Accelerating Cosmologies and Inflation by Higher Order Corrections in String Theories

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Congratulation to 60th birthday, both Prof. Maeda and Prof. Nakamura.

It is my great pleasure and honor to be acquainted with both of you, and in particular to collaborate with Maeda-san on various interesting subjects.

As other people, I wish both of you a good health and future prosperity.

My encounter with Maeda-san was the occasion when I was invited as a speaker in a workshop on brane world (2002 Jan.)

which was organized by those people. (In retrospect, ...)

It was then one or two years later JGRG at Osaka City University (2003 Dec.) that we really discuss seriously on collaboration on cosmological solutions in superstring/M-theory.

It is appropriate to talk about this subject in this occasion.

Contents

1 Introduction — Models of inflation and no-go theorem

- **Check of superstring**

Circumstance where quantum effects of gravity are manifest

$\Rightarrow$ **Black hole (singularity)**
Early universe (singularity)

It is extremely urgent and important to study if these problems could be resolved within superstring and if there is any possibility that it gives realistic models

We focus on the early universe ————————————

- **Why inflation is necessary?**
  - **Horizon problem: homogeneity beyond causally connected region**
  - **flatness problem: the present universe is very flat, why?**

Inflation resolves these problems, producing scale invariant density perturbation, in agreement with observation

Moreover the expansion of the present universe is found to be accelerating! (late-time acceleration)

The correct theory of gravity must explain not only the early inflation but also the present accelerating expansion.

- **The first model of inflation:**
  - **positive cosmological constant (A. Guth, K. Sato, 1981)**
    scale factor in the FLRW universe expands exponentially
  - **higher order corrections such as R^2 (A.A. Starobinsky, 1980)**
    these also give similar expansion

Without introducing artificial potential, we would like to have this behavior as a prediction of the fundamental theory, the superstring.

However we find no-go theorem (Gibbons, ...)
Einstein eqs. give

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P), \qquad \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho,$$

ρ**: energy density,** P**: pressure**
To have inflation we must have $\ddot{a} > 0$ i.e. $\rho + 3P < 0$ $(w \equiv \frac{P}{\rho} < -\frac{1}{3})$。
anti-gravity ⇒ famous one is the cosmological constant

$$\rho = \Lambda, \qquad P = -\Lambda,$$

Theorem: If $D(>2)$-dimensional sugra is compactified on smooth manifold without boundary and the following is true

1. **gravitational interactions do not contain higher derivative terms than ordinary Einstein theory**
2. **all massless fields have positive kinetic term (not ghost)**
3. **d-dimensional Newton constant is finite**

we cannot obtain accelerating expansion.

How to avoid no-go theorem

- **additional dof such as D-branes ⇒ Dvali-Tye, KKLT, KKLMMT**
- **time-dependent internal space ⇒ S-brane**
- **higher order corrections existing in superstring/M-theory (like Starobinsky) ⇒ Maeda-Ohta**
- **scalar fields with negative kinetic terms ⇒ Phantom cosmology**
- **non-compact and/or with boundary ⇒ brane world**

2 S-brane

A Vacuum solution which exhibits accelerating expansion was found (Townsend-Wohlfarth, Phys. Rev. Lett. 91 (2003) 061302)
It was immediately identified as a special case (0 flux) of S-brane (Ohta, Phys. Rev. Lett. 91 (2003) 061303)
SM2-brane (S2-brane in M-theory (with 3-dim. space))

$$\begin{aligned} ds_d^2 &= [\cosh 3c(t-t_2)]^{2/(k-1)} \Big[- e^{2kg(t)-6c'/(k-1)} dt^2 \\ &\quad + e^{2g(t)-6c'/(k-1)} d\Sigma_{k,\sigma}^2 + [\cosh 3c(t-t_2)]^{-2(k+2)/3(k-1)} e^{2c'} d\mathbf{x}^2 \Big]. \end{aligned}$$

$d = 4+k$, $d\Sigma_{k,\sigma}^2$ is k-dim sphere ($\sigma = +1$), flat ($\sigma = 0$) or hyperbolic ($\sigma = -1$) spaces. In 4-dim. Einstein frame

$$ds^2 = \delta^{-k}(t) ds_E^2 + \delta^2(t) d\Sigma_{k,\sigma}^2, \qquad ds_E^2 = -a^6(t)dt^2 + a^2(t)d\mathbf{x}^2,$$

$$\begin{aligned} \Rightarrow \delta(t) &= [\cosh 3c(t-t_2)]^{1/(k-1)} e^{g(t)-3c'/(k-1)}, \\ a(t) &= [\cosh 3c(t-t_2)]^{(k+2)/6(k-1)} e^{kg(t)/2-(k+2)c'/2(k-1)}. \end{aligned}$$

cosmic time: $d\eta = a^3(t)dt$

$\frac{da}{d\eta} > 0 \Rightarrow$ **4-dim. universe expands**

$$\Rightarrow n(t) \equiv \frac{3}{4}\tanh[3c(t-t_2)] - \frac{\sqrt{21}}{4}\coth(3\sqrt{3/7}ct) > 0,$$

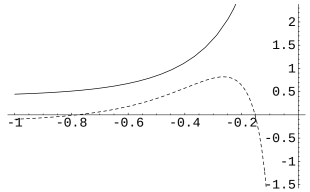


Figure 1: $\sigma = -1, k = 7$

$\frac{d^2a}{d\eta^2} > 0 \Rightarrow$ **accelerating expansion**

$$\Rightarrow \frac{9}{8}\left(\frac{1}{\cosh^2[3c(t-t_2)]} + \frac{1}{\sinh^2(3\sqrt{3/7}ct)}\right) - n^2(t) > 0.$$

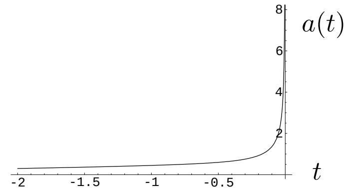


Figure 2: scale factor $a(t)$.

Alas, problem: obtained e-folding is around 2-3!!

Basic mechanism: consider the product of 4-dim. universe and k-dim. space

$$ds^2 = e^{-2\sum_i m_i\phi_i/(d-1)} ds_{d+1}^2 + \sum_{i=1}^{3} e^{2\phi_i(x)} d\Sigma_{m_i,\epsilon_i}^2,$$

The size of the internal space is determined by ϕ. The four-dim. effective potential is

$$V = \sum_{i=1}^{3}(-\epsilon_i)\frac{m_i(m_i-1)}{2} e^{-\frac{2}{d-1}\left((m_i+d-1)\phi_i + \sum_{j\neq i}^{1\le j\le 3} m_j\phi_j\right)} - \epsilon_0\frac{(d-1)^2}{2a^2}.$$

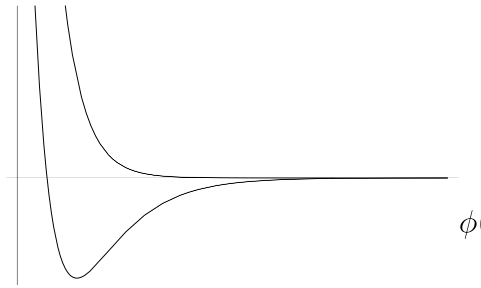


Figure 3: scalar potential

scalar fields comes in from the right, climbs up the slope and turn around. While the scalar is up on the slope, inflation occurs.
This happens always for S-branes even for $\epsilon_i > 0$ unlike solutions without flux (only for hyperbolic internal space).

If we use hyperbolic space for our space, there is accelerating ever-expanding solution.

(Chen-Ho-Neupane-Ohta-Wang, JHEP 0310(2003) 058, hep-th/0306291, JHEP 0611(2006) 044, hep-th/0609043.)

It was found for $m \geq 6 \Rightarrow$ M-theory or string theory! [$(4+m)$ **dims.**]

$$a(\tau) = \tau + A\tau^{-\sqrt{(m-6)/(m+2)}}$$

It may be useful to describe present accelerating expansion

3 Higher order corrections in M-theory (with Maeda-san)

S-brane $\cdots$ cannot be used for inflation at early universe

1. **not enough inflation**
2. **big internal space**

$\Rightarrow$ However higher order corrections in string theory $\Rightarrow$ must be important in the early universe

$$S = S_{\rm EH} + S_4,$$

$$S_{\rm EH} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-g}R, \qquad S_4 = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-g}\left[\alpha_4 \tilde{E}_8 + \gamma L_W + \delta R^4\right] .$$

$$\tilde{E}_8 = -\frac{1}{2^4\times 3!}\epsilon^{\alpha\beta\gamma\mu_1\nu_1\ldots\mu_4\nu_4}\epsilon_{\alpha\beta\gamma\rho_1\sigma_1\ldots\rho_4\sigma_4}R^{\rho_1\sigma_1}{}_{\mu_1\nu_1}\cdots R^{\rho_4\sigma_4}{}_{\mu_4\nu_4} ,$$

$$L_W = C^{\lambda\mu\nu\kappa}C_{\alpha\mu\nu\beta}C_\lambda{}^{\rho\sigma\alpha}C^\beta{}_{\rho\sigma\kappa} + \frac{1}{2}C^{\lambda\kappa\mu\nu}C_{\alpha\beta\mu\nu}C_\lambda{}^{\rho\sigma\alpha}C^\beta{}_{\rho\sigma\kappa}.$$

$C_{\lambda\mu\nu\kappa}$: **Weyl tensor,** $\alpha_4 = \frac{\kappa_{11}^2\ T_2}{3^2\times 2^9\times(2\pi)^4}, \gamma = \frac{\kappa_{11}^2\ T_2}{3\times 2^4\times(2\pi)^4}$, $T_2 = (2\pi^2/\kappa_{11}^2)^{1/3}$: **membrane tension**

$L_W(R) \sim L_W(C) + \frac{60}{(D-1)^2(D-3)^3}R^4 \Rightarrow \delta \sim 10^{-3}\gamma$ **(Otherwise R^4 is dominant and not interesting solution)**

(K. Maeda and N. Ohta, PLB 597 (2004) 400, hep-th/0405205; PRD 71 (2005) 063520, hep-th/0411093; K. Akune, K. Maeda and N. Ohta, PRD 73 (2006) 103506, hep-th/0602242.)

metric

$$ds_D^2 = -e^{2u_0(t)}dt^2 + e^{2u_1(t)}ds_p^2 + e^{2u_2(t)}ds_q^2, \qquad D = 1 + p + q.$$

σ_p, σ_q**: the signs of the $p = 3$-dim. and $q = 7$-dim spaces**

$$u_0 = \epsilon t, \qquad u_1 = \mu t + \ln a_0, \qquad u_2 = \nu t + \ln b_0,$$

$\epsilon = 0$**: generalized de Sitter solution**
$\epsilon = 1$**: power expanding solution**

Two out of three equations are independent:

$$F \equiv \sum_{n=1}^{4} F_n + F_W + F_{R^4} = 0 \ ,$$

$$F^{(p)} \equiv \sum_{n=1}^{4} f_n^{(p)} + X \sum_{n=1}^{4} g_n^{(p)} + Y \sum_{n=1}^{4} h_n^{(p)} + F_W^{(p)} + F_{R^4}^{(p)} = 0 \ ,$$

$$F^{(q)} \equiv \sum_{n=1}^{4} f_n^{(q)} + Y \sum_{n=1}^{4} g_n^{(q)} + X \sum_{n=1}^{4} h_n^{(q)} + F_W^{(q)} + F_{R^4}^{(q)} = 0 \ ,$$

$X = \ddot{u}_1 - \dot{u}_0\dot{u}_1 + \dot{u}_1^2$, $Y = \ddot{u}_2 - \dot{u}_0\dot{u}_2 + \dot{u}_2^2$.

Solutions are summarized below:

3.1 $\delta = \sigma_3 = \sigma_q = 0$

Generalized de Sitter solution

$$\mathbf{ME1}_{\pm}(\tilde{\mu}, \tilde{\nu}) = (\pm 0.10465, \mp 0.93666)$$

$a(\tau)$: **"super-inflation" in the Einstein frame.**

We do not find exact solution in power-law solutions but asymptotic solutions are found

3.2 $\delta = \sigma_3 = 0, \sigma_q \neq 0$ or the exchange

Generalized de Sitter solution

No exact solution

exact solution in power-law solution

$$\mathbf{ME12}(\mu, \nu, \sigma_q) = (0, 1, -1)$$

3.3 $\delta \neq 0$

More solutions for $\delta < 0 \Rightarrow$ See the next fig. and table

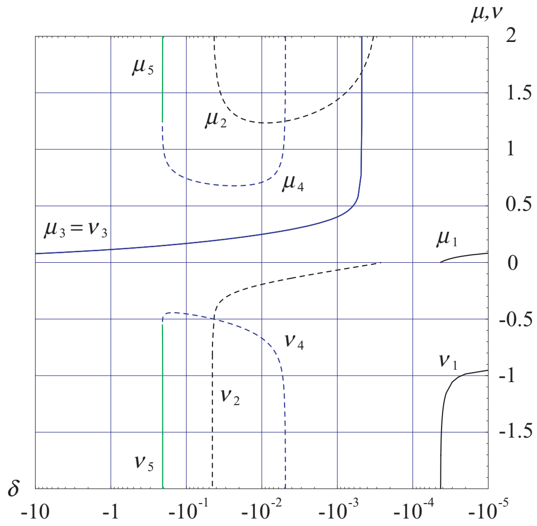


Figure 4: 5 de Sitter sol. for $\sigma_p = \sigma_q = 0, \delta < 0$

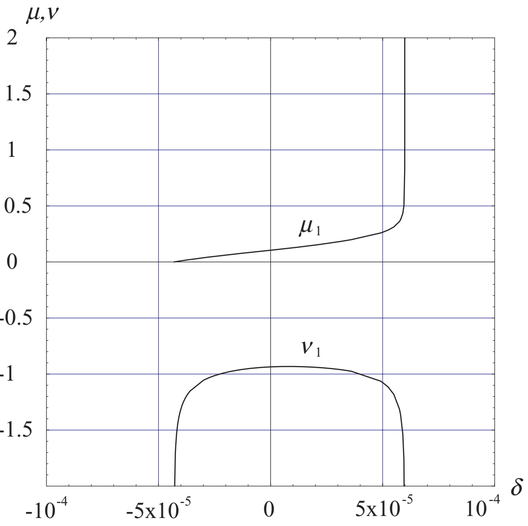


Figure 5: de Sitter sols. for $\delta \sim 0, \sigma_p = \sigma_q = 0$

Conditions for good inflation

1. $\mu > \nu, \mu \geq 0$

2. 60 e-folding

3. almost stable but small instability desirable

Closer study shows that enough inflation cannot be obtained only with the exact solution

Exact solutions have unstable modes, we should study the evolution after the exact solution decays in the direction of unstable mode

Numerical calculation ⇒ Solution which comes close to $\mathrm{ME6}_+$ and then goes away

Table 1: Solutions MEi_+ ($i = 1, \cdots, 5$) for various δ (ms,nu): m stable and n unstable modes Stable if 10-dim. volume expansion rate $3\mu + 7\nu$ is positive

Solution	Property	Range	Stability	$3\mu_i + 7\nu_i$
ME1$_+$	$\nu_1 < 0 < \mu_1$	$-0.000\,043\,11 < \delta < 0$	(0s,5u)	$-$
		$\delta = 0$	(1s,2u)	$-$
		$0 < \delta < 0.000\,059\,88$	(1s,4u)	$-$
ME2$_+$	$\nu_2 < 0 < \mu_2$	$-0.045\,20 < \delta < -0.002\,649$	(4s,1u)	$+$
ME3$_+$	$0 < \mu_3 = \nu_3$	$\delta < -0.000\,4732$	(3s,0u)	$+$
ME4$_+$	$\nu_4 < 0 < \mu_4$	$-0.2073 < \delta < -0.004\,852$	(1s,4u)	$-$
ME5$_+$	$\nu_5 < 0 < \mu_5$	$-0.2073 < \delta < -0.2056$	(2s,3u)	$-$
ME6$_+$	$\nu_6 = 0,\ 0 < \mu_6, \tilde\sigma_{q(6)}$	$\delta < -0.000\,5589$	(5s,1u)	$+$
ME7$_+$	$\nu_7 = 0,\ \tilde\sigma_{q(7)} < 0 < \mu_6$	$0.002\,999 < \delta$	(4s,2u)	$+$
ME8$_+$	$\mu_8 = 0,\ 0 < \tilde\sigma_{p(8)}, \nu_8$	$-0.003\,163 < \delta < -0.000\,5650$	(4s,2u)	$+$
ME9$_+$	$\mu_9 = 0,\ 0 < \tilde\sigma_{p(9)}, \nu_9$	$\delta < -0.000\,5657$	(5s,1u)	$+$
ME10$_+$	$\mu_{10} = 0,\ \tilde\sigma_{p(10)} < 0 < \nu_{10}$	$\delta < -0.000\,043\,49$	(5s,1u)	$+$
ME11$_+$	$\mu_{11} = 0,\ \tilde\sigma_{p(11)} < 0 < \nu_{11}$	$-0.085\,22 < \delta < -0.003\,164$	(4s,2u)	$+$

As a result, there is a case in which we can obtain large e-folding.

Feature of general sols: size of internal space is larger than the Planck scale

If we obtain 60 e-folding for MN1 (numerical solution in the next fig.)

$$R_0 \sim 4000 m_{11}^{-1}$$

This is rather big (large extra dimensions).

$$m_4^2 = R_0^7 m_{11}^9 \quad \Rightarrow \quad m_{11} \sim 2 \times 10^{-13} m_4 \sim 600 \ \textbf{TeV}$$

If true, we may achieve the energy scale of quantum gravity near future. But it may depend on the initial condition

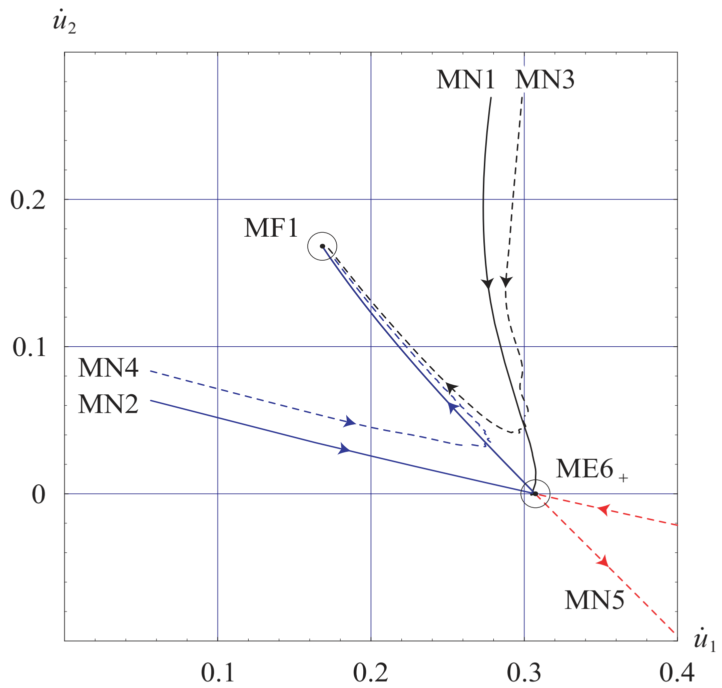


Figure 6: numerical results near ME6_+ on the $\dot{u}_1$-$\dot{u}_2$ plane for $\delta = -0.1$. Shown are ME6_+ and MF1

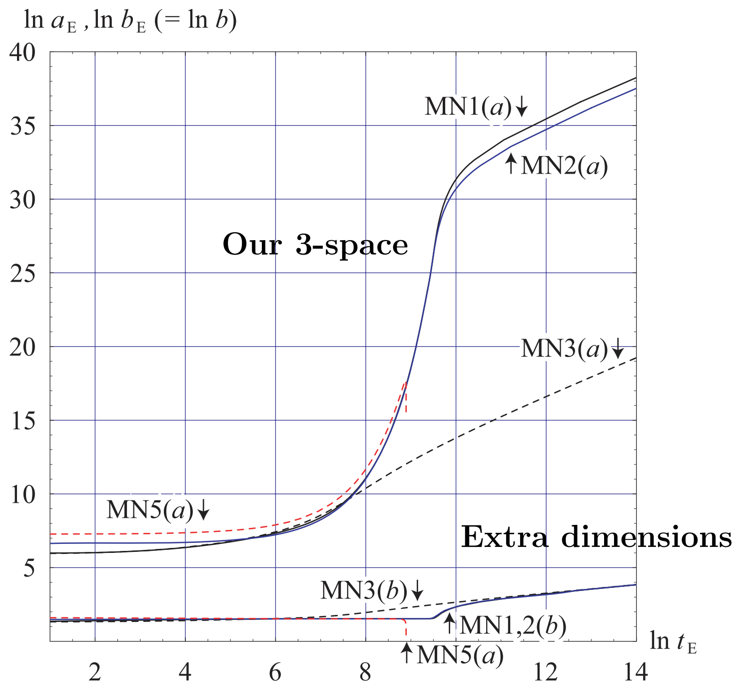


Figure 7: Sizes of the space for $\delta = -0.1$. MN$i(a)$ represents $\ln a_E$ of MNi solutions。

4 Gauss-Bonnet theory with dilaton

4.1 Solution space

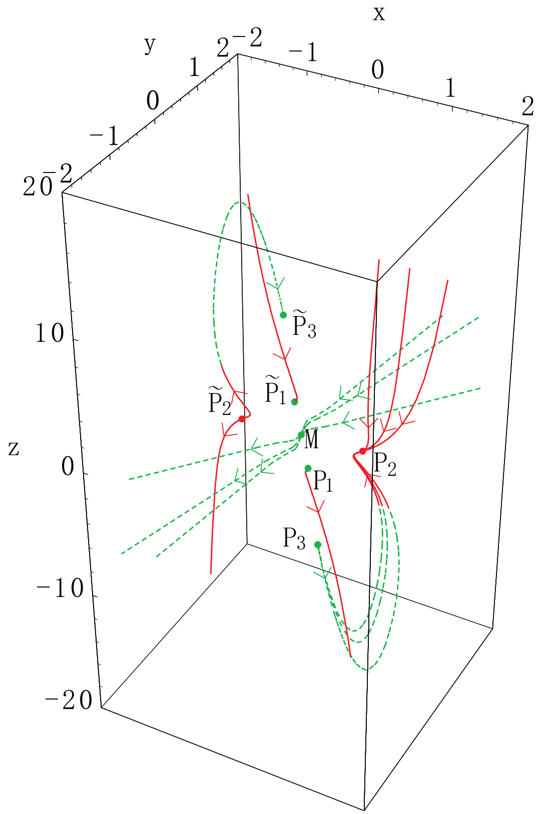
Similar solution in Gauss-Bonnet theory with dilaton (effective heterotic string)

(K. Bamba, Z.-K. Guo and N. Ohta, PTP 118 (2007) 879, arXiv:0707.4334 [hep-th].)

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-\tilde{g}}\, e^{-2\tilde{\phi}} \left[\tilde{R} + 4(\partial_\mu \tilde{\phi})^2 + \alpha_2 \tilde{R}^2_{\rm GB} \right],$$

Field equations gives autonomous system.

Fixed points in time-evolution ⇒ General solutions are those starting from one to another fixed points.



⇐ **Solution space with dilaton. solid (red) line for** $d^2a/d\tau^2 > 0$**, dotted (green) line for** $d^2a/d\tau^2 < 0$

7 fixed points

$$(x, y, z) = \mathbf{M}(0,0,0), \quad \mathbf{P}_1(\mp 0.292373, \pm 0.36066, \pm 0.954846),$$
$$\mathbf{P}_2(\pm 0.91822, \mp 0.080285, \pm 0.585906),$$
$$\mathbf{P}_3(\pm 0.161307, \pm 0.161307, \mp 9.30437),$$

Only P_2 gives accelerated expansion.

But actually P_2

$$a(\tau) = e^{u_1+3u_2} = e^{0.677T} \sim |\tau|^{-1.75} \quad (\tau : \textbf{cosmic time})$$

This approach is being reconsidered with Maeda-san in string frame, but the result seems not so much different.

With field redefinition ambiguity, the result changes much de Sitter solutions possible
$\Rightarrow$ poster by Wakebe

4.2 Density perturbation

Inflation by higher order corrections can produce scale invariant one?
(Z.-K. Guo, N. Ohta and S. Tsujikawa, PRD 75 (2007) 023520, astro-ph/0610336.)

Toy model: GB and dilaton higher derivative kinetic term

$$\mathcal{L}_c = -\frac{1}{2}\alpha' \xi(\phi)\left[c_1 R_{\rm GB}^2 + c_2 (\nabla\phi)^4\right], \quad \xi(\phi) = \lambda e^{\mu\phi} \ (\mu = \pm 1 \text{ at tree level.})$$

Typically $\lambda = -\frac{1}{4}, c_1 = 1, c_2 = -1$.

Our approach: no terms other than those from superstrings
See the next table for relevant solutions.

Result: If GB is dominant, NG. If higher kinetic term is dominant, G.

Parameters	$n_{\mathcal{R}}$	n_T	r
$\lambda = -1/4, c_1 = 1, c_2 = -1, \mu = 1$	3.28	2.28	–
$\lambda = -1/4, c_1 = 1, c_2 = -1, \mu = 10^{-2}$	1.0027	2.7×10^{-3}	–
$\lambda = -1/4, c_1 = 1, c_2 = 0, \mu = 10^{-2}$	1.011	0.011	–
$\lambda = 1/4, c_1 = 1, c_2 = -1, \mu = 1$	0.174	-0.826	46.4
$\lambda = 1/4, c_1 = 1, c_2 = -1, \mu = 10^{-2}$	0.9650	-0.0350	7.1×10^{-3}
$\lambda = 1/4, c_1 = 1, c_2 = -1, \mu = 10$	-12.4	-13.4	–
$\lambda = 1/4, c_1 = 1, c_2 = -1, \mu = 10$	-17.4	-18.4	–
$\lambda = 1/4, c_1 = 1, c_2 = -1, \mu = 10$	9.7×10^{-3}	-0.99	5.73
$\lambda = 1/4, c_1 = 1, c_2 = 0, \mu = 1$	0.37	-0.63	1.15×10^3
$\lambda = 1/4, c_1 = 1, c_2 = 0, \mu = 10^{-2}$	0.99	-0.01	–
$\lambda = 1/4, c_1 = 1, c_2 = 0, \mu = 10$	9.8×10^{-3}	-0.99	5.73

Table 2: de Sitter sols. for various λ, c_1, c_2, μ and $\omega_0 = -1$, spectral index $n_{\mathcal{R}}$ of scalar perturbation, n_T for tensor mode tensor-to-scalar ratio r

5 Conclusion

No-go theorem can be overcome.
Various models are possible, but fine tuning may be necessary.
In most models, quantum corrections seems to be important.
e.g. KKLT
S-brane

- **gives accelerating expansion**
- **small e-folding**

Higher order corrections are important in the early universe, and they can give generalized de Sitter solutions.

- **enough e-folding**
- **relatively large internal space which could be tested experimentally**
- **size of the internal space can be stabilized**
- **can produce density perturbation**

Outlook: Problems

- **Frame-dependence: Einstein or string (Jordan) frame**
  $\cdots$ K. Maeda, NO, R. Wakebe, in preparation
- **Fine tuning of the initial conditions?**
- **density perturbation $\Rightarrow$ Guo, Ohta, Tsujikawa, PRD 75 (2006) 023520, hep-th/0610336**
  Extend this GB case to M-theory and realization of graceful exit

KKLT need complicated and ad hoc settings. Does nature like that?
$\Rightarrow$ String Landscape, anthropic principle ?

How to derive 4-dim
We have 4-dim. as a solution, but it is not clear if it is natural. $\Rightarrow$ 4-form in M-theory important?
We then need higher order corrections in 4-form?

We hope that our solutions have useful applications!!

Thank you!