

**Probing the space-time
around astrophysical black hole candidates
with future VLBI experiments**

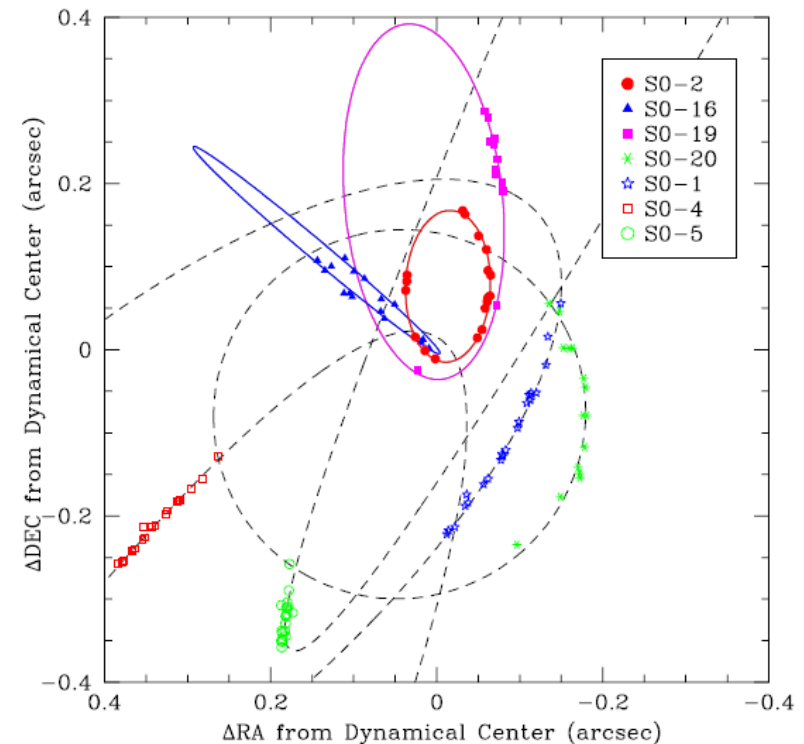
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**11 August 2010
Summer Institute 2010, Fuji-yoshida**

Observational evidences of super-massive black holes

- Super-massive object in the Galactic Center
- Study of the orbits of individual stars
- Mass about 4×10^6 Solar masses
- Radius < 45 AU ($600 R_{\text{Sch}}$)

The possibility that it is a cluster of some non-luminous bodies sounds very unlikely, because the cluster lifetime due to evaporation or physical collisions is too short (see e.g. Maoz, 1998)



Observational evidences of stellar-mass black holes

- **Dark objects in X-ray binary systems**

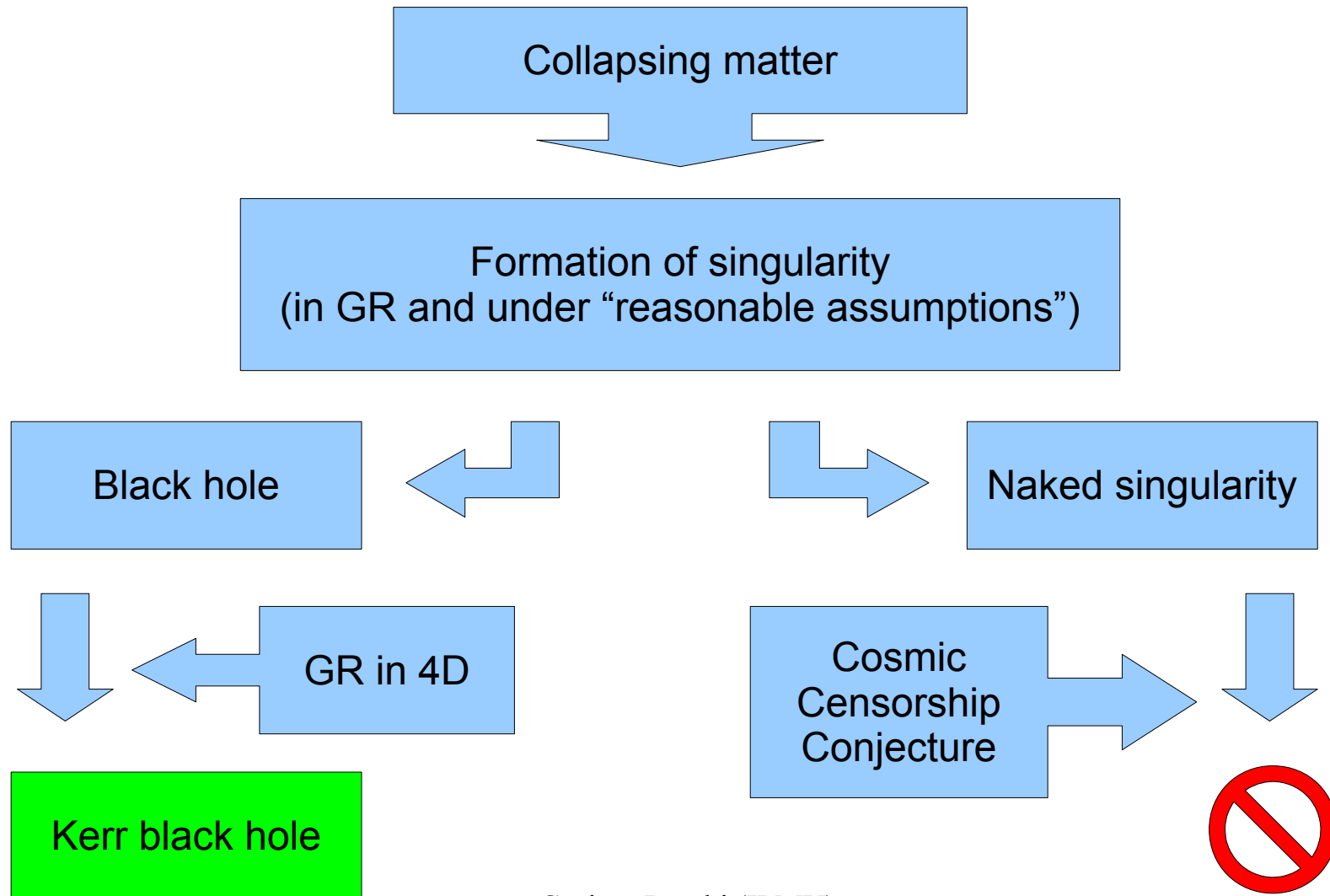
- **Mass function:**
$$f(M_{BH}) = \frac{K^3 T}{2\pi G_N} = \frac{M_{BH}^3 \sin^3 i}{(M_{BH} + M_c)^2}$$
$$K = v \sin i$$

- **In general, a good estimate of M_c and i is necessary**
- **Maximum mass of relativistic stars about 3 Solar masses
(see Rhoades & Ruffini 1974 and Kalogera & Baym 1996)**

Coordinate Name	Common Name/Prefix	Year	Spec.	P _{orb} (hr)	f(M) (M _⊙)	M ₁ (M _⊙)
0422+32	(GRO J)	1992/1	M2V	5.1	1.19±0.02	3.7–5.0
0538–641	LMC X–3	–	B3V	40.9	2.3±0.3	5.9–9.2
0540–697	LMC X–1	–	O7III	93.8 ^d	0.13±0.05 ^d	4.0–10.0: ^e
0620–003	(A)	1975/1 ^f	K4V	7.8	2.72±0.06	8.7–12.9
1009–45	(GRS)	1993/1	K7/M0V	6.8	3.17±0.12	3.6–4.7: ^e
1118+480	(XTE J)	2000/2	K5/M0V	4.1	6.1±0.3	6.5–7.2
1124–684	Nova Mus 91	1991/1	K3/K5V	10.4	3.01±0.15	6.5–8.2
1354–64 ^g	(GS)	1987/2	GIV	61.1 ^g	5.75±0.30	–
1543–475	(4U)	1971/4	A2V	26.8	0.25±0.01	8.4–10.4
1550–564	(XTE J)	1998/5	G8/K8IV	37.0	6.86±0.71	8.4–10.8
1650–500 ^h	(XTE J)	2001/1	K4V	7.7	2.73±0.56	–
1655–40	(GRO J)	1994/3	F3/F5IV	62.9	2.73±0.09	6.0–6.6
1659–487	GX 339–4	1972/10 ⁱ	–	42.1 ^{j,k}	5.8±0.5	–
1705–250	Nova Oph 77	1977/1	K3/7V	12.5	4.86±0.13	5.6–8.3
1819.3–2525	V4641 Sgr	1999/4	B9III	67.6	3.13±0.13	6.8–7.4
1859+226	(XTE J)	1999/1	–	9.2: ^e	7.4±1.1: ^e	7.6–12.0: ^e
1915+105	(GRS)	1992/Q ^l	K/MIII	804.0	9.5±3.0	10.0–18.0
1956+350	Cyg X–1	–	O9.7Iab	134.4	0.244±0.005	6.8–13.3
2000+251	(GS)	1988/1	K3/K7V	8.3	5.01±0.12	7.1–7.8
2023+338	V404 Cyg	1989/1 ^f	K0III	155.3	6.08±0.06	10.1–13.4

From Remillard & McClintock, ARAA 44 (2006) 49

Carter-Israel Conjecture



Counterexamples violating the Cosmic Censorship Conjecture

- **D. Christodoulou, Commun. Math. Phys. 93 (1984) 171**
- **A. Ori & T. Piran, Phys. Rev. D 42 (1990) 1068**
- **P.S. Joshi & I.H. Dwivedi, Phys. Rev. D 47 (1993) 5357**
- **D. Christodoulou, Annals Math. 140 (1994) 607**
- **I.H. Dwivedi & P.S. Joshi, Commun. Math. Phys. 166 (1994) 117**
- **S.S. Deshingkar, I.H. Dwivedi & P.S. Joshi, Phys. Rev. D 59 (1999) 044018**

Chronology Protection in Supergravity

- **E.K. Boyda et al., Phys. Rev. D 67 (2003) 106003**
- **D. Israel, JHEP 01 (2004) 042**
- **N. Drukker, Phys. Rev. D 70 (2004) 084031**
- **E.G. Gimon & P. Horava, arXiv:hep-th/0405019**

CLAIM: Formation of domain wall. Across the domain wall, the metric is non-differentiable and the expected region with closed time-like curves arises from the naive continuation of the metric ignoring the domain wall.

Tomimatsu-Sato space-times

[A. Tomimatsu & H. Sato, PTP 50 (1973) 95]

- 3 parameters: mass (M), spin (J), and deformation parameter (delta)
- delta = 1 (Kerr space-time)
- Quadrupole moment:

$$Q = \left(q^2 + \frac{\delta^2 - 1}{3\delta^2} p^2 \right) M^3$$

Delta = 2 Tomimatsu-Sato space-time

$$ds^2 = -f (dt - \omega d\phi)^2 + \frac{1}{f} [e^{2\gamma} (d\rho^2 + dz^2) + \rho^2 d\phi^2]$$

$$p^2 + q^2 = 1, \quad q = \frac{J}{M^2}, \quad \sigma = \frac{Mp}{\delta}$$

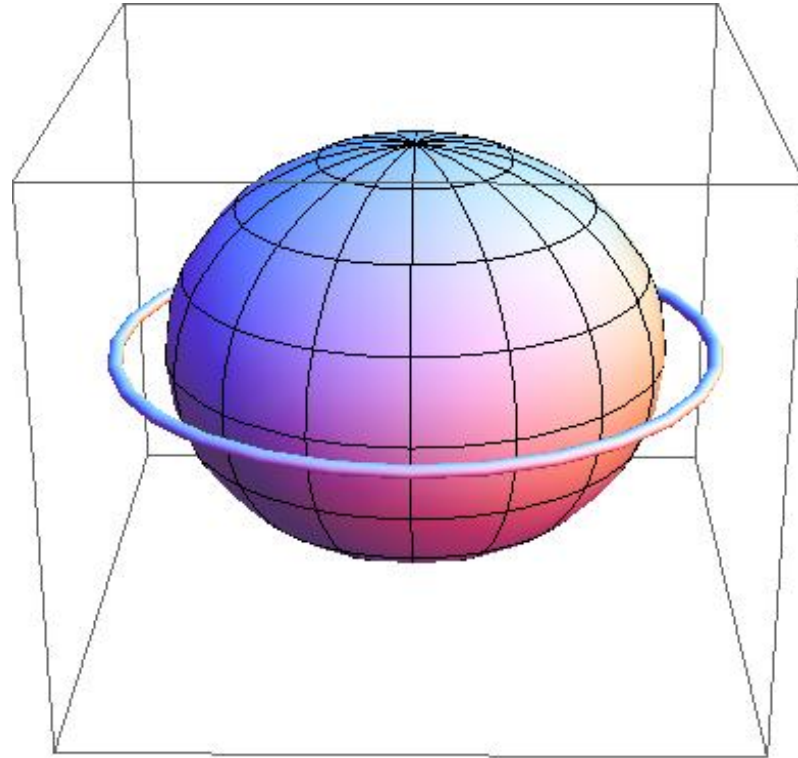
$$f = \frac{A}{B}, \quad \omega = \frac{2Mq(1-y^2)C}{A}, \quad e^{2\gamma} = \frac{A}{p^{2\delta}(x^2 - y^2)^{\delta^2}}$$

$$A = p^4(x^2 - 1)^4 + q^4(1 - y^2)^4 - 2p^2q^2(x^2 - 1)(1 - y^2) [2(x^2 - 1)^2 + 2(1 - y^2)^2 + 3(x^2 - 1)(1 - y^2)]$$

$$B = [p^2(x^2 + 1)(x^2 - 1) - q^2(1 + y^2)(1 - y^2) + 2px(x^2 - 1)]^2 + 4q^2y^2 [px(x^2 - 1) + (px + 1)(1 - y^2)]^2$$

$$C = -p^3x(x^2 - 1) [2(x^2 + 1)(x^2 - 1) + (x^2 + 3)(1 - y^2)] - p^2(x^2 - 1) [4x^2(x^2 - 1) + (3x^2 + 1)(1 - y^2)] + q^2(px + 1)(1 - y^2)^3.$$

Delta = 2 Tomimatsu-Sato space-time



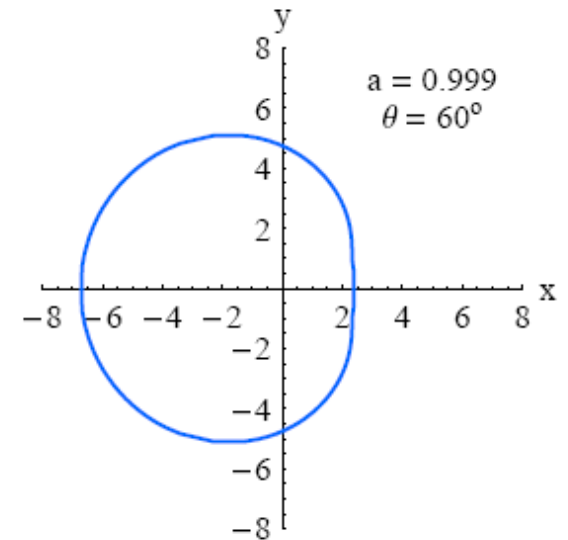
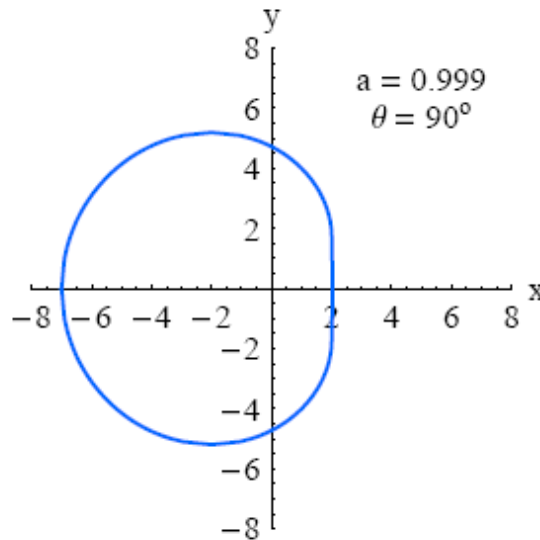
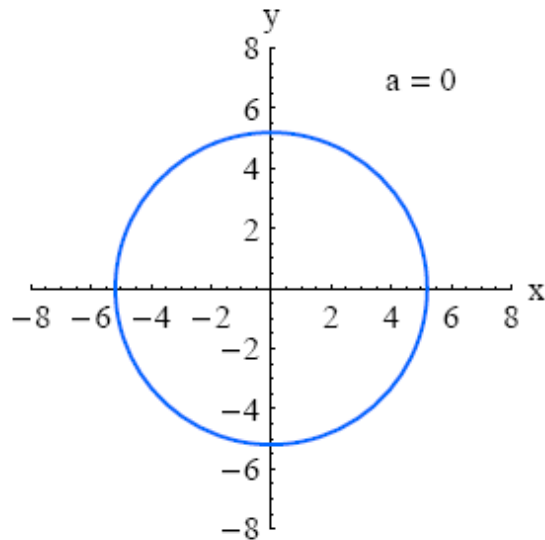
See Kodama & Hikida, CQG 20 (2003) 5121

Capture cross section

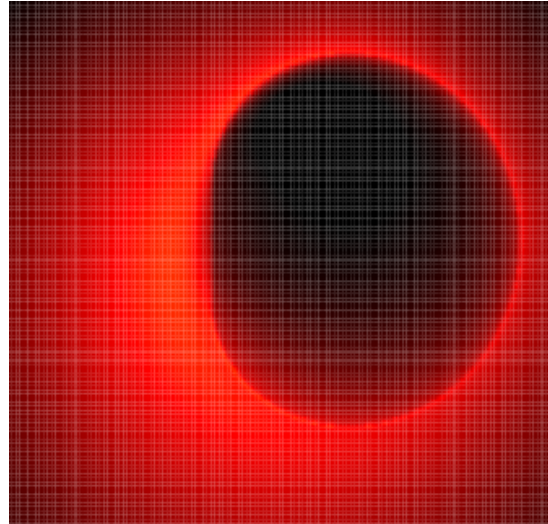
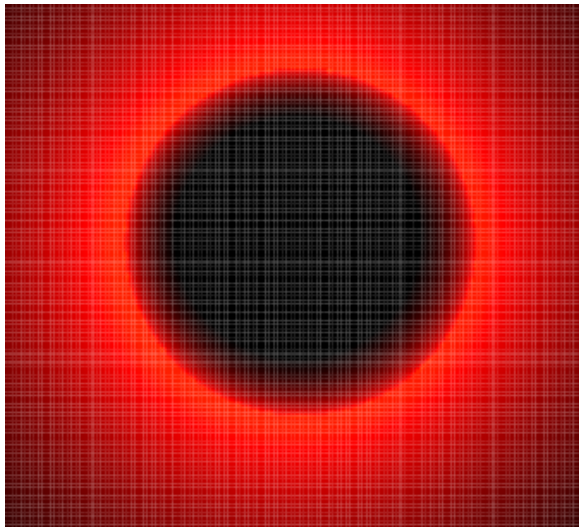
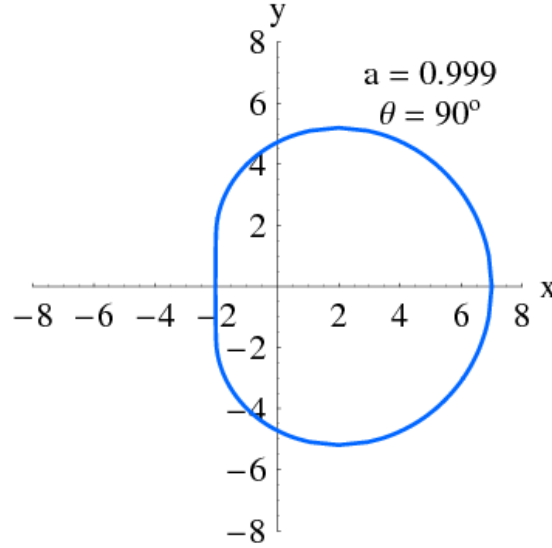
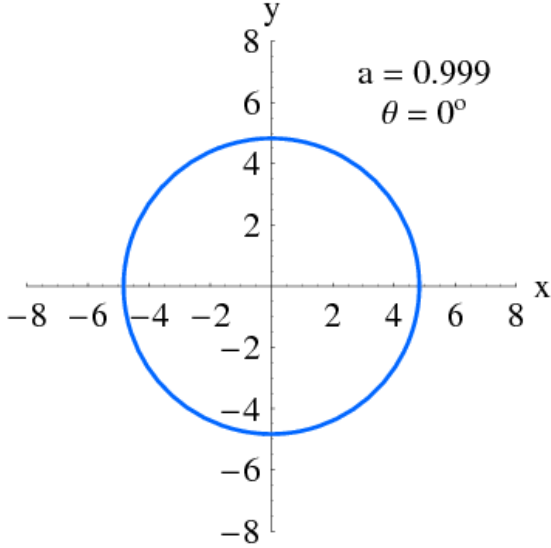
Apparent size:

Schwarzschild BH: about 10.4 M

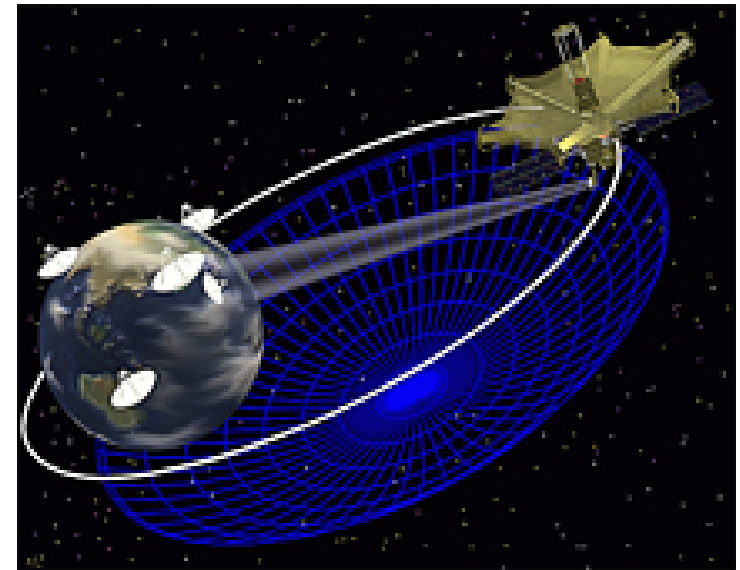
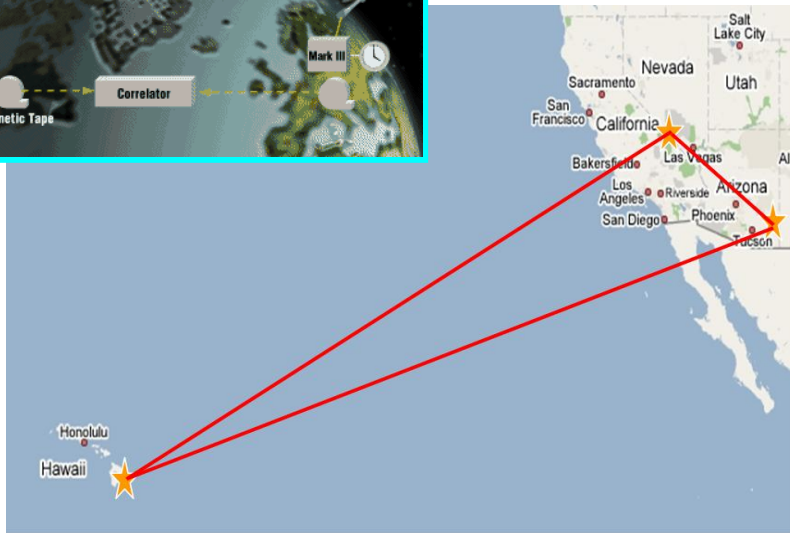
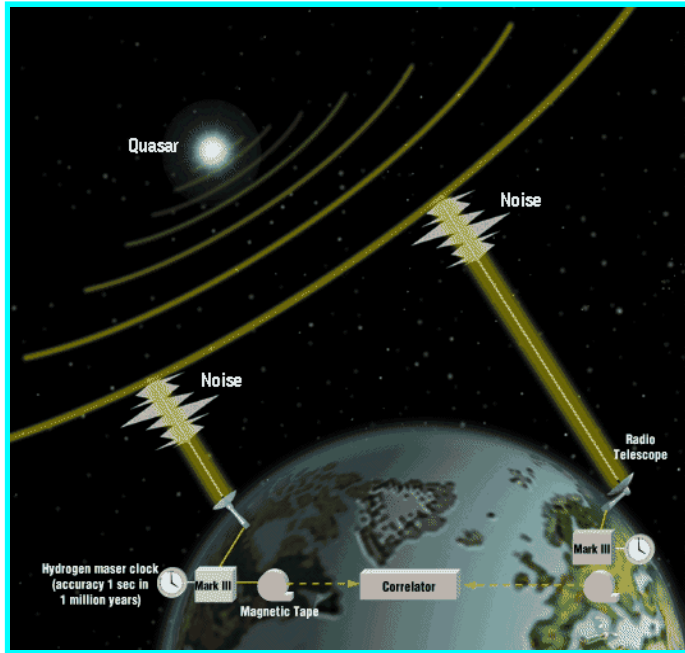
Extremal Kerr BH: 9 M (for an observer on the equatorial plane)



Direct Image



VLBI observatories

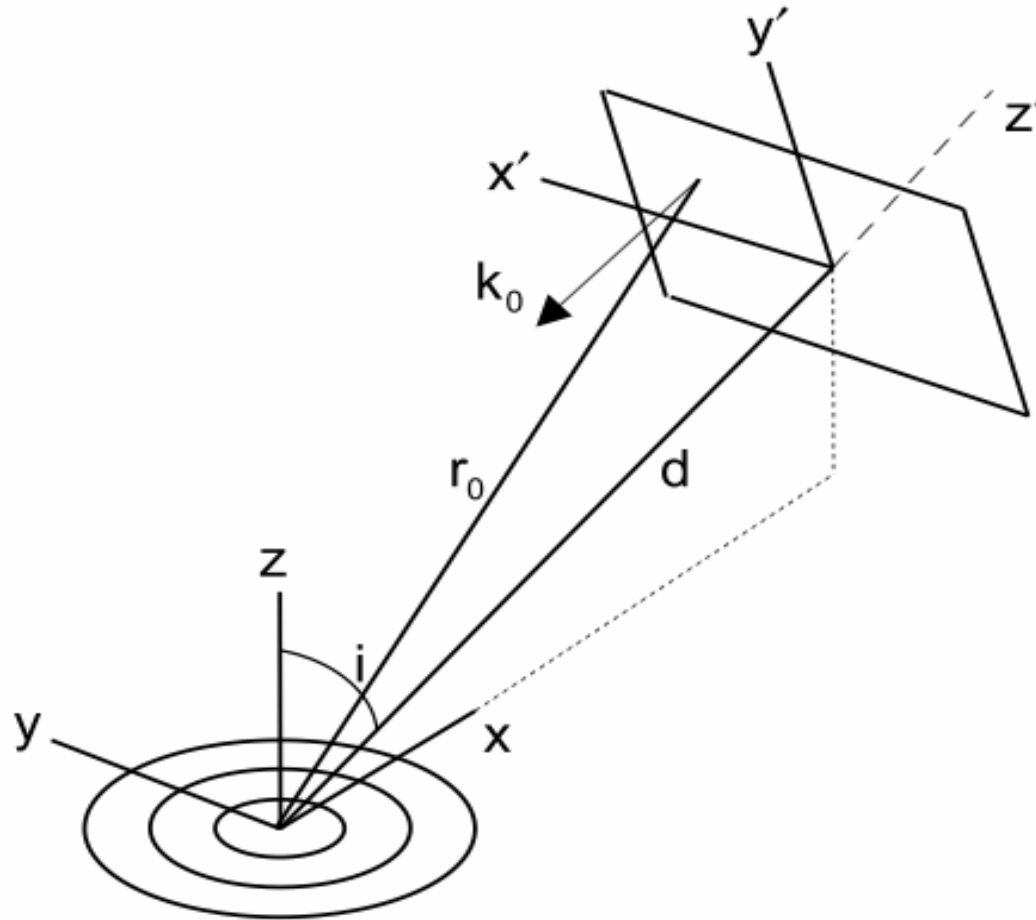


VSOP

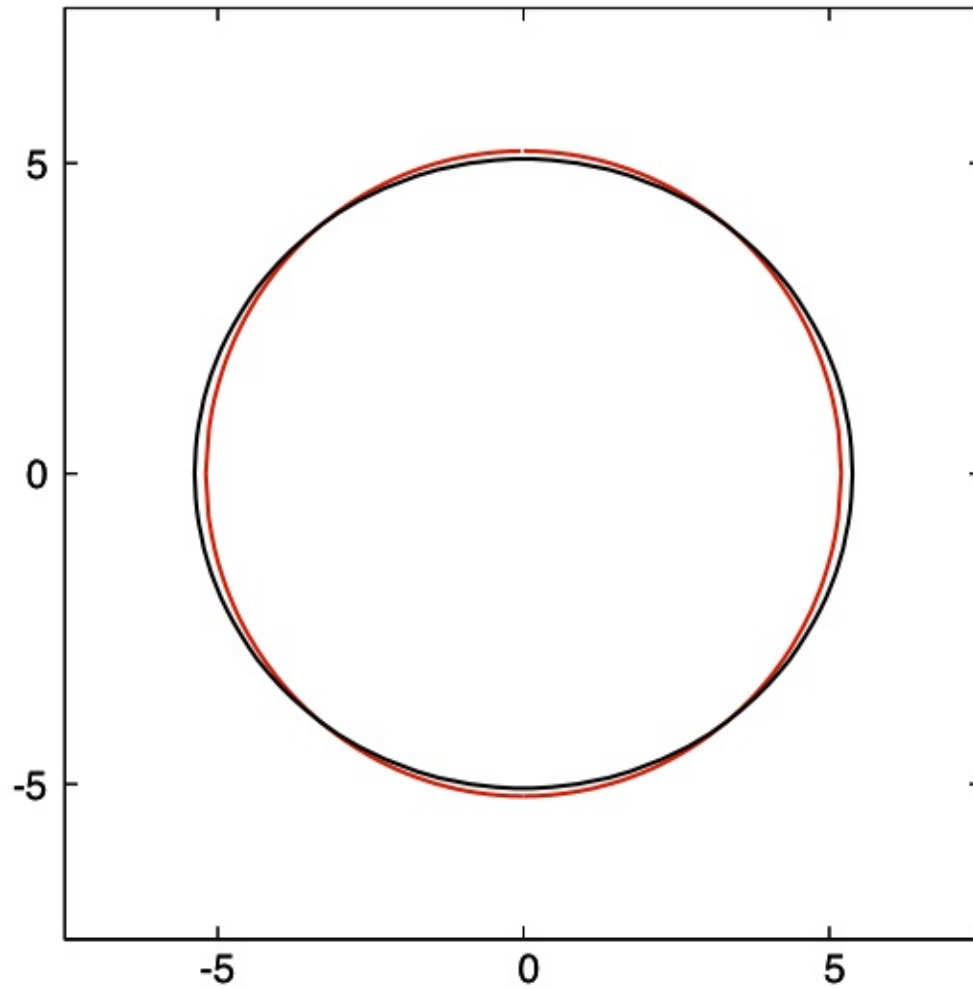
Doeleman's group

Cosimo Bambi (IPMU)

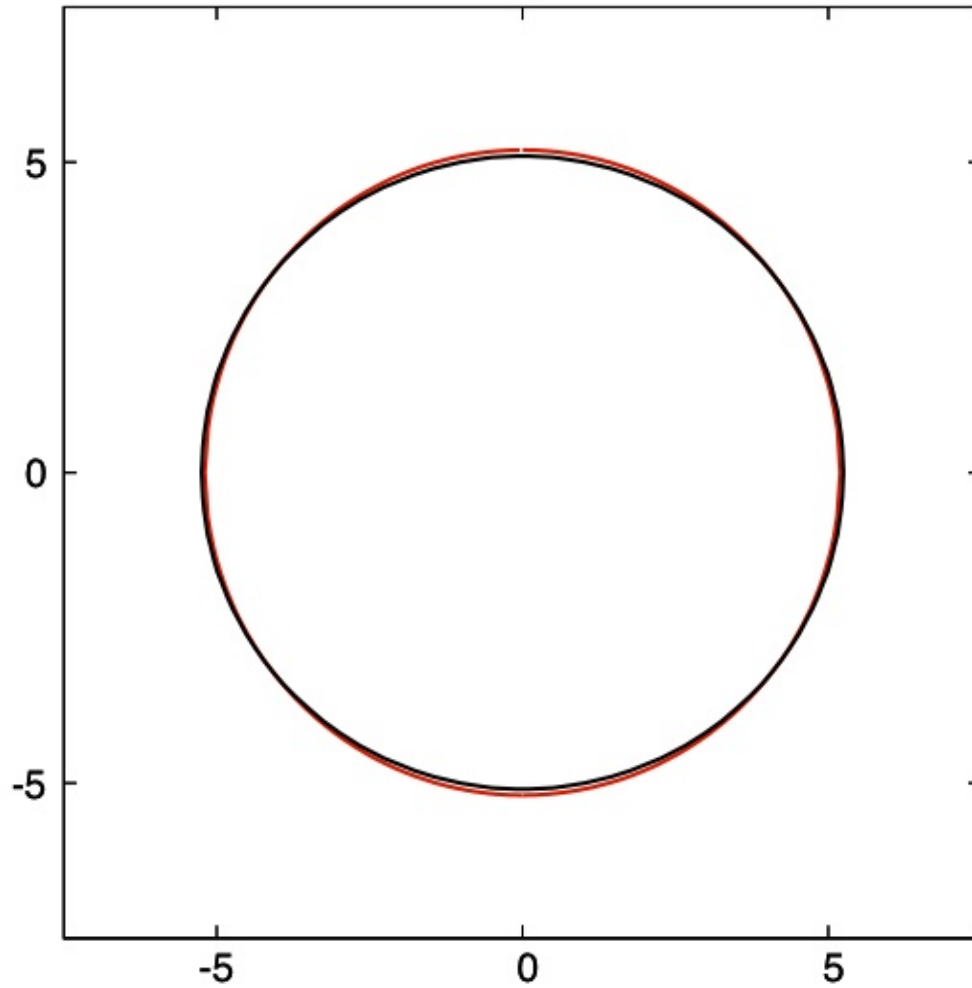
Set-up



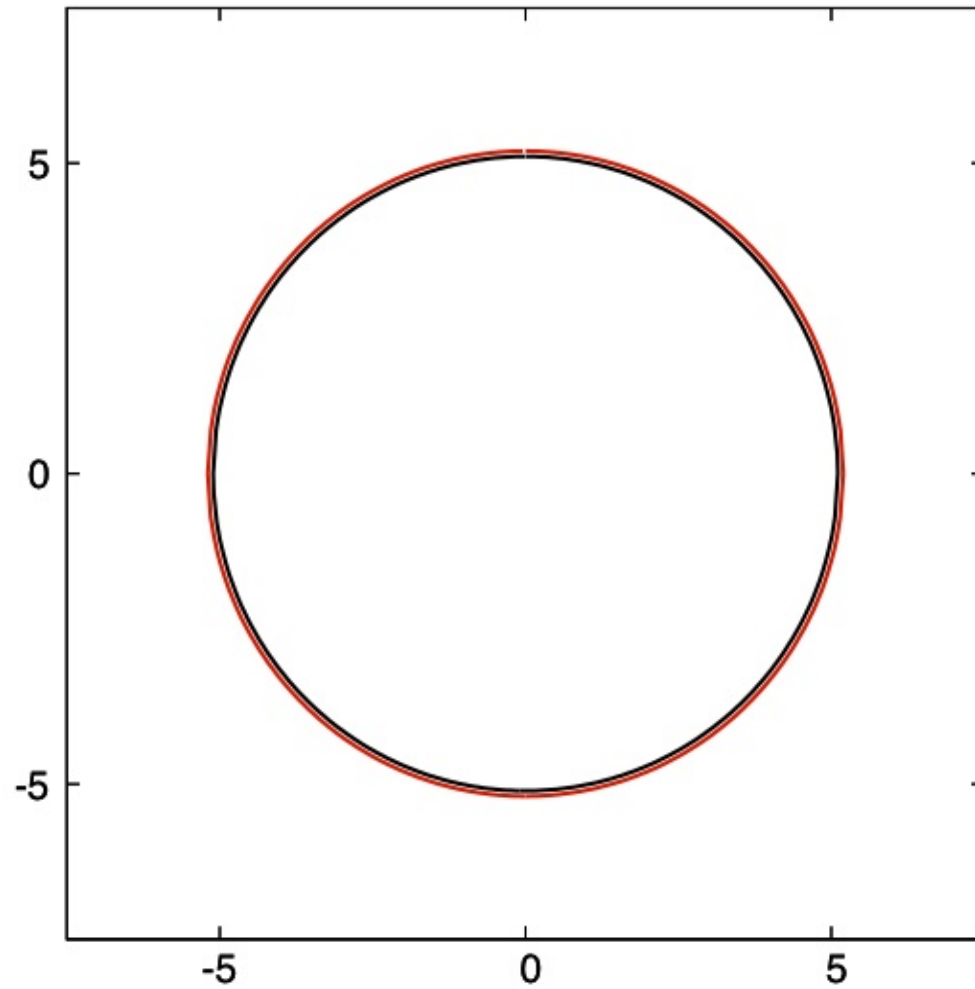
Shadow: $q=0, i=90^\circ$



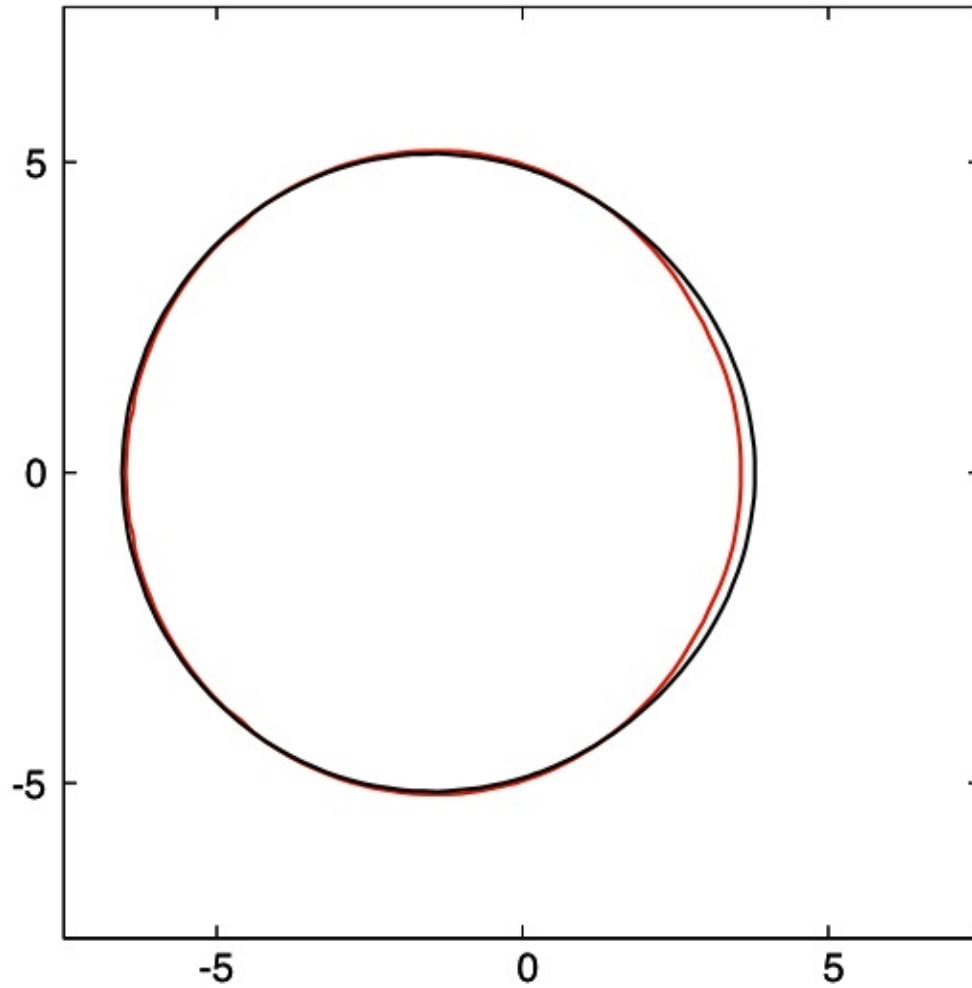
Shadow: $q=0, i=45^\circ$



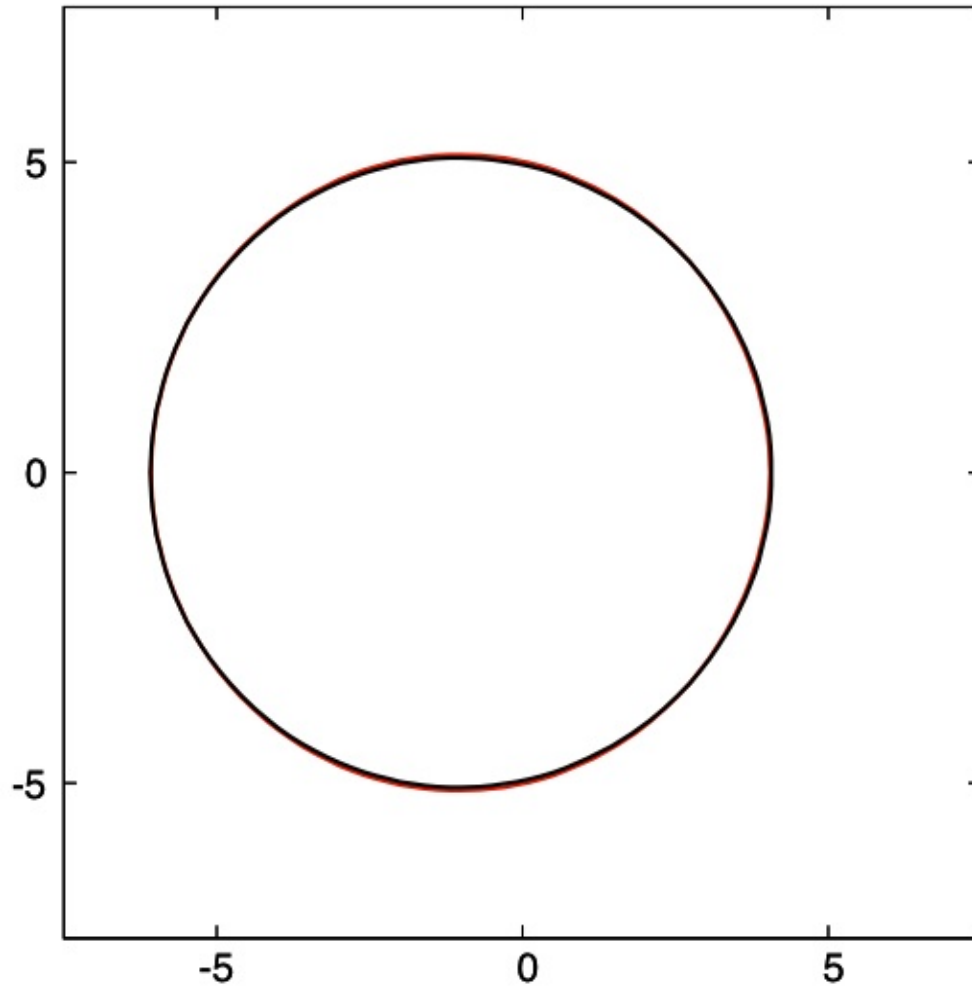
Shadow: $q=0, i=0^\circ$



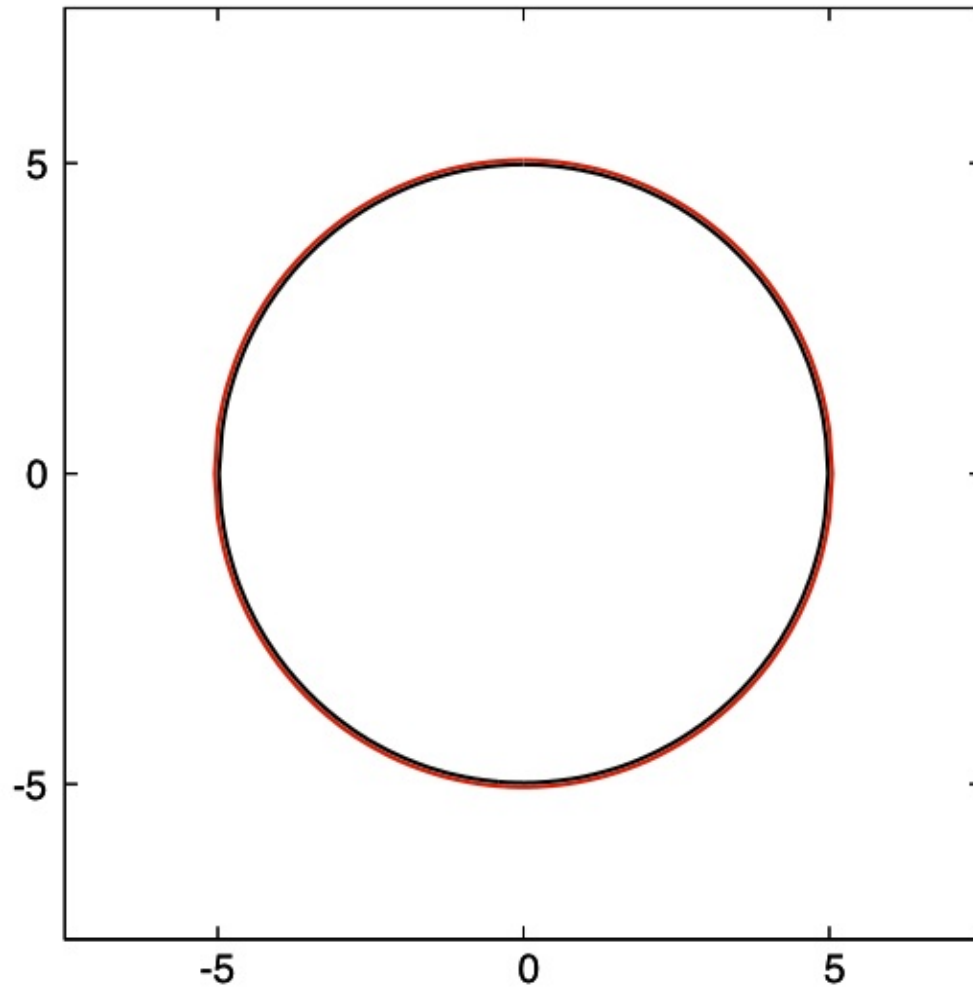
Shadow: $q=0.7, i=90^\circ$



Shadow: $q=0.7, i=45^\circ$

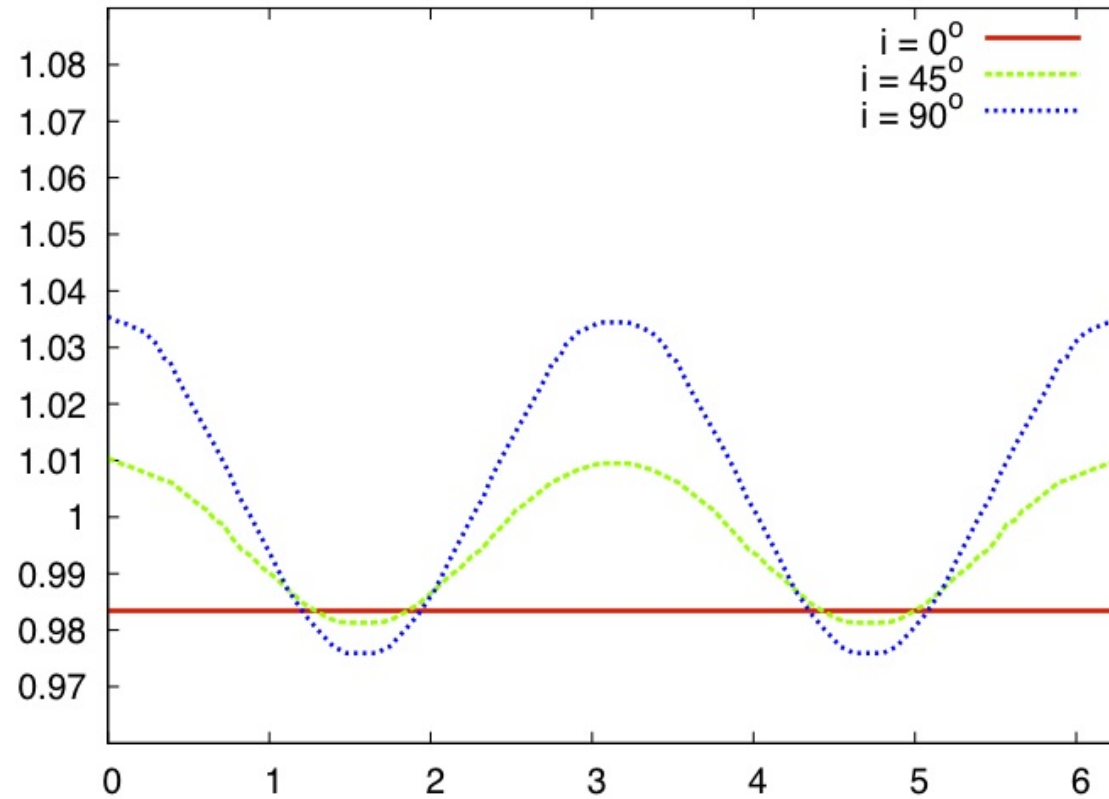


Shadow: $q=0.7$, $i=0^\circ$



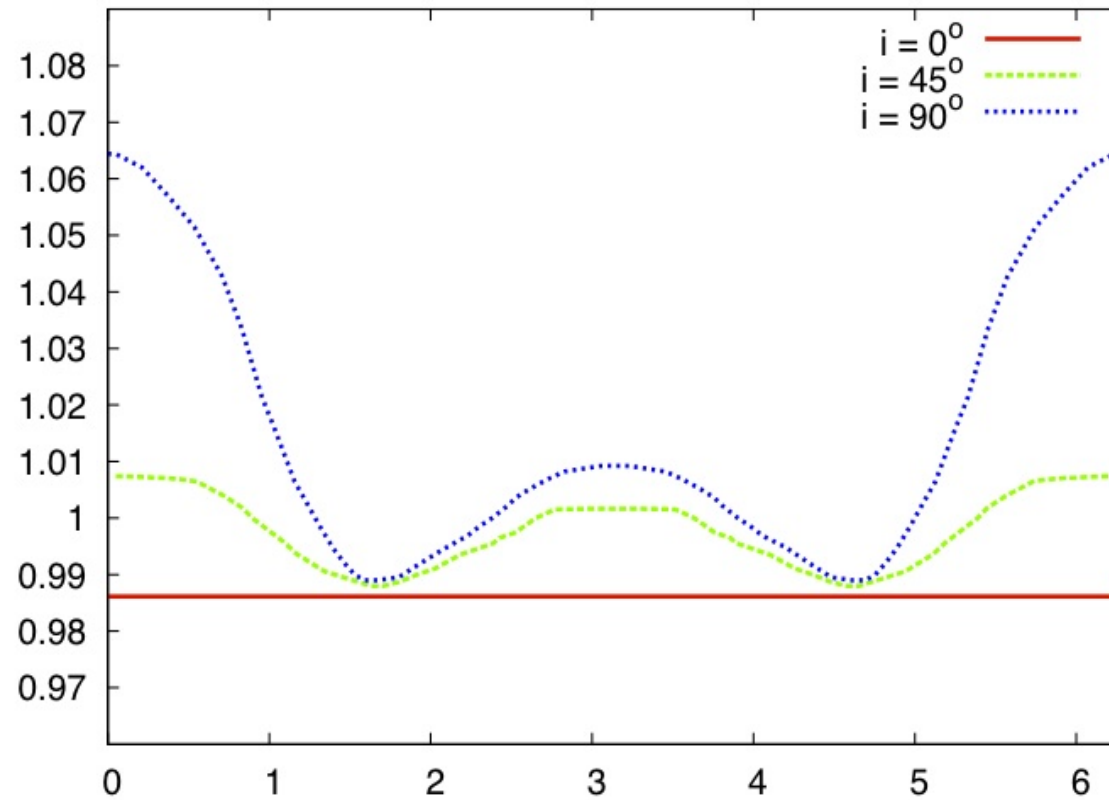
$$\frac{R_{\text{TS2}}}{R_{\text{Kerr}}}$$

$q = 0.0$



$$\frac{R_{\text{TS2}}}{R_{\text{Kerr}}}$$

$q = 0.7$



A couple of comments

- **The two shadows are quite similar**
- **No features associated to the ring singularity of TS2**
- **Deviations from the Kerr metric at the level of a few per cent**
- **Near future ground based experiments or space missions like VSOP-2 will not be able to detect any difference. With more advanced space missions, like VSOP-3, it is still challenging, but not out of reach**

Conclusions

- **At present there are no evidences that the final product of the gravitational collapse is a Kerr black hole**
- **To test the Kerr metric, we need to probe the space-time very close to the massive object. Deviations from the Kerr metric, if any, are small!**
- **Future VLBI experiments can be used to test strong gravity**
- **The Tomimatsu-Sato space-times are the simplest generalization of the Kerr metric. They represent the gravitational field of spinning and deformed mass**
- **Future space missions like VSOP-3 have the capability of testing the Kerr metric**