超新星におけるニュートリノ輸送方程式の解析解と観測への応用

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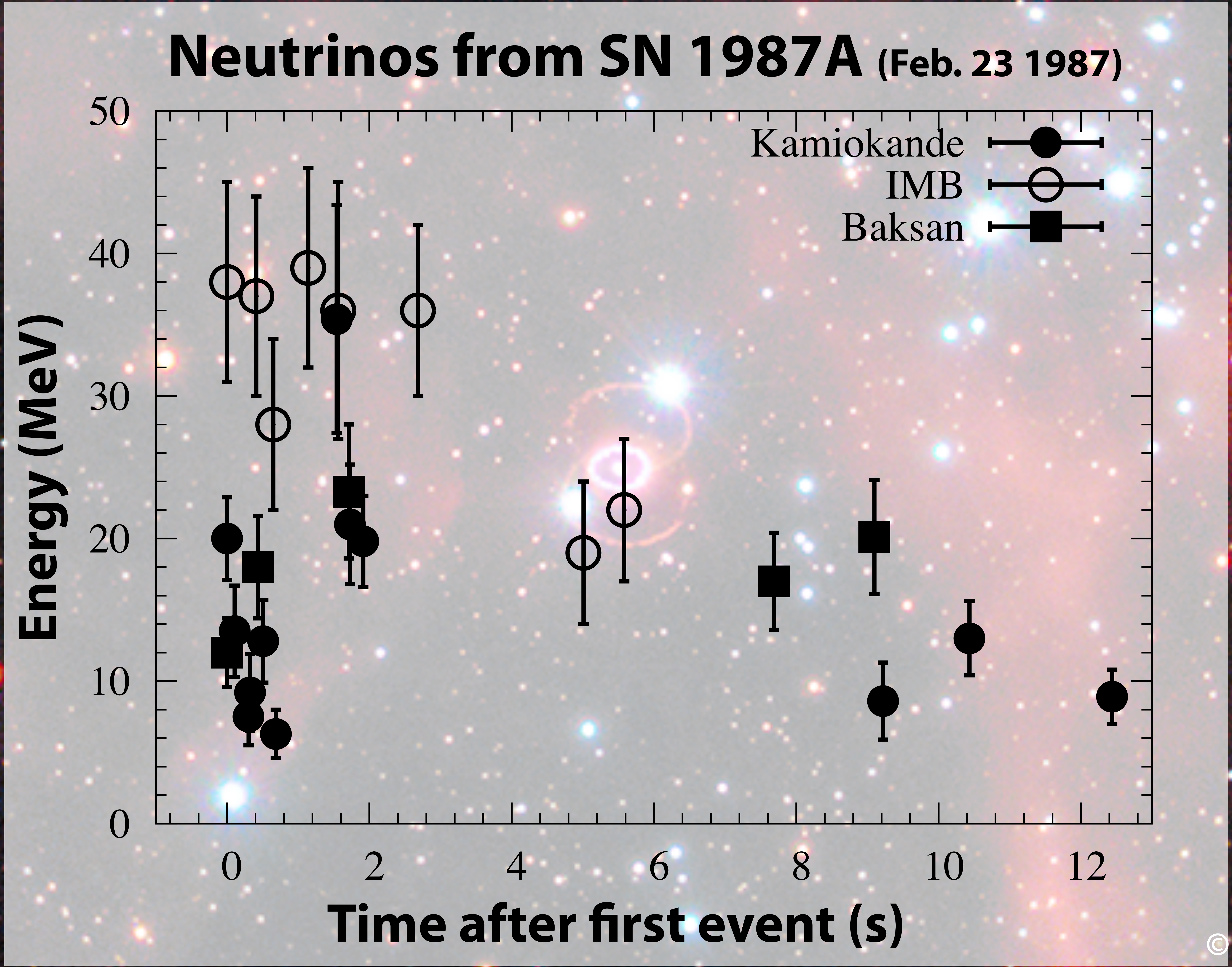
Supernova neutrinos



Why do we need an analytic model?

Forward problem

Protoneutron-star (PNS) physics

↓

Transport calculation

↓

Detector response

High fidelity, but expensive to explore and difficult to invert directly.

↔

Inverse problem

Observed events

↓

Luminosity and spectrum

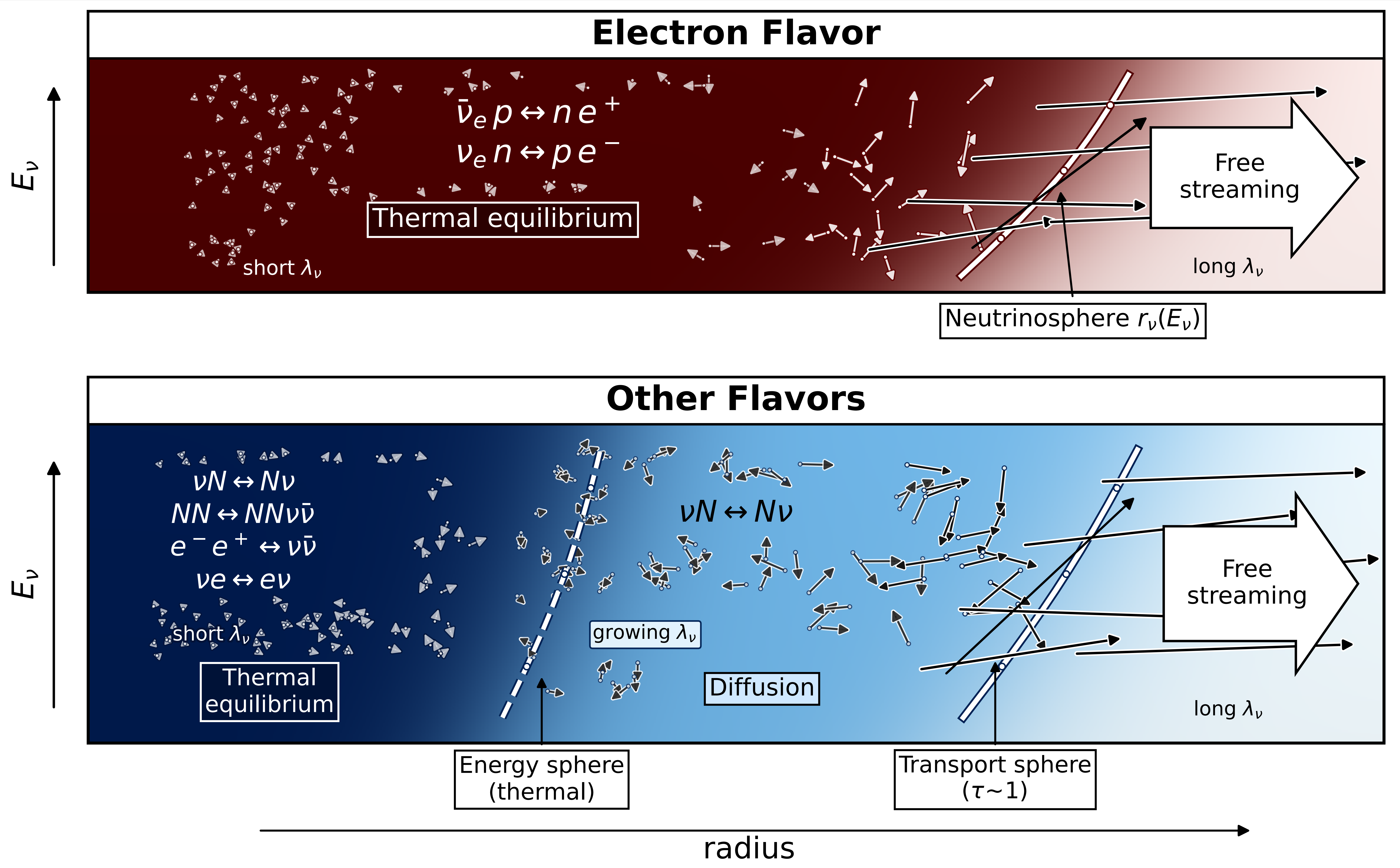
↓

NS parameters (M, R, etc.)

Finite counts, degeneracies, detector effects, and model uncertainty.

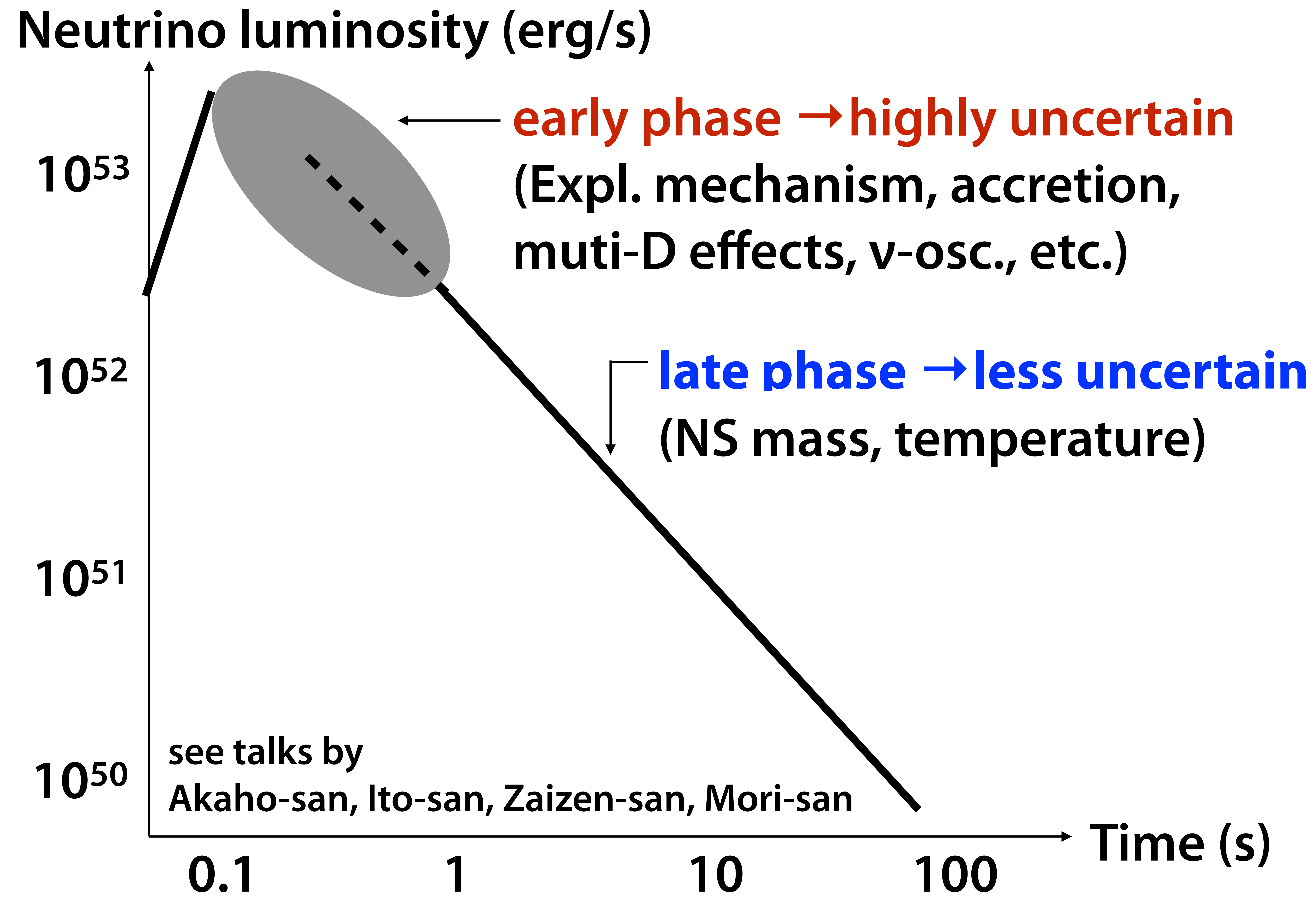
Analytic solution: an interpretable map between theory and observation

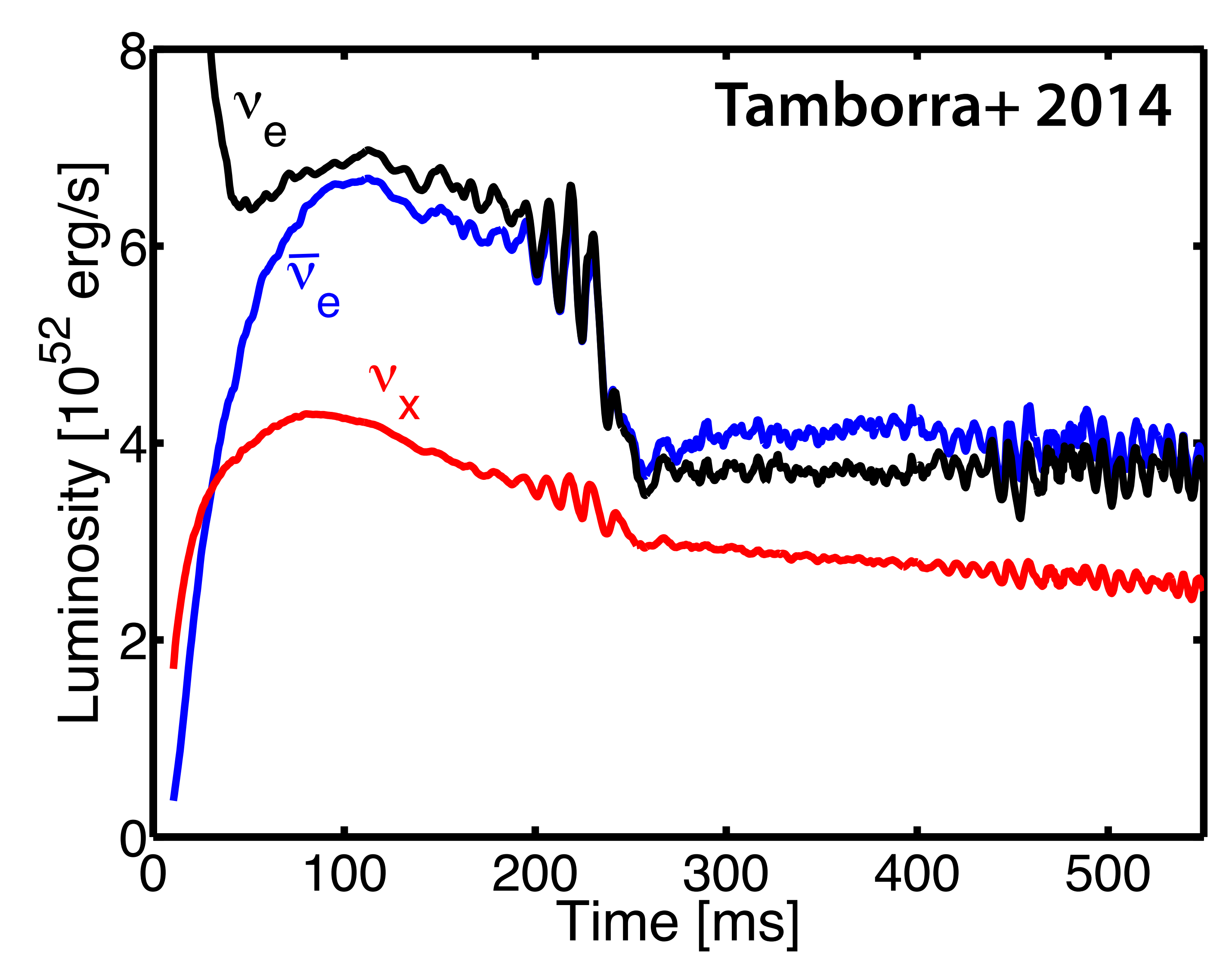
Neutrinosphere



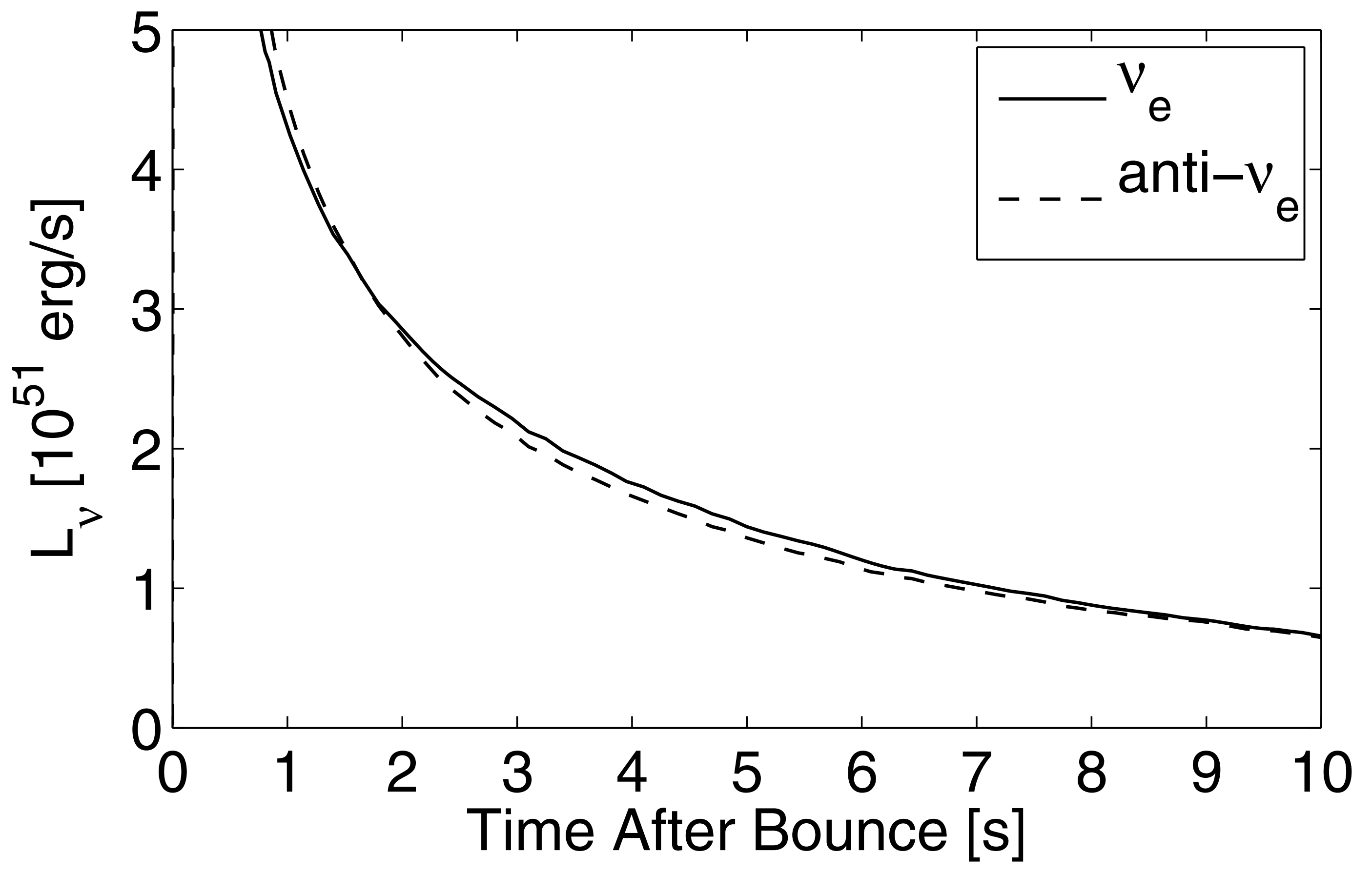
Suwa 2026

Late cooling phase is simple and understandable





18 solar mass



Fischer+ 2010

see also Burrows 1988, Fischer+ 2009, Hüdepohl+ 2010, Roberts+ 2012, Mirizzi+ 2016, Li+ 2021

3 Steps

step 1

NUMERICAL SIMULATIONS

- Cooling curves of PNS
- Detailed physics included
- Discrete grid of data set
- Computationally expensive

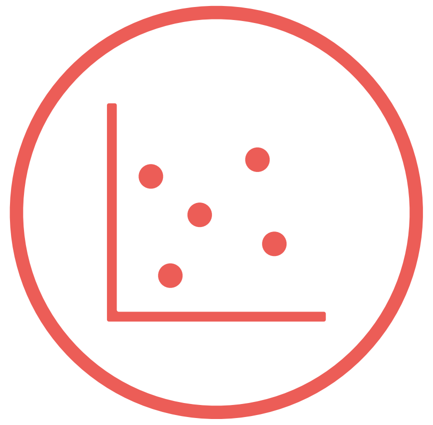
step 2

$f(x)$

ANALYTIC SOLUTIONS

- Analytic cooling curves
- Calibrated w/ numerical sol.
- Simplified but essential physics included
- Fast and continuous

step 3



DATA ANALYSIS

- Mock sampling
- Analysis pipeline for real data
- Error estimate for future observations

How to derive analytic solution

Neutrino Boltzmann eq.

$$\frac{df}{dt} = C[f]$$

Diffusion approx.

$$\frac{\partial \epsilon_\nu}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_\nu) = S_\nu$$

$$F_\nu = \frac{c}{3}\frac{d\epsilon_\nu}{d\tau}$$

Neutrinosphere (τ_ν=2/3)

$$L_\nu = 4\pi R_\nu^2 F_\nu \Big|_{r=R_\nu}$$

Time evolution

$$\frac{dE_{\mathrm{th}}}{dt} = -6L_\nu$$

Analytic solutions

[Suwa, Harada, Nakazato, Sumiyoshi, PTEP, 2021, 0130E01 (2021)]

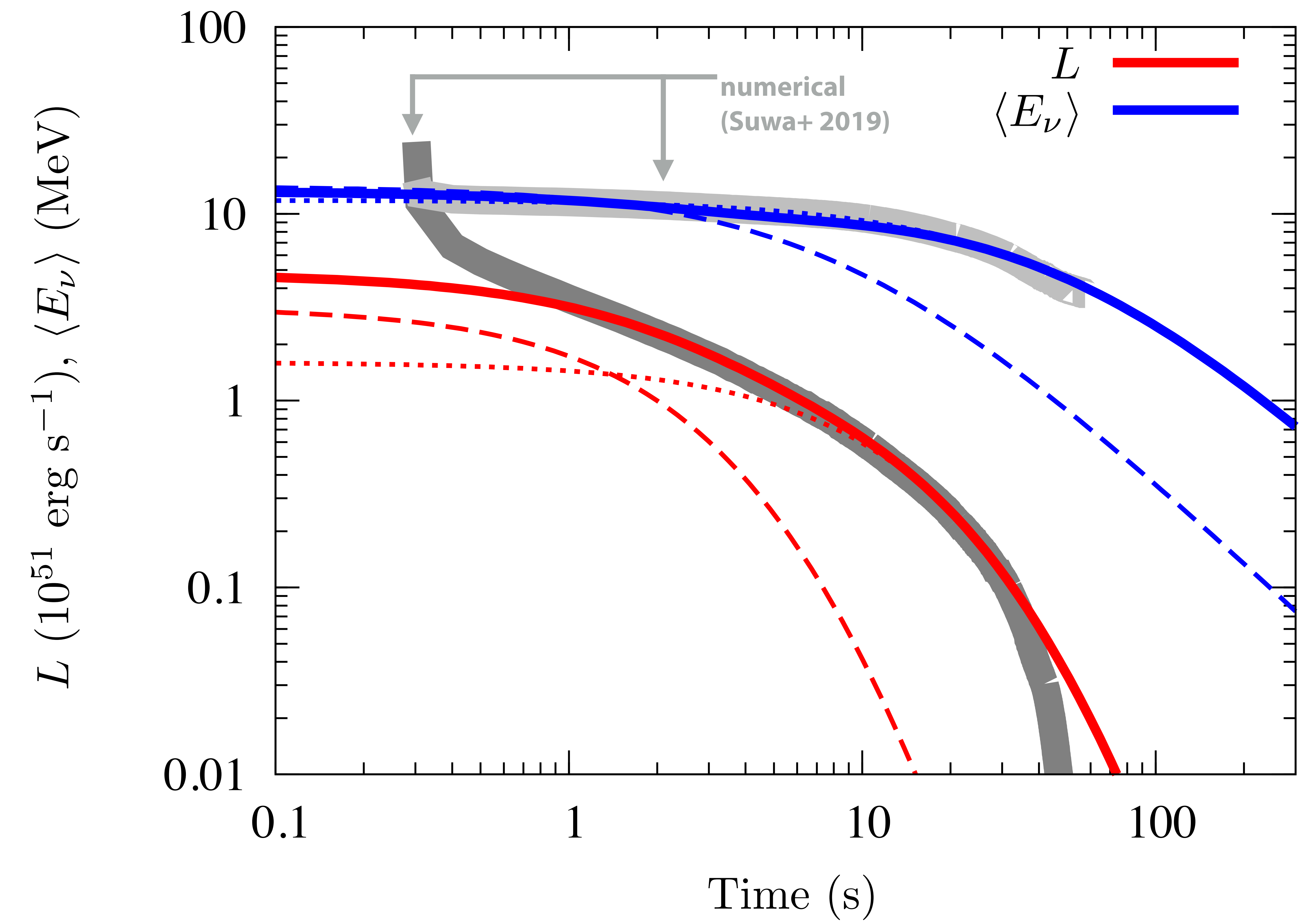
- Solve neutrino transport eq. analytically
  - **Neutrino luminosity**

$$L = 3.3\times10^{51}\,\mathrm{erg\,s^{-1}}\left(\frac{M_{\rm PNS}}{1.4M_\odot}\right)^{6}\left(\frac{R_{\rm PNS}}{10\,\mathrm{km}}\right)^{-6}\left(\frac{g\beta}{3}\right)^{4}\left(\frac{t+t_0}{100\,\mathrm{s}}\right)^{-6}$$

  - **Neutrino average energy**

$$\langle E_\nu\rangle = 16\,\mathrm{MeV}\left(\frac{M_{\rm PNS}}{1.4M_\odot}\right)^{3/2}\left(\frac{R_{\rm PNS}}{10\,\mathrm{km}}\right)^{-2}\left(\frac{g\beta}{3}\right)\left(\frac{t+t_0}{100\,\mathrm{s}}\right)^{-3/2}$$

  - **two-component model**
    - early cooling phase (β=3)
    - late cooling phase (β=O(10))



The parameter dependence remains explicit

- **Event rate**

$$\mathscr{R}(t) \propto \frac{L_\nu \left\langle E_\nu \right\rangle}{D^2} \propto \underbrace{D^{-2}}_{\textit{distance}} \quad \underbrace{M_{\rm PNS}^{15/2} R_{\rm PNS}^{-8}}_{\textit{structure}} \quad \underbrace{(g\beta)^5}_{\textit{opacity}} \quad (t+t_0)^{-15/2}$$

- **Average energy of positrons**

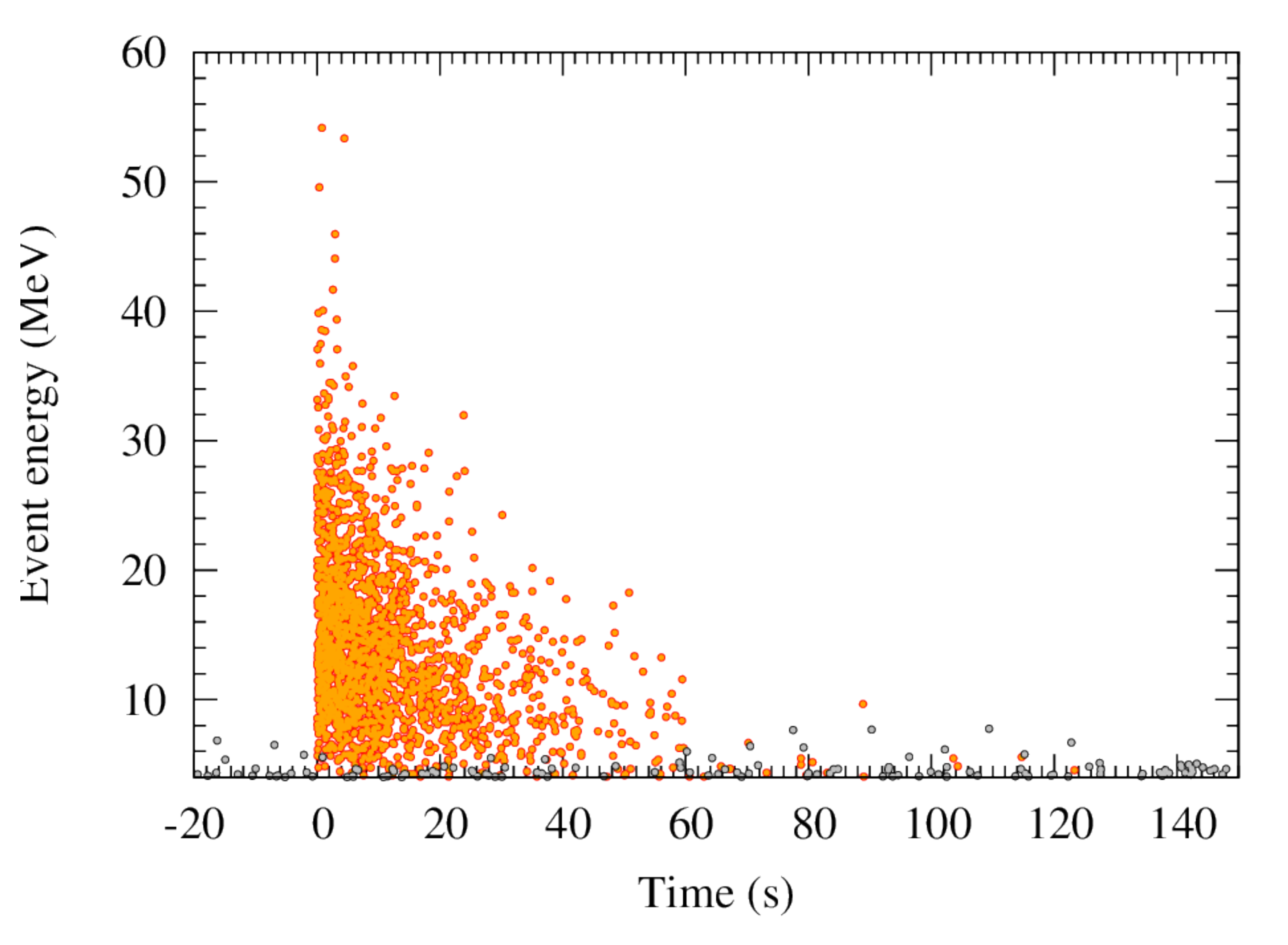
$$\left\langle E_{e^+} \right\rangle(t) \propto \left\langle E_\nu \right\rangle \propto \underbrace{M_{\rm PNS}^{3/2} R_{\rm PNS}^{-2}}_{\textit{structure}} \quad \underbrace{(g\beta)}_{\textit{opacity}} \quad (t+t_0)^{-3/2}$$

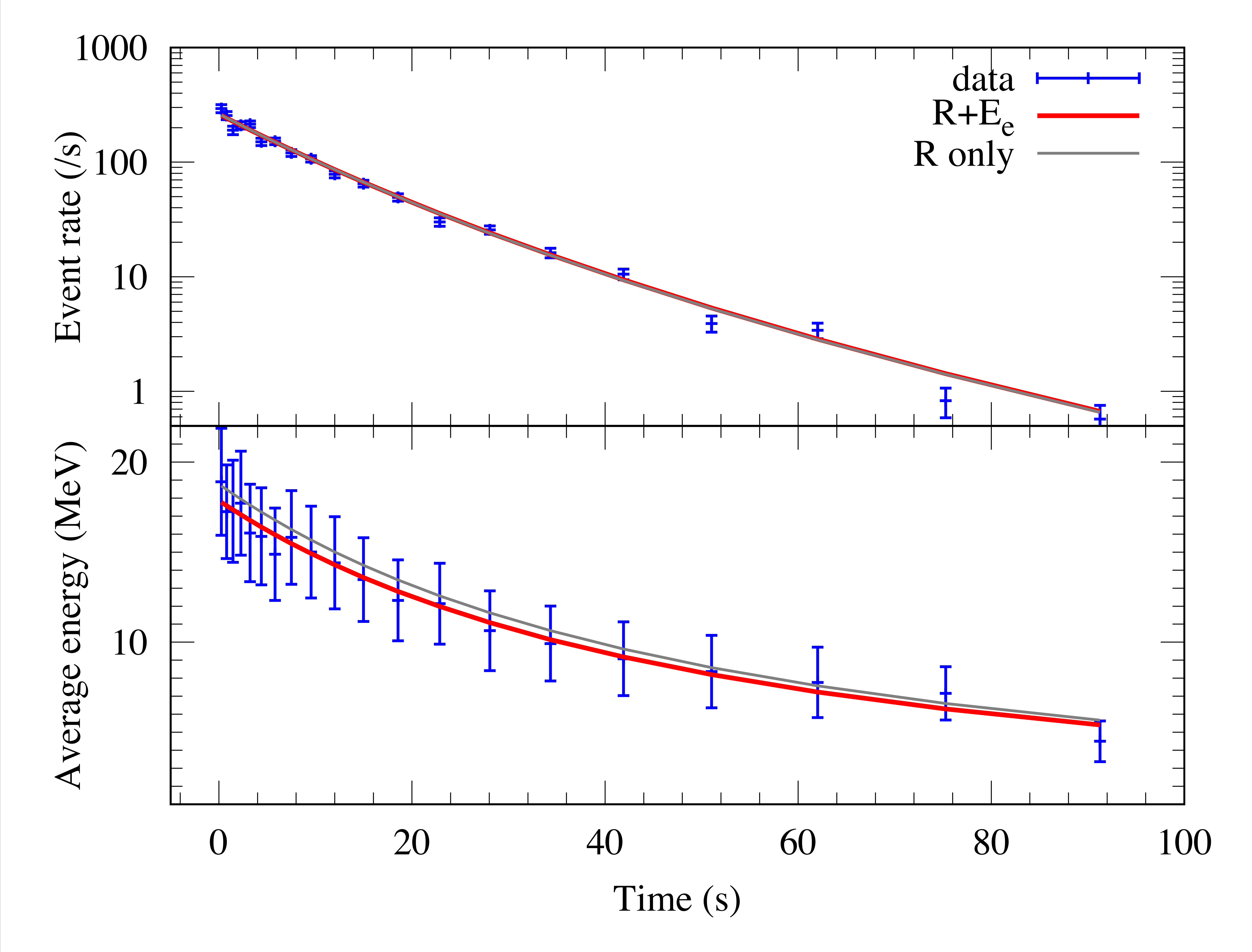
- **Timescale**

$$t_0 \propto \underbrace{M_{\rm PNS}^{6/5} R_{\rm PNS}^{-6/5}}_{\textit{structure}} \quad \underbrace{(g\beta)^{4/5}}_{\textit{opacity}} \quad \underbrace{E_{\rm tot}^{-1/5}}_{\textit{energetics}}$$

From an analytic light curve to detector events

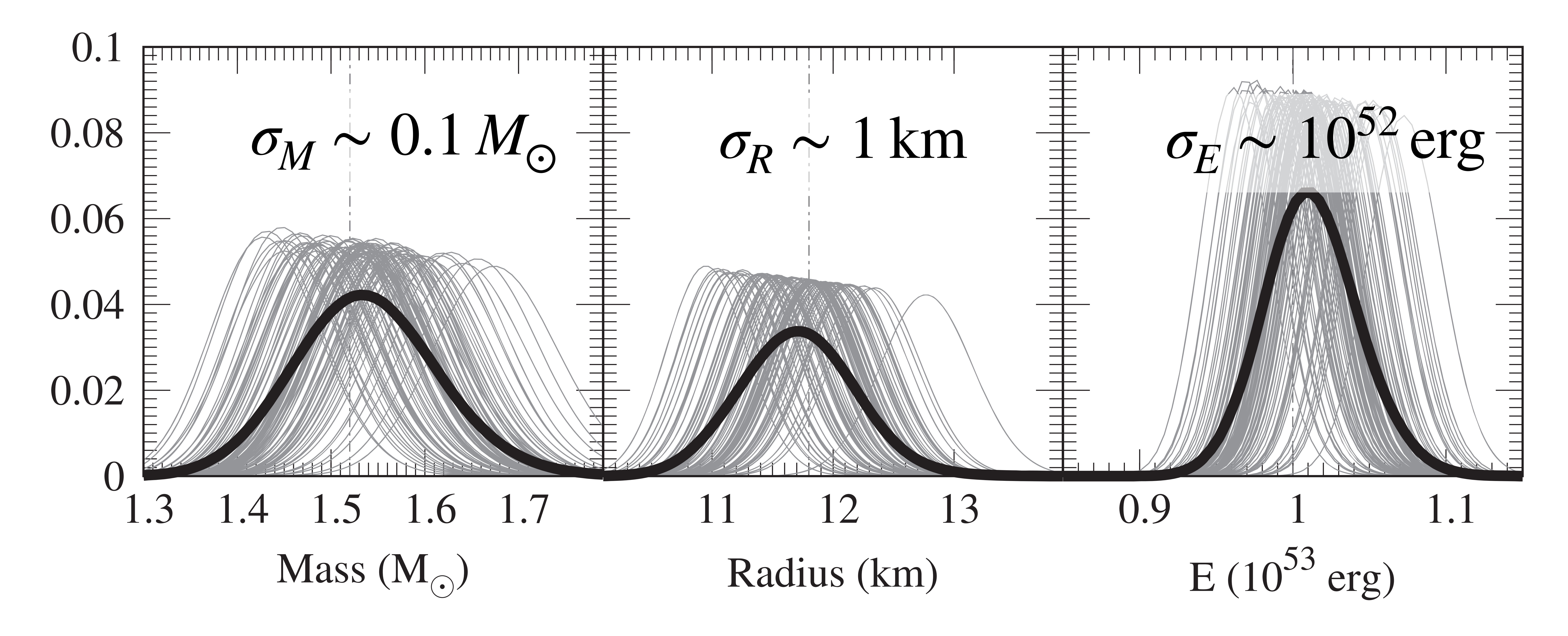
[Suwa, Harada, Harada, Koshio, Mori, Nakanishi, Nakazato, Sumiyoshi, Wendell, ApJ, 934, 15 (2022)]





What can a Galactic supernova measure?

[Suwa, Harada, Harada, Koshio, Mori, Nakanishi, Nakazato, Sumiyoshi, Wendell, ApJ, 934, 15 (2022)]



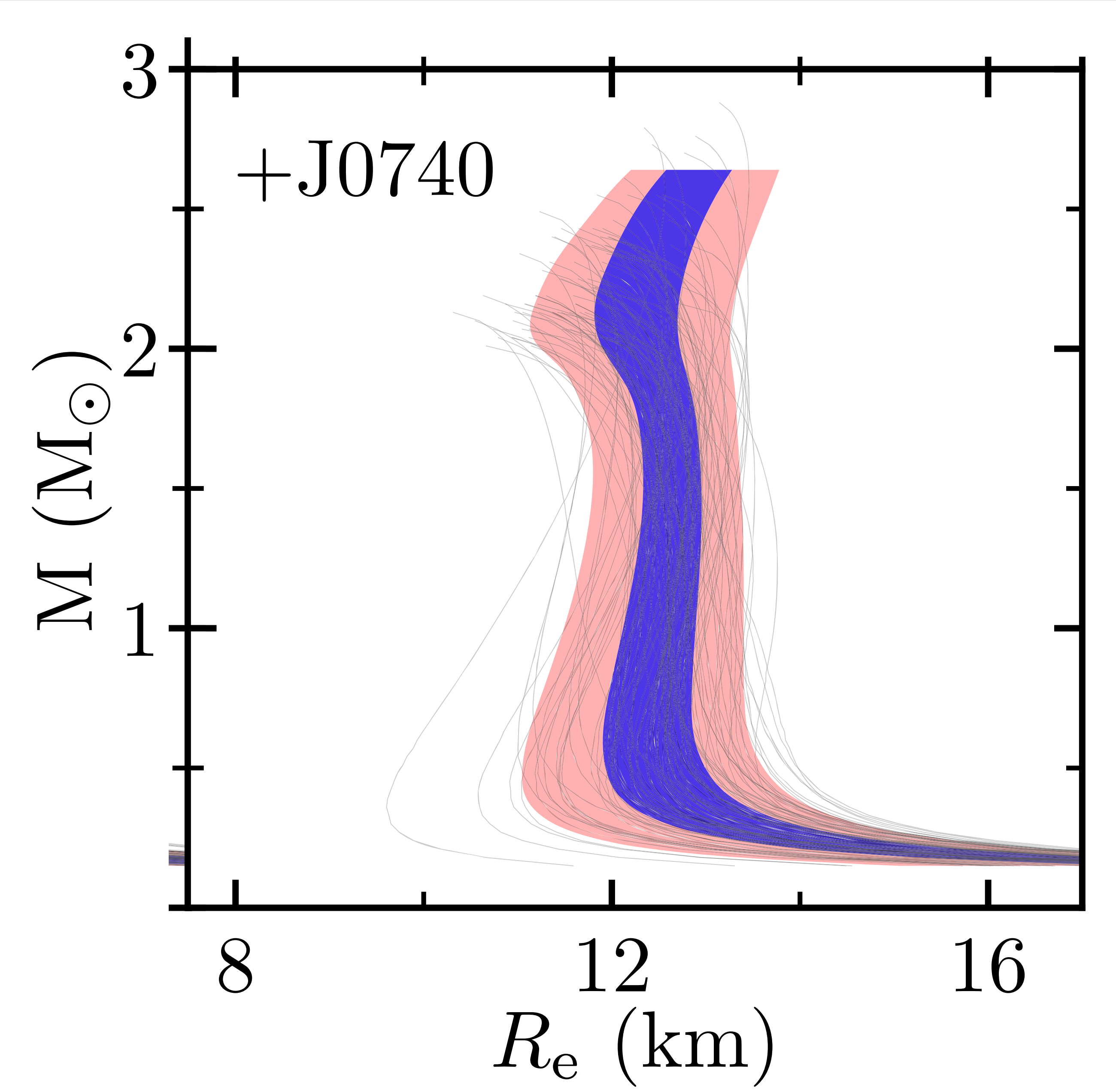
Distance estimation with neutrinos

[Suwa, Harada, Mori, Nakazato, Akaho, Harada, Koshio, Nakanishi, Sumiyoshi, Wendell, ApJ, 980, 117 (2025)]

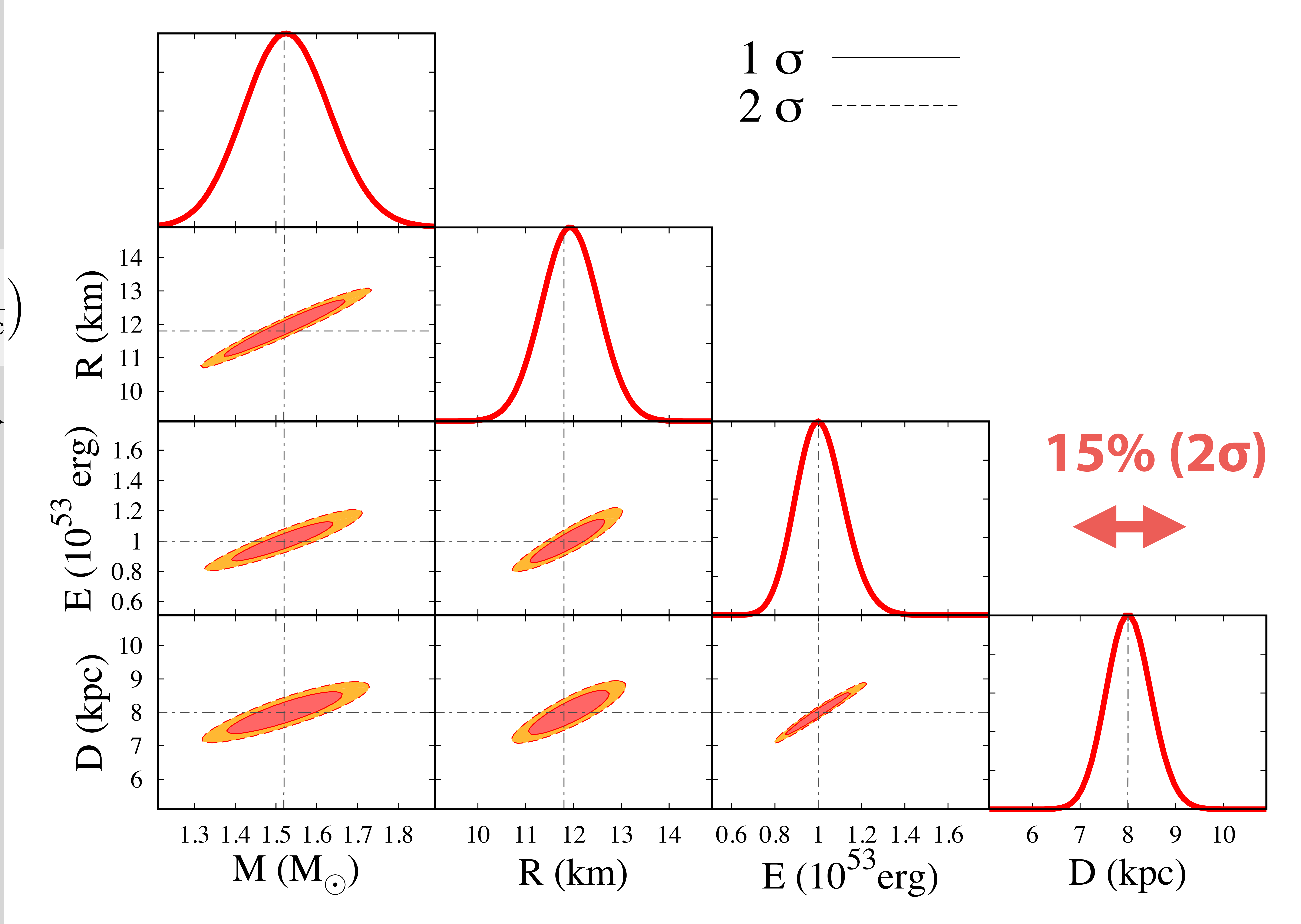
Analytic solution tells us (measured) PNS radius and source distance are correlated

$$R_{\rm PNS} = 10\,{\rm km}\left(\frac{\mathcal{R}}{720\,{\rm s}^{-1}}\right)^{1/2}\left(\frac{E_{e^+}}{25\,{\rm MeV}}\right)^{-5/2}\left(\frac{M_{\rm det}}{32.5\,{\rm kton}}\right)^{-1/2}\left(\frac{D}{10\,{\rm kpc}}\right)$$

with NS radius constraint



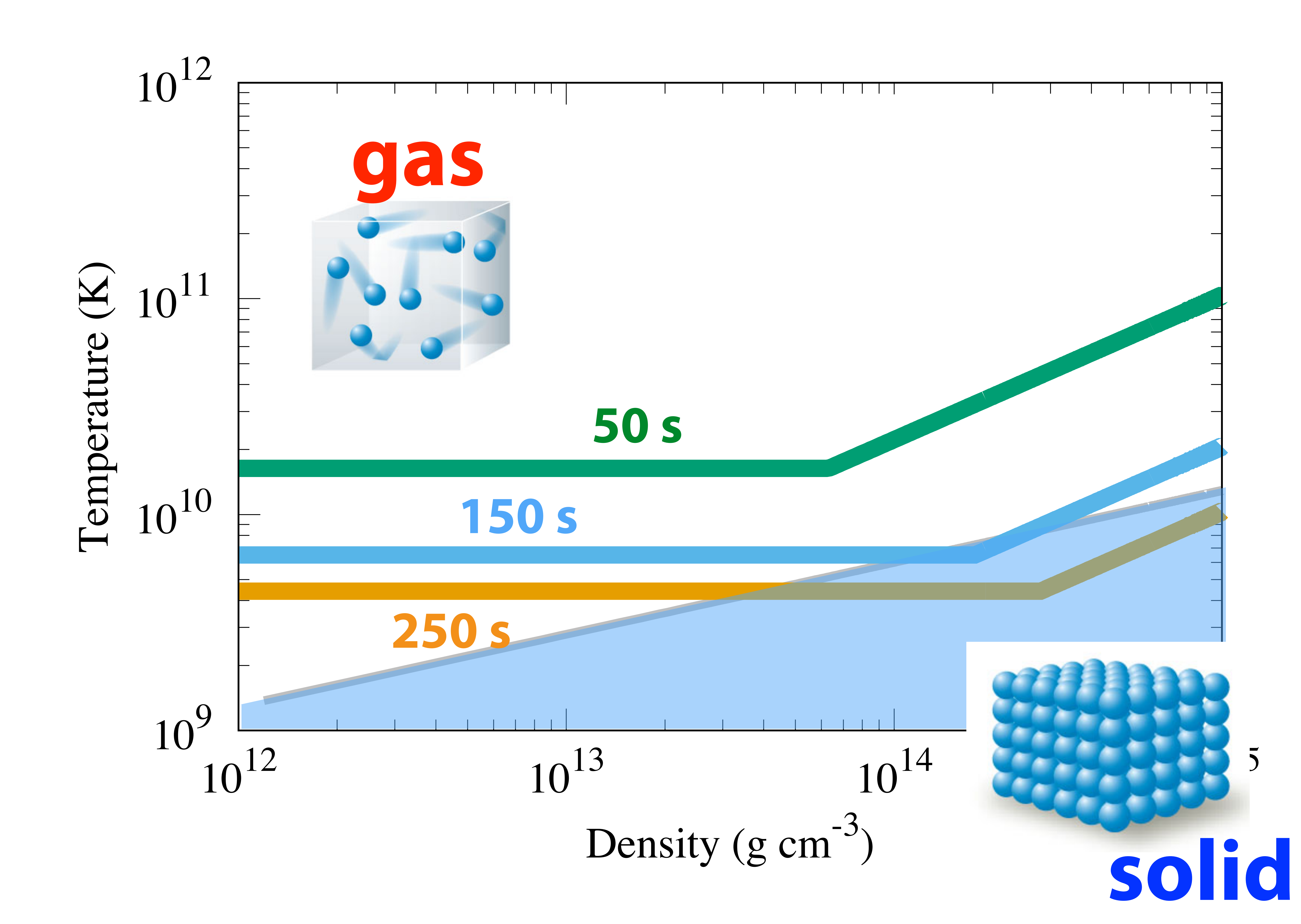
Miller+ 2021



NS mass NS radius total energy distance

Evolution in ρ-T plane and crust formation timescale

[Suwa & Nakazato, PASJ, 78, 1519 (2026)]



NS crust forms at ~100 s after SN

$$k_B T \lesssim \frac{(Ze)^2}{\langle r \rangle} \rightarrow T_{\mathrm{crystal}} \propto \rho^{1/3}$$

thermal　　Coulomb

Higher moments as spectral width probes transport physics

* **Moments of event spectrum**

$$M_n(t) \equiv \int dE\, E^n \frac{dN}{dE}(t)$$

0th ↔ rate

$$M_0 \propto \mathscr{R}$$

1st ↔ average energy

$$\frac{M_1}{M_0} \propto \left\langle E_{\mathrm{e}^+} \right\rangle$$

already included in analytic model

2nd ↔ spectral distortion

$$\frac{M_2}{M_0} - \left(\frac{M_1}{M_0}\right)^2 \propto \sigma_E^2$$

Do you know any good way?

Summary

* Analytic solutions as a common reduced form connecting detailed transport to observations
* Supernova neutrinos as an example
  * Forward model -> event generation (rate, average energy)
  * Inverse model -> parameter estimation (PNS mass, radius)
* Application
  * Distance estimation
  * NS crust formation time
* Next step
  * Higher moments and spectral distortion?