With the FRG towards the QCD Phase diagram

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RG Approach from Ultra Cold Atoms to the Hot QGP

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Helmholtz Alliance Extremes of Density and Temperature: Cosmic Matter in the Laboratory (EMMI)
in collaboration with Yukawa Institute for Theoretical Physics (YITP)

Kyoto, Japan
QCD → two phase transitions:

1. restoration of chiral symmetry

\[ SU_{L+R}(N_f) \rightarrow SU_L(N_f) \times SU_R(N_f) \]

order parameter:

\[ \langle \bar{q}q \rangle \begin{cases} > 0 \iff \text{symmetry broken, } T < T_c \\ = 0 \iff \text{symmetric phase, } T > T_c \end{cases} \]

2. de/confinement

order parameter: Polyakov loop variable

\[ \Phi \begin{cases} = 0 \iff \text{confined phase, } T < T_c \\ > 0 \iff \text{deconfined phase, } T > T_c \end{cases} \]

\[ \Phi = \left\langle \text{tr}_c \mathcal{P} \exp \left( i \int_0^\beta d\tau A_0(\tau, \vec{x}) \right) \right\rangle /N_c \]

alternative: dressed Polyakov loop (dual condensate)

relates chiral and deconfinement transition → spectral properties of Dirac operator

**effective models:**

1. Quark-meson model

2. Polyakov–quark-meson model

or other models e.g. NJL

or PNJL models
The conjectured QCD Phase Diagram

At densities/temperatures of interest only model calculations available

non-perturbative functional methods (FunMethods) → here: FRG

→ complementary to lattice

- no sign problem $\mu > 0$
- chiral symmetry/fermions (small masses/chiral limit) ...

Open issues:
related to chiral & deconfinement transition

- existence of CEP? Its location?
- additional CEPs?
  How many?
- coincidence of both transitions at $\mu = 0$?
- quarkyonic phase at $\mu > 0$?
- chiral CEP/deconfinement CEP?
- finite volume effects?
  → lattice comparison
- so far only MFA results
effects of fluctuations?
  → size of crit. region
The conjectured QCD Phase Diagram

Open issues:
related to chiral & deconfinement transition

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non-perturbative functional methods (FunMethods) → here: FRG
→ complementary to lattice

lattice versus FRG

$N_c = 2$ Polyakov-quark-meson-diquark (PQMD) model

[Strodthoff, BJS, von Smekal; in prep. '11]
Outline

- Three-Flavor Chiral Quark-Meson Model
- \textit{...with Polyakov loop dynamics}
- Taylor expansions and generalized susceptibilities
- Functional Renormalization Group
- Finite volume effects
$N_f = 3$ Quark-Meson (QM) model

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

Quark part with Yukawa coupling $h$:

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial - h\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

Meson part: scalar $\sigma_a$ and pseudoscalar $\pi_a$ nonet

Meson fields: $M = \sum_{a=0}^{8} \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$

$$\mathcal{L}_{\text{meson}} = \text{tr}[\partial_\mu M^\dagger \partial^\mu M] - m^2\text{tr}[M^\dagger M] - \lambda_1 (\text{tr}[M^\dagger M])^2 - \lambda_2 \text{tr}[(M^\dagger M)^2] + c[\det(M) + \det(M^\dagger)] + \text{tr}[H(M + M^\dagger)]$$

- explicit symmetry breaking matrix: $H = \sum_a \frac{\lambda_a}{2} h_a$

- $U(1)_A$ symmetry breaking implemented by 't Hooft interaction

→ talk by Mario Mitter
Phase diagram for $N_f = 2 + 1$ ($\mu \equiv \mu_q = \mu_s$)

- Model parameter fitted to (pseudo)scalar meson spectrum:
- one parameter precarious: $f_0(600)$ 'particle' (i.e. sigma) $\rightarrow$ broad resonance
  PDG: mass=$(400 \ldots 1200)$ MeV
  we use fit values for $m_\sigma$ between $(400 \ldots 1200)$ MeV

$\rightarrow$ existence of CEP depends on $m_\sigma$!

Example: $m_\sigma = 600$ MeV (lower lines), 800 and 900 MeV (here mean-field approximation)
Chiral critical surface \( (m_\sigma = 800 \text{ MeV}) \)

- standard scenario for \( m_\sigma = 800 \text{ MeV} \) (as expected)

Here: ’t Hooft coupling \( \mu \)-independent (might change if \( \mu \)-dependence is considered)

with \( U(1)_A \)

non-standard scenario in PNJL with (unrealistic) large vector int. \( \rightarrow \) bending of surface
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Polyakov-quark-meson (PQM) model

- Lagrangian \( \mathcal{L}_{PQM} = \mathcal{L}_{qm} + \mathcal{L}_{pol} \) with \( \mathcal{L}_{pol} = -\bar{q}\gamma_0 A_0 q - U(\phi, \bar{\phi}) \)


\[
\frac{U(\phi, \bar{\phi})}{T^4} = -\frac{b_2(T, T_0)}{2} \phi\bar{\phi} - \frac{b_3}{6} (\phi^3 + \bar{\phi}^3) + \frac{b_4}{16} (\phi\bar{\phi})^2
\]

\[
b_2(T, T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3
\]

2. logarithmic potential: Rößner et al. 2007

\[
\frac{U_{\text{log}}}{T^4} = -\frac{1}{2} a(T) \bar{\phi}\phi + b(T) \ln \left[ 1 - 6\bar{\phi}\phi + 4 (\phi^3 + \bar{\phi}^3) - 3 (\bar{\phi}\phi)^2 \right]
\]

\[
a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 \quad \text{and} \quad b(T) = b_3(T_0/T)^3
\]

3. Fukushima Fukushima 2008

\[
U_{\text{Fuku}} = -bT \left\{ 54e^{-a/T} \phi\bar{\phi} + \ln \left[ 1 - 6\bar{\phi}\phi + 4 (\phi^3 + \bar{\phi}^3) - 3 (\bar{\phi}\phi)^2 \right] \right\}
\]

- \( a \) controls deconfinement
- \( b \) strength of mixing chiral & deconfinement
Polyakov-quark-meson (PQM) model

- Lagrangian \( \mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}} \) with \( \mathcal{L}_{\text{pol}} = -\bar{q} \gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi}) \)

1. polynomial Polyakov loop potential:

\[
\frac{\mathcal{U}(\phi, \bar{\phi})}{T^4} = -\frac{b_2(T, T_0)}{2} \phi \bar{\phi} - \frac{b_3}{6} (\phi^3 + \bar{\phi}^3) + \frac{b_4}{16} (\phi \bar{\phi})^2
\]

\[
b_2(T, T_0) = a_0 + a_1 (T_0/T) + a_2 (T_0/T)^2 + a_3 (T_0/T)^3
\]

back reaction of the matter sector to the YM sector: \( N_f \) and \( \mu \)-modifications

in presence of dynamical quarks: \( T_0 = T_0(N_f, \mu, m_q) \)

<table>
<thead>
<tr>
<th>( N_f )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>2 + 1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_0 ) [MeV]</td>
<td>270</td>
<td>240</td>
<td>208</td>
<td>187</td>
<td>178</td>
</tr>
</tbody>
</table>

matter back reaction to the YM sector important at \( \mu \neq 0 \)

for \( \mu \neq 0 \): \( \bar{\phi} > \phi \) is expected

since \( \bar{\phi} \) is related to free energy gain of antiquarks

in medium with more quarks \( \rightarrow \) antiquarks are more easily screened.
QCD Thermodynamics $N_f = 2 + 1$

(P)QM models (three different Polyakov loop potentials) versus QCD lattice simulations

Stefan-Boltzmann limit:

$$\frac{p_{SB}}{T^4} = 2(N_c^2 - 1) \frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$$

- solid lines: PQM with lattice masses (HotQCD)
  - $m_\pi \sim 220$, $m_K \sim 503$ MeV
- dashed lines: (P)QM with realistic masses

data taken from: 

[Bazavov et al. ’09]
$N_f = 2 + 1$ (P)QM phase diagrams

Summary of QM and PQM models in mean field approximation

chiral transition and 'deconfinement' coincide at small densities

- for PQM model (upper lines)
- for QM model (lower lines)

$N_f = 2$: [BJS, Pawlowski, Wambach; 2007]
$N_f = 2 + 1$ (P)QM phase diagrams

Summary of QM and PQM models in mean field approximation

chiral transition and 'deconfinement' coincide at small densities

- for PQM model (upper lines)
  - with matter back reaction in Polyakov loop potential
    - → shrinking of possible quarkyonic phase

- for QM model (lower lines)

$N_f = 2$: [BJS, Pawlowski, Wambach; 2007]

[BJS, M. Wagner; in prep. '11]
Critical region

contour plot of **size of the critical region** around CEP
defined via fixed ratio of susceptibilities: \( R = \frac{\chi_q}{\chi_q} \)

\[ \rightarrow \text{compressed with Polyakov loop} \]

QM model (2+1)

PQM model (2+1)

\[ [\text{BJS, M. Wagner; in preparation} 11] \]
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- Functional Renormalization Group
- Finite volume effects
Finite density extrapolations $N_f = 2 + 1$

Taylor expansion:

$$\frac{p(T, \mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left( \frac{\mu}{T} \right)^n$$

with

$$c_n(T) = \left. \frac{1}{n!} \frac{\partial^n \left( \frac{p(T, \mu)}{T^4} \right)}{\partial \left( \frac{\mu}{T} \right)^n} \right|_{\mu=0}$$

convergence radii:

limited by first-order line?

$$\rho_{2n} = \left| \frac{c_2}{c_{2n}} \right|^{1/(2n-2)}$$

[C. Schmidt '09]
Finite density extrapolations $N_f = 2 + 1$

Taylor expansion:

$$\frac{p(T, \mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left( \frac{\mu}{T} \right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n \left( p(T, \mu) / T^4 \right)}{\partial (\mu / T)^n} \bigg|_{\mu=0}$$

Figure 1: Taylor coefficients of the pressure in terms of the up-quark chemical potential. Results are obtained with the $p4f$at3 action on $N_f = 4$ (full) and $N_f = 6$ (open symbols) lattices. We compare preliminary results of $(2+1)$-flavor a pion mass of $m_\pi \approx 220$ MeV to previous results of 2-flavor simulations with a corresponding pion mass of $m_\pi \approx 770$ [2].

1. Introduction

A detailed and comprehensive understanding of the thermodynamics of quarks and gluons, e.g. of the equation of state is most desirable and of particular importance for the phenomenology of relativistic heavy ion collisions. Lattice regularized QCD simulations at non-zero temperatures have been shown to be a very successful tool in analyzing the non-perturbative features of the quark-gluon plasma. Driven by both, the exponential growth of the computational power of recent super-computer as well as by drastic algorithmic improvements one is now able to simulate dynamical quarks and gluons on fine lattices with almost physical masses.

At non-zero chemical potential, lattice QCD is harmed by the "sign-problem", which makes direct lattice calculations with standard Monte Carlo techniques at non-zero density practically impossible. However, for small values of the chemical potential, some methods have been successfully used to extract information on the dependence of thermodynamic quantities on the chemical potential. For a recent overview see, e.g. [1].

2. The Taylor expansion method

We closely follow here the approach and notation used in Ref. [2]. We start with a Taylor expansion for the pressure in terms of the quark chemical potentials

$$p(T, \mu) = \sum_{n=0}^{\infty} c_n(T) \left( \frac{\mu}{T} \right)^n$$

The expansion coefficients $c_n(T)$ are computed on the lattice at zero chemical potential, using stochastic estimators. Some details on the computation are given in [3, 4]. Details on our current data set and the number of random vectors used for the stochastic random noise method are summarized in Table 1.

In Fig. 1 we show results on the diagonal expansion coefficients with respect to the up-quark

$[\text{Miao et al. '08}]$
New method: based on algorithmic differentiation

Taylor coefficients $c_n$ numerically known to high order, e.g. $n = 22$
Taylor coefficients for $N_f = 2 + 1$ PQM model

![Graph showing Taylor coefficients](image)

- this technique applied to PQM model
- investigation of convergence properties of Taylor series
- properties of $c_n$
  - oscillating
  - increasing amplitude
  - no numerical noise
  - small outside transition region
  - number of roots increasing
  - 26th order

Can we locate the QCD critical endpoint with the Taylor expansion?
Taylor expansions

Findings:

- simply Taylor expansion: slow convergence
  high orders needed
  disadvantage for lattice simulations

- Taylor applicable within convergence radius
  also for \( \mu/T > 1 \)

\[
\begin{align*}
  r_{2n} &= \left| \frac{c_{2n}}{c_{2n+2}} \right|^{1/2}, \\
  r_{2n+2} &= \left| \frac{(2n+2)(2n+1)}{(2n+3)(2n+4)} \right|^{1/2}
\end{align*}
\]

Padé \([N/N]\)

[\(F.\) Karsch, BJS, M. Wagner, J. Wambach; arXiv:1009.5211]
Taylor expansions

Findings:

- simply Taylor expansion: slow convergence
  high orders needed
  disadvantage for lattice simulations

- Taylor applicable within convergence radius
  also for \( \frac{\mu}{T} > 1 \)

- but 1st order transition not resolvable
  expansion around \( \mu = 0 \)

Generalized Susceptibilities

- Can we use these coefficients to locate CEP experimentally?

Is there a memory effect that the system (HIC) has passed through the QCD phase transition?

- Probing phase diagram with fluctuations of e.g. net baryon number

- Example: Kurtosis $R_{4,2}^B = \frac{1}{9} \frac{c_4}{c_2} \rightarrow$ probe of deconfinement?

  It measures quark content of particles carrying baryon number $B$

- in HRG model $R_{4,2} = 1$ (always positive)

[BJS, M.Wagner; in prep. 11]
Generalized Susceptibilities

Fluctuations of higher moments (more sensitive to criticality) exhibit strong variation from HRG model

- turn negative

higher moments: $R_{n,m}^q = c_n / c_m$

regions where $R_{n,m}$ are negative along crossover line in the phase diagram

see talk by V. Skokov

see also [Friman, Karsch, Redlich, Skokov; 11] [Karsch, Redlich; 11] [BJS, M.Wagner; in prep. 11]

PQM with $T_0(\mu)$

QM model

With the FRG towards the QCD Phase Diagram

B.-J. Schaefer (KFU Graz)
Outline

- Three-Flavor Chiral Quark-Meson Model
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- Finite volume effects
$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$ ; $R_k$ regulators

FRG (average effective action) [Wetterich '93]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \partial_t R_k \left( \frac{1}{\Gamma_k^{(2)} + R_k} \right) ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi}$$

$$k \partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$

scale $q$

bare action $\Gamma_k \rightarrow \infty$

effective average action $\Gamma_k$

temperature

infrared

$0$

ultraviolet

momentum shell

effective action $\Gamma_k \rightarrow 0$
**$T_0(N_f, \mu)$ modification**

**full dynamical QCD FRG flow**: fluctuations of gluon, ghost, quark and meson (via hadronization) fluctuations

[Braun, Haas, Marhauser, Pawlowski; '09]

\[
\partial_t \Gamma_k[\phi] = \frac{1}{2} \quad - \quad - \quad + \frac{1}{2}
\]

in presence of dynamical quarks

→ pure Yang Mills flow + these modifications

**pure Yang Mills flow**

replaced by effective Polyakov loop potential:

(fit to YM thermodynamics)

\[
\partial_t \Gamma_k[\phi] = \frac{1}{2} \quad -
\]

$T_0 \leftrightarrow \Lambda_{QCD} \quad : \quad T_0 \rightarrow T_0(N_f, \mu, m_q)$

[BJS, Pawlowski, Wambach; 2007]
Functional Renormalization Group

\[ \partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \partial_t R_k \left( \frac{1}{\Gamma_k^{(2)} + R_k} \right) ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi} \]

\[ \partial_t \Gamma_k[\phi] = \frac{1}{2} \Gamma_k[\phi] - \Gamma_k[\phi] + \frac{1}{2} \Gamma_k[\phi] \]

\( \Gamma_k[\phi] \) scale dependent effective action \quad ; \quad t = \ln(k/\Lambda) ; \quad R_k \) regulators

PQM truncation \( N_r = 2 \)

\[ \Gamma_k = \int d^4x \left\{ \bar{\psi} (i\slashed{D} + \mu \gamma_0 + i\hbar (\sigma + i\gamma_5 \vec{\tau} \vec{\pi})) \psi + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 + \Omega_k[\sigma, \vec{\pi}, \Phi, \bar{\Phi}] \right\} \]

Initial action at UV scale \( \Lambda \):

\[ \Omega_\Lambda[\sigma, \vec{\pi}, \Phi, \bar{\Phi}] = U(\sigma, \vec{\pi}) + U(\Phi, \bar{\Phi}) + \Omega^{\infty}_\Lambda[\sigma, \vec{\pi}, \Phi, \bar{\Phi}] \]

\[ U(\sigma, \vec{\pi}) = \frac{\lambda}{4} \left( \sigma^2 + \vec{\pi}^2 - v^2 \right)^2 - c\sigma \]
Phase diagram $T_0 = 208$ MeV

[Herbst, Pawlowski, BJS; 2011]
Phase diagram $T_0(\mu), T_0(0) = 208$ MeV
Phase diagram $T_0(\mu), T_0(0) = 208$ MeV

[Herbst, Pawlowski, BJS; 2011]

→ CEP unlikely for small $\mu_B/T$ → baryons
Phase diagram (DSE - HTL)

With the FRG towards the QCD Phase Diagram

[Ch. S. Fischer, J. Luecker, J. A. Mueller; 2011]
Critical region

similar conclusion if fluctuations are included

fluctuations via Functional Renormalization Group

comparison: $N_f = 2$ QM model

<table>
<thead>
<tr>
<th>Mean Field</th>
<th>RG analysis</th>
</tr>
</thead>
</table>

![Graph](image)

[BJS, Wambach '06]
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Finite volume effects

Finite Euclidean Volume: $L^3 \times 1/T$

cf. talk by Bertram Klein

$$\int d^3 p \rightarrow \left( \frac{2\pi}{L} \right)^3 \sum_{n_1 \in \mathbb{Z}} \sum_{n_2 \in \mathbb{Z}} \sum_{n_3 \in \mathbb{Z}} \text{ with } p^2 \rightarrow \frac{4\pi^2}{L^2} \left( n_1^2 + n_2^2 + n_3^2 \right)$$

Grand potential ($N_f = 2$ quark-meson truncation)

$$\partial_t \Omega_k = B_p(k, L) \frac{k^5}{12\pi^2} \left[ -\frac{2N_fN_c}{E_q} \left\{ \tanh \left( \frac{E_q - \mu}{2T} \right) + \tanh \left( \frac{E_q + \mu}{2T} \right) \right\} 
+ \frac{1}{E_{\sigma}} \coth \left( \frac{E_{\sigma}}{2T} \right) + \frac{3}{E_{\pi}} \coth \left( \frac{E_{\pi}}{2T} \right) \right]$$

with $E_{\sigma,\pi,q} = \sqrt{k^2 + m_{\sigma,\pi,q}^2}$, $m_{\sigma}^2 = 2\Omega'_k + 4\sigma^2 \Omega''_k$, $m_{\pi}^2 = 2\Omega'_k$, $m_q^2 = h^2 \sigma^2$

Mode counting functions

$$B_p(k, L) : \text{ periodic}$$
$$B_{ap}(k, L) : \text{ antiperiodic}$$
Finite volume effects

mode counting functions (periodic and antiperiodic (lower) boundary)

\[ \mathcal{B}_p(k, L) = \frac{6\pi^2}{(kL)^3} \sum_{n_1, n_2, n_3 \in \mathbb{Z}} \Theta \left( k^2 - \frac{4\pi^2}{L^2} \left( n_1^2 + n_2^2 + n_3^2 \right) \right) \]

\[ \mathcal{B}_{ap}(k, L) = \frac{6\pi^2}{(kL)^3} \sum_{n_1, n_2, n_3 \in \mathbb{Z}} \Theta \left( k^2 - \frac{4\pi^2}{L^2} \left( \left( n_1 + \frac{1}{2} \right)^2 + \left( n_2 + \frac{1}{2} \right)^2 + \left( n_3 + \frac{1}{2} \right)^2 \right) \right) \]
Finite volume effects

mode counting functions (periodic and antiperiodic (lower) boundary)

\[ B_p(k, L) = \frac{6\pi^2}{(kL)^3} \sum_{n_1, n_2, n_3 \in \mathbb{Z}} \Theta \left( k^2 - \frac{4\pi^2}{L^2} \left( n_1^2 + n_2^2 + n_3^2 \right) \right) \]

\[ B_{ap}(k, L) = \frac{6\pi^2}{(kL)^3} \sum_{n_1, n_2, n_3 \in \mathbb{Z}} \Theta \left( k^2 - \frac{4\pi^2}{L^2} \left( \left( n_1 + \frac{1}{2} \right)^2 + \left( n_2 + \frac{1}{2} \right)^2 + \left( n_3 + \frac{1}{2} \right)^2 \right) \right) \]

periodic \( L = 2 \) fm

antiperiodic \( L = 2 \) fm
Finite volume effects

preliminary results for periodic BC

\[ L \to \infty \]
Finite volume effects

preliminary results for periodic BC

L = 5 fm
Finite volume effects

preliminary results for **periodic** BC

$L = 4.5$ fm

![Diagram showing contour lines for $T$ and $\mu$ for the finite volume effects study.](image)
Finite volume effects

preliminary results for periodic BC

\[ L = 4 \text{ fm} \]
Finite volume effects

preliminary results for **periodic** BC

\[ L = 3.5 \text{ fm} \]
Finite volume effects

preliminary results for periodic BC

L = 3 fm
Finite volume effects

preliminary results for periodic BC
Finite volume effects

preliminary results for periodic BC

\[ L = 2 \text{ fm} \]
Finite volume effects

preliminary results for **periodic** BC

movement of the CEP ($L \to \infty \ldots 2$ fm)

\[ T \text{ [MeV]} \]
\[ \mu \text{ [MeV]} \]
Finite volume effects

preliminary results for antiperiodic BC

![Graph showing the QCD Phase Diagram with color contours indicating finite volume effects.](image)

L → ∞
Finite volume effects

preliminary results for **antiperiodic** BC

\[ L = 5 \text{ fm} \]
Finite volume effects

preliminary results for antiperiodic BC

$L = 4.5$ fm

$T$ [MeV]

$\mu$ [MeV]

$>80$

$>60$

$>40$

$>20$
Finite volume effects

preliminary results for **antiperiodic** BC

\[ L = 4 \text{ fm} \]

![Graph showing the finite volume effects with temperature (T [MeV]) and chemical potential (µ [MeV]) axes.](image)
Finite volume effects

preliminary results for antiperiodic BC

$L = 3.5 \text{ fm}$
Finite volume effects

preliminary results for **antiperiodic** BC

![Graph showing finite volume effects in the QCD phase diagram]
Finite volume effects

[A. Tripolt, J. Braun, B. Klein, BJS, in preparation ’11]

preliminary results for **antiperiodic** BC

\[ L = 2.5 \text{ fm} \]

![Graph showing phase transitions in QCD with antiperiodic boundary conditions.](image)
Finite volume effects

preliminary results for antiperiodic BC

L = 2 fm

\[ T \text{ [MeV]} \]

\[ \mu \text{ [MeV]} \]
Finite volume effects

preliminary results for \textbf{antiperiodic} BC

movement of the CEP ($L \rightarrow \infty \ldots 2 \text{ fm}$)
Finite volume effects

[A. Tripolt, J. Braun, B. Klein, BJS, in preparation ’11]

- curvature $\kappa$

\[
\frac{T_X(L, \mu)}{T_X(L, 0)} = 1 - \kappa(L) \left( \frac{\mu}{\pi T_X(L, 0)} \right)^2 + \ldots
\]

- relative change

\[
\Delta \kappa(L) = \frac{\kappa(L) - \kappa(\infty)}{\kappa(\infty)}
\]
Summary

- $N_f = 2$ and $N_f = 2 + 1$ chiral (Polyakov)-quark-meson model study
  - Mean-field approximation and FRG
  - Fluctuations are important

Functional approaches (such as the presented FRG) are suitable and controllable tools to investigate the QCD phase diagram and its phase boundaries

Findings:

- **matter back-reaction to YM sector**: $T_0 \Rightarrow T_0(N_f, \mu)$

- **FRG with PQM truncation**: Chiral & deconfinement transition **coincide** for $N_f = 2$ with $T_0(\mu)$-corrections

- Similar conclusion for $N_f = 2 + 1$
  - Size of the critical region around CEP smaller

- Finite volume effects

- Higher moments

Outlook:

- FRG methods suitable → test lattice predictions (such as finite volume or $N_c = 2, \ldots$)

- Include **glue dynamics** with FRG → towards full QCD