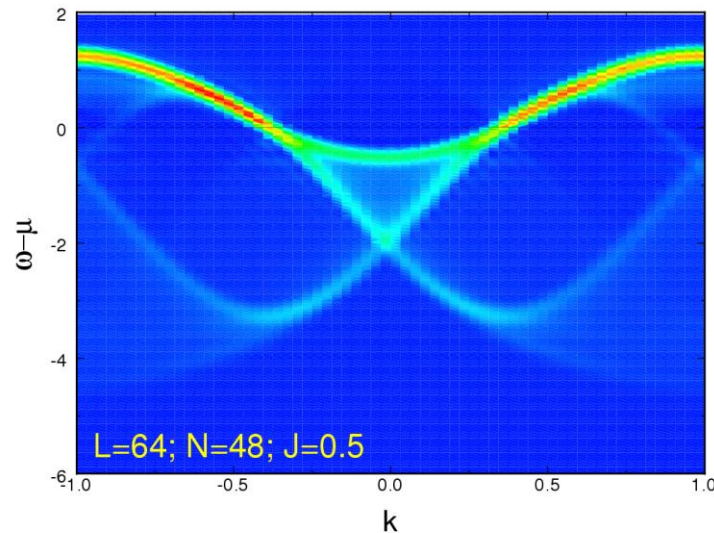
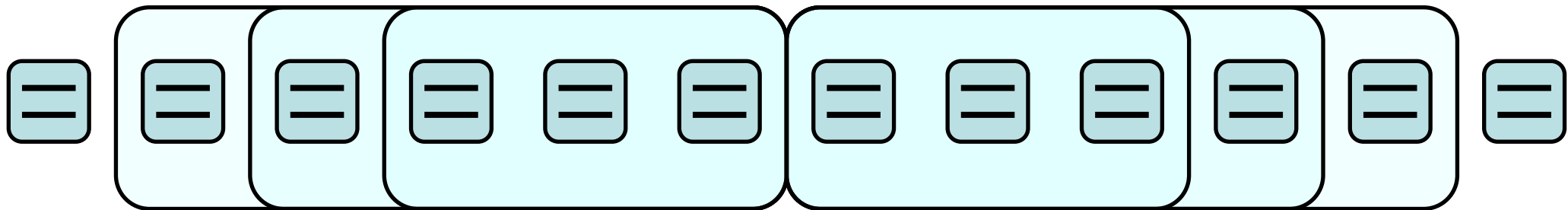


Spin incoherent behavior in strongly correlated systems

Adrian Feiguin



Spin incoherent behavior in strongly correlated systems

Outline:

- Luttinger liquids
- Spin-incoherent behavior
- The factorized wave function
- Thermofield/ancilla representation
- Finite Temperature state
- SILL behavior in the **ground state** of strongly correlated systems

Collaborator:

Greg Fiete (U.T. Austin)

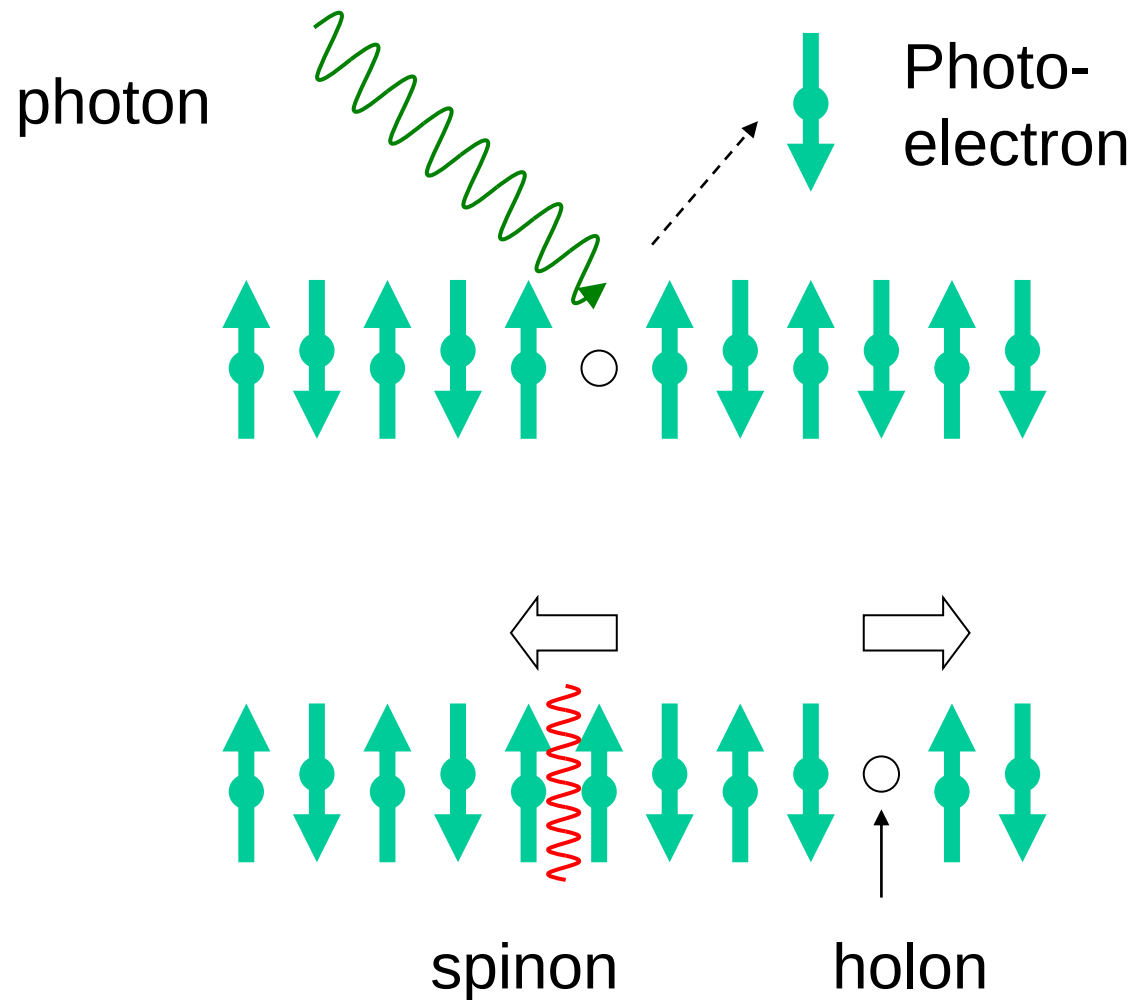
References:

AEF and G. Fiete: Phys. Rev. B 81, 075108 (2010)

AEF and G. Fiete: arXiv:1005.4707

Spin-charge separation

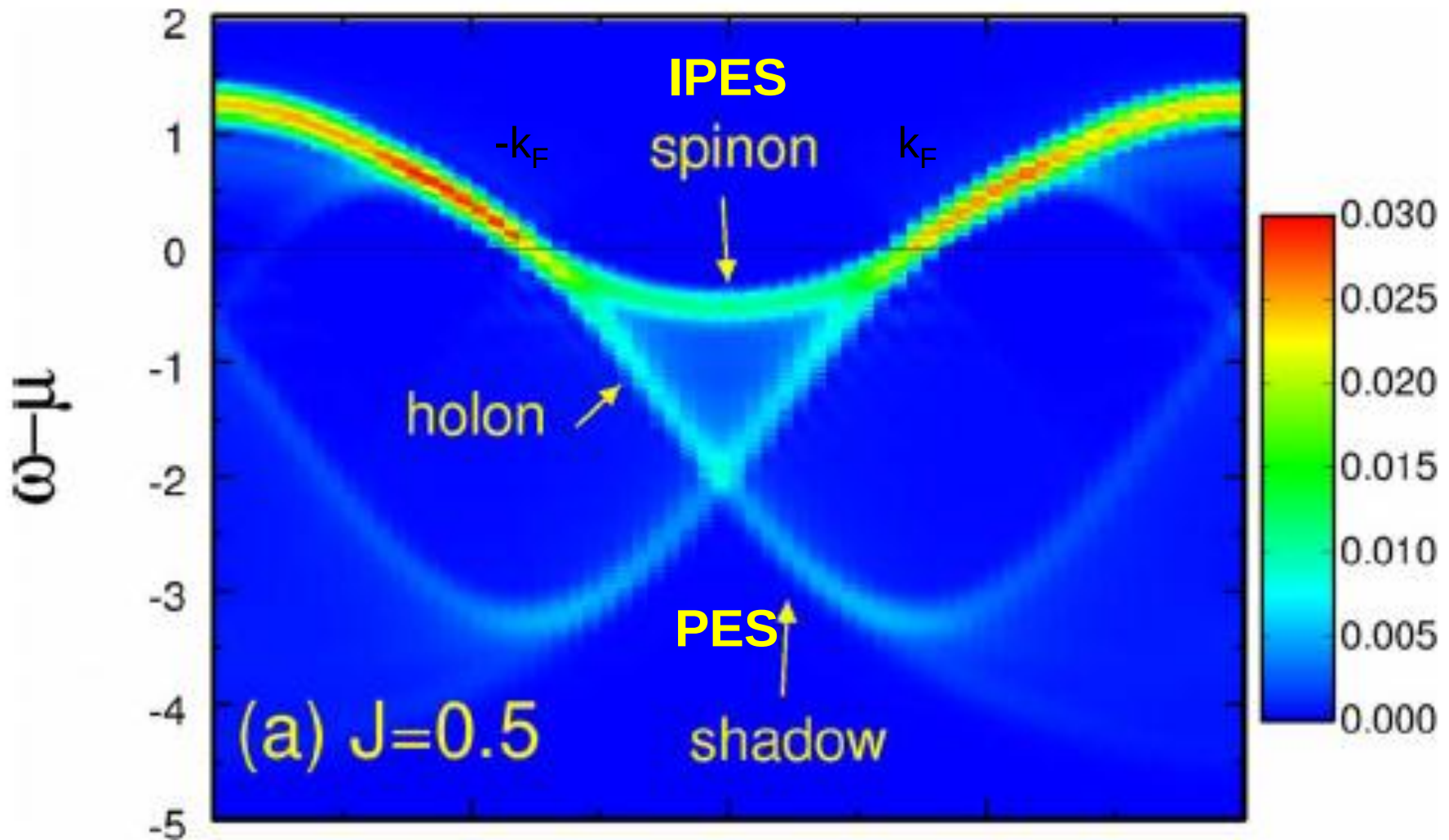
(seen in photoemission – ARPES)



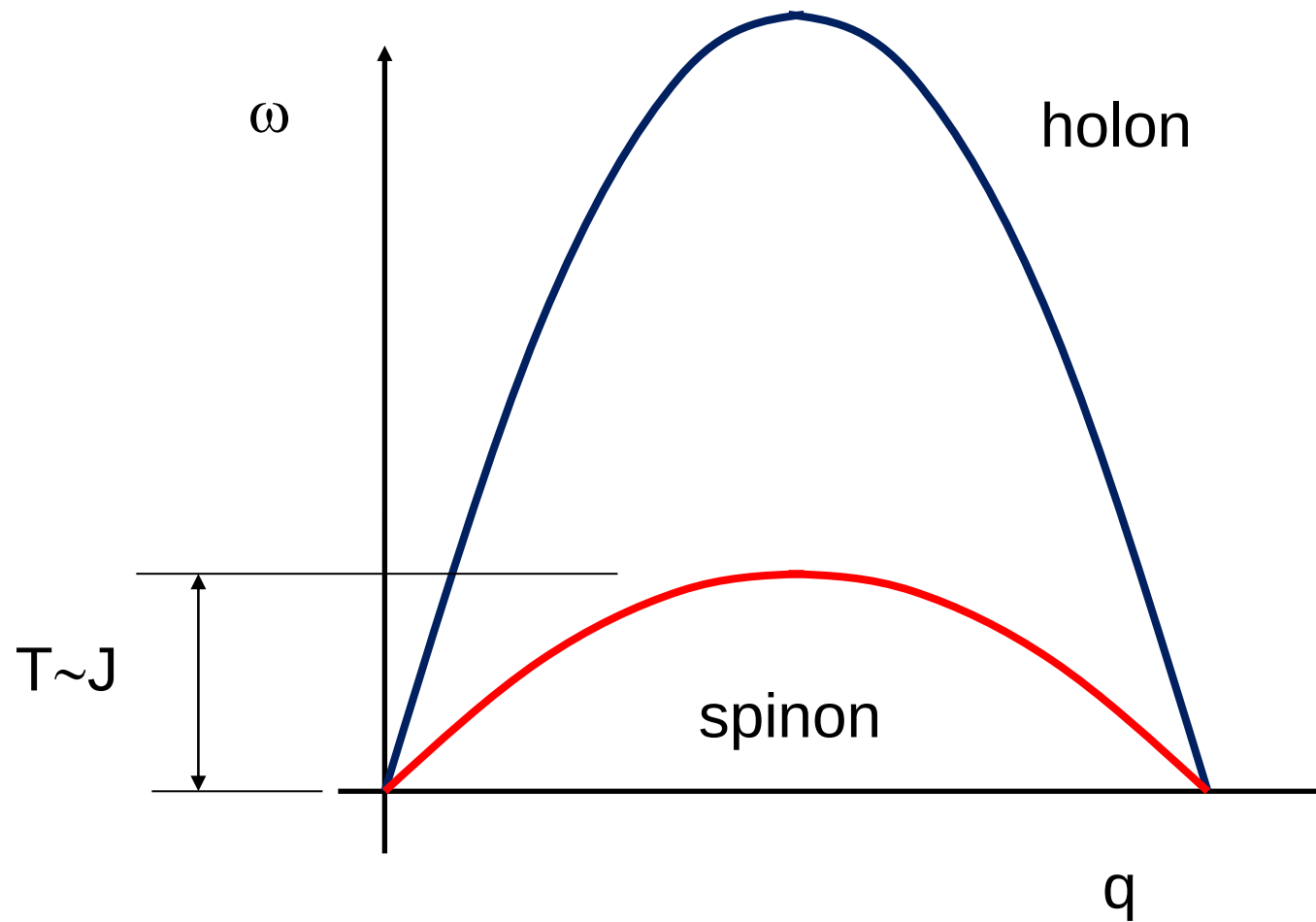
The excitations don't carry the same quantum numbers as the original electron → zero quasi-particle weight

ARPES at $T=0$

1D t-J model ($J=0.5$)



Spin incoherent behavior



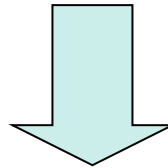
See G. Fiete, RMP (07)

Model Hamiltonian

Hubbard model:

$$H = \underbrace{-t \sum_{i=1, \sigma}^{L-1} (c_{i\sigma}^\dagger c_{i+1\sigma} + h.c.)}_{\text{Kinetic energy}} + \underbrace{U \sum_{i=1}^L n_{i\uparrow} n_{i\downarrow}}_{\text{On-site interaction}}$$

Large U



t-J model (no double occupancy):

$$H = -t \sum_{i=1, \sigma}^L (c_{i\sigma}^\dagger c_{i+1\sigma} + h.c.) + \underbrace{J \sum_{i=1}^L \vec{S}_i \vec{S}_{i+1}}_{\text{Heisenberg}}$$

Evolution in imaginary time: single spin

We introduce and **auxiliary spin (ancilla)**

$$|I_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle)$$

$\uparrow$: “physical” spin
 $\downarrow$: “ancilla”

We trace over **ancilla**:

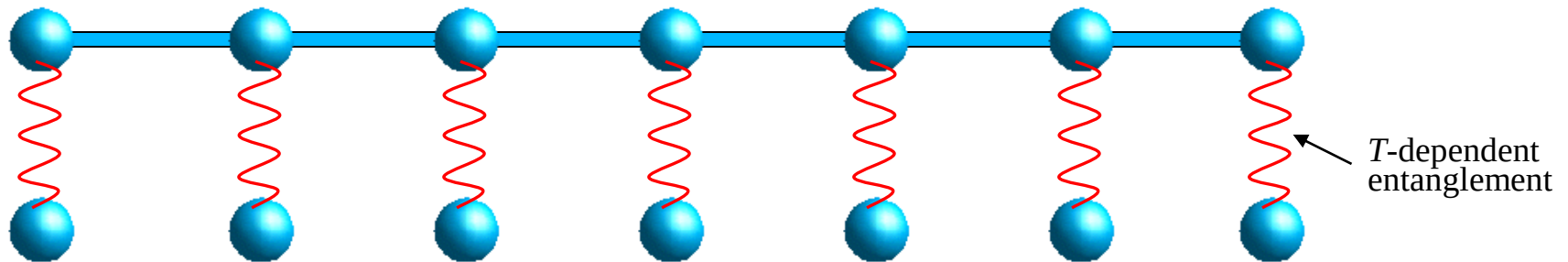
$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The density matrix corresponds to the physical spin
at infinite temperature!

Evolution in imaginary time

The thermal state is equivalent to evolving the maximally mixed state in imaginary time:

$$|\psi(\beta)\rangle = e^{-\beta H/2} |I\rangle$$



- The ancillas and the real sites **do not interact!**
- The **global** state is modified by the **action** of the Hamiltonian **on the real sites**, that are **entangled** with the ancillas.
- The **mixed state** can be written as a **pure state** in an enlarged Hilbert space (ladder-like).

Evolution in imaginary time: Thermal averages

A thermal average :

$$\langle A \rangle = Z^{-1}(\beta) \text{Tr}\{A e^{-\beta H}\}, \quad Z(\beta) = \text{Tr}\{e^{-\beta H}\}.$$

Can be obtained using a wave function instead of density matrices:

$$\langle A \rangle = \frac{\langle \psi(\beta) | A | \psi(\beta) \rangle}{\langle \psi(\beta) | \psi(\beta) \rangle} = Z^{-1}(\beta) \sum_n \langle n | A | n \rangle e^{-\beta E_n}$$

with $Z(\beta) = \langle \psi(\beta) | \psi(\beta) \rangle$

Green's functions

The finite temperature Green's function can be obtained as:

$$G(x - x_0, t, \beta) = \langle \psi(\beta) | e^{iH_{t-J}t} \hat{O}^\dagger(x) e^{-iH_{t-J}t} \hat{O}(x_0) | \psi(\beta) \rangle$$

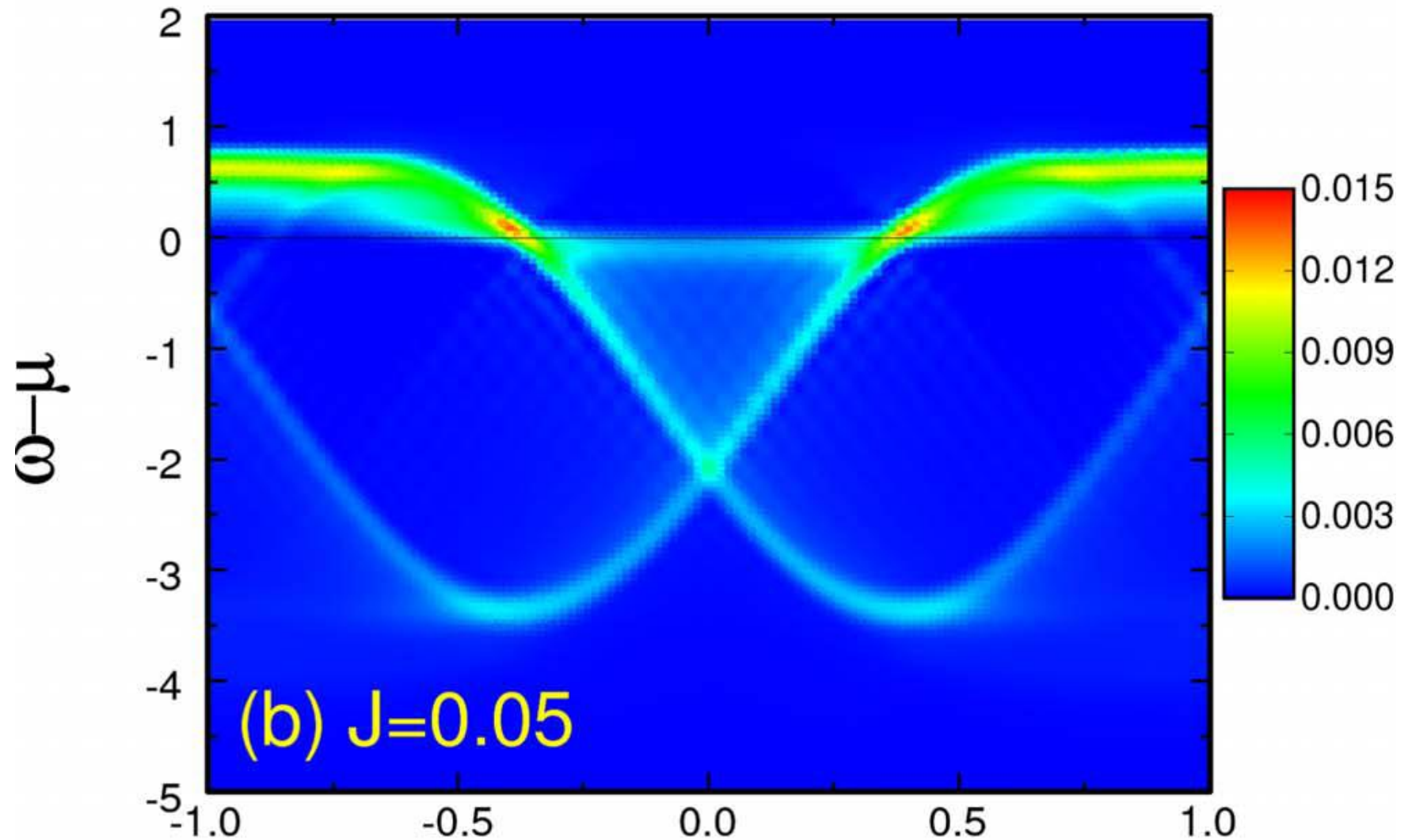
Since the thermal state is not an eigenstate, we need to evolve in time both:

$$e^{-iH_{t-J}t} | \psi(\beta) \rangle$$

$$e^{-iH_{t-J}t} \hat{O}(x_0) | \psi(\beta) \rangle$$

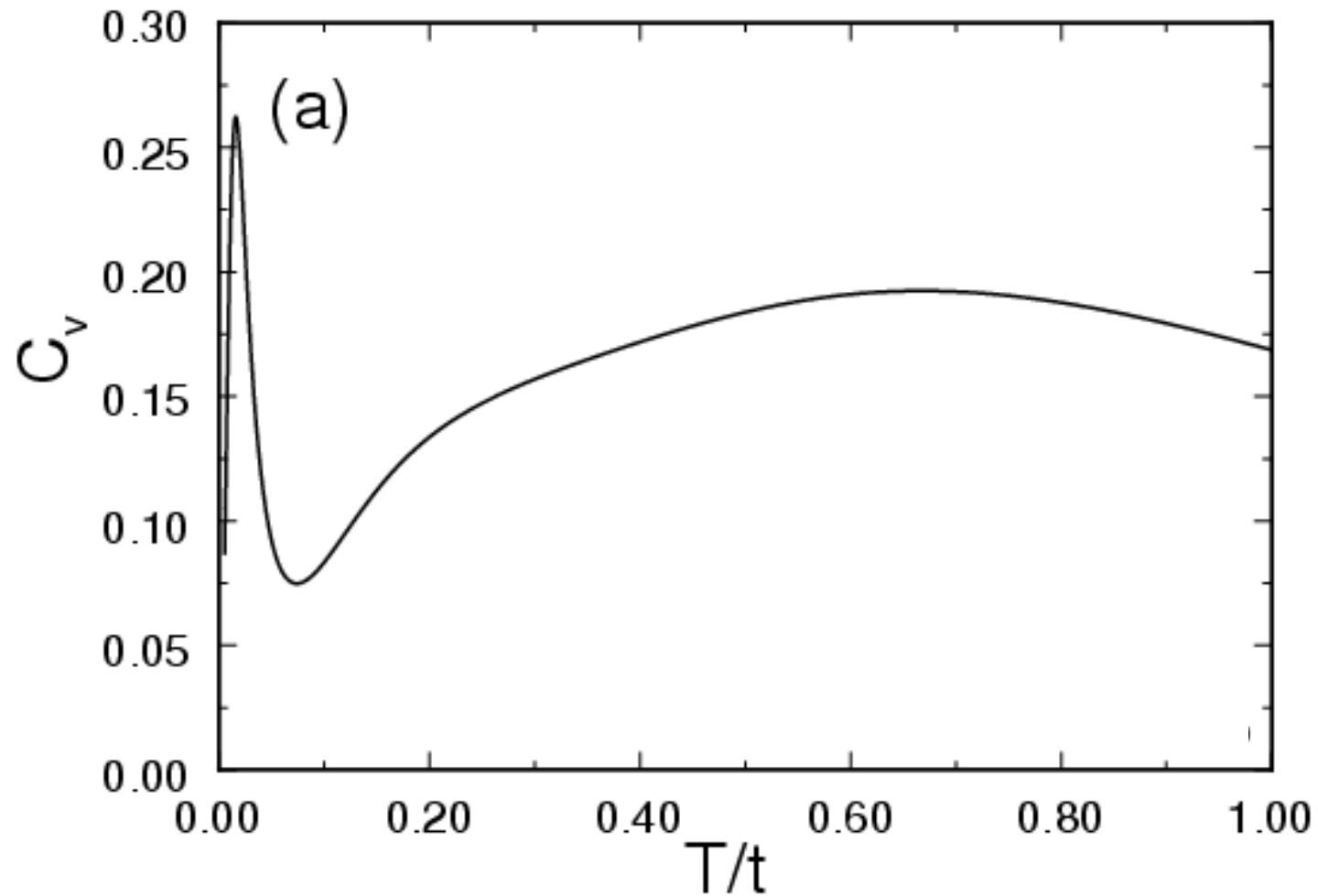
ARPES at $T=0$

1D t-J model ($J=0.05$)



Results: Thermodynamics

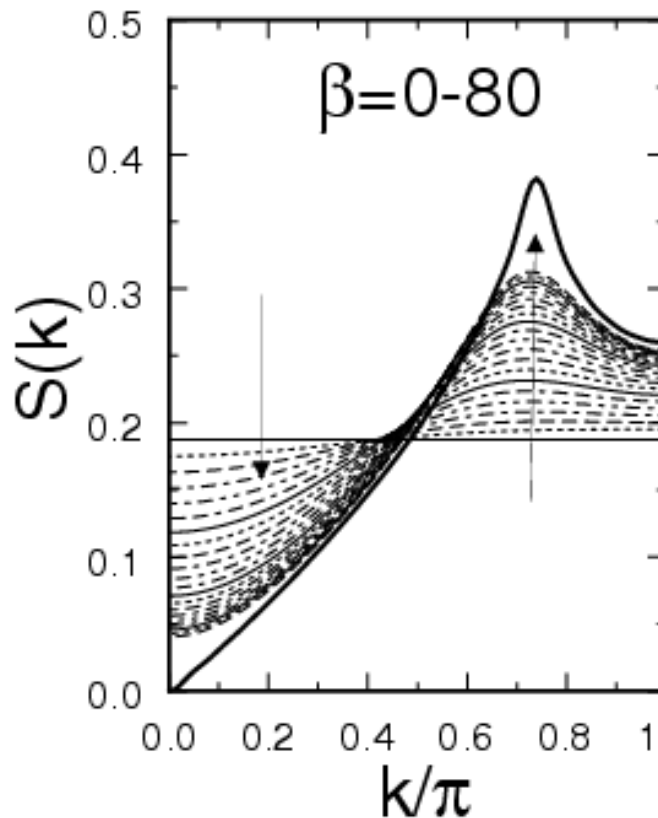
$J=0.05$; $L=32$, $N=24$; $n=0.75$



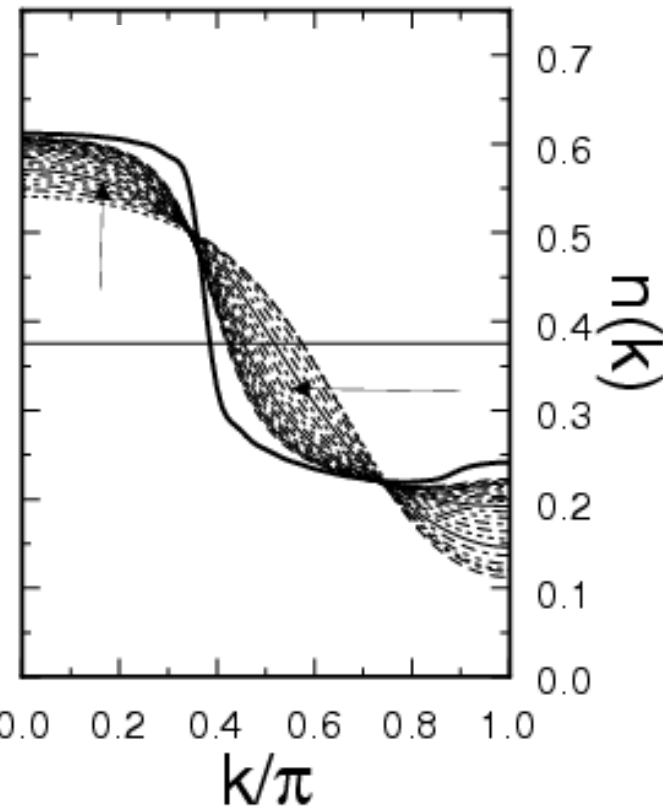
Correlation functions

$J=0.05$; $L=32$, $N=24$; $n=0.75$

Spin structure
factor

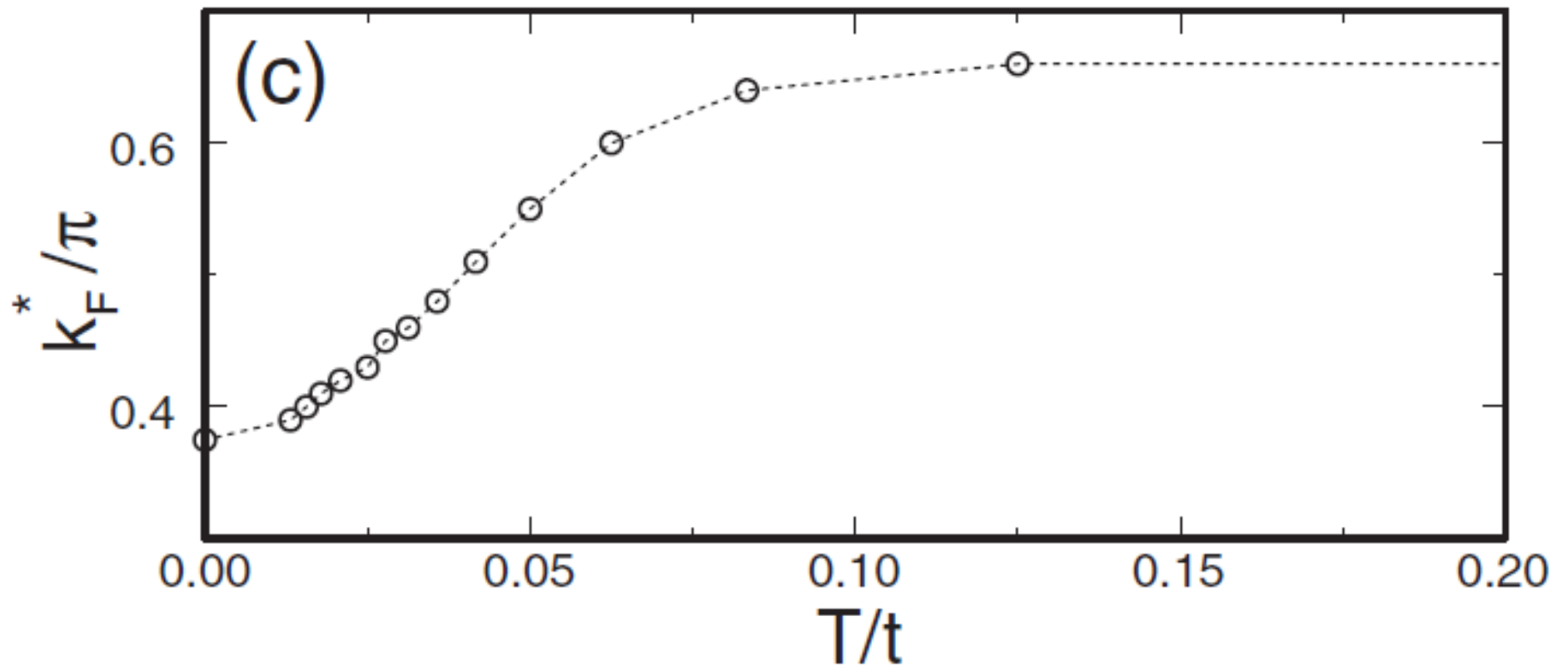


Momentum
distribution



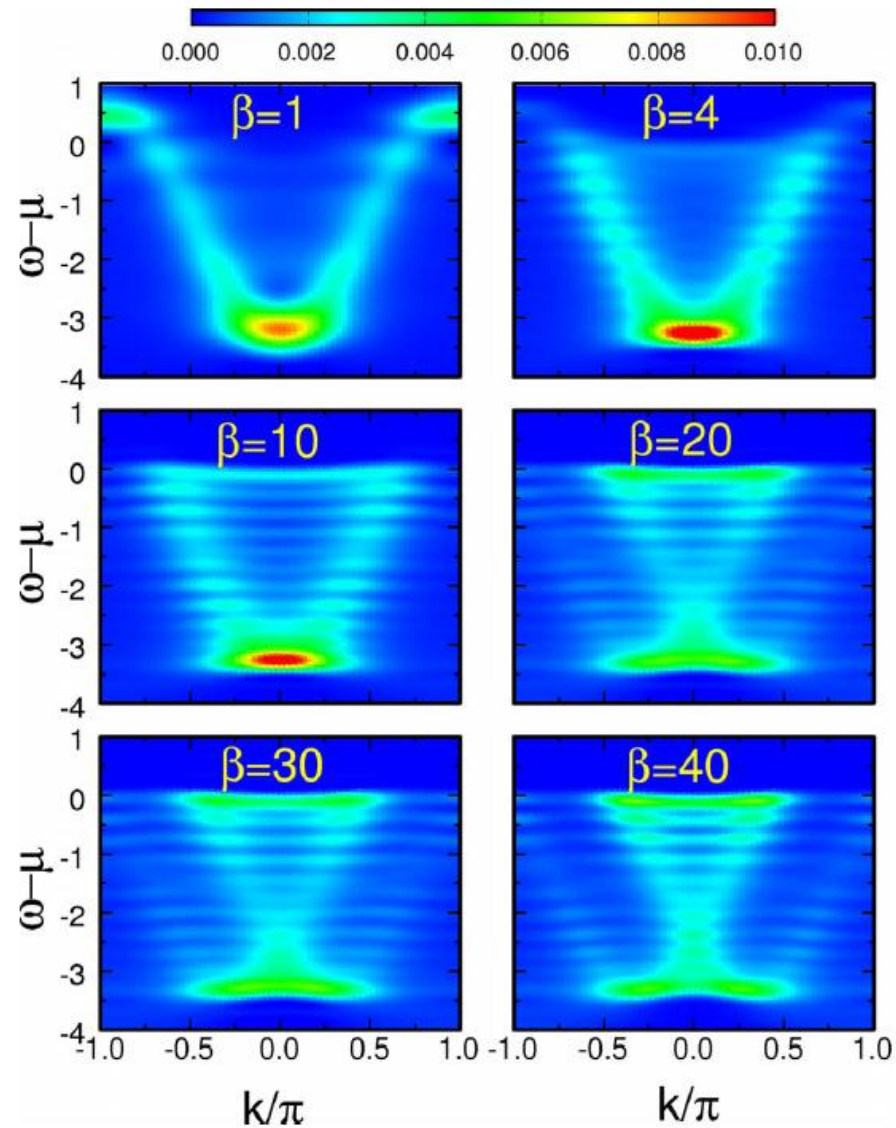
Fermi momentum

$J=0.05$; $L=32$, $N=24$; $n=0.75$



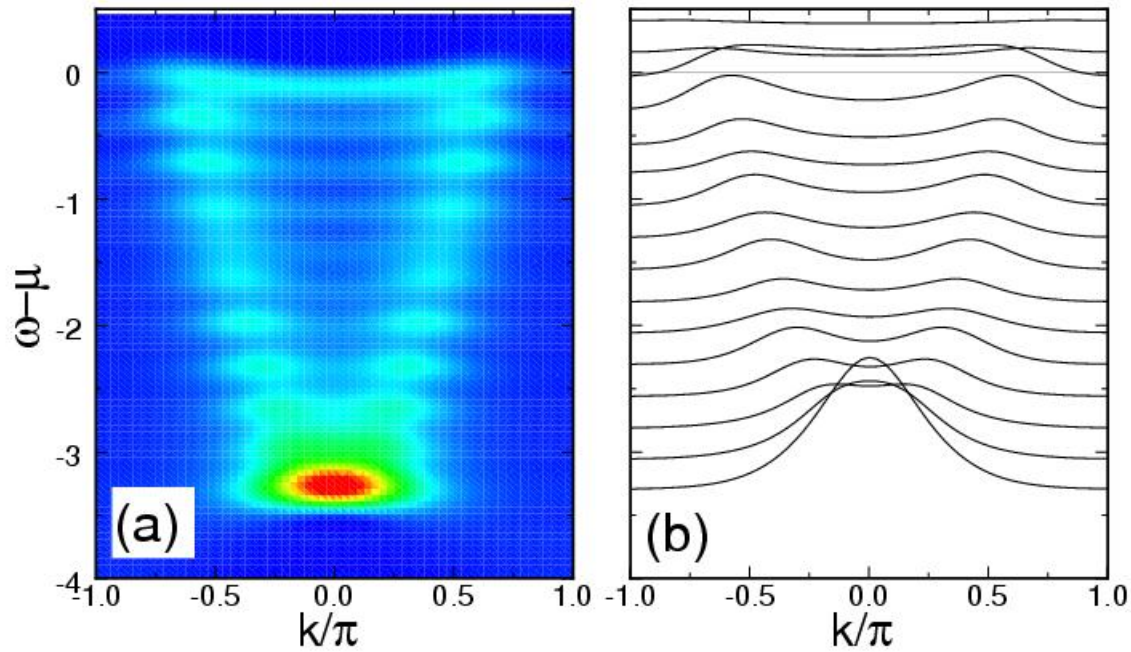
ARPES at finite T

$L=32, N=24, J=0.05$

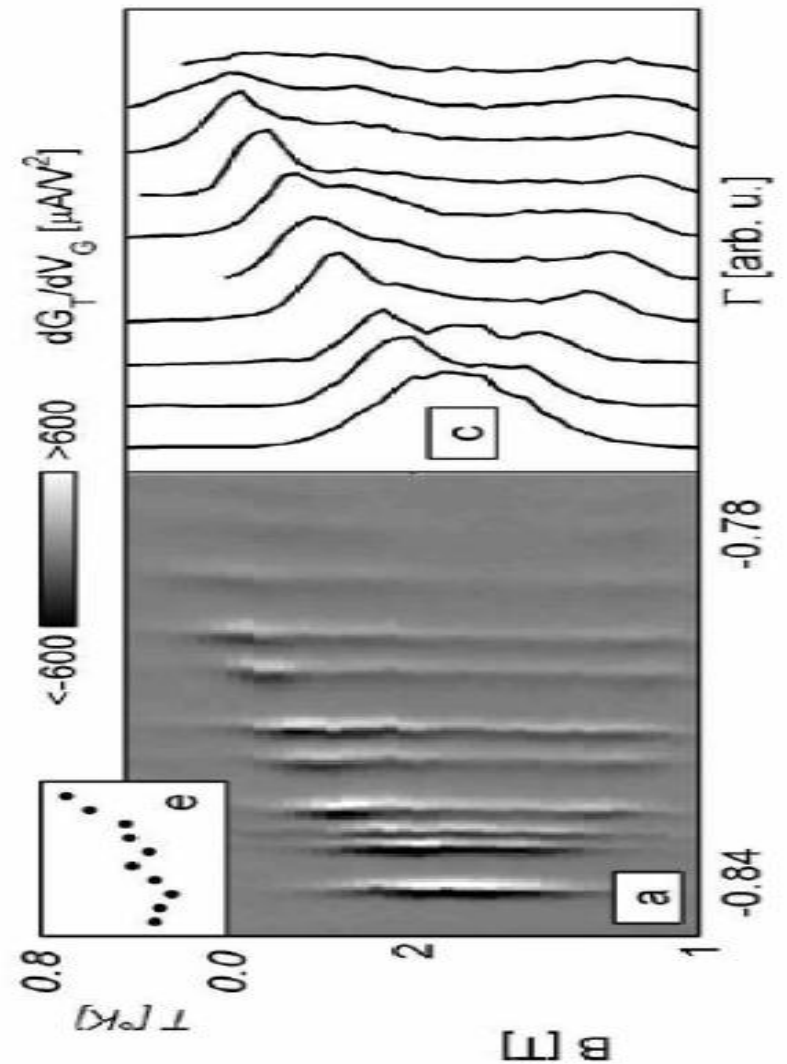


SILL regime

DMRG, $\beta=10$



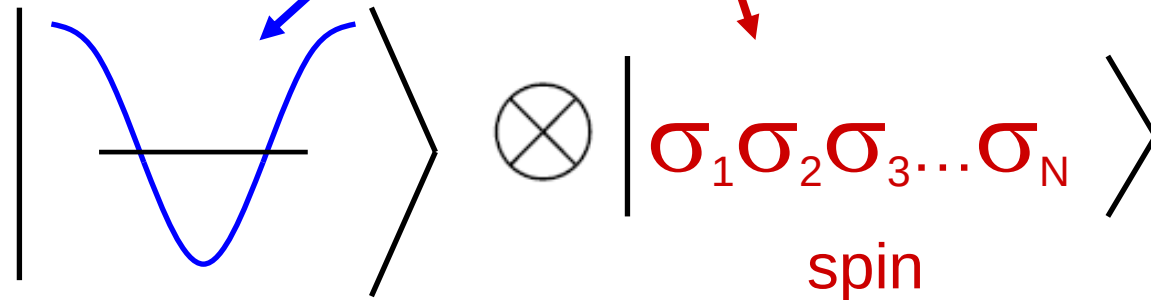
Experiment



SI behavior in the ground state of strongly interacting models

The factorized (Ogata and Shiba) wave function in the limit $U \rightarrow \infty, J \rightarrow 0$

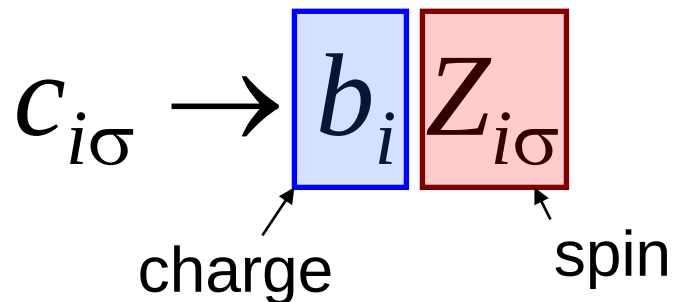
$$|\text{g.s.}\rangle = |\phi\rangle \otimes |\chi\rangle$$



charge

$$\epsilon(k) = -2t \cos(k)$$

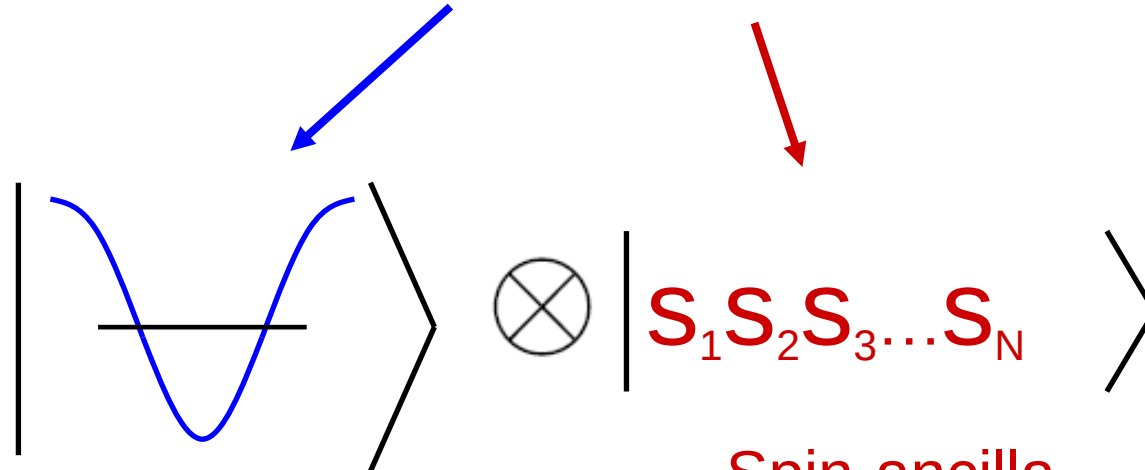
All configurations are degenerate



$$H = H_c + H_s$$

The factorized wave function (infinite *spin* Temperature)

$$|O(\beta)\rangle = |\phi\rangle \otimes |\chi\rangle$$

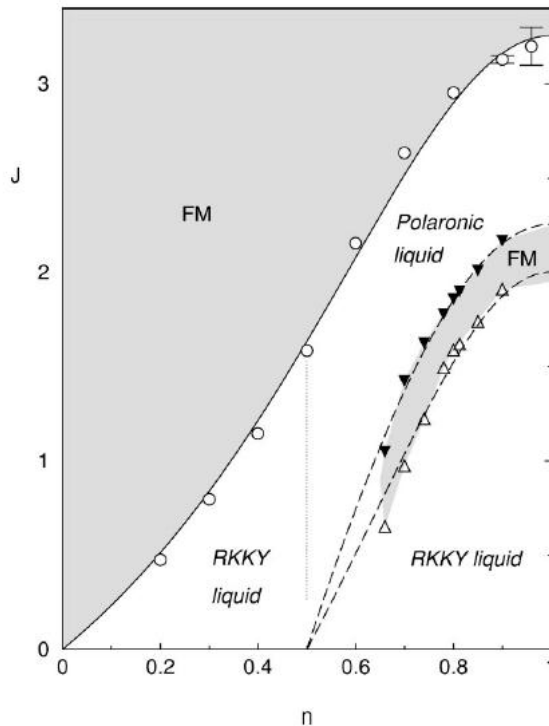
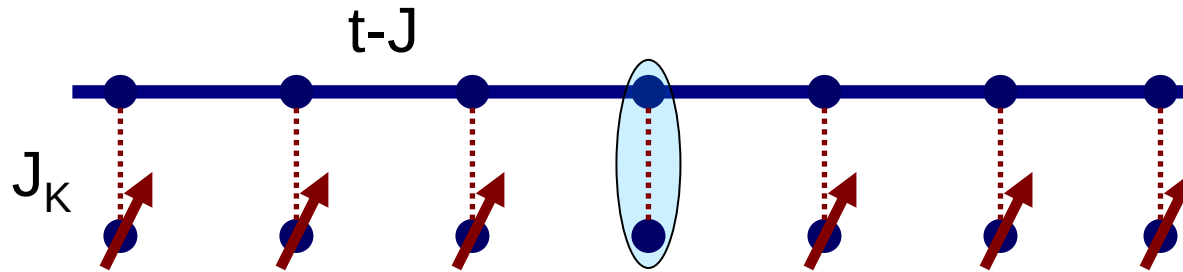


charge

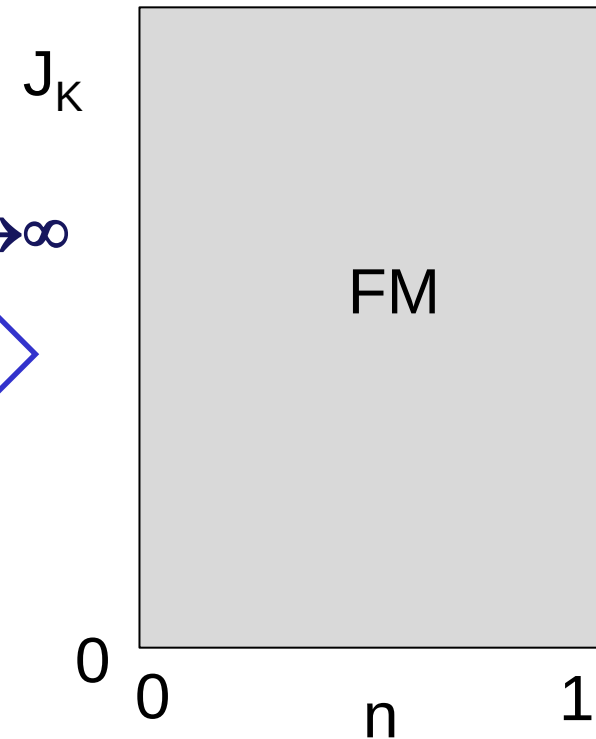
$$\epsilon(k) = -2t \cos(k)$$

Spin-ancilla
singlets

(I) Kondo lattice



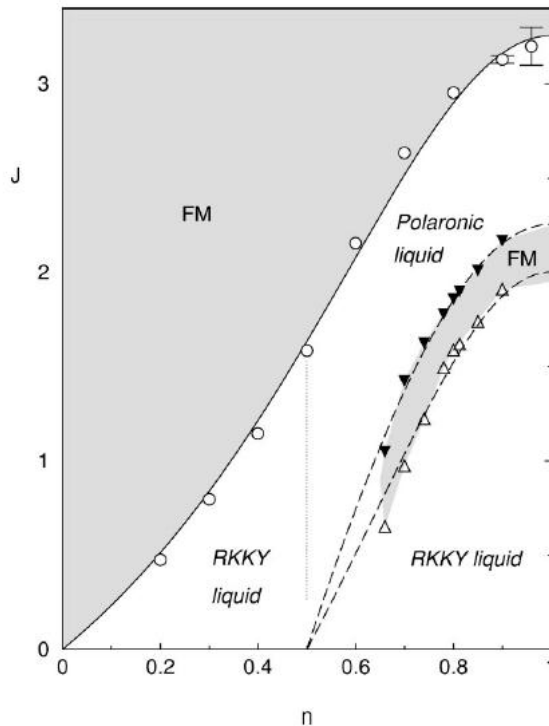
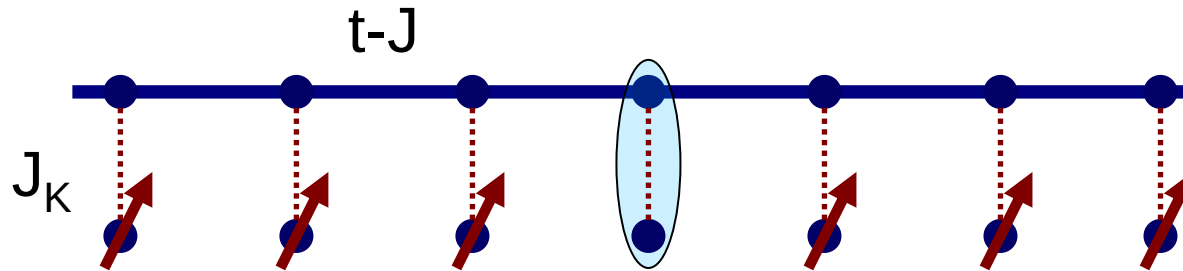
$J \rightarrow 0, U \rightarrow \infty$



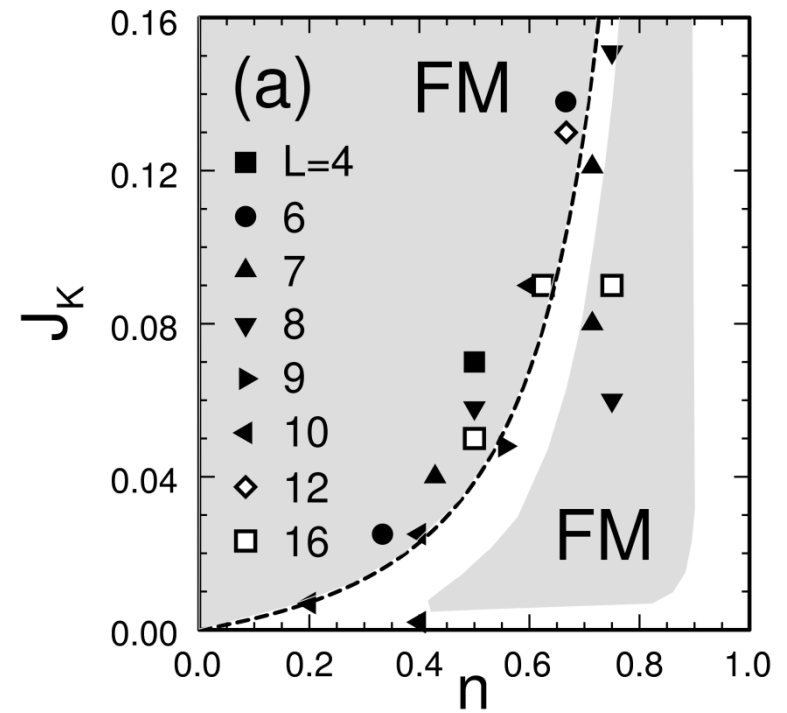
McCulloch et al, PRB '02, K. Hallberg et al, PRL '04

Tsunetsugu, Sigrist, Ueda, RMP '97

(I) Kondo lattice



$J=0.05$



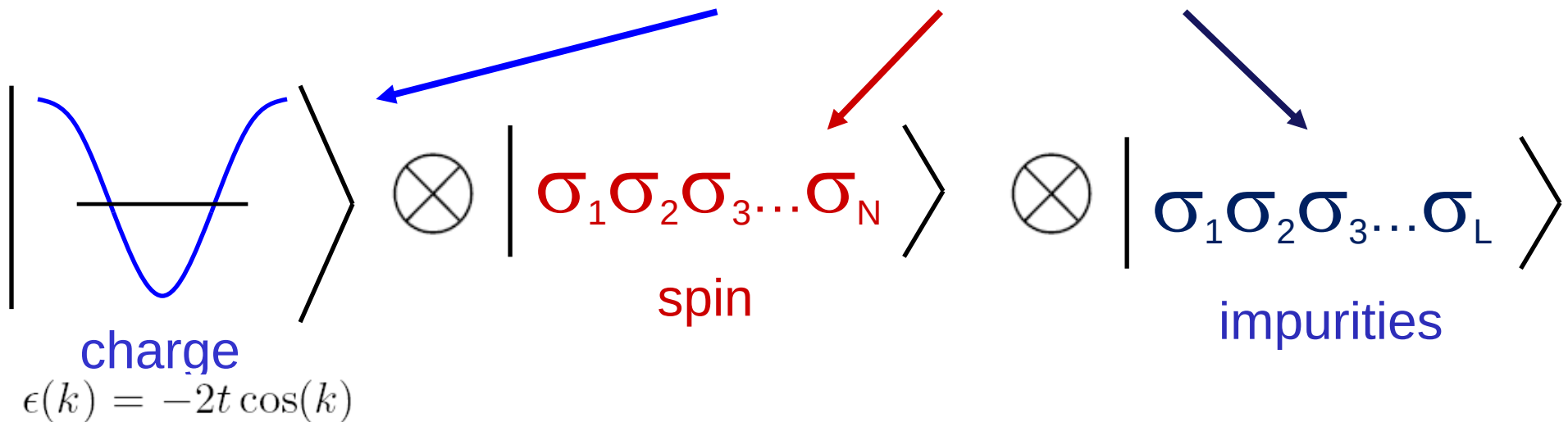
McCulloch et al, PRB '02, K. Hallberg et al, PRL '04

Tsunetsugu, Sigrist, Ueda, RMP '97

The factorized wave function in the limit

$$J \rightarrow 0, J_K = 0$$

$$|\text{g.s.}\rangle = |\varphi\rangle \otimes |\chi\rangle \otimes |\sigma\rangle$$



All spin configurations are degenerate.

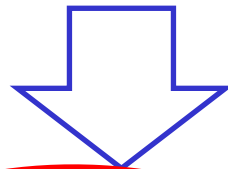
When we turn on the interactions with the impurities J_K :

- (i) The system becomes ferromagnetic,
- (ii) The conduction spins and the impurities get entangled
- (iii) A gap opens to break a pair

The factorized wave function in the limit

$$J \rightarrow 0, J_K \rightarrow \infty$$

$$|\text{g.s.}\rangle = |\varphi\rangle \otimes |\chi\rangle \otimes |\sigma\rangle$$



$$J_K \rightarrow \infty$$

$$|\text{g.s.}\rangle = |\varphi^*\rangle \otimes |S\rangle \otimes |\uparrow \dots \uparrow\rangle$$

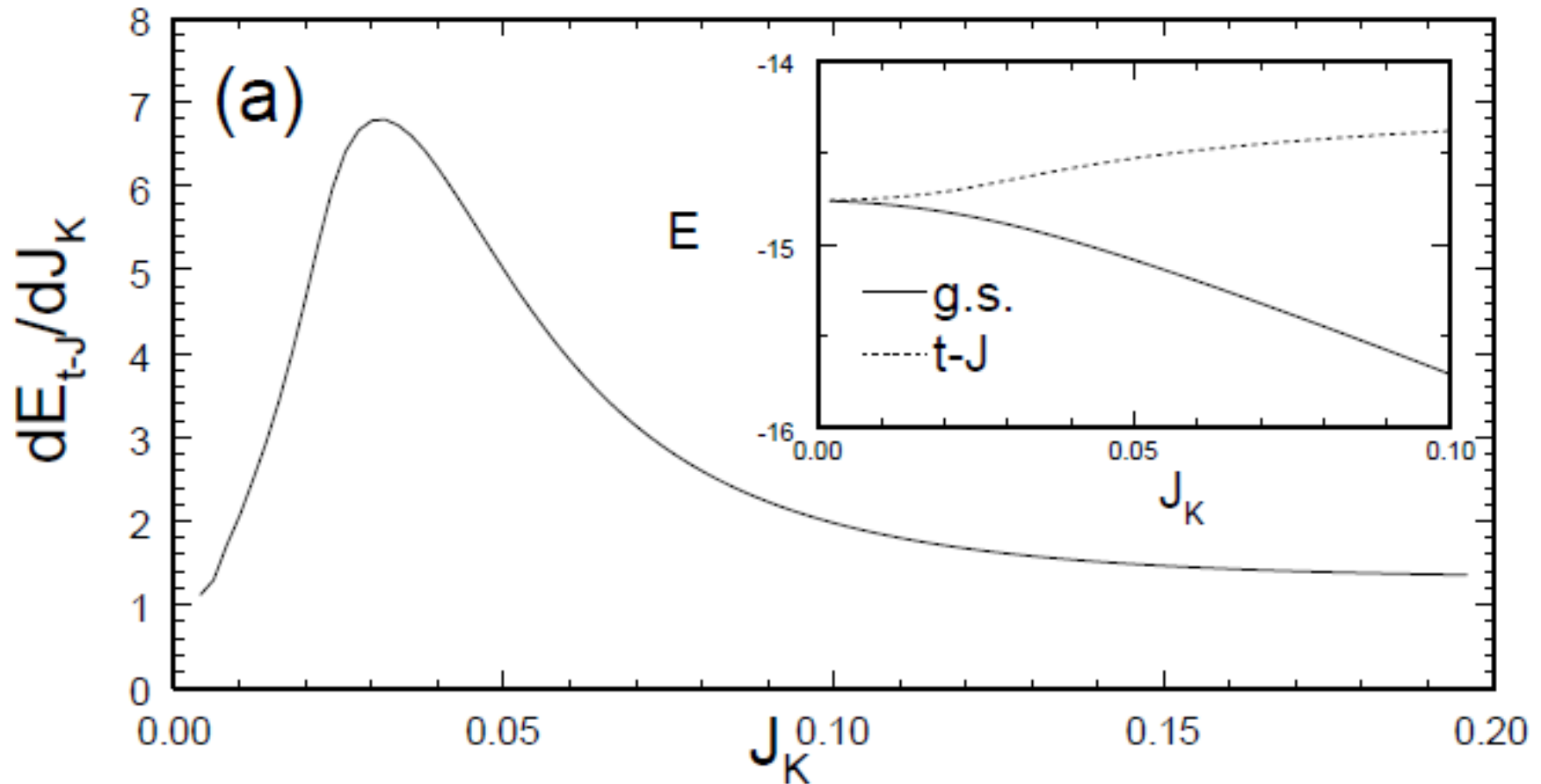
“heavy” charge
 $\varepsilon(k) = -t \cos(k)$

singlets

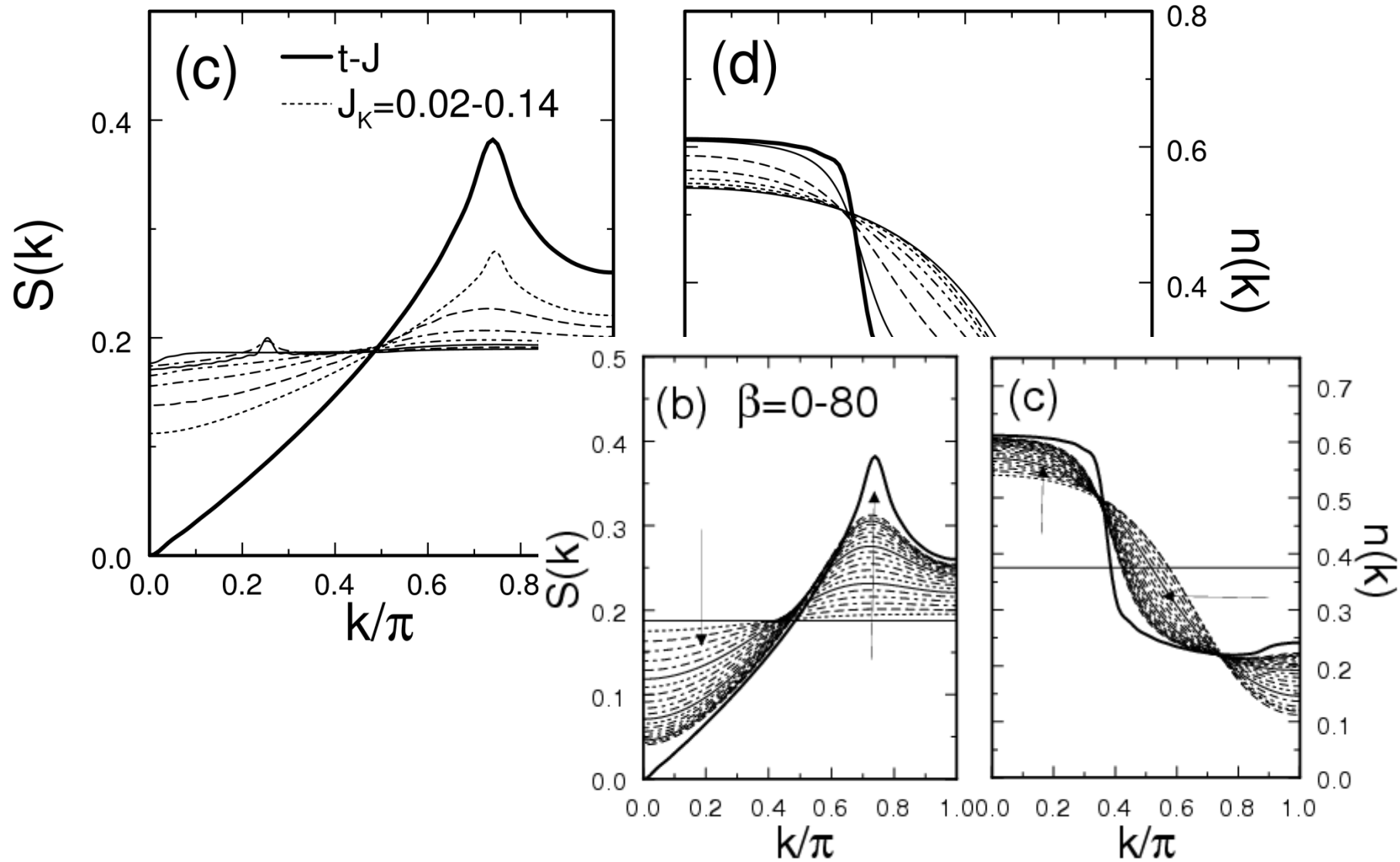
FM Unpaired
 impurities

Ogata and Shiba wave
 function at infinite spin T!!!

Ground state energy and effective “specific heat” ($J=0.05$)

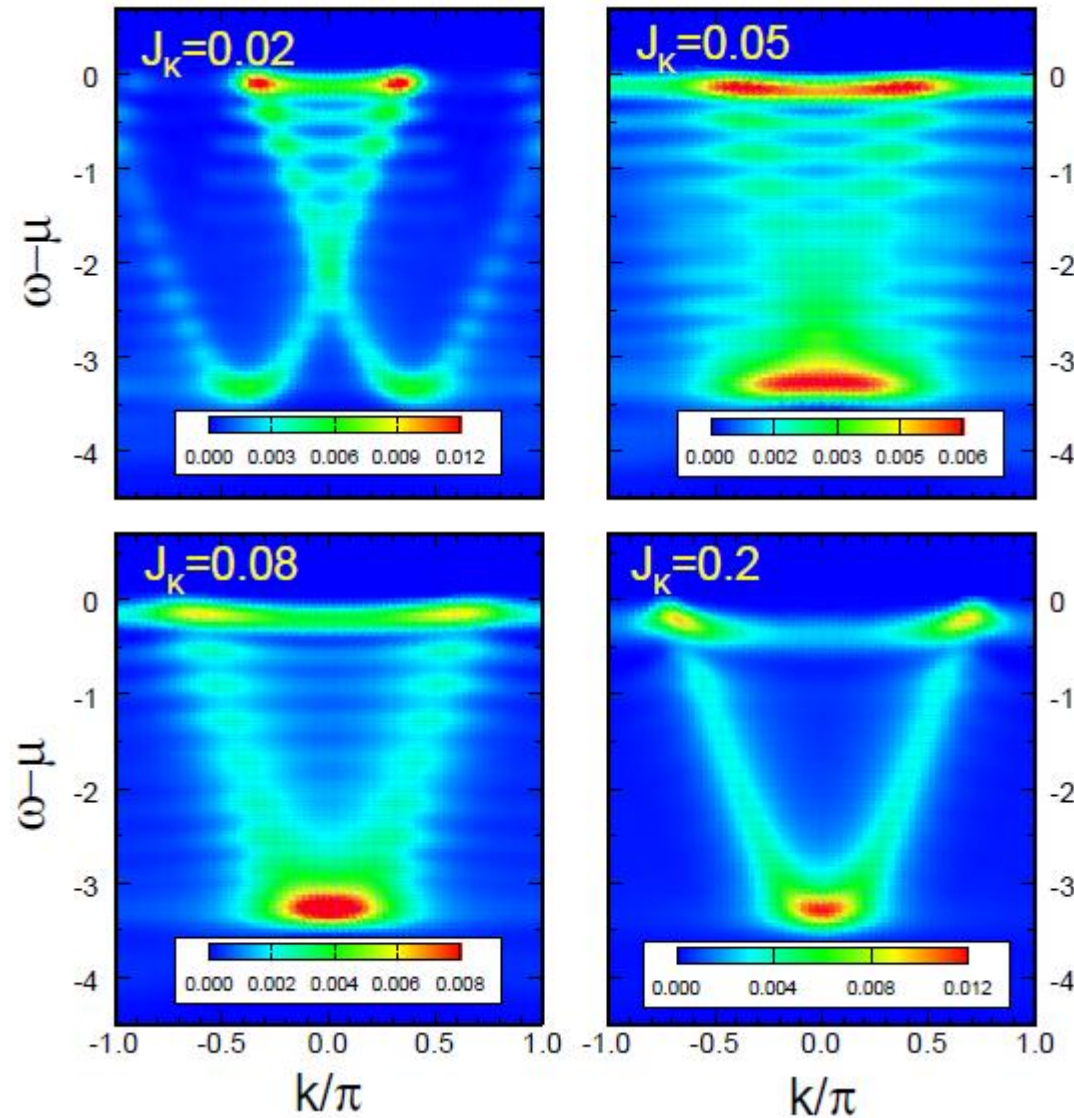


Correlation functions

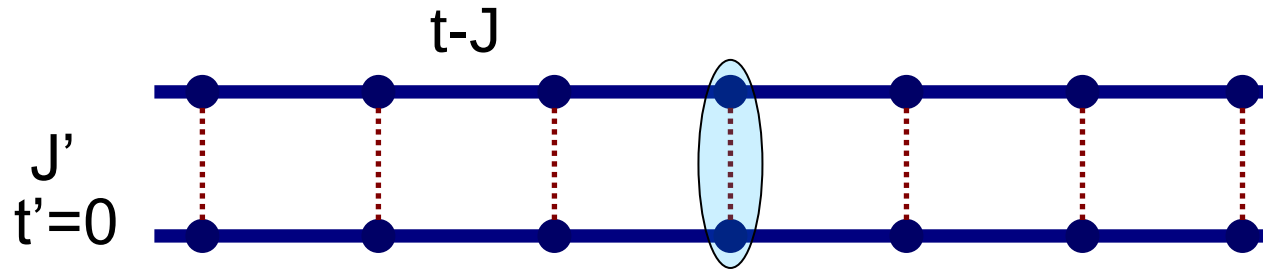


Kondo lattice at $T=0$

$L=32, N=24, J=0.05$



(II) t-J ladders



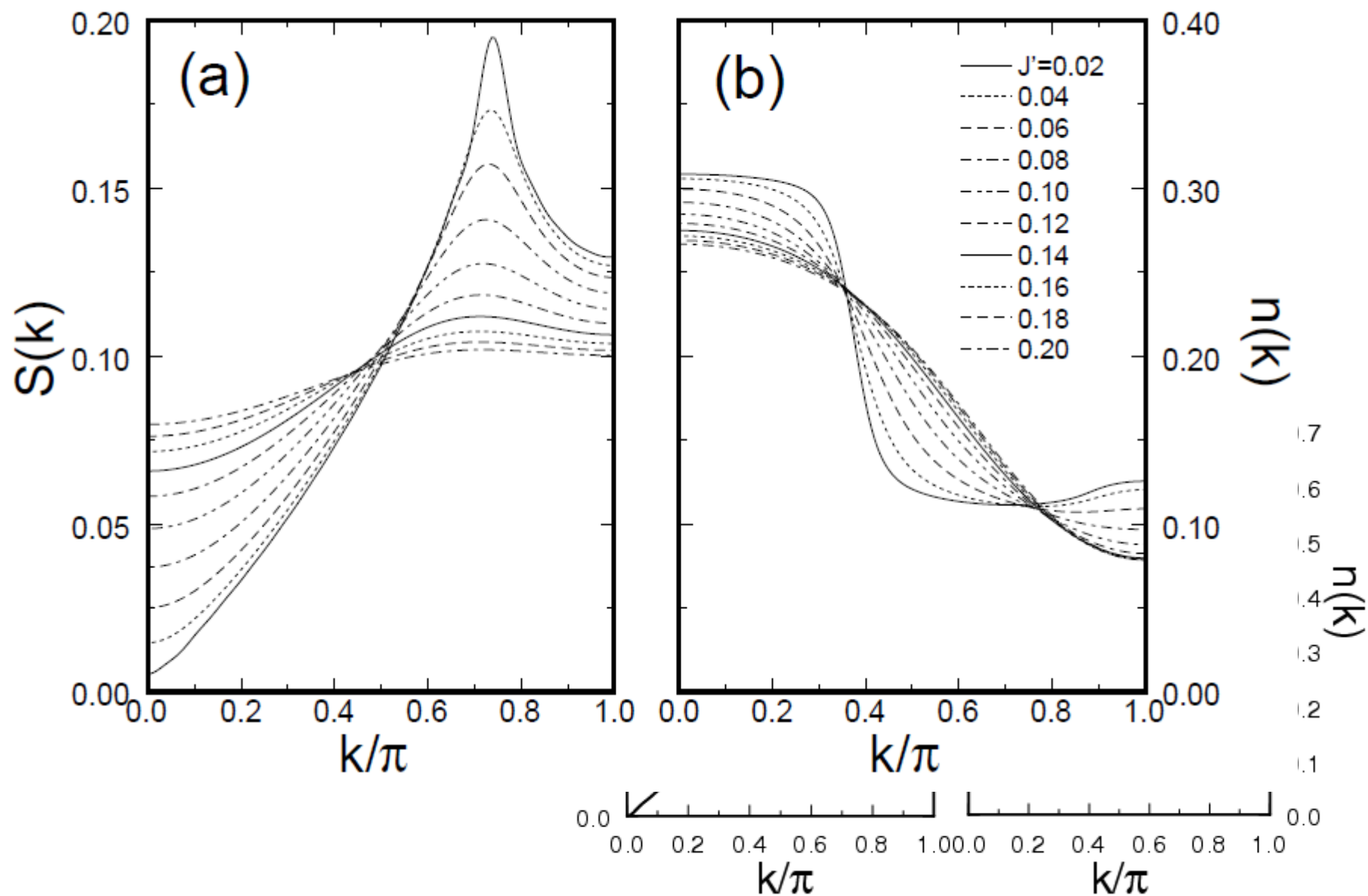
$$J=0, J'=0)$$

$$|\text{g.s.}\rangle = |\varphi_1\rangle \otimes |\varphi_2\rangle \otimes |\chi_1\rangle \otimes |\chi_2\rangle$$

$$J=0, J' \rightarrow \infty)$$

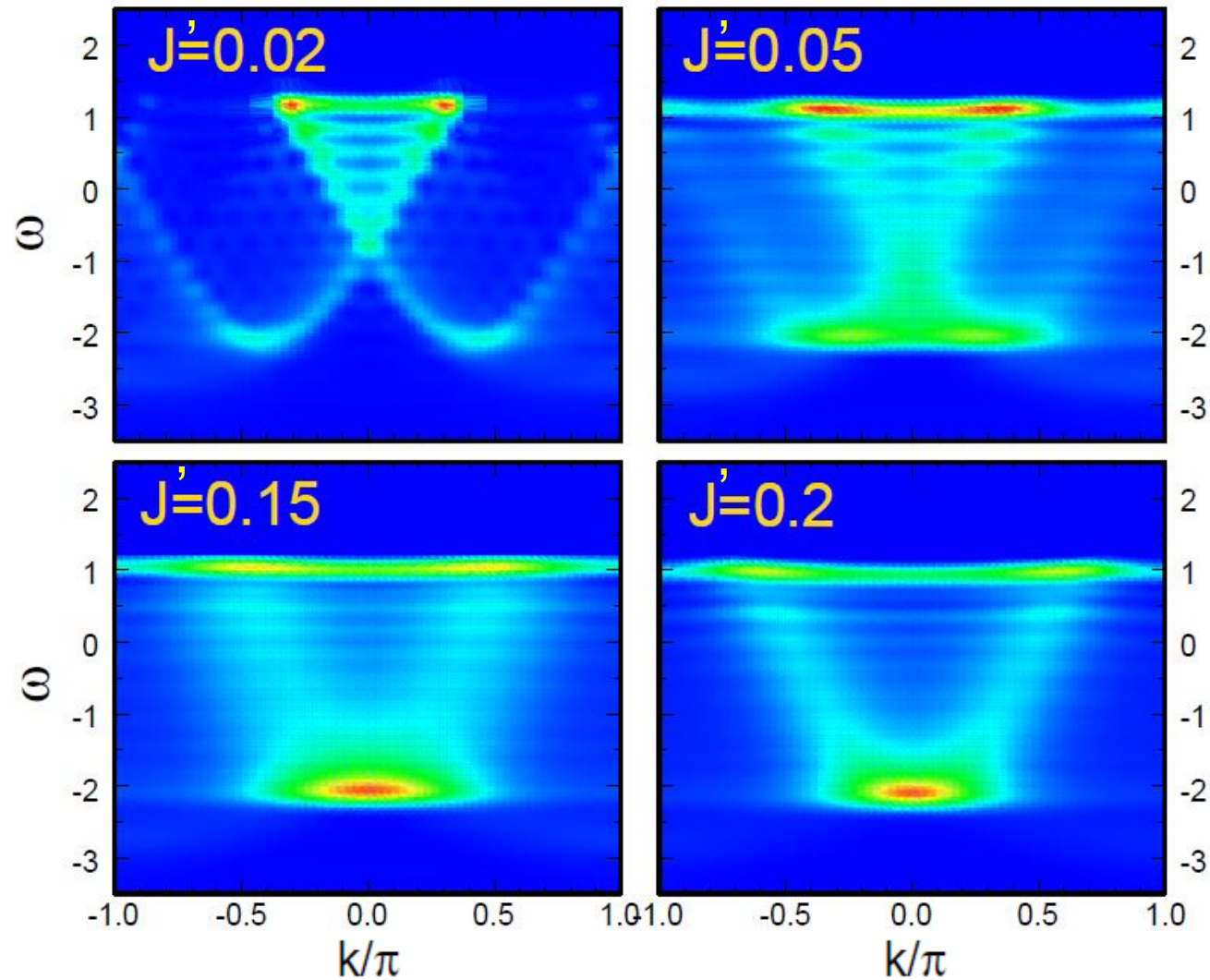
$$|\text{g.s.}\rangle = |\varphi^*\rangle \otimes |S\rangle$$

Correlation functions ($J=0.05$)



t-J ladder lattice at $T=0$

$L=32, N=24, J=0.05$



Conclusions

- We showed an application of the time dependent DMRG combining evolution in real-time and imaginary time.
- We studied the crossover from spin incoherent to spin coherent behavior
- We generalized the Ogata and Shiba's factorized wave function to finite *spin* temperatures
- We found that the t-J ladder in some regime of parameters and the Kondo lattice exhibit SI behavior in the *ground-state*.
- This SI behavior is not SILL, but results indicate that it might be possible to describe it within the same framework, and may present some universal features.

