Introduction to the Phenomenology of Neutrino Oscillation

and something more ...

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Preliminary

■What is the neutrino oscillation?

A neutrino is generated in the flavor base.
However,
the neutrino propagates in the mass base.

* It is similar to the spin precession in quantum mechanics and a coupled oscillator in classical dynamics.

▶Flavor mixing

$$|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle,$$

$$\alpha = e, \mu, \tau \cdots \text{flavor}, \qquad i = 1, 2, 3 \cdots \text{mass},$$

In 2-generation scheme, $U_{\alpha i} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$

▶Propagation in flavor basis

$$|\nu_\alpha(L)\rangle = \mathrm{T}\left[\mathrm{e}^{-\mathrm{i}\int_0^L (H+V(x))dx} \right] |\nu_\alpha\rangle$$

$$H_{\beta\alpha} \simeq p + \frac{m^2}{2E} = (p + m_1)\delta_{\alpha\beta} + \frac{1}{2E} U_{\beta i} \begin{pmatrix} 0 & \\ & \Delta m^2 \end{pmatrix} U^\dagger_{i\alpha}$$

$$V_{ee} = \frac{G_F}{\sqrt{2}} \left\{ \bar{\nu}_e \gamma^\mu (1-\gamma^5) e \right\} \left\{ \bar{e} \gamma^\mu (1-\gamma^5) \nu_e \right\} \simeq \sqrt{2} G_F (\nu_e^\dagger \nu_e) \underbrace{(e^\dagger e)}_{n_e(x)}, \qquad V_{e\mu} = V_{\mu e} = V_{\mu\mu} = 0$$

■Oscillation probability can be written as ...

▶Oscillation probability

$$P_{\nu_\alpha\to\nu_\beta}(L) = \left|\langle\nu_\beta|\nu_\alpha(L)\rangle\right|^2$$

$$P_{\nu_\alpha\to\nu_\beta}(L) = \delta_{\alpha\beta} - \sin^2 2\bar{\theta}\sin^2\frac{\Delta\lambda}{4E}L$$

$$\Delta\lambda = \sqrt{(\Delta m^2\cos\theta - a)^2 + (\Delta m^2\sin\theta)^2} \xrightarrow{a\to 0} \Delta m^2$$

$$\tan 2\bar{\theta} = \frac{\Delta m^2\sin 2\theta}{\Delta m^2\cos 2\theta - a} \xrightarrow{a\to 0} \tan 2\theta,$$

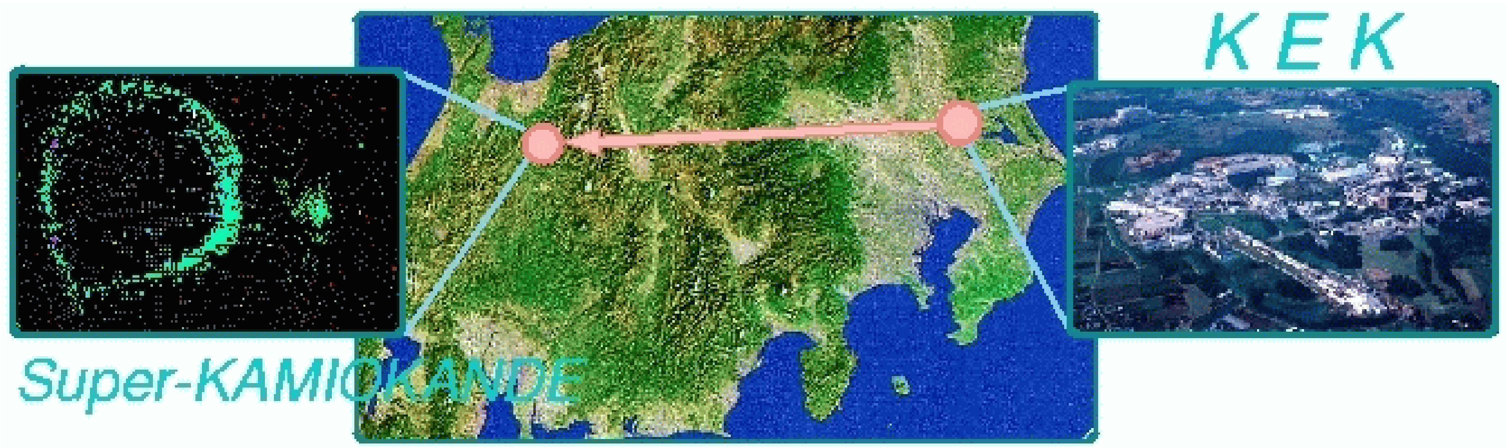
where $a \equiv 2EV_{ee}$

■In 3-generation scheme, the parameters concerning neutrino oscillation phenomena are, 2 squared mass differences, 3 mixing angles, 1 phase (and the effect induced by the matter and its profile).

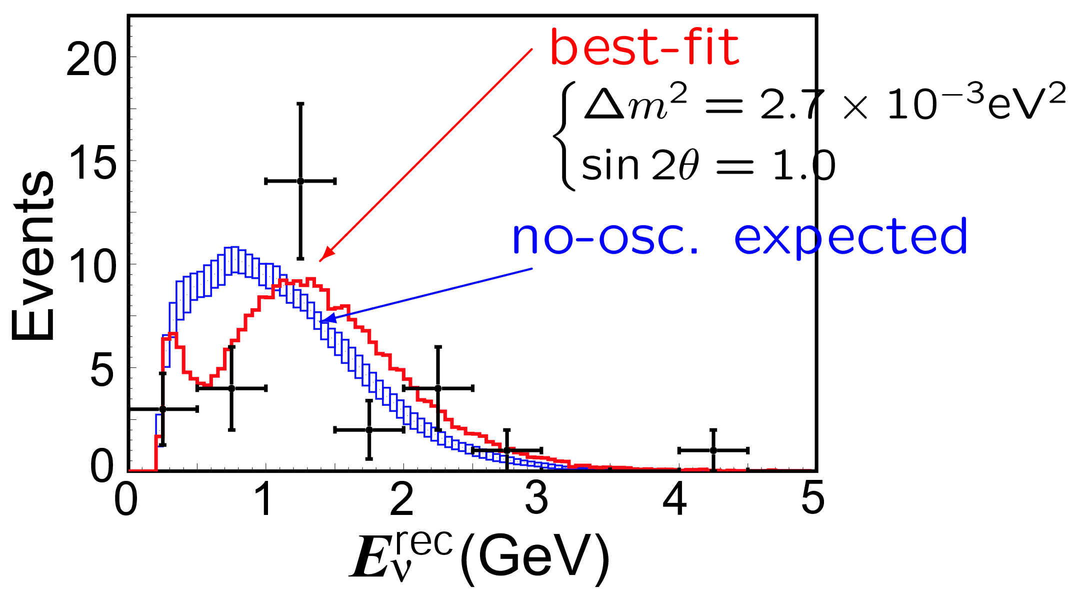
$$U_{\alpha i} \equiv \begin{pmatrix} 1 & & \\ & c_\psi & s_\psi \\ & -s_\psi & c_\psi \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta} \end{pmatrix} \begin{pmatrix} c_\phi & & s_\phi \\ & 1 & \\ -s_\phi & & c_\phi \end{pmatrix} \begin{pmatrix} c_\omega & s_\omega & \\ -s_\omega & c_\omega & \\ & & 1 \end{pmatrix}$$

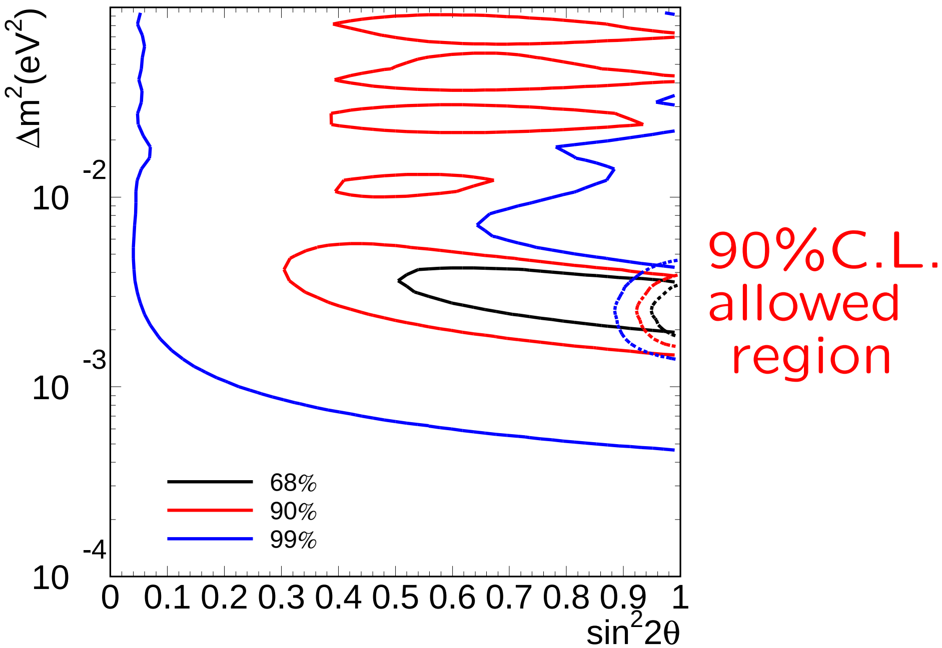
$$H(x)_{\beta\alpha} \equiv \frac{1}{2E}\left\{U_{\beta i}\begin{pmatrix} 0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31} \end{pmatrix} U^\dagger_{i\alpha} + \begin{pmatrix} a(x) & & \\ & 0 & \\ & & 0 \end{pmatrix}_{\beta\alpha}\right\}.$$

■K2K to confirm the atmospheric neutrino oscillation



$$L = 250 \text{ km}, \; E_\nu = 1 \sim 2 \text{ GeV} \rightarrow \Delta m^2 \sim 3 \times 10^{-3} \text{ eV}^2$$





■Current status of the experiments in 3-generation scheme

■Two kinds of reactor experiment:

In both of them, we can ignore the matter effect.

▶ Chooz and Palo Verde

$$L \simeq 1.0 \text{ km}, \quad \frac{\Delta m^2_{21} L}{4E} \ll 1,$$

$$P_{\bar{\nu}_e \to \bar{\nu}_e} \sim 1 - \sin^2 2\phi \sin^2 \frac{\Delta m^2_{31} L}{4E}.$$

$$\boxed{\sin\phi \lesssim 0.16 \text{ at } \Delta m^2_{31} \sim 3 \times 10^{-3} \ [\text{eV}^2].}$$

▶ KamLAND

$$\langle L \rangle \simeq 180 \text{ km}, \quad \frac{\Delta m^2_{31} L}{4E} \gg 1,$$

$$P_{\bar{\nu}_e \to \bar{\nu}_e} \sim c^4_\phi \left(1 - s^2_\omega \sin^2 \frac{\Delta m^2_{21} L}{4E} \right) + s^4_\phi.$$

$$\boxed{\sin^2 2\omega \sim 1.0, \ \Delta m^2_{21} \sim 6.9 \times 10^{-5} \ [\text{eV}^2].}$$

■Current status of the experiments in 3-generation scheme

■Atmospheric neutrino

$$\frac{\Delta m^2_{21} L}{4E} \ll 1,\ s_\phi \ll 1,$$

$$P_{\nu_\mu \to \nu_\tau} \sim s^2_{2\psi} c^4_{\tilde{\phi}} \sin^2 \frac{\lambda_+ - \lambda_-}{4E} L, \qquad \lambda_+ - \lambda_- = \sqrt{(\Delta m^2_{31} - a)^2 + 4 s^2_\phi \Delta m^2_{31} a}.$$

$$\sin 2\psi \simeq 1.0,\ \Delta m^2_{31} \simeq 2.5 \times 10^{-3}\ [\mathrm{eV}^2]$$

SK-II preliminary result
$\Delta m^2_{31} \simeq 2.0 \times 10^{-3} [\mathrm{eV}^2]$

■Solar neutrino

We have to consider the MSW effect in the solar interior,

$$P_{\nu_e \to \nu_e} = \left| A^{\oplus}_{ei} \exp\left[-\mathrm{i} \frac{\Delta m^2_{i1}(L - R_\odot)}{4E} \right] A^{\odot}_{ie} \right|^2$$

$$\simeq c^4_\phi \left\{ \frac{1}{2} + \left(\frac{1}{2} - P_{\mathrm{LZ}} \right) c_{2\omega} c_{2\tilde{\omega}(x_{\mathrm{core}})} \right\} + s^4_\phi .$$

This constrains ω and Δm^2_{21} (and ϕ).

$$\text{LMA: } \tan^2 \omega \sim 0.4,\ \Delta m^2_{21} \sim 5 \times 10^{-5}\ [\mathrm{eV}^2]$$

■What comes next?: Promised future

2007 JHF (J-PARC) to SK (Japan):

Long baseline experiment L =295 km

Precision measurement of Δm^2_{31}, 5% level

Search for $|U_{e3}| \gtrsim 0.03$ using ν_e appearance channel

▶5 year running

200? Borexino (EU):

Observation of the ^{7}Be solar neutrino

▶stop now??

2006 CNGS-ICARUS, OPERA (EU):

Long baseline experiment L =732 km

Observation of the ν_τ appearance event

Search for $|U_{e3}| \gtrsim 0.08$ using ν_e appearance channel

▶5 year running

2005 NuMI-MINOS (USA):

Long baseline experiment L =732 km

Precision measurement of Δm^2_{31}, $\mathcal{O}(10)$%

Search for $|U_{e3}| \gtrsim 0.1$ using ν_e appearance channel

▶5 year running

2003 KamLAND (Precise background study)

SK-II, K2K-II, SNO (NC update)

Mini-BooNE (USA):Check the result of LSND

■What comes next to the next?

- ▶Atmospheric neutrino oscillation is established.
- ▶Solar neutrino exps. favor the LMA. KamLAND strongly supports LMA.
- ▶Reactor experiments constrain $|U_{e3}|$.

$$|U_{MNS}| = \begin{pmatrix} 0.73 \sim 0.89 & 0.45 \sim 0.66 & < 0.24 \\ 0.23 \sim 0.66 & 0.24 \sim 0.75 & 0.52 \sim 0.87 \\ 0.06 \sim 0.57 & 0.40 \sim 0.82 & 0.48 \sim 0.85 \end{pmatrix}$$

$$1.4 \times 10^{-3} < |\Delta m^2_{31}| < 6.0 \times 10^{-3} [\mathrm{eV}^2]$$
$$2.4 \times 10^{-5} < \Delta m^2_{21} < 2.4 \times 10^{-4} [\mathrm{eV}^2]$$

∼M.C.Gonzalez-Garcia and Y.Nir, hep-ph/0202058.

▶We would like to know,

$|U_{e3}|$: Reactor with low backgrounds or a nuFact with long baseline

$\delta \equiv$CP phase: LMA !! We have a good chance to observe.

Sign of Δm^2_{31}: long baseline over 1,000 km

and more ...

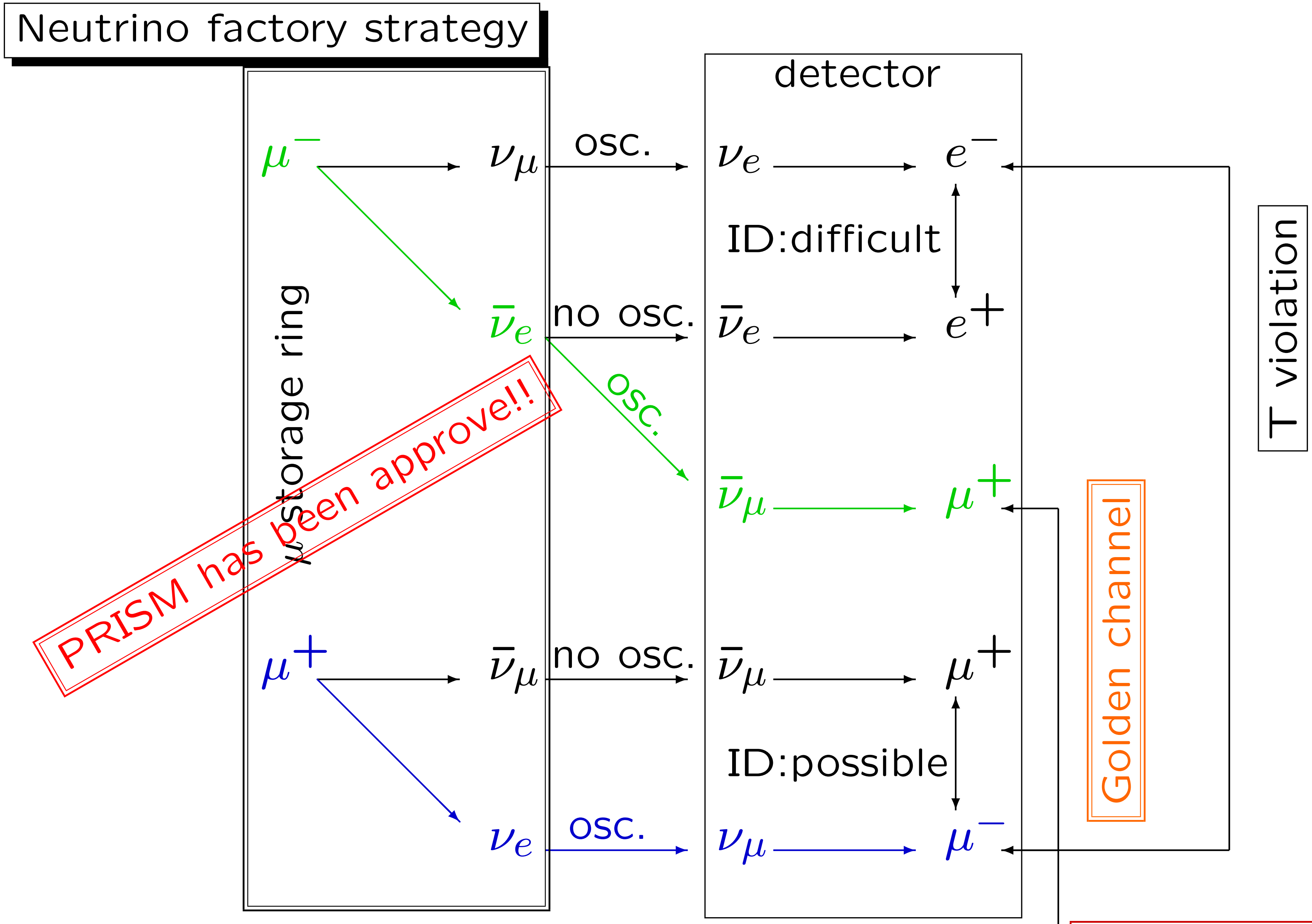
e.g., exotic interaction (=LFV with neutrinos) search

Leptonic CP-violation search using a neutrino factory

based on

Phys.Rev.**D65** (2002) 053015, M. Koike, T. O., J. Sato
Phys.Rev.**D63** (2001) 093004, T. O., J. Sato
Phys.Lett.**B520** (2001) 289, Y. Kuno, T. O., J. Sato
Phys.Rev.**D67** (2003) 053003, T. O., J.Sato

■CP violation search with a neutrino factory: *Golden channel*



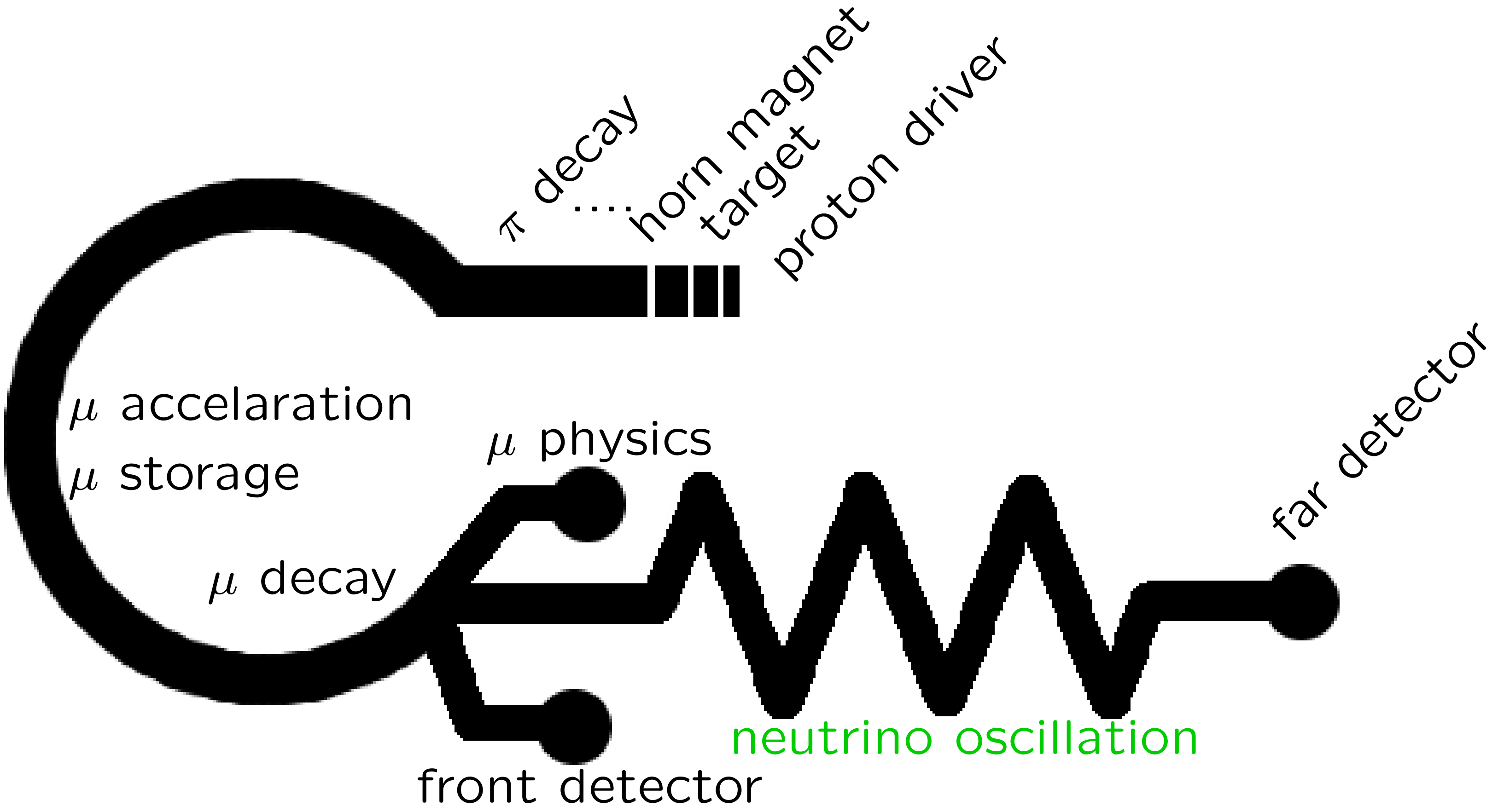
theory:$\delta_0 \in \{0, \pi\}$

$\mu^+ \longrightarrow \nu_e(\delta_0) \xrightarrow{\text{osc.}} \nu_\mu(\delta_0) \longrightarrow \mu^-$

indirect CP

■Neutrino factory in *Sansya*

■The relation between Sansya and a neutrino factory …



▶High energy experimental physicists ··· accelerator, detector etc...

▶Nuclear physicists ··· neutrino-nucleus cross-section, form factor

▶Elementary particle physicist ··· oscillation physics

and now I am studying...

■Our point:What is the differece between "works so far" with ours?

■Is the matter effect not so serious?

▶(I think) it is quite serious. We should consider the short baseline option if we want to know about the CP violation.

■Is the θ_{13}-δ correlation so serious?

▶(I think) it is not so serious when we want to measure the CP-violation effect. Because we measure the Jarlskog parameter directly. $J_{\rm CP} \propto s_{2\psi} s_{2\omega} \boxed{s_{2\phi} c_\phi s_\delta}$

■Is the real part information equivalent to the Jarlskog parameter?

▶In principle, we can re-construct the unitarity triangle using the real part information, but it is difficult since the signal is easily smeared by the parameter correlation effect.

▶Strictly speaking, it does not mean the CP violation effect since it is based on the 3-generation assumption → exotic interaction search

■Problem for CP violation search in Golden channel

■When we search for the CP-violation effect, the estimation of the matter effect is important since the matter effect make the fake CP signal.

$$P_{\nu_e \to \nu_\mu} \xrightarrow{\delta \to -\delta,\ a \to -a} P_{\bar{\nu}_e \to \bar{\nu}_\mu}$$

■Parameter correlations (= parameter degeneracies) smear the information on each parameter.

■We found out the correlation between the matter "profile" effect and ordinary (= average) matter effect.

▶We must know not only the averaged matter density but also the matter "profile" however it is difficult.

▶Therefore, we should become more conservative to estimate the error.

■Parameter correlation: Practical viewpoint

$P_{\nu_e\to\nu_\mu}$ $\mathcal{O}\{(\Delta m^2_{21}/\Delta m^2_{31})^2, |U_{e3}|^2\}$

$$
\begin{aligned}
P_{\nu_e\to\nu_\mu} \simeq\; & 4\frac{|\Delta m^2_{31}|^2}{(\lambda_+-\lambda_-)^2}|U_{e3}|^2|U_{\mu3}|^2 \sin^2\frac{\lambda_+-\lambda_-}{4E}L \\
& + 4\frac{\Delta m^2_{21}(\Delta m^2_{31})^2}{\lambda_-\lambda_+(\lambda_+-\lambda_-)}\mathrm{Re}[U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3}]\sin^2\frac{\lambda_-}{4E}L \\
& - 4\frac{\Delta m^2_{21}(\Delta m^2_{31})^2}{\lambda_-\lambda_+(\lambda_+-\lambda_-)}\mathrm{Re}[U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3}]\sin^2\frac{\lambda_+}{4E}L \\
& + 4\frac{\Delta m^2_{21}\Delta m^2_{31}}{(\lambda_+-\lambda_-)^2}\left\{2\mathrm{Re}[U_{e2}U^*_{\mu2}U^*_{e3}U_{\mu3}] + \frac{\Delta m^2_{31}(\lambda_++\lambda_-)}{\lambda_+\lambda_-}\mathrm{Re}[U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3}]\right\}\sin^2\frac{\lambda_+-\lambda_-}{4E}L \\
& + 8\frac{\Delta m_{21}(\Delta m^2_{31})^2}{\lambda_+\lambda_-(\lambda_+-\lambda_-)}\mathrm{Im}(U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3})\sin\frac{\lambda_-}{4E}L\sin\frac{\lambda_+}{4E}L\sin\frac{\lambda_+-\lambda_-}{4E}L \\
& + 4\frac{(\Delta m^2_{21})^2\Delta m^2_{31}}{\lambda_-\lambda_+(\lambda_+-\lambda_-)}\left(\frac{\Delta m^2_{31}}{\lambda_-}|U_{e1}|^2|U_{\mu1}|^2 + \mathrm{Re}[U_{e1}U^*_{\mu1}U^*_{e2}U_{\mu2}]\right)\sin^2\frac{\lambda_-}{4E}L \\
& - 4\frac{(\Delta m^2_{21})^2\Delta m^2_{31}}{\lambda_-\lambda_+(\lambda_+-\lambda_-)}\left(\frac{\Delta m^2_{31}}{\lambda_+}|U_{e1}|^2|U_{\mu1}|^2 + \mathrm{Re}[U_{e1}U^*_{\mu1}U^*_{e2}U_{\mu2}]\right)\sin^2\frac{\lambda_+}{4E}L \\
& + 4\frac{(\Delta m^2_{21})^2}{(\lambda_+-\lambda_-)^2}\left\{|U_{e2}|^2|U_{\mu2}|^2 + \frac{(\Delta m^2_{31})^2}{\lambda_-\lambda_+}|U_{e1}|^2|U_{\mu1}|^2 + \frac{\Delta m^2_{31}(\lambda_++\lambda_-)}{\lambda_-\lambda_+}\mathrm{Re}[U_{e1}U^*_{\mu1}U^*_{e2}U_{\mu2}]\right\}\sin^2\frac{\lambda_+-\lambda_-}{4E}L,
\end{aligned}
$$

where $\quad \lambda_\mp \equiv \frac{1}{2}\left(\Delta m^2_{31} + a \mp \sqrt{(\Delta m^2_{31}-a)^2 + 4\Delta m^2_{31}a|U_{e3}|^2}\right).$

We only observe P then we only know the entangled information on the parameters. ▶parameter correlation.

■Analytical study: Parametrize the profile effect

■We investigated the matter density profile effect.

▶We showed the conditions to enlarge the matter profile effect, analytically.

Hamiltonian in flavor base: 2-generation

$$H = \frac{1}{2E_\nu}\left\{U\begin{pmatrix}0 & \\ & \Delta m^2\end{pmatrix}U^\dagger + \begin{pmatrix}a_0+\delta a(x) & \\ & 0\end{pmatrix}\right\}$$

$$= \frac{1}{2E_\nu}\left\{\tilde{U}_0\begin{pmatrix}\lambda_- & \\ & \lambda_+\end{pmatrix}\tilde{U}_0^\dagger + \begin{pmatrix}\delta a(x) & \\ & 0\end{pmatrix}\right\}$$

*mass-square eigenvalues in const.(a_0) matter

$$\delta a(x) = \sum_{n=-\infty}^{\infty} a_n e^{-ip_n x}, \quad p_n \equiv \frac{2n\pi}{L}$$

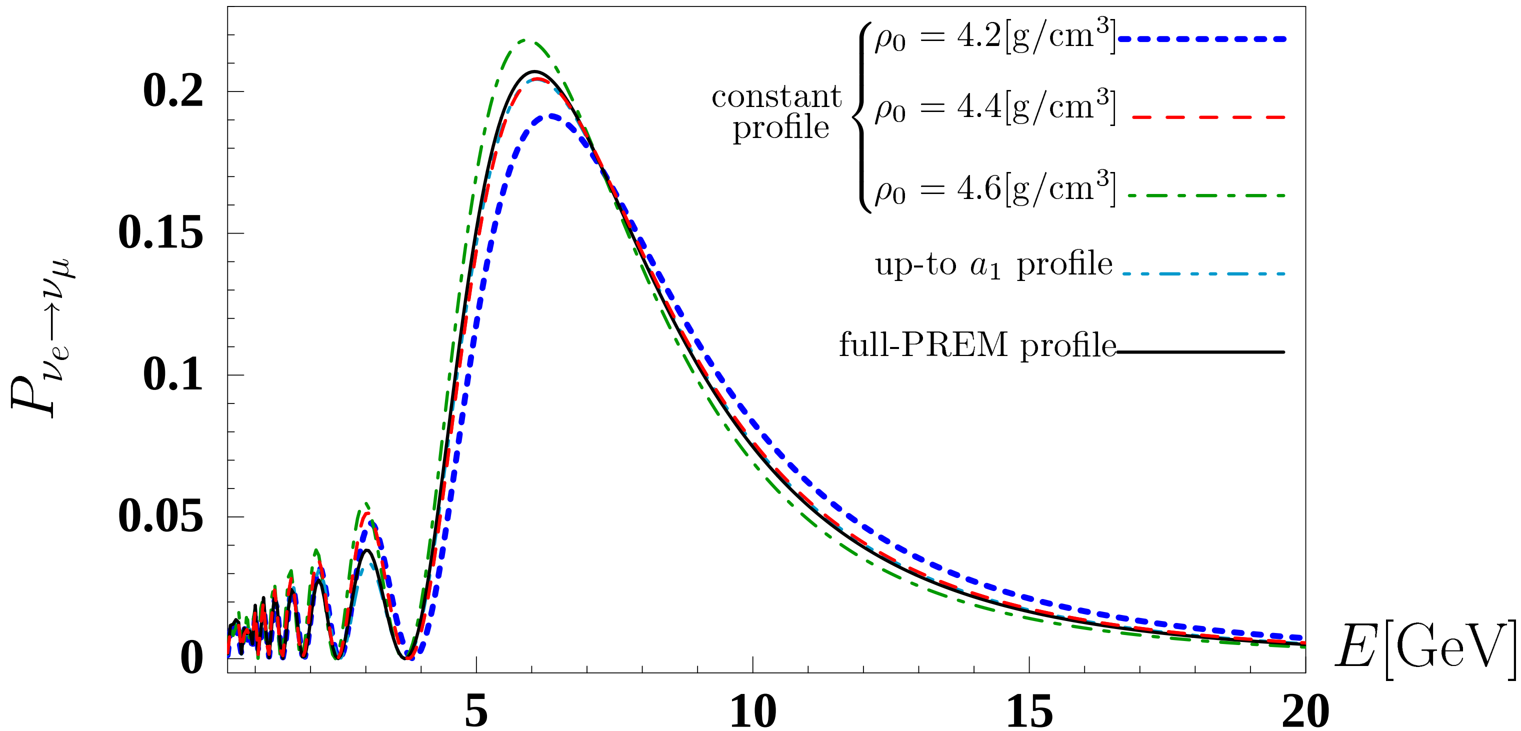
(i) Fourier coefficients are large.

(ii) The resonance conditions are satisfied.

$$\lambda_+ - \lambda_- = 2p_n E_\nu$$

▶ Only first few modes are important.

■Correlation between the "profile" and the average

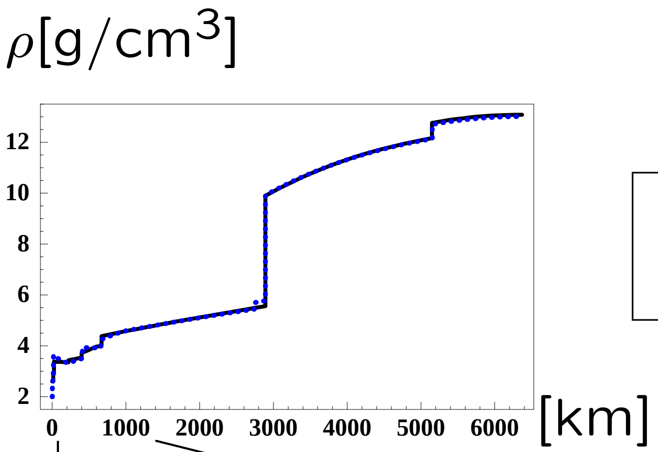


The probability calculated by constant profile with $\rho_0 = 4.42[\text{g/cm}^3]$ coincides with that of PREM (and $\rho_0 + \rho_1$), perfectly!

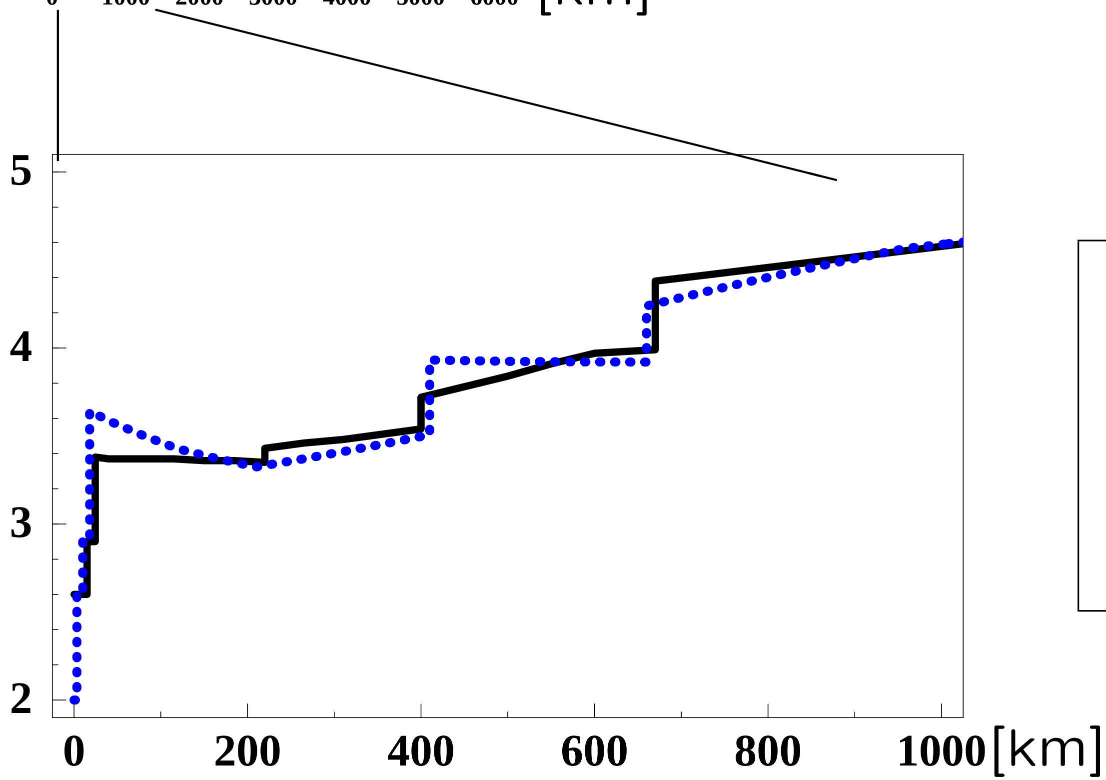
$$P_0(a_0 + \Delta a_0) \simeq P_0(a_0) + P_1(a_1).$$

This correlation is not accident in this example. It holds in wide energy and baseline region!

■Earth model: Error estimation for the profile



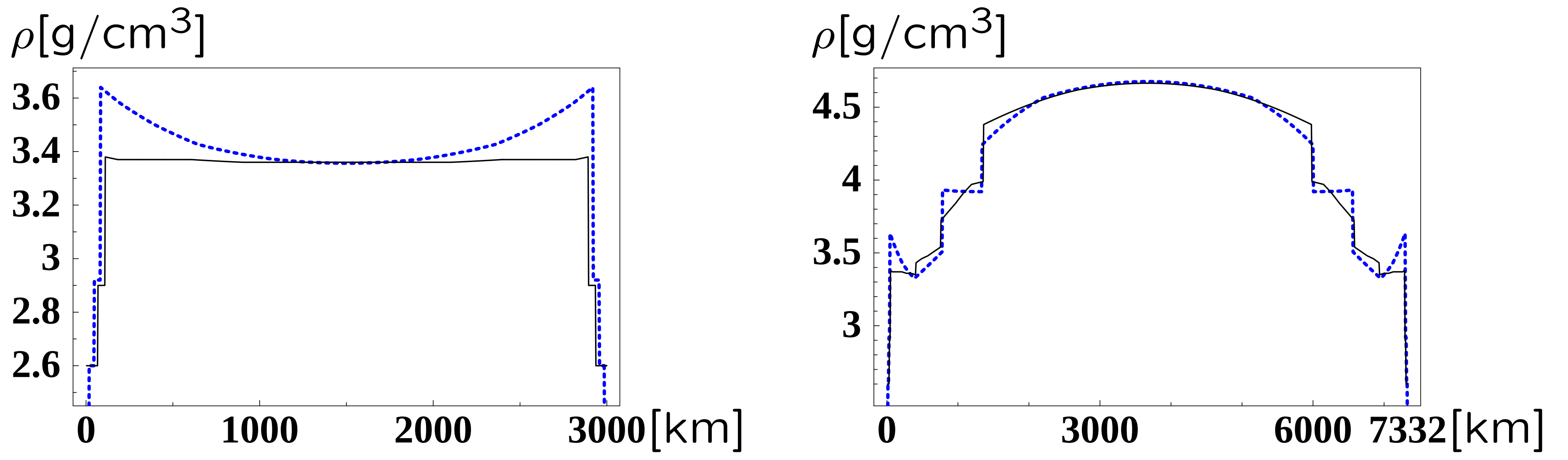
■Two Earth models, **PREM** and **ak135f**, are almost the same.



■However, they are different in the lower mantle and transition zone. This difference become relevant in the case where the baseline length is 3,000 km

■Earth model: Error estimation for the profile

■The matter profile functions in the case of 3,000 km and 7,332 km which are calculated using **PREM (solid line)** and **ak135-f (dotted line)** .

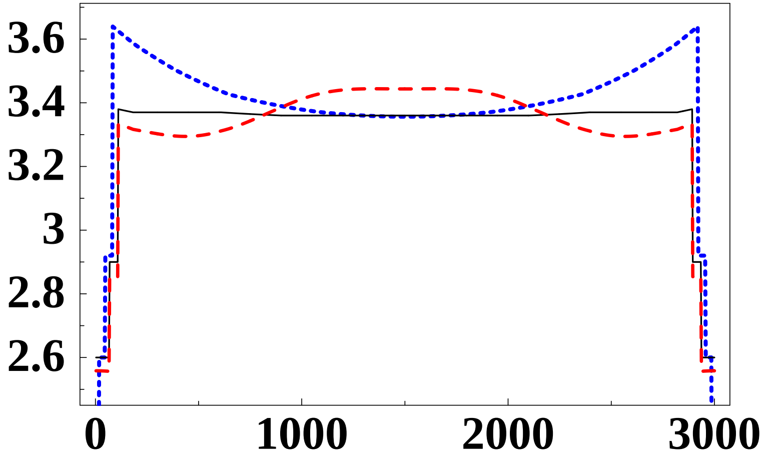


▶In 3,000 km, the uncertainty of the matter profile might be large.

■The CERN group (*et al.*) made a optimization on the experimental set up and they concluded that the L =3,000 km is best.

▶Is that true? Did they take account of this large uncertainty?

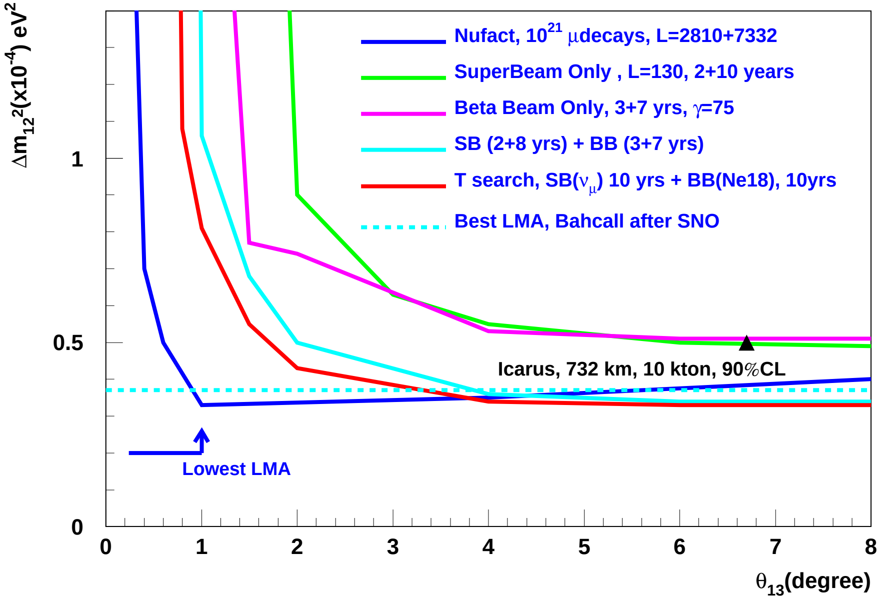
■Is L =3,000 km the best choice for CP-violation search?



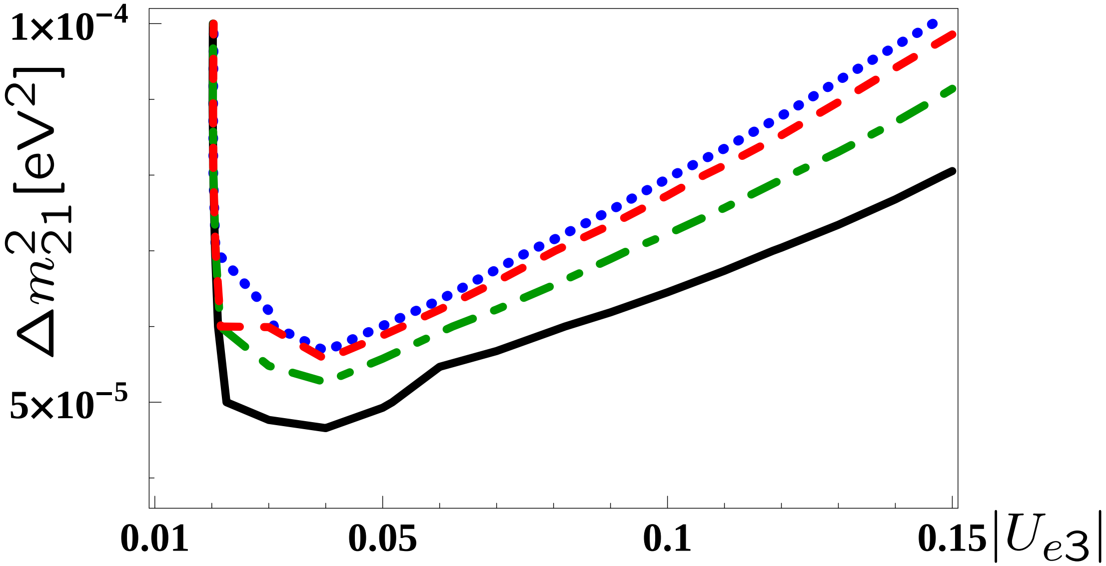
The first Fourier coefficient can include the 100% or more error.
This error propagates to the averaged matter density as additional 3% $\sim$ error.

■When nature takes the value of $\delta = \pi/2$, can we observe the CP-violation effect using a neutrino factory with L =3,000 km, E_μ = 30 GeV?

■CERN group:



■Our study:



■Test statistics

■The test statistics which have been used so far is

$$\chi_1^2 \equiv \sum_i^{\text{bin}} \frac{|N_i(\delta) - N_i(\delta_0)|^2}{N_i(\delta)} + (N \to \bar{N}) \propto (\cos\delta + \text{matter})\frac{1}{E^2} + \sin\delta\frac{1}{E^3}.$$

▶We propose the other test statistic:

$$\chi_3^2 \equiv \sum_i^{\text{bin}} \frac{|\bar{N}_i(\delta_0)N_i(\delta) - N_i(\delta_0)\bar{N}_i(\delta)|^2}{\bar{N}_i^2(\delta_0)N_i(\delta) + N_i^2(\delta_0)\bar{N}_i(\delta)} \propto (\sin\delta + \text{matter} + \cos\delta)\frac{1}{E^3}.$$

■Taking the correlation into account is equivalent to minimize the test statistic with adjustment of the oscillation parameters.

■Condition whether we can observe or not:

If these test statistics satisfy the condition,

$$\min_{\text{osc.para.}} \chi_{1,3}^2 > \chi_{90\%}^2(\text{d.o.f})$$

then we can say that we will observe the signal with statistical significance.

■Conclusion

■ Through the correlation between a_0 and a_1 the uncertainty of profile gives an extra uncertainty to a_0.

▼

■ We consider the short baseline option with the test statistic which is mainly sensitive to $\sin\delta$.

■ We re-consider the method of the hypothesis testing.

▼

■ If $|U_{e3}|$ is as large as the value just under the current bound,

then the matter effect is enhanced.

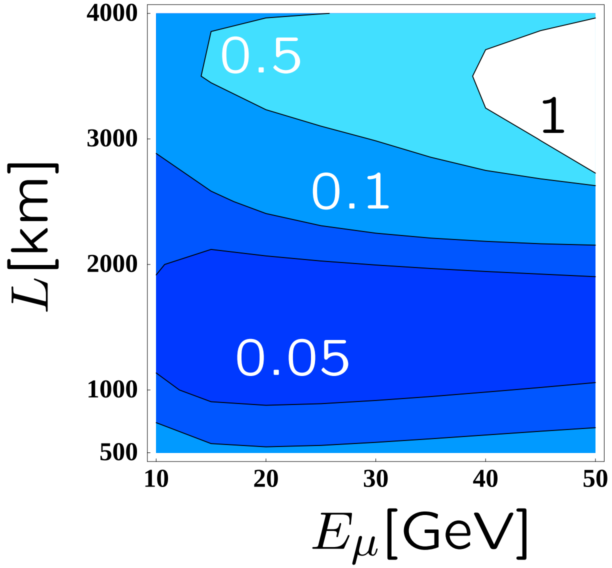
▶ The short baseline with χ_3^2 is advantageous.

■ If $|U_{e3}|$ is small,

then the matter effect is suppressed.

▶ The long baseline with χ_1^2 is advantageous.

$\sin\phi = 0.1$
$\Delta(\sin\phi) = 10\%$
$\Delta(a_0) = 10\%$
E_{th} =5 GeV



Exotic interaction search with a neutrino factory

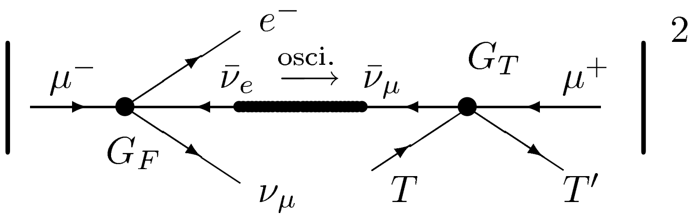
based on

Phys.Lett.**B545** (2002) 367, T.O., and J. Sato

Phys.Rev.**D65** (2002) 093015, T.O., J. Sato, and N. Yamashita

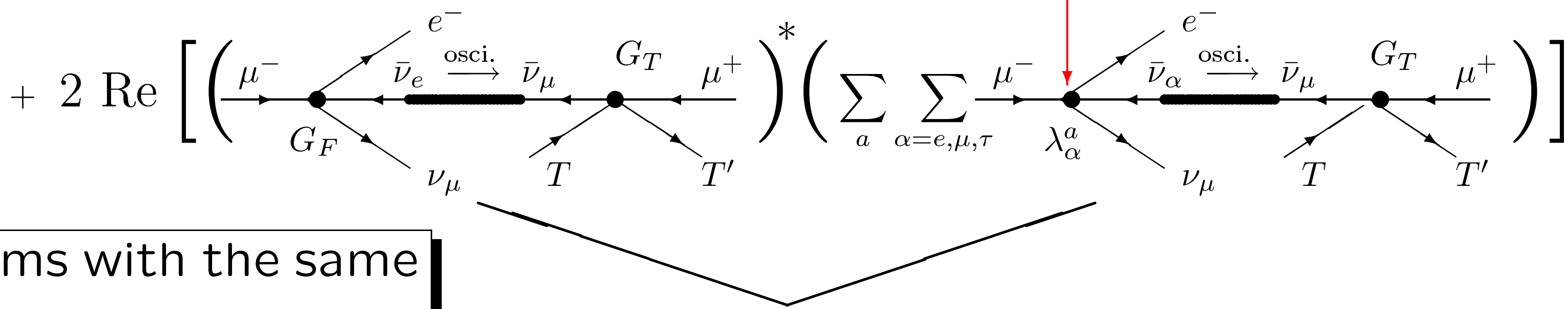
Sorry!! preliminary version!!

Basic Idea



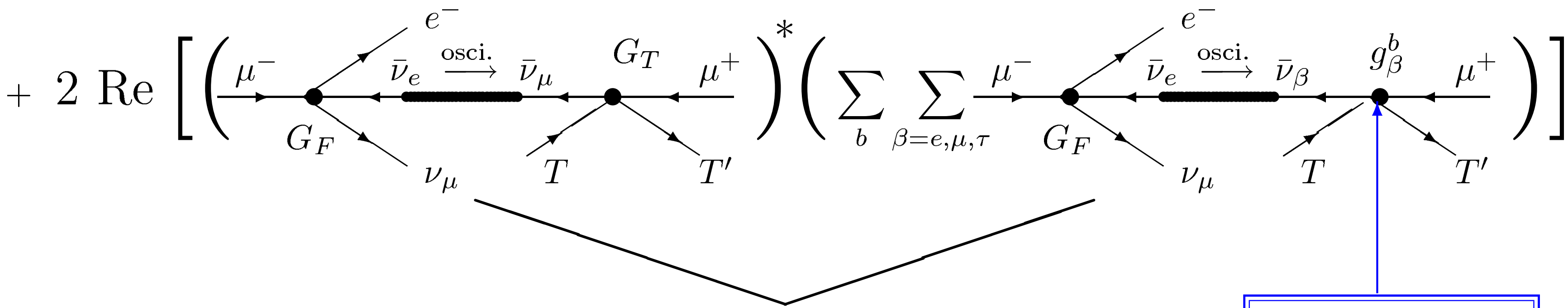
main term

n-LFV in muon decay



The diagrams with the same initial and final states interfere each other.

Interference $\sim \mathcal{O}(\lambda)$



Interference $\sim \mathcal{O}(g)$

n-LFV in detection

■ Parametrize the exotic interactions

▶The neutrinos (ν^s) which are generated by the muon decay are not the flavor eigenstates (ν_α): Y. Grossman, Phys.Lett.**B359** (1995) 141.

$$|\nu_e^s\rangle = |\nu_e\rangle + \epsilon^s_{e\mu}|\nu_\mu\rangle + \epsilon^s_{e\tau}|\nu_\tau\rangle, \qquad \epsilon^s_{e\alpha} \equiv \lambda_\alpha / G_F.$$

▶The exotic interactions also modify the Hamiltonian which describes the propagation of neutrino: A. Gago *et al*, Phys.Rev.**D64** (2001) 073003. etc...

$$\frac{1}{2E}\left\{U\begin{pmatrix}0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31}\end{pmatrix}U^\dagger + a\begin{pmatrix}1+\epsilon^m_{ee} & \epsilon^m_{e\mu} & \epsilon^m_{e\tau} \\ \epsilon^{m*}_{e\mu} & \epsilon^m_{\mu\mu} & \epsilon^m_{\mu\tau} \\ \epsilon^{m*}_{e\tau} & \epsilon^{m*}_{\mu\tau} & \epsilon^m_{\tau\tau}\end{pmatrix}\right\}$$

▶The rate for $\mu^+ + \mathcal{I} \to \mu^- + \mathcal{F}$ can be calculated as

$$\left| \mathcal{A}^D_{\mu\mu}\mathcal{A}^{P:\mathrm{St}}_{\mu e}\mathcal{A}^S_{ee} + \overbrace{\mathcal{A}^D_{\mu\mu}\mathcal{A}^{P:\mathrm{St}}_{\mu\mu}\mathcal{A}^S_{\mu e}}^{\mathcal{O}(\epsilon^s_{e\mu})} + \overbrace{\mathcal{A}^D_{\mu\mu}\mathcal{A}^{P:\mathrm{Ex}}_{\mu e}\mathcal{A}^S_{ee}}^{\mathcal{O}(\epsilon^m_{\alpha\beta})} + \cdots \right|^2$$

* In the detection process, it is possible to introduce the exotic interaction effect using the same way as the muon decay process. However, we have to introduce the Parton distribution and the hadoronaization process etc...

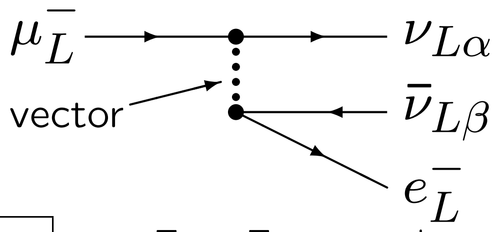
■ Model independent analyses

■The exotic interactions can be categorized into **2 types** in their Lorentz properties.

►*e.g.*, neutrino production in a neutrino factory

► $(V-A)(V-A)$ type

$SU(2)_L$ singlet and triplet type



$$\{\bar{\nu}_\alpha\gamma^\rho(1-\gamma^5)\mu\}\{\bar{e}\gamma_\rho(1-\gamma^5)\nu_\beta\} \supset (\bar{l}_\alpha\epsilon C\bar{l}_e)(l_\beta\epsilon C^\dagger l_\mu) \;\; (\alpha \neq e,\; \beta \neq \mu) \cdots \textbf{singlet type}$$

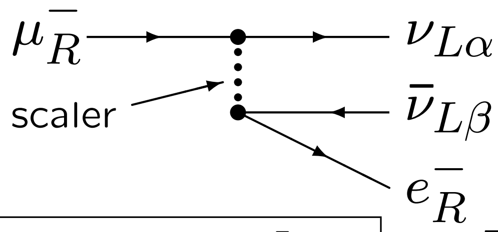
$$\cup$$

$$(\bar{l}_\alpha\tau^a\epsilon C\bar{l}_e)(l_\beta\tau^a\epsilon C^\dagger l_\mu) \subset \{\bar{e}_\alpha\gamma^\rho(1-\gamma^5)\mu\}\{\bar{e}_\alpha\gamma_\rho(1-\gamma^5)e_\beta\} \cdots \textbf{c-LFV process}$$

$\cdots$ **triplet type**

► $(V-A)(V+A)$ type

$SU(2)_L$ doublet type



$$(\bar{\nu}_\alpha\mu_R)(\bar{e}_R\nu_\beta) = \{\bar{\nu}_\alpha\gamma^\rho(1-\gamma^5)\nu_\beta\}\{\bar{e}\gamma_\rho(1+\gamma^5)\mu\} \supset (\bar{l}_\alpha\mu_R)(\bar{e}_R l_\beta) \cdots \textbf{doublet type}$$

$$\cap$$

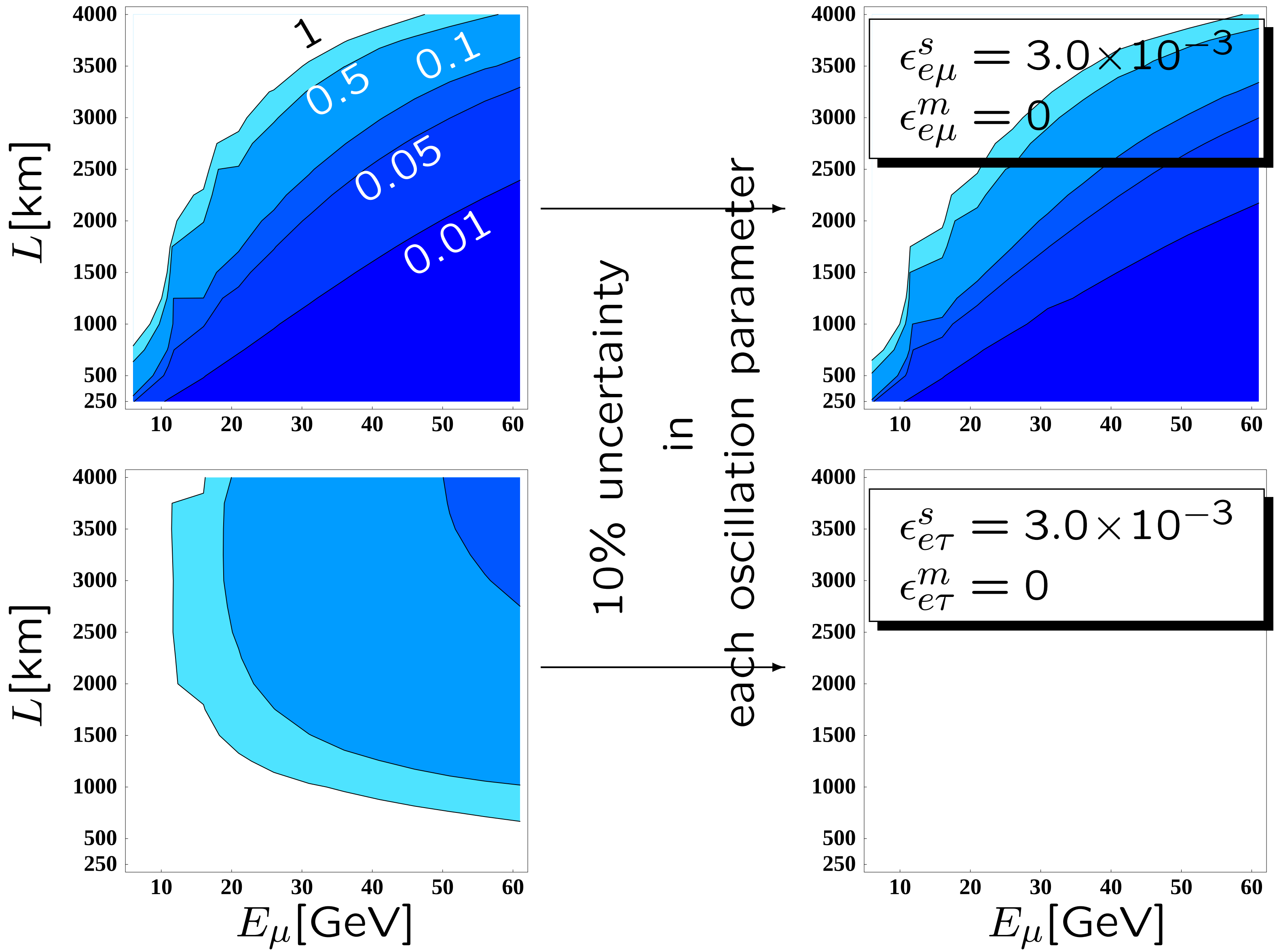
c-LFV process $\cdots \{\bar{e}_\alpha\gamma^\rho(1-\gamma^5)e_\beta\}\{\bar{e}\gamma_\rho(1+\gamma^5)\mu\}$

■The constraints come from the counter parts for doublet and triplet types:

$$\epsilon^s_{e\tau} \lesssim 3.1\times10^{-3}, \quad \epsilon^s_{e\mu} \lesssim 5\times10^{-5}, \quad \epsilon^s_{\mu\tau} \lesssim 3.2\times10^{-3}.$$

■ $\nu_e \to \nu_\mu$ channel, $\left|\epsilon^{s,m}_{\alpha\beta}\right| = 3 \times 10^{-3}$, $(V-A)(V-A)$ type

- The effects induced by $\epsilon^{s,m}_{e\mu}$ and $\epsilon^{s,m}_{e\tau}$ are large enough to be observed.
- However, the errors of the oscillation parameters absorb the $\epsilon^{s,m}_{e\tau}$ effects.



■Why $\epsilon^s_{e\tau}$ signal disappears?

$$
\begin{aligned}
\Delta P_{\nu_e\to\nu_\mu}\{\epsilon_{e\tau}\} &= 2 s_{23} s_{2\times 23} s_{2\times 13} \\
&\times \Bigg[c_{13}^2 \left(s_\delta \mathrm{Re}[\epsilon^s_{e\tau}] - c_\delta \mathrm{Im}[\epsilon^s_{e\tau}]\right) \left(\frac{\bar{a}}{4E_\nu} L\right) \left(\frac{\Delta m^2_{31}}{4E_\nu} L\right)^2 \\
&\quad + c_{13}^2 \left(c_\delta \mathrm{Re}[\epsilon^s_{e\tau}] + s_\delta \mathrm{Im}[\epsilon^s_{e\tau}]\right) \\
&\qquad \left\{ 1 - \frac{1}{2}\left(\frac{\bar{a}}{4E_\nu} L\right)^2 - s_{13}^2 \left(\frac{\bar{a}}{4E_\nu} L\right) \left(\frac{\Delta m^2_{31}}{4E_\nu} L\right) \right\} \left(\frac{\Delta m^2_{31}}{4E_\nu} L\right)^2 \\
&\quad - c_{13}^2 \left(s_\delta \mathrm{Re}[\epsilon^m_{e\tau}] + c_\delta \mathrm{Im}[\epsilon^m_{e\tau}]\right) \left(\frac{\bar{a}}{4E_\nu} L\right) \left(\frac{\Delta m^2_{31}}{4E_\nu} L\right)^2 \\
&\quad - \frac{1}{3} s_{13}^2 \left(c_\delta \mathrm{Re}[\epsilon^m_{e\tau}] - s_\delta \mathrm{Im}[\epsilon^m_{e\tau}]\right) \\
&\qquad \left\{ \left(\frac{\bar{a}}{4E_\nu} L\right) + 2\left(\frac{\Delta m^2_{31}}{4E_\nu} L\right) \right\} \left(\frac{\bar{a}}{4E_\nu} L\right) \left(\frac{\Delta m^2_{31}}{4E_\nu} L\right)^2 \Bigg]
\end{aligned}
$$

■All terms are proportional to $1/E_\nu^2$. This energy dependence is the same as the main oscillation term.

$$
\text{main term} = s_{23}^2 s_{2\times 13}^2 \left(\frac{\Delta m^2_{31}}{4E_\nu} L\right)^2
$$

■Why $\epsilon^s_{e\mu}$ signal survives?

$$\Delta P_{\nu_e\to\nu_\mu}\{\epsilon_{e\mu}\} = 2s_{23}s_{2\times 13}$$
$$\times\Bigg[\left(s_\delta \mathrm{Re}[\epsilon^s_{e\mu}] - c_\delta \mathrm{Im}[\epsilon^s_{e\mu}]\right)\left\{1-\frac{2}{3}\left(\frac{\bar{a}}{4E_\nu}L\right)^2\right\}\left(\frac{\Delta m^2_{31}}{4E_\nu}L\right)$$
$$-\left(c_\delta \mathrm{Re}[\epsilon^s_{e\mu}] + s_\delta \mathrm{Im}[\epsilon^s_{e\mu}]\right)\left\{1-\frac{1}{3}\left(\frac{\bar{a}}{4E_\nu}L\right)^2\right\}\left(\frac{\bar{a}}{4E_\nu}L\right)\left(\frac{\Delta m^2_{31}}{4E_\nu}L\right)$$
$$+2c^2_{23}\left(s_\delta \mathrm{Re}[\epsilon^m_{e\mu}] + c_\delta \mathrm{Im}[\epsilon^m_{e\mu}]\right)\left(\frac{\bar{a}}{4E_\nu}L\right)\left(\frac{\Delta m^2_{31}}{4E_\nu}L\right)^2$$
$$+2\left(c_\delta \mathrm{Re}[\epsilon^m_{e\mu}] - s_\delta \mathrm{Im}[\epsilon^m_{e\mu}]\right)\left\{1-\frac{1}{3}\left(\frac{\bar{a}}{4E_\nu}L\right)^2\right\}\left(\frac{\bar{a}}{4E_\nu}L\right)\left(\frac{\Delta m^2_{31}}{4E_\nu}L\right)\Bigg] + \mathcal{O}(1/E^2)$$

■Some of them depend on $1/E_\nu$. No standard interaction makes such signal.

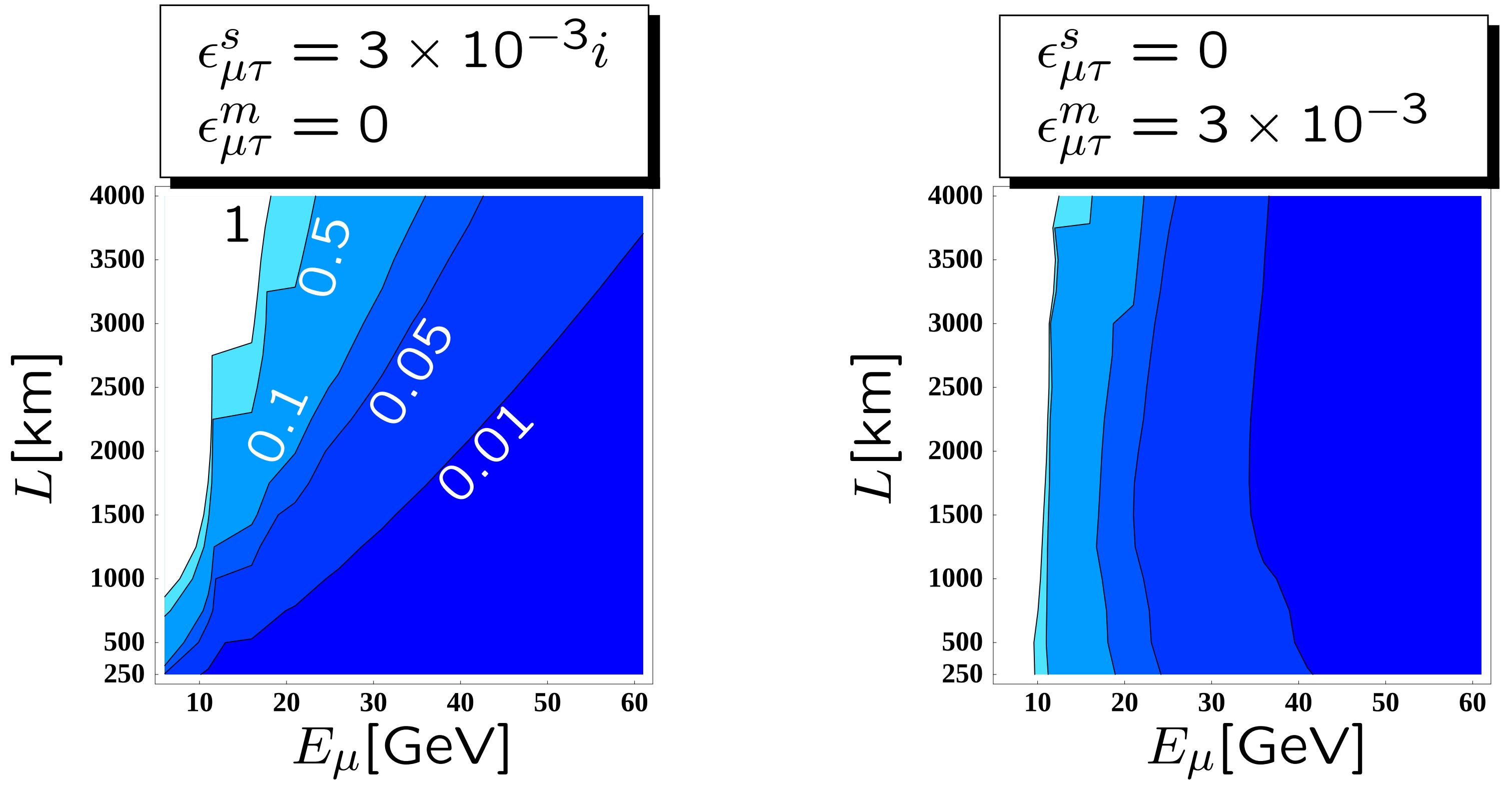
■However, in this channel, the $\epsilon^s_{e\mu}$ interaction induced by the singlet type is forbidden.

■Furthermore, $\epsilon^s_{e\mu}$ effects are strongly constrained by the counter part process.

This example makes it clear that we can search for $\epsilon^{s,m}_{\alpha\beta}$ in $\nu_\alpha \to \nu_\beta$.

■ $\nu_\mu \to \nu_\tau$ **channel,** $\left|\epsilon^{s,m}_{\mu\tau}\right| = 3 \times 10^{-3}$**,** $(V-A)(V-A)$ **type**

■The constraints for these interactions from the charged lepton sector are not so strong.



■ The technologies for ν_τ detection are under R&D.

▼

★ The CERN is now putting two experiments which aim at $\nu_\mu \to \nu_\tau$ into practice. **ICARUS** and **OPERA**

Summary: Model independent analyses

★For $(V-A)(V-A)$ interaction type

■In $\nu_\alpha \to \nu_\beta$, the effects induced by $\epsilon^{s,m}_{\alpha\beta}$ can be observed. The others are too small or easy to be absorbed into the error of the oscillation parameters.

■The expected sensitivity is $\epsilon \gtrsim \mathcal{O}(10^{-4})$.

	$\epsilon^{s,m}_{e\mu}$ $(\epsilon^{s}_{\mu e})$	$\epsilon^{s,m}_{e\tau}$	$\epsilon^{s,m}_{\mu\tau}$
$\nu_e \to \nu_\mu$	△	△	×
$\nu_\mu \to \nu_\mu$	×	×	○
$\nu_e \to \nu_\tau$	×	○	△
$\nu_\mu \to \nu_\tau$	×	△	○
$\nu_\mu \to \nu_e$	△	×	×
$\nu_e \to \nu_e$	×	×	×

★For $(V-A)(V+A)$ type, this method has no merit against the direct measurement.

★The CNGS experiments will be able to detect the new interactions with $\epsilon^{s,m,d}_{\mu\tau} \gtrsim \mathcal{O}(10^{-2})$, depending on their phases.

■Assuming a certain model in the high energy scale...

■First step: we've already done ...

We have investigated what kinds of exotic interaction ($\epsilon^{s,m}_{\alpha\beta}$) can make a detectable signal depending on the oscillation channel.

■If we will observe the signals of the exotic interaction, we will be able to explore the physics in the high energy scale using this information.

■Next step: now we are doing ...

▶We would like to know the correlation between the parameters in the high energy scale and those in the scale we live in ($= \epsilon^{s,m,d}_{\alpha\beta}$).

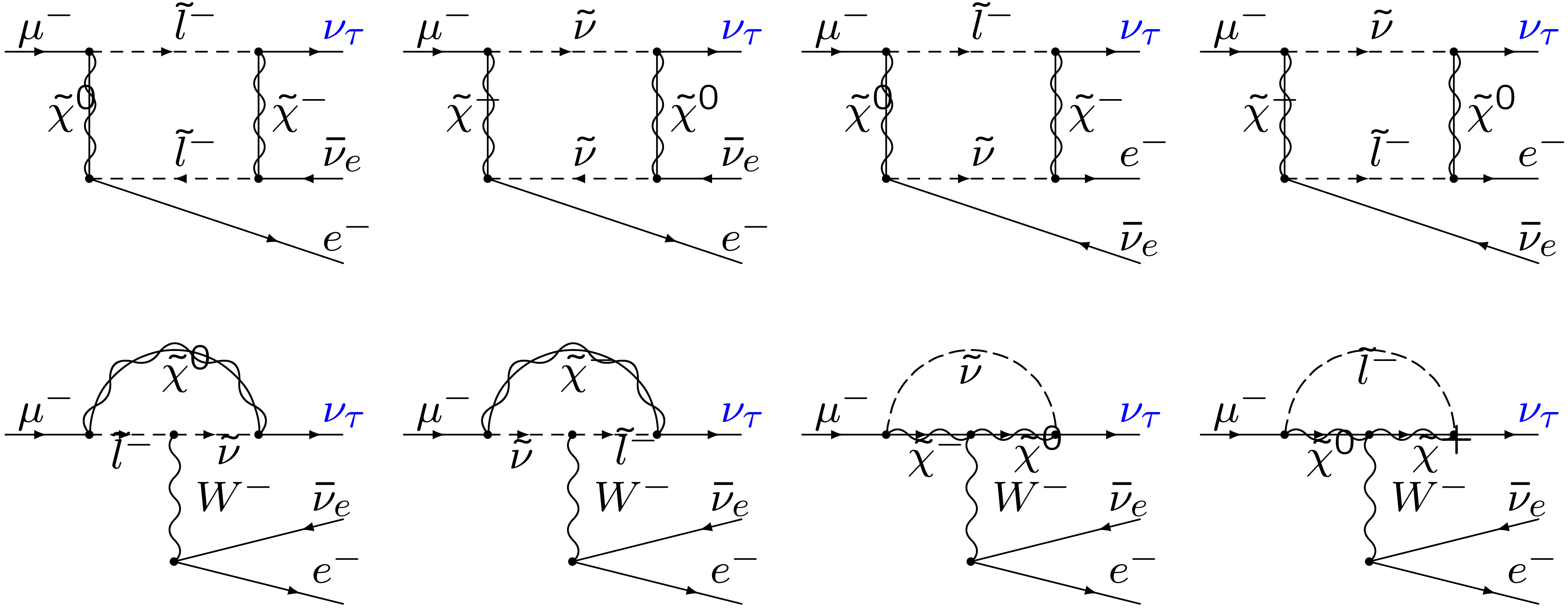
▶The parameters, $\epsilon^{s,m,d}_{\alpha\beta}$, are not independent each other, once we assume a certain model, e.g., MSSM + ν_R.

We will investigate the conditions on the high energy scale to make a detectable signal with the long baseline experiments.

■Flavor violating interaction in a muon decay

■One-loop diagrams which contribute to the muon decay in the case of MSSM + ν_R.

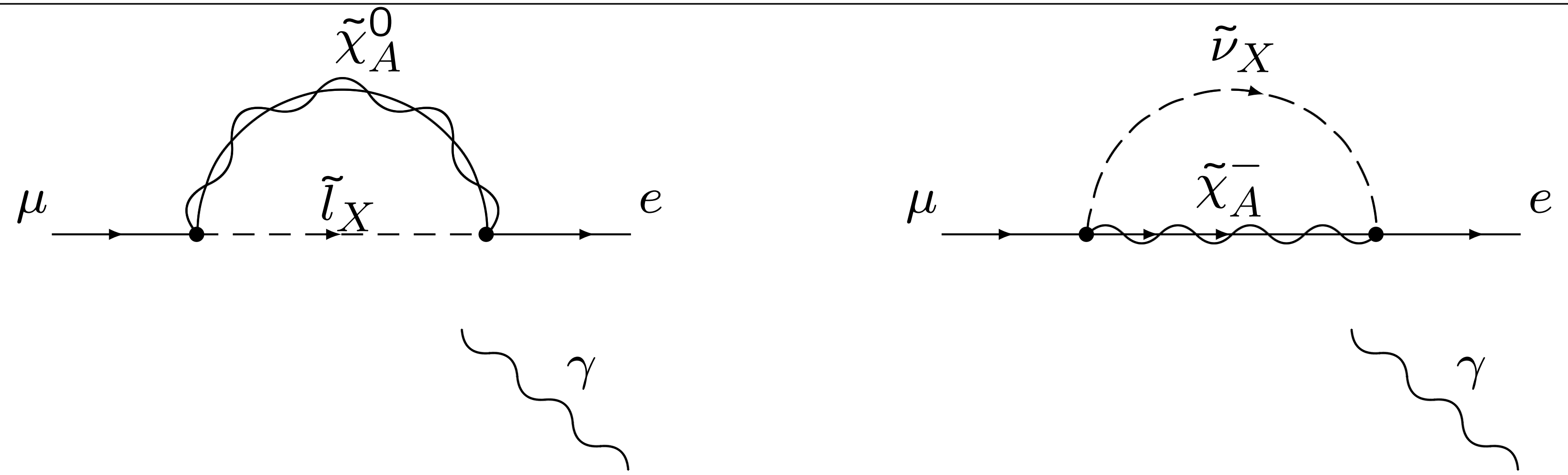
These are parametrized as $\epsilon^s_{\mu\tau}$.



■Flavor violating interaction in the charged lepton sector

■At the same time, the model, MSSM + ν_R, induces the c-LFV.

►*e.g.*, $\mu^{\pm} \to e^{\pm} + \gamma$



J. Hisano *et al*, Phys.Rev.**D53** (1996) 2442.

■Now, we search for the parameter regions which pass the experimental tests on c-LFV and can make the large signal of n-LFV simultaneously.