Reaction mechanism of fusion-fission process in superheavy mass region

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1. Model

Coupled-channels method + Dynamical Langevin calculation

Trajectory analysis ← Langevin equation
Two center shell model

2. Results

$^{36}\text{S}+^{238}\text{U}$ and $^{30}\text{Si}+^{238}\text{U}$

Mass distribution of Fission fragments
Capture Cross-section
Fusion Cross-section

3. Mechanism of Dynamical process

Potential energy surface on scission line
Analysis of trajectory behavior
Analysis of probability distribution

4. Summary
Exp. by K. Nishio et al.

$^{30}\text{Si} + ^{238}\text{U}$ $\text{Zcn}=106$ $\leftarrow$

$^{36}\text{S} + ^{238}\text{U}$ $\text{Zcn}=108$ $\rightarrow$

$E_{\text{cm}} = 169.0 \text{ MeV}$ $E^* = 75.5 \text{ MeV}$

$E_{\text{cm}} = 159.0 \text{ MeV}$ $E^* = 65.5 \text{ MeV}$

$E_{\text{cm}} = 154.0 \text{ MeV}$ $E^* = 60.5 \text{ MeV}$

$E_{\text{cm}} = 149.0 \text{ MeV}$ $E^* = 55.5 \text{ MeV}$

$E_{\text{cm}} = 144.0 \text{ MeV}$ $E^* = 50.5 \text{ MeV}$

$E_{\text{cm}} = 139.0 \text{ MeV}$ $E^* = 45.5 \text{ MeV}$

$E_{\text{cm}} = 134.0 \text{ MeV}$ $E^* = 40.5 \text{ MeV}$

$E_{\text{cm}} = 129.0 \text{ MeV}$ $E^* = 35.5 \text{ MeV}$

$E_{\text{cm}} = 180.0 \text{ MeV}$ $E^* = 65.5 \text{ MeV}$

$E_{\text{cm}} = 176.0 \text{ MeV}$ $E^* = 61.5 \text{ MeV}$

$E_{\text{cm}} = 170.0 \text{ MeV}$ $E^* = 55.5 \text{ MeV}$

$E_{\text{cm}} = 166.0 \text{ MeV}$ $E^* = 51.5 \text{ MeV}$

$E_{\text{cm}} = 160.0 \text{ MeV}$ $E^* = 45.5 \text{ MeV}$

$E_{\text{cm}} = 154.0 \text{ MeV}$ $E^* = 39.5 \text{ MeV}$

$E_{\text{cm}} = 150.0 \text{ MeV}$ $E^* = 35.5 \text{ MeV}$

$E_{\text{cm}} = 146.0 \text{ MeV}$ $E^* = 31.5 \text{ MeV}$
What we can obtain under the conditions

Phenomenalism
Dynamical Model based on Fluctuation-dissipation theory
(Langevin eq, Fokker-Plank eq, etc) ← Classical trajectory analysis

We can obtain:
- Mass and TKE distribution of fission fragments
- Neutron multiplicity
- Charge distribution
- Cross section (capture, mass symmetric fission, fusion)
- Angle of ejected particle, Kinetic energy loss (← two body)

Fission, Synthesis of SHE
A_{CN} : 200~300

Conditions
- Nuclear shape parameter
- Potential energy surface (LDM, shell correction energy, LS force)
- Transport coefficients (friction, inertia mass) ← Linear Response Theory
- Dynamical equation (memory effect, Einstein relation)
1. Model

1-1. Potential
1-2. Dynamical Equation
1-3. Simulation of Experiment and Cross Sections
Estimation of cross sections

Capture Cross Section

\[ \sigma_{\text{cap}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{cap}}(E; \theta), \]

\[ \sigma_{\text{cap}}(E; \theta) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) T_{\ell}(E; \theta), \]

Coupled-channel method

Fusion Cross Section

\[ \sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta), \]

\[ \sigma_{\text{fus}}(E; \theta) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) T_{\ell}(E; \theta) P_{\text{CN}}(E, \ell, \theta), \]

Formation probability \( P_{\text{CN}} \)

Dynamical calculation

Langevin eq.
Model: Outlook of calculation methods

Time-evolution of nuclear shape in fusion-fission process

1. Potential energy surface

2. Trajectory → described by equations
Nuclear shape

two-center parametrization \((z, \delta, \alpha)\)

(Maruhn and Greiner, Z. Phys. 251(1972) 431)

\[ q(z, \delta, \alpha) \]

\[ z = \frac{z_0}{BR} \]

\[ B = \frac{3 + \delta}{3 - 2\delta} \]

\( R \): Radius of the spherical compound nucleus

\[ \delta = \frac{3(a - b)}{2a + b} \]

\((\delta_1 = \delta_2)\)

\[ \alpha = \frac{A_1 - A_2}{A_{CN}} \]

Trajectory which enters into the spherical region = fusion trajectory
\[ V(q, \ell, T) = V_{DM}(q) + \frac{\hbar^2 \ell (\ell + 1)}{2I(q)} + V_{SH}(q, T) \]

\[ V_{DM}(q) = E_S(q) + E_C(q) \]

\[ V_{SH}(q, T) = E_{shell}^0(q) \Phi(T) \]

\( T \): nuclear temperature
\( E^* = aT^2 \quad a \): level density parameter

Toke and Swiatecki

\( E_S \): Generalized surface energy (finite range effect)
\( E_C \): Coulomb repulsion for diffused surface
\( E_{shell}^0 \): Shell correction energy at \( T=0 \)

\( I \): Moment of inertia for rigid body

\( \Phi(T) \): Temperature dependent factor

\[ \Phi(T) = \exp \left\{ -\frac{aT^2}{E_d} \right\} \]

\( E_d = 20 \text{ MeV} \)
\[
\frac{dq_i}{dt} = (m^{-1})_{ij} p_j \\
\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2} \frac{\partial}{\partial q_i} (m^{-1})_{jk} p_j p_k - \gamma_{ij} (m^{-1})_{jk} p_k + g_{ij} R_j(t)
\]

\[
\langle R_i(t) \rangle = 0, \quad \langle R_i(t_1) R_j(t_2) \rangle = 2 \delta_{ij} \delta(t_1 - t_2) : \text{white noise (Markovian process)}
\]

\[
\sum_k g_{ik} g_{jk} = T \gamma_{ij}
\]

\( q_i \): deformation coordinate  \( (\text{nuclear shape}) \)  
\( \text{two-center parametrization} (z, \delta, \alpha) \)  
\( \text{(Maruhn and Greiner, Z. Phys. 251(1972) 431)} \)

\( p_i \): momentum

\( m_{ij} \): Hydrodynamical mass  \( (\text{inertia mass}) \)

\( \gamma_{ij} \): Wall and Window (one-body) dissipation  \( (\text{friction}) \)

\[
E_{\text{int}} = E^* - \frac{1}{2} (m^{-1})_{ij} p_i p_j - V(q)
\]

\( E_{\text{int}} \): intrinsic energy,  \( E^* \): excitation energy
Two Center Shell Model

\[ \hat{H} = -\frac{\hbar^2}{2m_0} \nabla^2 + V(\mathbf{r}) + V_{\text{LS}}(\mathbf{r}, p, s) + V_{L^2}(\mathbf{r}, p). \]

Neck parameter is the ratio of smoothed potential height to the original one where two harmonic oscillator potential cross each other.

\[ V(\rho, z) = \frac{1}{2} m_0 \begin{cases} \omega_{z_1}^2 z'^2 + \rho^2 \omega_p^2, & z < z_1 \\ \omega_{z_1}^2 z'^2 \left(1 + c_1 z' + d_1 z'^2\right) + \rho^2 \left(1 + g_1 z'^2\right), & z_1 < z < 0 \\ \omega_{z_2}^2 z'^2 \left(1 + c_2 z' + d_2 z'^2\right) + \rho^2 \left(1 + g_2 z'^2\right), & 0 < z < z_2 \\ \omega_{z_2}^2 z'^2 + \rho^2 \omega_p^2, & z > z_2, \end{cases} \]

\[ z' = \begin{cases} z - z_1, & z < 0 \\ z - z_2, & z > 0 \end{cases} \]

Neck parameter \( \varepsilon = 1.0 \)

J. Maruhn and W. Greiner, Z. Phys, 1972
Time dependent adiabatic fusion-fission potential

$V_{\text{adiab}}(r, \delta, \alpha, \varepsilon; t) = V_{\text{adiab}}(r, \delta, \alpha, \varepsilon = 1) \cdot \exp\left(-\frac{t}{\tau_\varepsilon}\right) + V_{\text{adiab}}(r, \delta, \alpha, \varepsilon = \varepsilon_{\text{out}}) \cdot \left[1 - \exp\left(-\frac{t}{\tau_\varepsilon}\right)\right]$

$\tau_\varepsilon = 10^{-20} \text{ sec}$

V. Zagrebaev, A. Karpov, Y. Aritomo, M. Naumenko and W. Greiner,
2. Results

1. Capture and Fusion Cross sections
2. Mass distribution of Fission Fragments

$^{34, 36}S + ^{238}U$ and $^{30}Si + ^{238}U$

Touching probability $\leftarrow$ CC method

Perform a trajectory calculation starting from the touching distance between target and projectile to the end each process.
Fusion box and Sample trajectory $^{36}\text{S} + ^{238}\text{U}$

- $Z-\alpha$
- $\delta = 0$
- $E^* = 40$ MeV
- $L = 0$
- $\theta = 0$

- $Z-\delta$
- $\alpha = 0$

(a) and (b) diagrams with various labels and coordinates.
Results \( ^{36}\text{S} + ^{238}\text{U} \) MDFF and Cross sections

**FF process**

[Graph showing cross-sections for different energies with legends for individual cross-section categories]
Results $^{30}\text{Si}+^{238}\text{U}$ MDFF and Cross sections

![Graph showing cross-sections for different energies](image)

![Graph showing cross-sections as a function of energy](image)
3. Mechanism of Dynamical process

MDFF at Low incident energy

$^{30}\text{Si} + ^{238}\text{U}$

Clarification of the mechanism of Fusion-fission process
(a) 1-dim Potential energy on scission line

\[ V(A) = \begin{cases} \text{energy expression} & \text{for } z=1.5, \delta=0.22, \epsilon=1.0 \\ \text{energy expression} & \text{for } z=2.35, \delta=0.22, \epsilon=0.35 \end{cases} \]

(b) [Potential energy graph for \( ^{274}\text{Hs} + ^{238}\text{U} \)]
(b) Trajectory Analysis on Potential Energy Surface \( z-A \) plane

\[ ^{30}\text{Si}+^{238}\text{U} \quad E^* = 35.5 \text{ MeV} \]
\[ L=0, \theta=0 \]

\[ \delta=0.22, \varepsilon=1.0 \]

\[ ^{36}\text{S}+^{238}\text{U} \quad E^* = 39.5 \text{ MeV} \]
\[ L=0, \theta=0 \]

\[ \delta=0.22, \varepsilon=1.0 \]
Trajectory Analysis on Potential Energy Surface $^{30}\text{Si} + ^{238}\text{U}$

$E^* = 35.5$ MeV
$L=0, \theta=0$
Trajectory Analysis on Potential Energy Surface $^{36}\text{S}+^{238}\text{U}$

$E^* = 39.5$ MeV
$L=0, \theta=0$
Trajectory Analysis ⇐ using **ALL** trajectories

![Diagram showing trajectory analysis with ~40,000 points]
(c) Trajectory Analysis → “Probability Distribution”

\[ E^* = 35.5 \text{ MeV} \]
\[ L=0, \theta=0 \]

\[ E^* = 39.5 \text{ MeV} \]
\[ L=0, \theta=0 \]
1. In order to analyze the fusion-fission process in superheavy mass region, we apply the Couple channels method + Langevin calculation.

2. **Incident energy dependence** of mass distribution of fission fragments (MDFF) is reproduced in reaction $^{36}\text{S}+^{238}\text{U}$ and $^{30}\text{Si}+^{238}\text{U}$.

3. The shape of the MDFF is analyzed using
   (a) 1-dim potential energy surface on the scission line
   (b) sample trajectory on the potential energy surface
   (c) *probability distribution*

4. The relation between the touching point and the ridge line is very important to decide the process $\rightarrow$ fusion hindrance leading to synthesize SHE