Boundary Operator in the Matrix Product States

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The direct product state is written as $\prod_i |\psi\rangle_i$, where $|\psi\rangle_i$ is a local state at the ith cluster. To represent quantum entanglement between decoupled clusters, one of natural generalizations is the matrix product state (MPS) $|\Psi\rangle = \mathrm{Tr} \prod_i A_i$ with matrix elements $(A_i)_{mm'} = |\psi_{i;mm'}\rangle_i$. The translationally invariant MPS under the periodic boundary condition in one dimensional systems is written as $|\Psi\rangle = \mathrm{Tr} \prod_i A$ with a single uniform matrix A. To include the boundary effect, one can consider the boundary matrix Q with matrix elements $(Q)_{mm'} = |\phi_{mm'}\rangle_0$ and $|\Psi\rangle = \mathrm{Tr}\,[Q \prod_i A]$, where the artificial Hilbert space $|\phi\rangle_0$ is set to be one-dimensional generally [1]. Does not the translationally invariant MPS have the boundary operator Q?

Our studies show the importance of Q for the MPS. We have derived a MPS representation of the Bethe ansatz state for spin-1/2 Heisenberg chain [2] and the Lieb-Liniger model [3], from the algebraic Bethe ansatz using the factorizing F-matrices. The uniform matrix A obtained for the Heisenberg chains is the same as that in the matrix product ansatz [4] apart from normalization factors. For the Lieb-Liniger model describing the Bose gas with delta-function interaction in one-dimension, a "continuous" extension of the matrix product state is obtained. The exact MPS has both translationally invariance and the boundary operator Q. The latter comes from the domain wall boundary conditions [5]. In fact, for the MPS in the Bose gas, Q plays a role in fixing the number of particles. From a numerical point of view, Q is also important to consider the spontaneous symmetry breaking of the translational symmetry and long-period super-lattices for the magnetic plateau [6].

References

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