Single-ion anisotropy in Haldane chains and the form factor of the O(3) nonlinear sigma model

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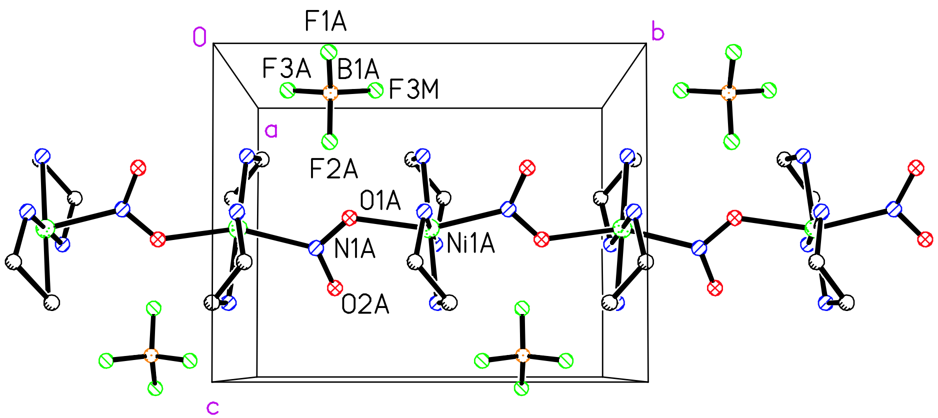
single-ion anisotropy in S=1 Haldane chains

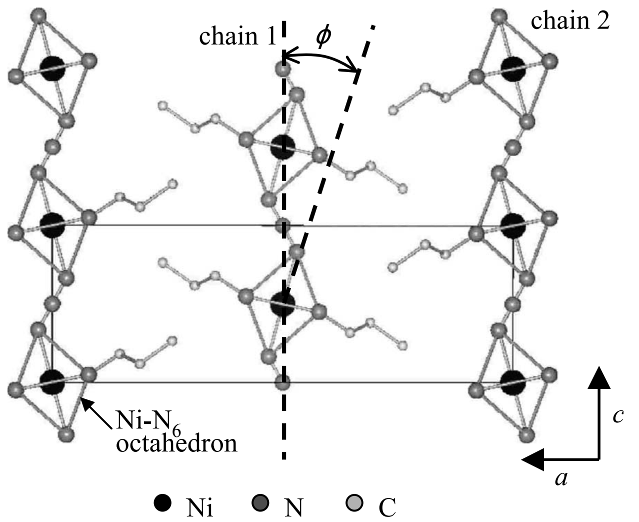
$$\mathcal{H} = J\sum_j \vec{S}_j \cdot \vec{S}_{j+1} + D\sum_j (S_j^z)^2 + E\sum_j \{(S_j^x)^2 - (S_j^y)^2\}$$

✓ NENB ($D/J = 0.17, |E|/J = 0.016$)



E. Čižmár et al., New J. Phys. (2008)

✓ NDMAP ($D/J = 0.25$)



T. Kashiwagi et al., PRB (2009)

S=1 **isotropic** Haldane chains

⟶ O(3) nonlinear sigma model $\mathcal{A} = \int dtdx \, \frac{1}{2g}(\partial_\mu \vec{n})^2 + i\theta Q$

$\theta = 2\pi S$

integrable

exact form factors are available

⟶ static & dynamical correlations

$$\langle 0|n^a|Z_b^\dagger(\theta)\rangle = \delta_{ab}\sqrt{Z}$$

E. S. Sørensen and I. Affleck, PRB (1994)

$$\langle 0|l^a|Z_{a_1}^\dagger(\theta_1)Z_{a_2}^\dagger(\theta_2)\rangle = i\frac{\Delta_0\pi^2}{4}\epsilon^{aa_1a_2}\psi(\theta_1-\theta_2), \qquad \psi(\theta) = \frac{\theta - i\pi}{\theta(\theta-2\pi i)}\tanh^2\frac{\theta}{2}$$

I. Affleck and R. A. Weston PRB (1992);
R. M. Konik PRB (2003)

“form factor perturbation theory”

D. Controzzi and G. Mussardo, PRL (2004)

cf.:

$$|\Psi_n\rangle = |\Phi_n\rangle + \sum_{m\neq n} \frac{\boxed{\langle\Phi_m|\mathcal{H}'|\Phi_n\rangle}}{E_n - E_m}|\Phi_m\rangle + \cdots$$

The unperturbed system is strongly interacting,
but integrable.

Two-particle form factor of the single-ion anisotropy

$$\langle 0|\mathcal{H}'|Z^{\dagger}_{a_1}(\theta_1)Z^{\dagger}_{a_2}(\theta_2)\rangle = \quad ?$$

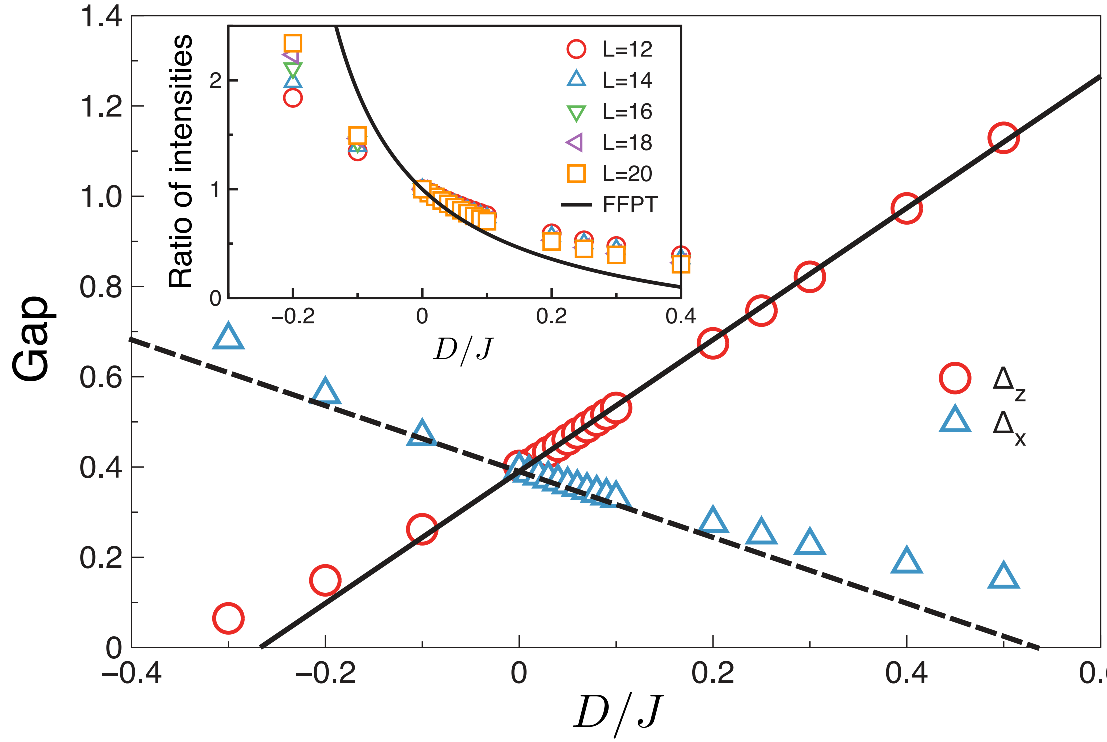
exact solution + iTEBD method

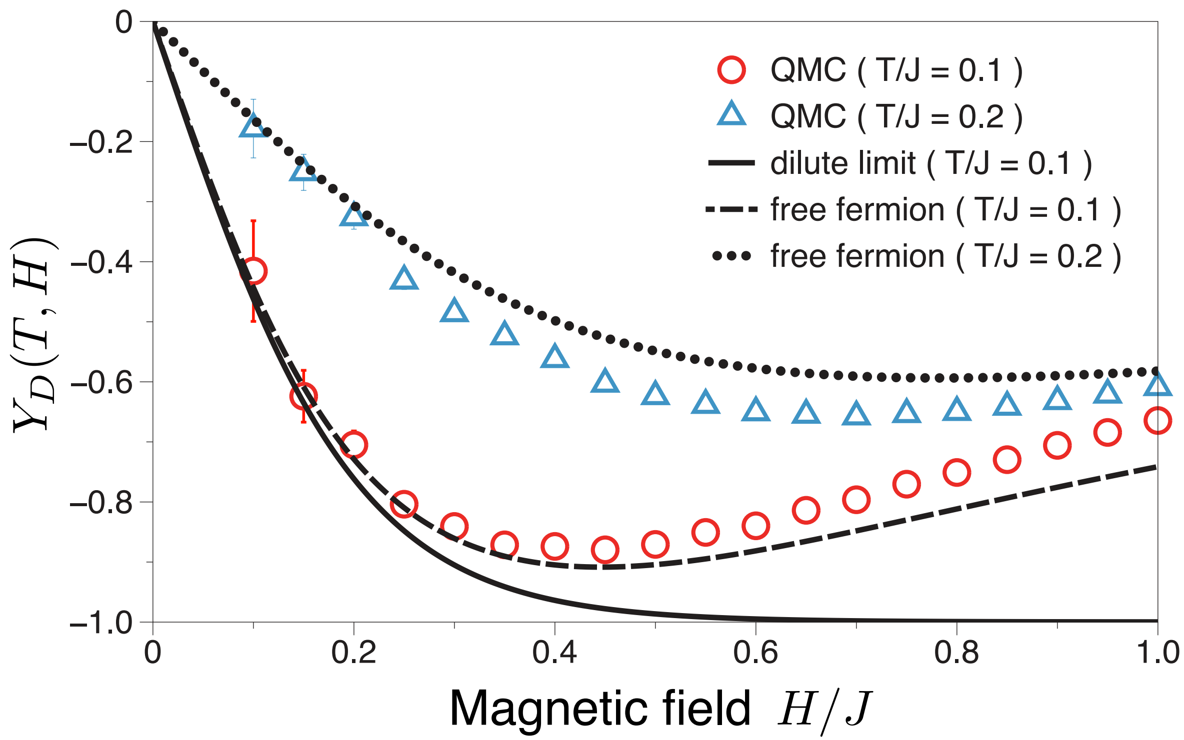
✓ excitation gaps

$$\Delta_a = \Delta_0 + \Delta_a^{(1)} + \cdots$$

$$\Delta_a^{(1)} \sim \frac{\langle Z_a(\theta)|\mathcal{H}'|Z_a^\dagger(0)\rangle}{\langle Z_a(\theta)|Z_a(0)\rangle}$$

FFPT vs diagonalization





✓ ESR shifts

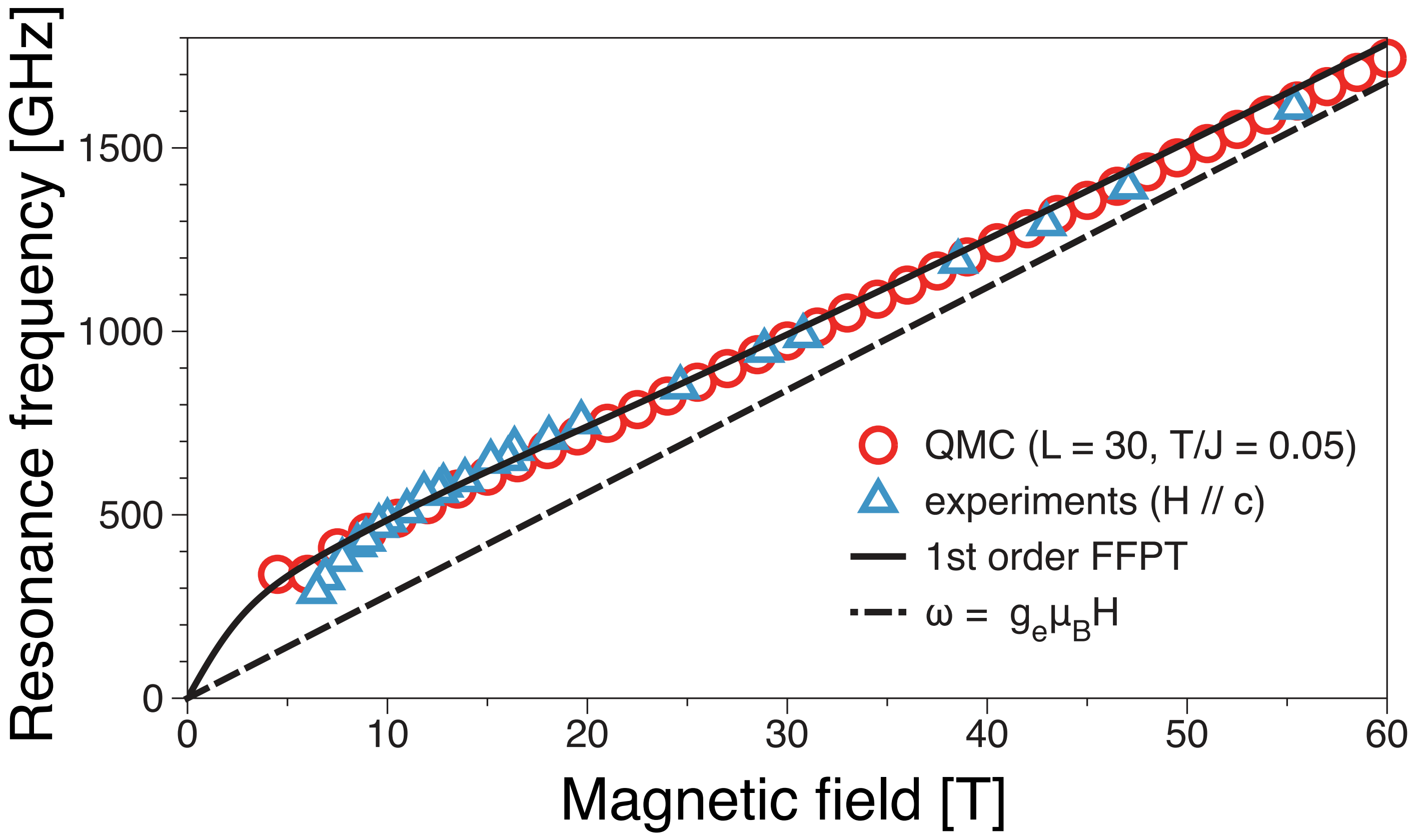
FFPT vs QMC

$$\delta\omega = \omega_r - g_e\mu_B H$$

dynamics under a magnetic field

✓ ESR frequency of NDMAP

FFPT, QMC vs experiments



PRB **84**, 180410(R); arXiv:1109.2607