# Floquet theory of photo-induced topological phase transitions: Application to graphene


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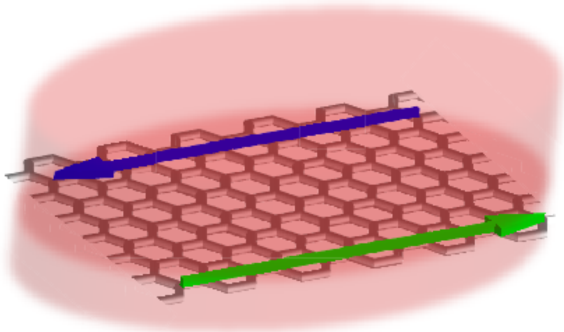


graphene (2d Dirac) + circularly polarized light
= quantum Hall state with edge states
(Haldane model + $\alpha$)

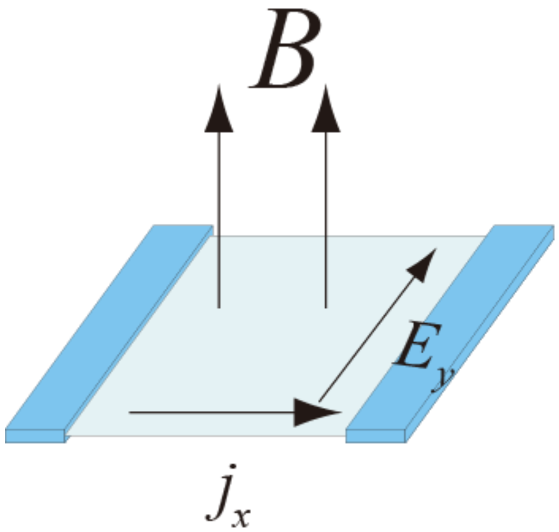
T. Oka and H. Aoki, Phys. Rev. B **79**, 081406 (R) (2009)
T. Kitagawa, T. Oka, A. Brataas, L. Fu, E. Demler: arXiv1104.4636.

# Introduction: Hall effect

Hall conductivity

$$j_x = \sigma_{xy} E_y$$

Hall effect in magnetic field




Hall effect in systems with a Chern number

$$\sigma_{xy} = \frac{e^2}{h} n$$

$n$:integer

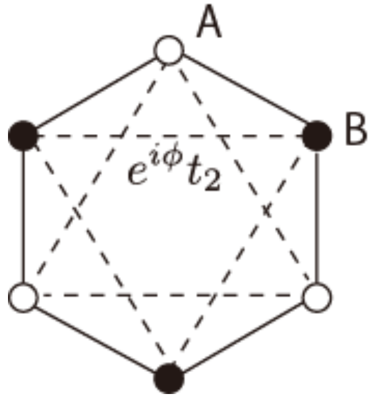
no-magnetic field

# Hall effect in graphene without magnetic fields

**Quantum Hall Effect**

Haldane PRL(1988)

local magnetic field
AB-level offset




$\sigma_{xy}$=1, 0,-1

**Quantum Spin Hall Effect**

Kane-Mele PRL(2005)

spin-orbit coupling

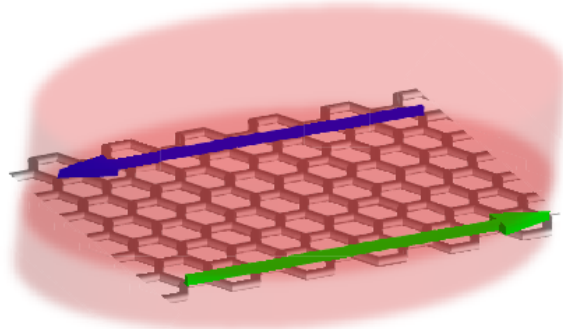
$$H = t\sum_{\langle ij\rangle} c_i^\dagger c_j + i\lambda_{SO}\sum_{\langle\langle ij\rangle\rangle} \nu_{ij} c_i^\dagger s^z c_j$$
$$+ i\lambda_R \sum_{\langle ij\rangle} c_i^\dagger (\mathbf{s}\times\hat{\mathbf{d}}_{ij})_z c_j + \lambda_v \sum_i \xi_i c_i^\dagger c_i.$$

$\sigma_{xy}{}^{\uparrow} - \sigma_{xy}{}^{\downarrow} > 0$

**Photo-induced Hall Effect**

TO-Aoki (2009)
Kitagawa et al. (2011)

circularly polarized light



$\sigma_{xy}$ depends on polarization

sometimes quantized

Why honeycomb lattice is important?

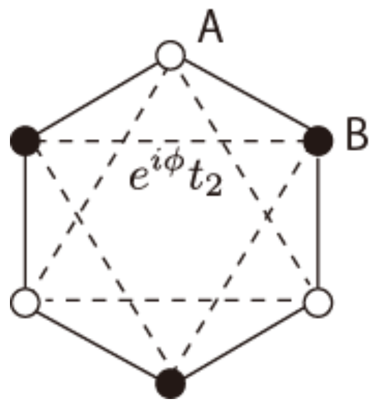
**Dirac model with a gap= simplest topological state**

# Hall effect in graphene without magnetic fields

Quantum Hall Effect

Haldane PRL(1988)

local magnetic field
AB-level offset



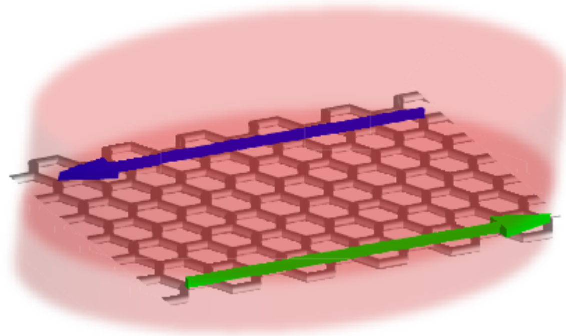

$\sigma_{xy}$=1, 0,-1

**same topological class**

Photo-induced Hall Effect

TO-Aoki (2009)
Kitagawa et al. (2011)

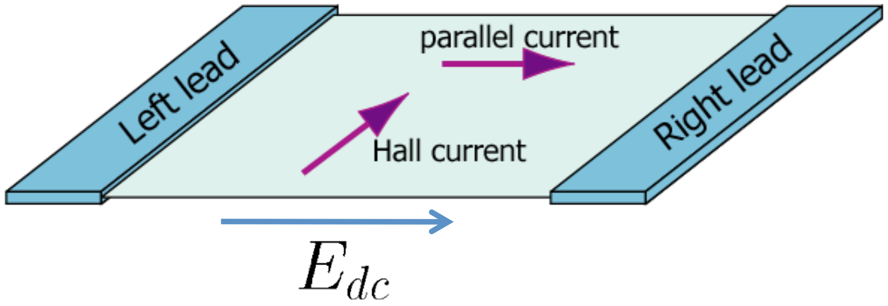
circularly polarized light



$\sigma_{xy}$ depends on polarization

sometimes quantized

# Kubo formula and the TKNN formula




$$J^i_{\mathrm{dc}} = \sigma_{ij} E^j_{\mathrm{dc}}$$

## Kubo formula（linear response）

$$\sigma_{ab} = i \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_{\alpha,\beta\neq\alpha} \frac{[f_\beta(\boldsymbol{k}) - f_\alpha(\boldsymbol{k})]}{E_\beta(\boldsymbol{k}) - E_\alpha(\boldsymbol{k})} \frac{\langle\Phi_\alpha(\boldsymbol{k})|J_b|\Phi_\beta(\boldsymbol{k})\rangle\langle\Phi_\beta(\boldsymbol{k})|J_a|\Phi_\alpha(\boldsymbol{k})\rangle}{E_\beta(\boldsymbol{k}) - E_\alpha(\boldsymbol{k}) + i\eta}$$

$\Phi_\alpha(\boldsymbol{k})$ Bloch wave function

$$f_\alpha(\boldsymbol{k}) = (\exp(\beta E_\alpha(\boldsymbol{k})) + 1)^{-1}$$

# TKNN formula in a 2d Dirac system

Thouless, Kohmoto, Nightingale, Nijs 1982

$$\sigma_{xy} = e^2 \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_\alpha f_\alpha(\boldsymbol{k}) \left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z$$

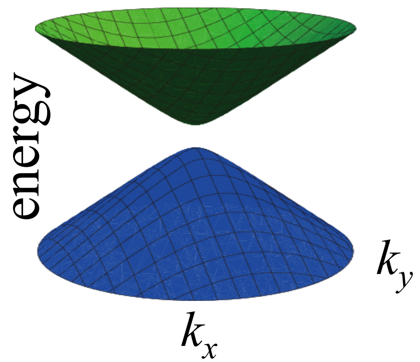
Berry curvature

artificial gauge field

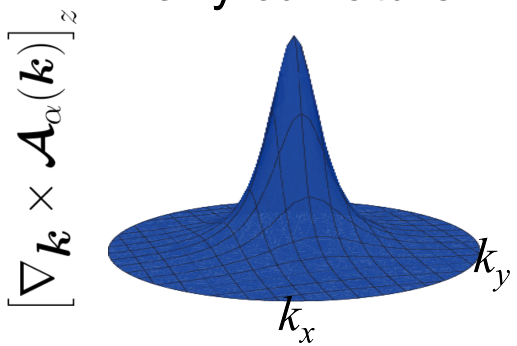
$$\boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k}) \equiv -i\langle \Phi_\alpha(\boldsymbol{k})|\nabla_{\boldsymbol{k}}|\Phi_\alpha(\boldsymbol{k})\rangle$$

## 2d Dirac system has a non-trivial Chern number

Niemi Semenoff '83, Redlich '84, Ishikawa '84




Berry curvature




$$H = \begin{pmatrix} m & \pm k_x - ik_y \\ \pm k_x + ik_y & -m \end{pmatrix}$$

$$\sigma_{xy} = e^2 \int \frac{d\boldsymbol{k}}{(2\pi)^d} \left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_1(\boldsymbol{k})\right]_z$$

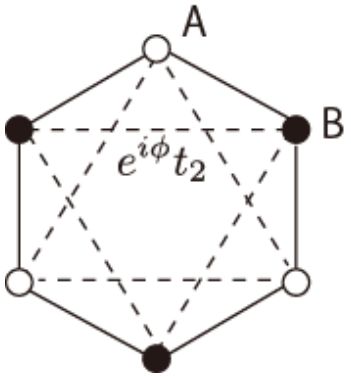
$$= \pm\frac{1}{2}\frac{e^2}{h}\boxed{\frac{m}{|m|}}$$

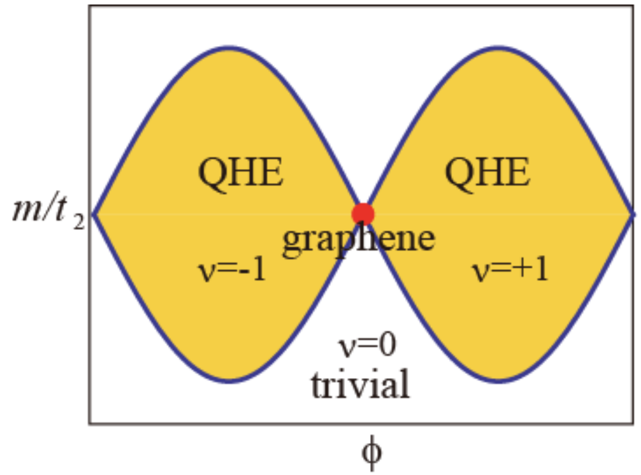
1. Sign of the mass = direction of the Hall current
2. Dirac cone = half quantum unit
3. parity anomaly in field theory

# Haldane's model of quantum Hall effect

Haldane (1988)

local magnetic field $\phi$
AB-level offset $m$

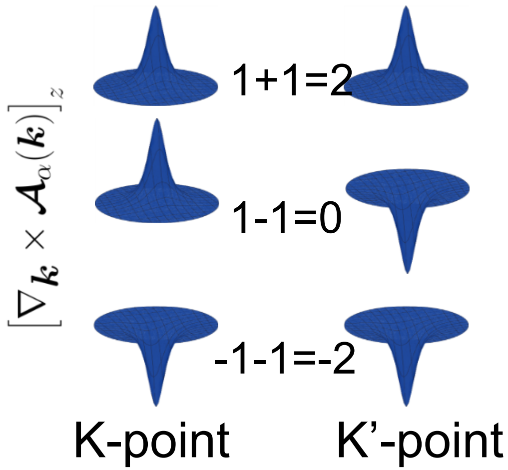


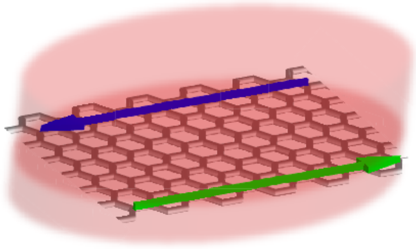

phase $\phi$ breaks time reversal symmetry

$$\sigma_{xy} = e^2 \int \frac{d\boldsymbol{k}}{(2\pi)^d} \left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_1(\boldsymbol{k})\right]_z$$
$$= \frac{1}{2}\frac{e^2}{h} \times 2,\ 0,\ -2$$

Berry curvature



# Floquet topological states

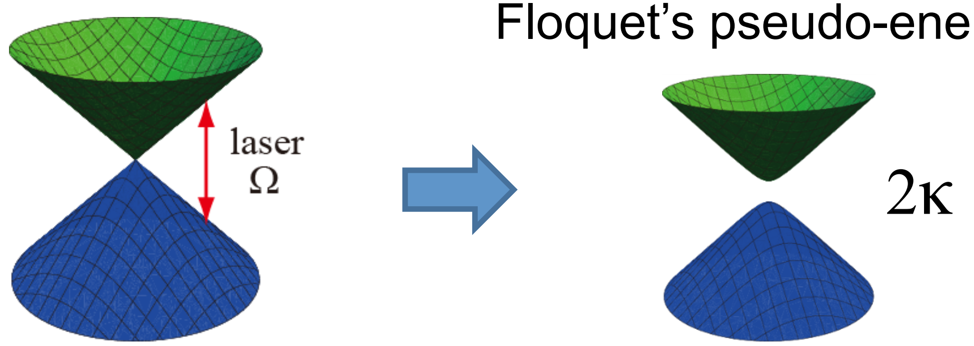


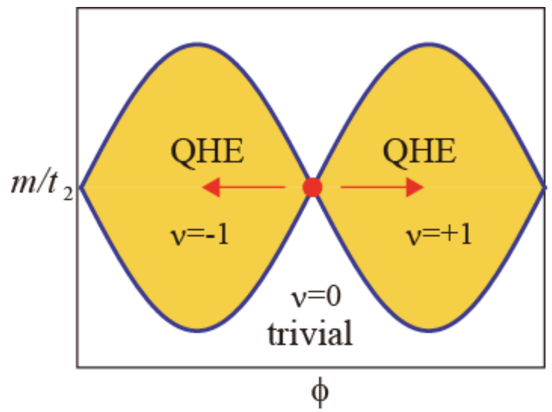
We apply circularly polarized light to graphene.

1. Change of distribution （heating）
2. Change of state （photon-dressed state）

Below, I will show that a *gap opens* at the Dirac point when circularly polarized light is applied.

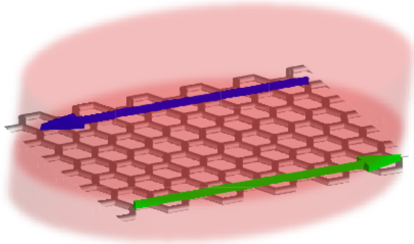
Floquet's pseudo-energy




opposite polarization leads to opposite Hall effect

# Honeycomb lattice + circularly polarized light



tight binding model a with a time dependent phase

$$H(t) = \sum_{ij} t_{ij} e^{-i\hat{e}_{ij}\cdot \boldsymbol{A}_{\rm ac}(t)} c_i^\dagger c_j$$

$$\boldsymbol{A}_{\rm ac}(t) = \frac{F}{\Omega}(\cos\Omega t, \sin\Omega t)$$

near K, K' point

td-Schrodinger equation of a Dirac system

$$i\partial_t \psi_k = \begin{pmatrix} 0 & k + Ae^{i\Omega t} \\ \bar{k} + Ae^{-i\Omega t} & 0 \end{pmatrix} \psi_k$$

$$k = k_1 - ik_2 \quad A = F/\Omega$$

# Floquet analysis (simplest example)

TO, H. Aoki (2009)

$$i\partial_t \psi_k = \begin{pmatrix} 0 & k + Ae^{i\Omega t} \\ \bar{k} + Ae^{-i\Omega t} & 0 \end{pmatrix} \psi_k$$

$$k = k_1 - ik_2 \quad A = F/\Omega$$

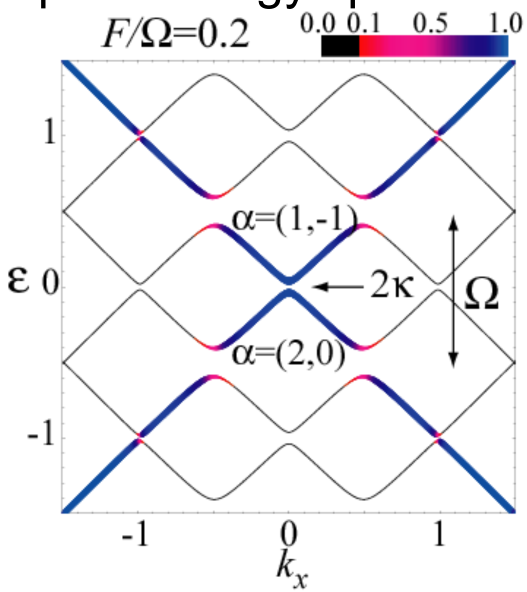
One obtains a eigenvalue problem after Fourier transformation

$$\sum_n \hat{H}^{mn} |u_\alpha^n\rangle = (\varepsilon_\alpha + m\Omega)|u_\alpha^m\rangle \qquad H^{mn} = \frac{1}{T}\int_0^T dt H(t) e^{i(m-n)\Omega t} + m\delta_{mn}\Omega I$$

$$H^{\mathrm{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

* truncated to m=0,+1, -1 for display

quasi-energy spectrum



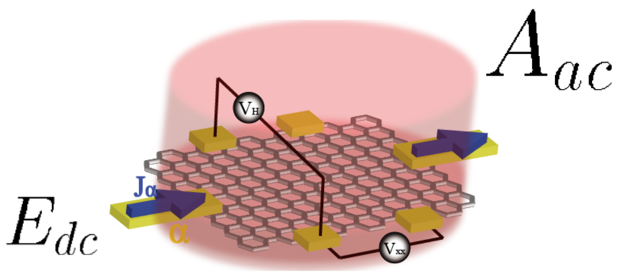

1. Dynamical topological gap

$$\kappa = \frac{\sqrt{4A^2 + \Omega^2} - \Omega}{2} \sim A^2/\Omega$$

2. Resonant gaps

# Non-equilibrium Kubo formula for photo-induced transport

TO and H. Aoki, Phys. Rev. B 79, 081406 (R) (2009)




Large $A_{ac}$ small $E_{dc}$

$$J^i_{\rm dc} = \sigma_{ij}(\boldsymbol{A}_{\rm ac})E^j_{\rm dc}$$

$$\sigma_{ab}(\boldsymbol{A}_{ac}) = i\int\frac{d\boldsymbol{k}}{(2\pi)^d}\sum_{\alpha,\beta\neq\alpha}\frac{[f_\beta(\boldsymbol{k})-f_\alpha(\boldsymbol{k})]}{\varepsilon_\beta(\boldsymbol{k})-\varepsilon_\alpha(\boldsymbol{k})}\frac{\langle\langle\Phi_\alpha(\boldsymbol{k})|J_b|\Phi_\beta(\boldsymbol{k})\rangle\rangle\langle\langle\Phi_\beta(\boldsymbol{k})|J_a|\Phi_\alpha(\boldsymbol{k})\rangle\rangle}{\varepsilon_\beta(\boldsymbol{k})-\varepsilon_\alpha(\boldsymbol{k})+i\eta}$$

$\varepsilon_\alpha$ Floquet's quasi-energy

$f_\alpha$ occupation fraction

inner product = time average

$$\langle\langle\Phi_\alpha|\Phi_\beta\rangle\rangle = \frac{1}{T}\int_0^T\langle\Phi_\alpha(t)|\Phi_\beta(t)\rangle$$

## Extended TKNN formula

$$\sigma_{xy}(\boldsymbol{A}_{\rm ac}) = e^2\int\frac{d\boldsymbol{k}}{(2\pi)^d}\sum_\alpha f_\alpha(\boldsymbol{k})\left[\nabla_{\boldsymbol{k}}\times\boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z$$

$$\boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k}) \equiv -i\langle\langle\Phi_\alpha(\boldsymbol{k})|\nabla_{\boldsymbol{k}}|\Phi_\alpha(\boldsymbol{k})\rangle\rangle$$

# Photo-induced Berry curvature (Chern density)

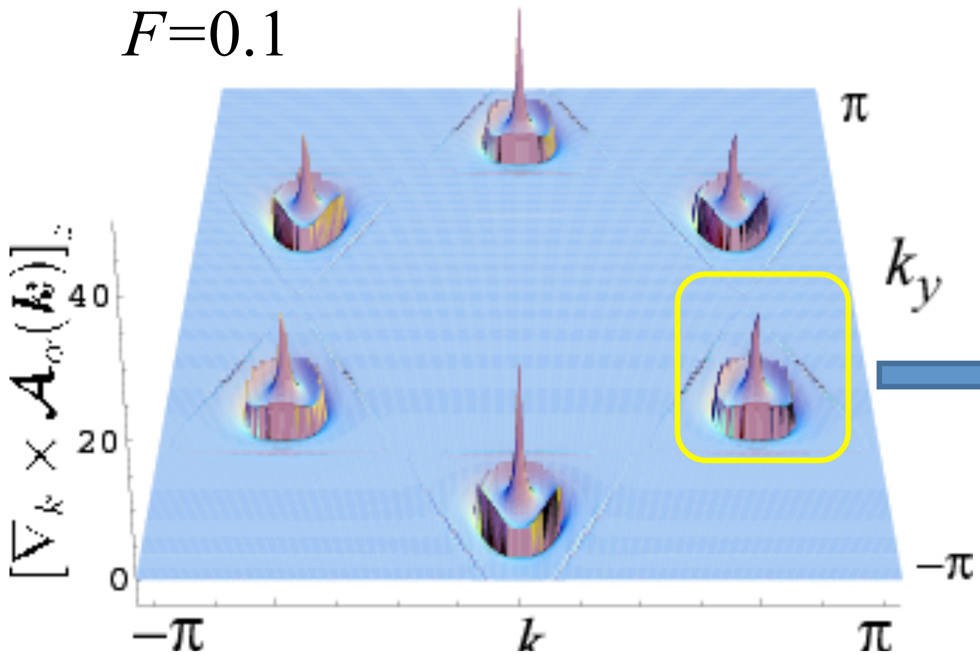
**Berry`s curvature**

$$\sigma_{xy}(\boldsymbol{A}_{\rm ac}) = e^2 \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_\alpha f_\alpha(\boldsymbol{k}) \left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z$$

$$\boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k}) \equiv -i\langle\langle\Phi_\alpha(\boldsymbol{k})|\nabla_{\boldsymbol{k}}|\Phi_\alpha(\boldsymbol{k})\rangle\rangle$$

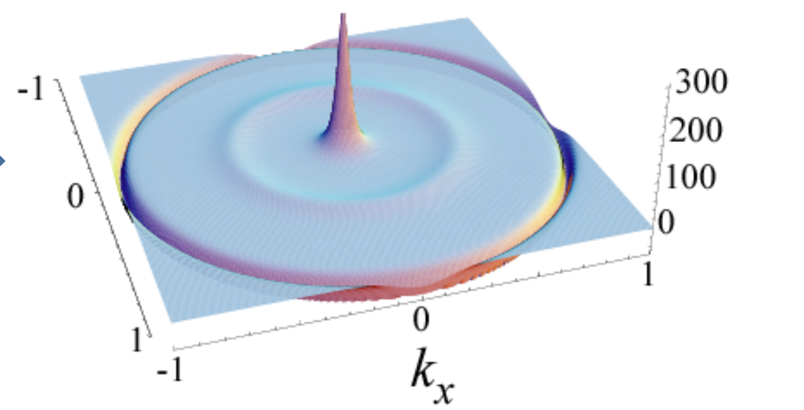
Floquet states

**Photo-induced Berry curvature**




Peaks at the Dirac cone

$$\left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z \sim \pm\tau_z \frac{1}{2}\kappa(|\boldsymbol{k}|^2 + \kappa^2)^{-3/2}$$




**NOTE: NO cancelation between K and K'**

TO and H. Aoki, Phys. Rev. B 79, 081406 (R) (2009)

# Floquet theories

## Transport

**Extension of Kubo, TKNN formula**

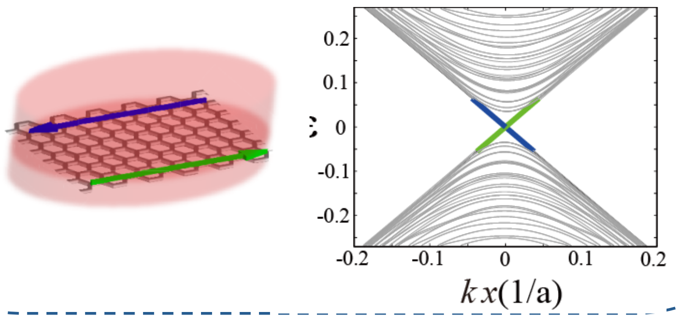
TO, H. Aoki (2009)

**Floquet + Landauer formula**

T. Kitagawa, et al. arXiv1104.4636.
Z. Gu, et al., PRL 2011

## edge state

Kitagawa et al. PRB(2010)
Lindner, Rafael, Galitski Nat.Phys.(2011)




**Relation to topological insulators**

classification by homotopy

Kitagawa et al. PRB(2010)

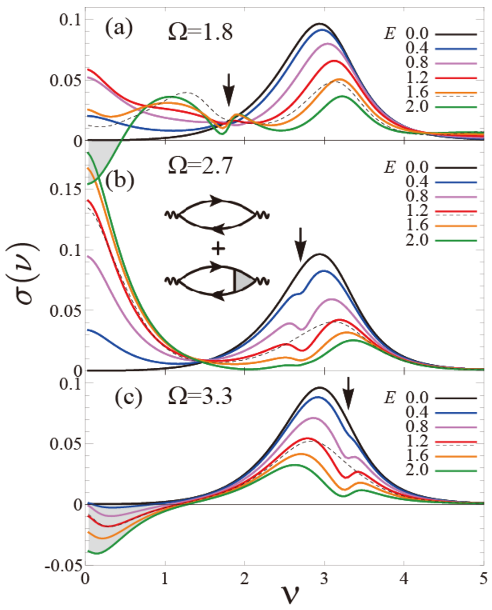
## Topological insulator

## many-body

Floquet + DMFT (Hubbard, Falikov-Kimbal)

Tsuji, TO, H. Aoki (2008),(2009)

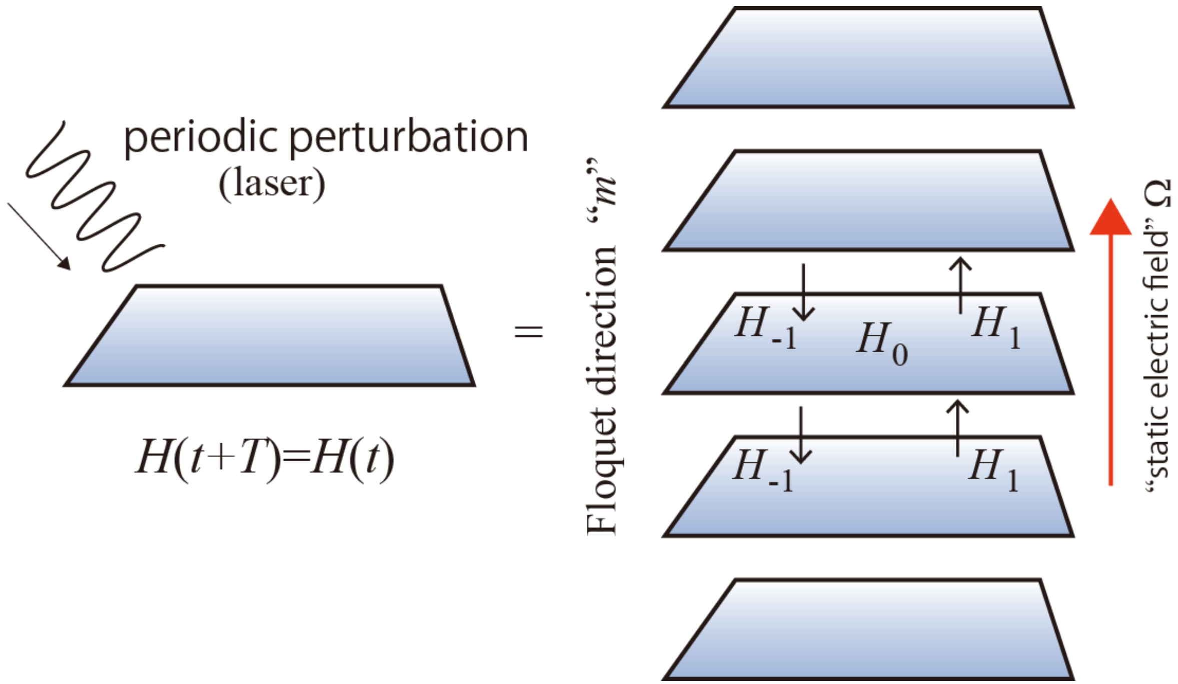
Optical response of Mott insulator in strong AC fields




Tsuji, TO, H. Aoki prl (2009)

# Floquet picture (Adding a dimension)

$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$




We add a new dimension $m$ (=Fourier exponent)

$$e^{im\Omega t}$$

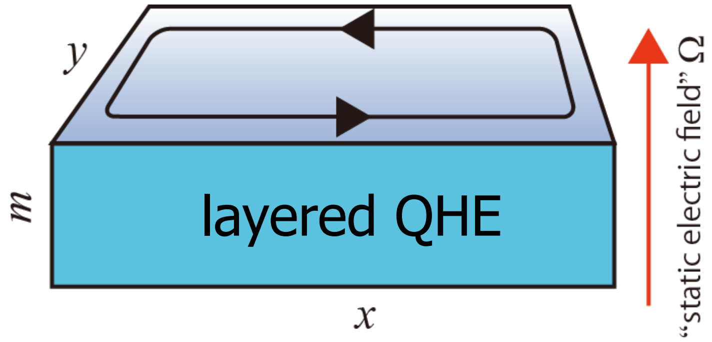
Static electric field Ω in the $m$-direction

# Floquet picture (Adding a dimension)

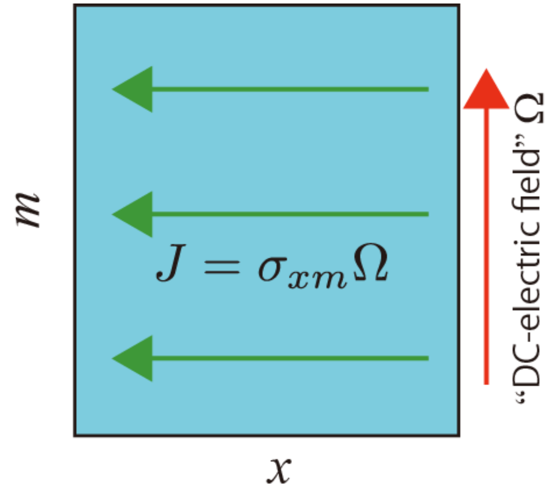
Classification scheme of ordinary topological insulators can be applied.

cf) Schnyder et al. (2008), Kitaev (2009)

Photo-induced Hall effect

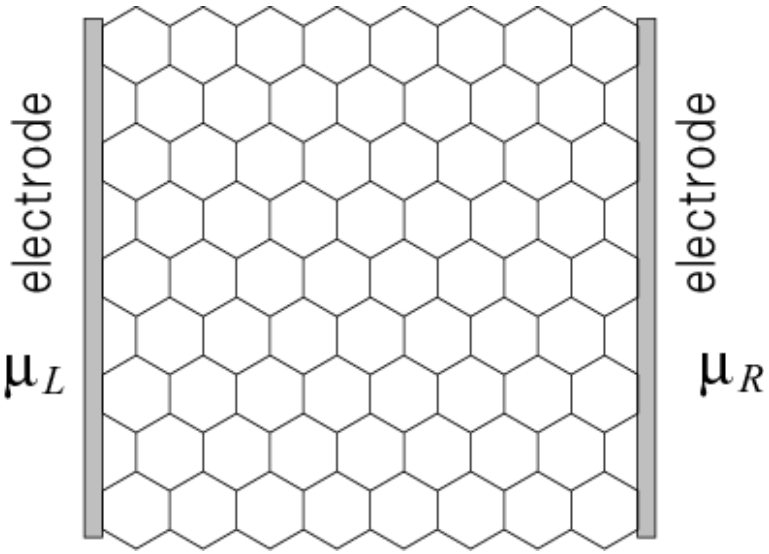



Thouless pumping




universality class = 2d quantum Hall state

# photo-induced transport (Keldysh approach)




Graphene ribbon attached to electrodes with circularly polarized light

$$S = \int_C dt \, (\mathcal{L}_{\rm graphene} + \mathcal{L}_{\rm mix} + \mathcal{L}_{\rm electrodes})$$

tight-binding model

$$\mathcal{L}_{\rm graphene} = \sum_{i \neq j} c_i^\dagger (i\partial_t - t_{ij} e^{iA_{ij}^{\rm ac}(t)}) c_j$$

$$\boldsymbol{A}^{\rm ac}(t) = (F/\Omega)(\cos \Omega t, \sin \Omega t)$$

Keldysh Green's function + Floquet method

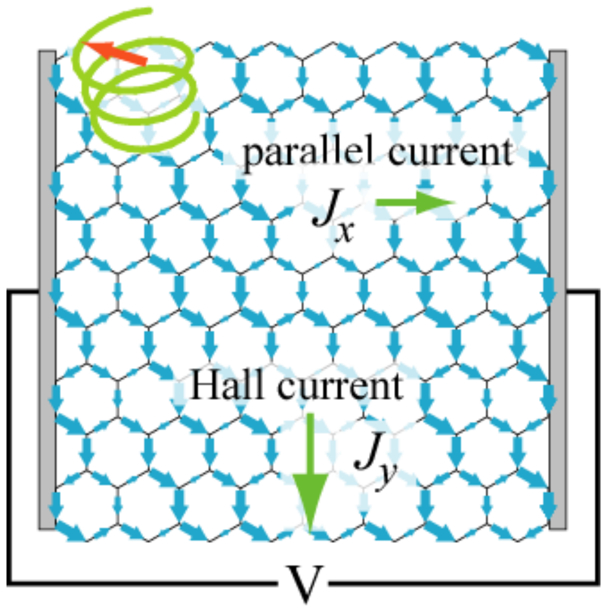
$$\begin{pmatrix} G_{k_y}^R(\omega) & G_{k_y}^K(\omega) \\ 0 & G_{k_y}^A(\omega) \end{pmatrix}_{ij;mn}^{-1} = \begin{pmatrix} (\omega + n\Omega + i\eta)\delta_{mn}\delta_{ij} - (\hat{H})_{ij}^{mn}(k_y) & 0 \\ 0 & (\omega + n\Omega - i\eta)\delta_{mn}\delta_{ij} - (\hat{H})_{ij}^{mn}(k_y) \end{pmatrix}$$
$$+\delta_{i1}\delta_{mn} \begin{pmatrix} i\Gamma_L/2 & -i\Gamma_L(1 - 2f_L(\omega + m\Omega)) \\ 0 & -i\Gamma_L/2 \end{pmatrix} + \delta_{iN}\delta_{mn} \begin{pmatrix} i\Gamma_R/2 & -i\Gamma_R(1 - 2f_R(\omega + m\Omega)) \\ 0 & -i\Gamma_R/2 \end{pmatrix},$$

m,n: Floquet index

TO and H. Aoki, Phys. Rev. B 79, 081406 (R) (2009)

# DC transport in AC field back ground

DC-component of the current




*IV*-characteristics

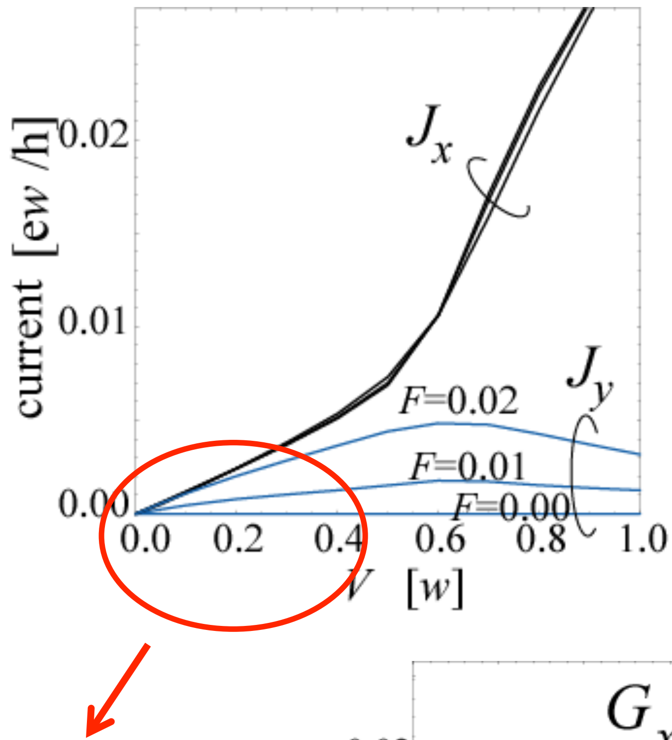
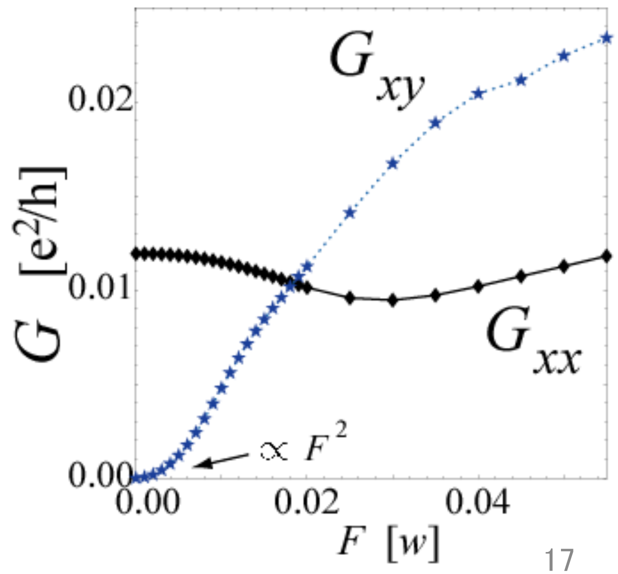



Photo-induced Hall conductivity

$$J_y = G_{xy} V$$

$$G_{xy} \propto F^2$$





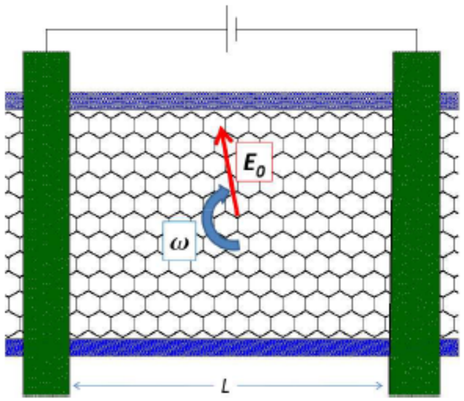

TO and H. Aoki, Phys. Rev. B 79, 081406 (R) (2009)

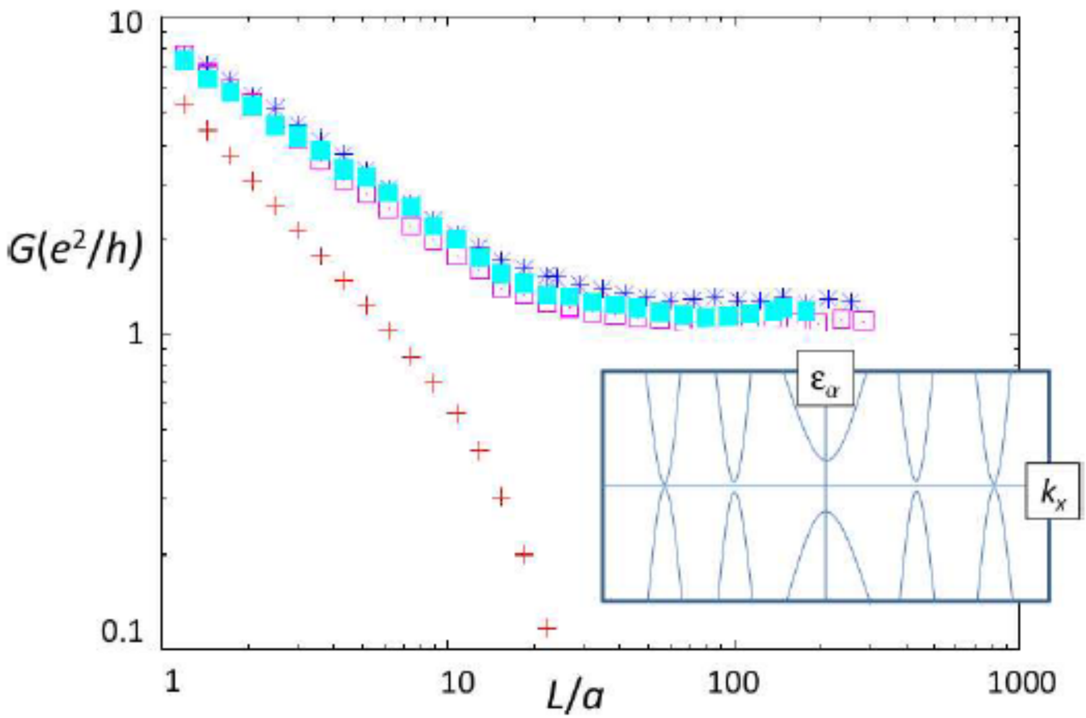
# Quantization of the current

Z. Gu, H.A. Fertig, D. P. Arovas, A. Auerbach, PRL 2011

open boundary condition (ribbon geometry)

Floquet Landauer formalism




Quantized transport when size is *large* (Edge state)

# Proposals for experimental detection

necessary field strength

$$\Omega \sim 1 - 3eV \qquad F \sim 10^7 V/m$$

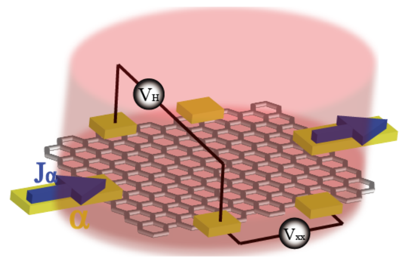
graphene, multilayer-graphene, graphite, surface of TI, etc.

TO, Aoki, arXiv:1007.5399

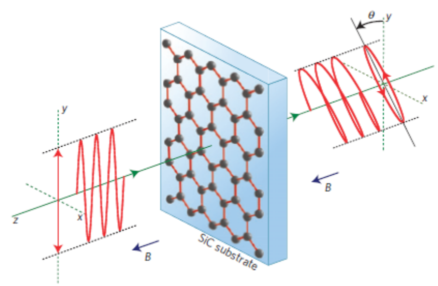
Gapless is best, but gapped system is OK

Possible detection

1. Photo-induced Transport




2.Pump-probe (Kerr effect, MOKE)




3.Pump-probe (photoemission)

# Summary

1. Photo-induced Hall effect:
   multiband+light = quantum Hall system
2. Applicable to various systems
   graphene, graphite, TI, etc.
3. Floquet topological states