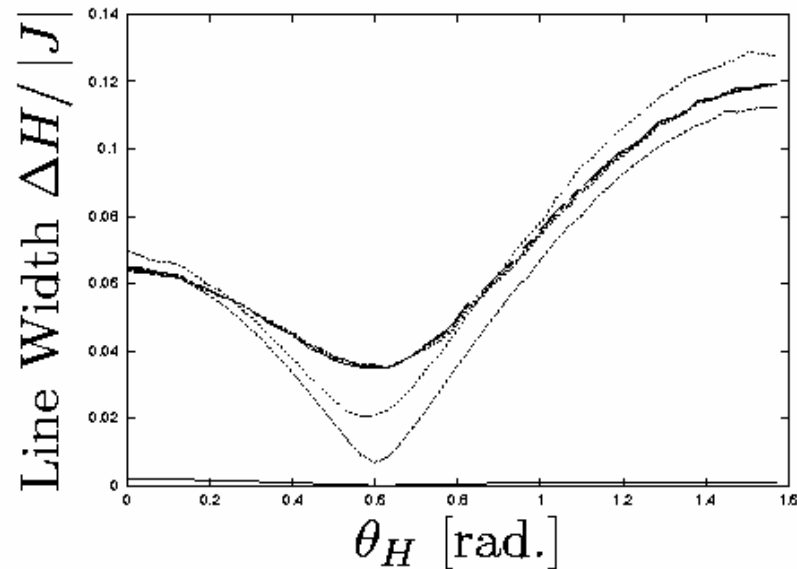
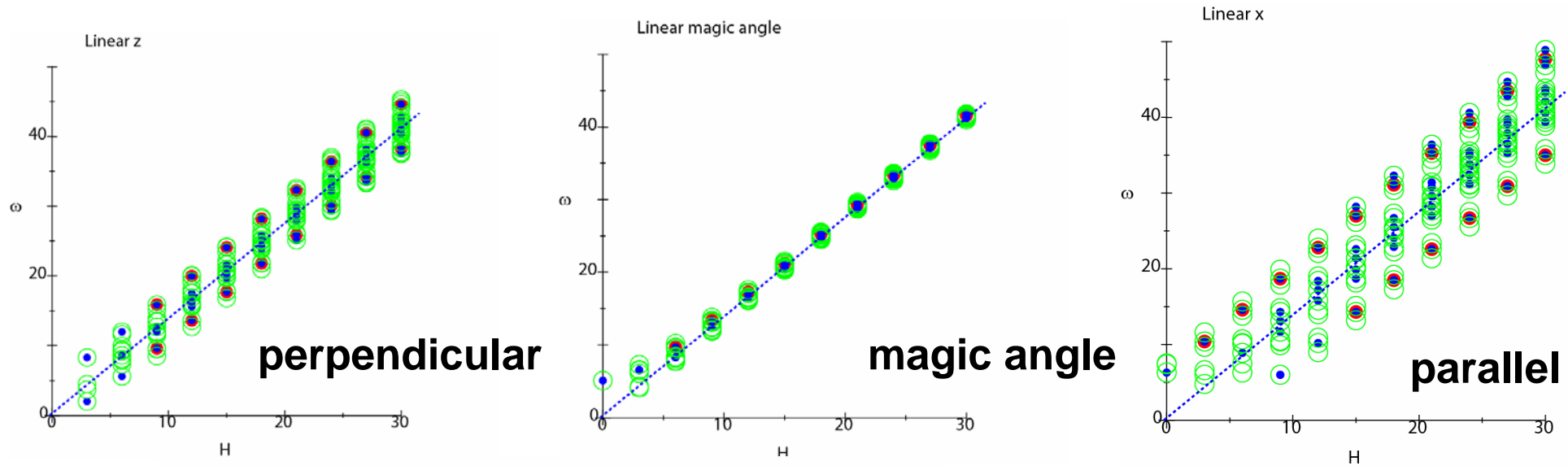


**SM, T. Yoshino, A. Ogasahara
JPSJ 68 (1999) 655**

Width

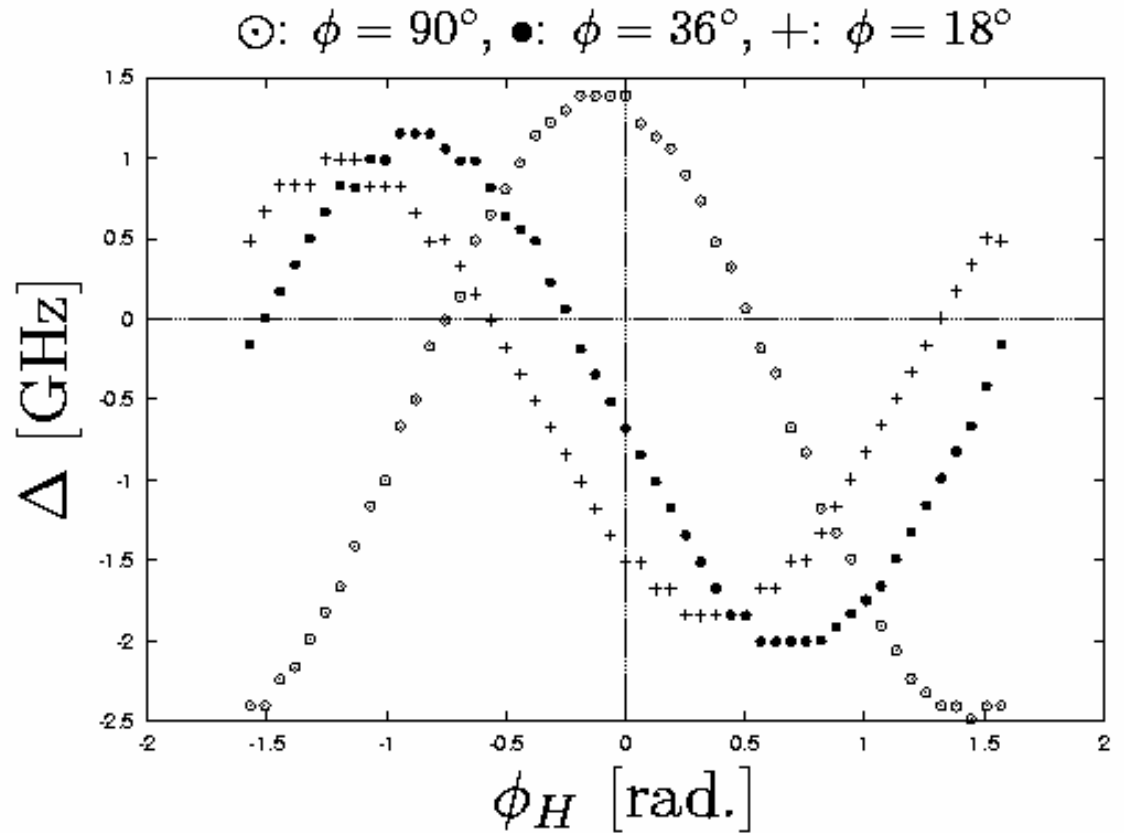
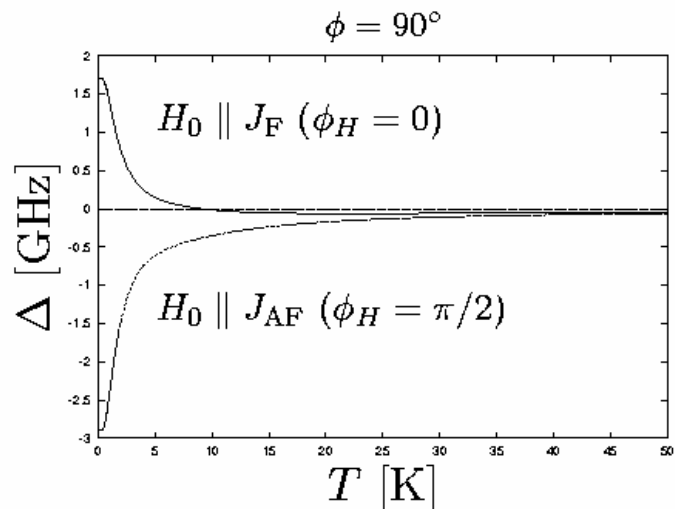
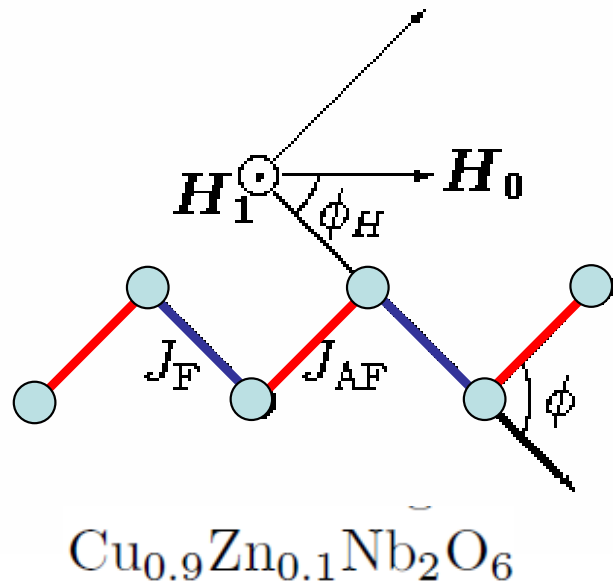
Magic Angle

R.E. Dietz, et al. PRL 26 (1971) 1186.
T.T. Cheung, et al. PRB 17 (1978) 1266



SM, T. Yoshino, A. Ogasahara
JPSJ 68 (1999) 655

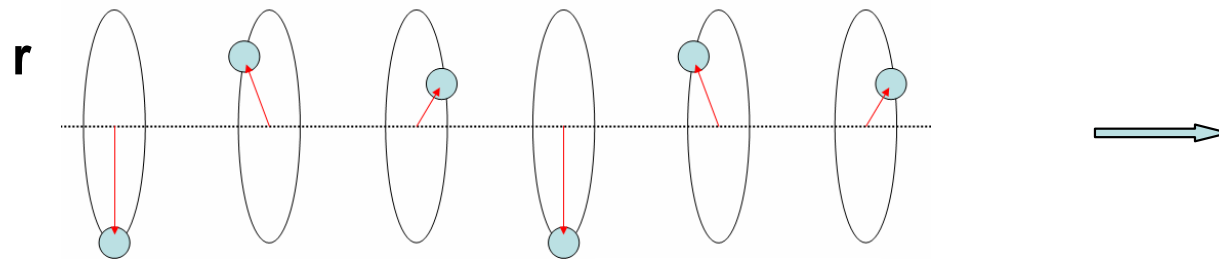
Zigzag Chain



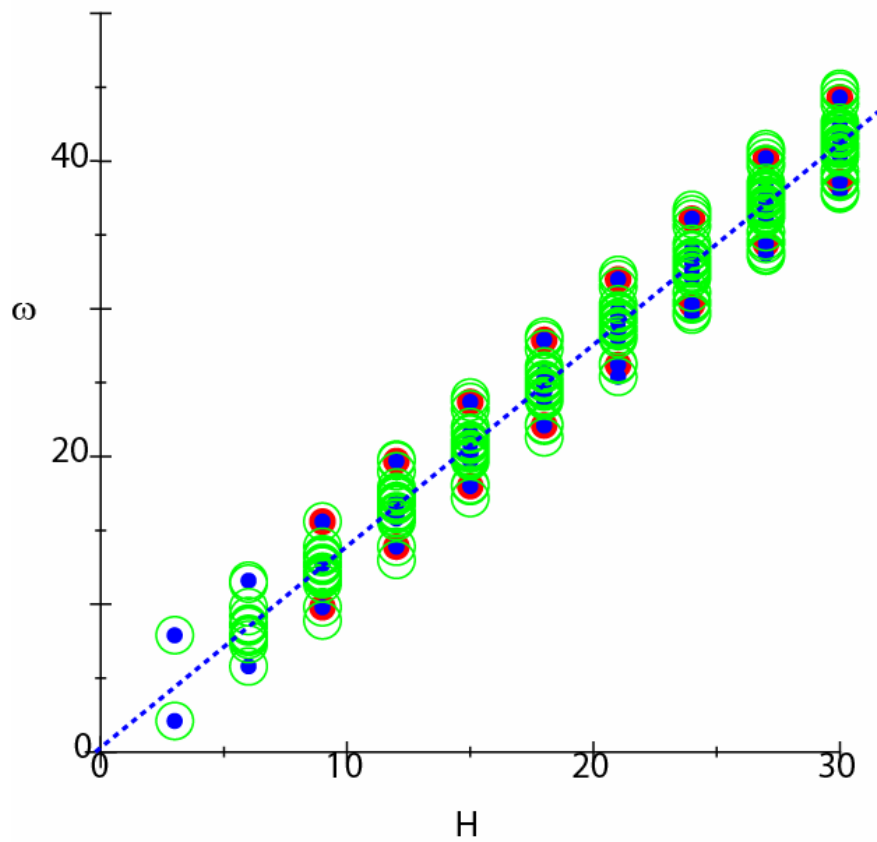
A. Ogasahara and S. Miyashita
J. Phys. Soc. Jpn. Suppl. B 72,44-52 (2003).

Spiral structure

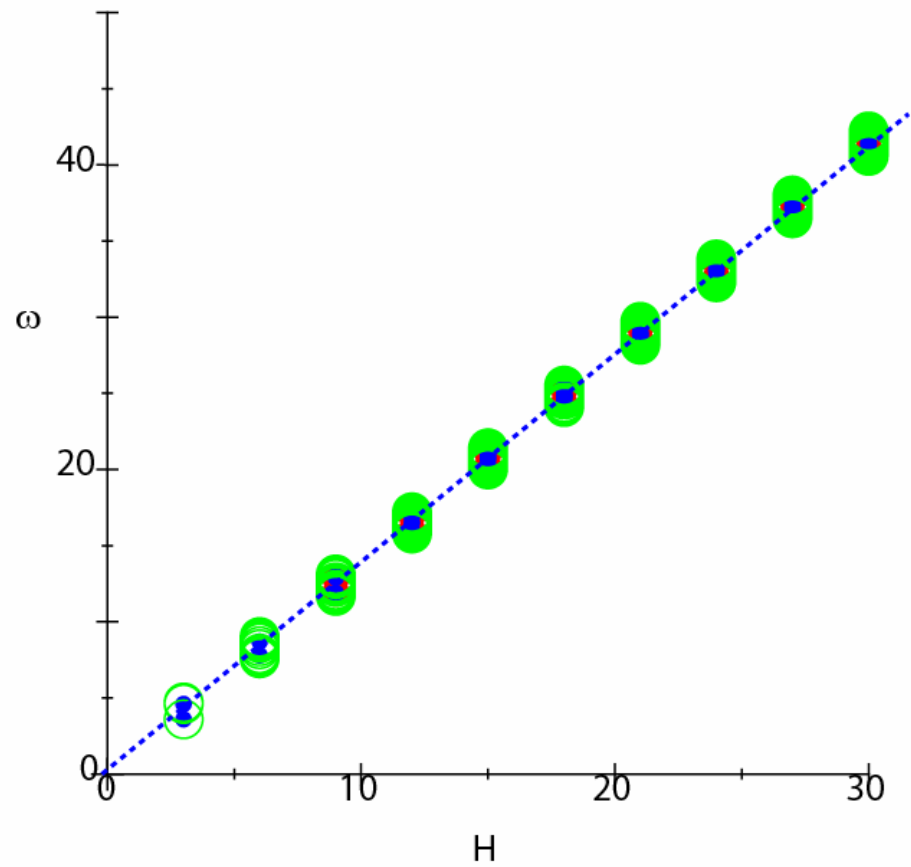
Dipole-dipole interaction



r=0.1 parallel



r=0.2

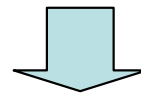


Response in dissipative dynamics

$$\chi_{xx}''(\omega) = \frac{1}{2} (1 - e^{-\beta\omega}) \int_{-\infty}^{\infty} \langle M^x(0) M^x(t) \rangle e^{-i\omega t} dt$$

pure quantum dynamics

$$M^x(t) = e^{iHt/\eta} M^x e^{-iHt/\eta}$$



quantum dynamics with dissipation

Relaxation effects:

I. Solomon: PR 99 (1955) 559.

Relaxation processes in a system of two spins

F. Bloch: PR 105 (1957) 1206.

General theory of relaxation

Y. Hamano and F. Shibata: JPSJ 51 (1982) 1727,2721,2728.

M. Saeki: Prog. Theor. Phys. 67 (1982) 1313. : relaxation method

Prog. Theor. Phys. 115 (2006) 1. : TCL method

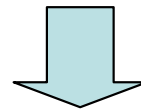
Dissipative dynamics

Quantum Master equation method

$$\chi_{xx}''(\omega) = \frac{1}{2} (1 - e^{-\beta\omega}) \int_{-\infty}^{\infty} \langle M^x(0) M^x(t) \rangle e^{-i\omega t} dt$$

↑
quantum dynamics with dissipation

$$M^x(t) = \text{Tr}_B e^{i(H_S + H_I + H_B)t/\eta} M^x e^{i(H_S + H_I + H_B)t/\eta}$$



Quantum master equation

$$\frac{d\rho}{dt} = -\frac{i}{\eta} [H, \rho] - \frac{\pi\lambda^2}{\eta} \left([X, R\rho] + [X, R\rho]^+ \right)$$

F. Bloch: PR 105 (1957) 1206.

S. Nakajima: PTP 20 (1958) 987, R. Zwanzig: J. Chem. Phys. 33 (1960) 1338.

A. G. Redfield: Adv. Magn. Reson. 1 (1965) 1.

H. Mori: PTP 33 (1965) 423. M. Tokuyama and H. Mori: PTP 55 (1976) 411.

N. Hashitsume, F. Shibata and M. Shingu: J. Stat. Phys. 17 (1977) 155 & 171.

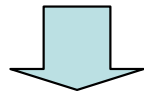
T. Arimitsu and H. Umezawa: PTP 77 (1987) 32.

Formulation of line-shape with dissipative dynamics

$$\langle A(t)A \rangle = \text{Tr} e^{iHt/\eta} A e^{-iHt/\eta} A \rho(t_0) = \text{Tr} A e^{-iHt/\eta} A \rho(t_0) e^{iHt/\eta}$$

cf. $\rho(t + t_0) = e^{-iHt/\eta} \rho(t_0) e^{iHt/\eta}$

$$\frac{\partial}{\partial t} \rho = \frac{1}{i\eta} [H, \rho] - \gamma \left([X, R(t), \rho] + [X, R(t), \rho]^+ \right) \equiv L\rho$$



$$\frac{\partial}{\partial t} [A\rho(t_0)](t) = L [A\rho(t_0)]$$

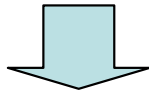
$$[A\rho(t_0)](t) = e^{Lt} [A\rho(t_0)](0)$$

**K. Saito, S. Takesue and SM.
Phys. Rev. B61 (2000) 2397.**

Eigenmode of time-evolution operator

$$\frac{\partial}{\partial t} \rho = \frac{1}{i\eta} [H, \rho] - \gamma ([X, R(t), \rho] + [X, R(t), \rho]^+)$$

matrix $\rho(i, j)$, $(i, j = 1, \Lambda, N)$



vector $\vec{\rho}$, $(\rho(k), k = 1, \Lambda, N^2)$

$$\frac{\partial}{\partial t} \vec{\rho}(t) = L \vec{\rho}(t) \quad \vec{\rho}(t) = e^{Lt} \vec{\rho}(0)$$

$$L \vec{\phi}_m = i \varepsilon_i \vec{\phi}_m$$

$$\vec{\phi}_m(t) = e^{i \varepsilon_i t} \vec{\phi}_m$$

$$\vec{\rho}(t) = \sum_m c_m e^{i \varepsilon_i t} \vec{\phi}_m$$

$$[\vec{\phi}_1, \vec{\phi}_2, \Lambda, \vec{\phi}_M] c = \vec{\rho}(0)$$

**I. Knezevic and D. K. Ferry: Phys. Rev. E66(2003) 016131,
Phys. Rev.A 69 (2004) 012104.**

S. Miyashita and K. Saito: Physica B 329-333 (2003) 1142.

Explicit form of the autocorrelation

$$[A\vec{\rho}(t_0)](t) = \sum_m c_m e^{i\varepsilon_i t} \vec{\phi}_m \quad \left[\vec{\phi}_1, \vec{\phi}_2 \Lambda \vec{\phi}_M \right] \vec{c} = [A\vec{\rho}(t_0)]$$

$$\langle A(t)A \rangle = \sum_{ik} A_{ik} \left([A\vec{\rho}(t_0)](t) \right)_{ki} = \sum_{ik} A_{ik} \sum_m c_m \left(e^{i\varepsilon_i t} \vec{\phi}_m \right)_{M(k-1)+i}$$

$$\int_0^\infty \langle A(t)A \rangle e^{-i\omega t} dt = \sum_{ik} \sum_m \frac{1 - e^{i(\varepsilon_i - \omega)\infty}}{i(\varepsilon_i - \omega)} A_{ik} c_m \left(\vec{\phi}_m \right)_{M(k-1)+i}$$

$$= -i \sum_{ik} \sum_m \frac{1}{(\varepsilon_i - \omega)} A_{ik} c_m \left(\vec{\phi}_m \right)_{M(k-1)+i}$$

Line shape

$$\chi_{AA} = \int_0^\infty \frac{i}{\eta} \left[\langle A(t)A \rangle - \langle AA(t) \rangle \right] e^{-i\omega t} dt$$

$$\int_0^\infty \langle AA(t) \rangle e^{-i\omega t} dt = -i \sum_{ik} \sum_m \frac{1}{(\varepsilon_i - \omega)} A_{ik} d_m \left(\vec{\phi}_m \right)_{M(k-1)+i}$$

$$\text{Im}(\chi_{AA}) = \text{Re} \sum_{ik} \sum_m \frac{1}{(\varepsilon_i - \omega)} A_{ik} (c_m - d_m) \left(\vec{\phi}_m \right)_{M(k-1)+i}$$

Paramagnetic Resonance

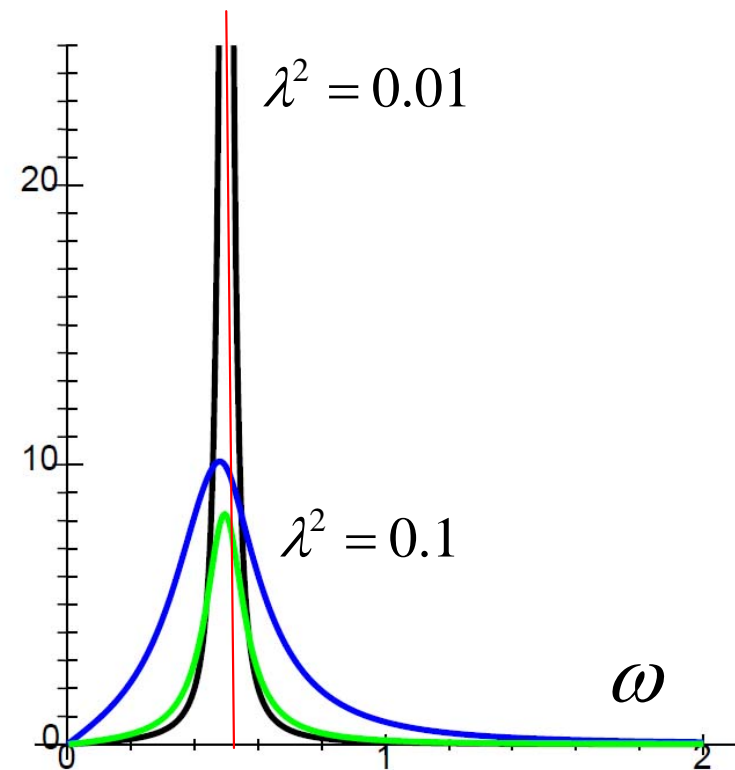
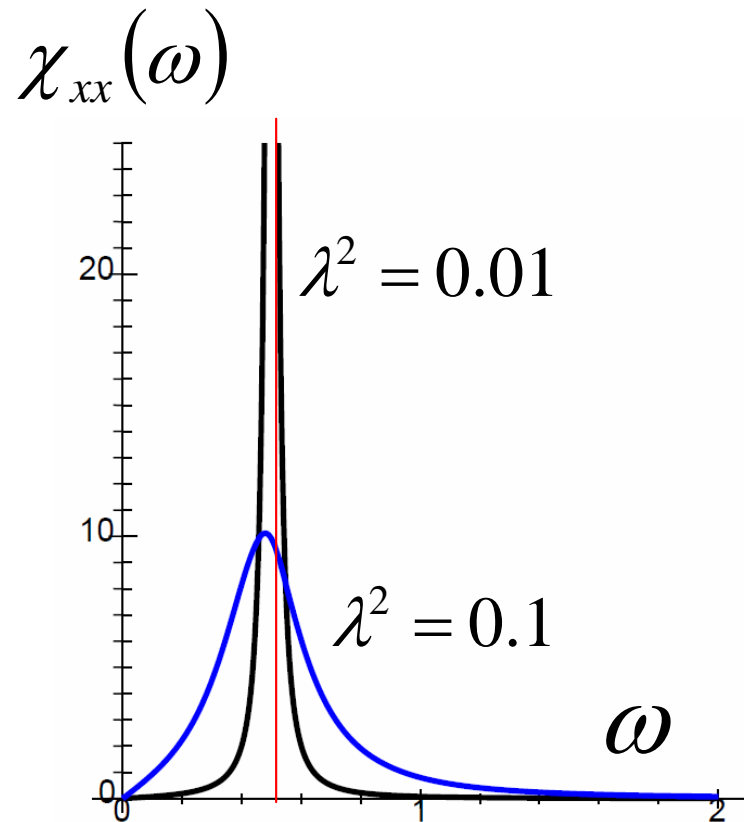
$$H = -H \sum_i S_i^z$$

$$X = \sum_i \alpha_i S_i^x + \beta_i S_i^y + \gamma_i S_i^z$$

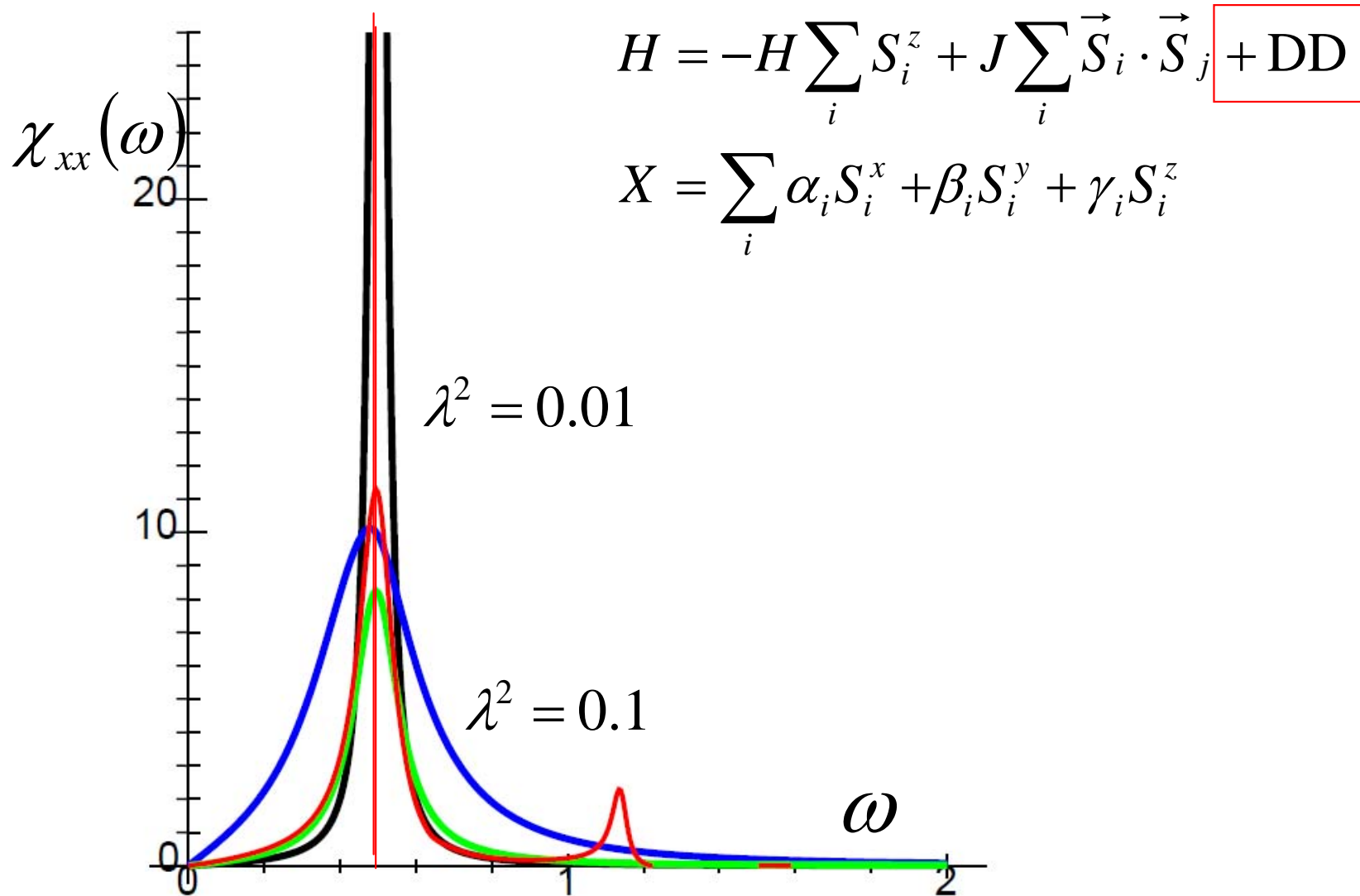
$$\Phi(\omega) = \frac{I_0 \omega^2}{|e^{\beta\omega} - 1|} = e^{-\beta\omega} \Phi(\chi_{xx} \omega)(\omega)$$

Exchange narrowing

$$H = -H \sum_i S_i^z + J \sum_i \vec{S}_i \cdot \vec{S}_j$$



Dipole-dipole interaction

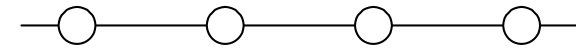


(Motional narrowing) Quantum narrowing effect

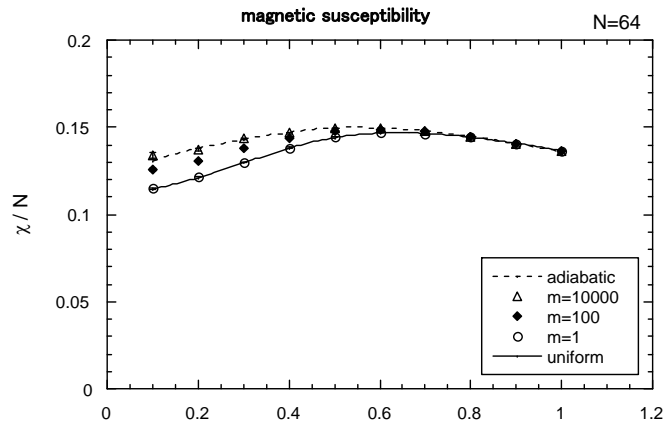
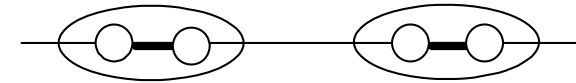
Spin-Peierls systems

$$H = \sum_{i=1}^N J [1 + \alpha(u_{i+1} - u_i)] S_i \cdot S_{i+1} + \sum_{i=1}^N \left[\frac{1}{2m} p_i^2 + \frac{k}{2} (u_{i+1} - u_i)^2 \right]$$

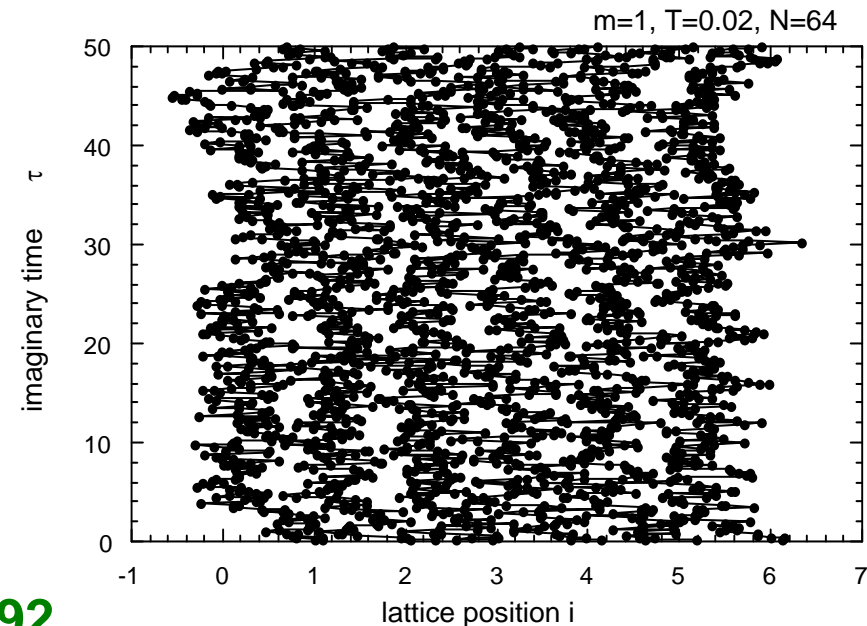
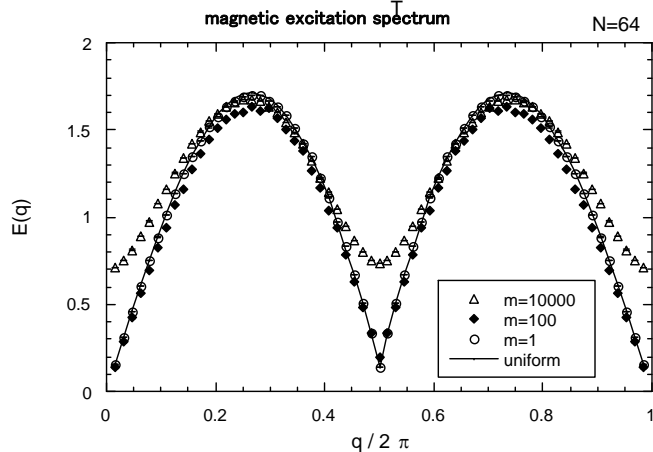
uniform



dimerization



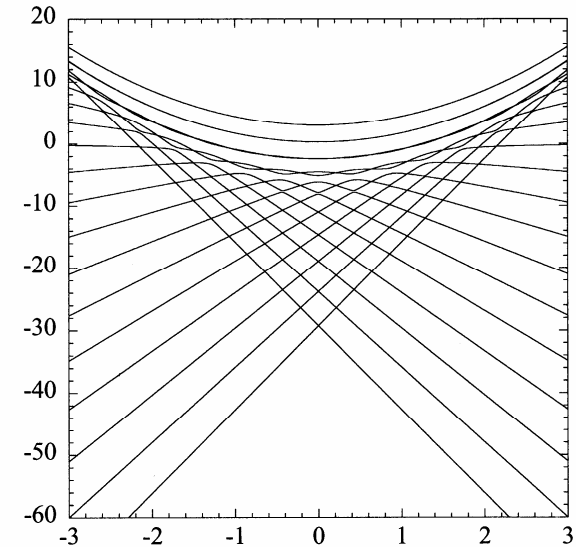
◆ effects of quantum lattice fluctuation becomes small when m small



Origin of the adiabatic change

S: even Large S (S=10) Mn12, Fe8

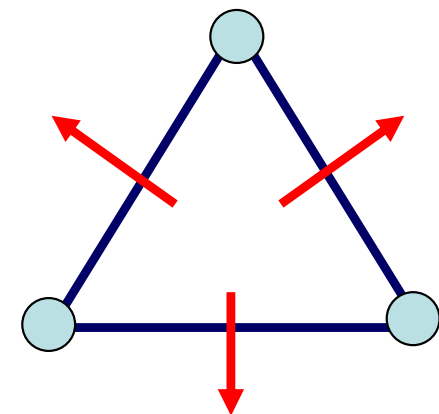
$$\begin{aligned}
 H = & -D(S^z)^2 - hS^z \\
 & + E((S^x)^2 - (S^y)^2) \\
 & + C((S^+)^4 - (S^-)^4) + \text{etc.}
 \end{aligned}$$



S: odd (S=1/2) V15 No anisotropy & Kramers doublet

Dzyloshinskii-Moriya interaction

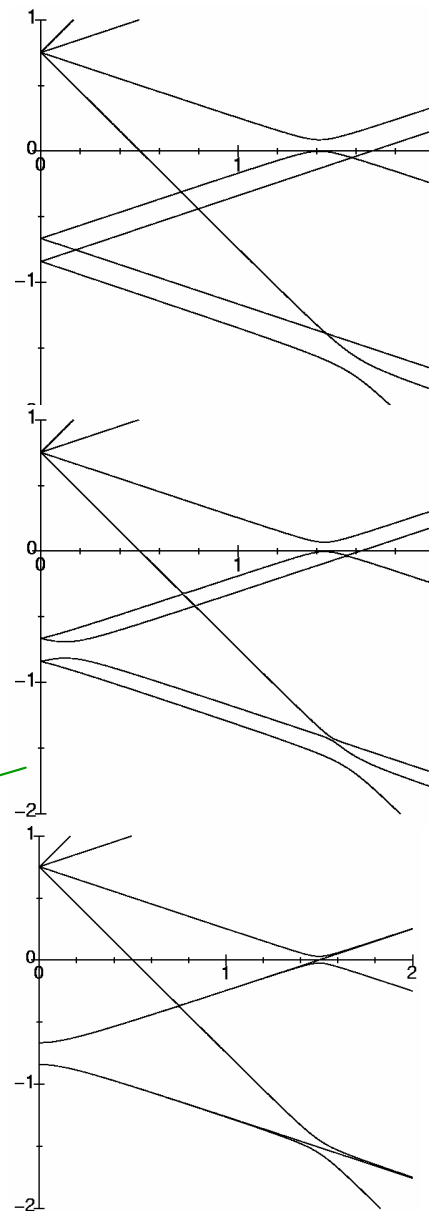
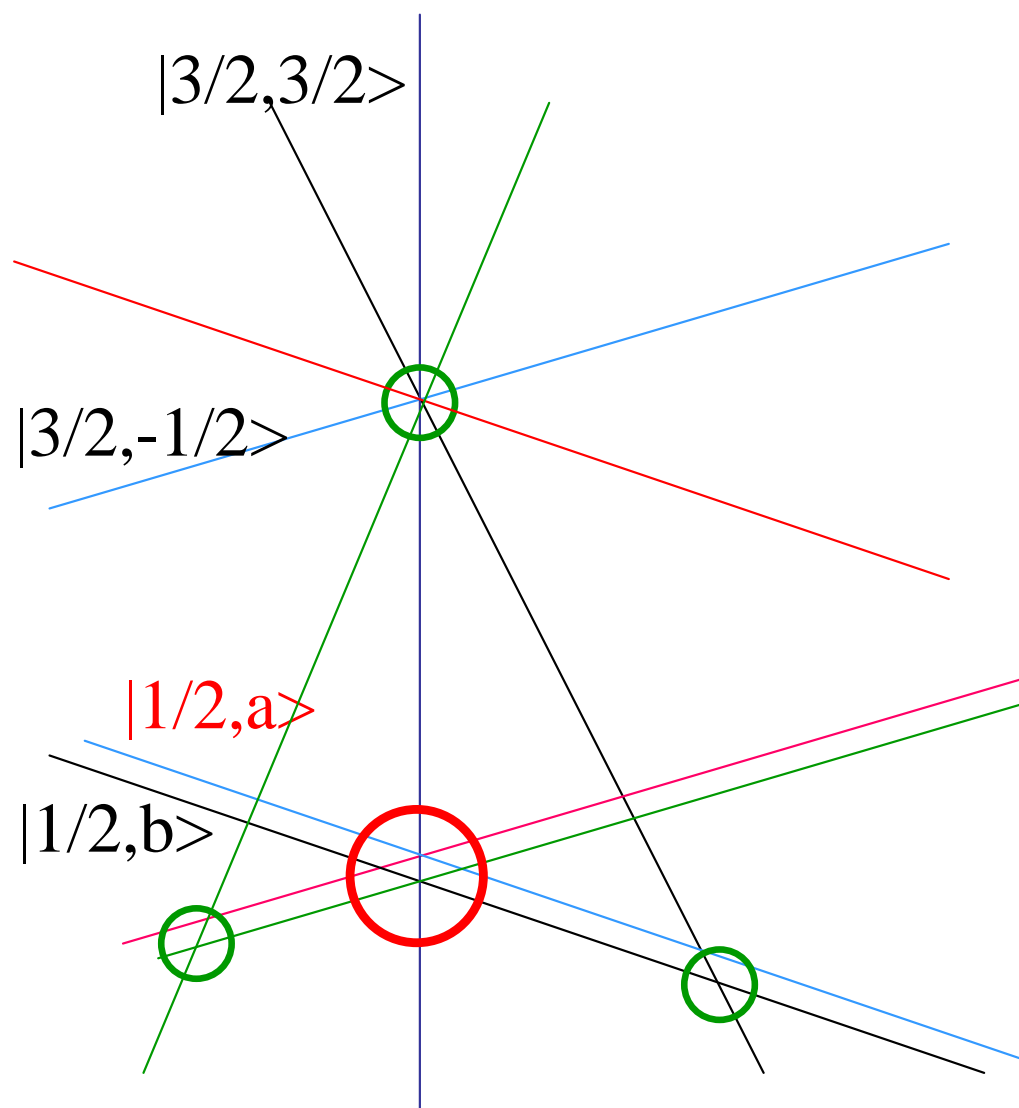
$$H_D = \sum_{ij} \vec{D}_{ij} (\vec{S}_i \times \vec{S}_j)$$



SM, & N. Nagaosa, Prog. Theor. Phys. 106 (2001) 533

Energy structure with DM

Anisotropy of DM interaction



$$\theta = 0^\circ$$

No adiabatic change
at $H=0$

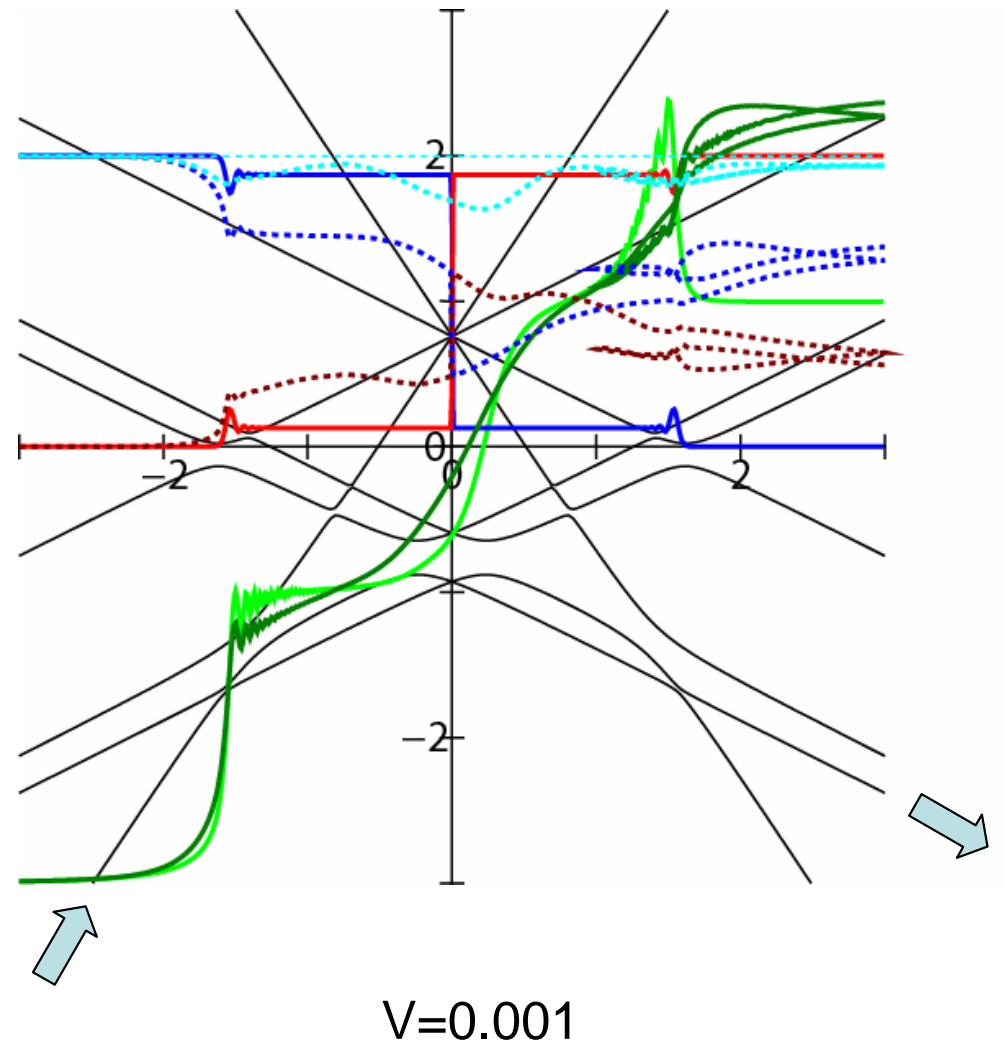
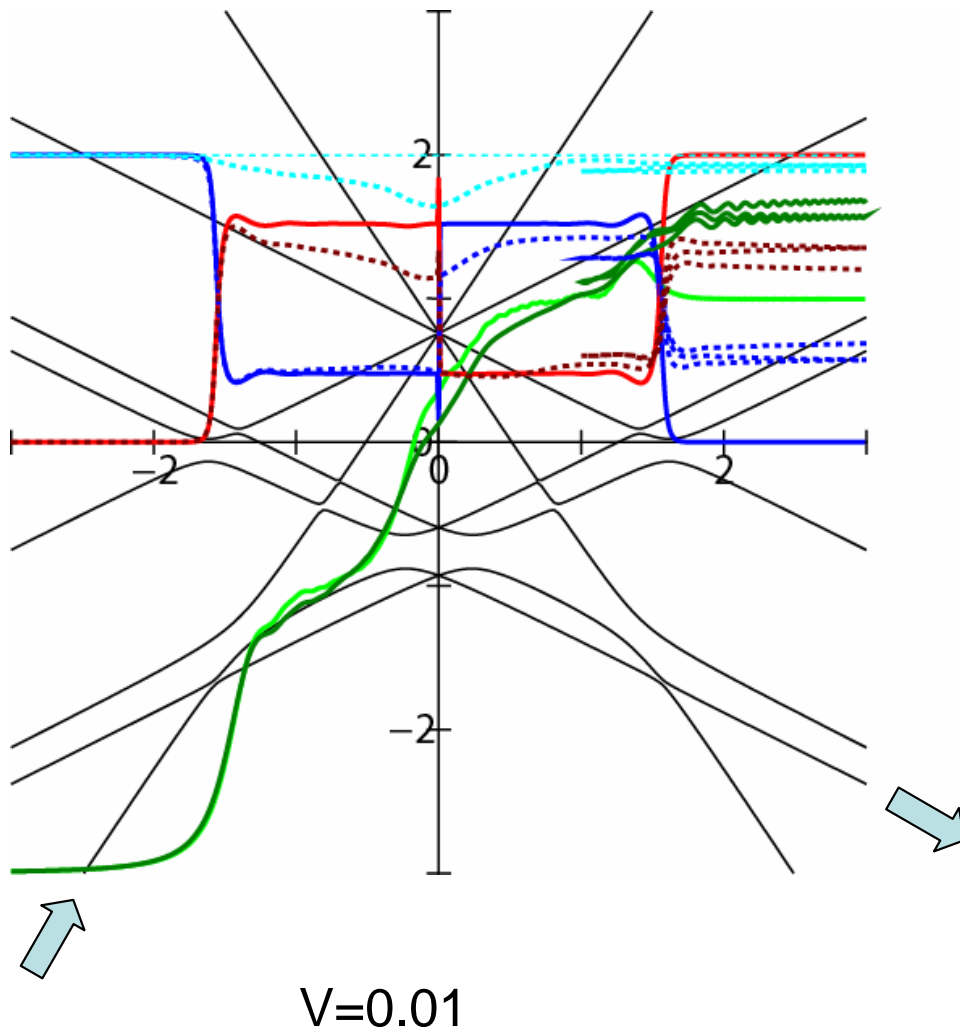
$$\theta = 45^\circ$$

$$\theta = 90^\circ$$

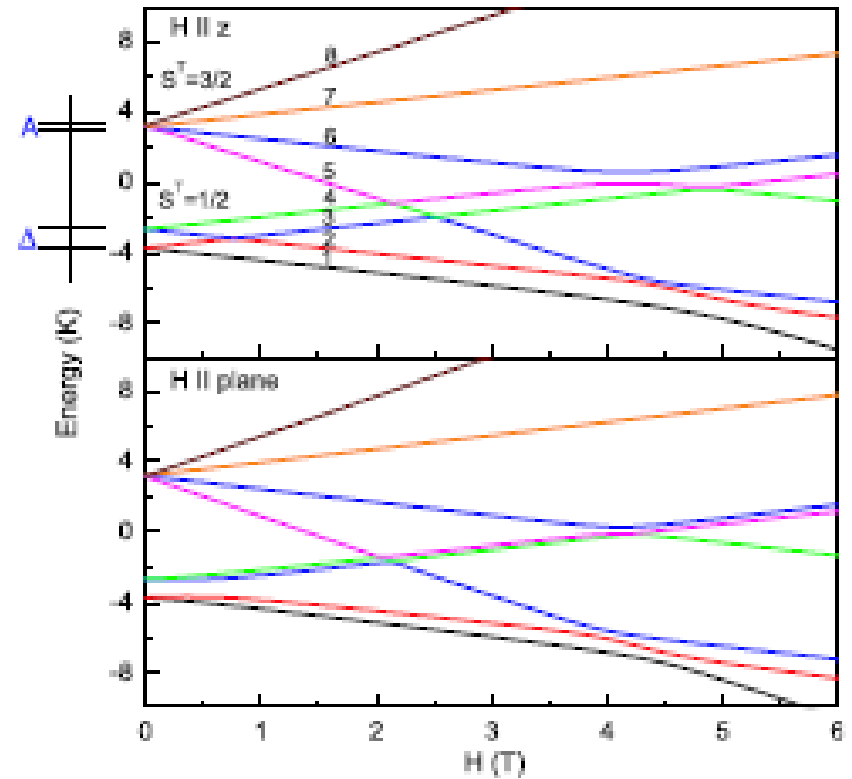
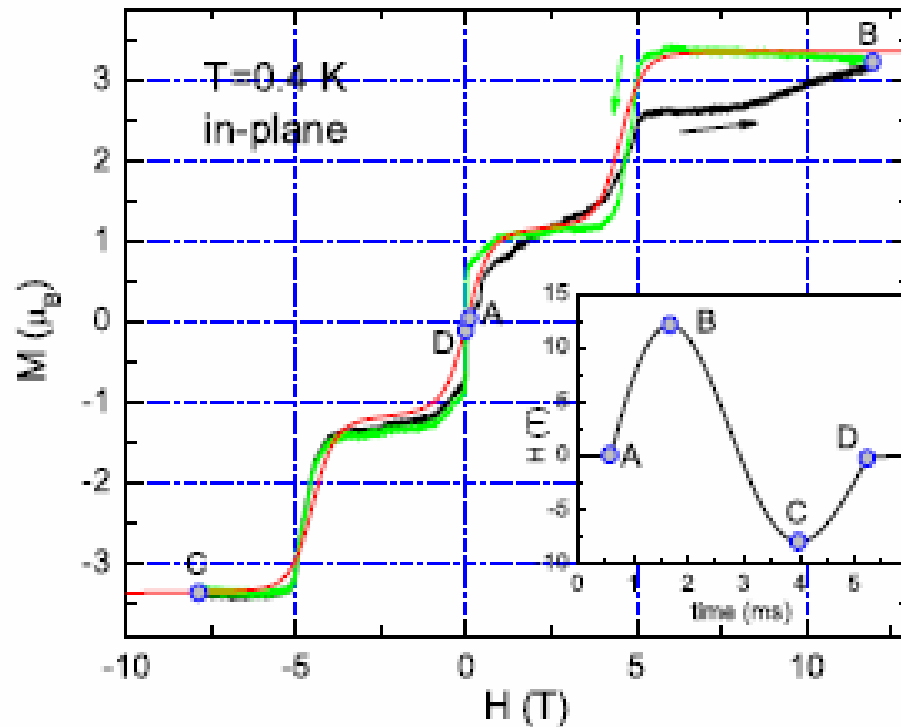
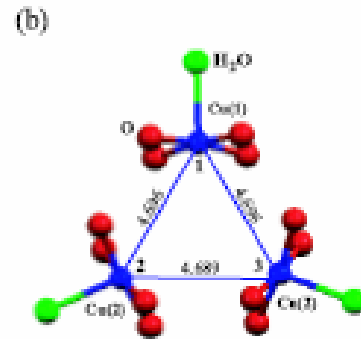
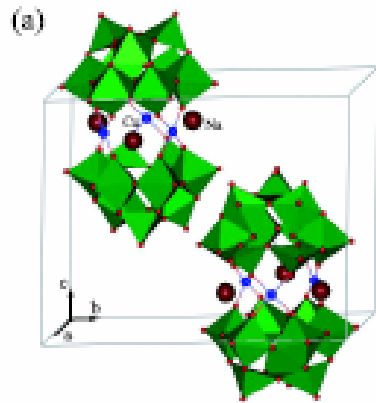
I.Chiorescu, W. Wernsdorfer, A. Mueller, SM, and B. Barbara:
PRB 67 (2003) 020402

H. De Raedt, SM, K. Michielsen & M. Machida: PRB 70 (2004) 064401

Nontrivial coherence



Cu3



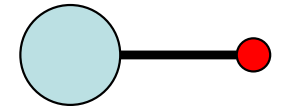
K.Y. Choi, et al. PRL 96 (2006) 107202

$$\mathcal{H} = \sum_{l=1}^3 \sum_{\alpha=x,y,z} J_{ll+1}^{\alpha} \mathbf{S}_l \cdot \mathbf{S}_{l+1} + \sum_{l=1}^3 \mathbf{D}_{ll+1} \cdot [\mathbf{S}_l \times \mathbf{S}_{l+1}] + \mu_B \sum_{l=1}^3 \mathbf{S}_l \cdot \mathbf{g}_{ll} \cdot \mathbf{H}_l$$

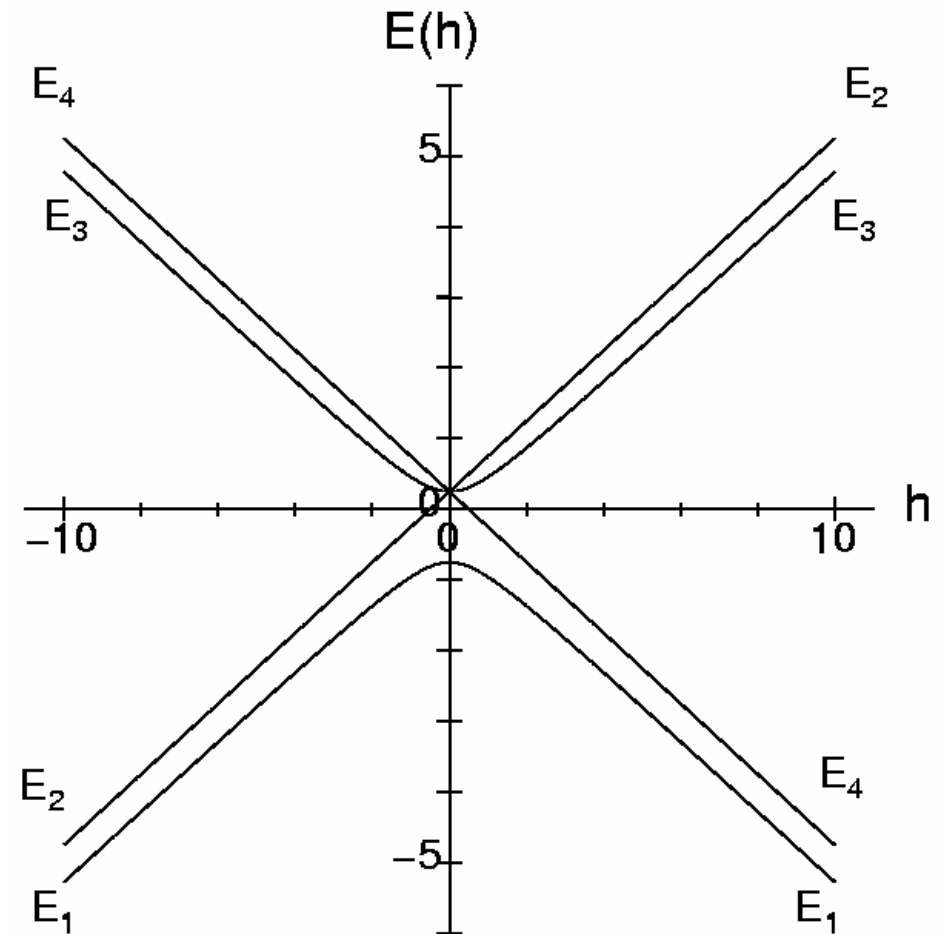
H. De Raedt, SM, K. Michielsen, M. Machida: PRB 70 (2004) 064401

Directionally independent energy gap due to Hyperfine interaction

$$H = -A\vec{S} \cdot \vec{\sigma} - (g\mu_B S^z + g'\mu_{NB}\sigma^z)h$$



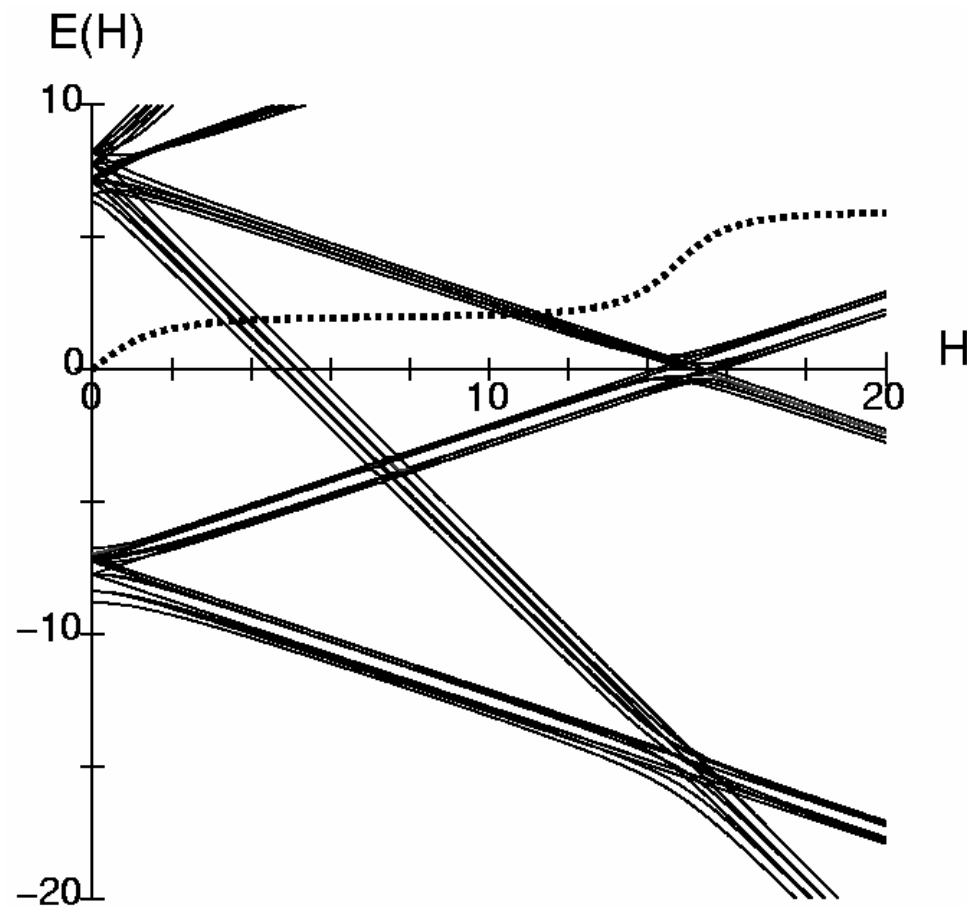
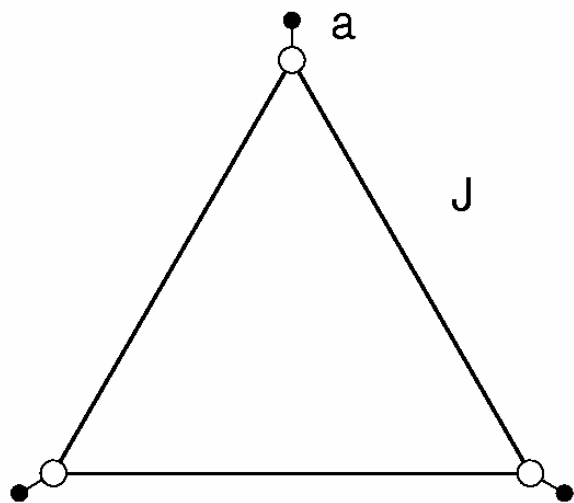
$$\begin{aligned} E_1 &= -\frac{1}{4}a - \frac{1}{2}\sqrt{a^2 + h^2} \\ E_2 &= \frac{1}{4}a + \frac{1}{2}h \\ E_3 &= -\frac{1}{4}a + \frac{1}{2}\sqrt{a^2 + h^2} \\ E_4 &= \frac{1}{4}a - \frac{1}{2}h \end{aligned}$$



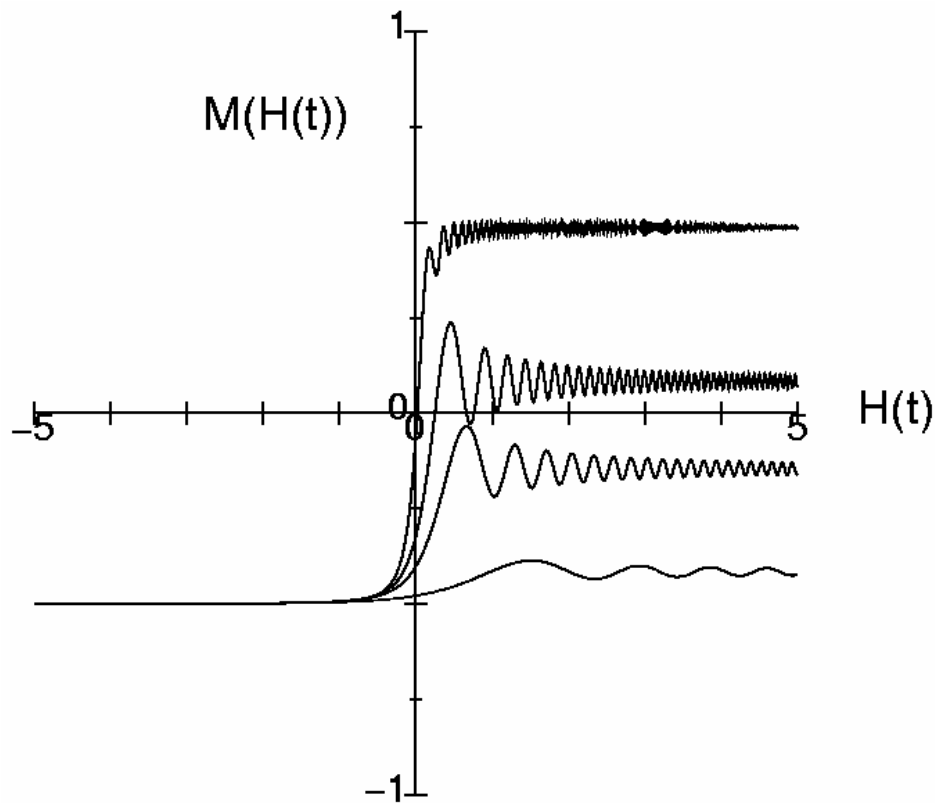
SM, H.de Raedt and K. Michielsen:
Prog. Thor. Phys.. 110 (2003) No.11

Triangle case

$$H = -A\vec{S} \cdot \vec{\sigma} - (g\mu_B S^z + g' \mu_{NB} \sigma^z)h$$



M(t) from the ground state



$$P = \left| \langle G(H_0) | \Psi(t_f) \rangle \right|^2$$

$$\Delta E = \sqrt{-2\nu \log(1-P) / \pi}$$

ν	P	ΔE
0.500	0.08371	0.16681
0.250	0.16067	0.16697
0.100	0.35481	0.16702
0.050	0.58378	0.16704
0.025	0.82678	0.16704
0.010	0.98751	0.16704

Apparent LZS relation

Gap control using hidden symmetries

Transverse field

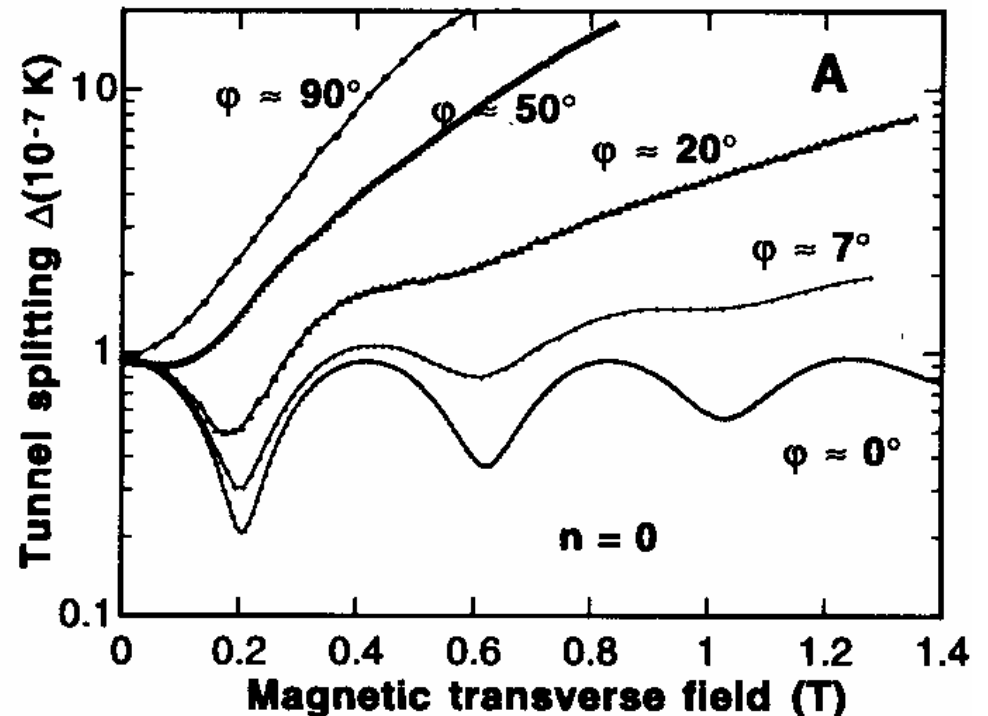
$$H = -DS_z^2 - h_x S_x \Rightarrow \Delta E \propto h_x^{|m-m'|}$$

Nontrivial control

$$H = -DS_z^2 + E(S_x^2 - S_y^2) + C\left((S^+)^4 + (S^-)^4\right) - h_x S_x$$

Quantum interference Berry phase

W. Wernsdorfer & R. Sessoli:
Science 284 (1999) 133



Non-monotonic gap due to H_x

$$H = -DS_z^2 + 2E(S_x^2 - S_y^2) + C(S_+^4 + S_-^4) - \vec{H}(t) \cdot \vec{S}$$

$$E = 0.5D, \quad C = 0$$

$$H = -D(S_z^2 - S_x^2 + S_y^2) - H_x S_x$$

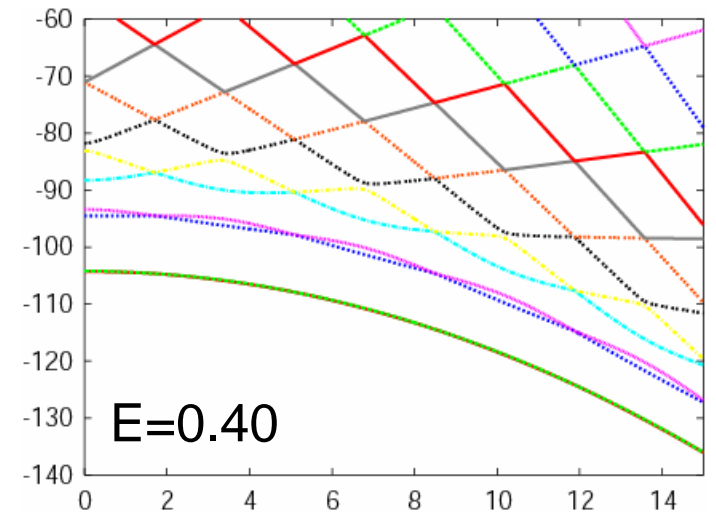
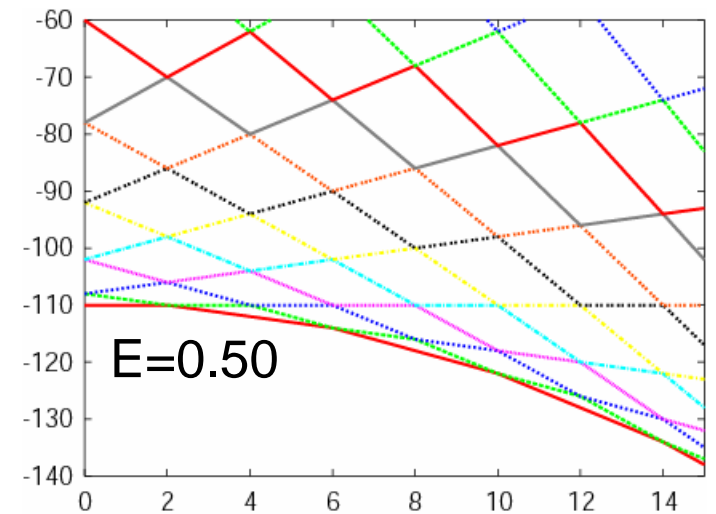
$$= 2DS_x^2 - H_x S_x - DS(S+1)$$

$$E = (0.5 + \delta)D, \quad C = 0$$

$$H = 2(D + \delta)S_x^2 - H_x S_x - 2\delta S_y^2$$

$$\Delta M_x = 0, \pm 2$$

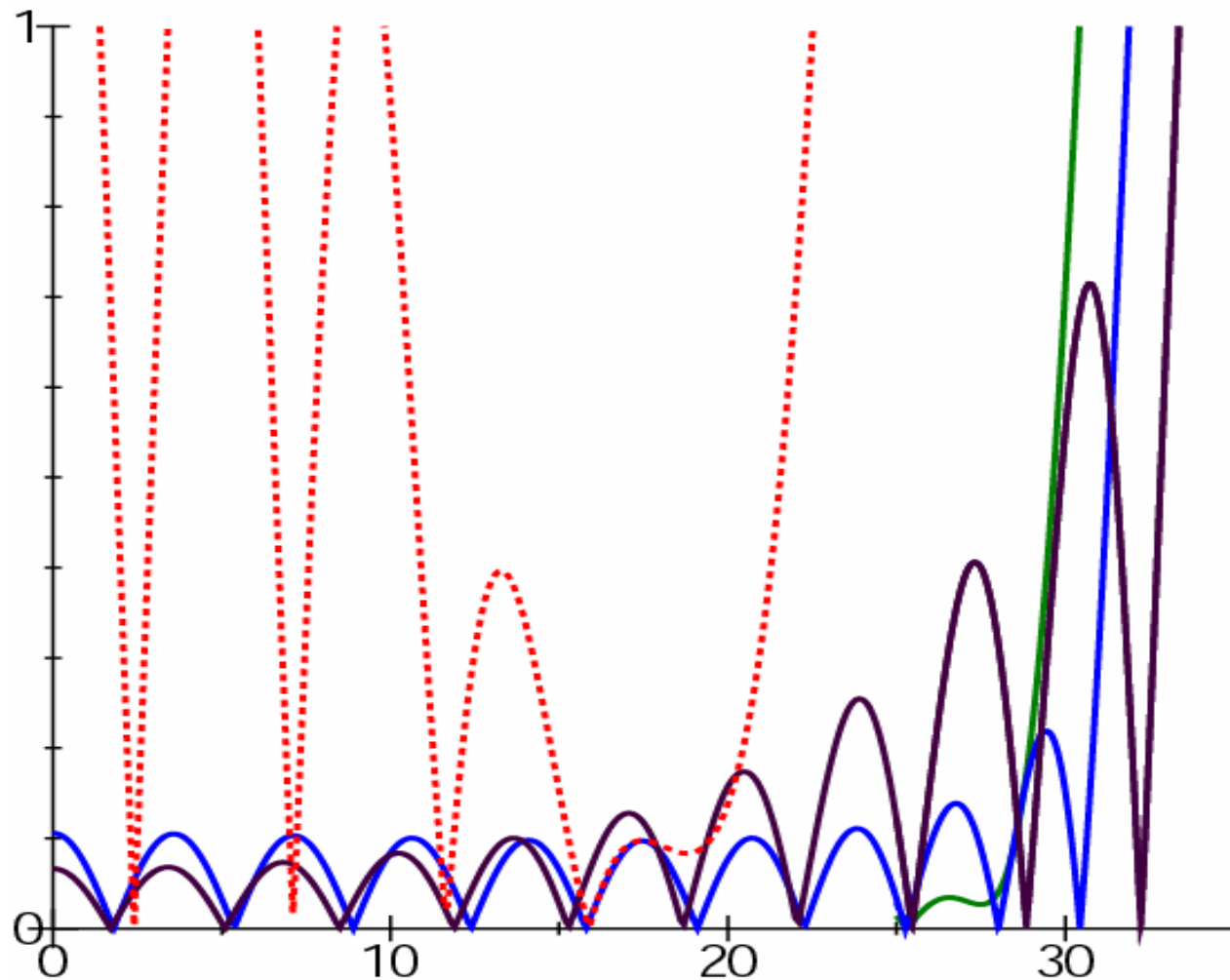
Gaps open at
crossing remain at $(M, M+1)$ etc. \Rightarrow $2S$ crossings



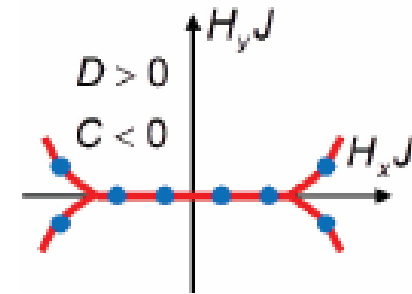
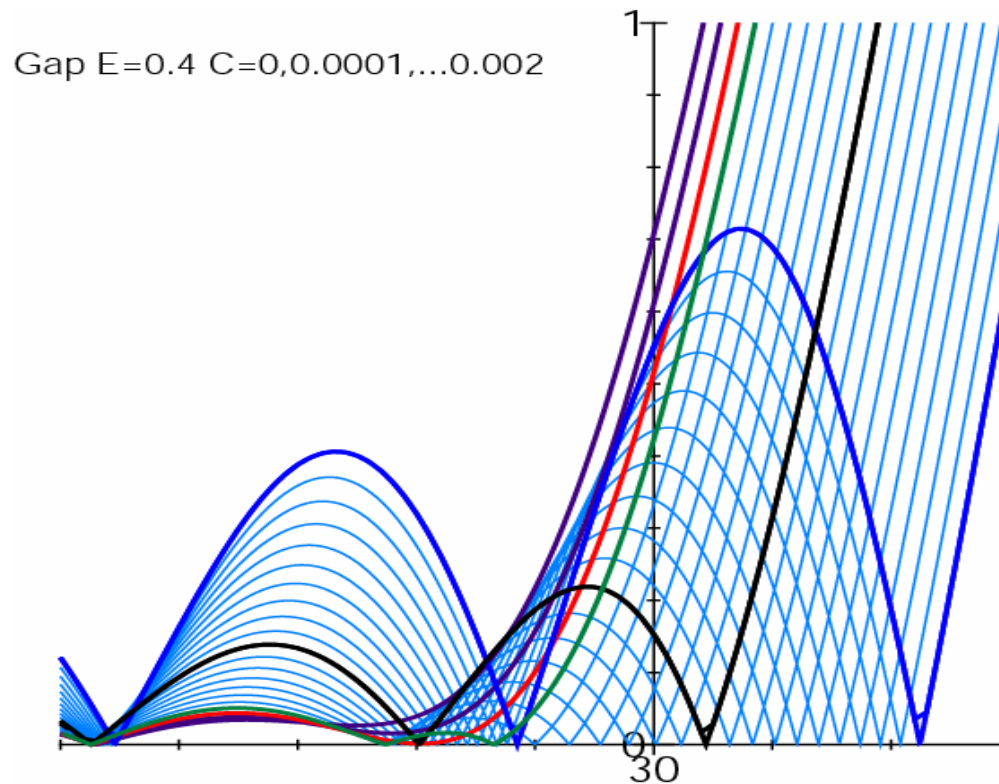
Gap with the C term

$$H = -DS_z^2 + 2E(S_x^2 - S_y^2) + C(S_+^4 + S_-^4) - \vec{H}(t) \cdot \vec{S}$$

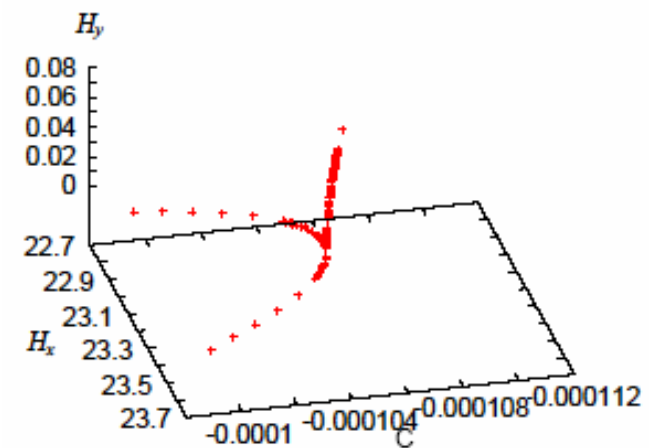
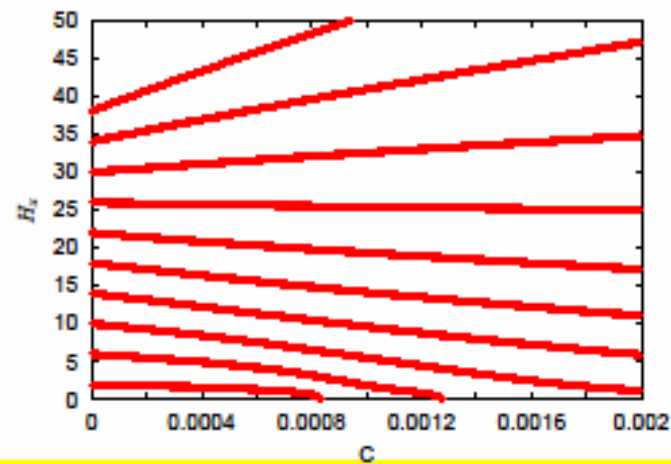
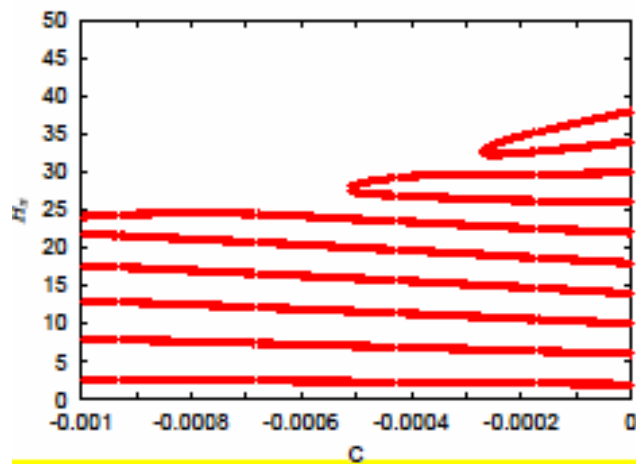
Gap $E=0.4$ $C=0, 0.0001, \dots, 0.01$



Collapse of degeneracy?



P. Bruno: PRL 96
(2006) 117208



Temporal symmetry-breaking induced DM interaction

NaV₂O₅ : charge fluctuation reduces the symmetry
=> virtual DM ESR

Nojiri, et al.: JPSJ 69 (2000) 2291

Fe₁₂ : configuration fluctuation reduces the symmetry
=> virtual DM M(H)

H. Nakano and SM: JPSJ 71 (2002) 2580

SrCu₂(BO₃)₂ : configuration fluctuation reduces
the symmetry => Raman, ESR

Cepas and Zimann cond-mat 0401240

SM & Ogasahara: JPSJ 72 (2003) 2350

**Charge transfer, Phonon,
Orbital degree of freedom, etc.**

Fluctuating DM interaction model

$$H = J \vec{S}_1 \cdot \vec{S}_2 + \vec{d} \cdot (\vec{S}_1 \times \vec{S}_2) + H(S_1^z + S_2^z) + \frac{k}{2} x^2 + \frac{1}{2m} p^2$$

$$\vec{d} = \vec{d}_0 x$$

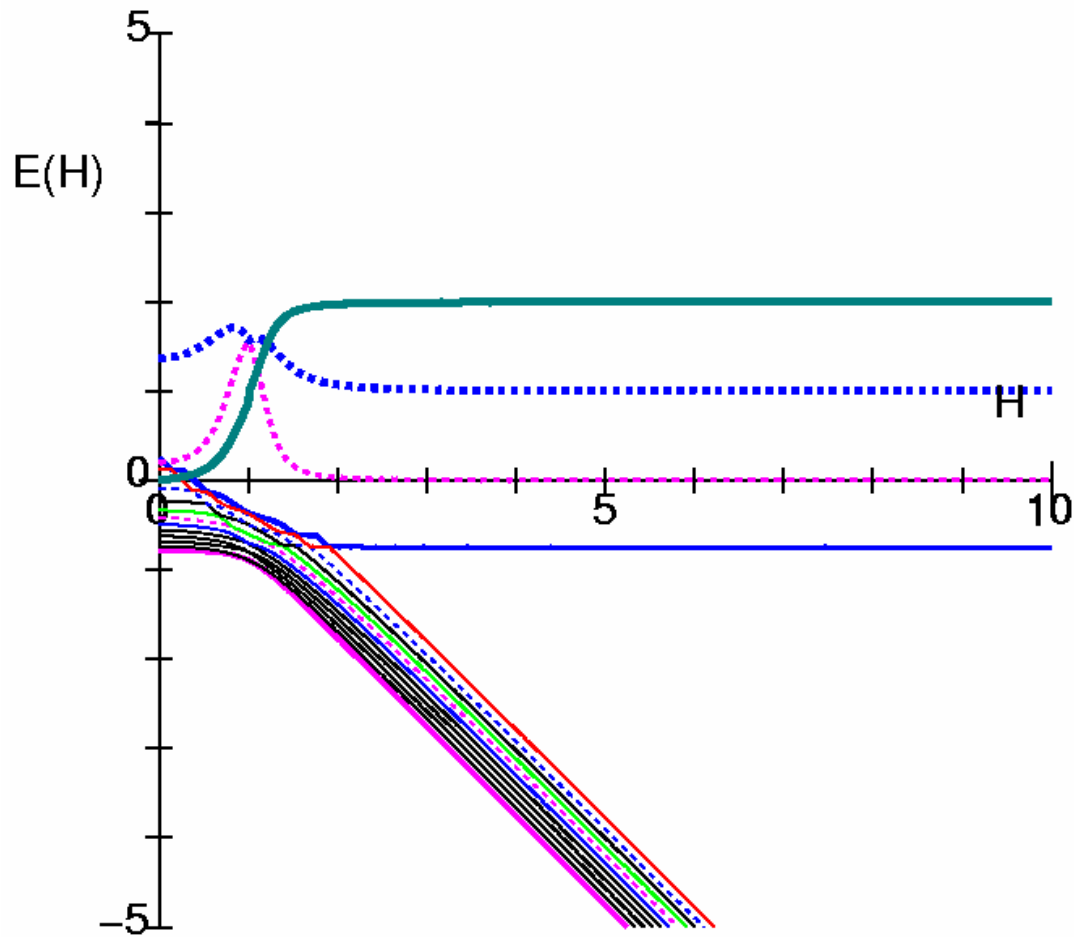
$$[x, p] = i\eta$$

$$\langle x \rangle = 0$$

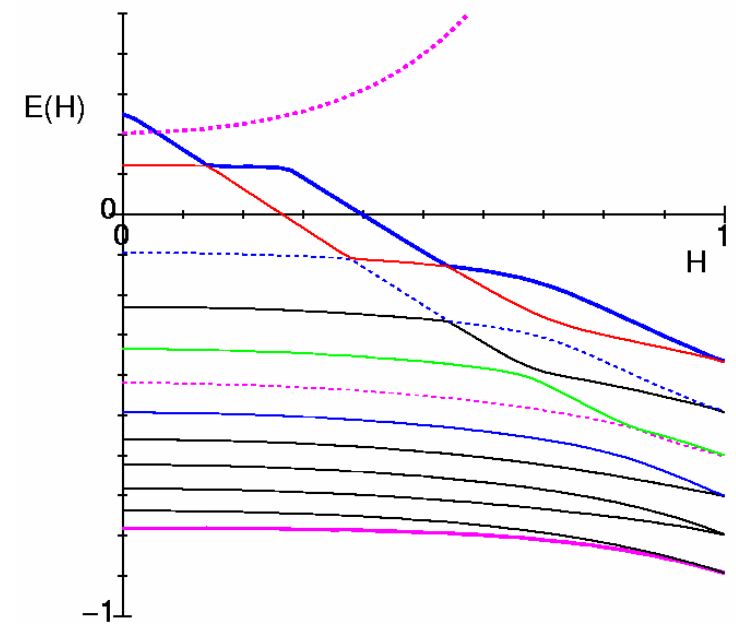
$$\langle \text{singlet} | e^{iH} | \text{triplet} \rangle \neq 0 \quad ?$$

Smooth magnetization process

$m=10, \omega=0.1, D_x=0.1$

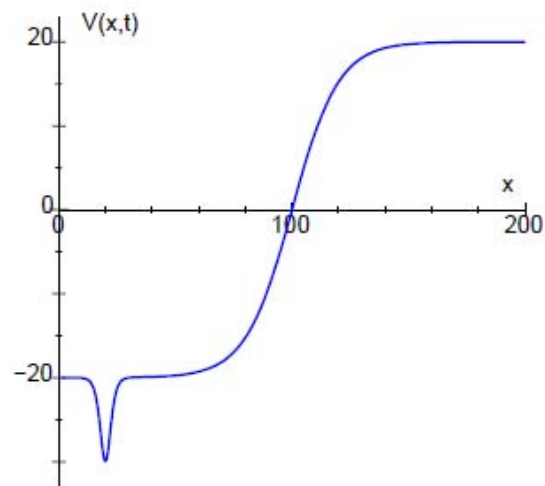


$m=10, \omega=0.1, D_x=0.1$

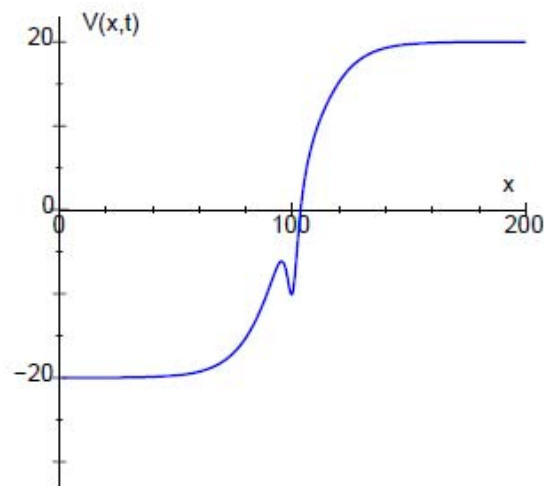


ポテンシャル移動による粒子運搬における量子効果

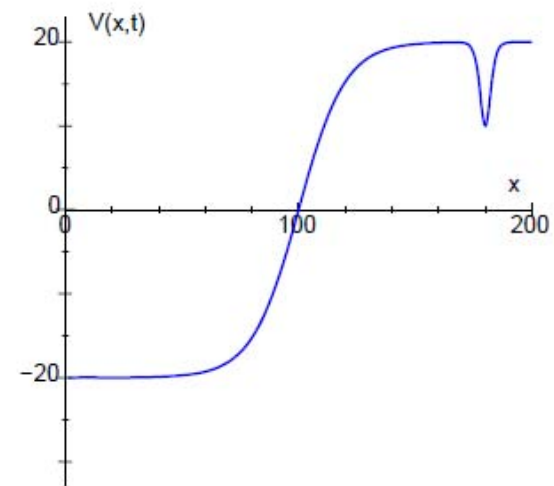
Particle trap by potential well --quantum dynamics for particle conveyance--



(a)



(b)



(c)

S. Miyashita,
Conveyance of quantum particles by a moving potential-well
J. Phys. Soc. Jpn. {¥bf 76} (2007) 104003.

Eigenstates in moving frame

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \mathcal{H}(t) \Psi(x, t) \quad \mathcal{H}(t) = \frac{p^2}{2m} + V(x - ct)$$

$$e^{ipct/\hbar} x e^{-ipct/\hbar} = x + ct$$

$$e^{ipct/\hbar} \mathcal{H}(t) e^{-ipct/\hbar} = \frac{p^2}{2m} + V(x) \equiv \mathcal{H}_0$$

$$\Phi(x, t) = e^{ipct/\hbar} \Psi(x, t)$$

$$i\hbar \frac{\partial}{\partial t} \Phi(x, t) = (\mathcal{H}_0 - cp) \Phi(x, t)$$

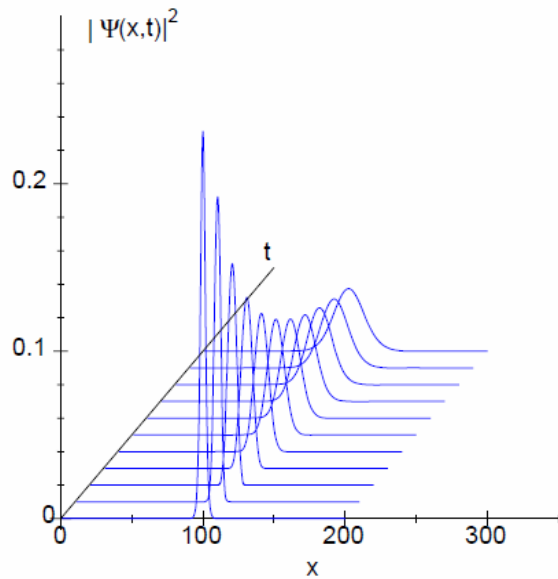
$$\mathcal{H}_0 - cp = \frac{1}{2m}(p - mc)^2 - \frac{1}{2}mc^2 + V(x)$$

$$e^{icmx/\hbar} p e^{-icmx/\hbar} = p - cm \quad f(x, t) \equiv e^{-icmx/\hbar} \Phi(x, t)$$

$$i\hbar \frac{\partial}{\partial t} f(x, t) = \left(\mathcal{H}_0 - \frac{1}{2}mc^2 \right) f(x, t)$$

Sweep velocity dependence (flat)

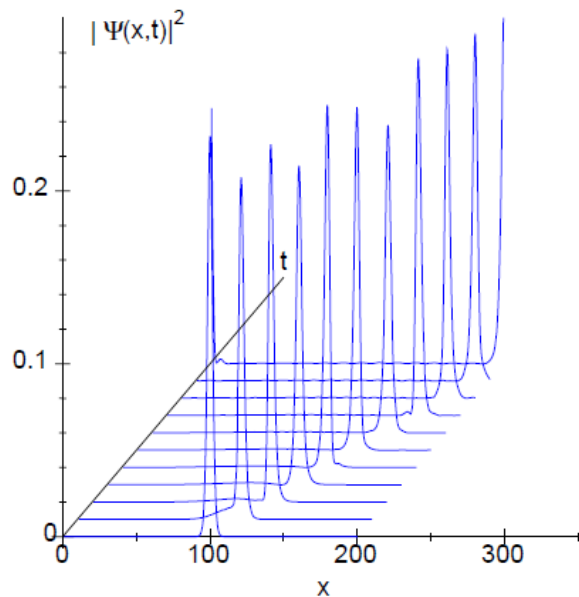
fast



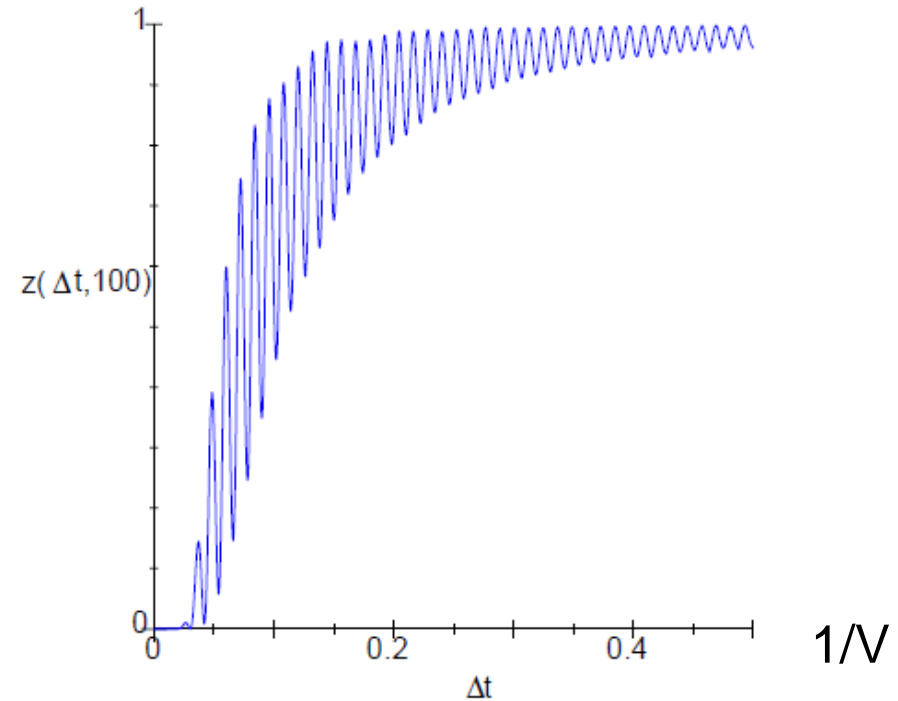
$$\Psi(x, t) = e^{-ictp/\hbar} e^{icmx/\hbar} e^{-i\mathcal{H}_0 t/\hbar} e^{-icmx/\hbar} |0\rangle$$

$$|\langle G(t) | \Psi(x, t) \rangle|^2 = \left| \langle 0 | e^{icmx/\hbar} e^{-i\mathcal{H}_0 t/\hbar} e^{-icmx/\hbar} | 0 \rangle \right|^2$$

Slow

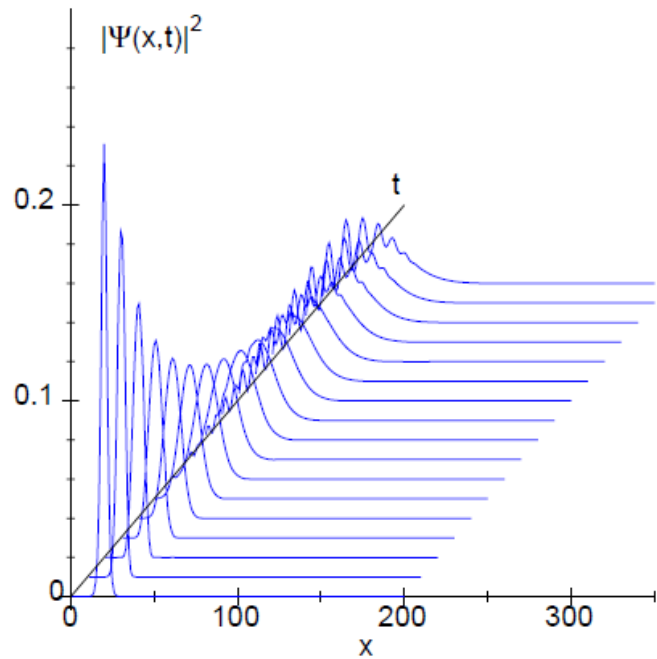


Trap probability

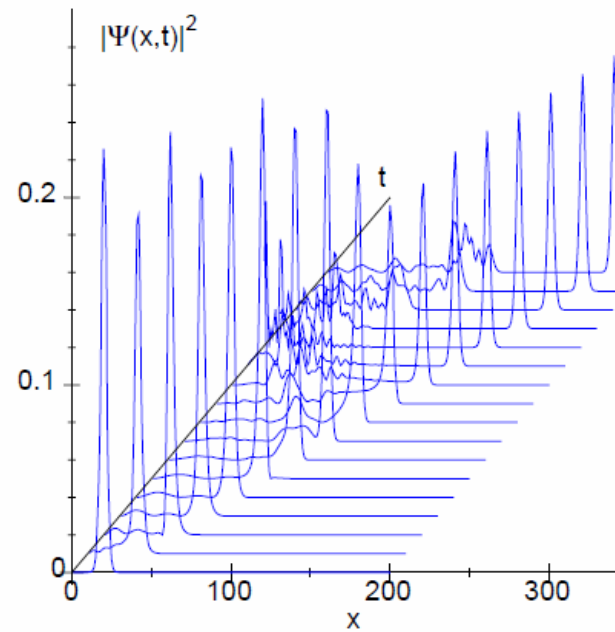


Sweep velocity dependence (carry-up)

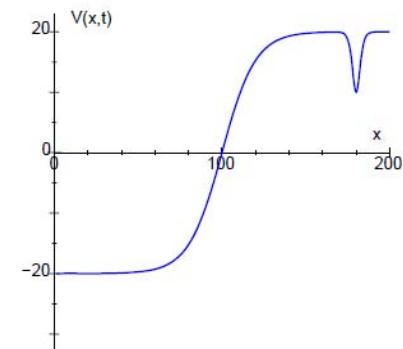
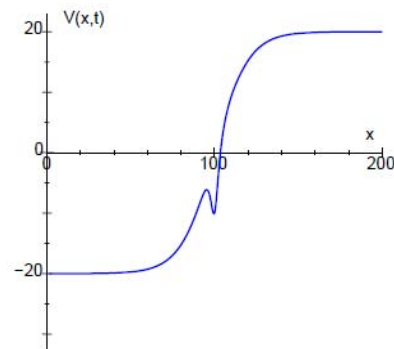
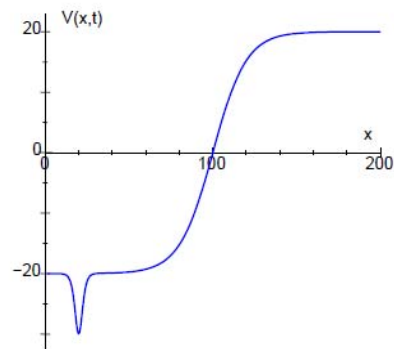
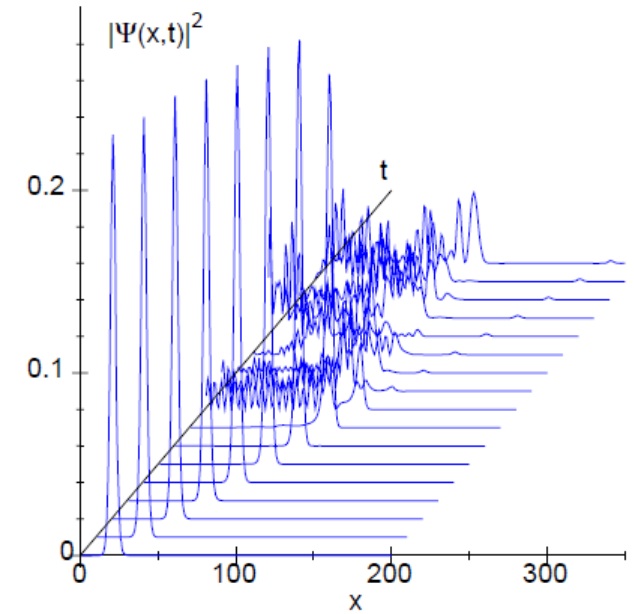
fast



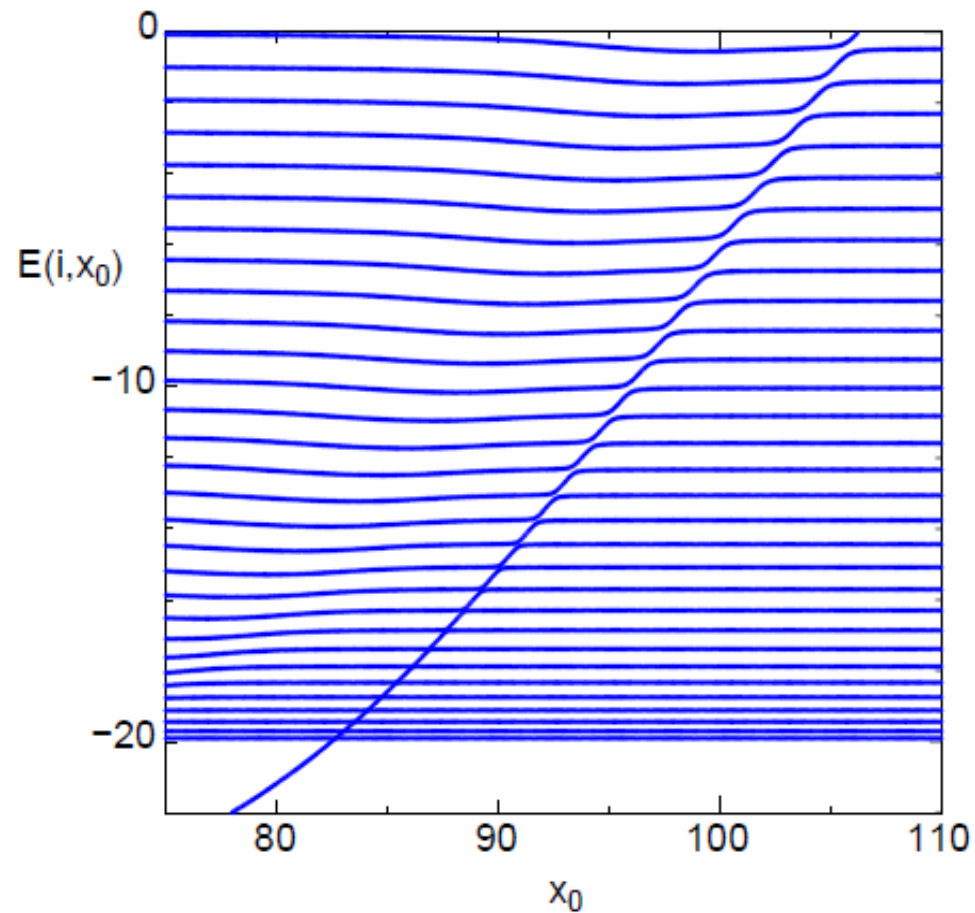
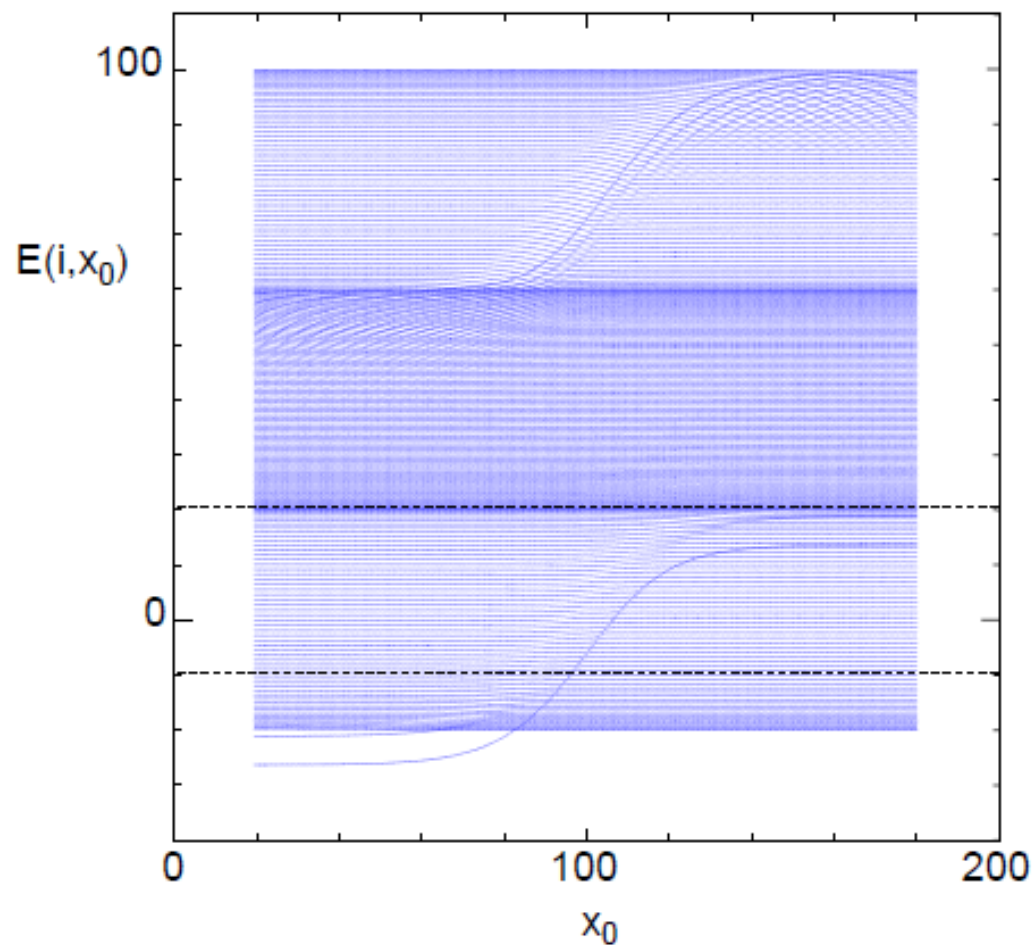
medium



slow

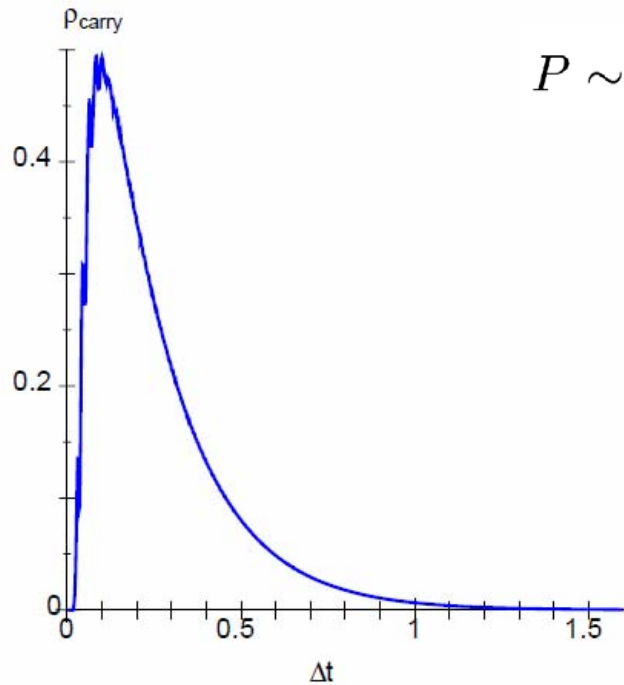


Adiabatic energy level as a function of the potential well

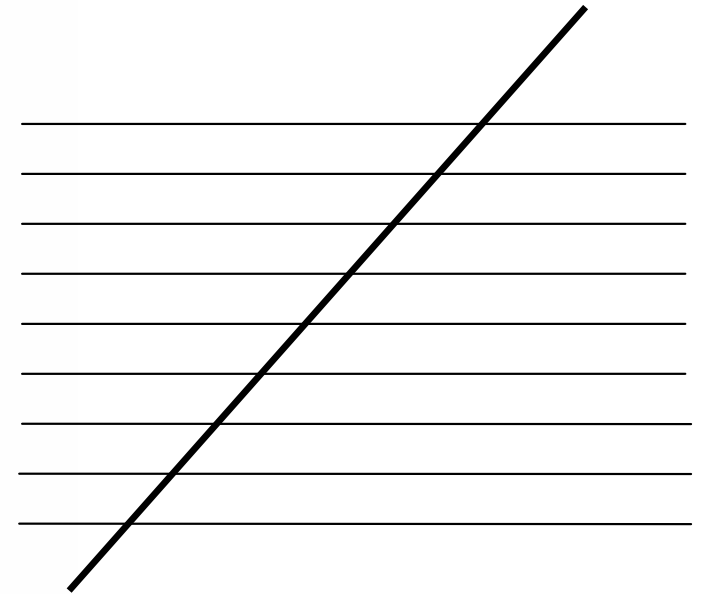
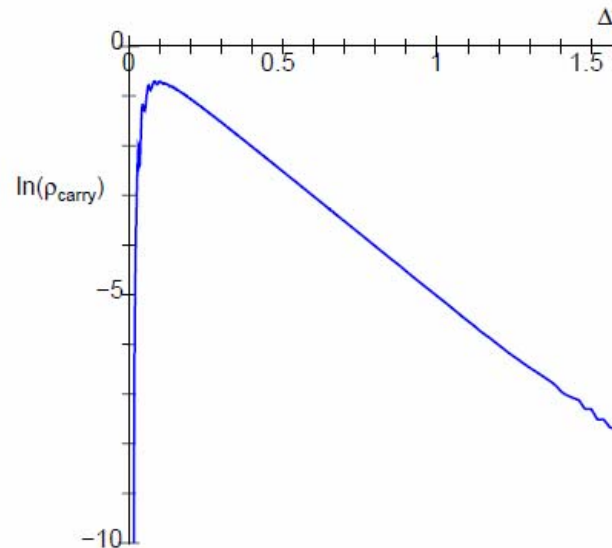


Successive Landau-Zener scattering

Adiabatic trap vs. tunneling



$$P \sim \exp(-\gamma/c) = \exp(-\gamma\Delta t)$$



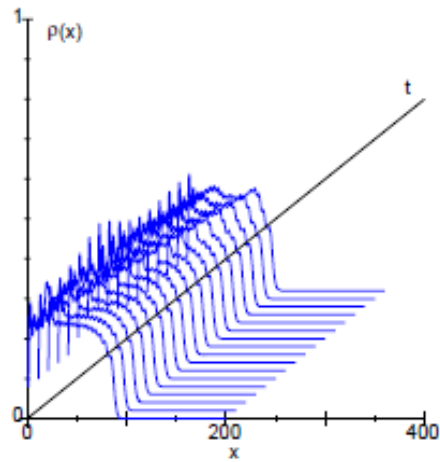
$$\rho_{\text{carry}} = \int_{x_0 - 2d_{\text{pw}}}^{x_0 + 2d_{\text{pw}}} |\Psi(x)|^2 dx$$

$$P = \exp\left(-\frac{2\pi}{\hbar} \sum_i J_i^2 |v|^{-1}\right)$$

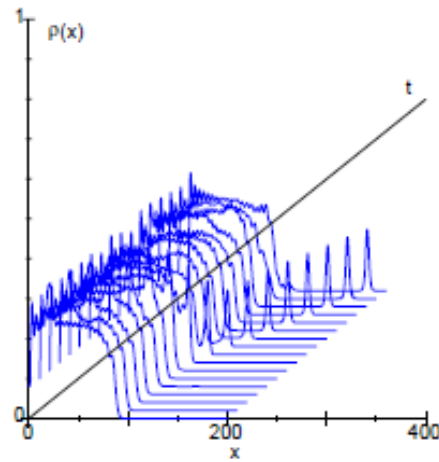
- Y. N. Demkov and V. I. Osherov,
Sov. Phys.-JEPT 26 1211 (1968)
- Y. Kayanuma and S. Fukuchi,
JPSJ 53 (1985) 1869,
J. Phys. B18 (1985) 4089.
- S. Tsuneyuki, et al. Surface
Sci. 186 (1987) 26.
- K. Kobayashi, et al.
Physica A 265 (1999) 565.
- Keiji Saito, et al., Phys. Rev. B75 (2007)
75, 214308.

Multiple free particles (fermion)

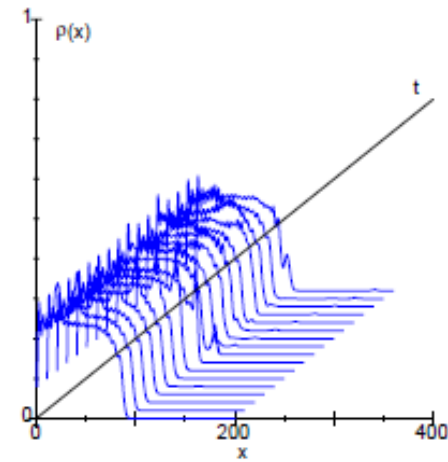
$N=0.1L$



(a)

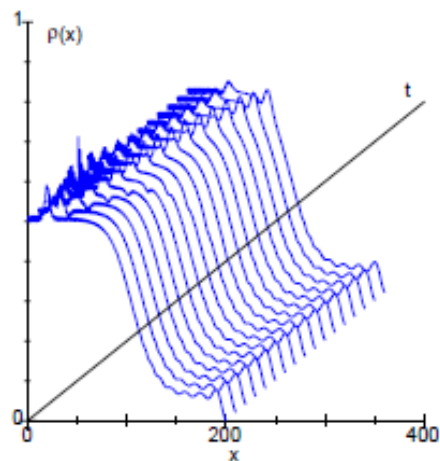


(b)

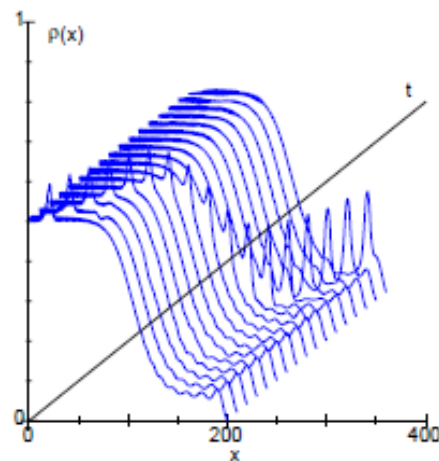


(c)

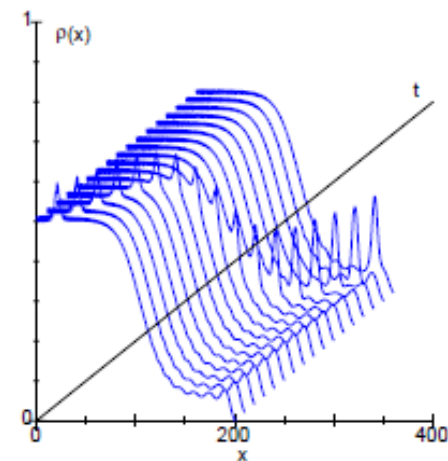
$N=0.3L$



(d)



(e)



(f)

Uniform acceleration

$$\mathcal{H}(t) = \frac{p^2}{2m} + V\left(x - \frac{1}{2}at^2\right) \quad c = at$$

$$U_1 = e^{-i\frac{1}{2}at^2 p/\hbar} \quad U_1^{-1} \mathcal{H}(t) U_1 = \mathcal{H}_0$$

$$\Phi(x, t) = U^{-1} \Psi(x, t)$$

$$i\hbar \frac{\partial}{\partial t} \Phi(x, t) = \left(\frac{1}{2m} (p - mat)^2 - \frac{1}{2} m (at)^2 + V(x) \right) \Phi(x, t) \equiv \mathcal{H}_1 \Phi(x, t)$$

$$V = e^{-imatx/\hbar} \quad V^{-1} p V = p - mat$$

$$i\hbar \frac{\partial}{\partial t} \Phi'(x, t) = \left(\mathcal{H}_0 + amx - \frac{1}{2} m (at)^2 \right) \Phi'(x, t) \equiv \mathcal{H}_2 \Phi(x, t)$$

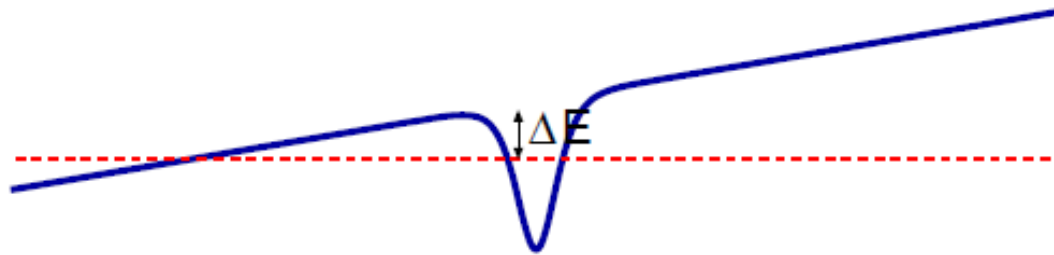
Adiabatic acceleration

$$V'(x) = V(x) + amx$$

$$c = at$$

$$t = c/a$$

$$a \rightarrow 0$$



$$x = |\langle G(0) | \Psi(x, t) \rangle|^2 = \left| \langle 0 | e^{-i\mathcal{H}_2 t / \hbar} | 0 \rangle \right|^2$$

$$P = \exp\left(-\frac{2\sqrt{2mS}}{\hbar}\right) \quad S \simeq \frac{\Delta E}{2am}$$

$$Pt = \frac{c}{a} \exp\left(-\frac{2\sqrt{\Delta E/a}}{\hbar}\right)$$

Thank you very much