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Stochastic efficiencies

G. Verley, M. Esposito, T. Willaert and C. Van den Broeck
The unlikely Carnot efficiency
Nature Communications DOI: 10.1038/ncomms5721 (2014)

G. Verley, M. Esposito, T. Willaert and C. Van den Broeck
Universal theory of efficiency fluctuations
Phys. Rev. E 90, 052145 (2014).

K. Proesmans, B. Cleuren and C. Van den Broeck
Stochastic efficiency for effusion as a thermal engine
EPL 109, 20004 (2015).

K. Proesmans and C. Van den Broeck
Stochastic efficiency: five case studies
NJP 17, 065004 (2015).

T. R. Gingrich, G. M. Rotskoff, S. Vaikuntanathan, and P. L. Geissler, New Journal of Physics 16, 102003 (2014).

S. Rana, P. Pal, A. Saha, and A. Jayannavar, Physical Review E 90, 042146 (2014).

M. Polettini, G. Verley, and M. Esposito, Physical Review Letters 114, 050601 (2015).

M. Esposito, M. A. Ochoa, and M. Galperin, Physical Review B 91, 115417 (2015).

J.-H. Jiang, B. K. Agarwalla, and D. Segal, arXiv:1503.04310 (2015).

S. Rana, P. Pal, A. Saha, and A. Jayannavar, arXiv:1503.02559 (2015).

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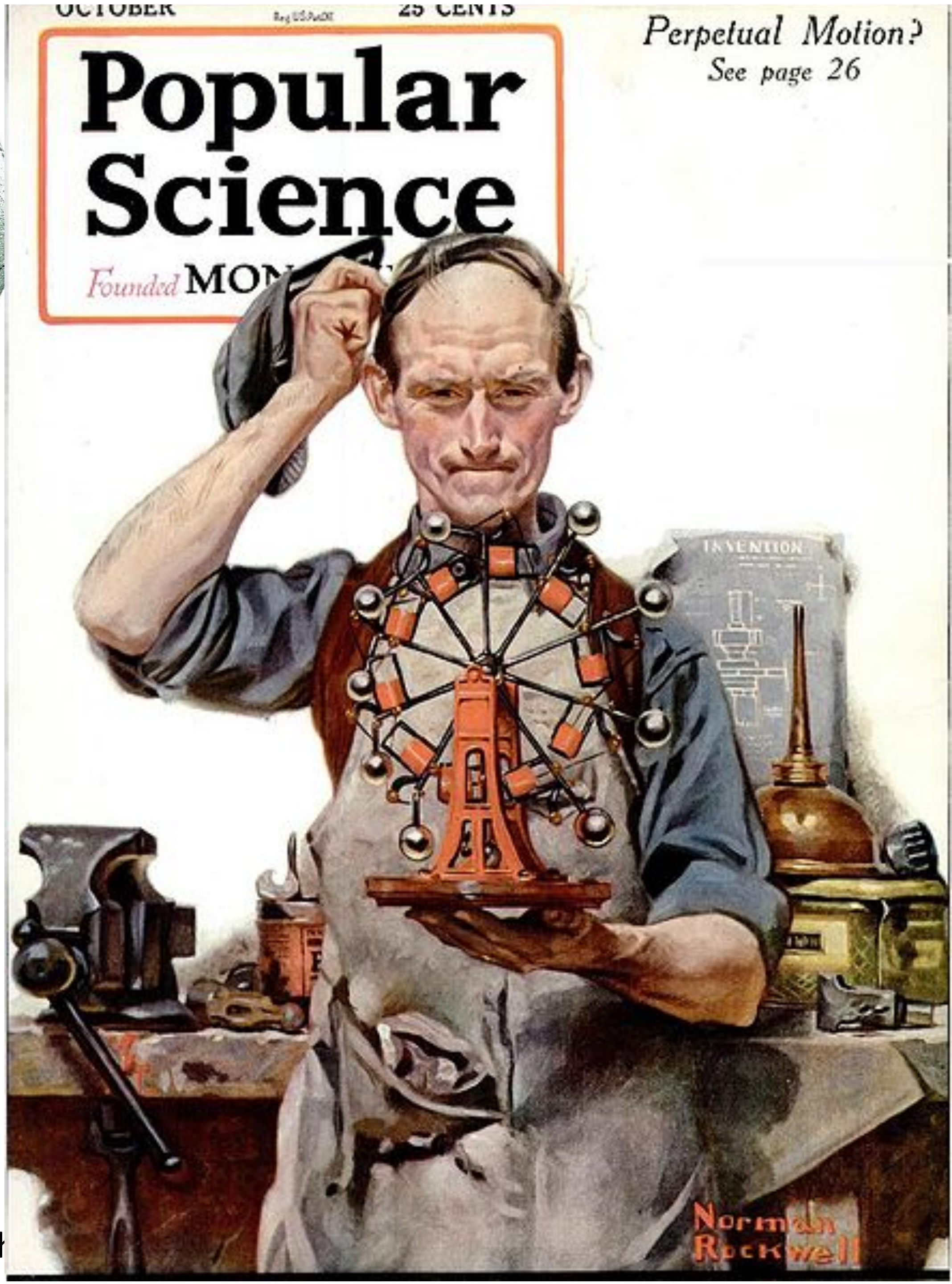
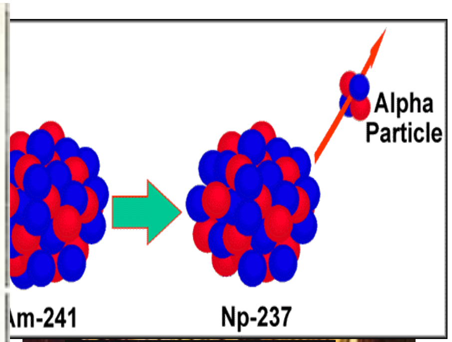
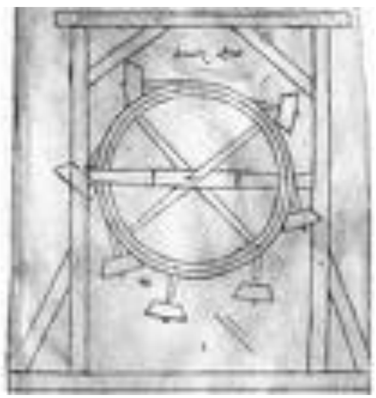
elixir
of life

Pierre de Maricourt
(about 1270)



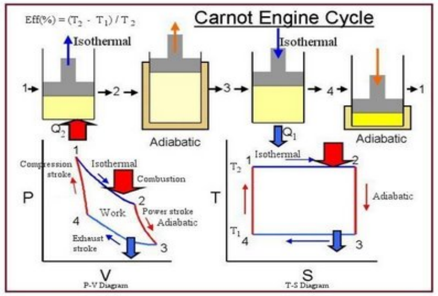


transmutation
elements



Villard de Honnecourt
(about 1230)

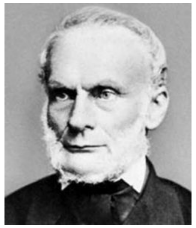
Efficiency: heat to work



Réflexions sur la puissance motrice du feu et sur les machines propres a developer cette puissance.

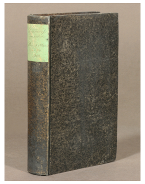
$$\eta = \frac{output\ \ work\ \ W}{input\ \ heat\ \ Q_h} \le \eta_c = 1 - \frac{T_c}{T_h}$$

Carnot Efficiency



$$\oint_{qs} \frac{dQ}{T} = 0$$

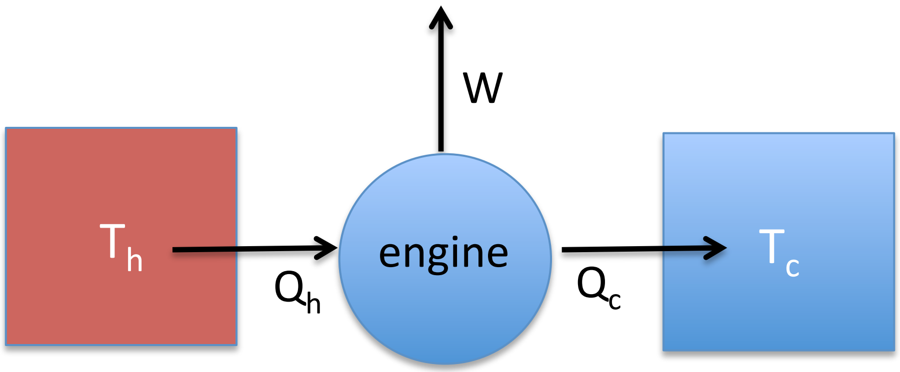
$$\Delta S = \int_{qs} \frac{dQ}{T}$$



Ueber die bewegende Kraft der Wärme und die Gesetze, welche sich daraus für die Wärme selbst ableiten lassen

$$\Delta S_{tot} \ge 0$$

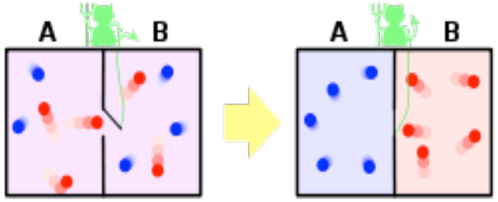
2nd law



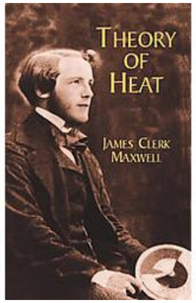
$$1^{st}\ \text{law} \qquad W = Q_h - Q_c - \cancel{\Delta E}$$

$$2^{nd}\ \text{law} \qquad \Delta S_{tot} = \frac{-Q_h}{T_h} + \frac{Q_c}{T_c} + \cancel{\Delta S} \ge 0$$

$$\eta = \frac{W}{Q_h} \overset{1^{st}}{=} 1 - \frac{Q_c}{Q_h} \overset{2^{nd}}{\le} \eta_C = 1 - \frac{T_c}{T_h}$$



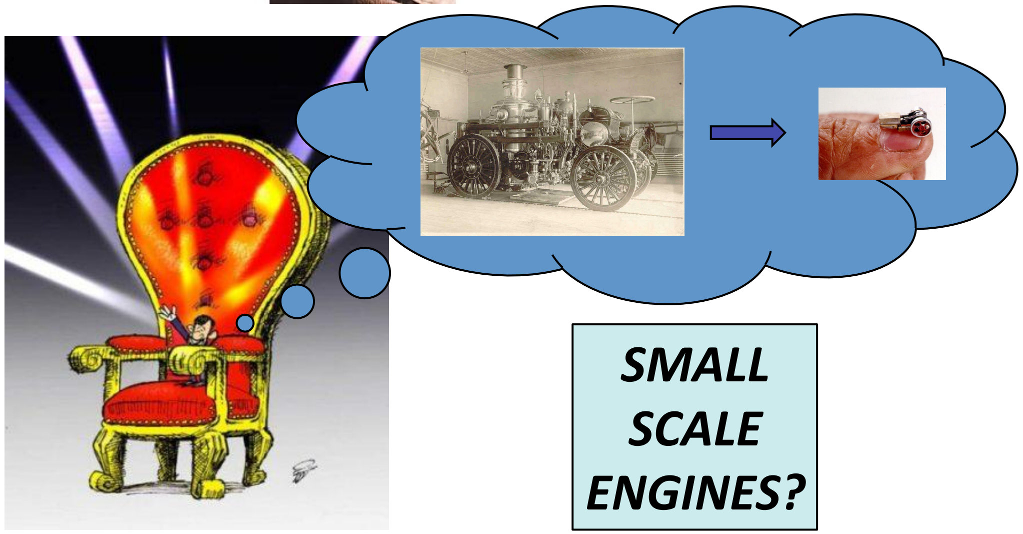
Maxwell demon



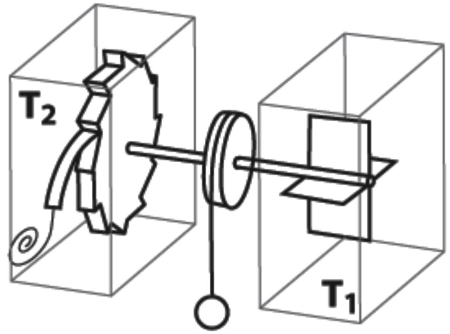
Physik. Zeitschr. XIII, 1912. v. Smoluchowski, Molekularphänomene 1069

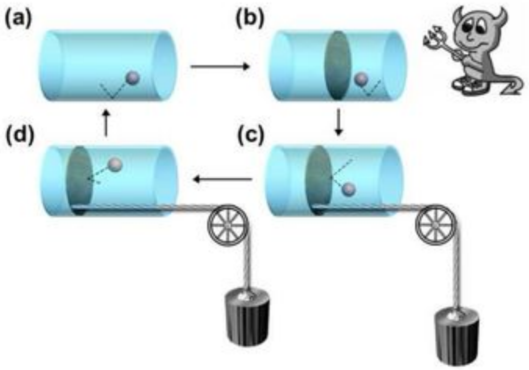
M. v. Smoluchowski (Lemberg), *Experimentell nachweisbare, der üblichen Thermodynamik widersprechende Molekularphänomene*

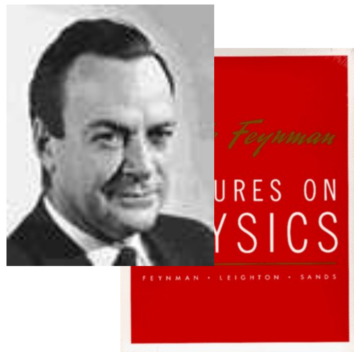
.... Theoretisch noch einfacher und übersichtlicher ist das Beispiel des Torsionsfadens. Bringen wir anstatt des Spiegels unten ein Zahnrad mit einer Sperrklinke (mit Zwangsführung) an, welche nur einseitige Drehung zuläßt. Infolge der fortwährenden Schwankungen wird das Zahnrad eine Drehung, der Faden eine Torsion erfahren, welche dauernd zu nutzbarer Arbeit am Aufhängepunkt verwendet werden könnte. Es wäre diese Vorrichtung analog einer Spielbank, welche die Gesetze des Zufalls systematisch korrigiert.
Die Schwierigkeit der technischen Ausführung bildet da keinen Einwand, wenn die Sache prinzipiell möglich ist.



SMALL SCALE ENGINES?

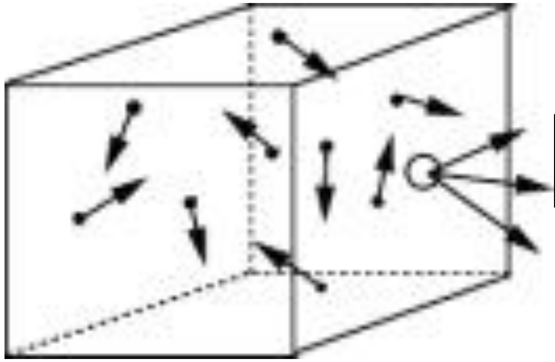






Szilard engine 1 bit=kTln2

Thomas Graham



Flux $1/\sqrt{M}$ **enrichment**

Energy $2k_BT$ **cooling**

Cosine law **deposition**

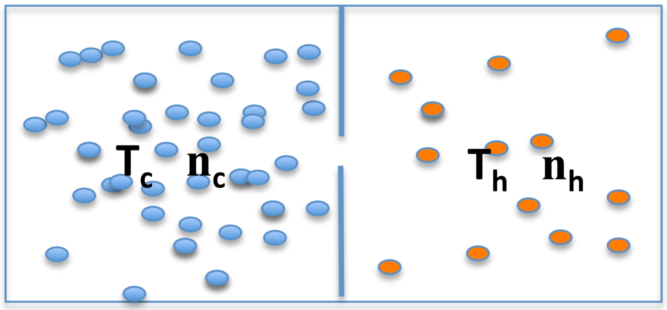
Martin Knudsen

EFFUSION
as a thermal engine

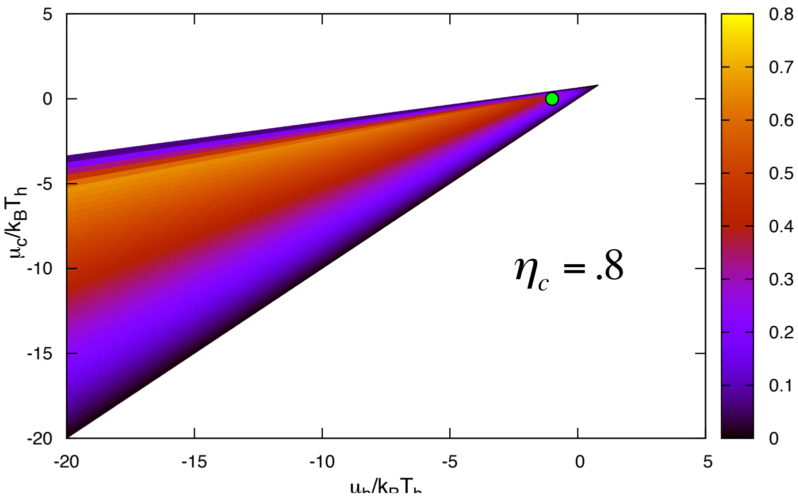
$$\Delta E = Q + W$$
$$W = \mu\ \Delta N$$

efficiency η

$$\eta = \frac{W}{Q_h} = \frac{(\mu_c - \mu_h)\langle \Delta N\rangle}{\langle \Delta e\rangle - \mu_h \langle \Delta N\rangle} \le 1 - \frac{T_c}{T_h} = \eta_C$$



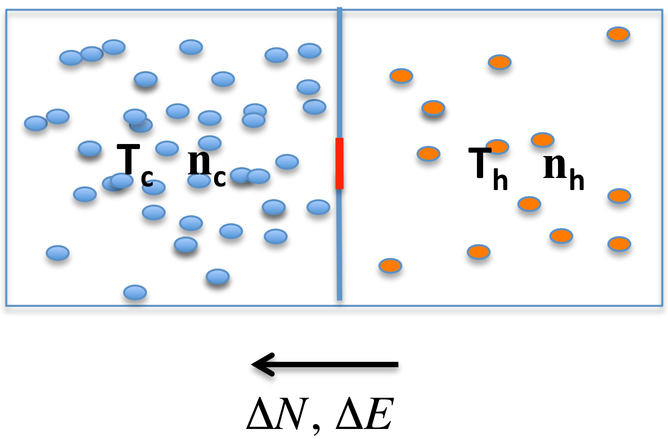
$$\overleftarrow{\Delta N,\ \Delta E}$$



EFFUSION
as a thermal engine
reaching Carnot efficiency?

reversible operation: *filter specific speed v*

particle density with speed v same left and right



$$\frac{n_c}{T_c^{3/2}}\exp\left(-\frac{mv^2}{2kT_c}\right)=\frac{n_h}{T_h^{3/2}}\exp\left(-\frac{mv^2}{2kT_h}\right)$$

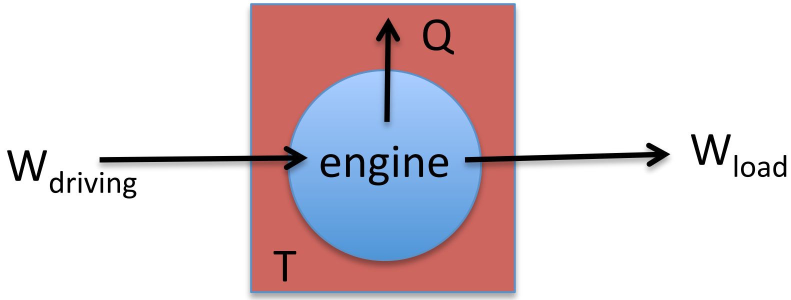
$$\mu = kT\ln n\sqrt{h^2/2\pi mkT}^{\,3}$$

$$e^{-\beta_c\left(\frac{mv^2}{2}-\mu_c\right)} = e^{-\beta_h\left(\frac{mv^2}{2}-\mu_h\right)} \qquad \mu_c-\mu_h=\eta_c\left(\frac{mv^2}{2}-\mu_h\right)$$

$$\Delta E = \Delta N\,\frac{mv^2}{2}$$

$$\eta=\frac{W}{Q_h}=\frac{(\mu_c-\mu_h)\,\Delta N}{\left\langle \Delta e\right\rangle-\mu_h\,\Delta N}=\frac{\mu_c-\mu_h}{mv^2/2-\mu_h}=\boxed{\eta_C=1-\frac{\mathrm{T}_c}{\mathrm{T}_h}}$$

Efficiency: Work to Work Engine

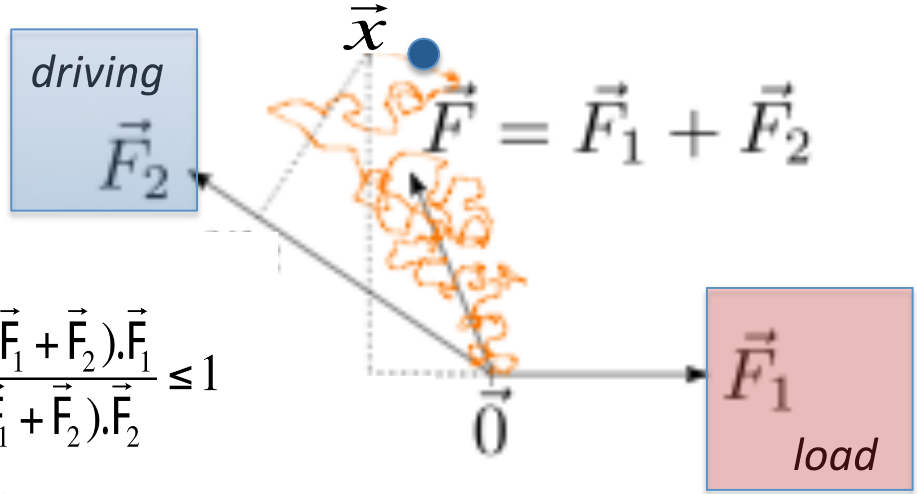


$$1^{st}\ \text{law} \qquad W_{load} = W_{driving} - Q - \cancel{\Delta E}$$

$$2^{nd}\ \text{law} \qquad \Delta S_{tot} = \frac{Q}{T} + \cancel{\Delta S} \geq 0$$

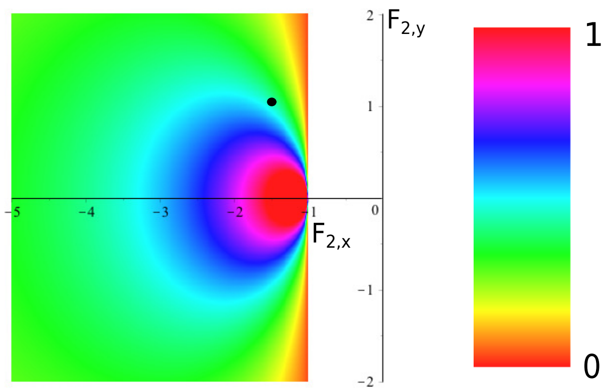
$$\eta = \frac{W_{load}}{W_{driving}} \overset{1^{st}}{=} 1 - \frac{Q}{W_{driving}} \overset{2^{nd}}{\leq} \eta_{rev} = 1$$

Brownian particle
F_1 (load) F_2 (driving)



$$efficiency\ \eta = \frac{W_{load}}{W_{driving}} = \frac{\langle w_{load} \rangle}{\langle w_{driving} \rangle} = \frac{-\langle \vec{x} \rangle . \vec{F}_1}{\langle \vec{x} \rangle . \vec{F}_2} = \frac{-(\vec{F}_1 + \vec{F}_2).\vec{F}_1}{(\vec{F}_1 + \vec{F}_2).\vec{F}_2} \leq 1$$

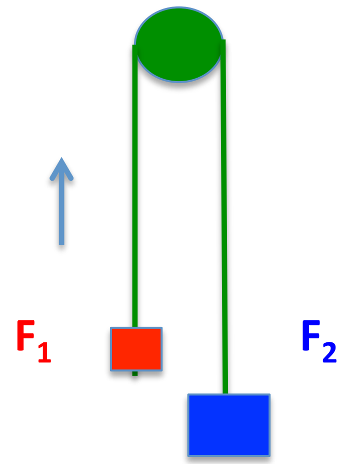
$$\langle \vec{x} \rangle = \frac{t(\vec{F}_1 + \vec{F}_2)}{\gamma}$$



reversible operation

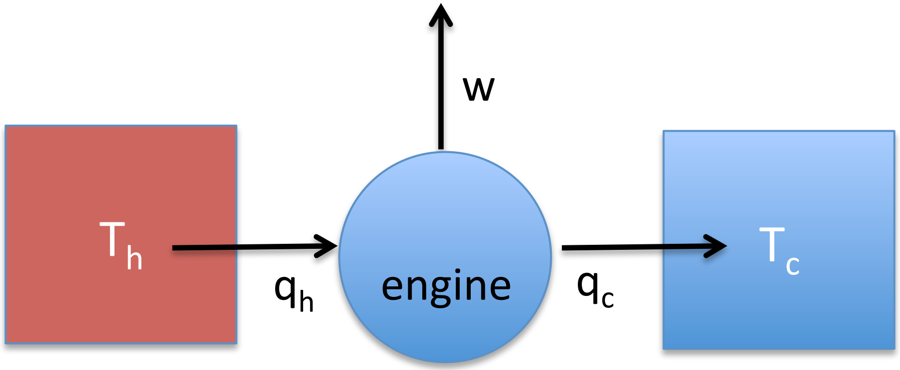
$$\vec{F}_1 \rightarrow -\vec{F}_2$$

$$\eta = 1 = \eta_{rev}$$



Stochastic efficiency of small engines?

$$\bar{\eta} = \frac{W}{Q_h} = \frac{\langle w \rangle}{\langle q_h \rangle} \quad \textit{versus} \quad \eta = \frac{w}{q_h}$$



$$1^{\text{st}}\ \text{law} \qquad w = q_h - q_c - \cancel{\Delta e}$$

$$2^{\text{nd}}\ \text{law} \qquad \Delta s_{tot} = \frac{-q_h}{T_h} + \frac{q_c}{T_c} + \cancel{\Delta s}$$

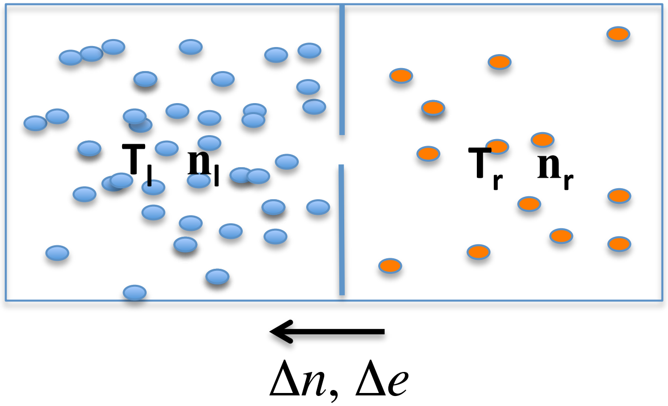
$$\langle \Delta s_{tot} \rangle \geq 0$$

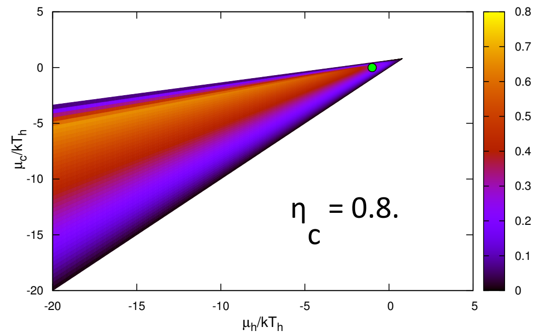
$$\eta = \frac{w}{q_h} \overset{1st\ \ law}{=} 1 - \frac{q_c}{q_h} \overset{2nd\ \ law}{=} \eta_C - \frac{T_c \Delta s_{tot}}{q_h}$$

$$\Delta s_{tot} = 0 \longrightarrow \eta = \eta_C$$

$$\left. \begin{array}{l} P_t(\Delta s_{tot}) \\ P_t(w, q_h) \end{array} \right\} \rightarrow P_t(\eta)$$

Heat to work:
effusion engine



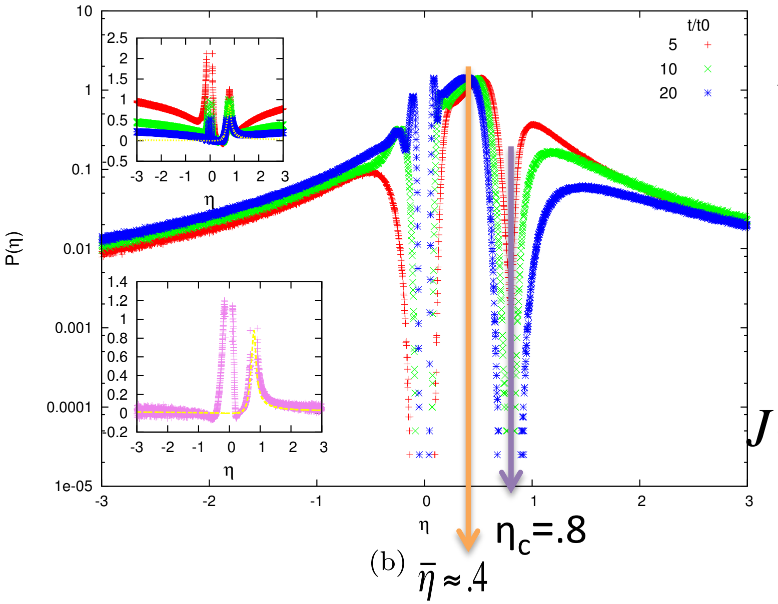


$$\eta = \frac{w}{q_h} = \frac{(\mu_c - \mu_h)\,\Delta n}{\Delta e - \mu_h \Delta n}$$

kinetic theory $\rightarrow P_t(\Delta n, \Delta e) \rightarrow P_t(\eta)$

$P_t(\eta)$

$J(\eta)$

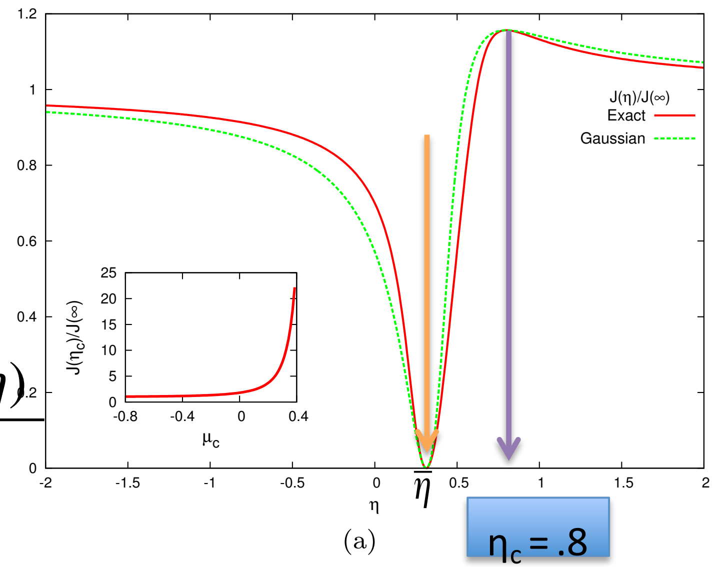


(b)

$$P_t(\eta) \propto e^{-tJ(\eta)}$$

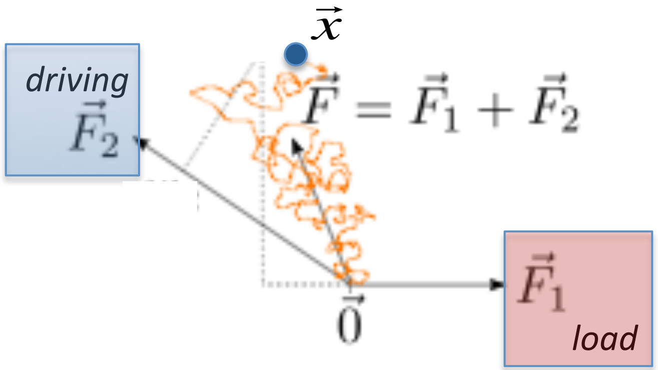
large deviation function

$$J(\eta) = -\lim_{t\to\infty} \frac{\ln P_t(\eta)}{t}$$



(a)

Work to work:
Brownian particle
F_1 (load) F_2 (driving)



$\vec{x}$: Brownian motion in force field $\vec{F}=\vec{F}_1+\vec{F}_2$

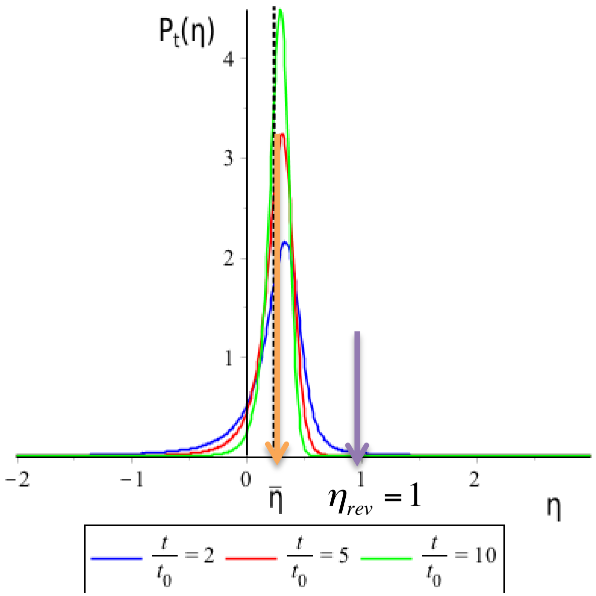
$\vec{x}$ bi-Gaussian $\langle \vec{x} \rangle = \frac{\vec{F}}{\gamma} t \quad \langle \delta\vec{x} \,.\, \delta\vec{x} \rangle = 2Dt\ \vec{\vec{1}}$

$$\eta = \frac{-\vec{F}_1 \,.\, \vec{x}}{\vec{F}_2 \,.\, \vec{x}}$$ ratio of correlated Gaussians

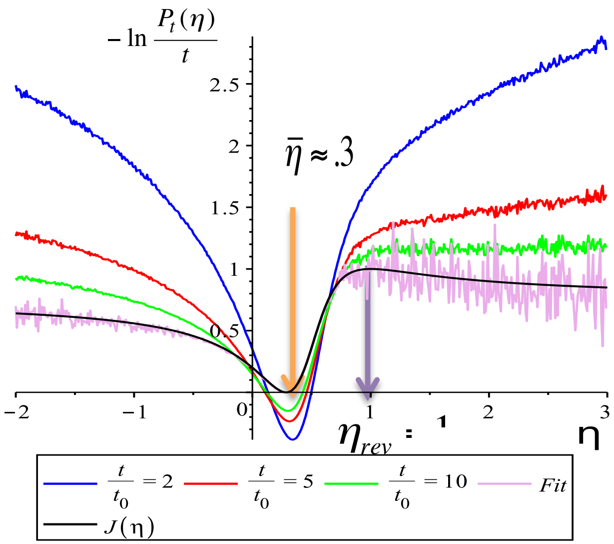
$$P_t(\eta) = \frac{\left|\vec{F}_1 \times \vec{F}_2\right| e^{-\frac{t}{t_0}}}{\pi(\vec{F}_1 + \eta\vec{F}_2)^2} \left(1 + \sqrt{\pi g(\eta)}\,\mathrm{Erf}\left[\sqrt{g(\eta)}\right] e^{g(\eta)}\right)$$

$$g(\eta) = \frac{t}{t_0} \frac{(1-\eta)^2 \left(\vec{F}_1 \times \vec{F}_2\right)^2}{\vec{F}^2 \left(\vec{F}_1 + \eta\vec{F}_2\right)^2}$$

$$J(\eta) = -\lim_{t\to\infty} \frac{1}{t} \ln P_t(\eta)$$
$$= \frac{\mu^2}{4D} \frac{\left[(\vec{F}_1 + \eta\vec{F}_2)\cdot(\vec{F}_1 + \vec{F}_2)\right]^2}{(\vec{F}_1 + \eta\vec{F}_2)^2}$$

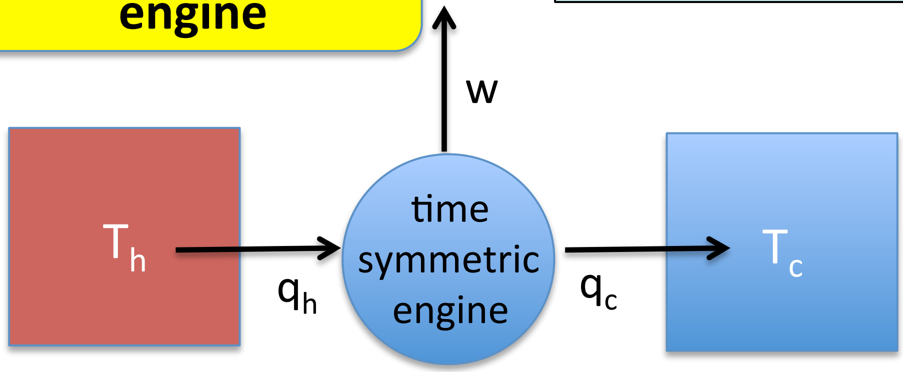


$$P_t(\eta) \propto e^{-tJ(\eta)}$$



time symmetric engine

Stochastic thermodynamics



1st law $\quad w = q_h - q_c \cancel{- \Delta e}$

2nd law $\quad \Delta s_{tot} = \frac{-q_h}{T_h} + \frac{q_c}{T_c} \cancel{+ \Delta s}$

$$\frac{P(q_h, w)}{P(-q_h, -w)} \propto e^{\Delta s_{tot}}$$

$$\boxed{P(q_h, w) \propto e^{-tI(\dot{q}_h, \dot{w})}}$$

$$I(\dot{q}_h, \dot{w}) - I(-\dot{q}_h, -\dot{w}) = -\Delta s_{tot} / t$$

$$\eta = w / q_h = \dot{w} / \dot{q}_h$$

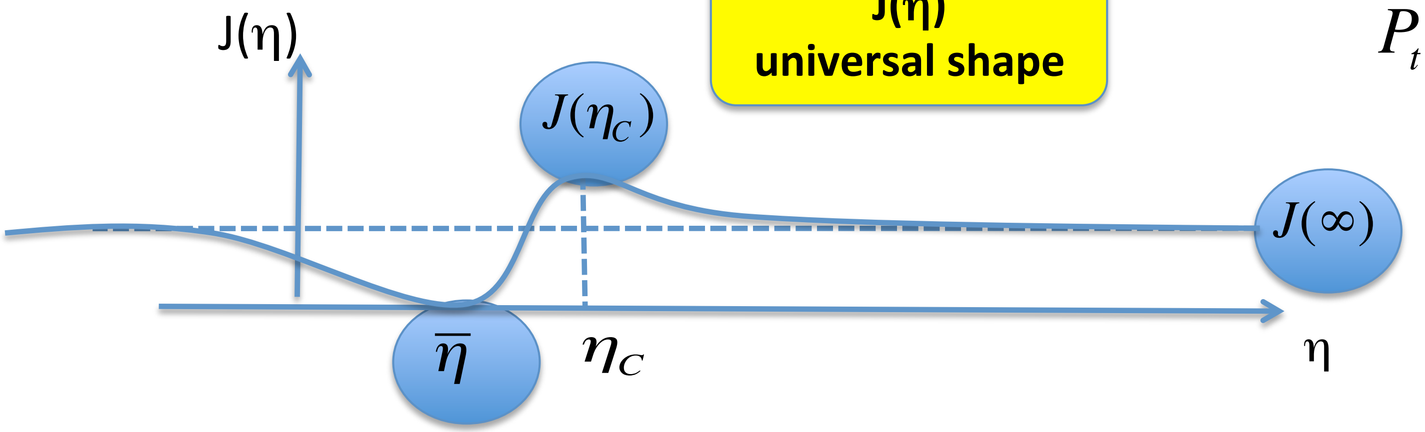
$$\boxed{P_t(\eta) \propto e^{-tJ(\eta)}}$$

$$\mathrm{J}(\eta) = \min_{\dot{q}_h, \dot{w} \;\; \eta = \dot{w}/\dot{q}_h} I(\dot{q}_h, \dot{w})$$

$$\mathrm{J}(\eta) = \min_{\dot{q}_h, \dot{w} \;\; \eta = \dot{w}/\dot{q}_h} I(\dot{q}_h, \dot{w}) = \min_{\dot{q}_h} I(\dot{q}_h, \eta \dot{q}_h) \le I(0,0)$$

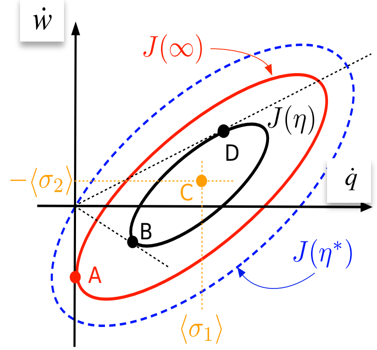
$$\Delta s_{tot} = 0 \rightarrow \begin{array}{c} \eta = w / q_h = \eta_c \\ I(\dot{q}_h, \dot{w}) \text{ symmetric} \end{array} \rightarrow J(\eta_C) = \min I(\dot{q}_h, \eta_c \dot{q}_h) = I(0,0) \ge J(\eta)$$

The Carnot efficiency is the least likely to be observed in the long time limit!



J(η)
universal shape

$$P_t(\eta) \propto e^{-tJ(\eta)}$$



close to equilibrium
universal scaling form

featuring Onsager coefficients L_{11} L_{22} $L_{12}=L_{21}$

$$\frac{J(\eta)}{J(\eta_C)} = \frac{(\overline{\eta}-\eta)^2}{(\overline{\eta}-2\eta+\eta_C)(\overline{\eta}-\eta_C)+\frac{J(\eta_C)}{J(\infty)}(\eta-\eta_C)^2}$$

J(η) unusual
large deviation function

$$P(\eta) \propto \frac{\int P(w,0)\,|w|\,dw}{\eta^2}$$

$$C_{qq} = \frac{8J(\eta_C)^2 T_c^2}{(\bar{\eta}-\eta_C)^2 J(\infty)},$$

$$\frac{1}{\det C}\begin{bmatrix} C_{qq} & -C_{wq} \\ -C_{wq} & C_{ww}\end{bmatrix}\begin{pmatrix}\dot{W}\\ \dot{Q}\end{pmatrix} = \begin{pmatrix} 1/2T_c \\ \eta_C/(2T_c)\end{pmatrix}$$

$$C_{wq} = -8J(\eta_C)T_c^2\frac{J(\infty)\bar{\eta}-J(\infty)\eta_C+\eta_C J(\eta_C)}{J(\infty)(\bar{\eta}^2-2\bar{\eta}\eta_C+\eta_C^2)},$$

$$C_{ww} = 8J(\eta_C)T_c^2\frac{J(\infty)\bar{\eta}^2+\eta_C^2 J(\eta_C)-J(\infty)\eta_C^2}{J(\infty)(\bar{\eta}^2-2\bar{\eta}\eta_C+\eta_C^2)}.$$

$$\begin{aligned} P_t(\eta) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_t(w,q)\,\delta(\eta-\frac{w}{q})\,dw\,dq \\ &= \int_{-\infty}^{\infty} P_t(w,\frac{w}{\eta})\left|\frac{w}{\eta^2}\right| dw, \end{aligned}$$

time asymmetric engine

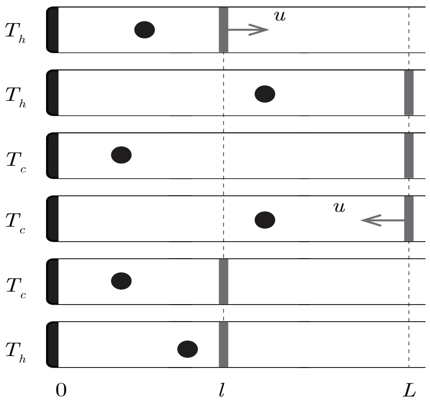
1st law $\quad w = q_h - q_c - \cancel{\Delta e}$

2nd law $\quad \Delta s_{tot} = \dfrac{-q_h}{T_h} + \dfrac{q_c}{T_c} + \cancel{\Delta s}$

$$P_t(\eta) \propto e^{-tJ(\eta)} \qquad \tilde{P}_t(\eta) \propto e^{-t\tilde{J}(\eta)} \qquad \frac{P(q_h, w)}{\tilde{P}(-q_h, -w)} \propto e^{\Delta s_{tot}}$$

J and $\tilde{\text{J}}$ cross in η_c same asymptotes   same maximum

Single particle Carnot engine



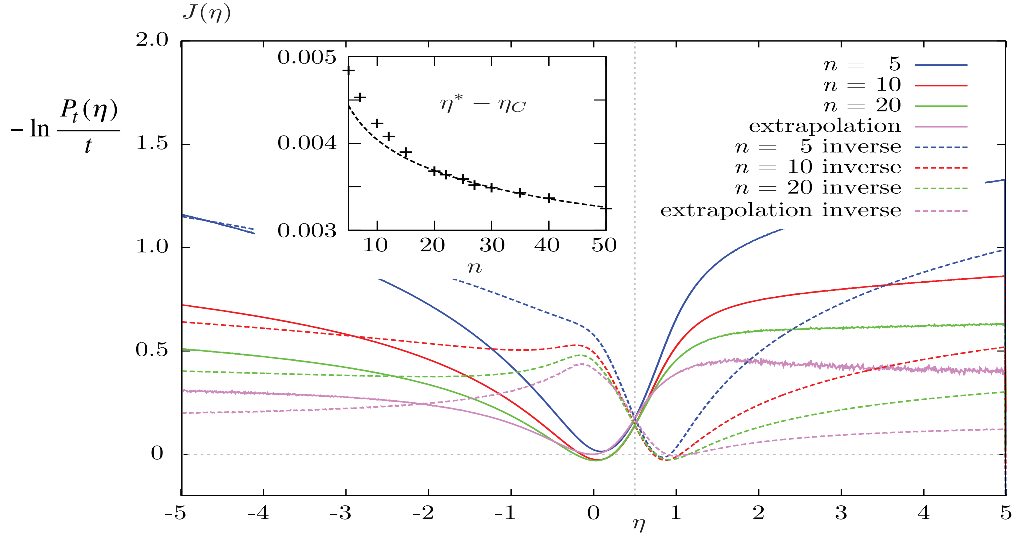
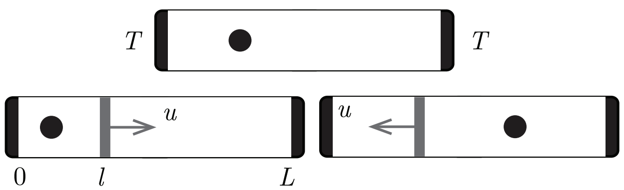


Figure 6: $-(1/n) \ln P_n(\eta)$ for 5 (blue), 10 (red) and 20 (green) cycles of the heat engine and its time-inverse, with $T_h = 2T_c$ (i.e. $\eta_C = 1/2$), $u = 0.3$ and $x = 0.5$. The purple curve is the extrapolation to the LDF. The macroscopic efficiency is given by $\bar{\eta} = -0.02$ Inset: convergence of the intersections efficiency η^* of forward and time-reverse curves to η_C as the number n of cycles increases. The dashed line is a power law fit of the form α/n^β, with $\alpha = 5.49 \cdot 10^{-3}$ and $\beta = 0.13$

Information to work
Szilard engine

$$\eta = \frac{w}{k_B T \Delta i}.$$



$\Delta i = -\ln \dfrac{l}{L}$ $\qquad$ $\Delta i = -\ln \dfrac{L-l}{L}$

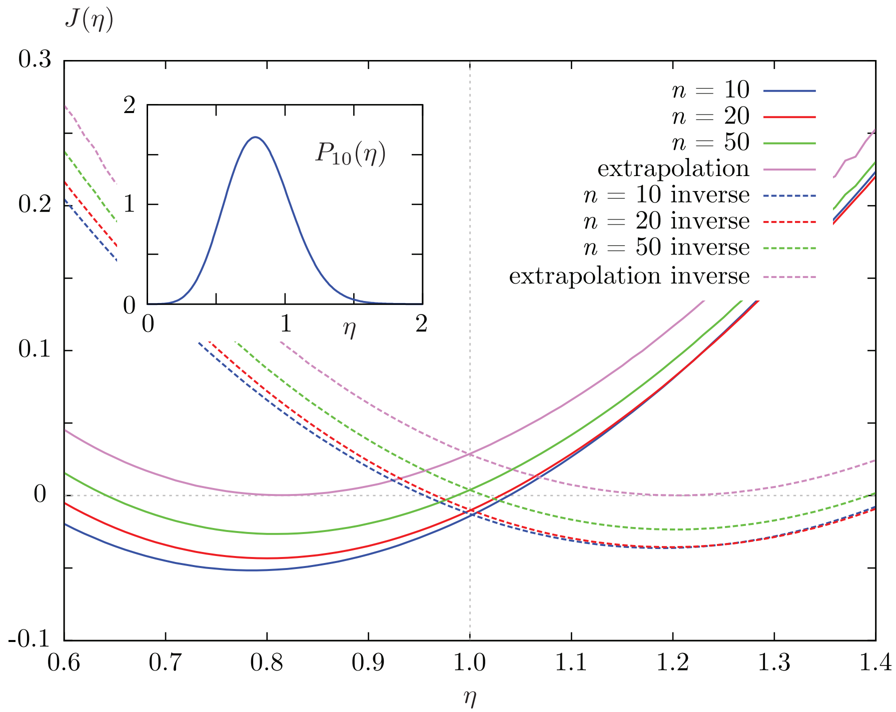
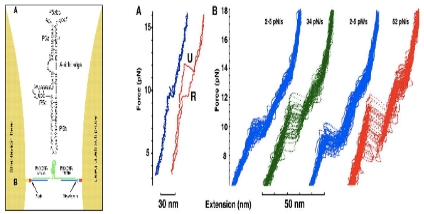


Figure 5: $-(1/n)\ln P_n(\eta)$ for 10 (blue), 20 (red) and 50 (green) cycles of the Szilard engine and its time-inverse, with $u = 0.1$ and $x = 0.7$. The purple curve is the extrapolation to the LDF. The macroscopic efficiency is $\bar{\eta} = 0.80$. The inset shows $P_{10}(\eta)$.

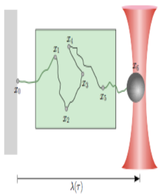
Stochastic work w and heat q hence stochastic efficiency η=w/q can be measured!

[Liphardt et al, Science **296** 1832, 2002.

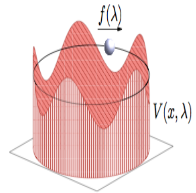
V. Blickle, T. Speck, L. Helden, U.S., C. Bechinger, PRL 96, 070603, 2006

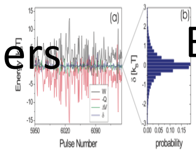


biomolecules

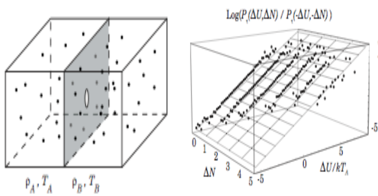


polymers





Brownian particles



PHYSICAL REVIEW E 74, 021117 (2006)

Fluctuation theorem for the effusion of an ideal gas

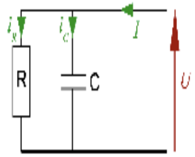
B. Cleuren* and C. Van den Broeck

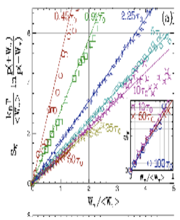
PHYSICAL REVIEW E 71, 060101(R) (2005)

Nonequilibrium fluctuations in a resistor

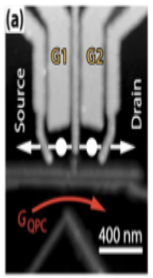
N. Garnier* and S. Ciliberto

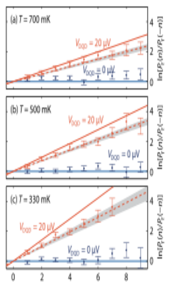
gas (molecular dynamics)





electric network





JOURNAL OF APPLIED PHYSICS 113, 136507 (2013)

Test of the fluctuation theorem for single-electron transport

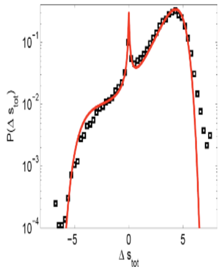
B. Küng, C. Rössler, M. Beck, M. Marthaler, D. S. Golubev, Y. Utsumi, T. Ihn, and K. Ensslin

PRL 109, 180601 (2012) PHYSICAL REVIEW LETTERS week ending 2 NOVEMBER 2012

Test of the Jarzynski and Crooks Fluctuation Relations in an Electronic System

O.-P. Saira, Y. Yoon, T. Tanttu, M. Möttönen, D. V. Averin, and J. P. Pekola

quantum dot



single electron box

Brownian Carnot engine

Ignacio A. Martínez*,[1,2] Édgar Roldán*,[1,3,4] Luis Dinis,[5,4] Dmitri Petrov,[1] J. M. R. Parrondo,[5,4] and Raúl A. Rica[†1]

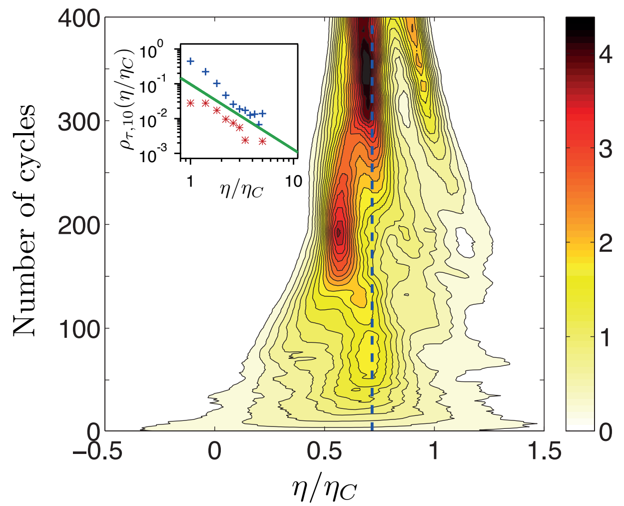
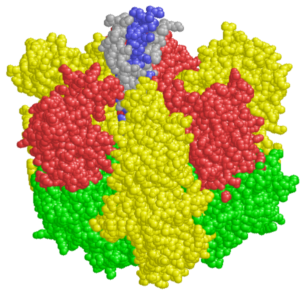


FIG. 3: **Efficiency fluctuations at maximum power.** Contour plot of the probability density function of the efficiency $\rho_{\tau=40\,\mathrm{ms},i}(\eta)$ computed summing over $i = 1$ to 400 cycles (left axis). The long-term efficiency (averaged over $\tau_{\mathrm{exp}} = 50\,\mathrm{s}$) is shown with a vertical blue dashed line. Super Carnot efficiencies appear even far from quasistatic driving. *Inset*: Tails of the distribution for $\rho_{\tau=40\,\mathrm{ms},10}(\eta)$ (blue squares, positive tail; red circles, negative tail). The green line is a fit to a power-law to all the data shown, whose exponent is $\gamma = (-1.9 \pm 0.3)$.

bimodality

P(η)~1/η



$$\frac{P(\Delta s_{tot})}{\tilde{P}(-\Delta s_{tot})} \propto e^{\Delta s_{tot}/k_B}$$

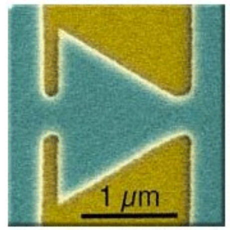
stochastic efficiency
first + second law

$$P(\eta) \propto e^{-tJ(\eta)}$$

time symmetric engine:
reversible efficiency is least likely
universal scaling form near equilibrium

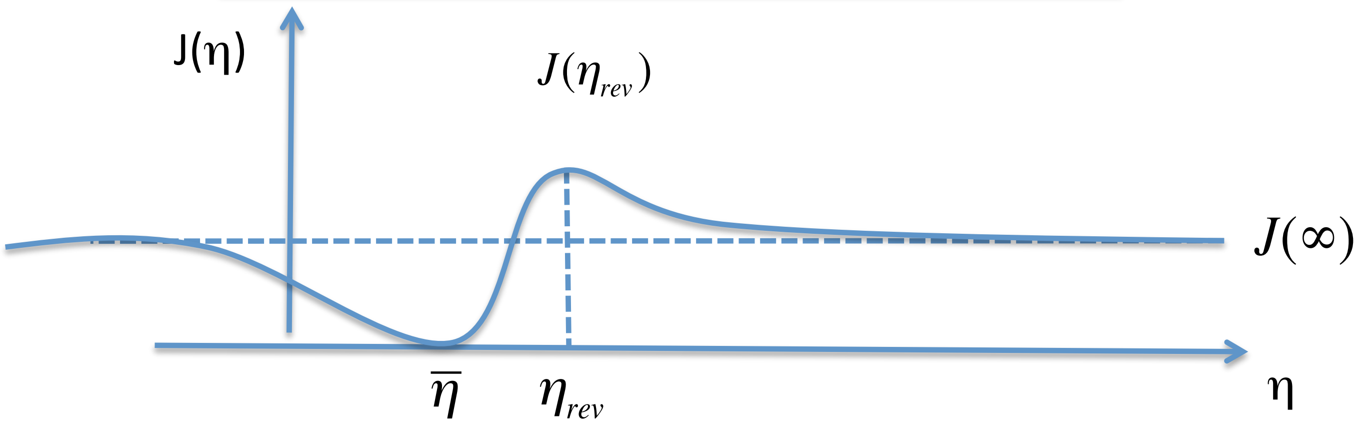
time asymmetric engine:
crossing at reversible efficiency
maxima equally unlikely, same asymptotes

absolute temperature measurement
in nonequilibrium experiment



Electron microscopic image of a quantum dot ratchet

$$\left\langle e^{-\Delta s_{tot}/k_B} \right\rangle = 1$$

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