Analysis of 2D Hyperbolic Surfaces by Corner Transfer Matrix Renormalization Group

Andrej Gendiar[1] and Tomotoshi Nishino[2]

[1] Research Center for Quantum Information, Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, SK-845 11 Bratislava, SLOVAKIA
[2] Department of Physics, Graduate School of Science, Kobe University, Kobe 657-8501 JAPAN

We consider an infinite 2D discrete surface with a negative curvature (the hyperbolic geometry) which is represented by a spin lattice. The lattice is constructed by a regular tessellation of either triangles or squares, pentagons, hexagons, etc., each of equal size and shape. Having generalized the DMRG algorithm, we found a way how to treat the Ising, Potts, clock, and frustrated models on the hyperbolic surfaces in a high accuracy by specifying their universality classes [1, 2, 3, 4]. Since the Hausdorff dimension of the hyperbolic geometry is infinite, the second order mean field universality is expected. Our numerical analysis supports this prediction, and the mean field critically is concluded by specifying the critical exponents. It should be noted that Monte Carlo simulations experience difficulties due to the exponential increase of the lattice sites. Nor any transfer matrix formulations have been available yet. We also found out that he Ising model in the transverse field applied to the related hyperbolically deformed 1D quantum chain exhibited a first order phase transition instead [5]. This discrepancy has not been clarified yet. We focus our interest on a corner transfer matrix analysis near the Euclidean plain geometry as we consider this to be the missing link to understand the origin of the mean field behavior on the hyperbolic surfaces. This feature may rest upon an appropriate formulation of the continuous limit toward the Euclidean geometry.

References

[1] K. Ueda, R. Krcmar, A. Gendiar, and T. Nishino, J. Phys. Soc. Jpn. **76** (2007), 084004.

[2] R. Krcmar, A. Gendiar, K. Ueda, and T. Nishino, J. Phys. A: Math. Theor. **41** (2008), 125001.

[3] A. Gendiar, R. Krcmar, K. Ueda, and T. Nishino, Phys. Rev. E **77** (2008), 041123.

[4] R. Krcmar, T. Iharagi, A. Gendiar, and T. Nishino, Phys. Rev. E **78** (2008), 061119.

[5] H. Ueda, A. Gendiar, V. Zauner, T. Iharagi, and T. Nishino, arXiv:1008.3458.