

Multiple- $Q$  states and skyrmion lattice of the triangular-lattice Heisenberg model under magnetic fields with an incommensurate helical structure

*Osaka Univ., Tsuyoshi Okubo, S. Chung, and H. Kawamura*

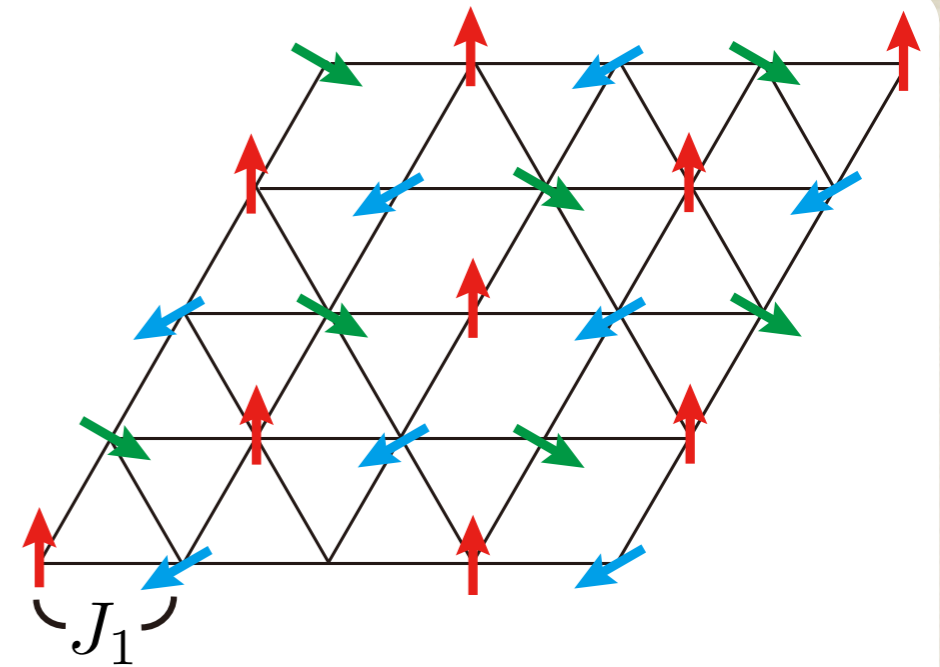
# Triangular-lattice Heisenberg antiferromagnets

$$\mathcal{H} = -J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

$$\vec{S}_i = (S_{i,x}, S_{i,y}, S_{i,z})$$

- Nearest neighbor model:  $J_1 < 0$

The ground state is the  $120^\circ$  structure



# Triangular-lattice Heisenberg antiferromagnets

$$\mathcal{H} = -J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - J_2 \sum_{\langle i,j \rangle_2} \vec{S}_i \cdot \vec{S}_j - J_3 \sum_{\langle i,j \rangle_3} \vec{S}_i \cdot \vec{S}_j$$

$$\vec{S}_i = (S_{i,x}, S_{i,y}, S_{i,z}) \quad J_2, J_3 < 0: \text{antiferromagnetic}$$

- Nearest neighbor model:  $J_1 < 0$

The ground state is **the 120° structure**

- Effects of **further neighbor interactions**

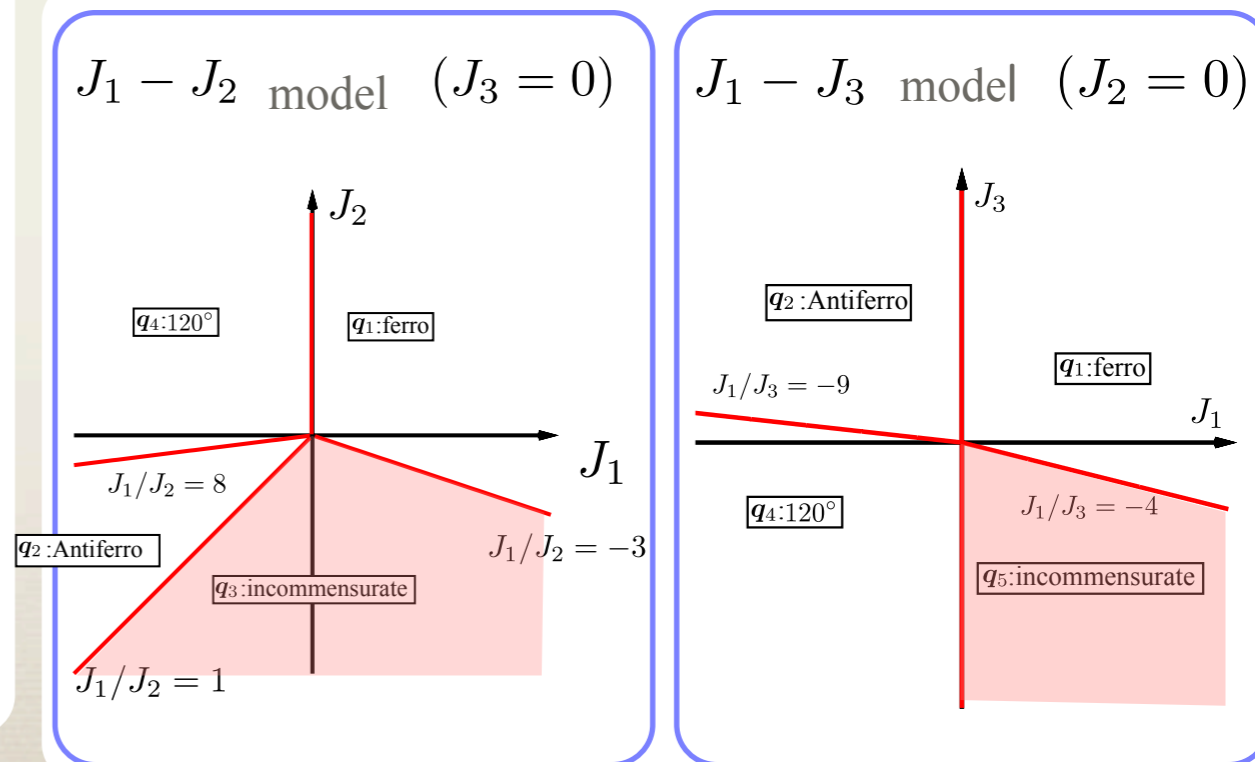
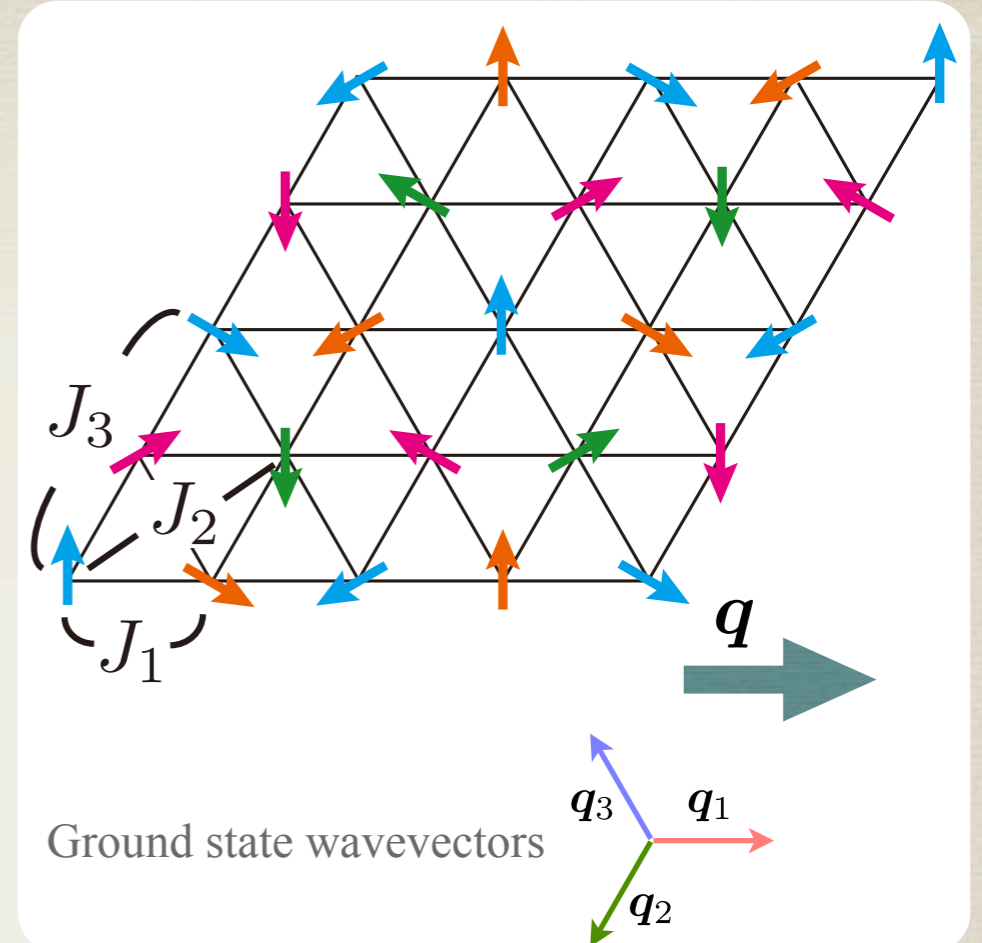
Large antiferromagnetic  $J_2, J_3$

The ground state is **an incommensurate spiral**



**Three-fold degeneracy**  
with respect to the choice of  
three equivalent directions on the lattice

c.f. NiGa<sub>2</sub>S<sub>4</sub> :  $J_1/J_3 \simeq -0.2$   
 $J_1 > 0$  : **Ferromagnetic**



# Ordering due to the three-fold degeneracy

1. Phase transition associated with breaking of the three-fold lattice symmetry

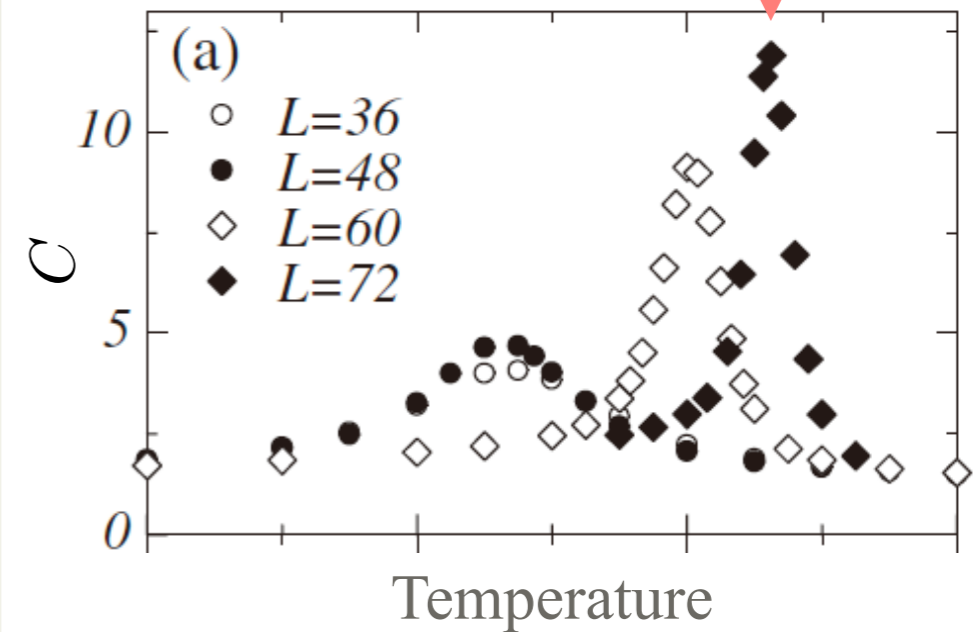
(R. Tamura and N. Kawashima, 2008)

- In zero magnetic fields, the ordered state is a single- $q$  spiral state.

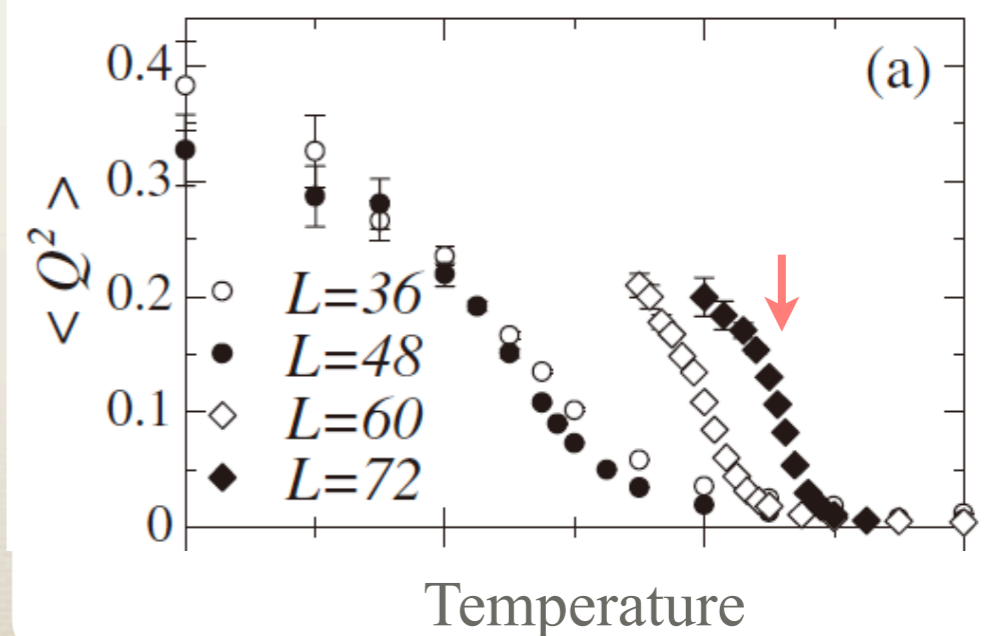
$J_1 - J_3$  model

(R. Tamura and N. Kawashima, JPSJ (2008))

Specific heat



Order parameter



# Ordering due to the three-fold degeneracy

1. **Phase transition** associated with **breaking of the three-fold lattice symmetry**

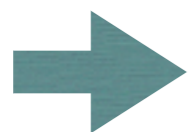
(R. Tamura and N. Kawashima, 2008)

- **In zero magnetic fields**, the ordered state is a **single- $q$**  spiral state.

2. Possibility of **multiple- $q$**  states

Multiple- $q$  state :

**More than one wavevectors** coexist

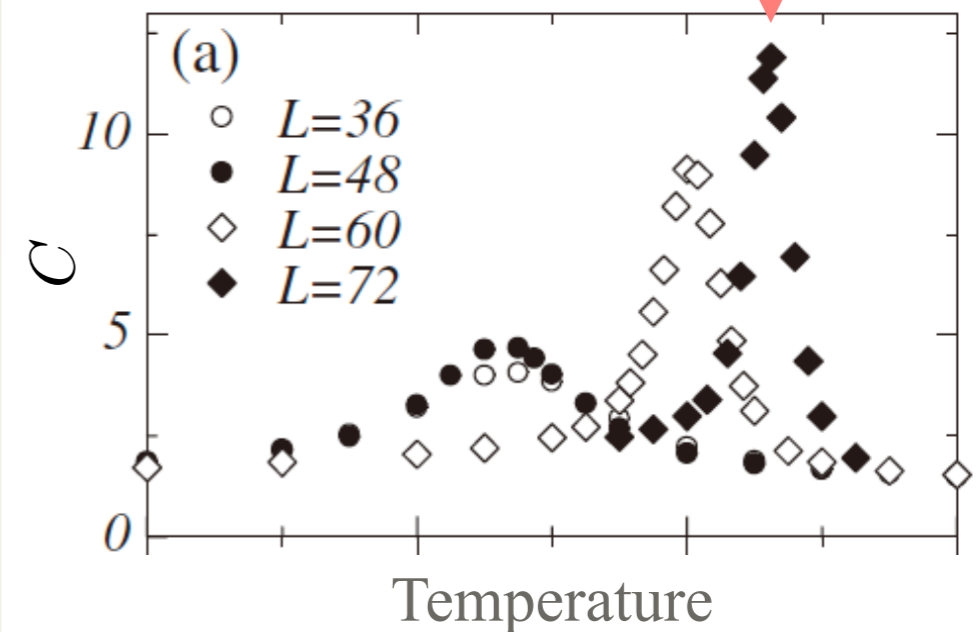


It might be realize **under applied fields**.

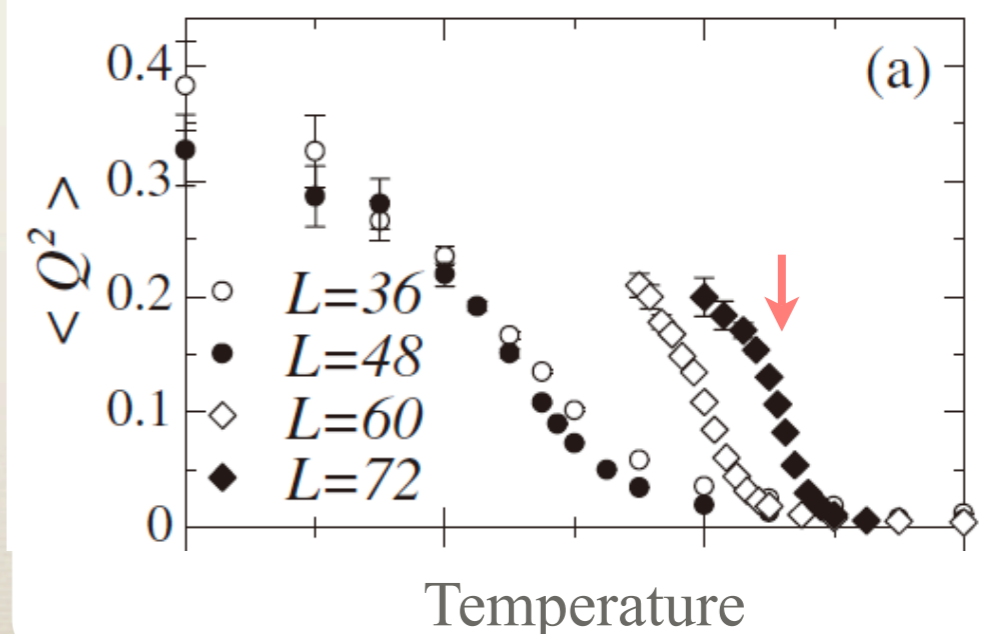
$J_1 - J_3$  model

(R. Tamura and N. Kawashima, JPSJ (2008))

Specific heat



Order parameter



# Model and methods

Triangular-lattice  $J_1$ - $J_3$  (or  $J_1$ - $J_2$ ) model:

$$\mathcal{H} = -J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - J_2 \sum_{\langle i,j \rangle_2} \vec{S}_i \cdot \vec{S}_j - J_3 \sum_{\langle i,j \rangle_3} \vec{S}_i \cdot \vec{S}_j - \underline{H \sum_i S_{i,z}}$$

$J_2, J_3 < 0$  : antiferromagnetic

Purpose :

To clear the ordering of the triangular-lattice Heisenberg model with **an incommensurate ground state** under magnetic fields

Question : Are **multiple- $q$  states** stabilized under applied fields?

Methods : Mean-field analysis  
Monte Carlo simulations

# Model and methods

Triangular-lattice  $J_1$ - $J_3$  (or  $J_1$ - $J_2$ ) model:

$$\mathcal{H} = -J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - J_2 \sum_{\langle i,j \rangle_2} \vec{S}_i \cdot \vec{S}_j - J_3 \sum_{\langle i,j \rangle_3} \vec{S}_i \cdot \vec{S}_j - H \sum S_{i,z}$$

Answer :

Purpose :

To clear the origin of the  
model with

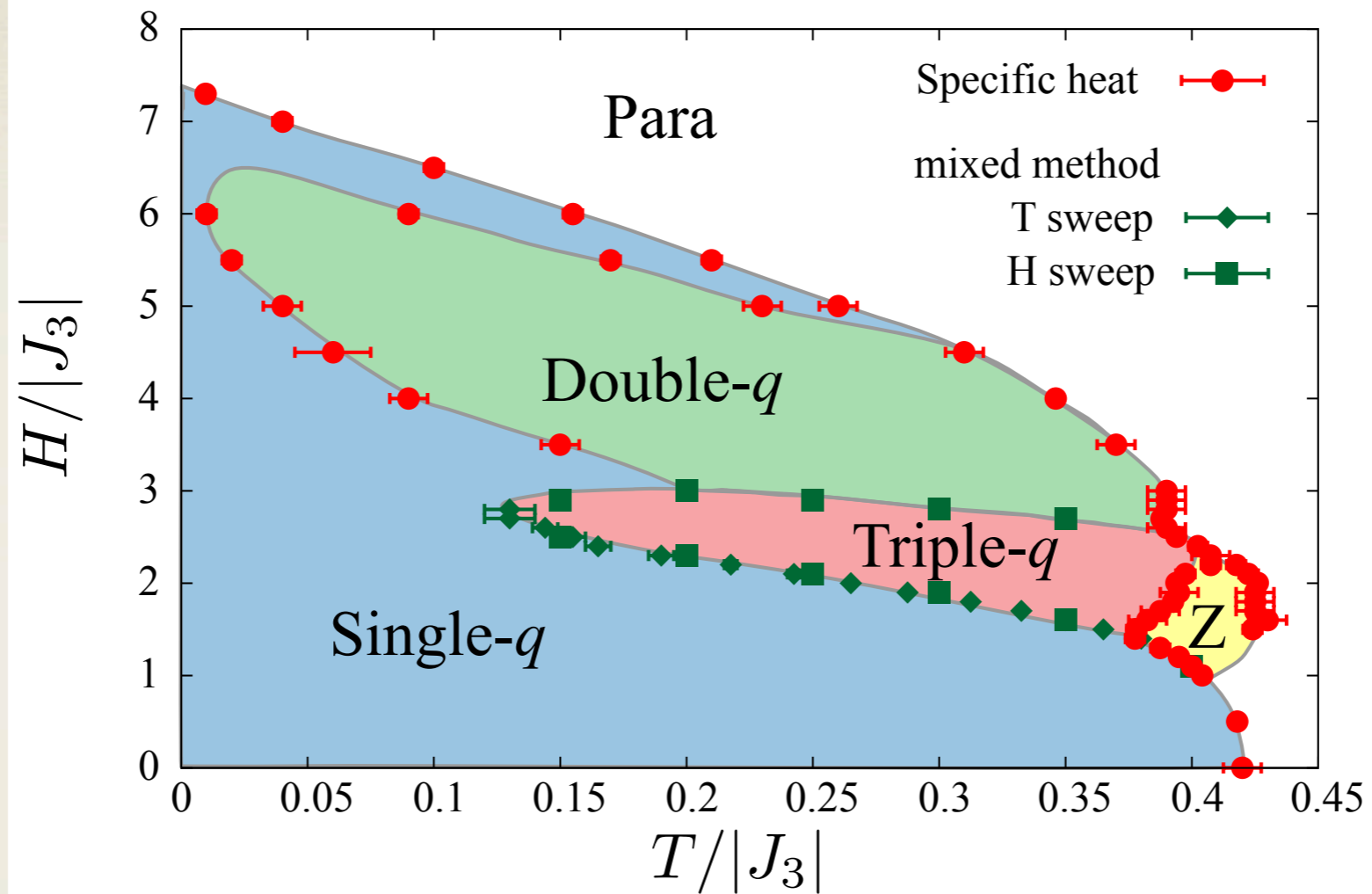
- Several types of multiple- $q$  states are indeed stabilized under magnetic fields.
- One of them corresponds to the so called “*skyrmion lattice*” state.

Question : Are **multiple- $q$  states** stabilized under applied fields?

Methods : Mean-field analysis  
Monte Carlo simulations

# Phase diagram

$J_1 - J_3$  model  $J_1/J_3 = -1/3$

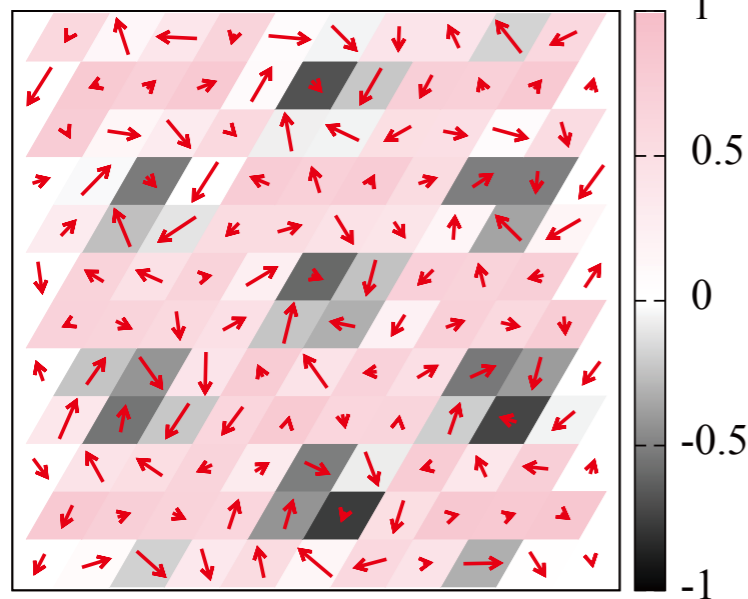


- In addition to the single- $q$  state, the double- $q$  and the triple- $q$  states are stabilized under magnetic fields.
- Triple- $q$  phase and Z phase are related to the skyrmion lattice

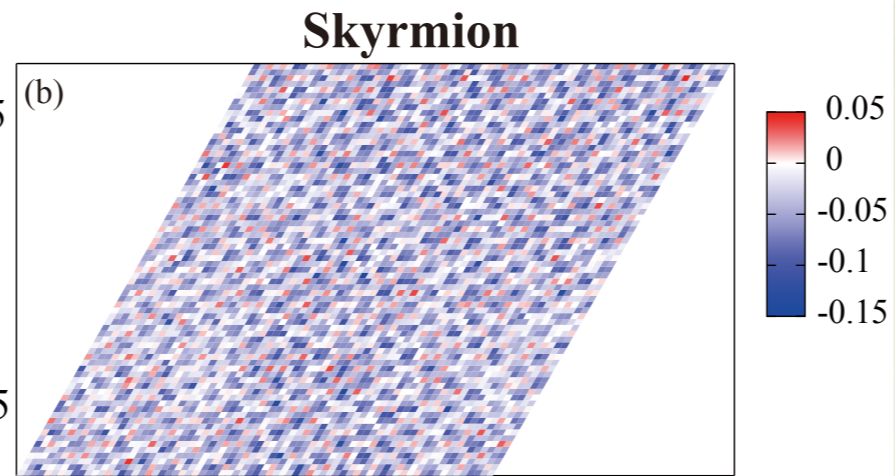


# Triple- $q$ phase: the skyrmion lattice

Snapshot

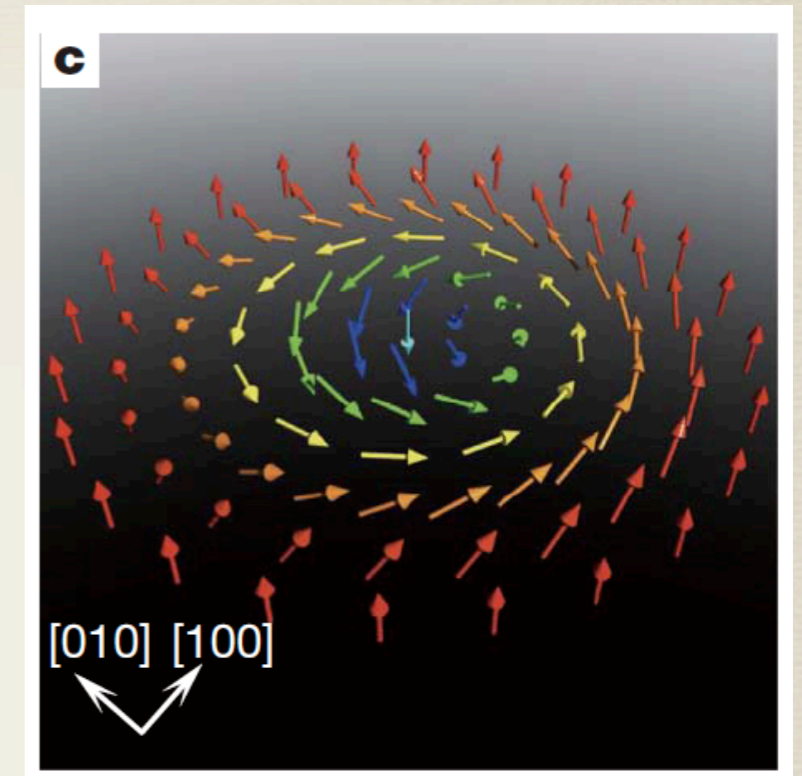


Skyrmion density



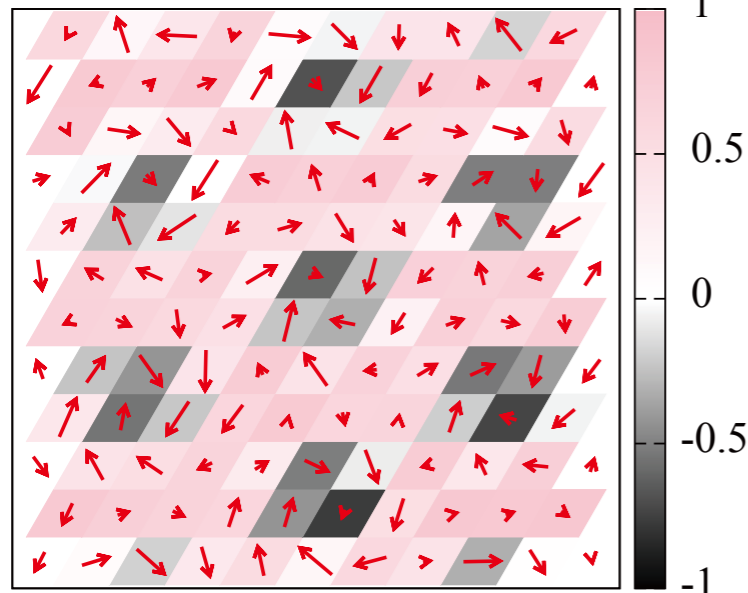
Skyrmion

X. Z. Yu et al., Nature (2010)



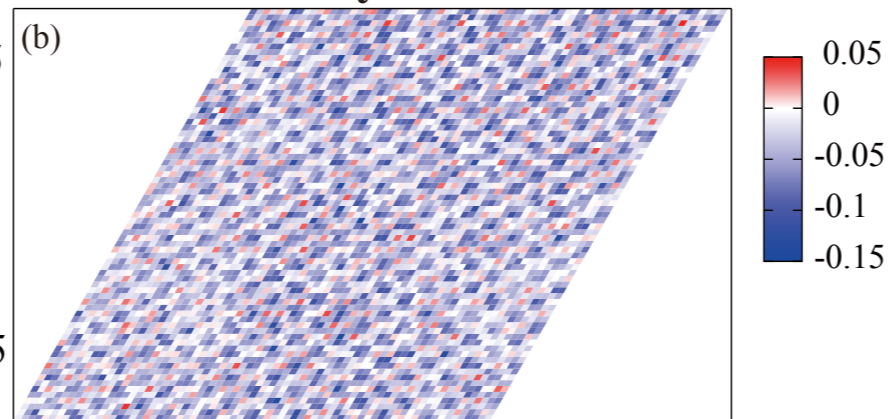
# Triple- $q$ phase: the skyrmion lattice

Snapshot

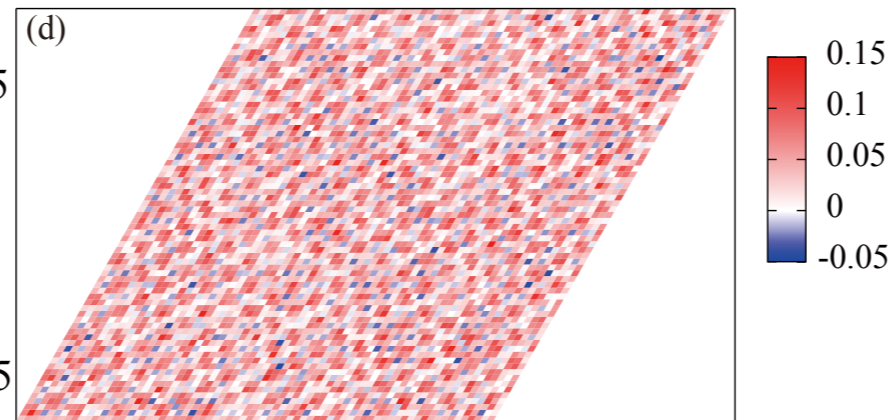
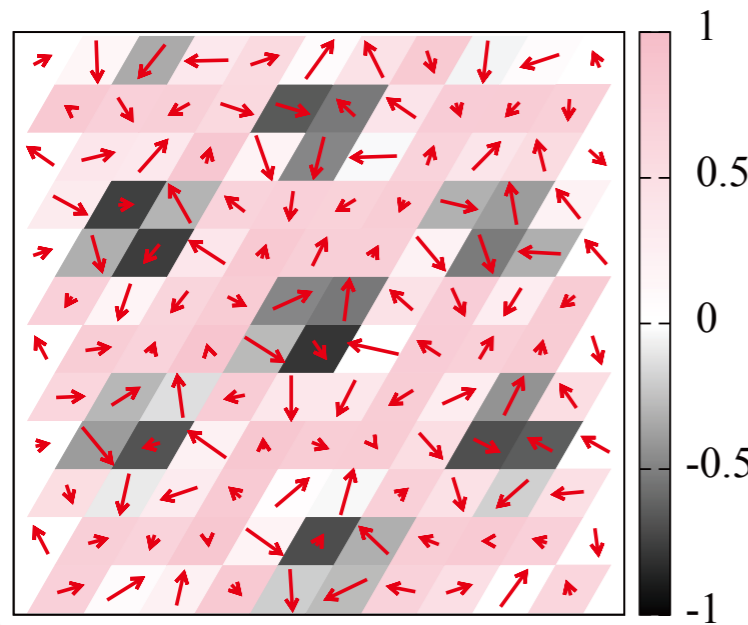


Skyrmion density

Skyrmion

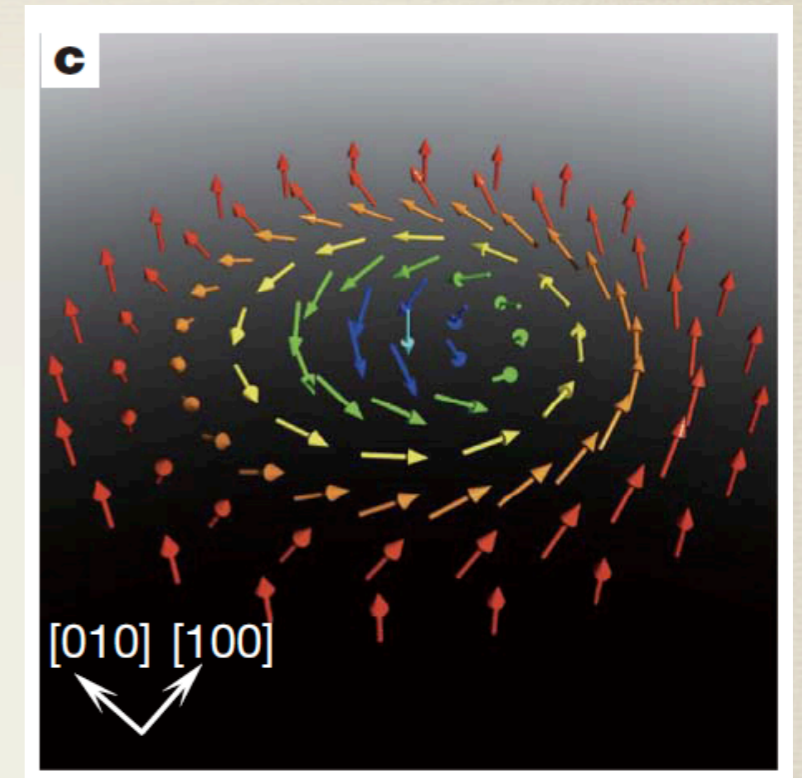


Anti-skyrmion

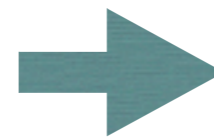


Skyrmion

X. Z. Yu et al., Nature (2010)



Both skyrmion and the anti-skyrmion lattices are possible.

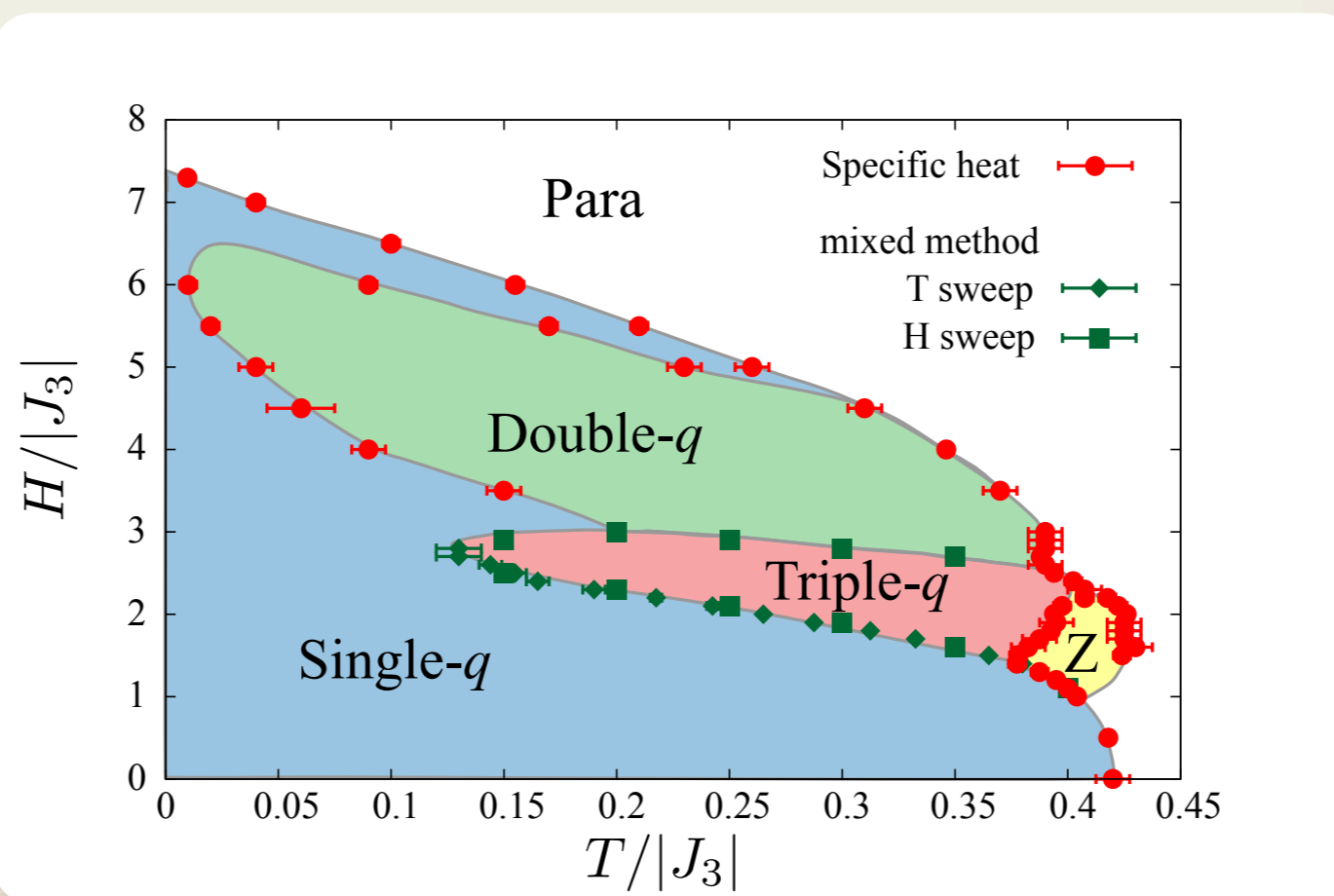


Z phase:

A domain state of skyrmion and anti-skyrmion lattices

# Summary

- \* We investigated the ordering of the classical Heisenberg model on the triangular-lattice with an incommensurate helical spin structure under magnetic fields.
- \* In addition to the standard single- $q$  phase, the double- $q$  and the triple- $q$  phases are stabilized under applied fields.
- \* The spin structure in the triple- $q$  phase is the **skyrmion lattice**.
- \* **A domain state** of skyrmion and anti-skyrmion lattices also appears.



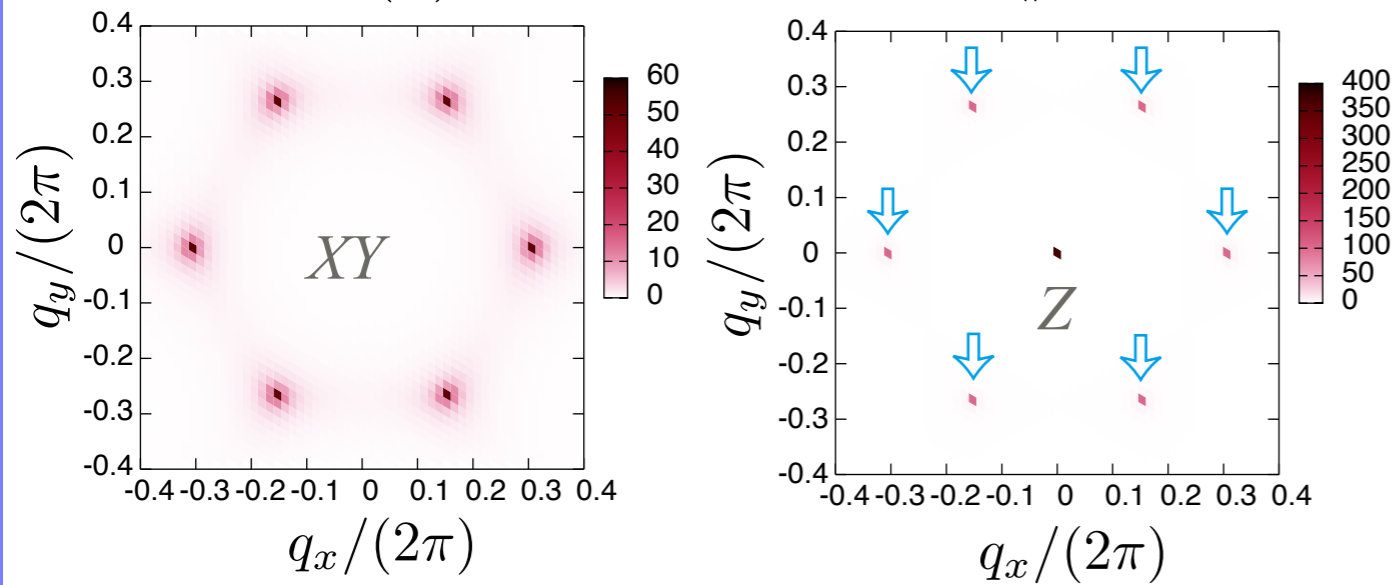
# “Z” Phase

A domain state of skyrmion and anti-skyrmion lattices

Spin structure factor  $T=0.41$

$S_{\perp}(\mathbf{q})$

$S_{\parallel}(\mathbf{q})$   $H=2.0$



XY: **broad** peaks

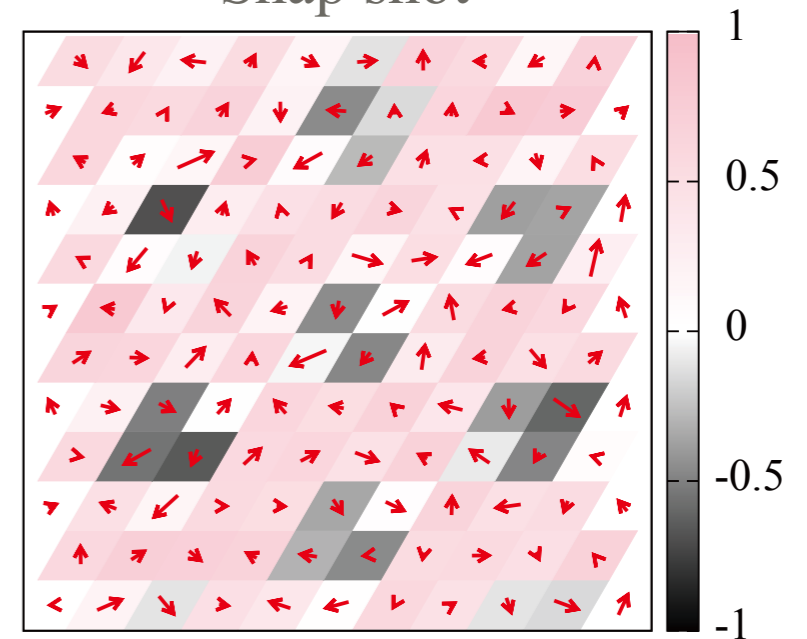


Z: **sharp** peaks

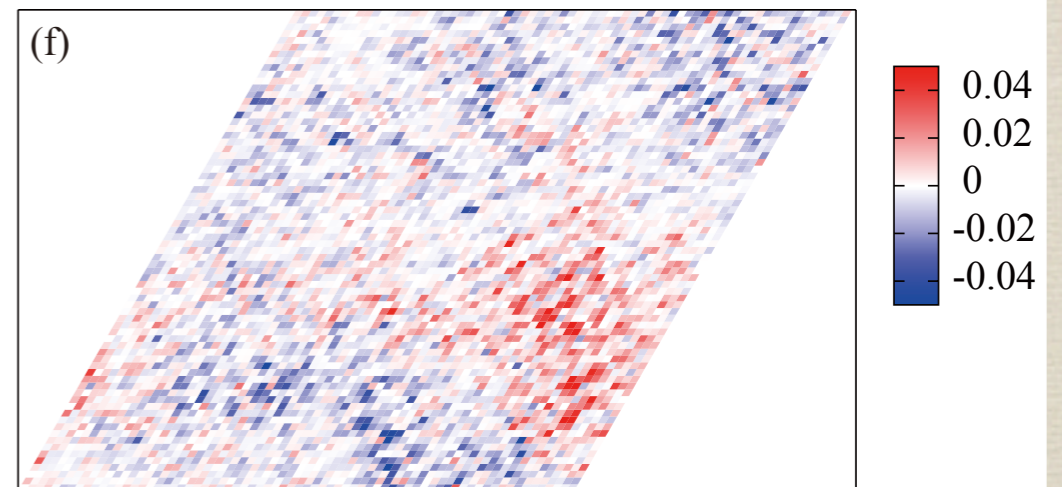
Only the Z component retains a quasi-long-range order similar to the triple- $q$  phase.

- A **domain state** of skyrmion and anti-skyrmion lattices

Snap shot

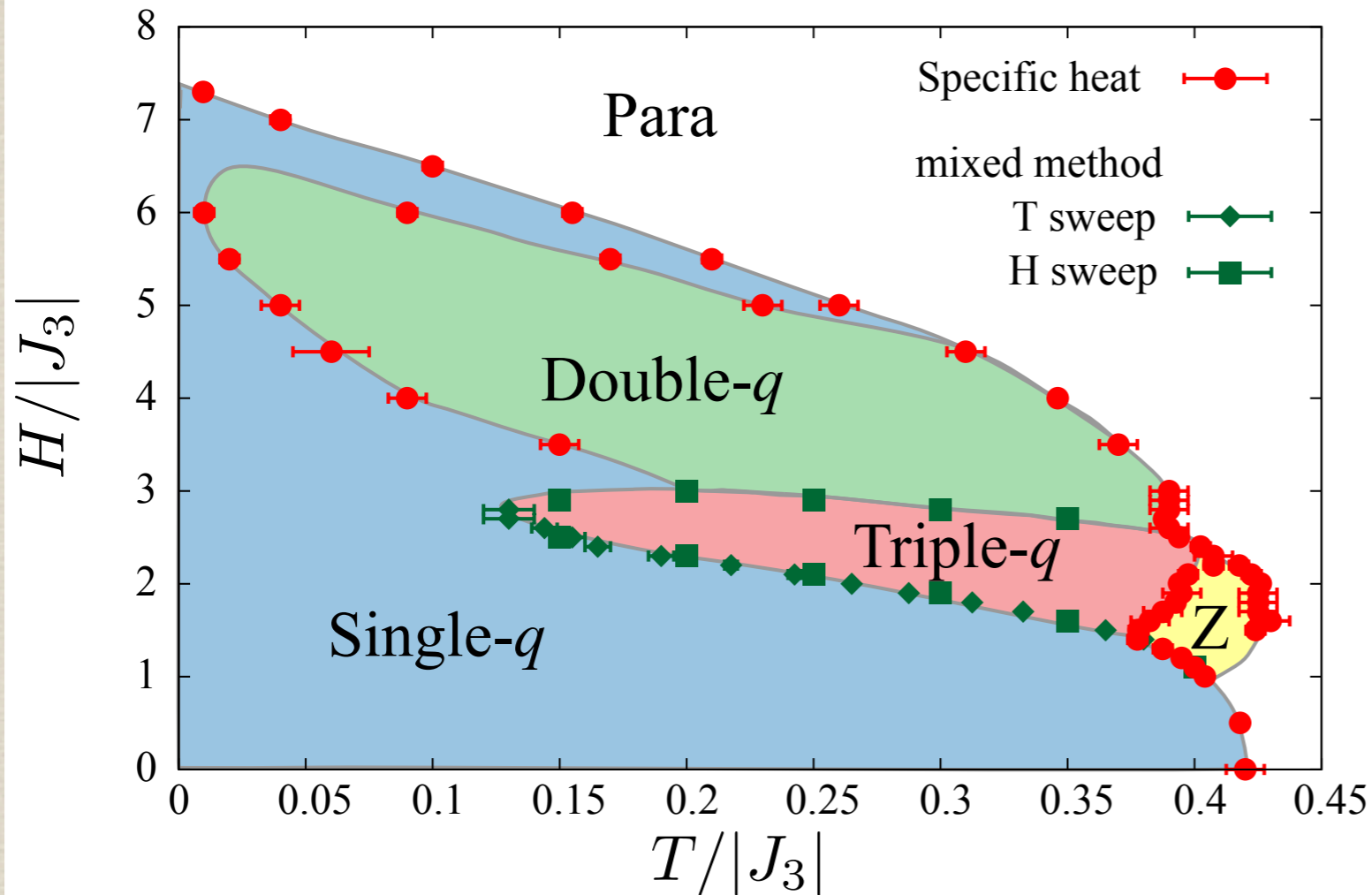


Skyrmion density

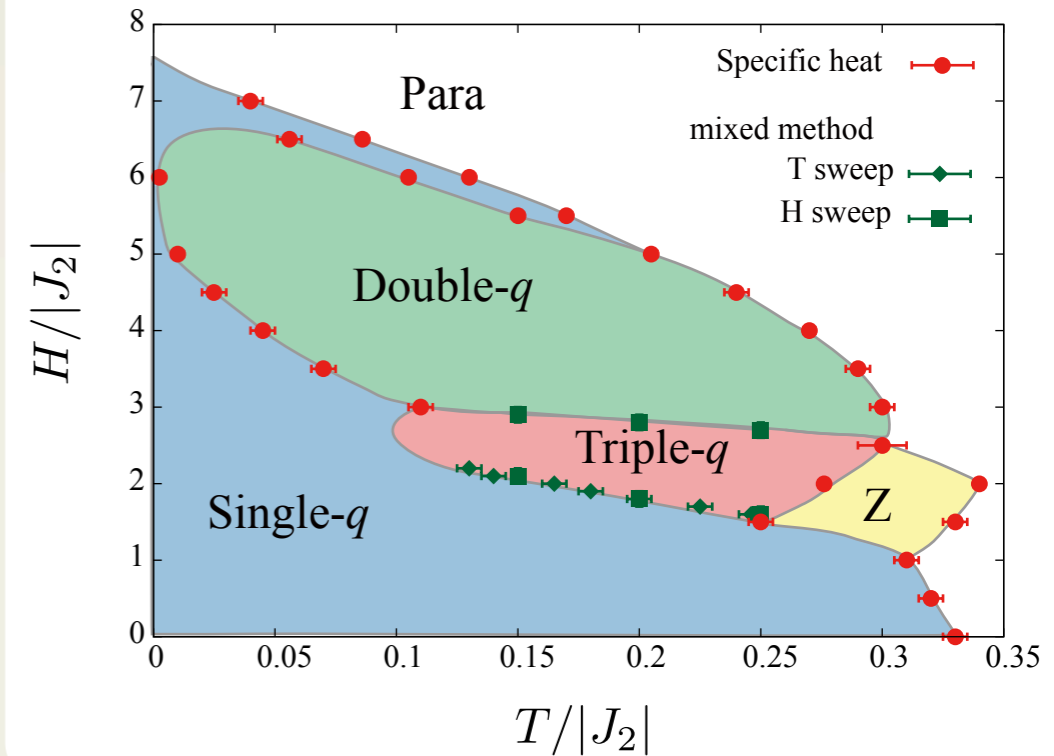


# Phase diagram

$J_1 - J_3$  Model  $J_1/J_3 = -1/3$



$J_1 - J_2$  Model  $J_1/J_2 = -1/4$



- In addition to the single- $q$  state, the double- $q$  and the triple- $q$  states are stabilized under magnetic fields.
- A similar phase diagram is obtained also for the  $J_1 - J_2$  model