Neutrino acceleration:
analogy with Fermi acceleration and Comptonization

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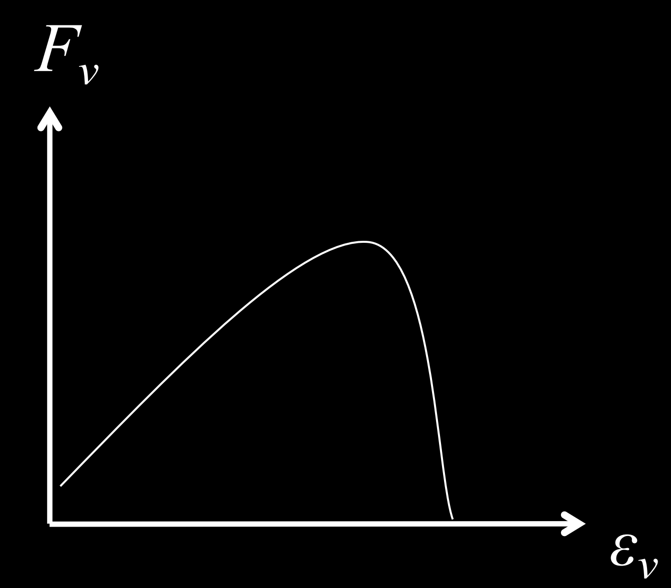
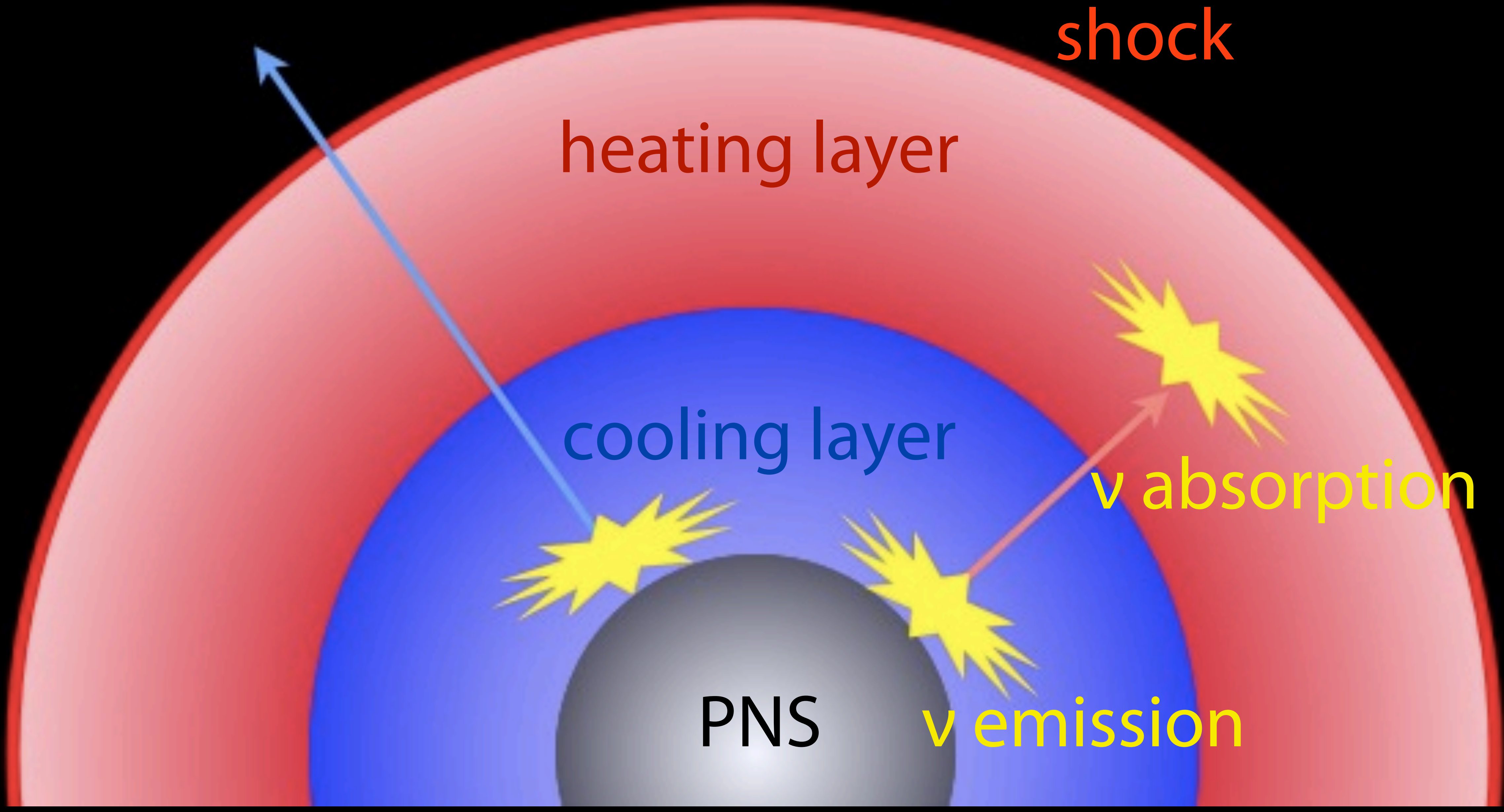
Neutrinos

- **Supernovae, collapsars, mergers**
  - High temperature (~10 MeV), high density ($>10^{12}$g cm^{-3})
  - Copious amount of neutrinos generated ($\sim 10^{53}$ erg)
  - Even neutrinos become optically thick
    - "neutrinospheres"
    - Thermal distribution (0-th approx.)



- **Cross section ($\sigma_\nu \propto \varepsilon_\nu^2$)**
  - Small change of distribution function can lead to significant difference of interaction rates

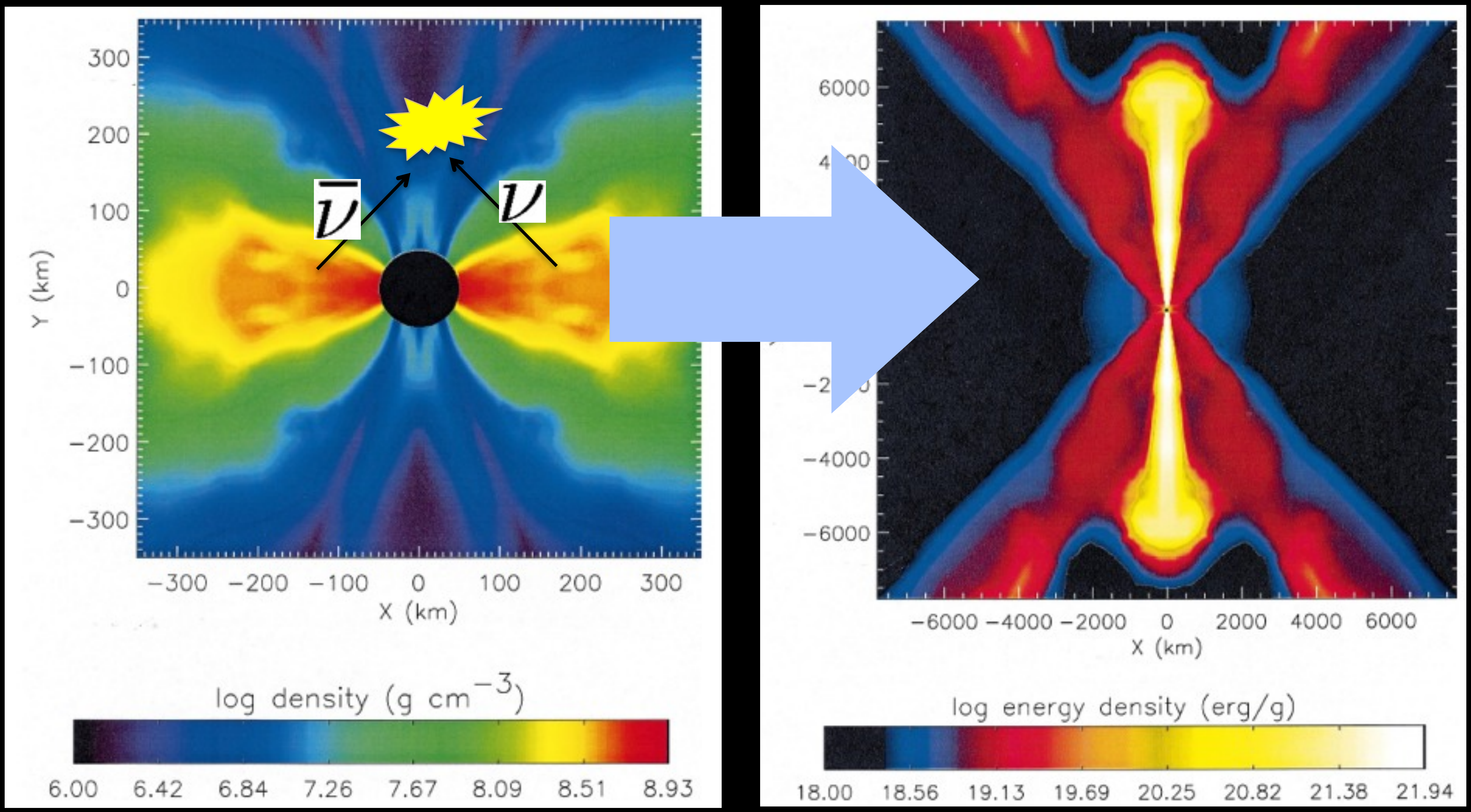
Supernova



see talks by Fischer, Takiwaki, Kuroda, Messer, Sumiyoshi, O'Connor, Pan

Neutrino-driven jet

McFadyen & Woosley 99



see talks by Just, Richers

Problems?

- ★ For supernovae, explosion energy in simulation ($E_{exp}=10^{49\text{-}50}$ erg) is much smaller than observation ($E_{exp}\sim10^{51}$ erg)
- ★ For collapsars, neutrino annihilation might not produce enough strong jet for GRBs
- ★ Is there something missing?
- ★ Let's reconsider about neutrino spectrum in more detail, beyond thermal spectrum

Courtesy of M. Liebendörfer

Neutrino-matter interactions

Bruenn (1985)
Raffelt (2001)

UNI BASEL

Description:	Number Sphere	Energy Sphere	Transport Sphere
Emission & Absorption	Inelastic Scattering	Isoenergetic Scattering	

high ρ — High matter density

low ρ — Low matter density

Can be Pauli-blocked in diffusive regime	Probability enhanced by isoenergetic scattering	Strong energy dependence, ~E^2	
Production/Annihilation	Thermalisation	Scattering	Streaming

Electron or Positron capture

$$\nu_e + n \rightleftharpoons e^- + p$$
$$\bar{\nu}_e + p \rightleftharpoons e^+ + n$$
$$\nu_e + (A,Z) \rightleftharpoons e^- + (A,Z+1)$$

NN bremsstrahlung (Thompson et al. 2002)
νe pair --> νμ pair (Buras et al. 2003)

Neutrino-electron scattering

$$\nu + e \rightleftharpoons \nu + e$$

Inelastic neutrino-nucleus scattering

Elastic coherent scattering of neutrinos on nuclei

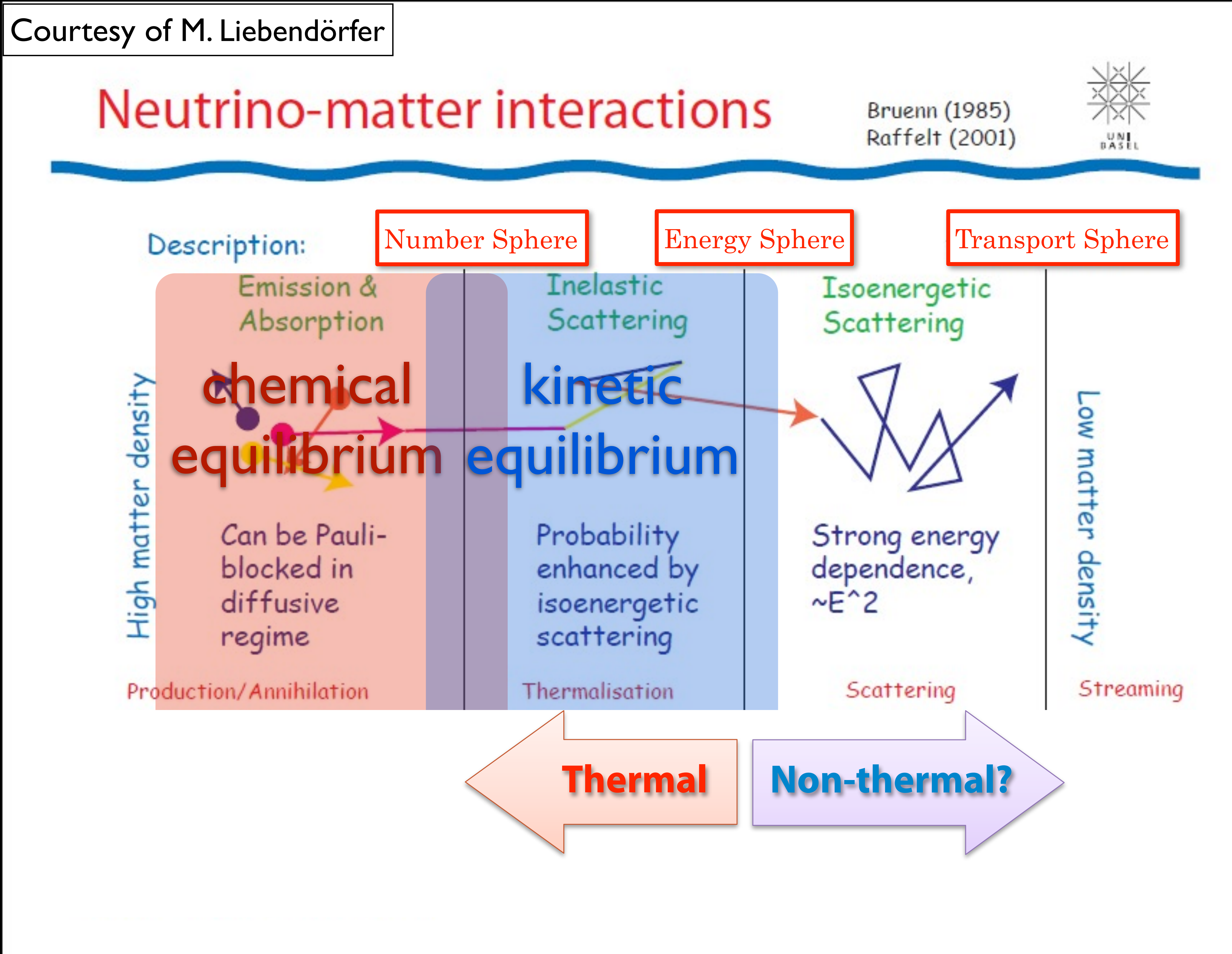
$$\nu + (A,Z) \rightleftharpoons \nu + (A,Z)$$

Neutrino-nucleon scattering

$$\nu + N \rightleftharpoons \nu + N$$

Analogy

- ★ ***Number*** and ***energy*** spheres can be called in different way
  - **chemical equilibrium**:
    => thermal equilibrium
    => inside ***number*** sphere
  - **kinetic equilibrium**:
    => does not change particle number
    => between ***number*** and ***energy*** spheres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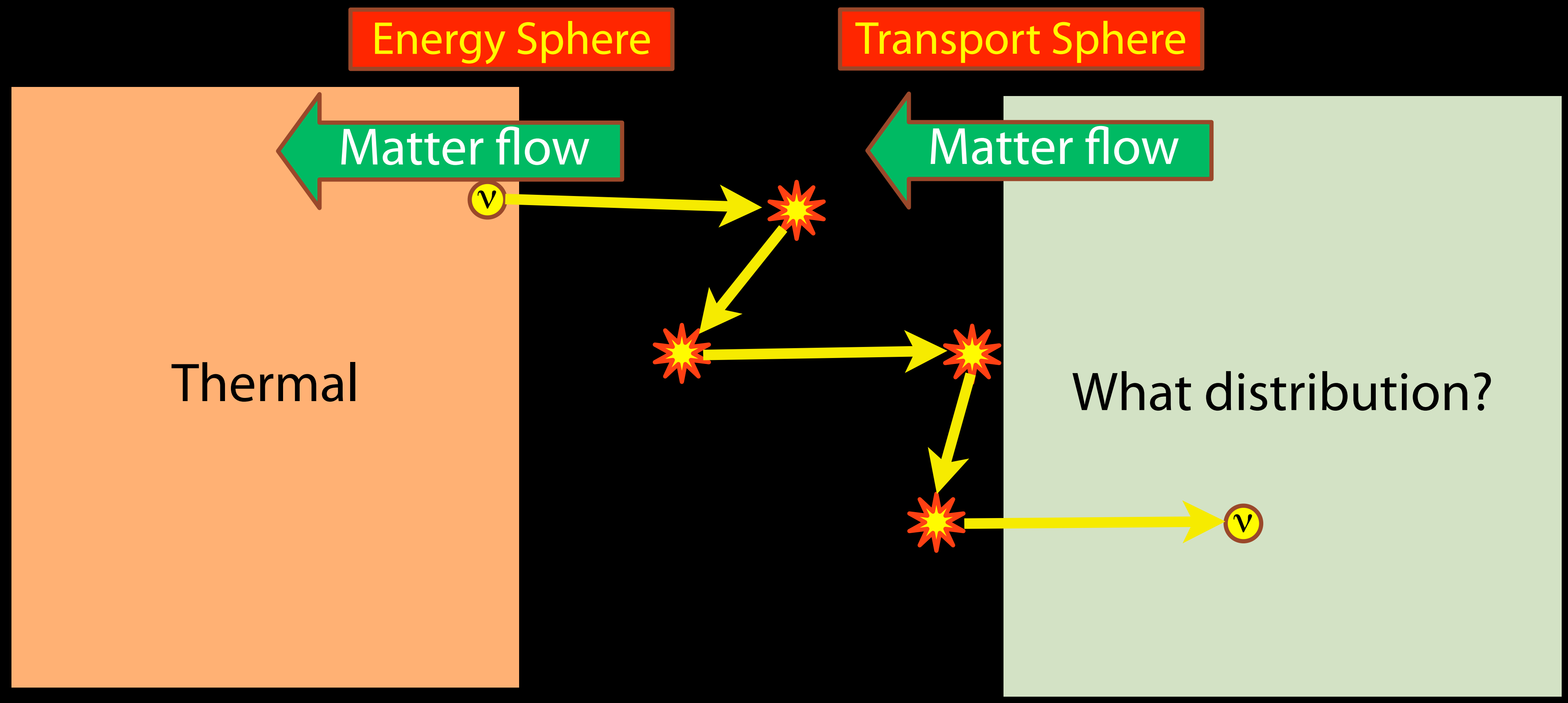
Thermal
Non-thermal?

Non-thermal neutrinos



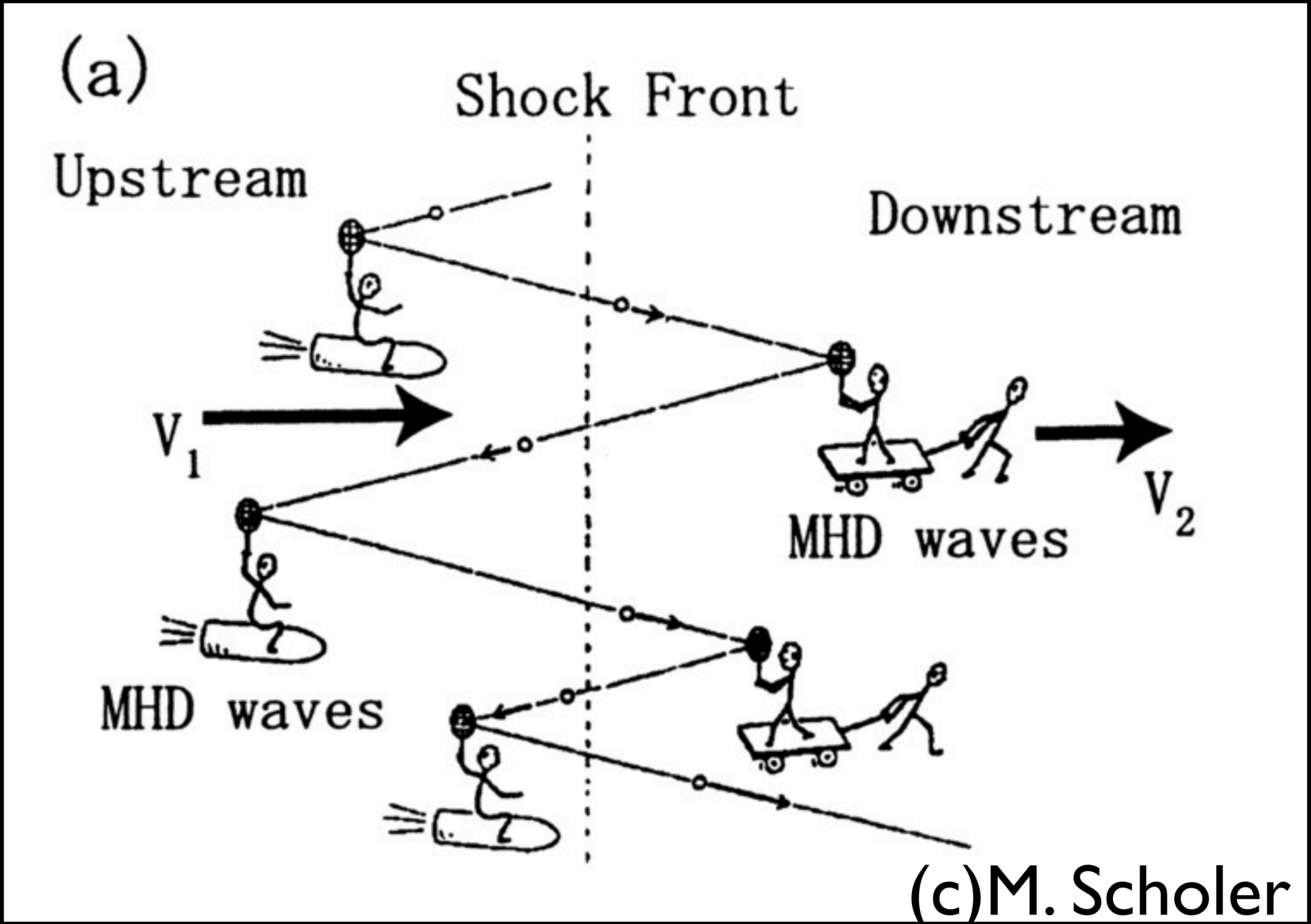
Gain energy by scattering bodies' kinetic energy

$$\langle \Delta E \rangle \sim \frac{\nabla \cdot \mathbf{u}}{\kappa} E$$

"Fermi acceleration" of ν

⇒ **Non-thermal neutrinos**

Fermi acceleration



e.g., Axford+ (1977), Blandford & Ostriker (1978), Bell (1977)

Bulk Comptonization

- ★ The application of Fermi acceleration to photons
- ★ Compressional flow ($\nabla . V<0$) leads to acceleration of photons
- ★ Compression is naturally realized for accretion flows onto black holes / neutron stars (**WITHOUT** shock!)
- ★ Non-thermal components are generated from thermal components

Blandford & Payne (1981), Payne & Blandford (1981)

Let's go to neutrinos

Boltzmann eq. w/ diffusion approx.

Blandford & Payne 1981, Titarchuk+ 1997, Psaltis 1997

$$k^{\mu}\partial_{\mu}n(\mathbf{k}) = \left(\frac{\partial f}{\partial t}\right)_{\rm coll}$$

$$n(\mathrm{l},\nu) = \bar{n}(\nu) + 3\mathrm{l}\cdot\mathrm{f}(\nu)$$

diffusion approx.

$$\langle \mathrm{u} \rangle = \mathrm{V}$$

bulk velocity

$$\langle \mathrm{u}^2 \rangle = \frac{3k_BT}{m} + V^2$$

thermal & turbulent vel.

$$\frac{\partial n}{\partial t} + \mathbf{V}\cdot\nabla n = \nabla\cdot\left(\frac{c}{3\kappa}\nabla n\right) + \frac{1}{3}(\nabla\cdot\mathbf{V})\epsilon_{\nu}\frac{\partial n}{\partial \epsilon_{\nu}} + \frac{1}{\epsilon_{\nu}^2}\frac{\partial}{\partial \epsilon_{\nu}}\left[\frac{\kappa}{mc^2}\epsilon_{\nu}^4\left(n + (k_BT + \frac{mV^2}{3})\frac{\partial n}{\partial \epsilon_{\nu}}\right)\right] + j(\mathbf{r},\epsilon_{\nu})$$

Transfer equation

Boltzmann equation with diffusion approx., up to $O((u/c)^2)$

$$\frac{\partial n}{\partial t} + \mathbf{V}\cdot\nabla n = \nabla\cdot\left(\frac{c}{3\kappa}\nabla n\right) + \frac{1}{3}(\nabla\cdot\mathbf{V})\epsilon_\nu\frac{\partial n}{\partial \epsilon_\nu} + \frac{1}{\epsilon_\nu^2}\frac{\partial}{\partial \epsilon_\nu}\left[\frac{\kappa}{mc^2}\epsilon_\nu^4\left(n + (k_BT + \frac{mV^2}{3})\frac{\partial n}{\partial \epsilon_\nu}\right)\right] + j(\mathbf{r},\epsilon_\nu)$$

diffusion term | bulk term | recoil term | thermal & turbulent terms | source term

n: ν's number density

ε_ν: ν energy

$\mathbf{V}$: velocity of matter

κ: opacity

T: temperature of matter

First order term

By neglecting $O((u/c)^2)$ terms and recoil term, we get

$$\frac{\partial n}{\partial t} + \mathbf{V}\cdot\nabla n = \nabla\cdot\left(\frac{c}{3\kappa}\nabla n\right) + \frac{1}{3}(\nabla\cdot\mathbf{V})\epsilon_\nu \frac{\partial n}{\partial \epsilon_\nu} + j(\mathbf{r},\epsilon_\nu)$$

This is exactly the same equation we are solving with MGFLD or IDSA

MGFLD
Bruenn (1985)

$$\frac{1}{c}\frac{\partial}{\partial t}\psi^{(0)} - \frac{1}{3r^2}\frac{d}{dr}\left\{r^2\lambda^{(t)}(\omega)\left[\frac{\partial}{\partial r}\psi^{(0)}(\omega) - A^{(1)}(\omega)\psi^{(0)} - C^{(1)}(\omega)\right]\right\} + \frac{1}{3c}\frac{\partial \ln\rho}{\partial t}\left(\omega\frac{\partial}{\partial \omega}\psi^{(0)}\right)$$
$$= X(\omega) + Y(\omega)\psi^{(0)} + Z(\omega)\frac{\partial}{\partial r}\psi^{(0)}, \quad \text{(A27)}$$

IDSA
Liebendörfer+ (2009)

$$\frac{df^{t}}{cdt} + \frac{1}{3}\frac{d\ln\rho}{cdt}E\frac{\partial f^{t}}{\partial E} = j - (j+\chi)f^{t} - \Sigma. \quad (5)$$

$$\Sigma = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{-r^2}{3(j+\chi+\phi)}\frac{\partial f^{t}}{\partial r}\right) + (j+\chi)\frac{1}{2}\int f^{s} d\mu. \quad (6)$$

Original ν-Boltzmann eq. (Lindquist 1966, Castor 1972)

$$\frac{df}{cdt} + \mu\frac{\partial f}{\partial r} + \left[\mu\left(\frac{d\ln\rho}{cdt} + \frac{3u}{cr}\right)\right](1-\mu^2)\frac{\partial f}{\partial \mu} + \left[\mu^2\left(\frac{d\ln\rho}{cdt} + \frac{3u}{cr}\right) - \frac{u}{cr}\right]\epsilon_\nu\frac{\partial f}{\partial \epsilon_\nu}$$
$$= j(1-f) - \chi f + \frac{E^2}{c(hc)^3}\left[(1-f)\int Rf' d\mu' - f\int R(1-f')d\mu'\right]$$

spherically symmetric
up to $O(u/c)$

$d\ln\rho/dt = \nabla.V$

Order of approx.

$$k^\mu \partial_\mu n(\mathbf{k}) = \left(\frac{\partial f}{\partial t}\right)_{\rm coll}$$

O(u/c)

$$\frac{df}{cdt} + \mu\frac{\partial f}{\partial r} + \left[\mu\left(\frac{d\ln\rho}{cdt} + \frac{3u}{cr}\right)\right](1-\mu^2)\frac{\partial f}{\partial \mu} + \left[\mu^2\left(\frac{d\ln\rho}{cdt} + \frac{3u}{cr}\right) - \frac{u}{cr}\right]\epsilon_\nu\frac{\partial f}{\partial \epsilon_\nu}$$
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diffusion approx.

diffusion approx.

$$\frac{\partial n}{\partial t} + \mathbf{V}\cdot\nabla n = \nabla\cdot\left(\frac{c}{3\kappa}\nabla n\right) + \frac{1}{3}(\nabla\cdot\mathbf{V})\epsilon_\nu\frac{\partial n}{\partial \epsilon_\nu} + \frac{1}{\epsilon_\nu^2}\frac{\partial}{\partial \epsilon_\nu}\left[\frac{\kappa}{mc^2}\epsilon_\nu^4\left(n + (k_B T + \frac{mV^2}{3})\frac{\partial n}{\partial \epsilon_\nu}\right)\right] + j(\mathbf{r},\epsilon_\nu)$$

O(u/c)

$$\frac{\partial n}{\partial t} + \mathbf{V}\cdot\nabla n = \nabla\cdot\left(\frac{c}{3\kappa}\nabla n\right) + \frac{1}{3}(\nabla\cdot\mathbf{V})\epsilon_\nu\frac{\partial n}{\partial \epsilon_\nu} + j(\mathbf{r},\epsilon_\nu)$$

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diffusion term

bulk term

recoil term

source term

thermal & turbulent terms

n: ν's number density

ε_ν: ν energy

$\mathbf{V}$: velocity of matter

κ: opacity

T: temperature of matter

Solve this equation with adequate boundary condition. The background matter is assumed to be free fall and stationary solution ($\partial/\partial t=0$) is obtained.

Analytic solutions

Nondimensional equation

$$\tau \frac{\partial^2 f_\nu}{\partial \tau^2} - \left(2\tau + \frac{3}{2}\right)\frac{\partial f_\nu}{\partial \tau} = \frac{1}{2} x \frac{\partial f_\nu}{\partial x}$$

$$V(r) = c\left(\frac{r_s}{r}\right)^{1/2}$$

$$\dot{m} = \frac{\dot{M}}{\dot{M}_{E,\nu}}$$

$$\tau_{sc}(r) = \int_r^\infty dr n(r)\sigma(\varepsilon_\nu) = \dot{m}\left(\frac{r_s}{r}\right)^{1/2}$$

$$\tau = \frac{3}{2}u(r)\tau_{sr}(r) = \frac{3}{2}\dot{m}\frac{r_s}{r}$$

$$x = \frac{\varepsilon_\nu}{kT}$$

$f_\nu(\tau,x)=R(\tau)\tau^{5/2}x^{-\alpha}$

(separation of variables)

Boundary conditions

1. flux $\propto \tau^2$ $(\tau \rightarrow 0)$
2. remain finite for $\tau >> 1$

$$R(\tau) = \sum_{n=0}^{\infty} c_n L_n^{5/2}(2\tau)$$

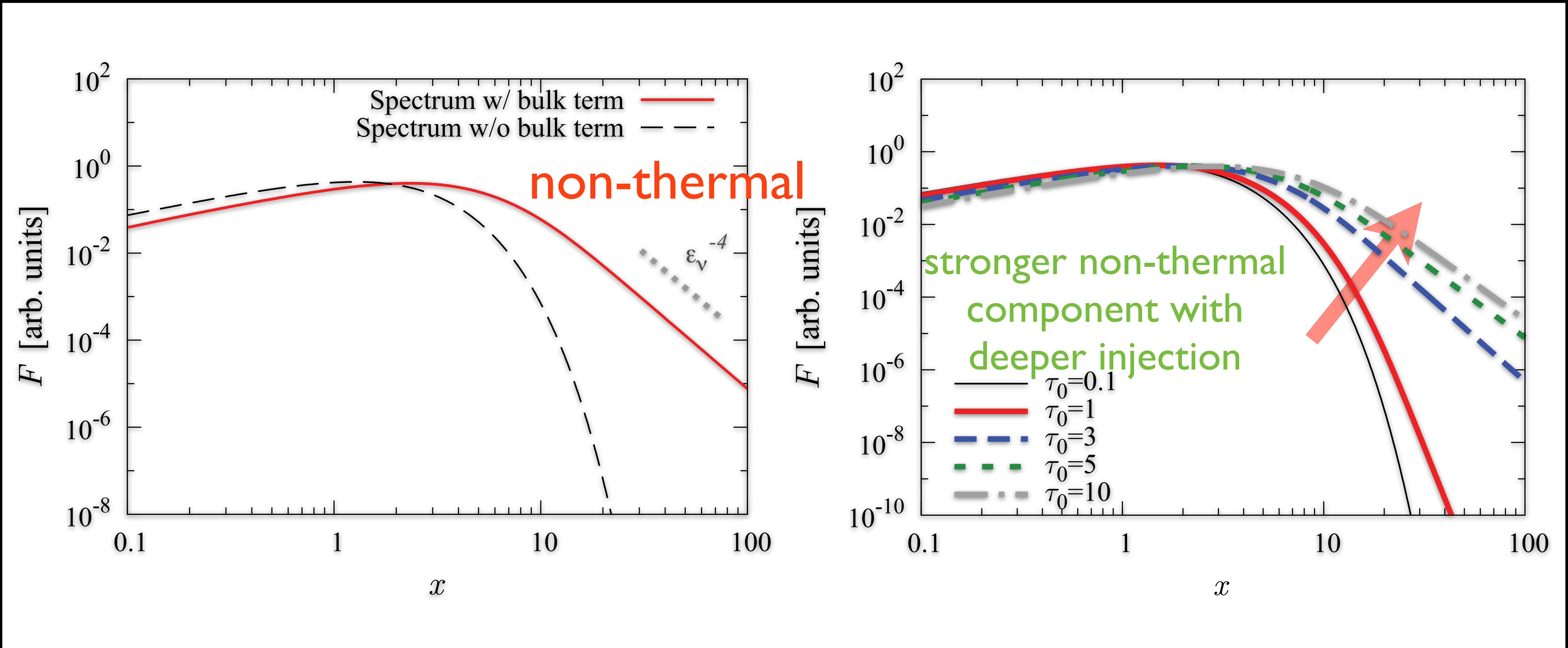
$$\alpha_n = 4n + 10 \quad (n=0,1,2\ldots)$$

spectral energy flux $F_\nu \propto \varepsilon_\nu^{-4}$

YS, MNRAS (2013)

Numerical solution

★ Solved the transfer equation using relaxation method
★ At $\tau=\tau_0$ (@energy sphere), thermal distribution is imposed



YS, MNRAS (2013)

Neutrino annihilation

Energy injection rate by neutrino pair annihilation

Goodman+ 87, Setiawan+06

$$\dot{E}_{\nu\bar{\nu}} = \mathcal{C} F_{3,\nu} F_{3,\bar{\nu}} \left(\frac{\left\langle \varepsilon_{\nu}^{2} \right\rangle \langle \varepsilon_{\bar{\nu}} \rangle + \left\langle \varepsilon_{\bar{\nu}}^{2} \right\rangle \langle \varepsilon_{\nu} \rangle}{\langle \varepsilon_{\nu} \rangle \langle \varepsilon_{\bar{\nu}} \rangle} \right)$$

$$F_{i,\nu} = \int f_{\nu} \varepsilon_{\nu}^{i} \mathrm{d}\varepsilon_{\nu} \quad \langle \varepsilon_{\nu} \rangle = F_{3,\nu}/F_{2,\nu} \quad \langle \varepsilon_{\nu}^{2} \rangle = F_{4,\nu}/F_{2,\nu}$$

$$\dot{E}_{\nu\bar{\nu}} \propto \frac{F_{3,\nu}^{2} \left\langle \varepsilon_{\nu}^{2} \right\rangle}{\langle \varepsilon_{\nu} \rangle} \propto \langle \varepsilon_{\nu} \rangle \langle \varepsilon_{\nu}^{2} \rangle .$$

τ_0	$\langle \varepsilon_\nu \rangle / \langle \varepsilon_\nu \rangle_{thermal}$	$\langle \varepsilon_\nu^2 \rangle / \langle \varepsilon_\nu^2 \rangle_{thermal}$	Amplification
0.1	1.01	1.02	1.03
0.2	1.03	1.05	1.08
0.5	1.07	1.16	1.24
1.0	1.16	1.37	1.59
2.0	1.37	1.99	2.73
3.0	1.60	2.83	4.52
5.0	1.95	4.49	12.5
10.0	2.43	7.12	17.3

Annihilation rate can be amplified by a factor of ~10 for the case of $\tau_0 = 10$

Does it work for supernova?

- ★ **Unfortunately, no**
- ★ **To accelerate radiations $\nabla . V$ need to be large at optically thick regime, but $\nabla . V$ is small in the vicinity of PNS**
- ★ **For a black-hole forming collapse, this mechanism naively works (competition of acceleration and advection times)**

Higher order effects?

- ★ Bulk Comptonization is $O(u/c)$ effect WITH compressional flow
- ★ Is there any effects from higher order? Let's learn from photon case again
  - Thermal Comptonization
  - Turbulent Comptonization

Turbulent Comptonization

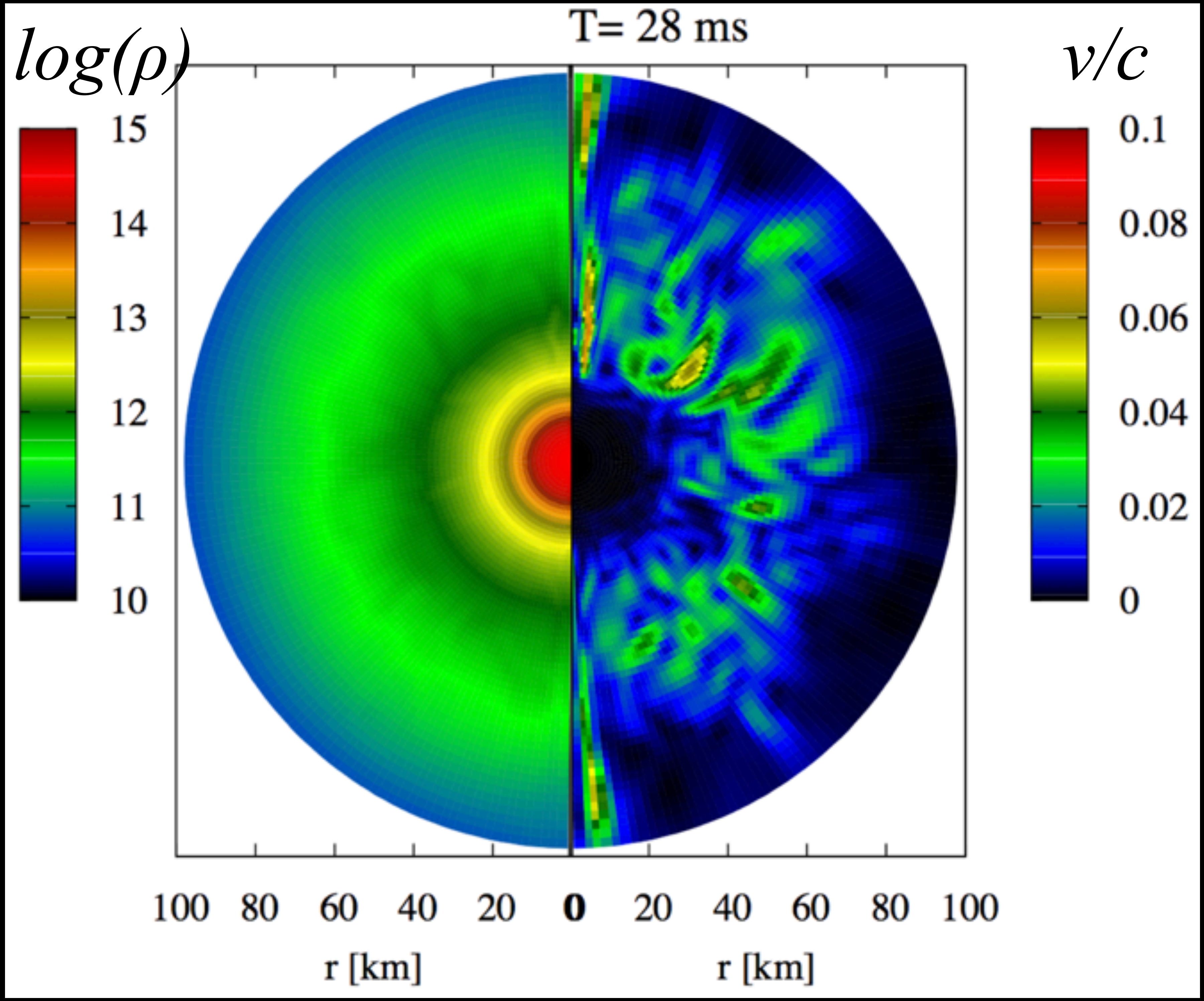
- ★ When there are turbulent flows, stochastic scattering can accelerate particles, like second order Fermi acceleration
- ★ Compressional flow is unnecessary, i.e., even when $\nabla . V=0$, particle acceleration is possible

e.g., Zel'dovich, Illarinov, Sunyaev (1972), Thompson (1994), Socrates (2004)

Neutrino transfer

	Boltzmann solver		$max(v/c)$ in PNS
	$O(u/c)$	$O((u/c)^2)$	
spherical symmetry (1D)	included	sometimes included	$\sim< 10^{-3}$
multi dimension (2D/3D)	sometimes included	not included	$\sim 0.1?$

Turbulent velocity



from neutrino-radiation hydro. simulation by Suwa+ (2014)

Summary

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diffusion term · **bulk term** · **recoil term** · **thermal & turbulent terms** · **source term**

- ★ Based on analogy of photons, neutrino acceleration is investigated
- ★ *O(u/c)*: bulk Comptonization for γ
  => non-thermal ν from collapsars
- ★ *O((u/c)²)*: thermal/turbulent Comptonization for γ
  => non-thermal ν from supernovae
- ★ Non-thermal ν can amplify neutrino interaction rate due to its high-energy tail