

Two parameter flow diagram in ac regime for the graphene quantum Hall system

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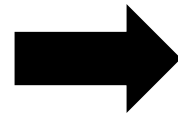
Ben Gurion

Purpose

Dirac QHE in graphene ($\sigma_{xy}(\omega) = 4(n+1/2)e^2/h$) is established with **static** transport.

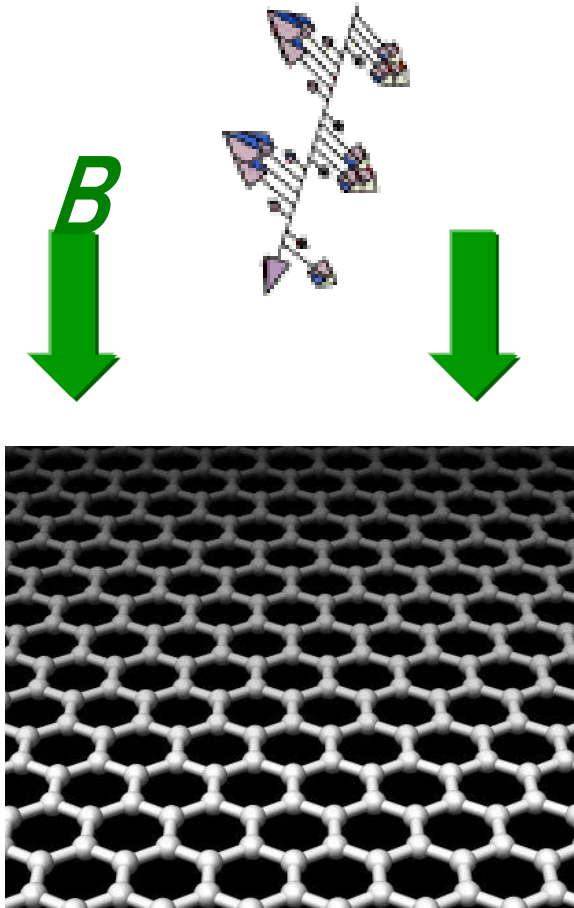
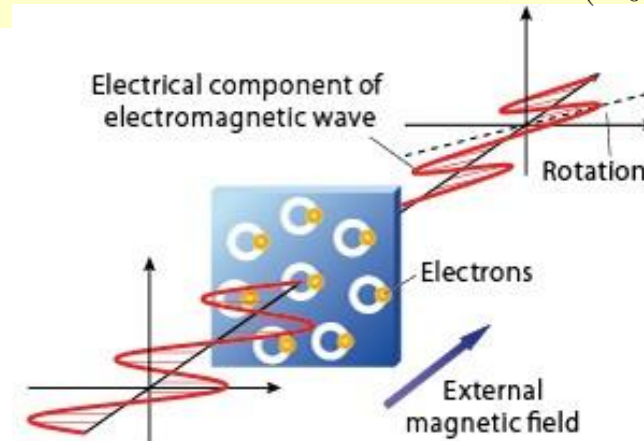


How about **dynamical** properties ?



Optical Hall conductivity $\sigma_{xy}(\omega)$ in graphene QHE system

\propto Faraday rotation $\Theta_H(\omega) \sim \frac{1}{(n_0 + n_s)c\epsilon_0} \sigma_{xy}(\omega)$




Model and method

● Explore Anderson localization effect for optical responses

→ Disordered system analyzed with exact diagonalization method

$$V(r) = \sum_j u_j \exp(-|r - R_j|^2/2d^2)/(2\pi d^2)$$

$$H = \underbrace{\sigma \cdot (p + eA)}_{\text{Free Dirac Hamiltonian +B}} + \underbrace{V(r)}_{\text{Impurity potential}}$$


■ Optical longitudinal and Hall conductivity from Kubo formula

$$\sigma_{xy}(\epsilon_F, \omega) = \frac{i\hbar e^2}{L^2} \sum_{\epsilon_a, \epsilon_b} \frac{f(\epsilon_b) - f(\epsilon_a)}{\epsilon_b - \epsilon_a} \frac{j_x^{ab} j_y^{ba}}{\epsilon_b - \epsilon_a - \hbar\omega - i\eta}$$

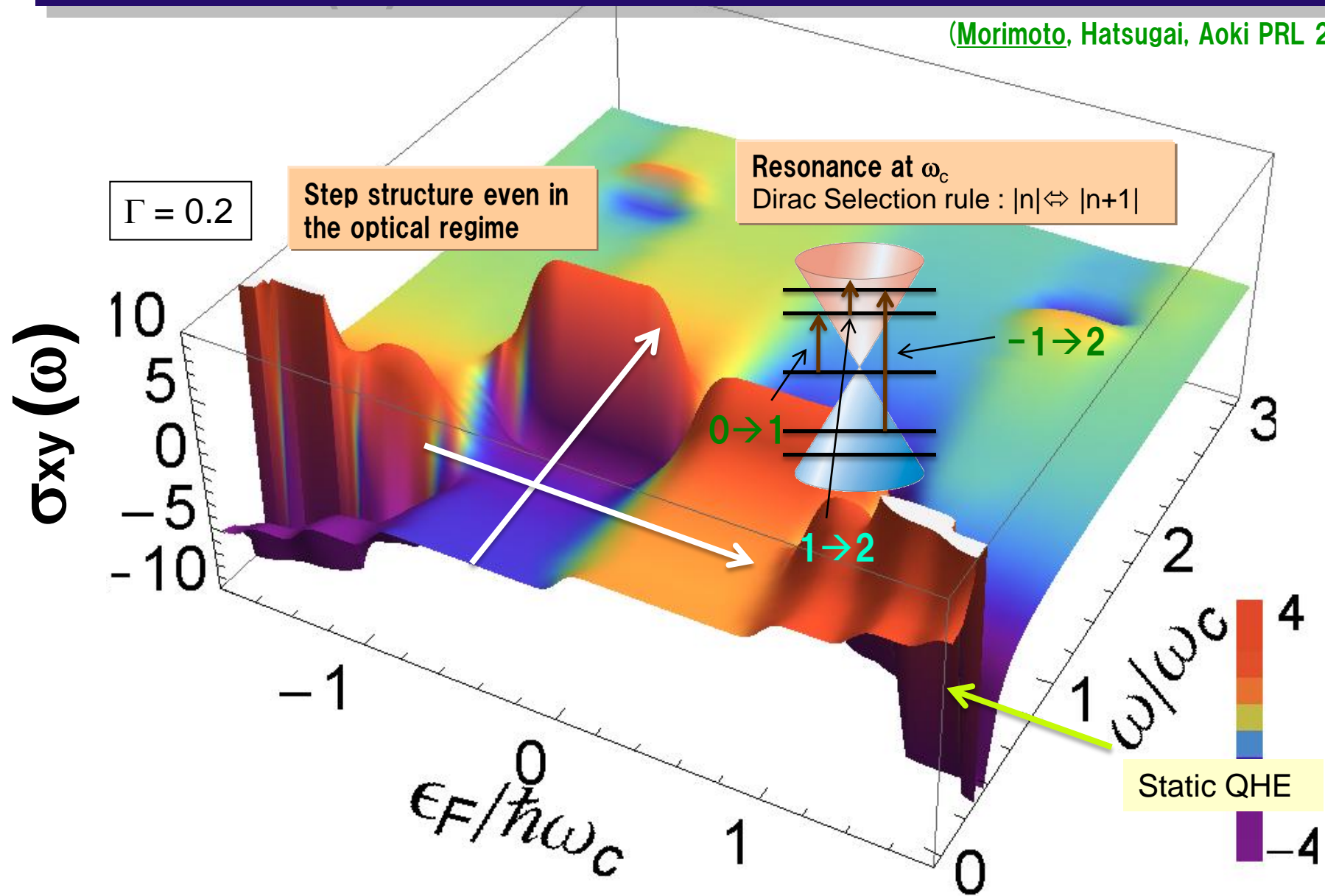
Energy cutoff
 $\eta \sim 1/L^2$

$$\text{Re}\sigma_{xx}(\omega) = \frac{\hbar e^2}{L^2} \sum_{\epsilon_a, \epsilon_b} \frac{f(\epsilon_b) - f(\epsilon_a)}{\epsilon_b - \epsilon_a} \frac{|j_x^{ab}|^2 \eta}{(\epsilon_b - \epsilon_a - \hbar\omega)^2 + \eta^2}$$

→ Average over disorder realizations

$\sigma_{xy}(\omega)$ from effective Dirac model

(Morimoto, Hatsugai, Aoki PRL 2009)



Robust plateaus even in ac \rightarrow Localization effect

Scaling of plateau to plateau transition

$$\sigma_{xy}(\varepsilon_F, \omega, L) = \frac{e^2}{h} F\left(\underbrace{(\varepsilon_F - \varepsilon_c)L^{1/\nu}}_{\text{Localization length}}, \underbrace{\omega L^z}_{\text{Dynamical length (scattering length)}}$$

Localization length

$\xi \sim 1/|\varepsilon_F - \varepsilon_c|^\nu$
size of wavefunction

Dynamical length
(scattering length)

distance electron diffuses in one cycle

$$L_\omega \sim 1/\omega^{1/z}$$

Scaling analysis for inverse of plateau transition width W

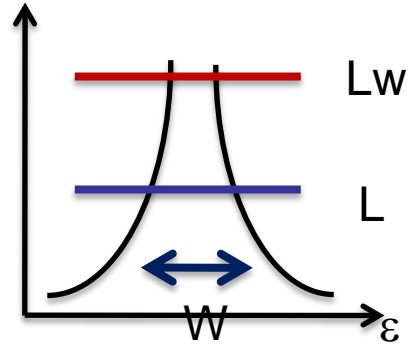
dc region

Morimoto, Avishai, Aoki PRB (R) (2010)

Localization length ξ

$$L < L_\omega$$

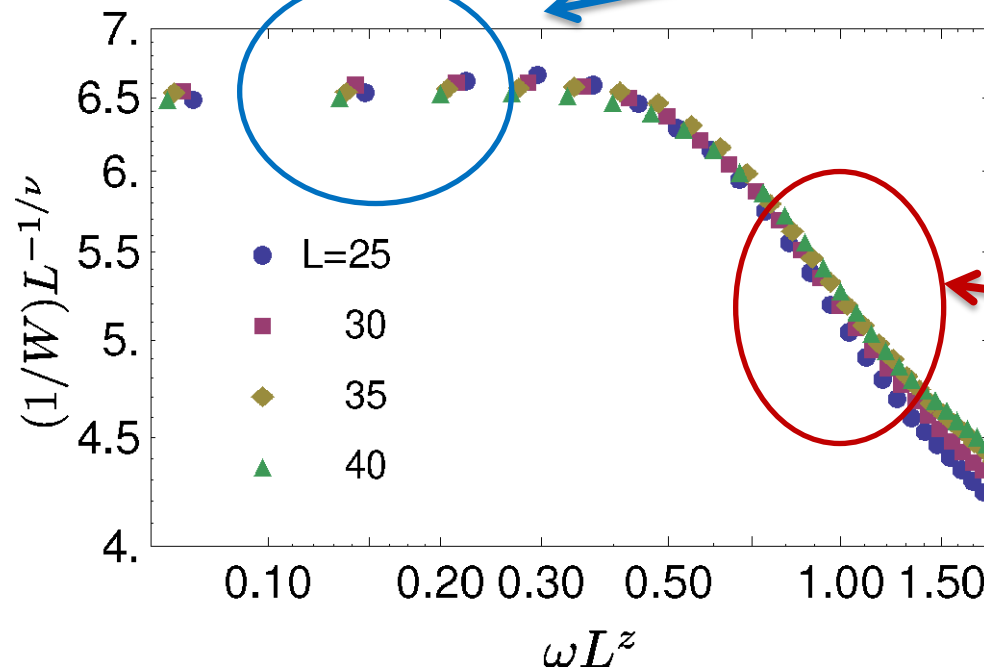
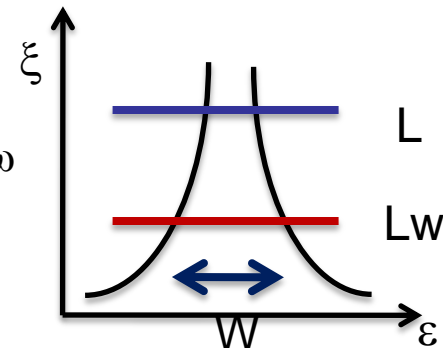
system size L
determines $\sigma_{xy}(\omega)$



ac region

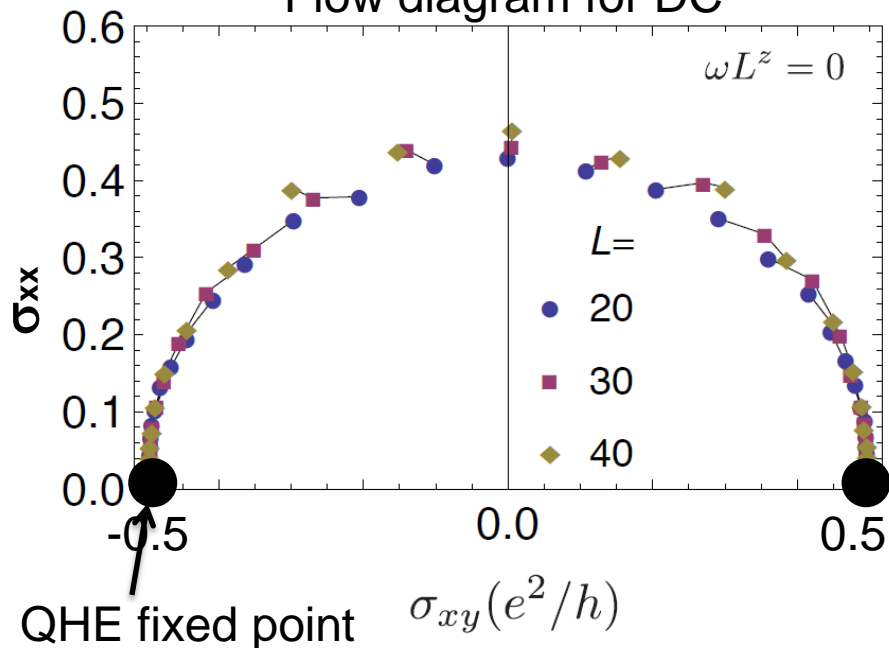
$$L > L_\omega$$

Dynamical length L_ω
determines $\sigma_{xy}(\omega)$
less L dependency.



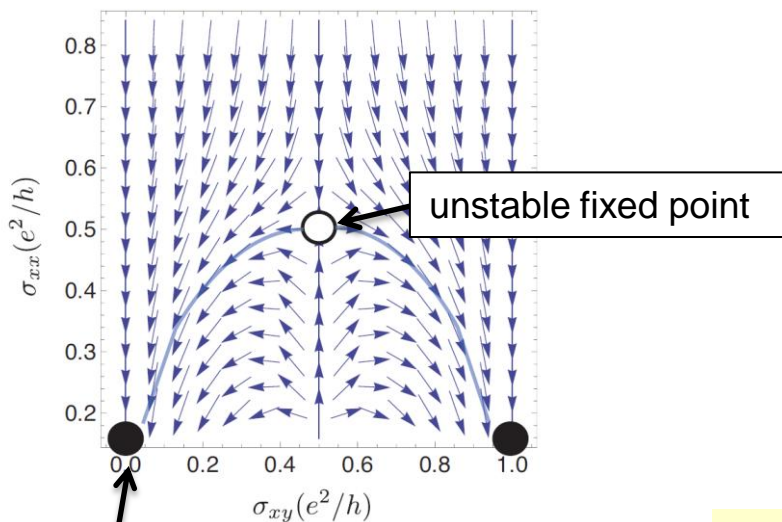
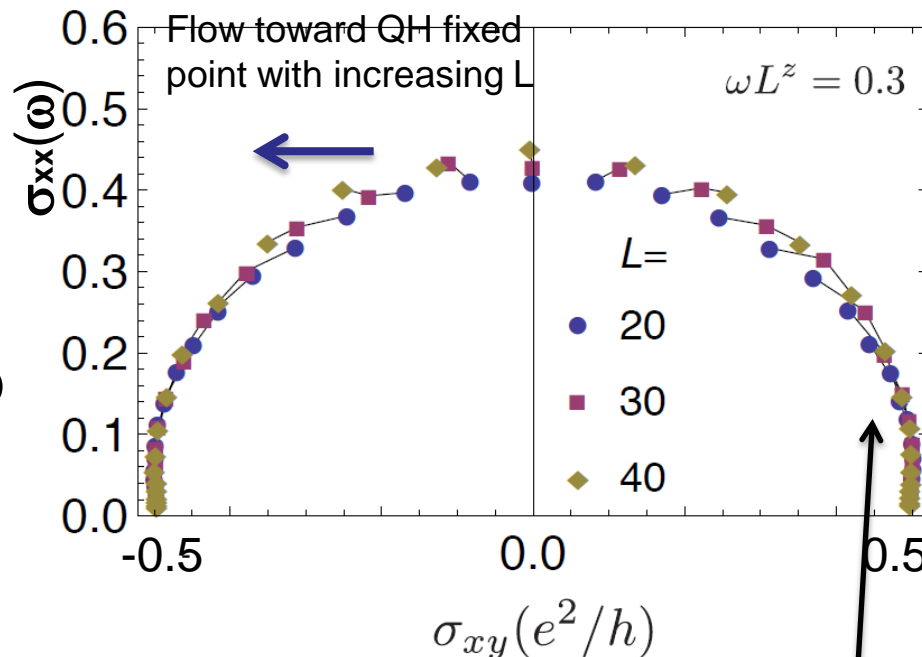
Two parameter flow extended to ac

Flow diagram for DC



Flow diagram for AC

Morimoto, Aoki,
arXiv:1108.1898



the flow tends to fall into a single curve as expected from dynamical scaling.

$$\sigma_{xx}(\varepsilon_F, \omega, L) = \frac{e^2}{h} F_{xx}((\varepsilon_F - \varepsilon_c) L^{1/\nu}, \omega L^z)$$

$$\sigma_{xy}(\varepsilon_F, \omega, L) = \frac{e^2}{h} F_{xy}((\varepsilon_F - \varepsilon_c) L^{1/\nu}, \omega L^z)$$

One parameter scaling

QHE fixed point **Pruisken PRL (1988)**

→ Origin of the plateau structure in ac region